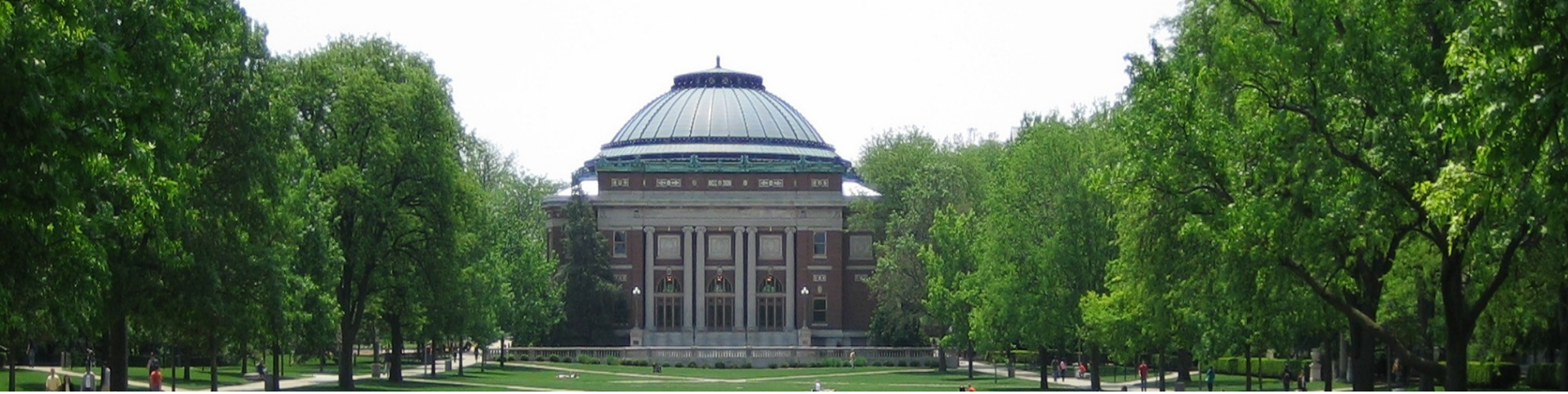


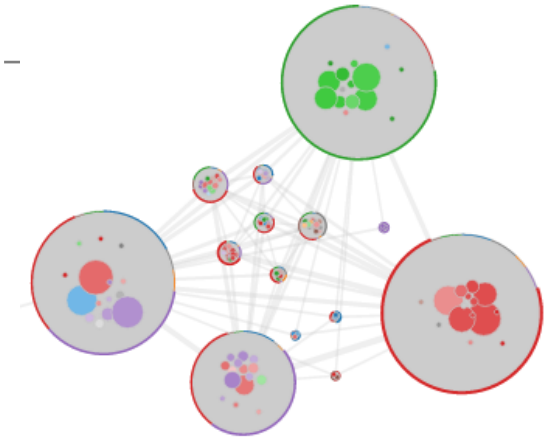
CS 512 Data Mining Principles

Network and Graph Connectivity

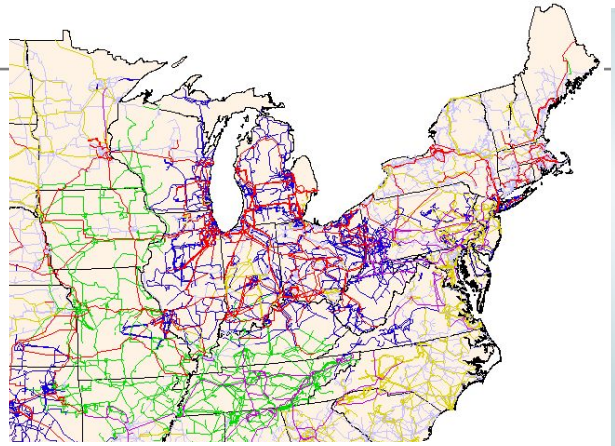
Hanghang Tong, Computer Science, Univ. Illinois at Urbana-Champaign, 2021

Suggested studying time: 3/26/2021-4/5/2021

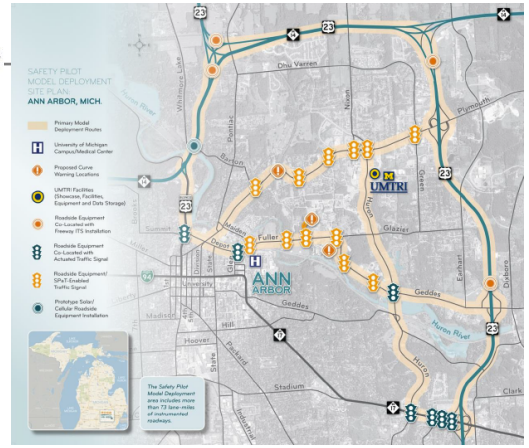
Observation: Networks & Graphs Are Everywhere!



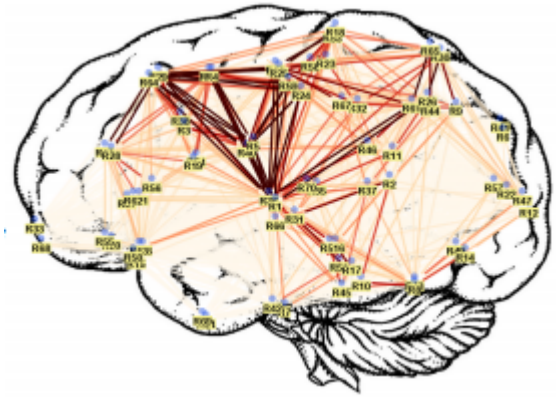
Collaboration Networks



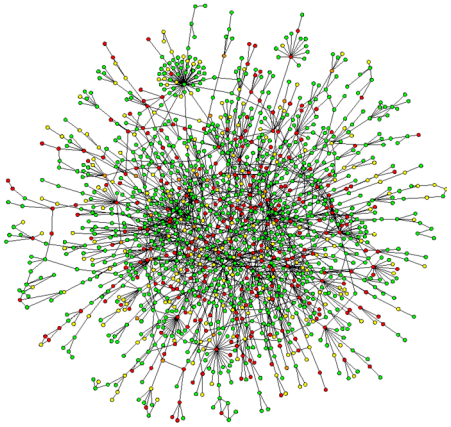
US Power Grid



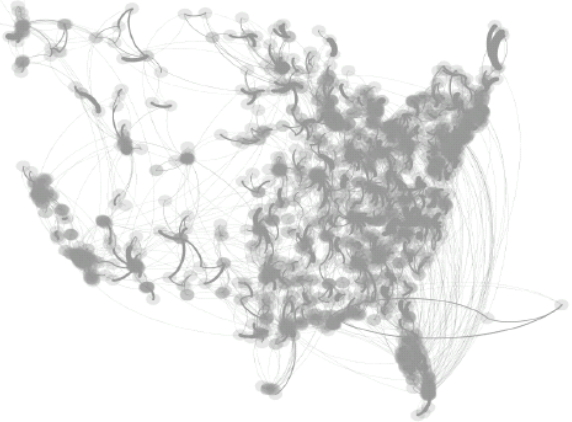
Traffic Network



Brain Networks



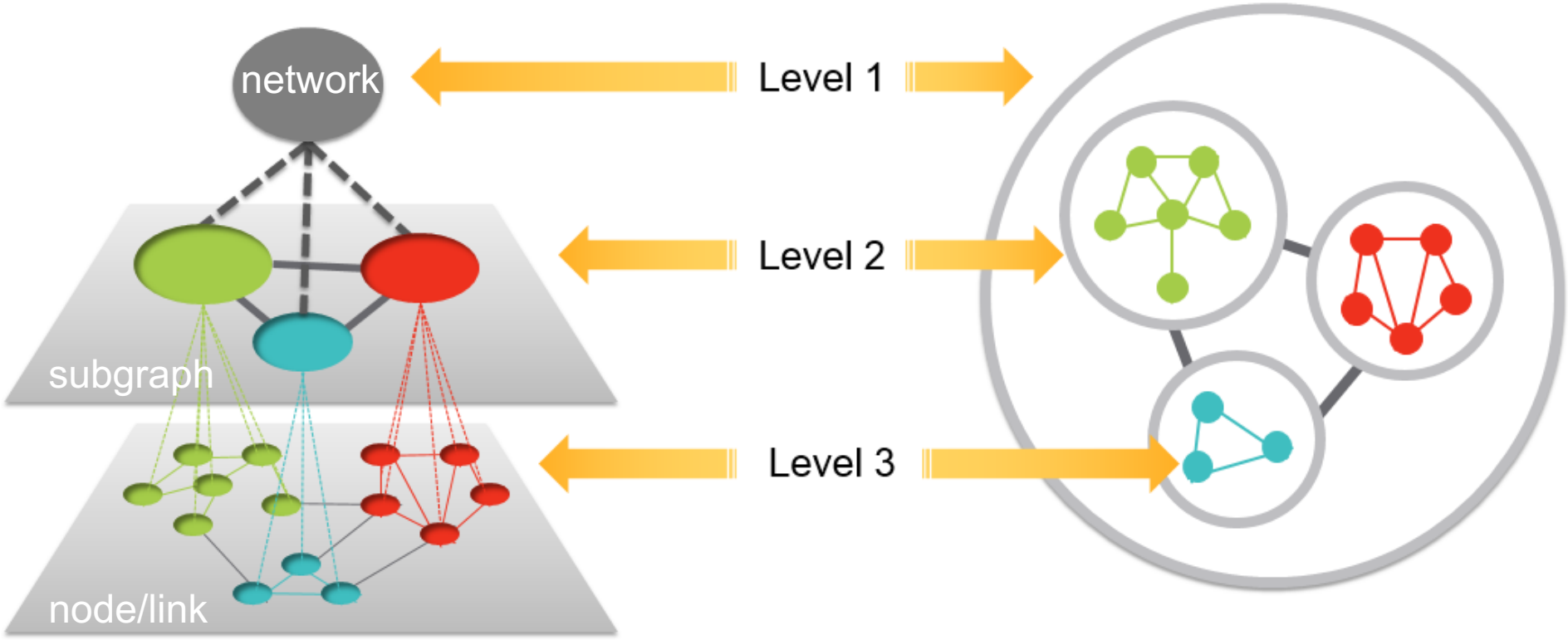
Biological Networks



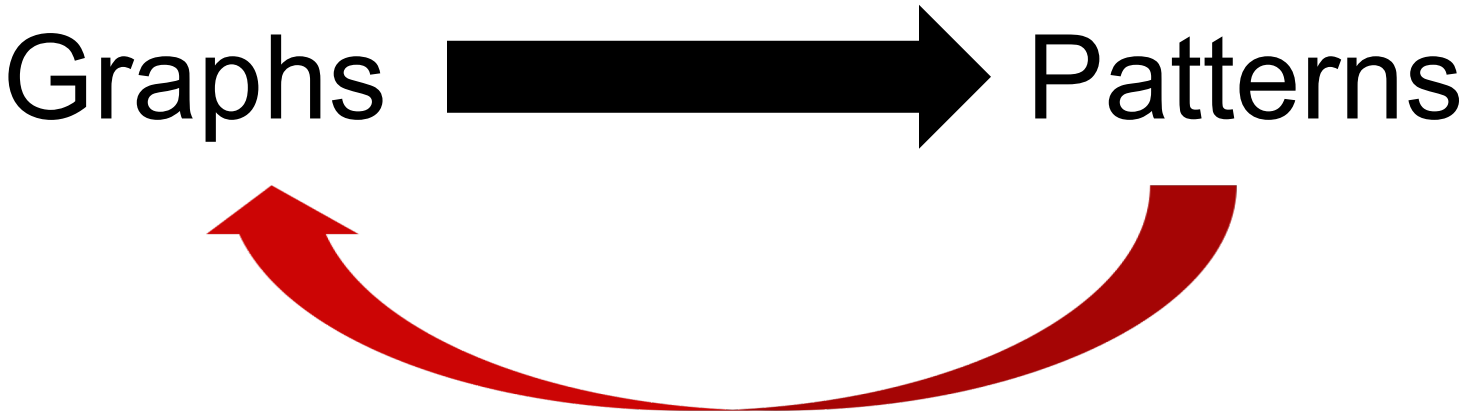
Hospital Networks

This Lecture: Networks = Graphs

Graph Mining: An Overview



A Typical Graph Mining Paradigm



Graph Connectivity Optimization (GCO) - This Lecture

Given:
(1) an initial graph
(2) a graph operation
(3) a mining task

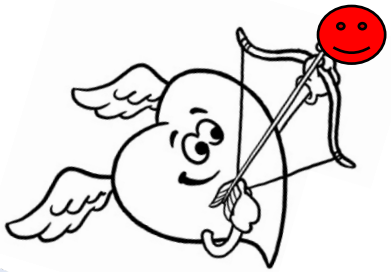
Find:
an 'optimal' graph

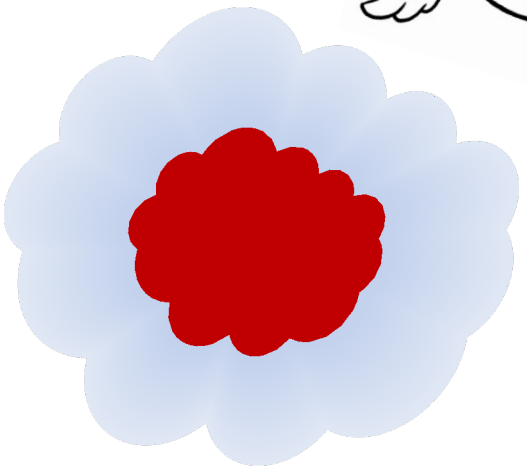
Graph operation: deleting 10 nodes; adding 5 links; etc.
Mining tasks: contain the virus; maximize the traffic flow

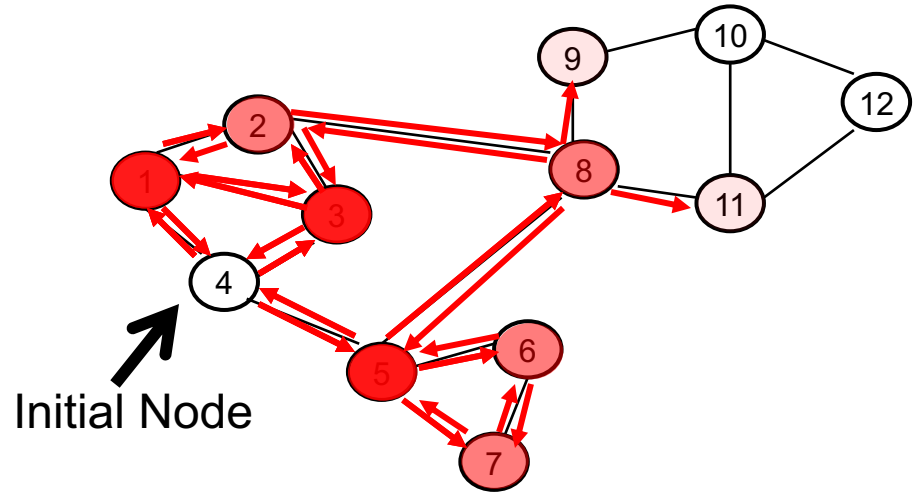
Dissemination: Think of it as Wine Spill

1. Spill a drop of wine on cloth
2. Spread/disseminate to the neighborhood

Dissemination: Wine Spill on a Graph



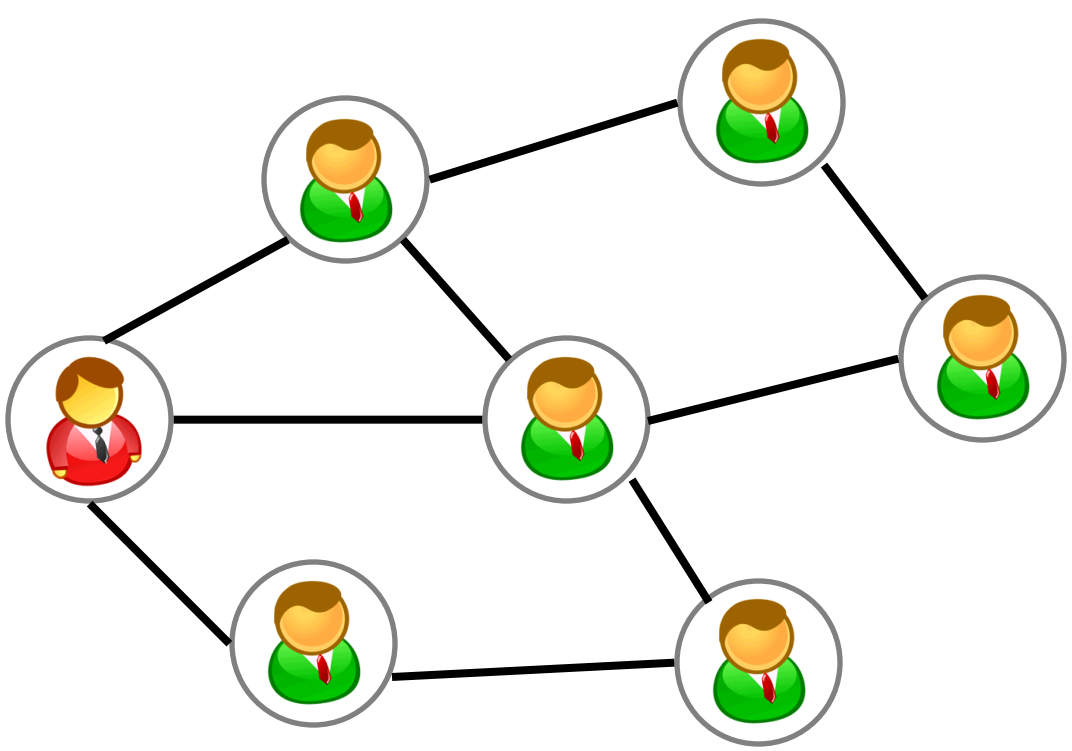




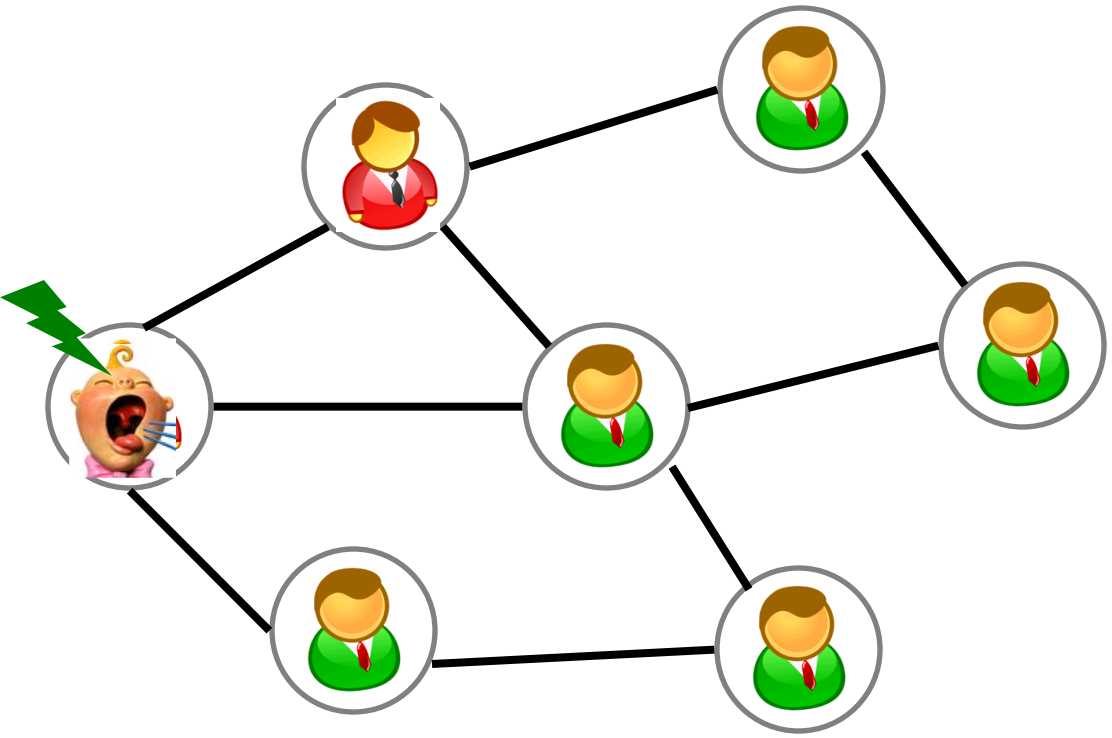
wine spill on cloth Dissemination on a graph

Same Diffusion Eq.

An Example: Virus Propagation/Dissemination



An Example: Virus Propagation/Dissemination




Sick


Healthy

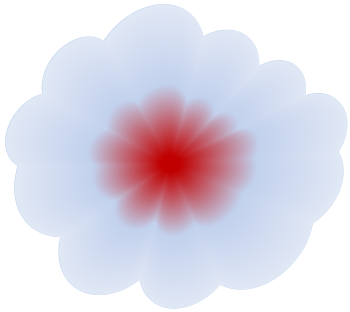
Contact

1: Sneeze to neighbors

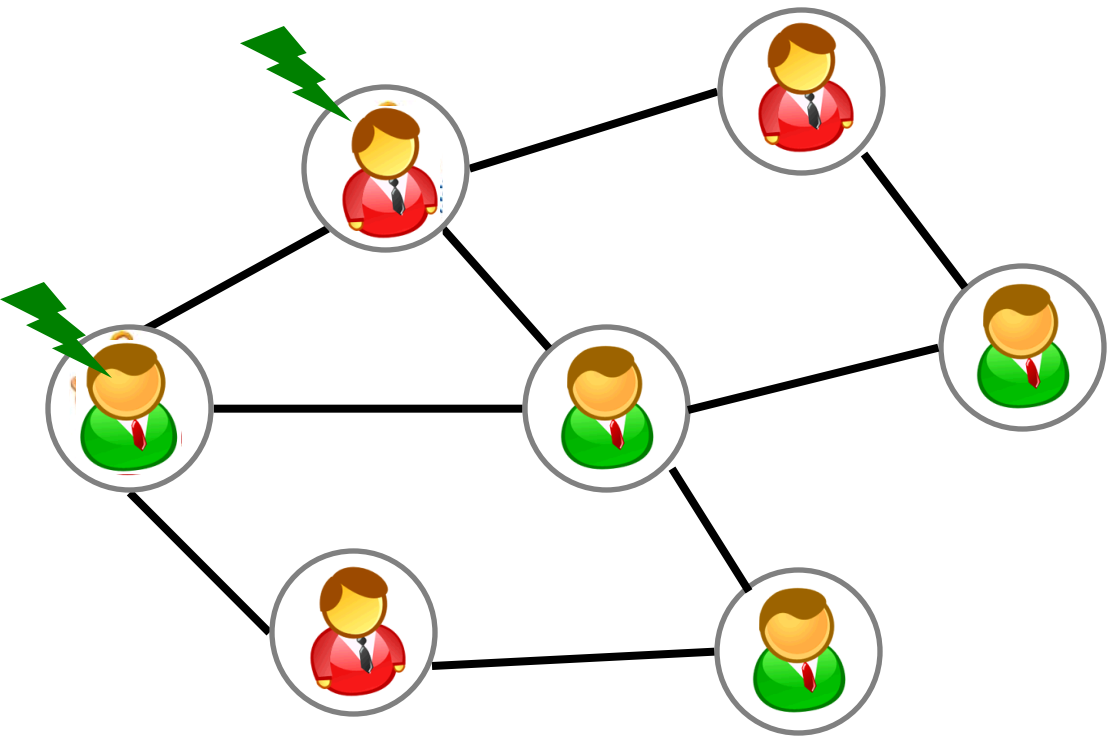
2: Some neighbors → Sick

3: Try to recover

Similar Diffusion Eq.



An Example: Virus Propagation/Dissemination



Contact

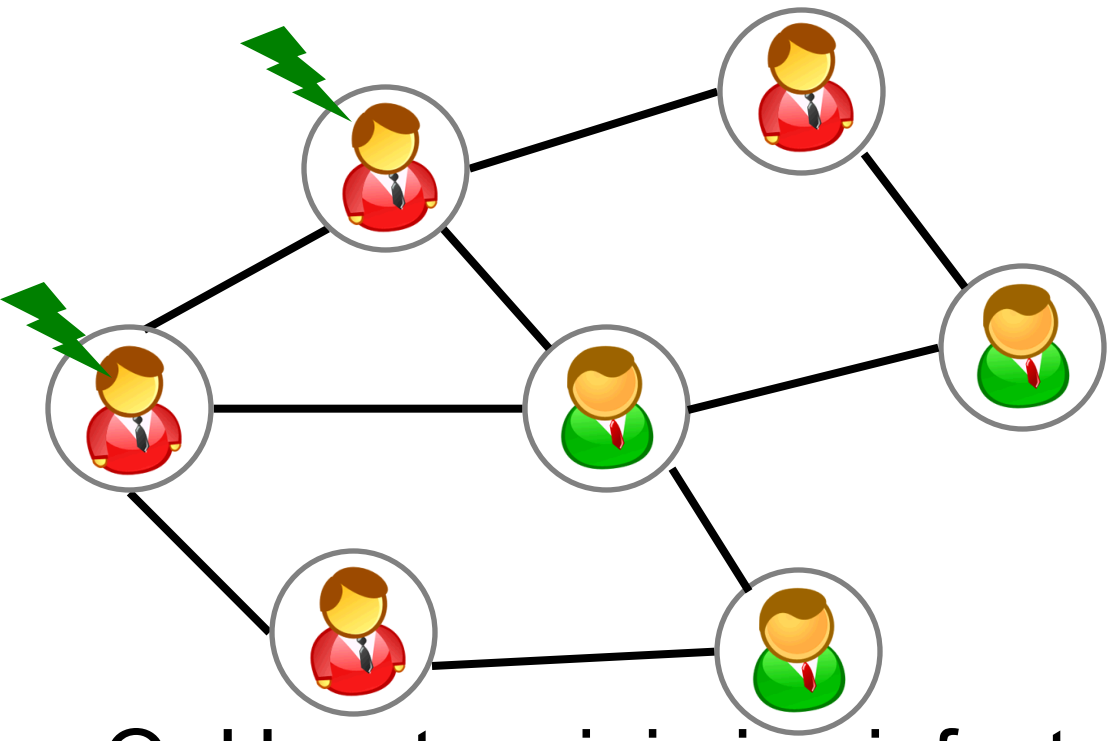
1: Sneeze to neighbors

2: Some neighbors → Sick

3: Try to recover

Q: How to minimize infected population?

