Algorithmic game theory HW0

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1 Problem 1

Let $a_1, a_2, ..., a_n$ be fixed real numbers and X be a random variable that takes value a_i with a probability p_i.

1.1 Define the set of probability distributions that maximize $E[X]$

2 Problem 2

Consider throwing n balls into n bins where each ball is thrown independently and uniformly into a bin

2.1 What is the probability that a given bin(say the first bin) is empty

2.2 What is the probability that it contains exactly K balls

2.3 What is the expected number of bins that are empty

3 Problem 3

Consider the following linear program

$$max\ c^T x \tag{1}$$
$$s.t\ \ Ax \leq b \tag{2}$$
$$x \geq 0 \tag{3}$$

3.1 Write the dual linear program of the above LP

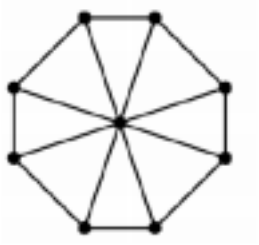
3.2 Write the corresponding complementary slackness conditions

3.3 Using the complementary slackness conditions, derive the strong duality theorem

.(If x^* is an optimal solution to the primal LP and y^* is an optimal solution to the dual LP then $c^T x^* = b^T y^*$)

4 Problem 4

A wheel of size k consists of a cycle on k vertices along with an additional vertex connected to every vertex in the cycle. As an example, you can see with a wheel the size 8 in the figure below.



The WHEEL problem is the following

4.1 Given an undirected graph $G = (V, E)$ and an integer k does G contain a wheel of size k as a sub graph?

4.2 Prove that Wheel is NP-Complete

reduce from Hamiltonian cycle problem