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A Simulation Model for Passive Attitude Control Systems Used in CubeSats

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CHAPTER 1

Objective

The objective of this document is to describe the components of a simulation model for a nanosatellite's passive attitude control system, derive the related equations and provide a software architecture for the integration of such components.

A simulation of this system must capture the phenomena related to orbital motion, attitude dynamics and the interaction between the geomagnetic field and the magnetic materials used for attitude control, hence the components of such simulation are chosen to be independent models of each phenomena containing some physically motivated interface between them, allowing for the integration of the components to provide the final model.

CHAPTER 2

Geomagnetic Field Model

2.1 Introduction

The International Geomagnetic Reference Field (IGRF) is a standard mathematical description of the Earth's magnetic field widely used in studies of the magnetosphere. The model is developed and maintained by the International Association of Geomagnetism and Aeronomy (IAGA) and it's going to be the model used in this simulation.

The objective of this section is to derive a mathematical expression that defines a magnetic flux density field over a Cartesian space described in spherical coordinates, hence by the end of the derivation it must be possible to assign a vector, representing the local magnetic field, to every point of an orbit around the Earth.

2.2 Model Description

Assuming that there are no sources of magnetic field, that is, free currents, outside the Earth's surface, a scalar potential description of the magnetic field becomes feasible, since the field becomes irrotational and the existence of the potential field can be proved given the existence of a solution to Poisson's equation. Given the linearity of free-space, a well known solution to Poisson's equation can be found by the expansion of Green's function in terms of spherical harmonics, but since the source of magnetic potential, mathematically found by deriving the Poisson equation for the problem, cannot be fully known for the case of the geomagnetic field, a statistical approach is used to find the coefficients of the spherical harmonics expansion, based on measurement data, providing an approximated model for the geomagnetic field. The final equation for the model, described in [1], is:

$$V(r, \theta, \phi, t) = a \sum_{n=1}^{N} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \left[g_n^m(t) \, cos\,(m\phi) + h_n^m(t) \, sin\,(m\phi)\right] P_n^m\,(cos\theta) \tag{2.1}$$

Where a is the Earth's radius, g_n^m,h_n^m are named Gauss Coefficients and P_n^m is the Schmidt Quasi-Normalized Associated Legendre Functions of Degree n and Order m, deeply described in [2].

Is this model, the convention used for the spherical coordinate system defines r as the radial distance from the center of the Earth, θ as the co-latitude and ϕ as the east longitude.

Based on the equation 2.1, it is possible to compute the magnetic flux density through the expression:

$$\vec{B} = -\nabla V \tag{2.2}$$

In this equation, the gradient operator is described in spherical coordinates, hence the components of the magnetic flux density vector are given by:

$$\begin{aligned} B_r &= -\frac{\partial V}{\partial r} \\ B_\theta &= -\frac{1}{r}\frac{\partial V}{\partial \theta} \\ B_\phi &= -\frac{1}{rsin\theta}\frac{\partial V}{\partial \phi} \end{aligned}$$

An explicit representation for the magnetic flux density vector with regard to it's basis vectors is given by:

$$\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} + B_\phi\hat{\phi} \tag{2.3}$$

Where $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are unit vectors forming a basis for a curvilinear space.

Since vectors are invariant to coordinate transformations (see [3] for a detailed explanation), a direct consequence of the fact that physical quantities must not change when it's mathematical representation changes, the use of the spherical coordinates representation of the magnetic flux density field should not be a problem, as long as the basis vectors are properly defined.

A definition for the basis vectors can be constructed based on the relation between Cartesian coordinates and spherical coordinates, given by:

$$\begin{aligned} x &= rsin\theta cos\phi \\ y &= rsin\theta sin\phi \\ z &= rcos\theta \end{aligned}$$

To arrive at a basis all it takes is to differentiate the vector (x,y,z) with respect to (r,ϕ,θ), arriving at a Jacobian matrix. The Jacobian matrix represents the amount of deformation caused by a transformation from Cartesian to spherical coordinates, establishing a relation between the systems. By simple differentiation rules, one gets:

$$\begin{aligned} \vec{R} &= (x, y, z) \\ \frac{\partial R}{\partial r} &= (sin\theta cos\phi, sin\theta sin\phi, cos\theta) \\ \frac{\partial R}{\partial \theta} &= (rcos\theta cos\phi, rcos\theta sin\phi, -rsin\theta) \\ \frac{\partial R}{\partial \phi} &= (-rsin\theta sin\phi, rsin\theta cos\phi, 0) \end{aligned}$$

After normalization:

$$\hat{r} = \frac{\partial R}{\partial r} = (sin\theta cos\phi, sin\theta sin\phi, cos\theta)$$
$$\hat{\theta} = \frac{1}{r}\frac{\partial R}{\partial \theta} = (cos\theta cos\phi, cos\theta sin\phi, -sin\theta)$$
$$\hat{\phi} = \frac{1}{rsin\theta}\frac{\partial R}{\partial \phi} = (-sin\phi, cos\phi, 0)$$

By using such basis, it is possible to verify the relation:

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} = B_r\hat{r} + B_\theta\hat{\theta} + B_\phi\hat{\phi} \tag{2.4}$$

And the spherical coordinates representation of vector field can be used directly in the computation of the magnetic flux density at every point of an orbit around the Earth.

CHAPTER 3

Orbital Motion Model

3.1 Introduction

Since most LEO satellite orbits are near-circular, for this analysis a circular orbit will be assumed, based on the orbital motion model used in [4].

The objective of this model is to associate every instant in time to a position contained in a feasible orbit around the Earth.