An Example: Virus Propagation/Dissemination



Contact

1: Sneeze to neighbors

2: Some neighbors → Sick

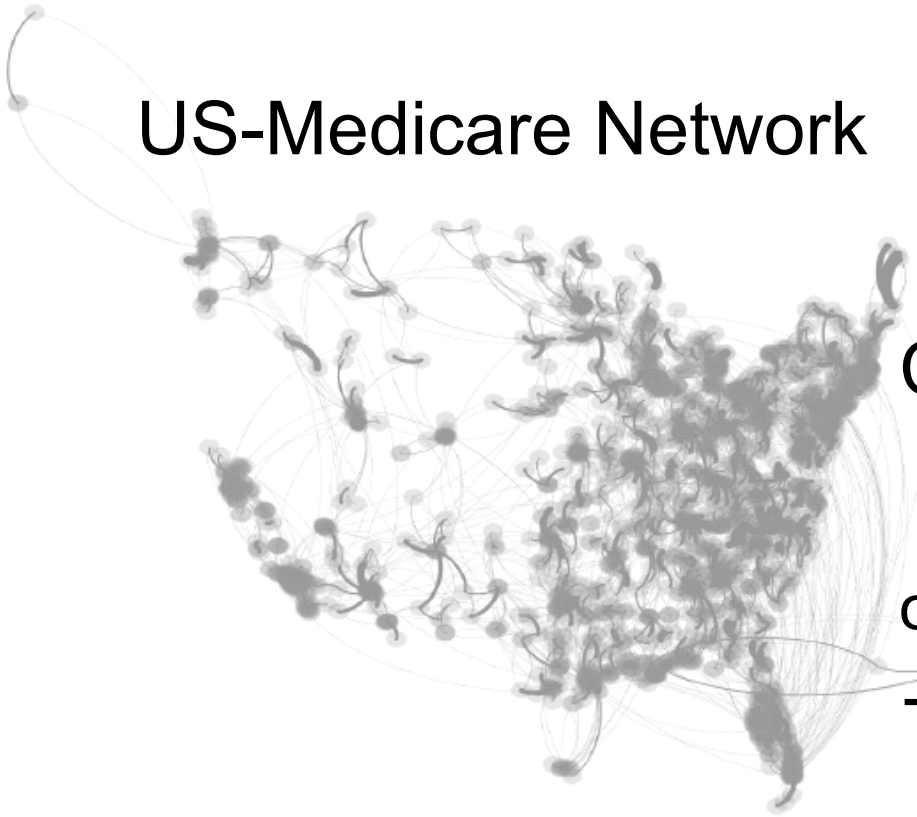
3: Try to recover

Q: How to minimize infected population?

- Q1: Understand tipping point
- Q2: Affecting algorithms

Why Do We Care? – Healthcare

US-Medicare Network



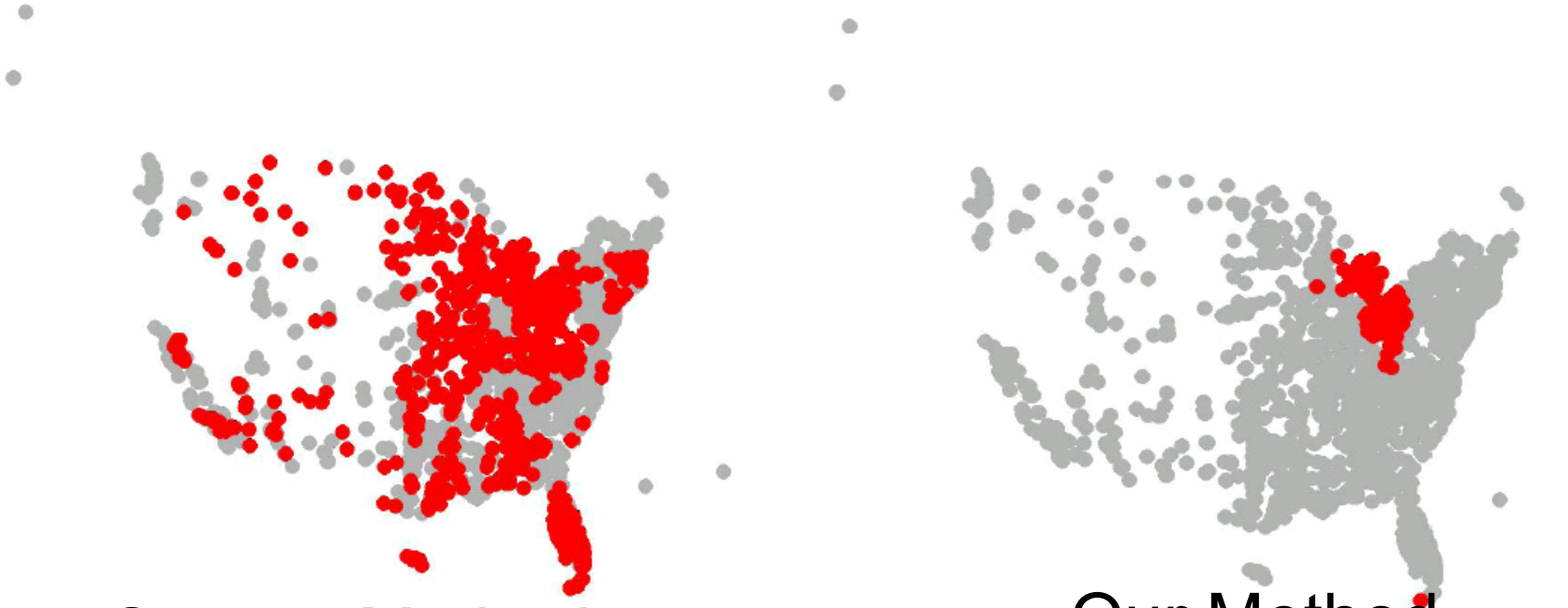
Critical Patient transferring

Move patients → specialized care → highly resistant micro-organism → Infection controlling → costly & limited

Q: How to allocate resource to minimize overall spreading?

SARS costs 700+ lives; $40+ Bn; H1N1 costs Mexico $2.3bn; Flu 2013: one of the worst in a decade, 105 children in US.

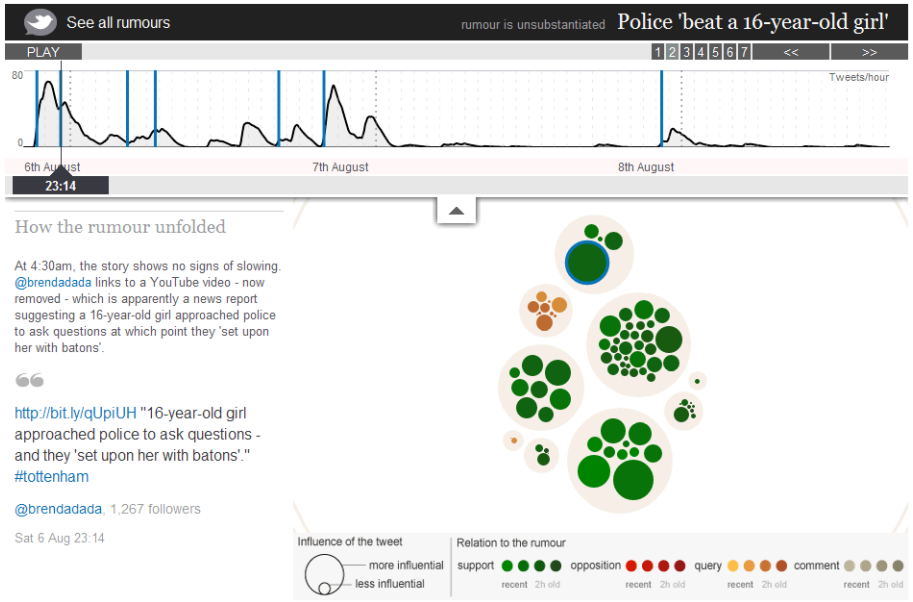
Why Do We Care? – Healthcare



Red: Infected Hospitals after 365 days

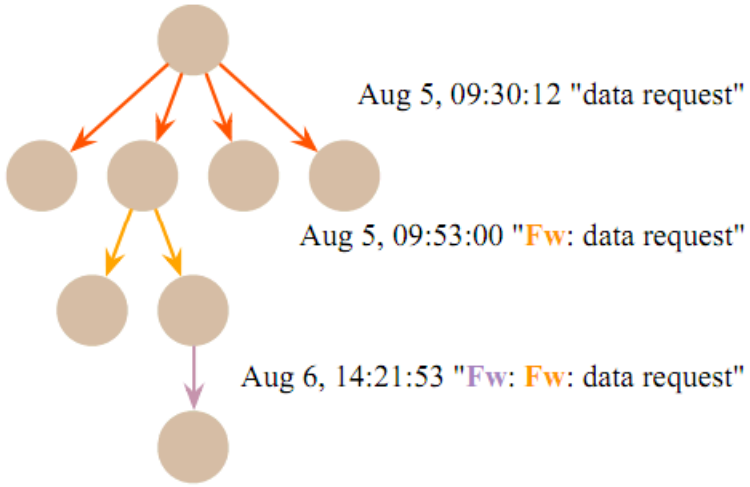
B. Aditya Prakash, Lada Adamic, Theodore Iwashnya, Hanghang Tong and Christos Faloutsos: Fractional Immunization on Networks. SDM 2013

Why Do We Care? (More)

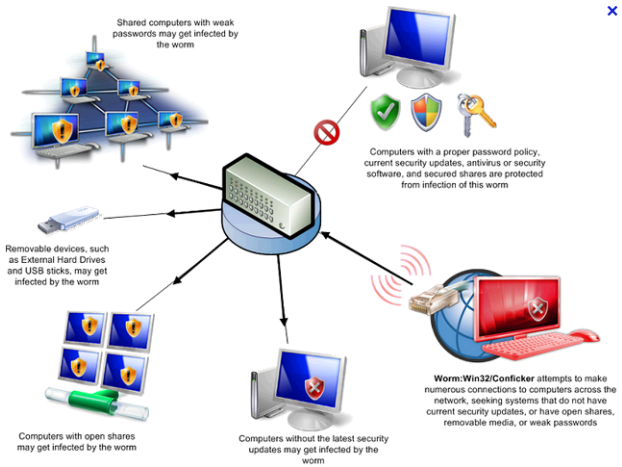


Rumor Propagation

Viral Marketing



Email Fwd in Organization



Malware Infection

Roadmap

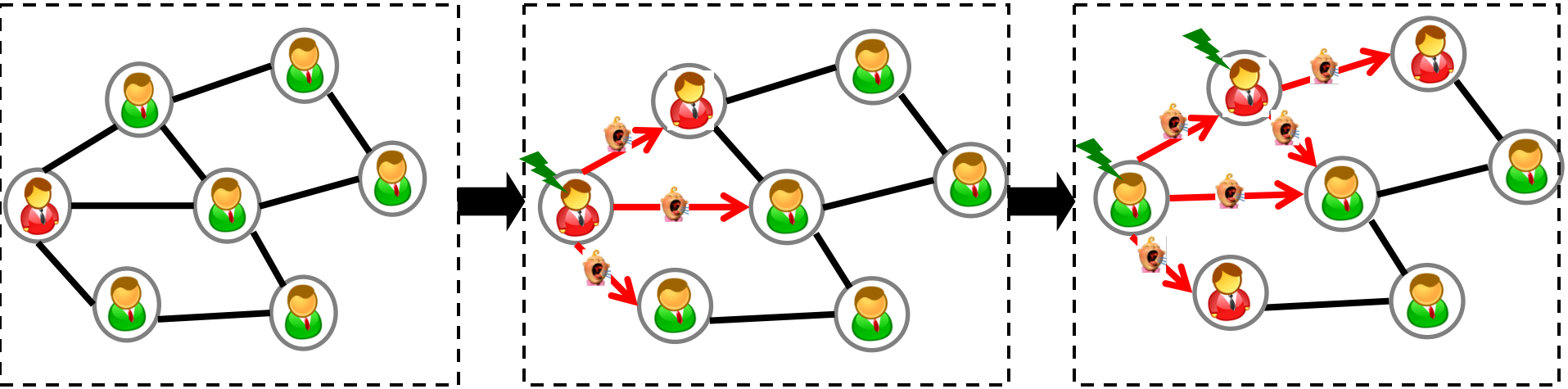
- ✔ Motivations and Background
- ➡ Part I: GCO Measures
- Part II: GCO Theories & Algorithms
- Part III: GCO Applications
- Part IV: Open Challenges & Future Trends

Part I: GCO Measures

- GCO Measure #1: Epidemic Threshold (λ)
- GCO Measure #2: Graph Robustness
- Other GCO Measures
- Comparison of GCO Measures
- Unification of GCO Measures

SIS Model (e.g., Flu) (Susceptible-Infected-Susceptible)

- Each Node Has Two Statuses: Sick Healthy
- β: Infection Rate (Prob (→ |))
- δ: Recovery Rate (Prob (→ |))

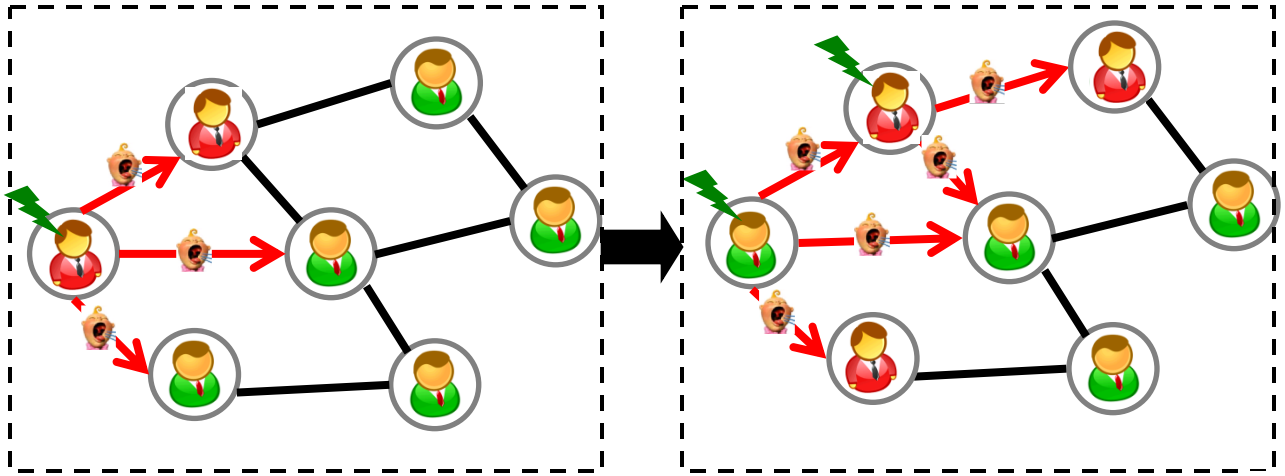


t = 1 *t* = 2 *t* = 3

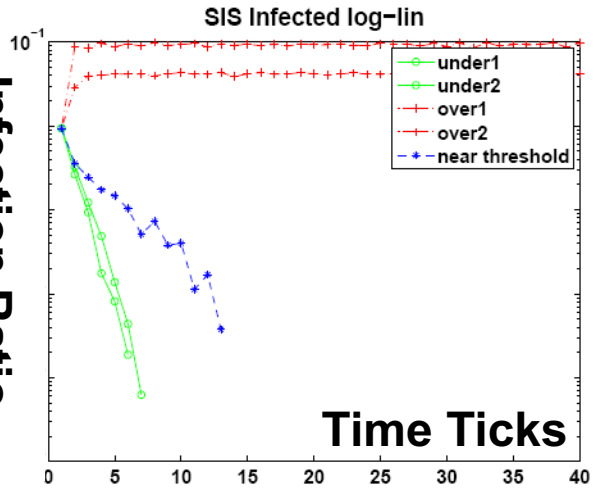
SIS Model (e.g., Flu)

β: Prob (→ |)

δ: Prob (→ |)



$p_{t+1} = H(p_t)$



Theorem [Chakrabarti+ 2003, 2007]:

If $\lambda \times (\beta/\delta) \leq 1$; no epidemic

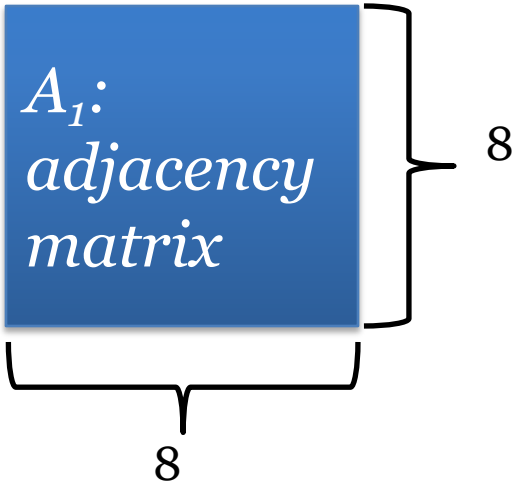
for any initial conditions

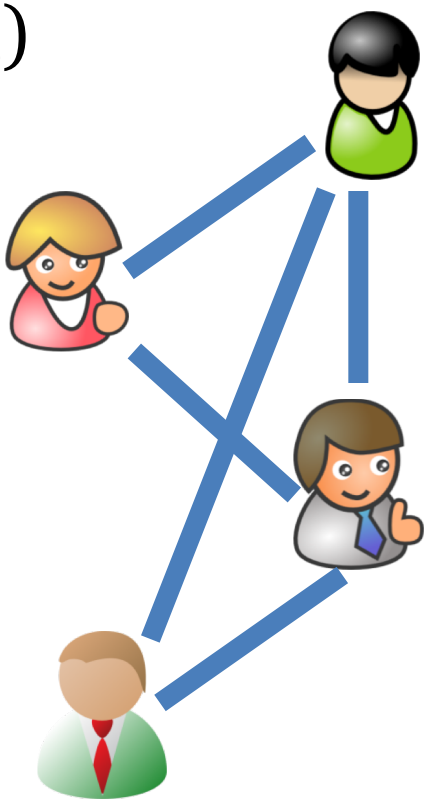
λ: largest eigenvalue of the graph (~ connectivity of the graph)

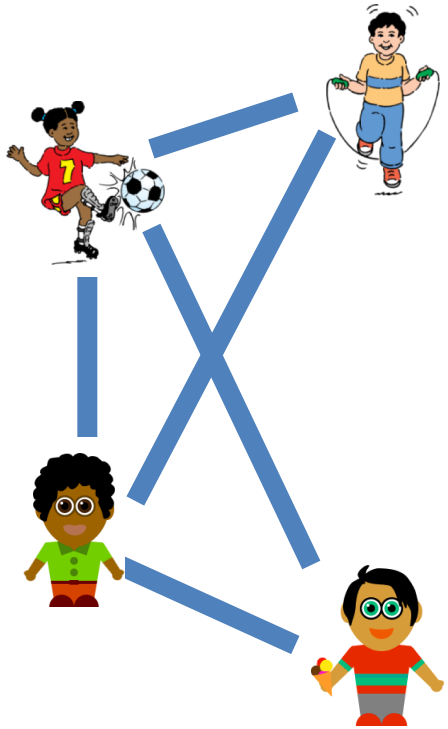
β, δ : virus parameters (~strength of the virus)

Beyond Static Graphs: Alternating Behavior

DAY
(e.g., work, school)



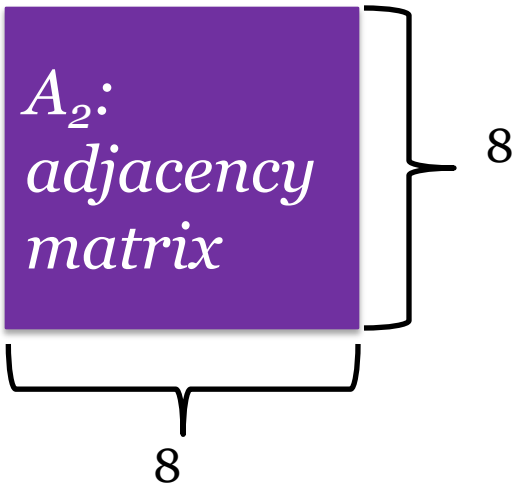


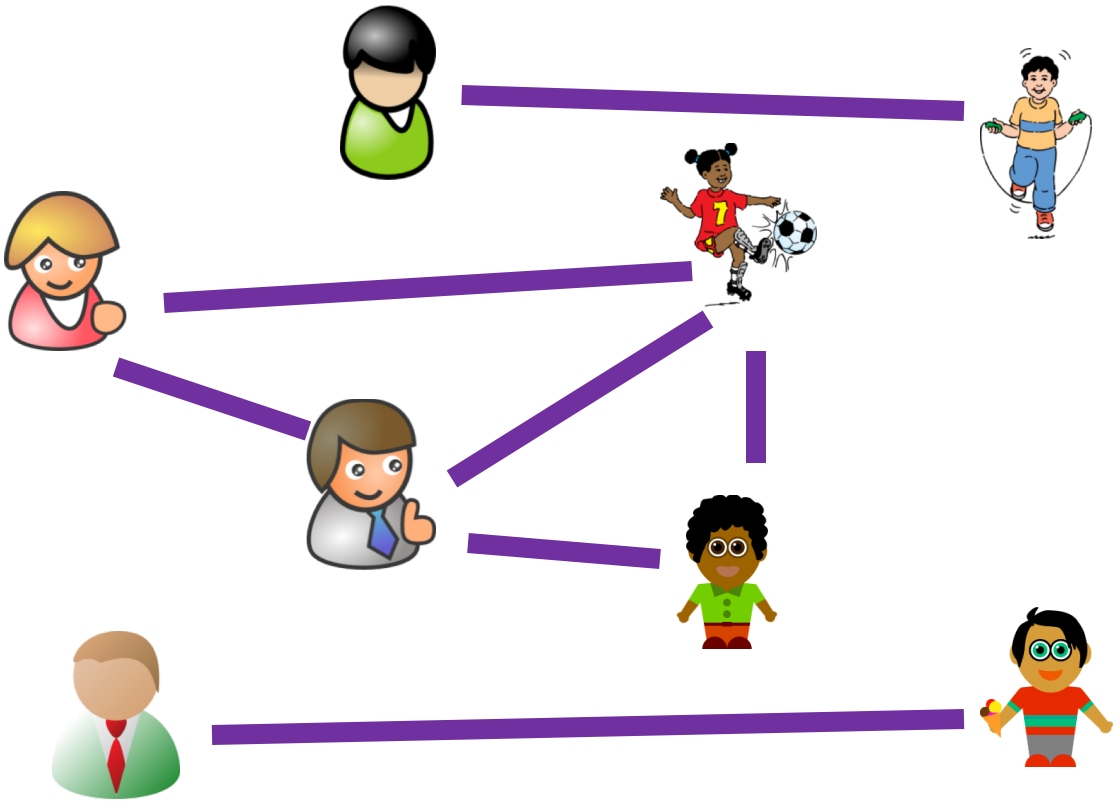


B. Aditya Prakash, Hanghang Tong, Nicholas Valler, Michalis Faloutsos, Christos Faloutsos:Virus Propagation on Time-Varying Networks: Theory and Immunization Algorithms. ECML/PKDD (3) 2010: 99-114
Nicholas Valler, B. Aditya Prakash, Hanghang Tong, Michalis Faloutsos, Christos Faloutsos:Epidemic Spread in Mobile Ad Hoc Networks: Determining the Tipping Point. Networking (1) 2011: 266-280

Beyond Static Graphs: Alternating Behavior

NIGHT
(e.g., home)



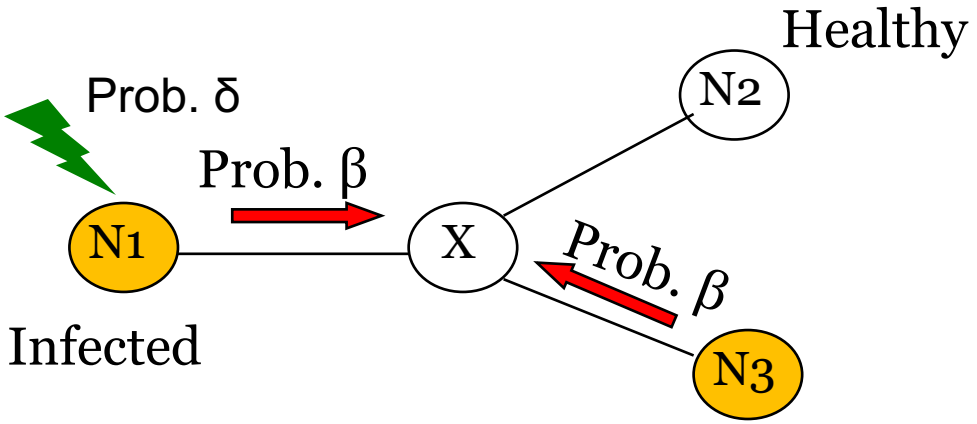


B. Aditya Prakash, Hanghang Tong, Nicholas Valler, Michalis Faloutsos, Christos Faloutsos:Virus Propagation on Time-Varying Networks: Theory and Immunization Algorithms. ECML/PKDD (3) 2010: 99-114
Nicholas Valler, B. Aditya Prakash, Hanghang Tong, Michalis Faloutsos, Christos Faloutsos:Epidemic Spread in Mobile Ad Hoc Networks: Determining the Tipping Point. Networking (1) 2011: 266-280

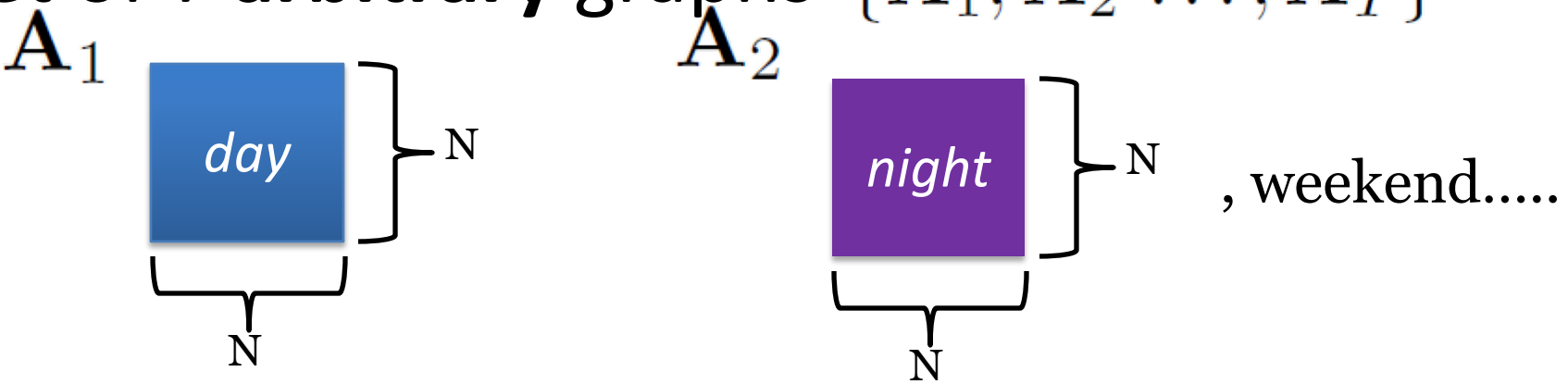
Formal Model Description

[PKDD 2010, Networking 2011]

- SIS model
  - recovery rate δ
  - infection rate β



- Set of *T* **arbitrary** graphs $\{\mathbf{A}_1, \mathbf{A}_2 \ldots, \mathbf{A}_T\}$



Epidemic Threshold for Alternating Behavior

[PKDD 2010, Networking 2011]

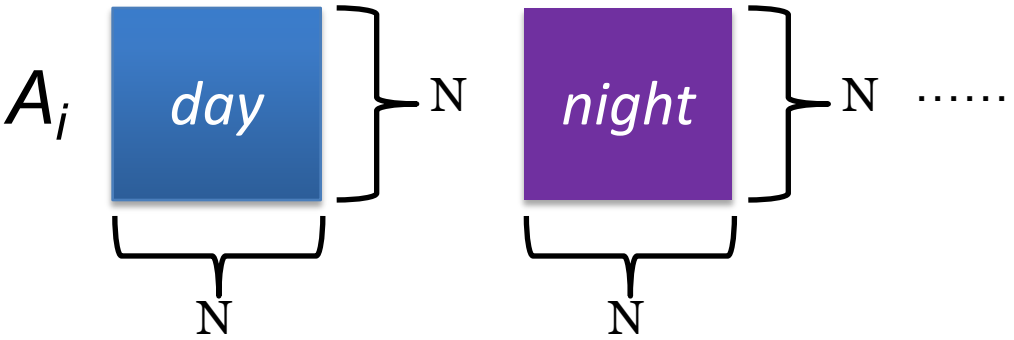
Theorem [PKDD 2010, Networking 2011]:
No epidemic If $\lambda(S) \leq 1$.

