CS 510 Assignment 1:A Poisson distribution

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1   Log Likelihood of Poisson distribution

1.1   Solution

First we find the likelihood function

$$L(u) = \prod_{i=1}^{n} \frac{u^x e^{-u}}{x!} \tag{1}$$

Next we find the log likelihood function

$$L(u) = \sum_{i=1}^{n} (x_i * log(u) - u - log(x_i)!) \tag{2}$$

$$L(u) = log(u) * \sum_{i=1}^{n} x_i - n * u - \sum_{i=1}^{n} log(x_i)! \tag{3}$$

2   Derivative of the log likelihood

2.1   Solution

$$\frac{d}{du} = \frac{\sum_{i=1}^{n} x_i}{u} - n \tag{4}$$

3   Solve for MLE of U

3.1   Solution

We take our prior derivative and set to 0

$$\frac{d}{du} = \frac{\sum_{i=1}^{n} x_i}{u} - n = 0 \tag{5}$$

$$n = \frac{\sum_{i=1}^{n} x_i}{u} \tag{6}$$

$$nu = \frac{\sum_{i=1}^{n} x_i}{1} \tag{7}$$

$$u = \frac{\sum_{i=1}^{n} x_i}{n} \tag{8}$$

$$u = \bar{x} \tag{9}$$