3.2 Model Description

The mathematical description of a circular orbit is defined by five parameters: three angles that define the orbital plane with respect to the xy plane, through successive rotations; a radius, defining the distance between the Earth's center to the satellite's center of mass; and an angle that defines the angular position of the satellite's center of mass in the orbit at an instant. Note that since the orbit is approximated as a circle, both the radius and the rotation of the orbital plane are constant, implying that only the angular position in the orbit changes with time.

The construction of the proposed representation of orbital motion can be realized by successive rotations of a circular orbit in the xy plane by a set of angles α, β, γ. Hence any point P contained in the orbit can be computed using the relation:

$$P = R_{\alpha\beta\gamma} P_0 \tag{3.1}$$

Where:

$$R_{\alpha\beta\gamma} = R_\alpha R_\beta R_\gamma \tag{3.2}$$

$$R_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\alpha) & -sin(\alpha) \\ 0 & sin(\alpha) & cos(\alpha) \end{bmatrix} \tag{3.3}$$

$$R_\beta = \begin{bmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & & cos(\beta) \end{bmatrix} \tag{3.4}$$

$$R_\gamma = \begin{bmatrix} cos(\gamma) & -sin(\gamma & 0 \\ sin(\gamma) & cos(\gamma) &) \\ 0 & 0 & 1 \end{bmatrix} \tag{3.5}$$

$$\tag{3.6}$$

Note that matrix multiplication is not a commutative operation, hence the order of rotations, α-β-γ, must be respected.

Since $P_0(\theta)$ is a point contained in a circular orbit defined in the xy plane, a convenient parameterization for the curve is:

$$P_0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} rcos\theta \\ rsin\theta \\ 0 \end{bmatrix} \tag{3.7}$$

Where r is the constant radius, and theta is the angular position parameter.

Finally, for the link between the orbital position and time to be established, an expression for the angular position as a function of time must be defined. Assuming that the there is no energy dissipation in the orbital motion, implying that the kinetic energy of the satellite's center of mass is constant, then the angular velocity of such motion is constant and the angular position evolves linearly in time. Mathematically:

$$\theta(t) = \omega_o t = \sqrt{\frac{Gm}{(R_E + h)^3}}\,t \tag{3.8}$$

Where G is the gravitational constant, m is Earth's mass, R_E is Earth's surface average radius and h is the altitude, using the Earth's surface average radius as reference, as established in [4].

CHAPTER 4

Attitude Representation

4.1 Introduction

Before building a concise attitude representation, there are two main topics that must discussed: the frames of reference that are going to be used in the computations and the chosen parameterization for spatial rotations.

Specifying an orientation in space only makes sense in a relative perspective, hence the objetive of this section is to define a procedure through which the numbers resulting from attitude computation can be traced back to it's physical interpretation.

4.2 Reference Frames

To mathematically describe the satellite's attitude in space, it is convenient to set an inertial reference frame, that is, a reference frame that does not, or at least approximatly not, experience acceleration, mostly because it helps at keeping the mathematics simple and clear.

In this work, based on what has been presented on [5], the chosen inertial reference frame is the Earth Centered Inertial Reference Frame. In this reference frame, the x-axis is defined by a unit vector pointing in the direction of the vernal equinox, the point of intersection between the equatorial plane and the eclipitc in which the Sun's trajectory crosses the Equator moving from the Southern Hemisphere to the Northern Hemisphere, as viewed by an observert at Earth. The z-axis is given by a unit vector in the direction of the geographic north pole axis, and, finally, the y-axis unit vector is the result of the cross-product between the x-axis unit vector and the z-axis unit vector. All equations describing attitude dynamics will be written with respect to this reference frame.

Next, an idea of how to the describe attitude based on the definition of an inertial frame will be developed. First, it is relevant to point out that the geometry of the satellite is understood mathematically as a collection of points that move with a moving reference frame attached to the satellite, that is, those points are stationary when observed from such a moving reference frame. This is intuitive. In the inside of a car, if a person is holding a cellphone it seems's like the cellphone is static from the point of view of another passenger, but it is clear that the cellphone's position is changing according to the car's motion, from the perspective of an observer standing outside of the car. It is tempting to the describe, then, a point contained in the satellite's set of points, by:

$$r_I^P = r_I^B + r_B^P \tag{4.1}$$

This equation reads: "the position of the point P with respect to the inertial reference frame, I, equals the position of the origin of the B reference frame with respect to the inertial reference frame, I, plus the position of the point P with respect to the B reference frame". Where B is a reference frame attached to the satellite. To clearify what this expression means, an intermediate reference frame, B', is defined as a reference frame whose origin coincides with the origin of the B reference frame, but has all of it's axis parallel to the inertial frame's axis, that is, it only differs from the inertial reference frame by a translation operation. This means that the relation between the reference frames B and B' is a rotation operation. Mathematically:

$$r_I^P = r_I^B + R_B^{B'} r_{B'}^P \tag{4.2}$$

The capital letter R is a rotation matrix that transforms the basis vectors of the B' reference frame into the basis vectors of the B reference frame and is the proposed definition of attitude. Attitude is the orientation of a body with respect to some reference placement in space that can be described by a the rotation that must take place in order to make the current placement of the body in space equal to the reference placement. In this case, the reference placement is the inertial reference frame, after the translation required by the position description, and the current placement is the so called Body Reference Frame, a reference frame that moves with the satellite and has two additional special properties. The first special property of the body reference frame is that it's origin is at the satellite's center of mass, making it invariant to rotational movements of the satellite. The second special property of the body reference frame is that in this reference frame the Inertia Tensor is constant and diagonal, which means that the basis vectors for this frame of reference are the normalized eigenvectors of the inertia tensor, that is, the body's principal axes of rotation.

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