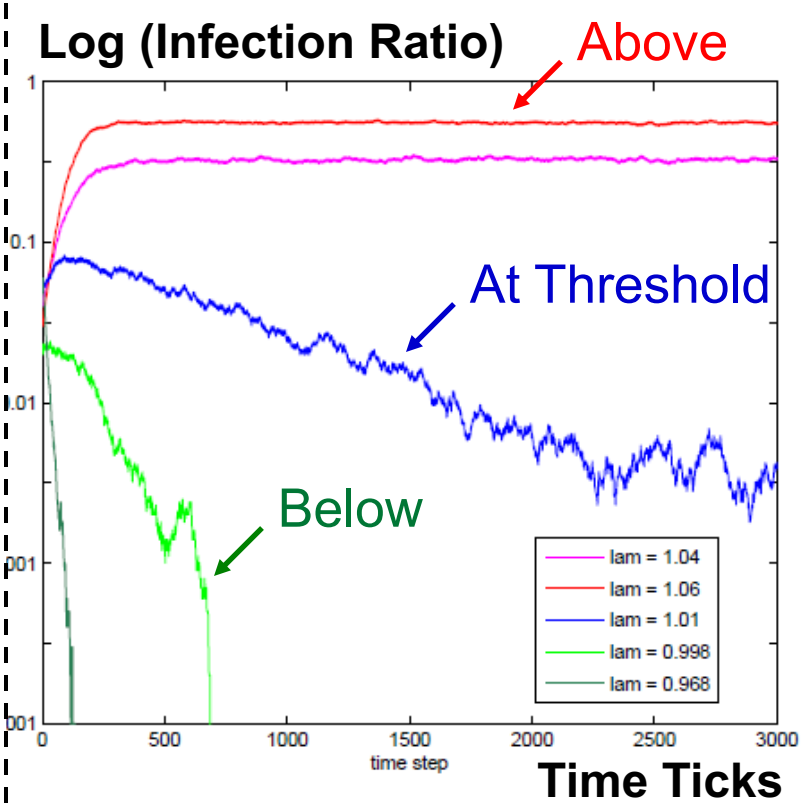
System matrix $S = \Pi_i S_i$

$S_i = (1-\delta)I + \beta A_i$

β: Prob (→ |)

δ: Prob (→ |)



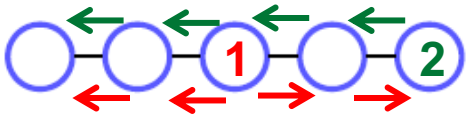


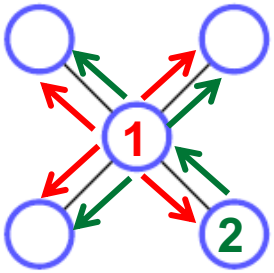
Also generalize to other 25 virus propagation models

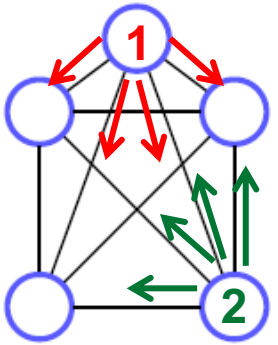
Why is λ So Important?

- λ → **Path Capacity** of a Graph:

$$\left(\vec{1}^{*} A^{k} \vec{1}\right)^{1/k} \xrightarrow[k \to \infty]{} \lambda$$







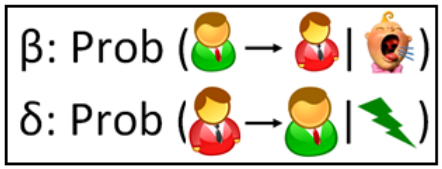
(a)Chain($\lambda_1 = 1.73$) **(b)Star($\lambda_1 = 2$)** **(c)Clique($\lambda_1 = 4$)**

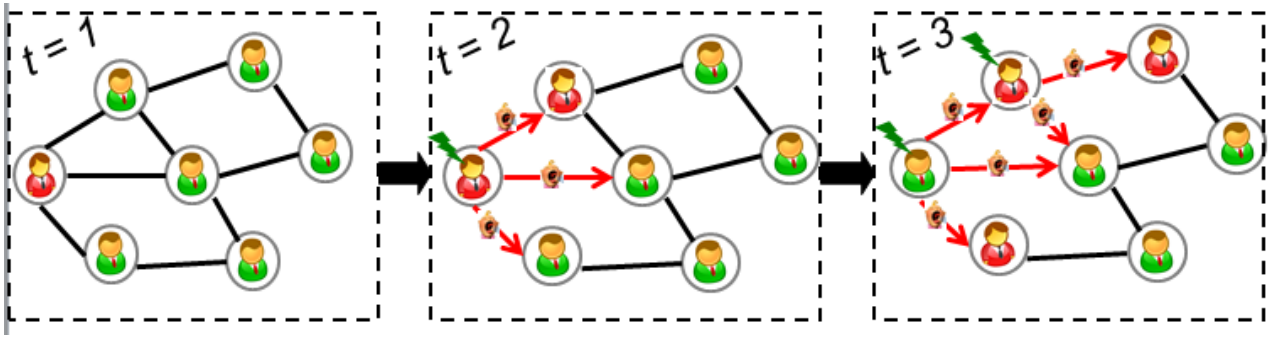
Larger λ → better connected

Why is λ So Important?

Details

- **Key 1: Model Dissemination as an NLDS:**





$p_{t+1} = g(p_t)$

p_t : Prob. vector: nodes being sick at t

g : Non-linear function (graph + virus parameters)

- **Key 2: Asymptotic Stability of NLDS:**

$p = p^* = 0$ is asymptotic stable if $|\lambda(J)| < 1$, where

$$J_{k,l} = [\nabla g(\mathbf{p}^*)]_{k,l} = \frac{\partial p_{k,t+1}}{\partial p_{l,t}}|_{\mathbf{p}_t=\mathbf{p}^*}$$

$$\frac{\partial \mathbf{p}_{2t+2}}{\partial \mathbf{p}_{2t+1}}|_{\mathbf{p}_{2t+1}=\mathbf{0}} = (1-\delta)\mathbf{I} + \beta\mathbf{A}_1 = \mathbf{S}_1$$

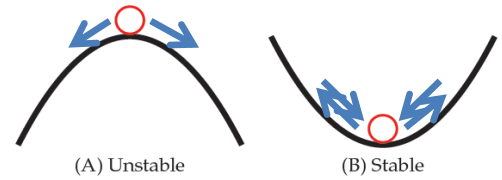
$$\frac{\partial \mathbf{p}_{2t+1}}{\partial \mathbf{p}_{2t}}|_{\mathbf{p}_{2t}=\mathbf{0}} = (1-\delta)\mathbf{I} + \beta\mathbf{A}_2 = \mathbf{S}_2$$

$$p_{i,2t+1} = 1 - \delta p_{i,2t} - (1 - p_{i,2t})\zeta_{2t}(i)$$

$$p_{i,2t+2} = 1 - \delta p_{i,2t+1} - (1 - p_{i,2t+1})\zeta_{2t+1}(i)$$

$$\zeta_{2t}(i) = \prod_{j \in \mathcal{NE}_2(i)} (p_{j,2t}(1-\beta) + (1 - p_{j,2t})) = \prod_{j \in \{1..n\}} (1 - \beta \mathbf{A}_2(i,j) p_{j,2t}))$$

$$\zeta_{2t+1}(i) = \prod_{j \in \mathcal{NE}_1(i)} (p_{j,2t+1}(1-\beta) + (1 - p_{j,2t+1})) = \prod_{j \in \{1..n\}} (1 - \beta \mathbf{A}_1(i,j) p_{j,2t+1}))$$



Beyond λ: Graph/Network Robustness

- Robustness is the ability of a network to continue performing well when it is subject to failures or attacks.
  - random failure (server down)
  - cascading failure (virus propagating)
  - targeted attack (carefully-chosen agents down)
- How to measure the robustness of a given network?
  - interpretable
  - (strictly) monotonic
  - captures redundancy

Hau Chan, Leman Akoglu, Hanghang Tong: Make It or Break It: Manipulating Robustness in Large Networks. SDM 2014: 325-333

Details

Beyond λ: Graph/Network Robustness

- Study of robustness:
  - mathematics, physics, computer science, biology
- A long (!) and profoundly diverse list of measures:
  - vertex/edge connectivity
  - avg. shortest distance
  - max. shortest distance (diameter)
  - efficiency
  - vertex/edge betweenness
  - clustering coefficient
  - largest component fraction/avg. component size
  - total pairwise connectivity
  - average available flows

Hau Chan, Leman Akoglu, Hanghang Tong: Make It or Break It: Manipulating Robustness in Large Networks. SDM 2014: 325-333

Beyond λ: Graph/Network Robustness

- …
- algebraic connectivity
- effective resistance
- number of spanning trees

eigenvalues of the Laplacian **L**

- principal eigenvalue λ_1
- spectral gap $\lambda_1 - \lambda_2$
- natural connectivity

eigenvalues of the adjacency **A**

- other (combinatorial) measures:

– toughness, scattering number, tenacity, integrity, fault diameter, isoperimetric number, min balanced cut, restricted connectivity, …

Beyond λ: Graph/Network Robustness

- …
- algebraic connectivity
- effective resistance
- number of spanning trees

eigenvalues of the Laplacian **L**

- principal eigenvalue λ_1
- spectral gap $\lambda_1 - \lambda_2$
- natural connectivity

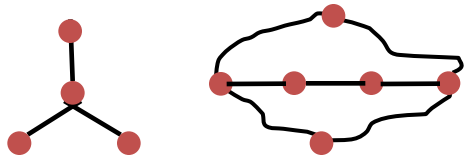
eigenvalues of the adjacency **A**

- other (combinatorial) measures:
  - toughness, scattering number, tenacity, integrity, fault diameter, isoperimetric number, min balanced cut, restricted connectivity, …

Hau Chan, Leman Akoglu, Hanghang Tong: Make It or Break It: Manipulating Robustness in Large Networks. SDM 2014: 325-333

A “guide” for “good” robustness measures

- Strict monotonicity
  - improves strictly when edges are added
  - *related: differentiates graphs



- Redundancy
  - accounts for alternative/back-up paths
- Stability
  - does not change drastically by small changes
  - *related: meaningful for disconnected graphs
- Interpretability
  - its meaning is intuitively clear

Details

A “guide” for “good” robustness measures

Measures	S. Monotone	Redundant	Stable	Interpretable
vertex / edge connectivity	✗		✗	✓
avg. shortest distance	✗	✗	✗	✓
diameter	✗	✗	✗	✓
efficiency	✓	✗	✓	✓
vertex / edge betweenness	✓	✗	✗	✓
clustering coefficient	✗		✓	✓
largest component fraction	✗	✗		✓
total pairwise connectivity	✗	✗		✓
avg. available flows		✓	✗	✓
algebraic connectivity	✗		✗	✗
effective resistance	✓	✓	✓	✓
number of spanning trees	✗		✗	
spectral radius / gap			✓	✗
natural connectivity	✓	✓	✓	✓

Hau Chan, Leman Akoglu, Hanghang Tong: Make It or Break It: Manipulating Robustness in Large Networks. SDM 2014: 325-333

Connectivity Unification: Single Networks

- **SUBLINE Connectivity Family**
  - **Key Idea**: graph connectivity as the **aggregation** over the subgraph connectivity

$$C(G, f) = \sum_{\pi \subseteq G} f(\pi)$$

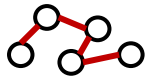
  - G : the given network
  - π: a non-empty valid subgraph in G
  - $f(\pi)$: connectivity of the subgraph π
  - $C(G, f)$: connectivity of graph G

Chen Chen, Jingrui He, Nadya Bliss, Hanghang Tong. On the Connectivity of Multi-layered Networks: Models, Measures and Optimal Control. ICDM 2015: 715-720
Chen Chen, Jingrui He, Nadya Bliss, Hanghang Tong, "Towards Optimal Connectivity on Multi-layered Networks", IEEE Transactions on Knowledge and Data Engineering 2017

Connectivity Unification: Single Networks

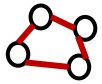
- **SUBLINE Connectivity Family**
  - Key idea $C(G, f) = \sum_{\pi \subseteq G} f(\pi)$
  - Examples
    - Epidemic threshold (Path Capacity)

$$f(\pi) = \begin{cases} \beta^{len(\pi)} & \text{if } \pi \text{ is a valid path of length } len(\pi) \\ 0 & \text{otherwise.} \end{cases}$$



    - Natural connectivity (Loop Capacity)

$$f(\pi) = \begin{cases} 1/len(\pi)! & \text{if } \pi \text{ is a valid loop of length } len(\pi) \\ 0 & \text{otherwise.} \end{cases}$$



    - Clustering coefficient (Triangle Capacity)

$$f(\pi) = \begin{cases} 1 & \text{if } \pi \text{ is a triangle} \\ 0 & \text{otherwise.} \end{cases}$$

    - …

Chen Chen, Jingrui He, Nadya Bliss, Hanghang Tong. On the Connectivity of Multi-layered Networks: Models, Measures and Optimal Control. ICDM 2015: 715-720
Chen Chen, Jingrui He, Nadya Bliss, Hanghang Tong, "Towards Optimal Connectivity on Multi-layered Networks", IEEE Transactions on Knowledge and Data Engineering 2017

Connectivity as Eigen-function

- Computation of Connectivity
  - Most connectivity measures can be expressed as eigen-functions

$$g(\Lambda, U) = \begin{cases} 1/\lambda_1 & \text{Epidemic Threshold} \\ u_1 & \text{Eigenvector Centrality} \\ \Delta(G) = \frac{1}{6}\sum_{i=1}^{n} \lambda_i^3 & \text{\#Triangles} \\ S(G) = \ln(\frac{1}{k}\sum_{i=1}^{n} e^{\lambda_i}) & \text{Natural Connectivity} \\ \lambda_1 - \lambda_2 & \text{Eigen-Gap} \end{cases}$$

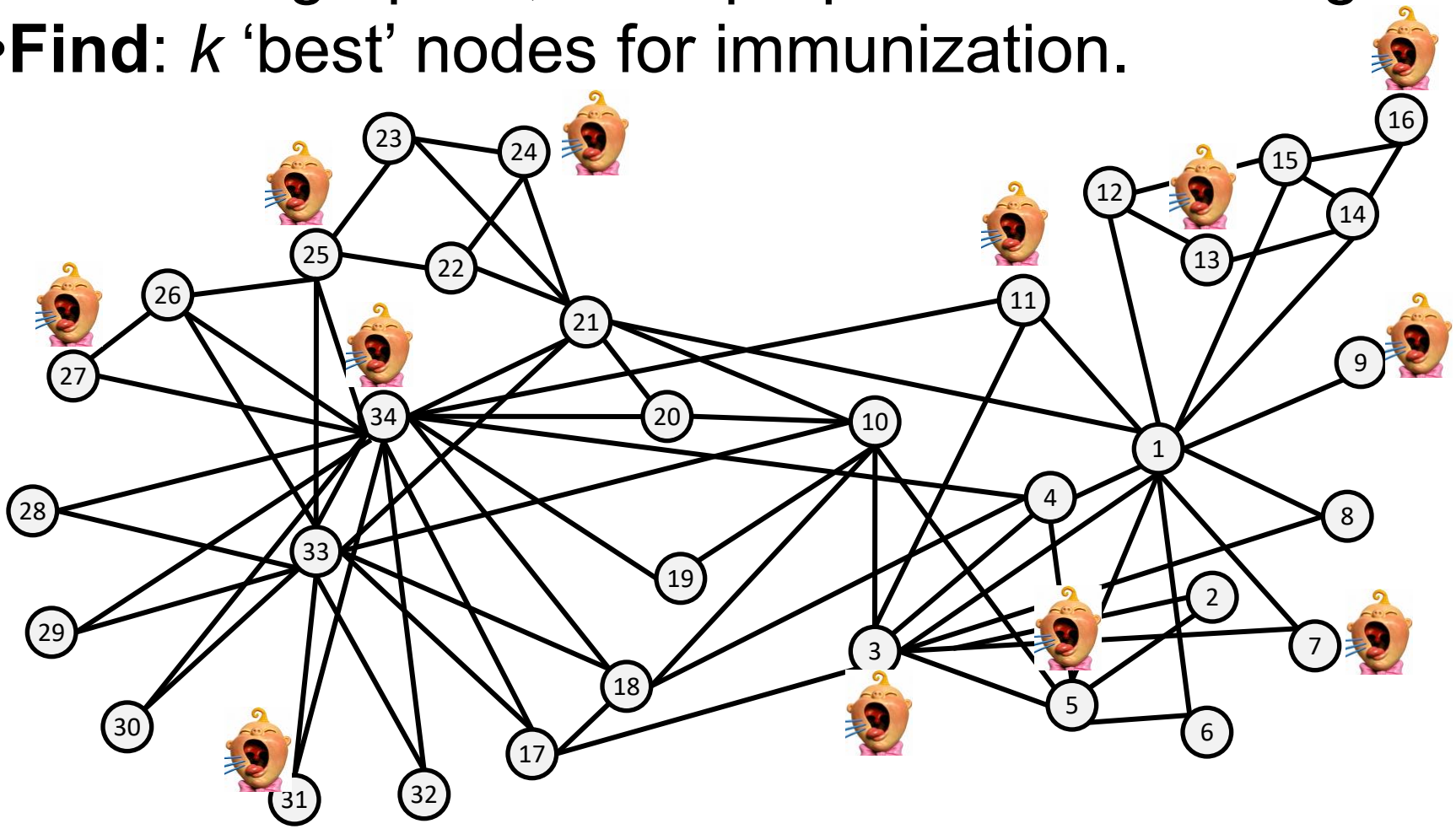
Chen Chen, Hanghang Tong. Fast Eigen-Functions Tracking on Dynamic Graphs. SDM 2015: 559-567
Chen Chen, Hanghang Tong, "On the Eigen-Functions of Dynamic Graphs: Fast Tracking and Attribution Algorithms", SAM Special Issue of "Best of SDM 2015"

Roadmap

- ✔ Motivations and Background
- ✔ Part I: GCO Measures
- ➡ Part II: GCO Theories & Algorithms
- Part III: GCO Applications
- Part IV: Open Challenges & Future Trends

Minimizing Dissemination: Immunization

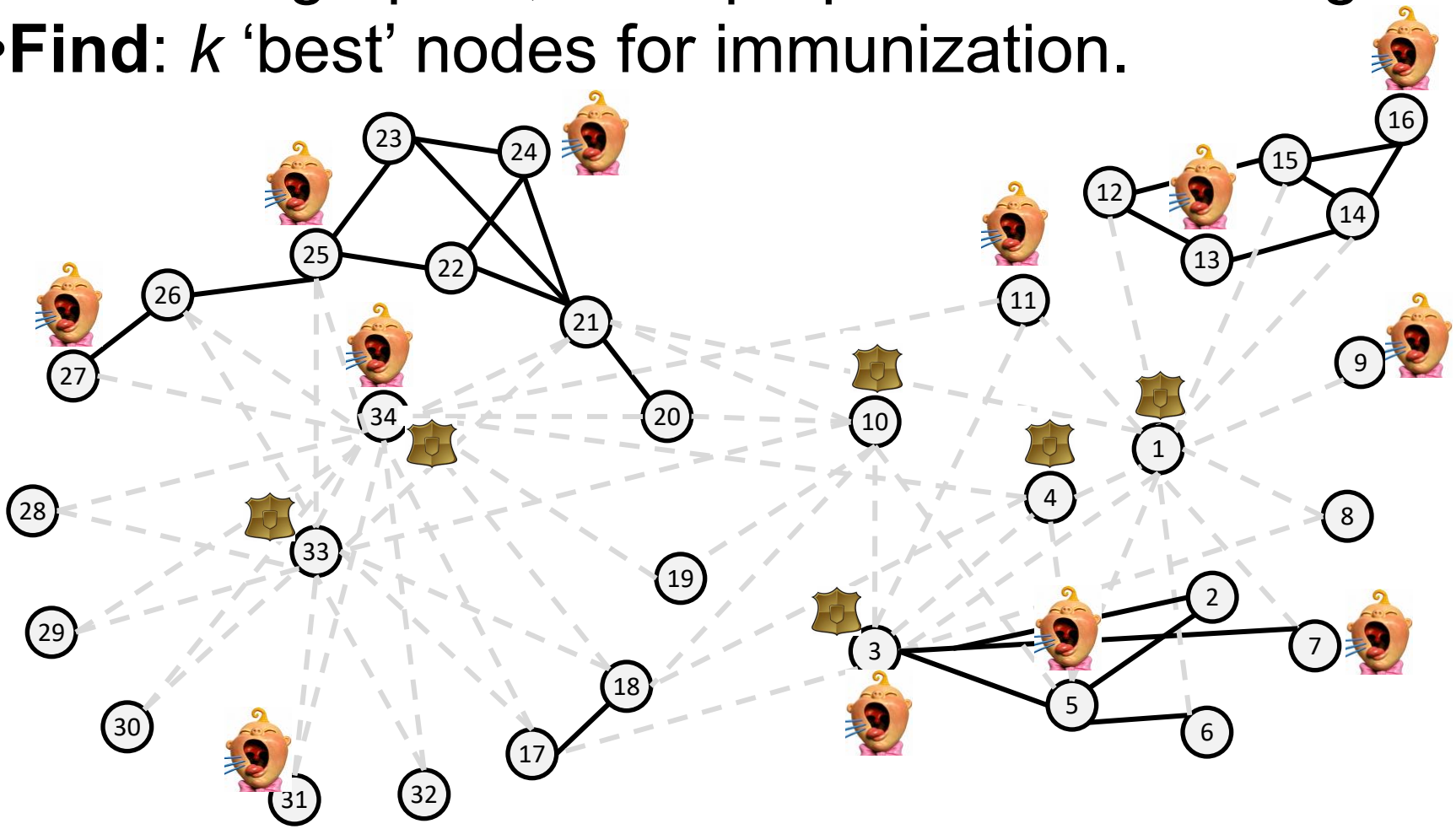
- **Given**: a graph *A*, virus prop model and budget *k*;
- **Find**: *k* 'best' nodes for immunization.



SARS costs 700+ lives; $40+ Bn; H1N1 costs Mexico $2.3bn

Minimizing Dissemination: Immunization

- **Given**: a graph *A*, virus prop model and budget *k*;
- **Find**: *k* 'best' nodes for immunization.

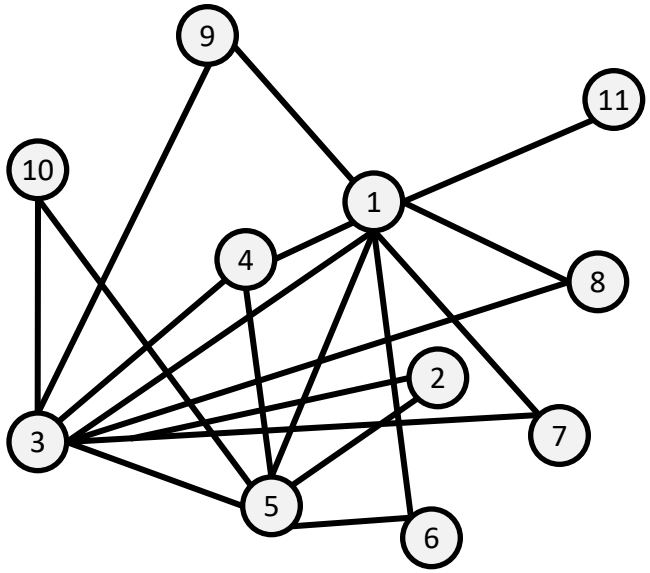


SARS costs 700+ lives; $40+ Bn; H1N1 costs Mexico $2.3bn

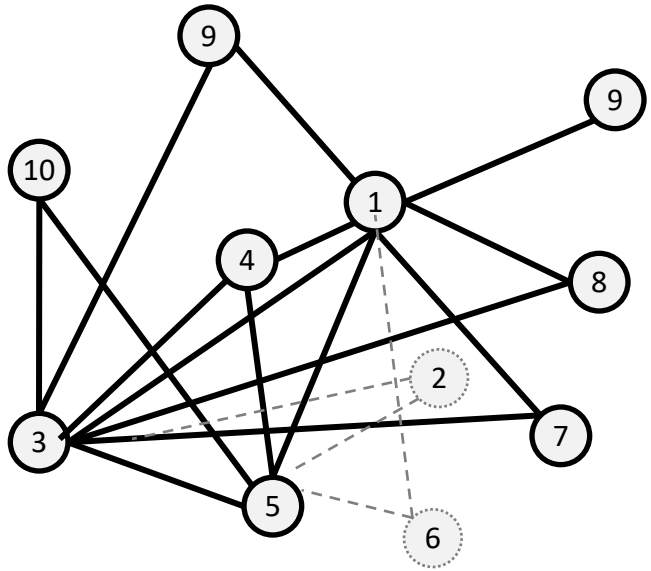
Optimal Method

- Select *k* nodes, whose absence creates the largest drop in λ

$$S = \arg\max_{|S|=k} \lambda - \lambda_S$$



Original Graph: λ



Without $\{2, 6\}$: λ_s

Optimal Method

- Select *k* nodes, whose absence creates the largest drop in λ

$$S = \arg\max_{|S|=k} \lambda - \lambda_S$$

Largest eigenvalue w/o subset of nodes *S*

- But, we need $O\left(\binom{n}{k} \cdot m\right)$ in time
  - Example: 1,000 nodes, with 10,000 edges
    - It takes 0.01 seconds to compute λ
    - It takes **2,615 years** to find best-*5* nodes !

Theorem: Find Optimal k-node Immunization is NP-Hard

C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015

Optimal *k*-node immunization is NP-Hard

- **Basic Idea:** Reduction from P1 (known NP-hard)

Given an undirected/unweighted graph *G*, and *k*

- **P1 (*k*-independent set problem)**: is there *k* nodes, no two of which are adjacent?
- **P2 (*k*-node immunization problem)**: is there k nodes, the deletions of which makes the leading eigenvalues ≤ 0

$$A = \begin{bmatrix} S_{kxk} & X_{(k)x(n-k)} \\ X_{(k)x(n-k)} & T_{(n-k)x(n-k)} \end{bmatrix}$$

- **Proof #1: If YES to P1(*G*,*k*)→ YES to P2(*G*, *n-k*)**

YES to P1 ➡ $S_{kxk} = \boldsymbol{0}$ $\xrightarrow[\text{Nodes in } T]{\text{Removing}}$ $\lambda(\tilde{A}) = \lambda(\boldsymbol{0}) = 0$ ➡ YES to P2

- **Proof #2: If NO to P1(*G*,*k*)→ NO to P2(*G*, *n-k*)**

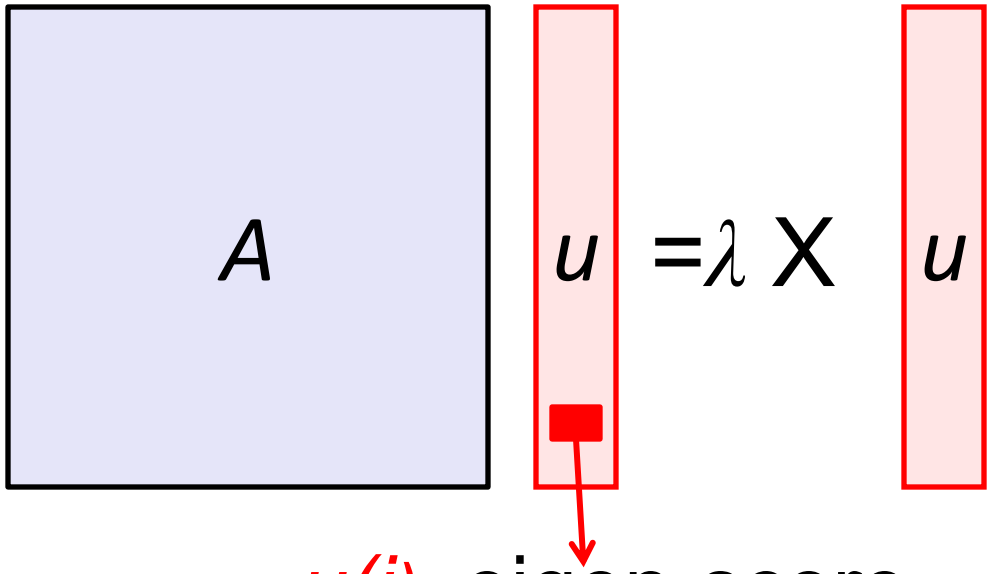
Suppose YES to P2 $\xrightarrow[\text{Nodes in } T]{\text{Removing}}$ $\lambda(\tilde{A}) = \lambda(\boldsymbol{0}) \leq 0$ $\xrightarrow{S(i,j)\geq 0}$

➡ $S_{kxk} = \boldsymbol{0}$ ⟷ Nodes in ***S*** being ind. set ➡ contradict

Netshield to the Rescue

Theorem:

(1) $\lambda - \lambda_s \approx Sv(S) = \sum_{i \in S} 2\lambda u(i)^2 - \sum_{i,j \in S} A(i,j)u(i)u(j)$



$u(i)$: eigen-score

$A_S = A - E \rightarrow \begin{bmatrix} 0 & \\ & 0 \end{bmatrix}$

$= A - (F + F' - G) \rightarrow \begin{bmatrix} 0 & \\ & 0 \end{bmatrix}$

$\lambda_s = \lambda - u' Eu/(u'u) + O(|E|^2)$

$= \lambda - 2u' Fu + 2u' Eu + O(|E|^2)$

$= \lambda - (\sum_{i \in S} 2\lambda u(i)^2 - \sum_{i,j \in S} A(i,j)u(i)u(j)) + O(|E|^2)$

Footnote:
$u(i)$ ~ PageRank(i) ~ in-degree(i)

C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015

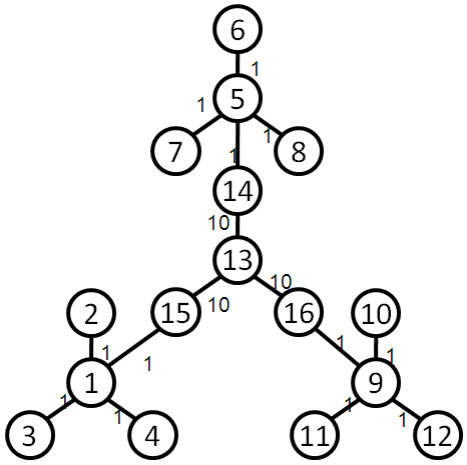
Netshield to the Rescue

Intuition

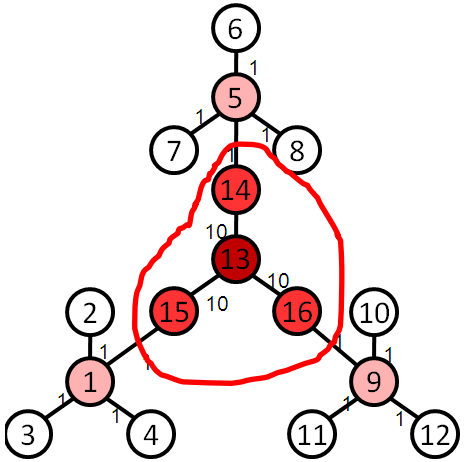
Theorem:

(1) $\lambda - \lambda_s \approx Sv(S) = \sum_{i \in S} 2\lambda u(i)^2 - \sum_{i,j \in S} A(i,j)u(i)u(j)$

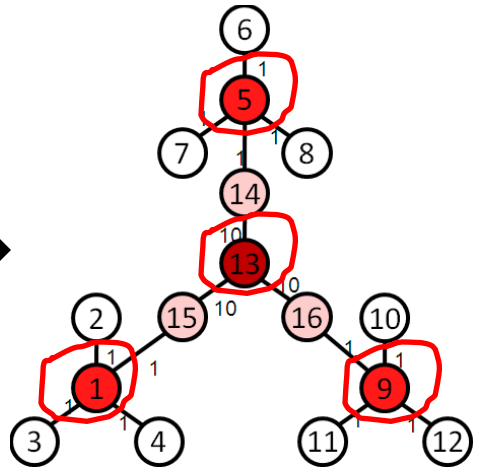
- find a set of nodes S *(e.g. k=4),* which
  - (C1) each has high eigen-scores
  - (C2) diverse among themselves



Original Graph



Select by C1



Select by C1+C2

(1) $\lambda - \lambda_s \approx Sv(S)$ ✓
(2) $Sv(S)$ is submodular
(3) *Netshield* is near-opt
(4) *Netshield* scales linearly

Netshield to the Rescue

Theorem:

(1) $\lambda - \lambda_s \approx Sv(S) = \sum_{i \in S} 2\lambda u(i)^2 - \sum_{i,j \in S} A(i,j)u(i)u(j)$

(2) Sv(S) is sub-modular (+monotonically non-decreasing)

Corollary:

(3) *Netshield* is near-optimal (wrt max $Sv(S)$)

(4) *Netshield* is $O(nk^2+m)$

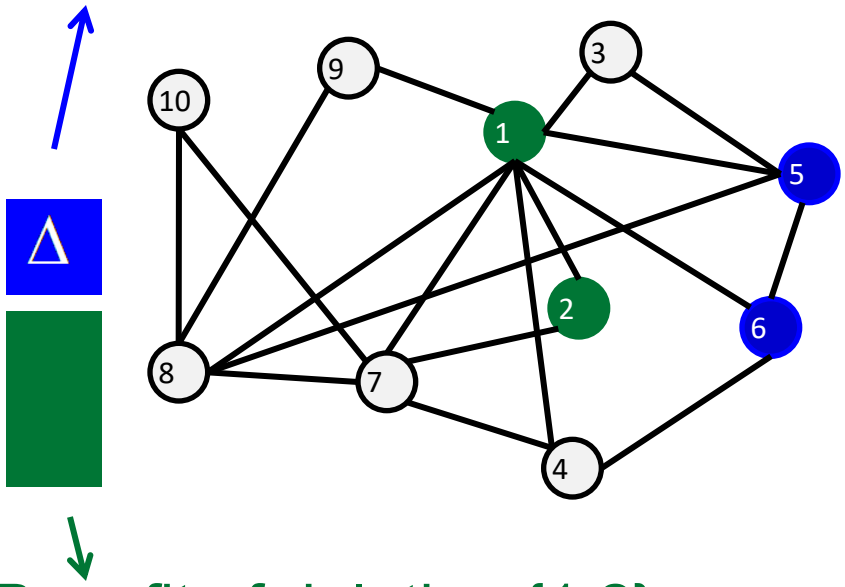
- Example: 1,000 nodes, with 10,000 edges
  - *Netshield* takes **< 0.1 seconds** to find best-*5* nodes !
  - … as opposed to **2,615 years**

Footnote: near-optimal means $Sv(S^{Netshield}) >= (1-1/e)\ Sv(S^{Opt})$

Why *Netshield* is Near-Optimal?

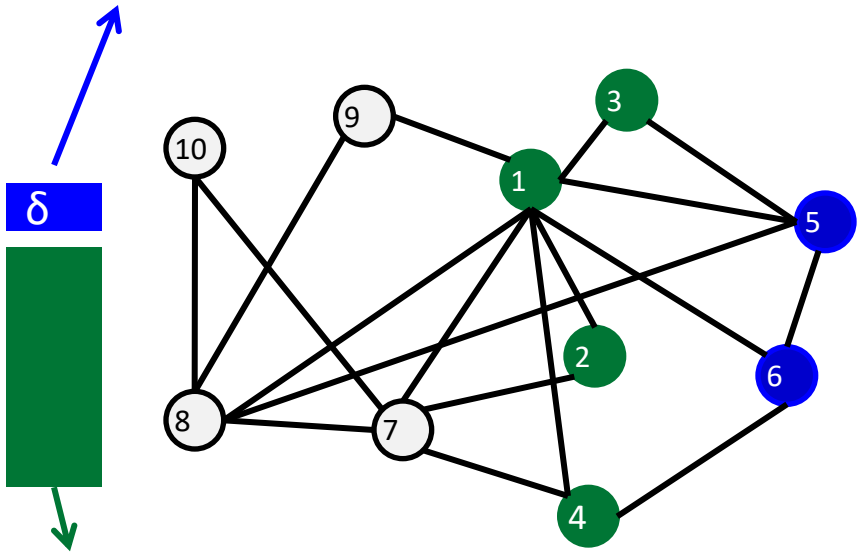
(1) $\lambda - \lambda_s \approx Sv(S)$ ✓
(2) $Sv(S)$ is submodular
(3) *Netshield* is near-opt
(4) *Netshield* scales linearly

Marginal benefit of deleting {5,6}



Benefit of deleting {1,2}

Marginal benefit of deleting {5,6}

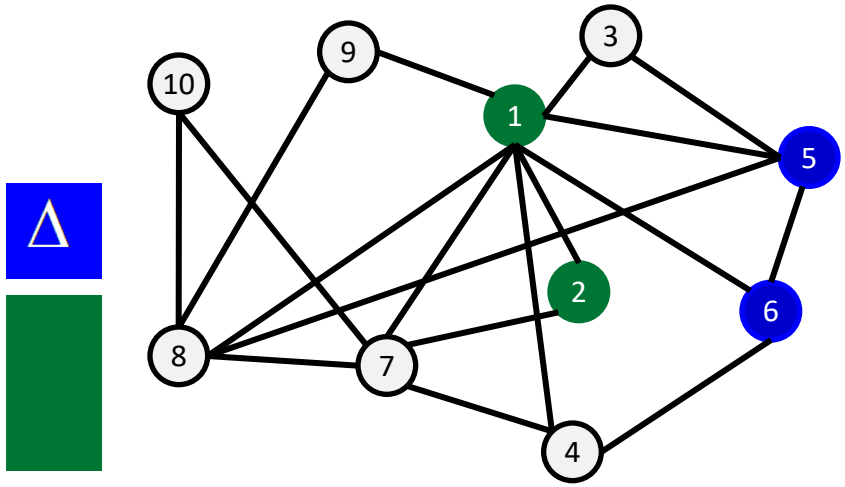


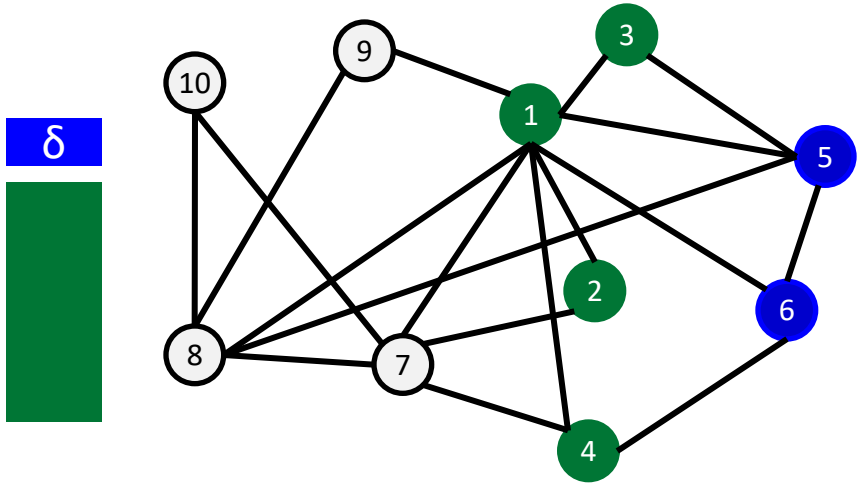
Benefit of deleting {1,2, 3,4}

Δ >= δ ⟺ Sub-Modular (i.e., Diminishing Returns)

Why *Netshield* is Near-Optimal?

(1) $\lambda - \lambda_s \approx Sv(S)$ ✓
(2) $Sv(S)$ is sub-modular
(3) *Netshield* is near-opt
(4) *Netshield* scales linearly



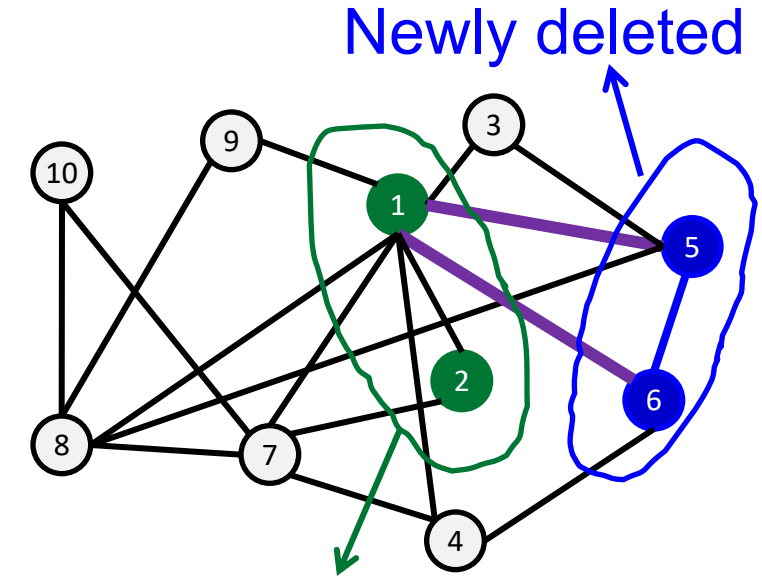


Δ >= δ ⟷ Sub-Modular (i.e., Diminishing Returns)

Theorem: *k*-step greedy alg. to maximize a sub-modular function guarantees (1-1/e) optimal [Nemhauster+ 78]

Why Sv(*S*) is sub-modular?

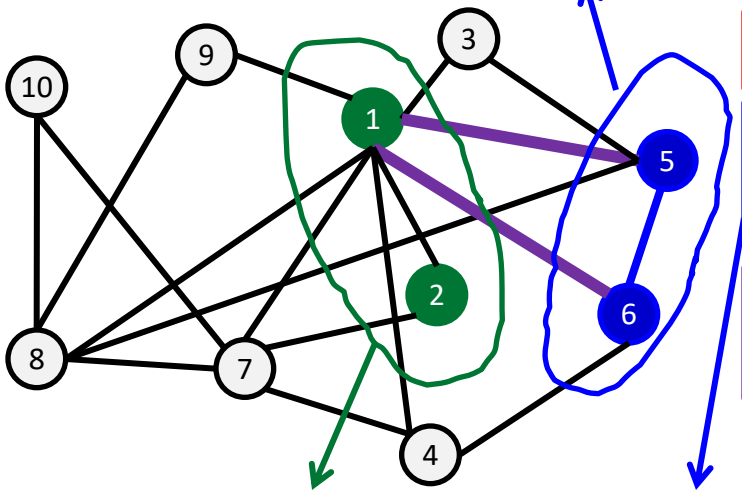
(1) $\lambda - \lambda_s \approx Sv(S)$ ✓
(2) Sv(*S*) is sub-modular
(3) *Netshield* is near-opt ✓
(4) *Netshield* scales linearly



- H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: On the Vulnerability of Large Graphs. ICDM 2010: 1091-1096
- C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015

Why Sv(*S*) is sub-modular?

(1) $\lambda - \lambda_s \approx Sv(S)$ ✓
(2) $Sv(S)$ is sub-modular ⋰
(3) *Netshield* is near-opt ✓
(4) *Netshield* scales linearly



Marginal Benefit of deleting {5,6}

$Sv(S_{green} \cup S_{blue}) - Sv(S_{green})$ =

$\sum_{i \epsilon S_{blue}} 2\lambda u(i)^2 - \sum_{i,j \epsilon S_{blue}} A(i,j)u(i)u(j)$ -

$(\sum_{i \epsilon S_{blue},\, j \epsilon S_{green}} A(i,j)u(i)u(j) + \sum_{i \epsilon S_{green},\, j \epsilon S_{blue}} A(i,j)u(i)u(j))$

Pure benefit from {5,6}

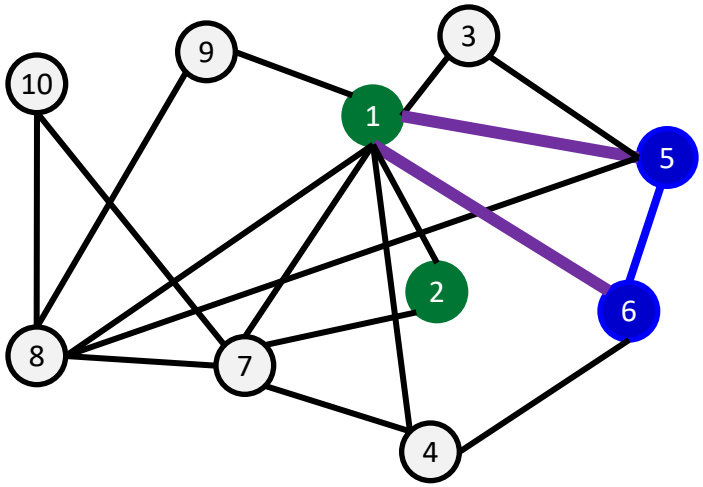
Interaction between {5,6} and {1,2}

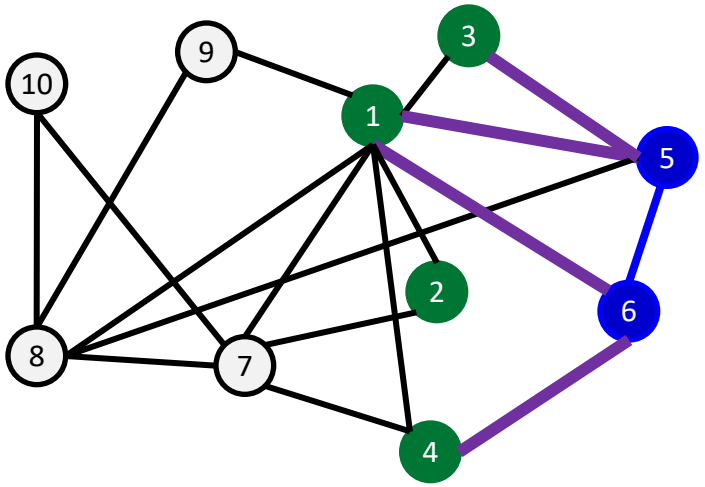
Only purple term depends on {1, 2}!

- H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: On the Vulnerability of Large Graphs. ICDM 2010: 1091-1096
- C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015

Why Sv(*S*) is sub-modular?

(1) $\lambda - \lambda_s \approx Sv(S)$ ✓
(2) Sv(*S*) is sub-modular ✓
(3) *Netshield* is near-opt ✓
(4) *Netshield* scales linearly





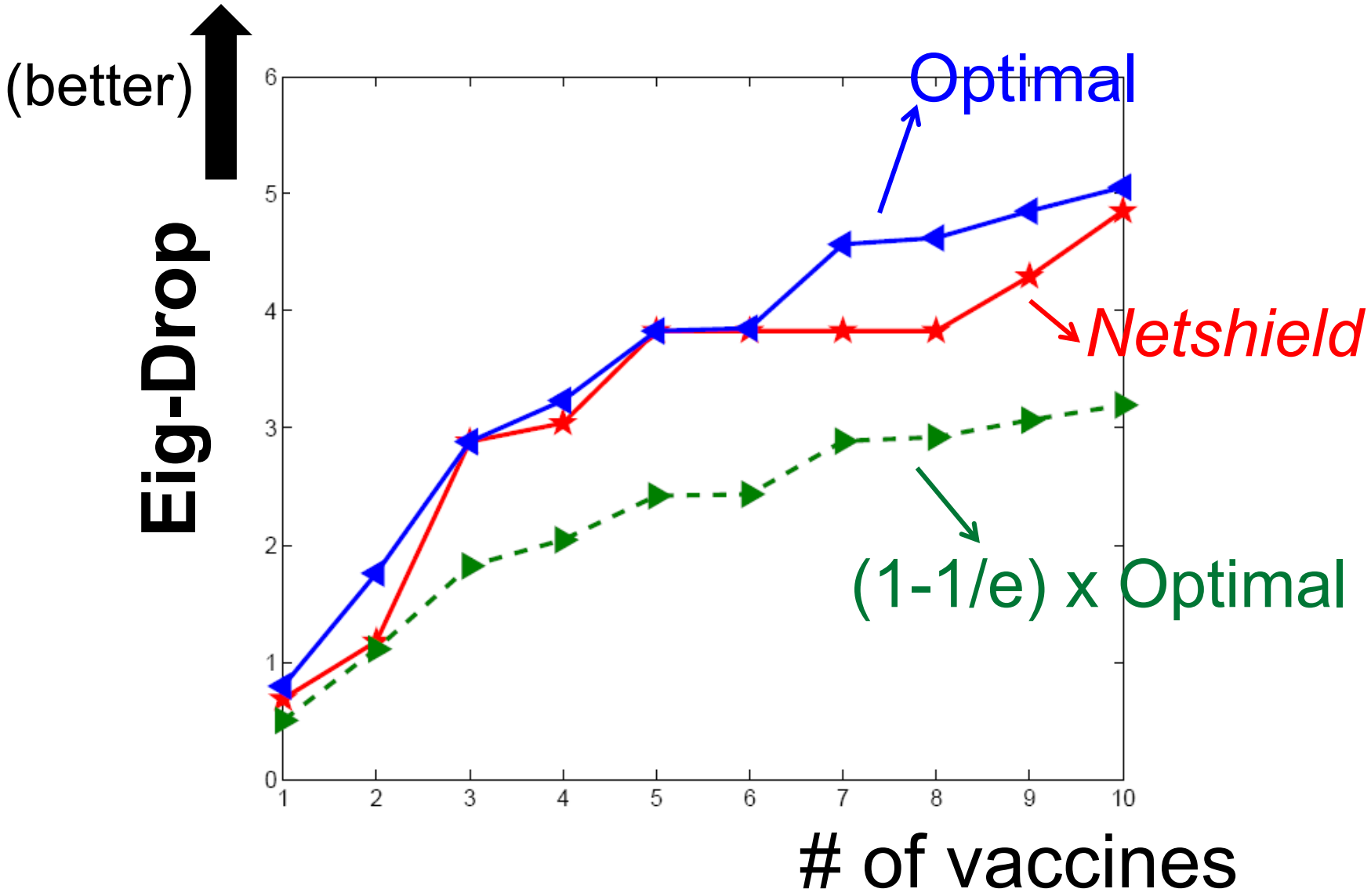
Marginal Benefit = Blue –Purple

More Green ⟷ More Purple ⟷ Less Red

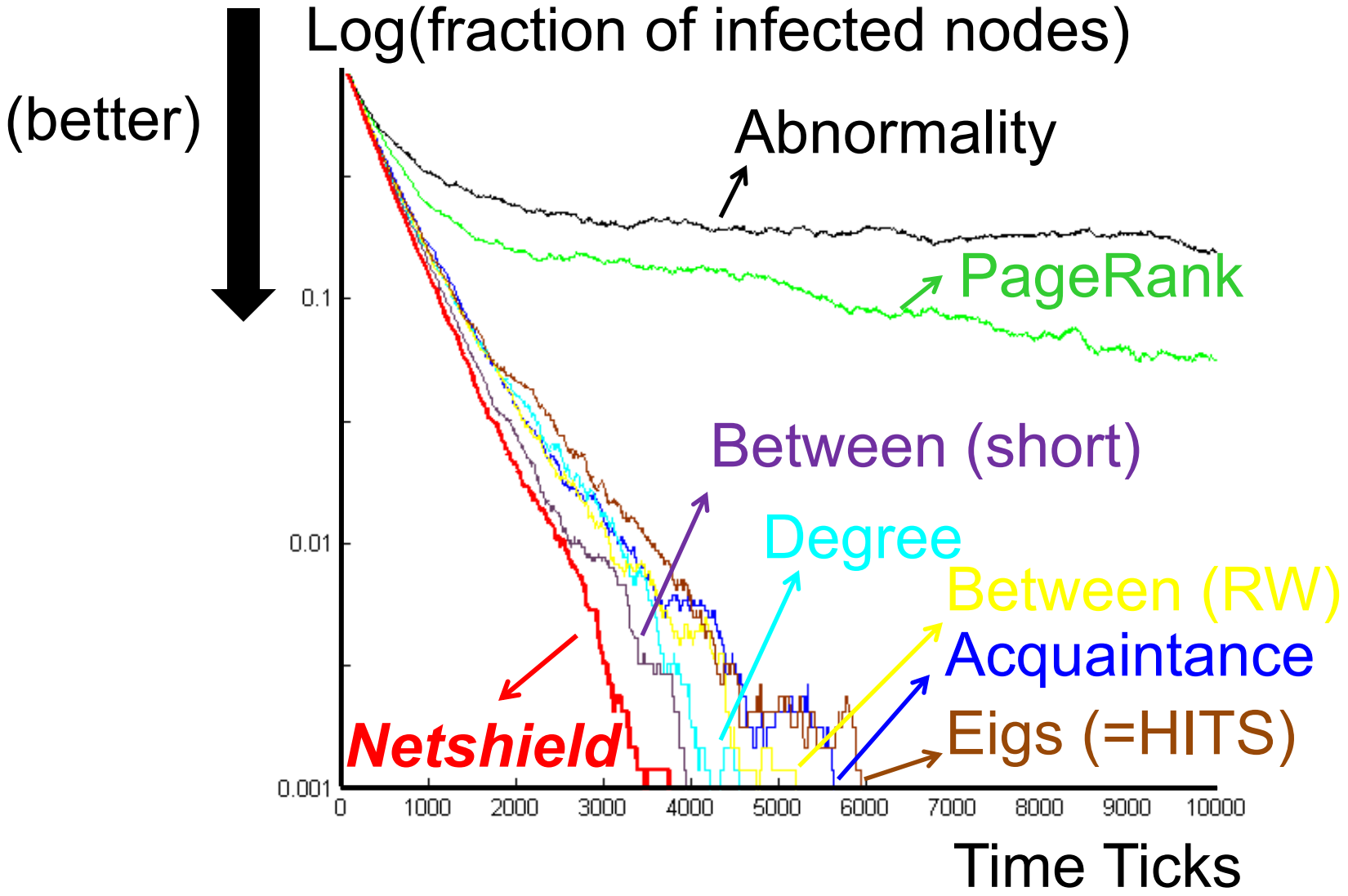
Marginal Benefit of Left >= Marginal Benefit of Right

Footnote: greens are nodes already deleted; blue {5,6} nodes are nodes to be deleted

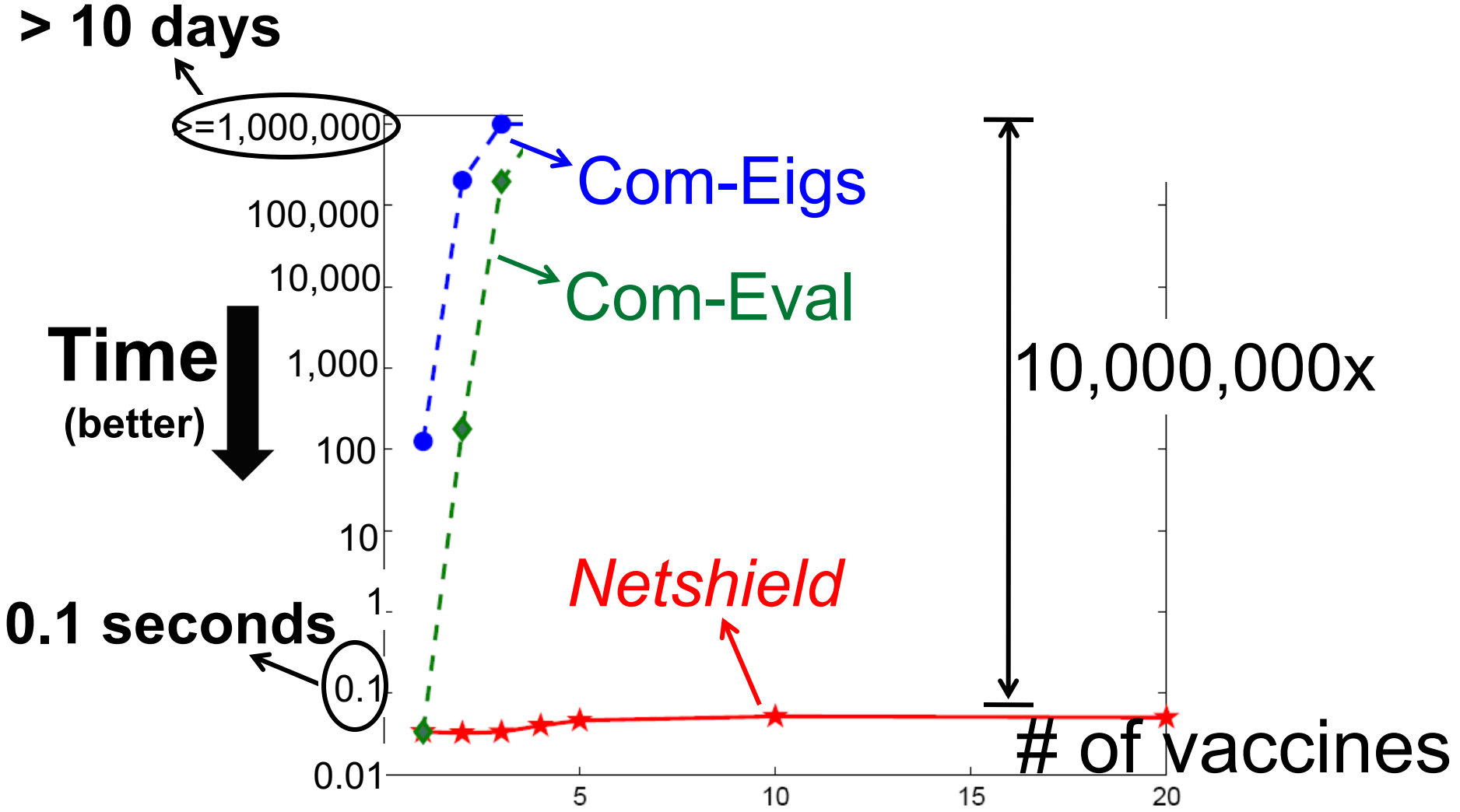
Quality of *Netshield*



Comparison of Immunization

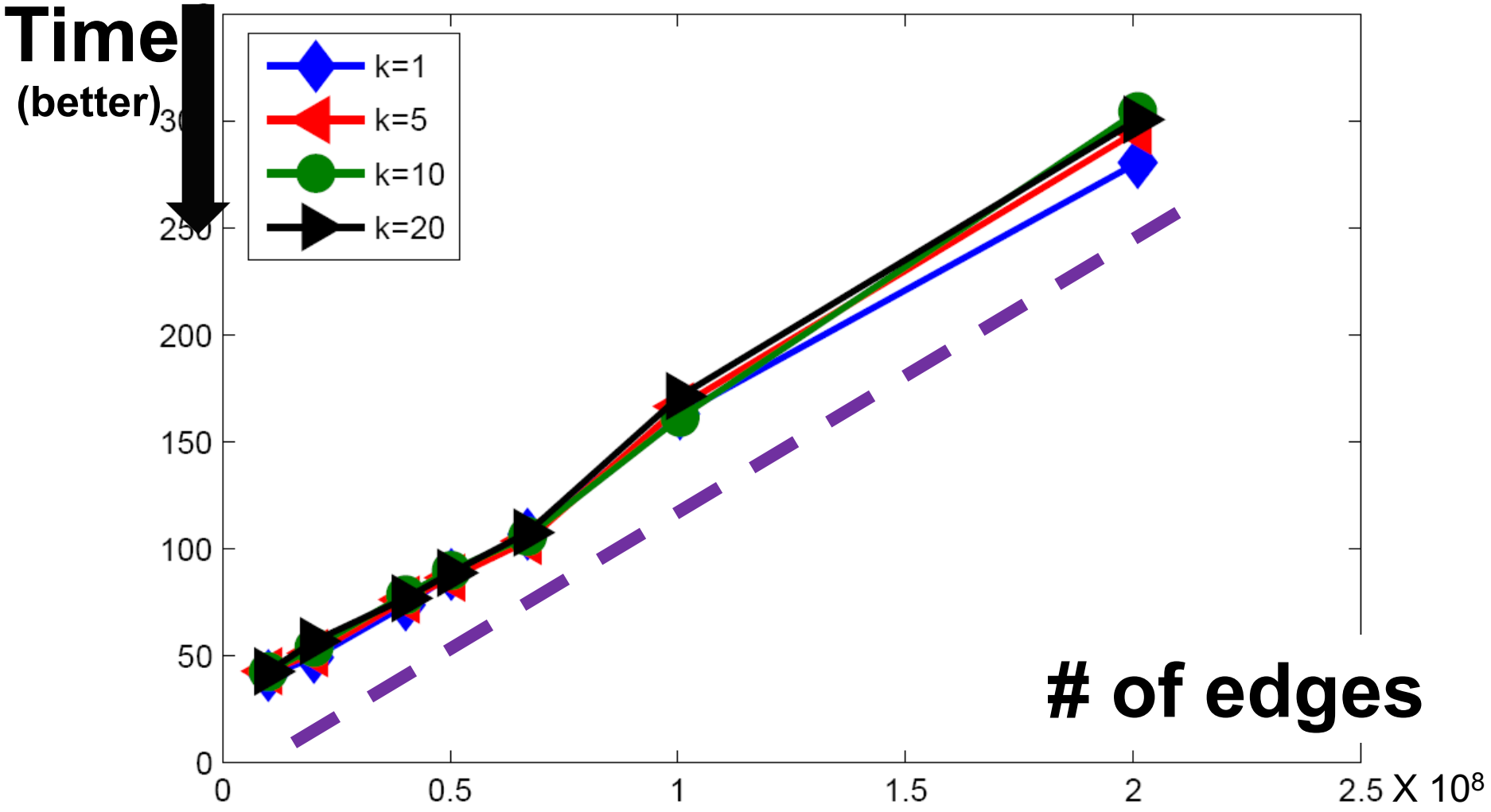


Speed of *Netshield*



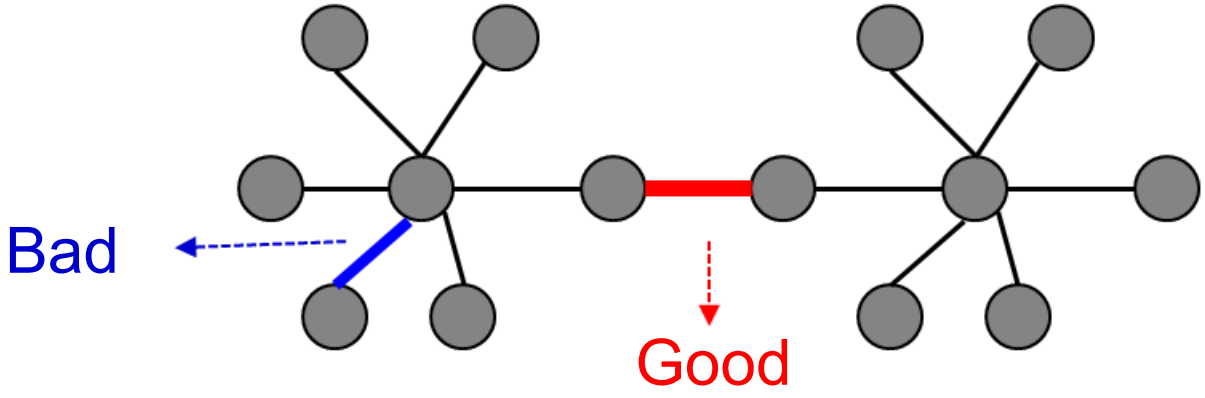
NIPS co-authorship Network: 3K nodes, 15K edges

Scalability of *Netshield*



More on GCO Algorithms: Node → Edge

- Given: a graph A, virus prop model and budget k;
- Find: delete k 'best' edges from A to minimize λ



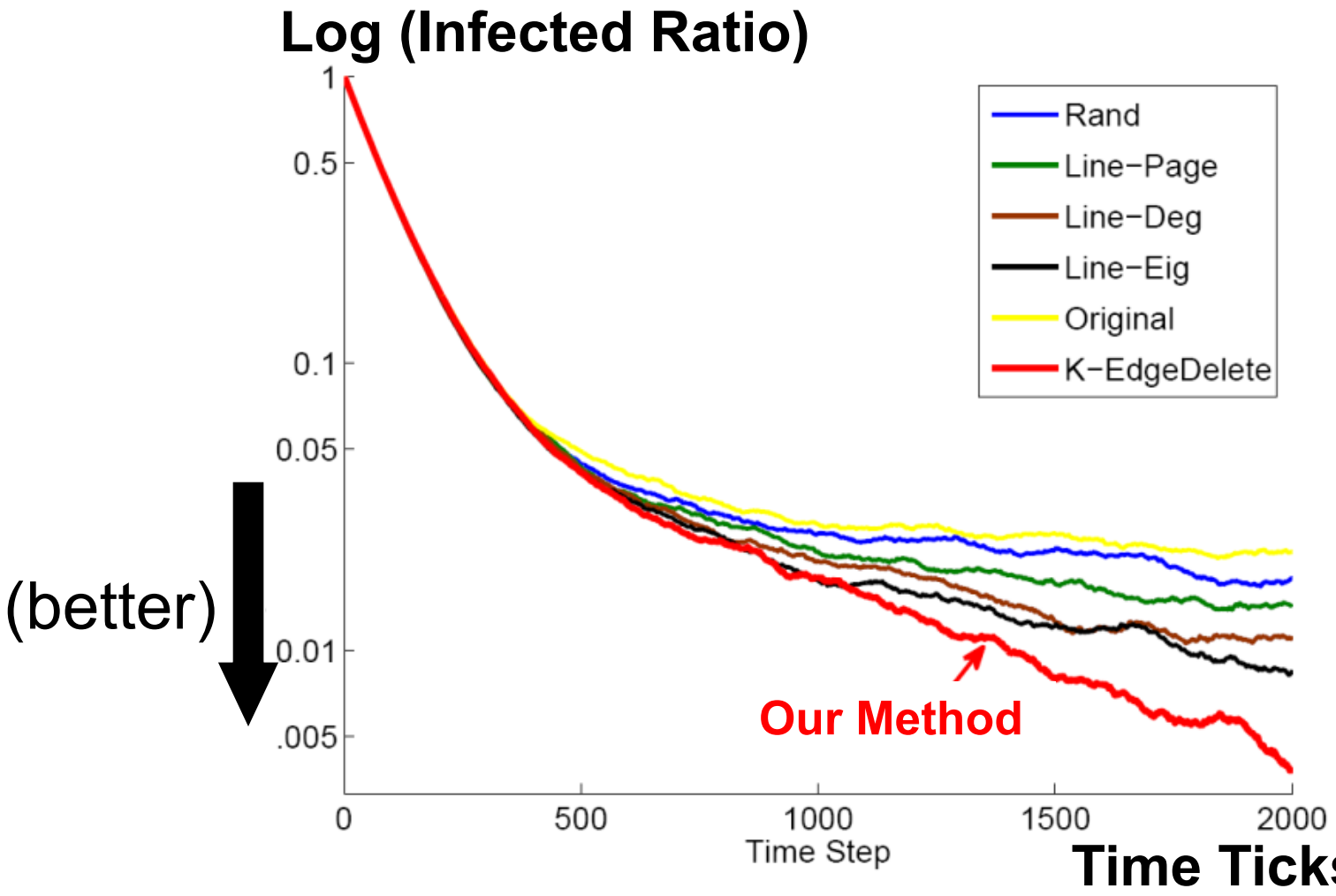
Our Solutions: 1st order matrix perturbation again!

$$\lambda - \lambda_s \approx Mv(S) = c \sum_{e \in S} u(i_e) v(j_e)$$

Left eigen-score of source

Right eigen-score of target

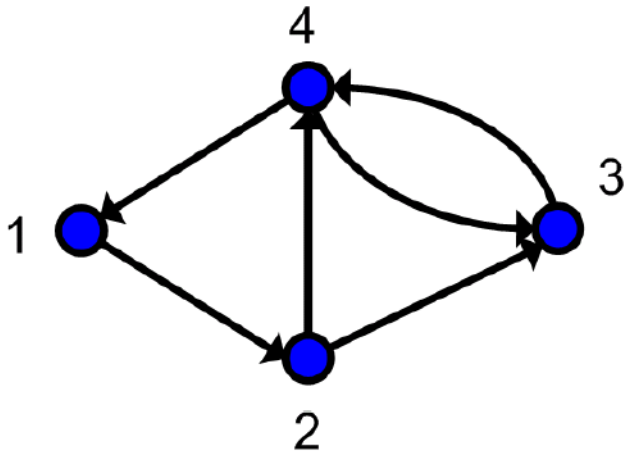
Minimizing Propagation: Evaluations



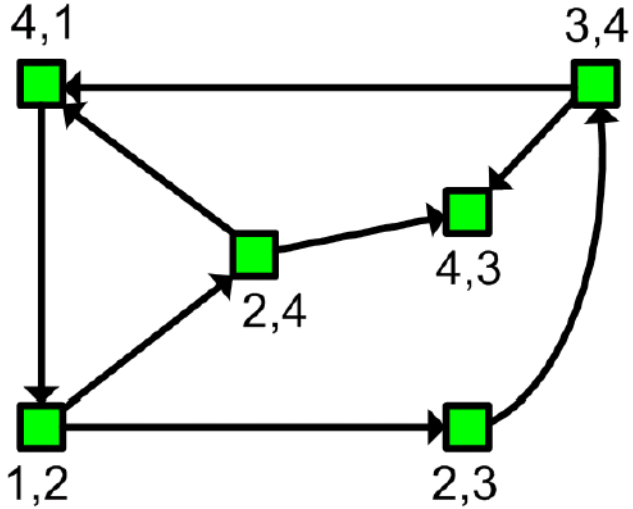
Data set: Oregon Autonomous System Graph (14K node, 61K edges)

Discussions: Node Deletion vs. Edge Deletion

- Observations:
  - Node or Edge Deletion → λ *Decrease*
  - Nodes on A = Edges on its line graph *L(A)*



Original Graph *A*



Line Graph *L(A)*

- Questions?
  - Edge Deletion on *A* = Node Deletion on *L(A)?*
  - Which strategy is better (when both feasible)?

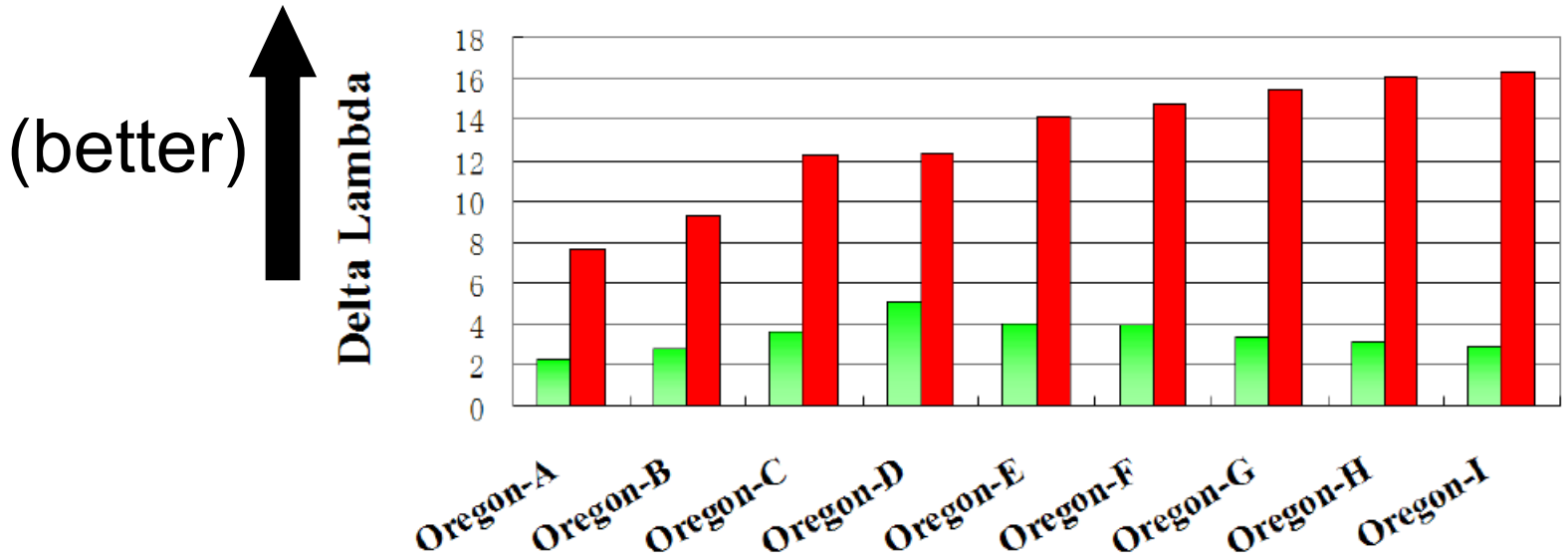
Discussions: Node Deletion vs. Edge Deletion

- Q: Is Edge Deletion on A = Node Deletion on $L(A)$?
- A: Yes!

Theorem: Line Graph Spectrum.
Eigenvalue of A → Eigenvalue of $L(A)$

Discussions: Node Deletion vs. Edge Deletion

- Q: Which strategy is better (when both feasible)?
- A: Edge Deletion > Node Deletion



Green: Node Deletion (e.g., shutdown a twitter account)
Red: Edge Deletion (e.g., un-friend two users)

Maximizing Dissemination: Edge Addition

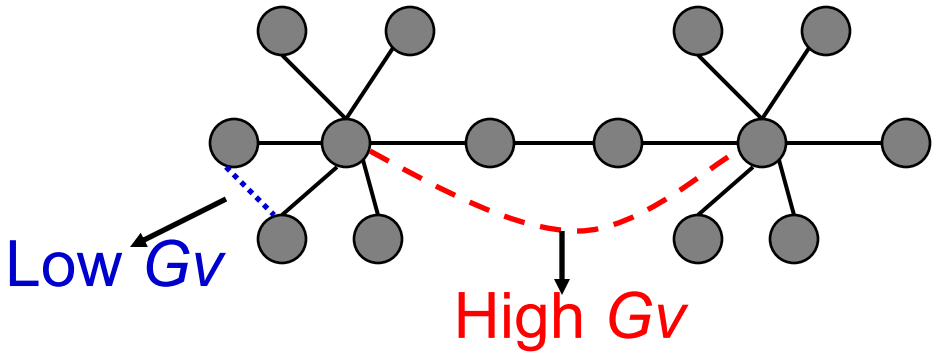
- **Given**: a graph A, virus prop model and budget k;
- **Find**: add k 'best' new edges into A.
  - By 1st order perturbation, we have

$$\lambda_s - \lambda \approx Gv(S) = c \sum_{e \epsilon S} u(i_e) v(j_e)$$

Left eigen-score of source

Right eigen-score of target

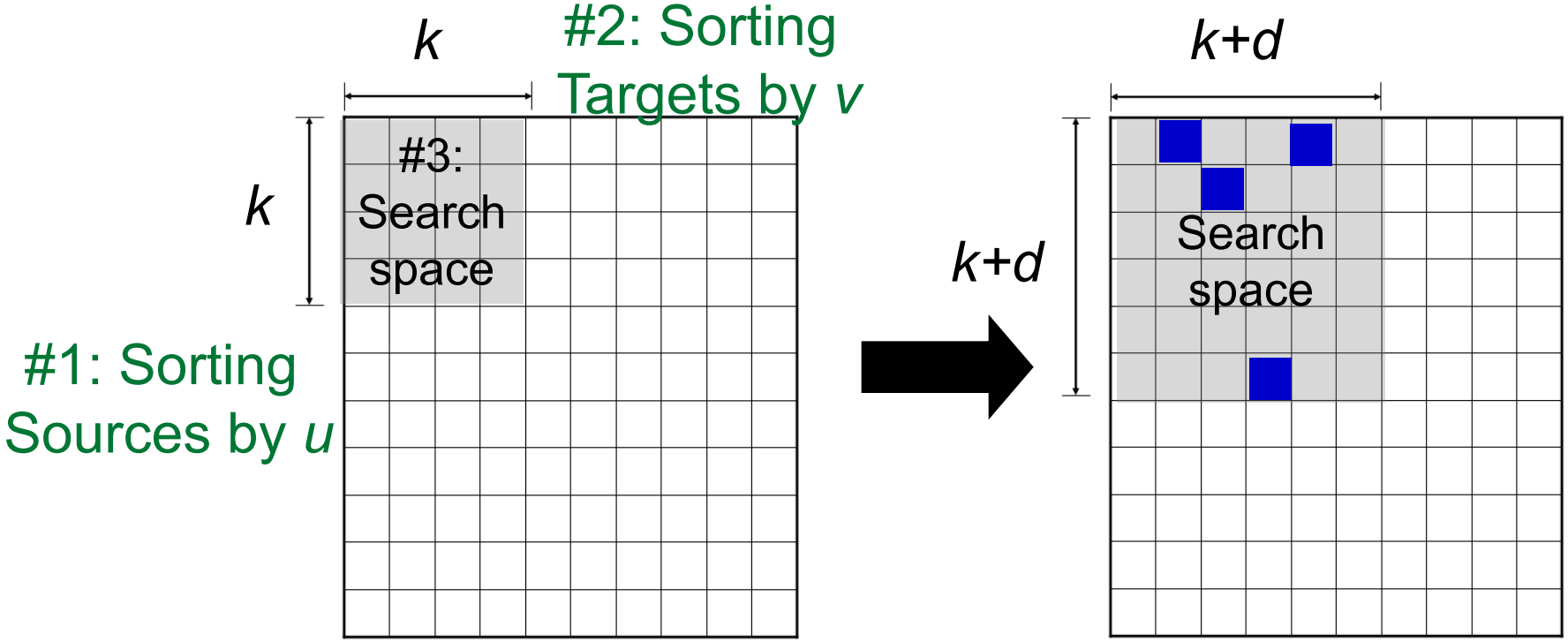
  - So, we are done → need $O(n^2-m)$ complexity



Maximizing Dissemination: Edge Addition

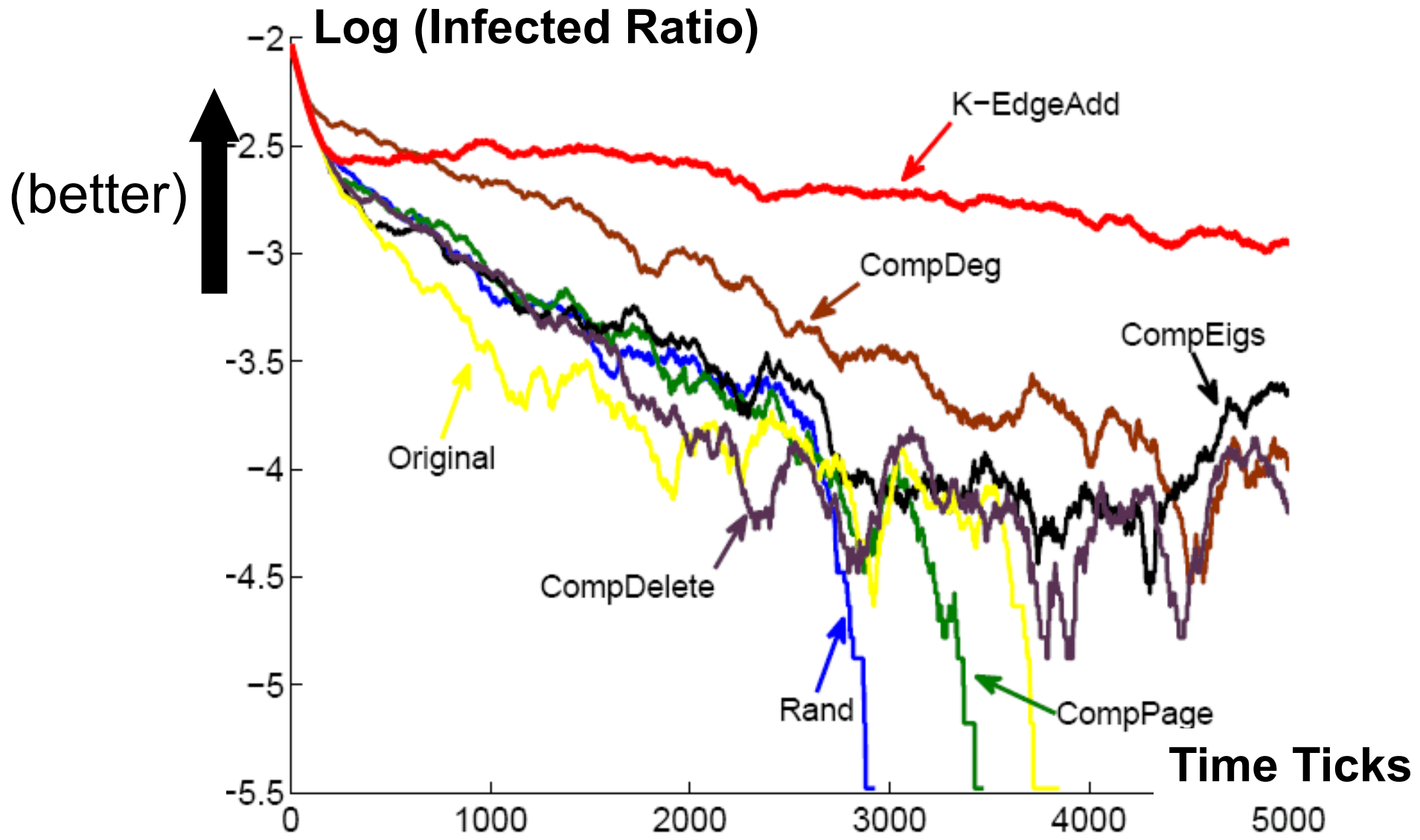
$$\lambda_s - \lambda \approx Gv(S) = c \sum_{e \epsilon S} u(i_e) v(j_e)$$

- Q: How to Find k new edges w/ highest $Gv(S)$?
- A: Modified Fagin's algorithm



Time Complexity: $O(m+nt+kt^2)$, $t = \max(k,d)$:existing edge

Maximizing Dissemination: Evaluation



More on GCO Algorithms (cont.)

- **M1: Higher Order Variants**
  - `Better' Matrix Perturbation → Better Approximation of Eigen-gap?
  - C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015
- **M2: Beyond Full & Symmetric Immunity**
  - Immunizing a node weakens (but not deleting) the incoming (but not the out-going) links
  - B. Aditya Prakash, Lada Adamic, Theodore Iwashnya, Hanghang Tong and Christos Faloutsos: Fractional Immunization on Networks. SDM 2013

More on GCO Algorithms (cont.)

- **M3: Immunization on Dynamic Graphs**
  - Optimize connectivity on Time-Varying Graphs (with alternating behavior)
  - B. Aditya Prakash, Hanghang Tong, Nicholas Valler, Michalis Faloutsos, Christos Faloutsos: Virus Propagation on Time-Varying Networks: Theory and Immunization Algorithms. ECML/PKDD (3) 2010: 99-114
- **M4: Manipulating Network Robustness**
  - Beyond λ: Optimizing an eigen-function of the underlying graph
  - Hau Chan, Leman Akoglu, Hanghang Tong: Make It or Break It: Manipulating Robustness in Large Networks. SDM 2014: 325-333

More on GCO Algorithms (cont.)

- **M5: Robust Network Construction**
  - How to building a `well-connected' network, that is robust to external intentional attack, with resource constraint?
  - Hui Wang, Wanyun Cui, Yanghua Xiao, Hanghang Tong:Robust network construction against intentional attacks. BigComp 2015: 279-286
- **M6: Vaccine Distribution with Uncertainty**
  - Optimizing the connectivity of a 'noisy', uncertain graph.
  - Yao Zhang and B. Aditya Prakash: Scalable Vaccine Distribution in Large Graphs given Uncertain Data. ICDM 2014
  - Code available at: http://people.cs.vt.edu/badityap/CODE/UDAV.zip

More on GCO Algorithms (cont.)

- **M7: Handling Small Eigen-Gap**
  - Optimal edge deletion strategy on a graph with small eigen-gap (e.g., social networks), where matrix-perturbation might collapse.
  - L. Le, T. Eliassi-Rad and H. Tong: MET: A Fast Algorithm for Minimizing Propagation in Large Graphs with Small Eigen-Gaps. SDM 2015
- **M8: Source/Target-Specific Connectivity Optimization**
  - Identifying most important nodes in connecting two nodes, or two groups of nodes
  - Hanghang Tong, Spiros Papadimitriou, Christos Faloutsos, Philip S. Yu, Tina Eliassi-Rad: Gateway finder in large graphs: problem definitions and fast solutions. Inf. Retr. 15(3-4): 391-411 (2012)

SUBLINE Optimization

optional

- Hardness
  - Max *k*-Hitting Set $\leq_p$ NETCOP

Max k-hitting set problem

Given: (1) a set of elements $\mathcal{U}$; (2) a collection $\mathcal{S} = \{\mathfrak{B}_1, \mathfrak{B}_2, \ldots, \mathfrak{B}_m\}$ of distinct subsets of $\mathcal{U}$; (3) budget *k*

Output: a set $\mathcal{H}$ of *k* elements from $\mathcal{U}$ that hits maximum number of subsets in $\mathcal{S}$

elements in $\mathcal{U} \rightarrow$ nodes in G
subsets in $\mathcal{S} \rightarrow$ valid subgraphs in G

k-node connectivity optimization problem

Given: (1) network G; (2) connectivity measure $C(G, f) = \sum f(\pi)$ where $f(\pi) = \begin{cases} 1. \text{ if } \pi \text{ is valid subgraph} \\ 0. \text{ otherwise} \end{cases}$; (3) budget *k*

Output: a set $\mathcal{H}$ of *k* nodes from G whose removal would minimize $C(G, f)$

NETCOP (SUBLINE Optimization) problem is NP-hard.

THEOREM.
The optimization problem for any SUBLINE connectivity measures with non-independent valid subgraphs is NP-hard. [Chen et al. KDD'18]

Chen Chen, Ruiyue Peng, Lei Ying, Hanghang Tong. **Network Connectivity Optimization: Fundamental Limits and Effective Algorithms.** In Proceedings of KDD 2018

optional

SUBLINE Optimization

- Approximability
  - Approximability of Max *k*-hitting set:
    - $(1 - 1/e)$ is the best approximation ratio for Max *k*-hitting set problem in polynomial time unless $NP \subseteq DTIME(n^{O(\log(\log n))})$
  - Max *k*-hitting set $\leq_p$NETCOP

Proof by Contradiction

$(1 - 1/e)$ is the best approximation ratio for NETCOP problem in polynomial time unless $NP \subseteq DTIME(n^{O(log(log n))})$

Samir Khuller, Anna Moss, and Joseph Seffi Naor. 1999. The budgeted maximum coverage problem. Inform. Process. Lett. 70, 1 (1999), 39–45.

optional

SUBLINE Optimization

- Optimality
  - Diminishing returns property for NETCOP

    Given $\mathcal{S}_1$ and $\mathcal{S}_2$ where $\mathcal{S}_1 \subseteq \mathcal{S}_2$, and a network element $o \notin \mathcal{S}_2$.
    Then we have $I(\mathcal{S}_1 \cup o) \geq I(\mathcal{S}_2 \cup o)$

  - Greedy strategy
    - Iteratively pick the highest impact network element from current network
    - Update the network by removing the selected element

Approximation Ratio: $1 - 1/e$ ⇓ Approximation Ratio Bound: $1 - 1/e$

Greedy algorithm is the best polynomial algorithm for network connectivity optimization problems unless $NP \subseteq DTIME(n^{O(log(log\ n))})$

optional

SUBLINE Optimization

- Greedy Strategy
  - Many iterations
  - Impact calculation

bottleneck

1: initialize $\mathcal{X}$ to be empty
2: **for** $i = 1$ to k **do**
3: **for** each valid network element o in G **do**
4: calculate $I(o) \leftarrow C(G) - C(G \setminus \{o\})$
5: **end for**
6: add the element $\tilde{o} = \text{argmax}_o I(o)$ to $\mathcal{X}$
7: remove the element $\{\tilde{o}\}$ from network G
8: **end for**

Many connectivity measures can be approximated with eigen-functions

$$g(\Lambda, U) = \begin{cases} 1/\lambda_1 & \text{Epidemic Threshold} \\ u_1 & \text{Eigenvector Centrality} \\ \Delta(G) = \frac{1}{6}\sum_{i=1}^{n} \lambda_i^3 & \text{\#Triangles} \\ S(G) = \ln(\frac{1}{k}\sum_{i=1}^{n} e^{\lambda_i}) & \text{Natural Connectivity} \\ \lambda_1 - \lambda_2 & \text{Eigen-Gap} \end{cases}$$

$$I(o) = C(G) - C(G\backslash\{o\}) = g(\Lambda) - g(\tilde{\Lambda})$$

Element Impact Calculation ⟷ **Updated Eigenvalues Calculation**

Updated Eigenvalues Calculation

optional

- Exact Method: Lanczos method $\sim O(m)$
  - Node-level optimization: $O(kmn)$
  - Edge-level optimization: $O(km^2)$

Pro:
- No error

Con:
- Not scalable
(at least quadratic complexity)

- Approximation Method: matrix perturbation $\sim O(r|\Delta G|)$
  - Node-level optimization: $O(k(nr^2+mr)+r^2|G_S|)$
  - Edge-level optimization: $O(k(nr^2+mr)+r^2|G_S|)$

Linear w.r.t. $|G|$

$$\tilde{\lambda}_i = \lambda_i + \Delta\lambda_i$$
$$\tilde{\boldsymbol{u}}_i = \boldsymbol{u}_i + \Delta\boldsymbol{u}_i$$

$$\begin{cases} \Delta\lambda_i = u_i' \Delta A u_i \\ \Delta u_i = \sum_{p=1}^{k} \alpha_{ip} u_p \end{cases} \quad \left(\text{where } \alpha_{ij} = \frac{u_j' \Delta A u_i}{\lambda_i - \lambda_j} \right)$$

Pro:
- Scalable

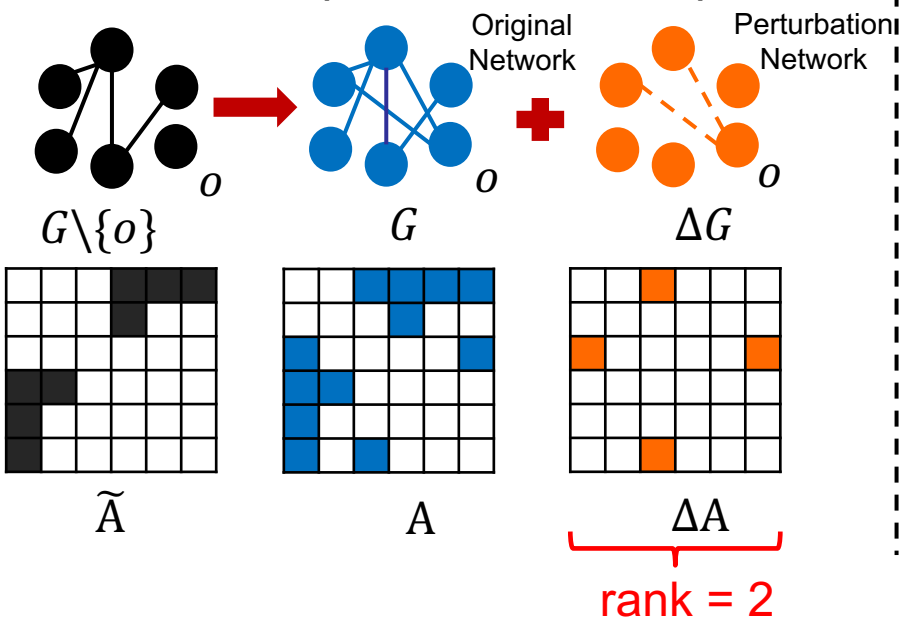
Con:
- Small eigen-gap / multiplicity>1 may introduce large approximation error

r: the number of top eigenvalues used for connectivity approximation; G_S: subgraph that incident to set S

optional

The CONTAIN Algorithm

- Key Problem
  - Approximate updated eigenvalues in $G\backslash\{o\}$ $(\widetilde{\Lambda})$ with the original eigenvalues (Λ)
- Method
  - Low-rank ΔA + partial QR decomposition



$$\mathbf{A} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}'$$

$$\Delta\mathbf{A} = \mathbf{U}_\Delta\boldsymbol{\Lambda}_\Delta\mathbf{U}_\Delta{}'$$

$$\widetilde{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A}$$

$$= [\mathbf{U}, \mathbf{U}_\Delta]\begin{bmatrix}\boldsymbol{\Lambda} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_\Delta\end{bmatrix}\begin{bmatrix}\mathbf{U}' \\ \mathbf{U}_\Delta{}'\end{bmatrix}$$

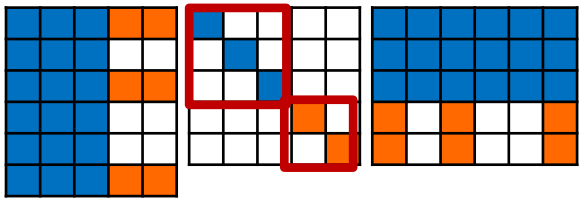
optional

Eigenvalue Approximation

Step 1: $\tilde{\mathbf{A}} = [\mathbf{U}, \mathbf{U}_\Delta] \begin{bmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_\Delta \end{bmatrix} \begin{bmatrix} \mathbf{U}' \\ \mathbf{U}_\Delta{}' \end{bmatrix}$

Step 2: $\tilde{\mathbf{A}} = \mathbf{Q}\mathbf{R} \underbrace{\begin{bmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_\Delta \end{bmatrix}}_{\mathbf{Z}} \mathbf{R}'\mathbf{Q}'$

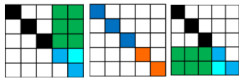
Almost Orthonormal



$[\mathbf{U}, \mathbf{U}_\Delta] = \mathbf{Q}\mathbf{R}$

Partial QR Decomposition
(Gram-Schmidt Process)

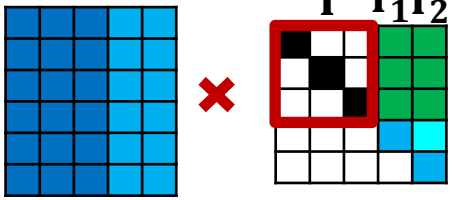
Eigen-Decomposition



$\tilde{\mathbf{A}} = \mathbf{Q}\mathbf{U}_\mathbf{z}\mathbf{\Lambda}_\mathbf{z}\mathbf{U}_\mathbf{z}{}'\mathbf{Q}'$

Orthonormal

Orthonormal



$\mathbf{I} \quad \mathbf{r}_1\mathbf{r}_2$

$\tilde{\mathbf{U}} = \mathbf{Q}\mathbf{U}_\mathbf{z}$
$\tilde{\mathbf{\Lambda}} = \mathbf{\Lambda}_\mathbf{z}$
No Extra Error!

$$\mathbf{Q} = \left[\mathbf{U} \quad \frac{\mathbf{q}_1}{\|\mathbf{q}_1\|} \quad \frac{\mathbf{q}_2}{\|\mathbf{q}_2\|}\right] \qquad \mathbf{R} = \begin{bmatrix} \mathbf{I} & \mathbf{r}_1 & \mathbf{r}_2 \\ 0 & \|\mathbf{q}_1\| & \frac{\mathbf{r}_1{}'\mathbf{r}_2}{\|\mathbf{q}_1\|} \\ 0 & 0 & \|\mathbf{q}_2\| \end{bmatrix}$$

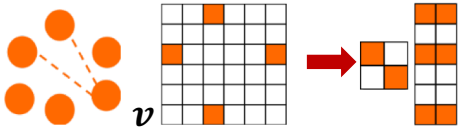
$$\begin{cases} \mathbf{r}_1 = \mathbf{U}'\mathbf{U}_\Delta(:,1) & \mathbf{q}_1 = \mathbf{U}_\Delta(:,1) - \mathbf{U}\mathbf{r}_1 \\ \mathbf{r}_2 = \mathbf{U}'\mathbf{U}_\Delta(:,2) & \mathbf{q}_2 = \mathbf{U}_\Delta(:,2) - \mathbf{U}\mathbf{r}_2 \end{cases}$$

Xilun Chen, K Selcuk Candan. LWI-SVD: low-rank, windowed, incremental singular value decompositions on time-evolving data sets. In Proceedings of KDD 2014
Liangyue Li, Hanghang Tong, Yanghua Xiao, and Wei Fan. 2015. Cheetah: fast graph kernel tracking on dynamic graphs. In Proceedings SDM 2015

optional

Complexity Analysis

- Step 1: Partial QR Decomposition
  - $\Delta \mathbf{A} \rightarrow \mathbf{U}_\Delta \mathbf{\Lambda}_\Delta \mathbf{U}_\Delta{}'$ $O(d_v)$
  - Calculate $\mathbf{R}$ $O(d_v r)$



$\Delta \mathbf{A}$: Rank-2 Matrix

$$\mathbf{\Lambda}_\Delta = \begin{bmatrix} \sqrt{n_v} & 0 \\ 0 & \sqrt{n_v} \end{bmatrix}$$

$$\mathbf{U}_\Delta(v,1) = \mathbf{U}_\Delta(v,2) = \frac{1}{\sqrt{2}}$$

$$\mathbf{U}_\Delta(\mathcal{N}_v,1) = -\frac{1}{\sqrt{2n_v}}$$

$$\mathbf{U}_\Delta(\mathcal{N}_v,2) = \frac{1}{\sqrt{2n_v}}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{I} & \mathbf{r}_1 & \mathbf{r}_2 \\ 0 & \| \mathbf{q}_1 \| & -\frac{\mathbf{r}_1{}'\mathbf{r}_2}{\| \mathbf{q}_1 \|} \\ 0 & 0 & \| \mathbf{q}_2 \| \end{bmatrix}$$

$$\begin{cases} \| \mathbf{q}_1 \| = \sqrt{1 - \| \mathbf{r}_1 \|^2} \\ \| \mathbf{q}_2 \| = \sqrt{1 - \| \mathbf{r}_2 \|^2 - \frac{(\mathbf{r}_1{}'\mathbf{r}_2)^2}{1 - \| \mathbf{r}_1 \|^2}} \end{cases}$$

- Step 2: Eigen Decomposition
  - Calculate $\mathbf{\Lambda}_\mathbf{z}$ ($\tilde{\mathbf{\Lambda}}$) on $\mathbf{R} \begin{bmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_\Delta \end{bmatrix} \mathbf{R}'$ $O(r^3)$
- Constant Complexity for Node v
  - $O(d_v r) + O(r^3) = O(d_v r + r^3)$
- Complexity for All Nodes $O(mr + nr^3)$

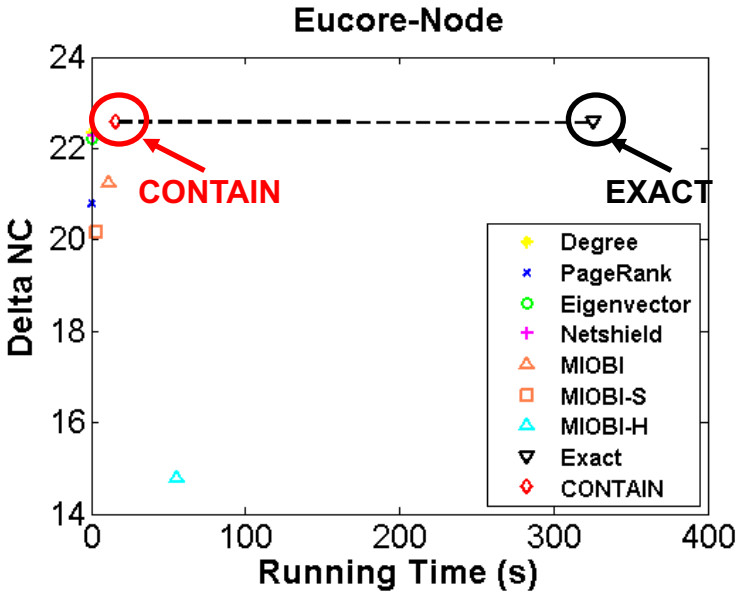
Overall Time Complexity:
The complexity for node-level and edge-level optimization are $O\big(k(mr + nr^3)\big)$ and $O(k(mr^3 + nr^2))$ respectively.

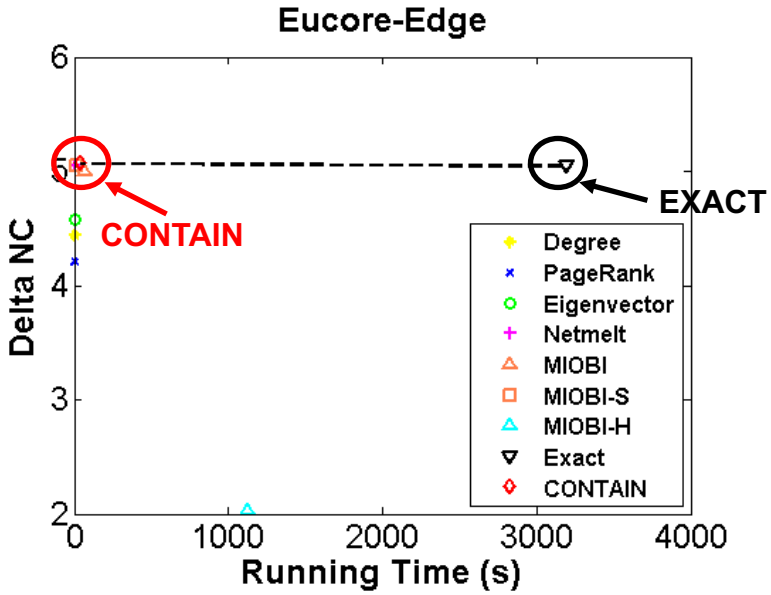
Chen Chen, Ruiyue Peng, Lei Ying, Hanghang Tong. **Network Connectivity Optimization: Fundamental Limits and Effective Algorithms.** In Proceedings of KDD 2018

optional

Efficiency of CONTAIN

- Quality vs. Running Time Trade-off





CONTAIN is orders of magnitudes faster than the Exact Algorithm while achieving similar results

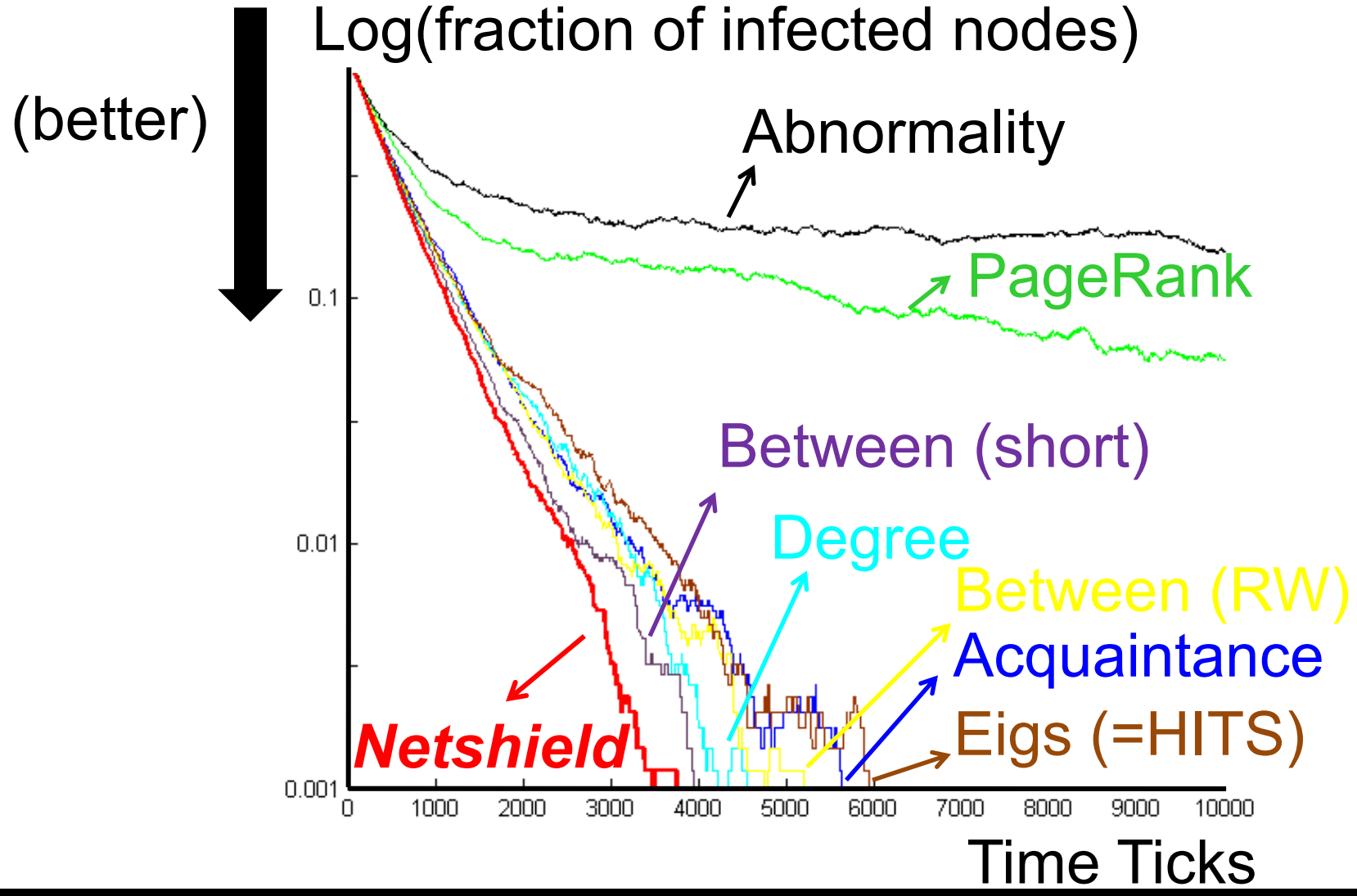
Roadmap

- ✔ Motivations and Background
- ✔ Part I: GCO Measures
- ✔ Part II: GCO Theories & Algorithms
- ➡ Part III: GCO Applications (optional, not required)
- Part IV: Open Challenges & Future Trends (optional, not required)

Part III: Applications

- A1: Immunization
- A2: Optimal Resource Allocation
- A3: Optimal Network Demolition: Collective Influence
- A4: Diversified Ranking on Graphs
- A5: Information Spreading in Context
- A6: Vulnerability of Cyber-Physical Systems
- A7: Team Member Replacement
- A8: Competitive Virus on Composite Networks
- A9: Gateway finder

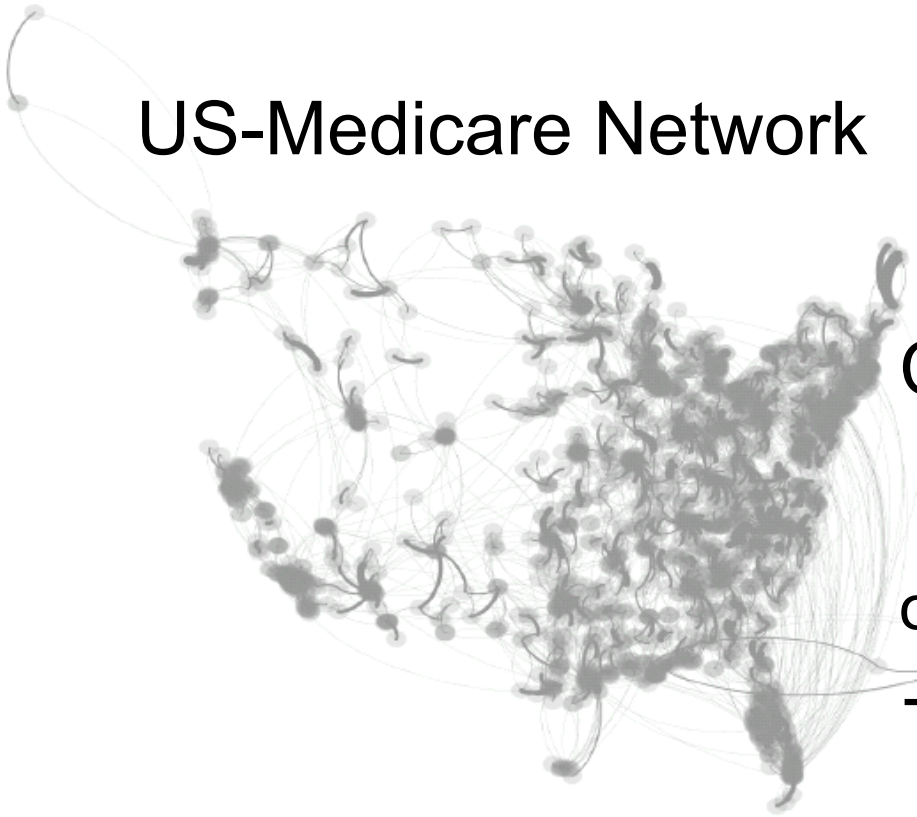
A1: Immunization



- H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: On the Vulnerability of Large Graphs. ICDM 2010: 1091-1096
- C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015

A2: Optimal Recourse Allocation

US-Medicare Network



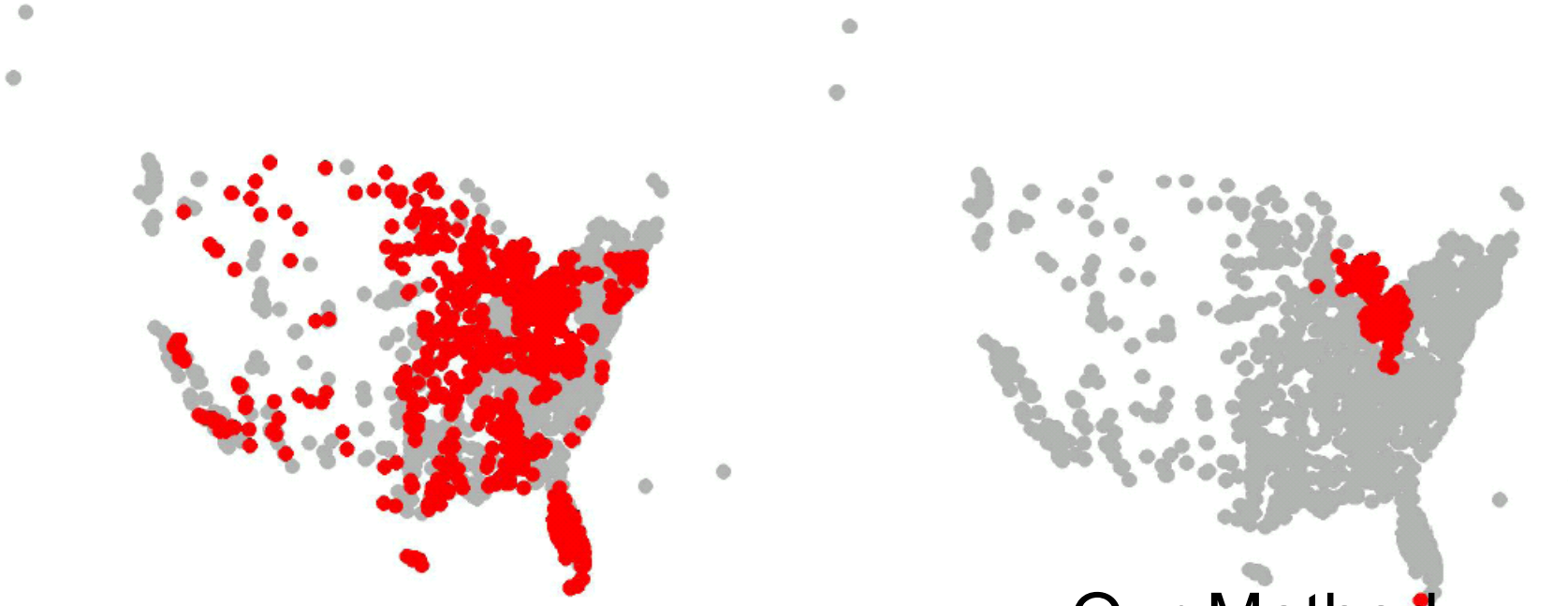
Critical Patient transferring

Move patients → specialized care → highly resistant micro-organism → Infection controlling → costly & limited

Q: How to allocate resource to minimize overall spreading?

SARS costs 700+ lives; $40+ Bn; H1N1 costs Mexico $2.3bn; Flu 2013: one of the worst in a decade, 105 children in US.

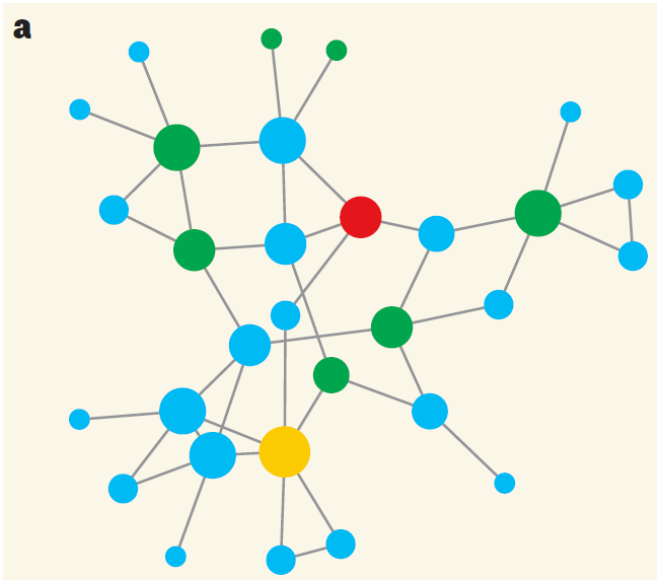
A2: Optimal Recourse Allocation

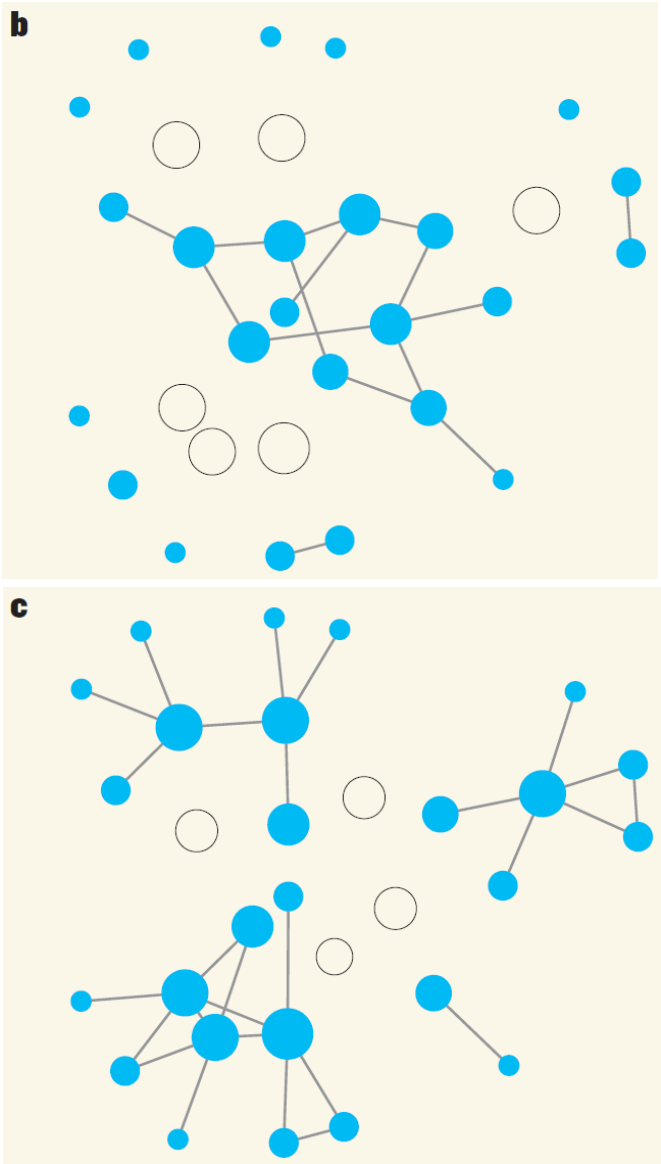


Red: Infected Hospitals after 365 days

B. Aditya Prakash, Lada Adamic, Theodore Iwashnya, Hanghang Tong and Christos Faloutsos: Fractional Immunization on Networks. SDM 2013

A3: Optimal Network Demolition: Collective Influence





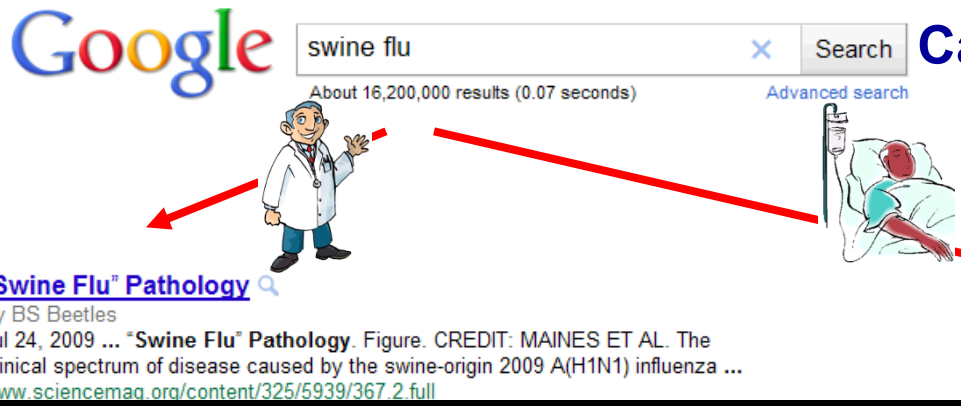
(a): the original input network.
(b): removing six (white) nodes w/ highest individual influence scores → GCC of size12.
(c): removing four (white) nodes with highest collective influence→ GCC of size 10.

István A. Kovács & Albert-László Barabási: Network science: Destruction perfected. Nature 524, 38–39, 2015

A4: Diversified Ranking on Large Graphs

- Q: Why Diversity?
- A1: Uncertainty & Ambiguity ***in*** an Information

Case 1: Uncertainty from the query



Case 2: Uncertainty from the user

Hanghang Tong, Jingrui He, Zhen Wen, Ravi Konuru, Ching-Yung Lin:
Diversified ranking on large graphs: an optimization viewpoint. KDD 2011: 1028-1036

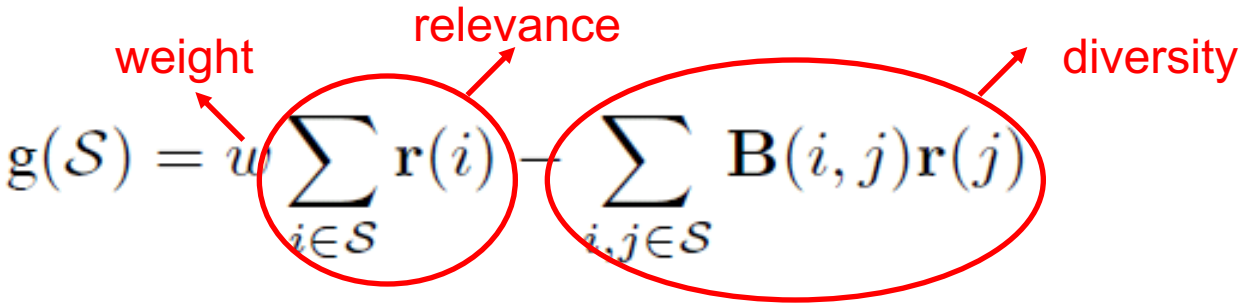
A4: Why Diversity? (cont.)

- A2: Address uncertainty & ambiguity ***of*** an information need
  - C1: Product search → want different reviews
  - C2: Political issue debate → desire different opinions
  - C3: Legal search → find ALL relevant cases
  - C4: Team assembling → find a set of relevant & diversified experts
- A3: Become a ***better*** and ***safer*** employee
  - ***Better***: A *1%* increase in diversity → an additional *$886* of monthly revenue
  - ***Safer***: A *1%* increase in diversity → an increase of *11.8%* in job retention

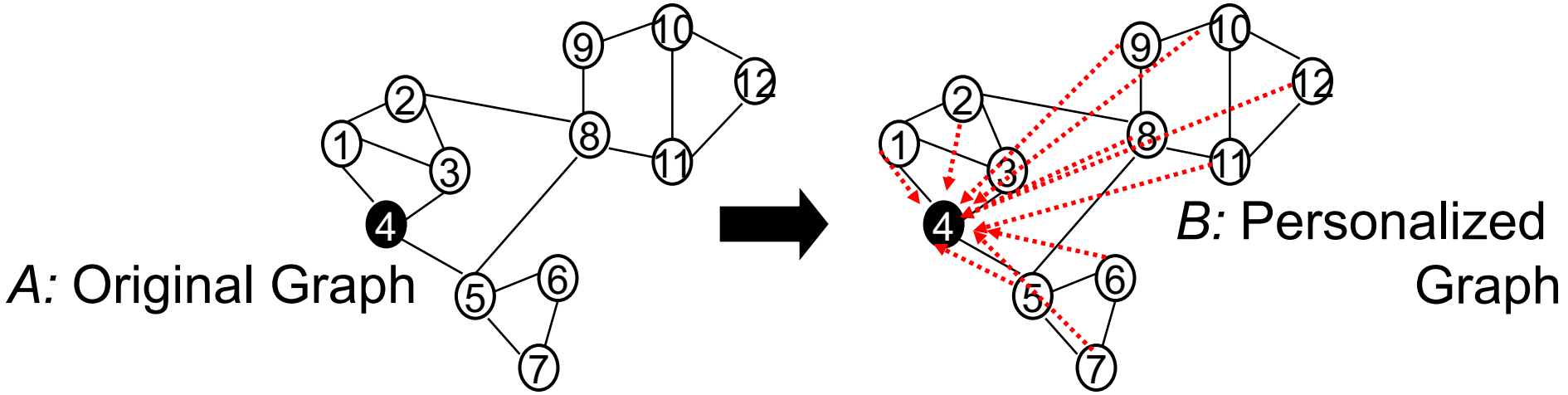
Hanghang Tong, Jingrui He, Zhen Wen, Ravi Konuru, Ching-Yung Lin:
Diversified ranking on large graphs: an optimization viewpoint. KDD 2011: 1028-1036

A4: Our Solutions (10 sec. introduction!)

- Problem 1 (Evaluate/measure a given top-k ranking list)
- A1: A weighted sum between relevance and similarity



- Problem 2 (Find a near optimal top-k ranking list)
- A2: A greedy algorithm (near-optimal, linear scalability)



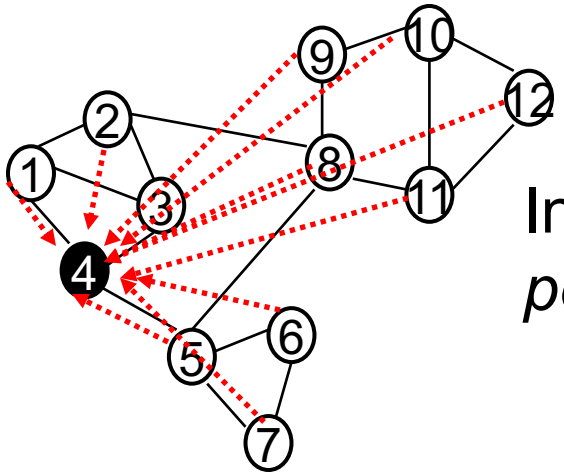
A Special Case of Dragon = Generalized *Netshild*

$r = B\,r$

- Fact 1: The largest eigenvalue of ***B*** is *1*
- Fact 2: ***r*** is the corresponding right eigenvector of ***B***
- Fact 3: The corresponding left eigenvector of ***B*** is ***1***

For *w=2, g(S)*~=drop in the largest eigenvalue of ***B***

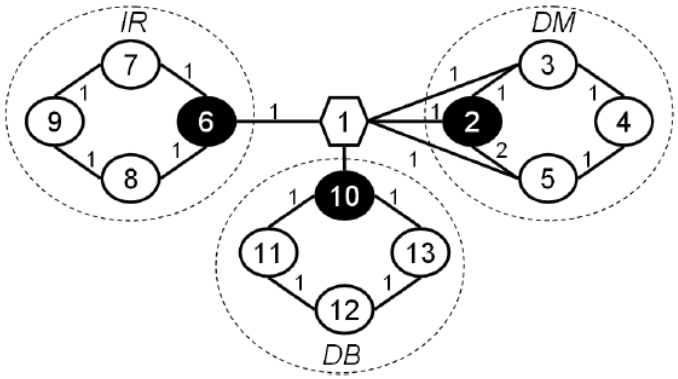
- Dragon (w=2) = Netshield on directed graphs



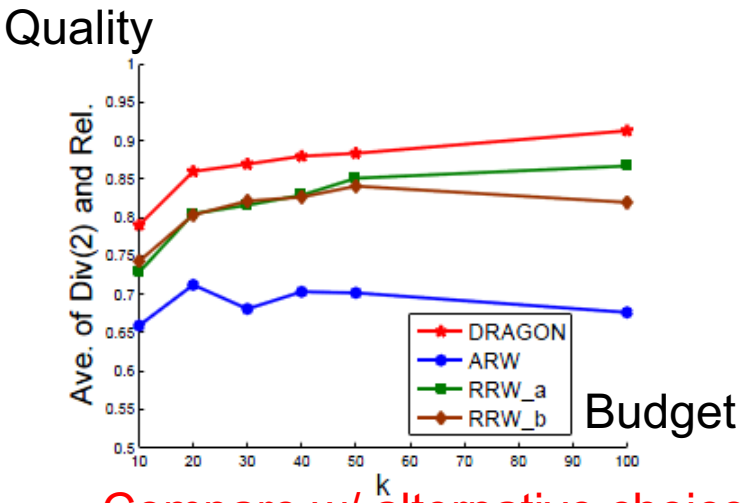
Intuition: find *k* nodes to disconnect *the personalized graph* ***B*** as much as possible

Hanghang Tong, Jingrui He, Zhen Wen, Ravi Konuru, Ching-Yung Lin:
Diversified ranking on large graphs: an optimization viewpoint. KDD 2011: 1028-1036

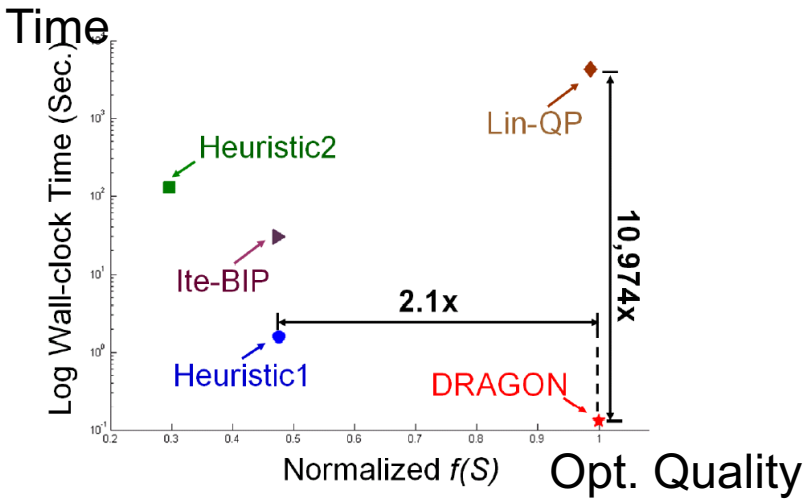
A4: Experimental Results



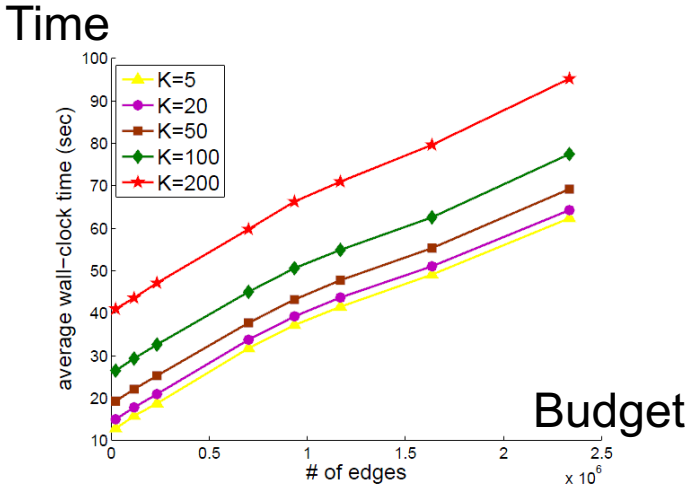
An Illustrative Example



Compare w/ alternative choices



Quality-Time Balance



Scalability

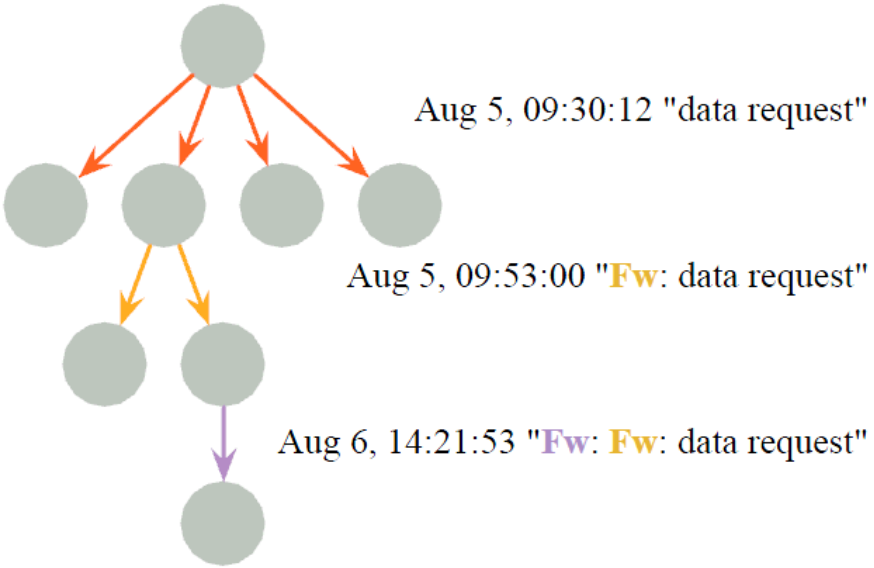
Hanghang Tong, Jingrui He, Zhen Wen, Ravi Konuru, Ching-Yung Lin:
Diversified ranking on large graphs: an optimization viewpoint. KDD 2011: 1028-1036

A5: Information Spreading in Context

- Micro-Behavior

Q1: What does information spreading depend on?

- Macro-Behavior

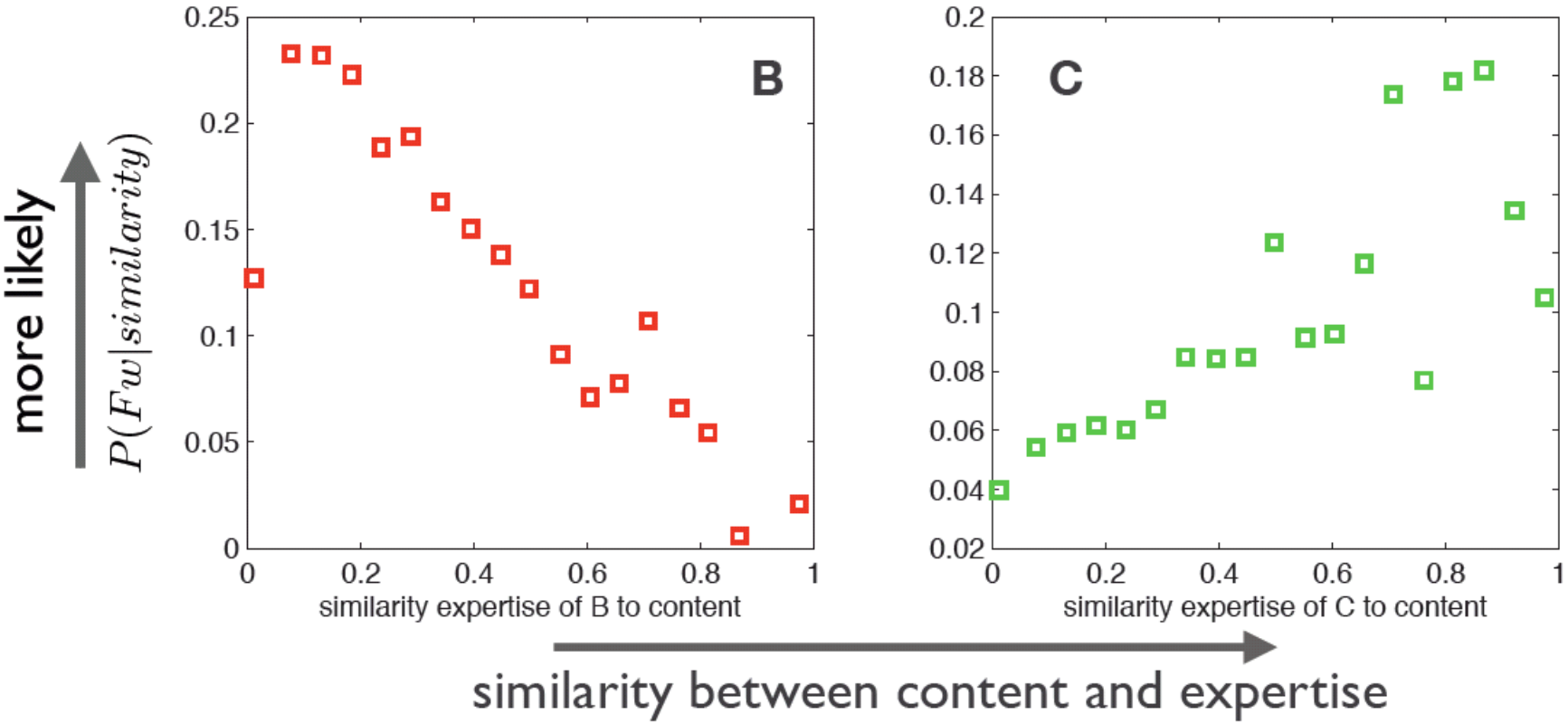


Q2: How does the tree look Like (depth, width, size), and why?

Data: 8000+ IBM employees emails, 2000+ Fw threads, information about the individuals (performance, dept, job role), content of emails

Dashun Wang, Zhen Wen, Hanghang Tong, Ching-Yung Lin, Chaoming Song, Albert-László Barabási: Information spreading in context. WWW 2011: 735-744

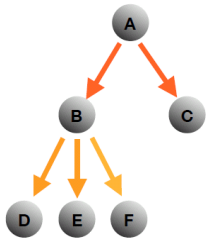
A5: Information Spread (*whether or not*) vs. Content

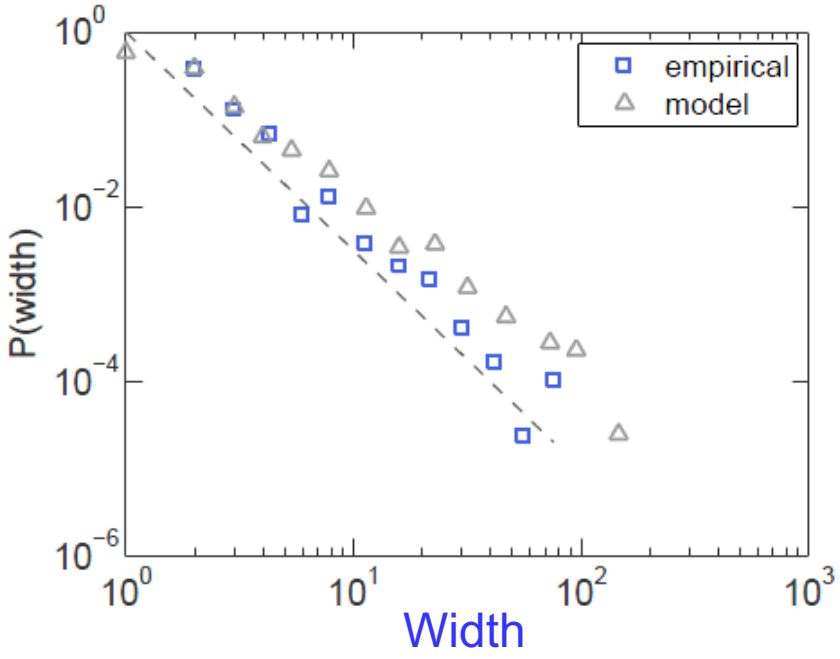


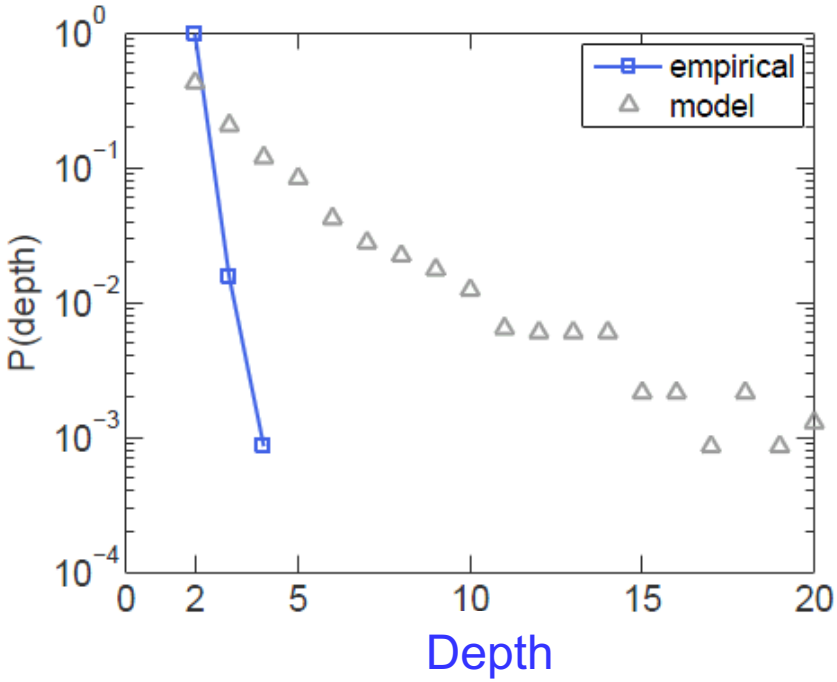
Information is more likely non-expert → expert

Dashun Wang, Zhen Wen, Hanghang Tong, Ching-Yung Lin, Chaoming Song, Albert-László Barabási: Information spreading in context. WWW 2011: 735-744

A5: The Structure of Information Spreading







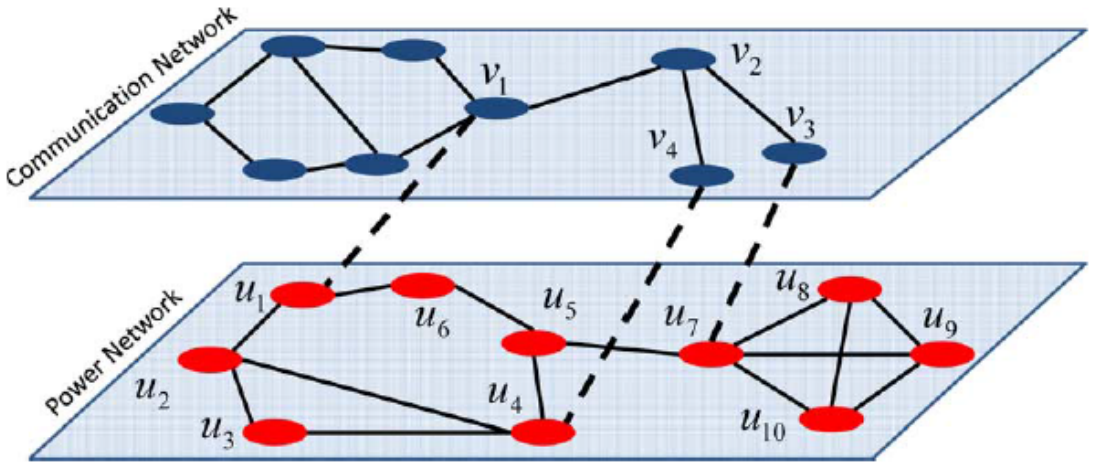
1) The trees are ***fat and shallow*** (instead of ***thin and deep*** as in Kleinberg's chain-letter setting)

2) Can be explained by a simple branch model (w/ decaying branching factors)

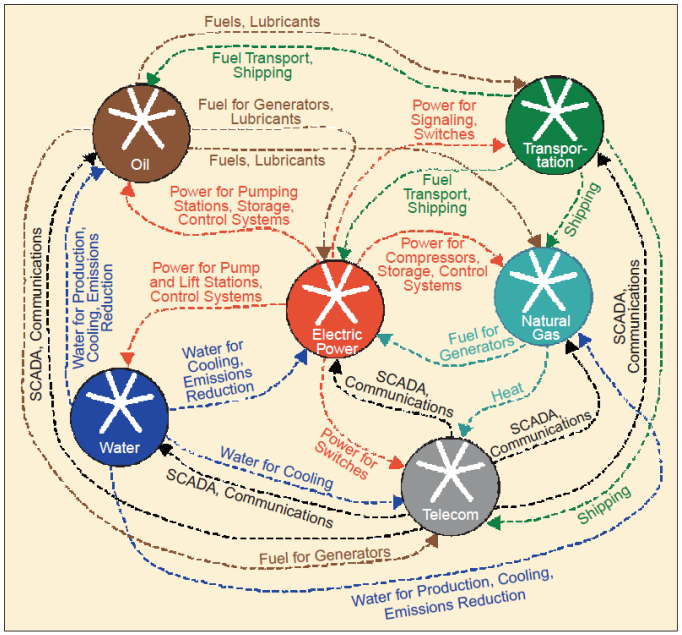
Dashun Wang, Zhen Wen, Hanghang Tong, Ching-Yung Lin, Chaoming Song, Albert-László Barabási: Information spreading in context. WWW 2011: 735-744

A6: Vulnerability of Cyber-Physical Systems

- A Two-layered CPS
  - Blue: communication networks
  - Red: Power grid
  - Dashed line: cross-layer inter-dependency



- Examples of Infrastructure Interdependencies



- Q: which node(s) and/or link(s) dysfunctions will lead to a catastrophic failure of the entire system?

- Rinaldi, Steven M., James P. Peerenboom, and Terrence K. Kelly. "Identifying, understanding, and analyzing critical infrastructure interdependencies." Control Systems, IEEE 21.6 (2001): 11-25.
- Nguyen, Duy T., Yilin Shen, and My T. Thai. "Detecting critical nodes in interdependent power networks for vulnerability assessment." Smart Grid, IEEE Transactions on 4.1 (2013): 151-159.
- Vespignani, Alessandro. "Complex networks: The fragility of interdependency." Nature 464.7291 (2010): 984-985.

A7: Team Member Replacement

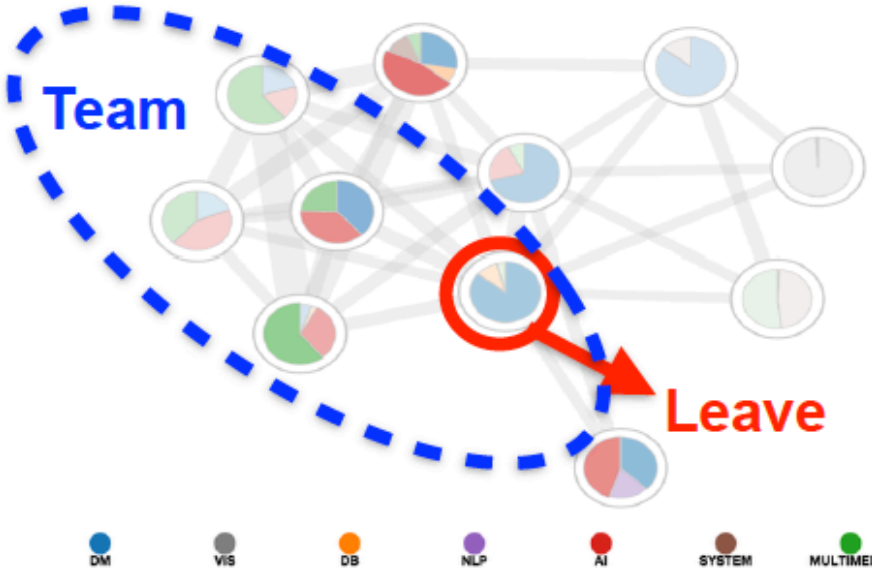
Problem Definition:

Given: (1) A labelled social network $G := \{A, L\}$
(2) A team $G(\mathcal{T})$
(3) A team member $p \in \mathcal{T}$

Adj. Matrix

Skill Indicator

Recommend: A "best" alternative $q \notin \mathcal{T}$ to replace the person p's role in the team $G(\mathcal{T})$

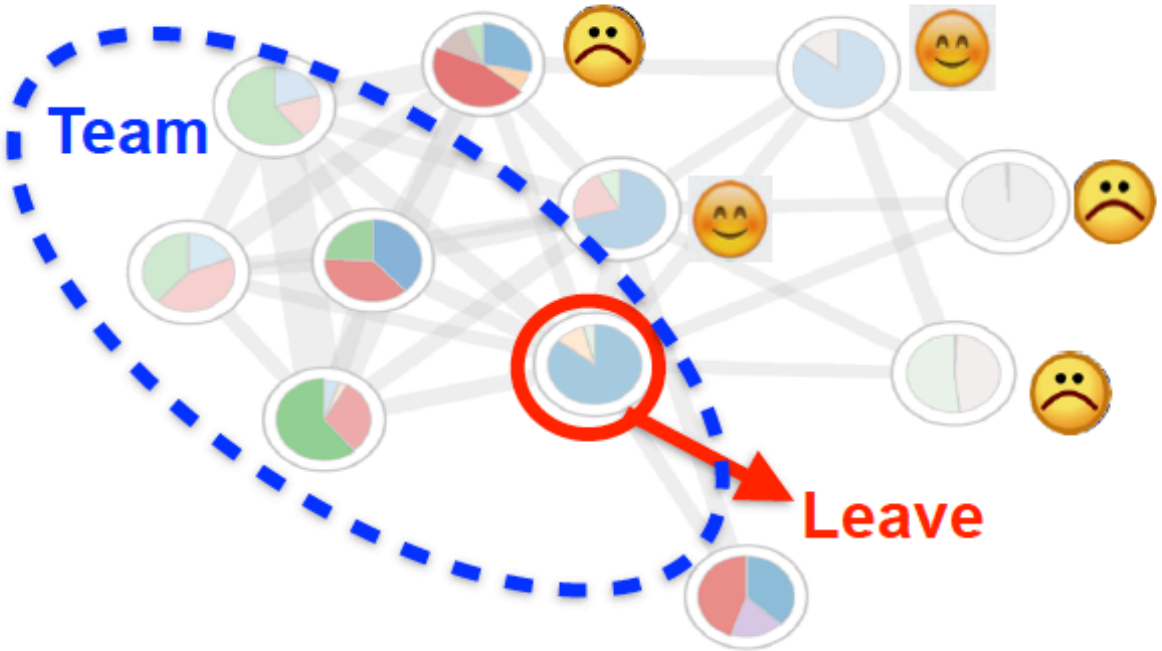


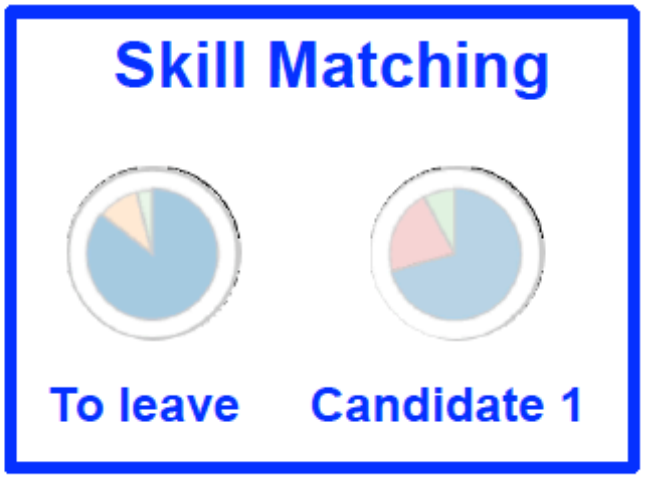
Q: who is a good candidate to replace the person to leave

- Liangyue Li, Hanghang Tong, Nan Cao, Kate Ehrlich, Yu-Ru Lin, Norbou Buchler:Replacing the Irreplaceable: Fast Algorithms for Team Member Recommendation. WWW 2015: 636-646

A7: Team Member Replacement

Objective 1: A good candidate should have a similar skill set



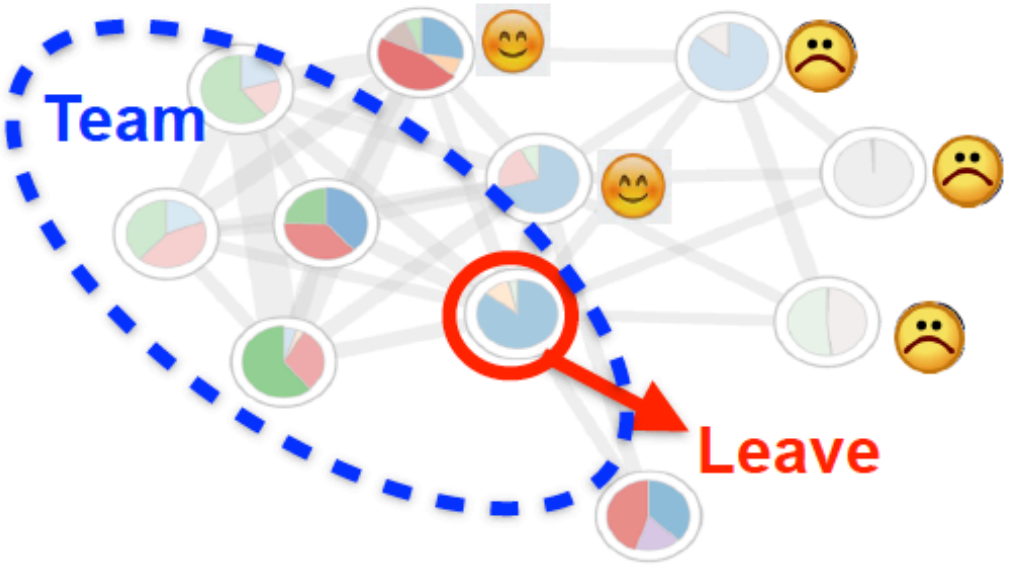


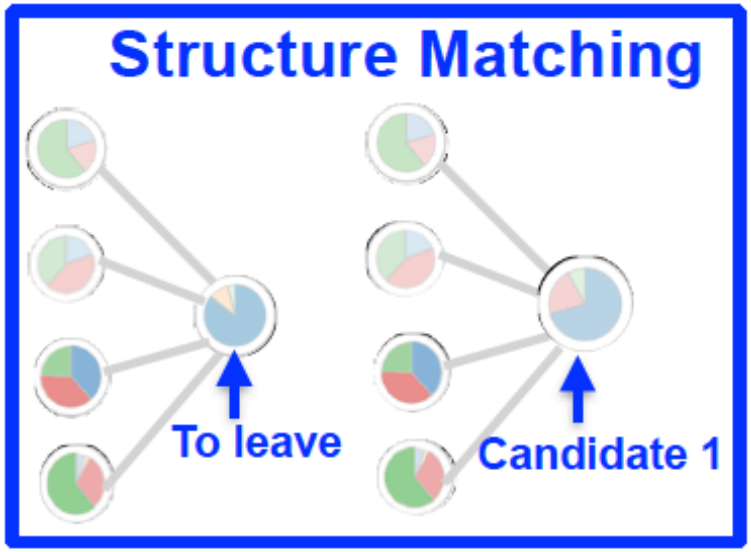
New team will have similar skill set as the old team to complete the task

- Liangyue Li, Hanghang Tong, Nan Cao, Kate Ehrlich, Yu-Ru Lin, Norbou Buchler:Replacing the Irreplaceable: Fast Algorithms for Team Member Recommendation. WWW 2015: 636-646

A7: Team Member Replacement

Objective 2: A good candidate should have a similar network structure





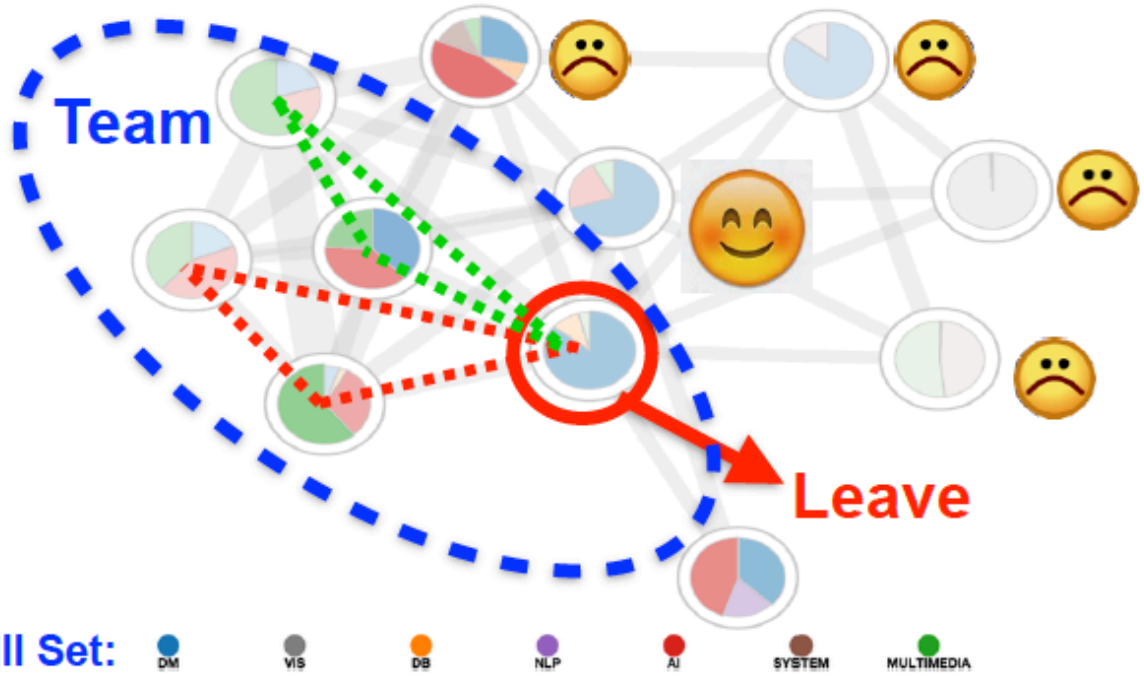
Skill Set: DM VIS DB NLP AI SYSTEM MULTIMEDIA

New team will have similar network structure as the old team to collaborate effectively

- Liangyue Li, Hanghang Tong, Nan Cao, Kate Ehrlich, Yu-Ru Lin, Norbou Buchler:Replacing the Irreplaceable: Fast Algorithms for Team Member Recommendation. WWW 2015: 636-646

A7: Team Member Replacement

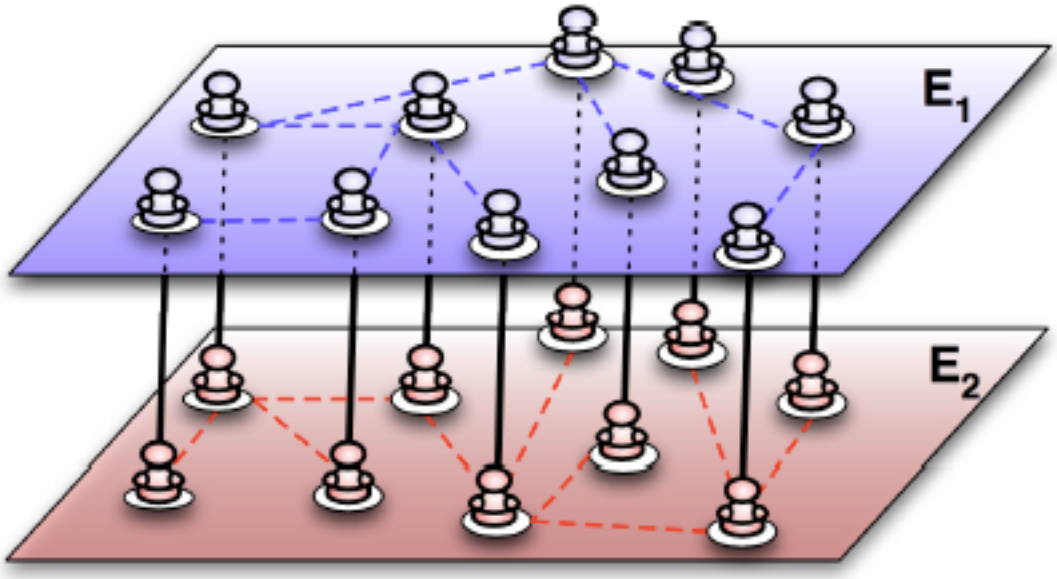
The two objectives should be fulfilled simultaneously!



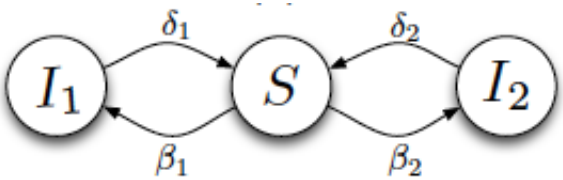
New team will have similar skill and communication configuration for each sub-task

- Liangyue Li, Hanghang Tong, Nan Cao, Kate Ehrlich, Yu-Ru Lin, Norbou Buchler:Replacing the Irreplaceable: Fast Algorithms for Team Member Recommendation. WWW 2015: 636-646

A8: Competitive Virus on Composite Networks

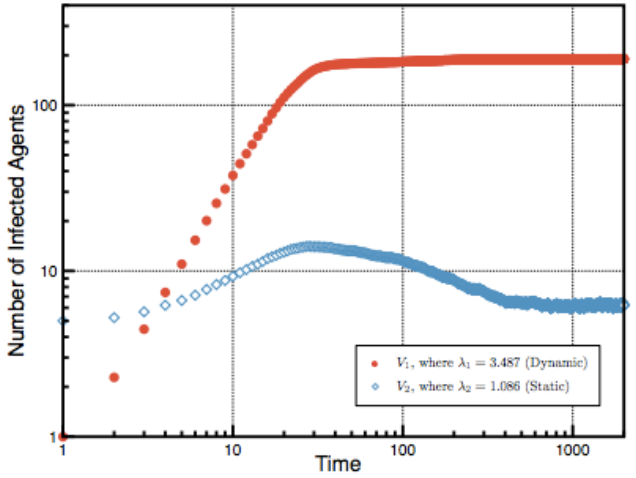


An example of composite network: a single set of nodes with two distinct sets of links



Virus Model: $S\ I_1\ I_2\ S$

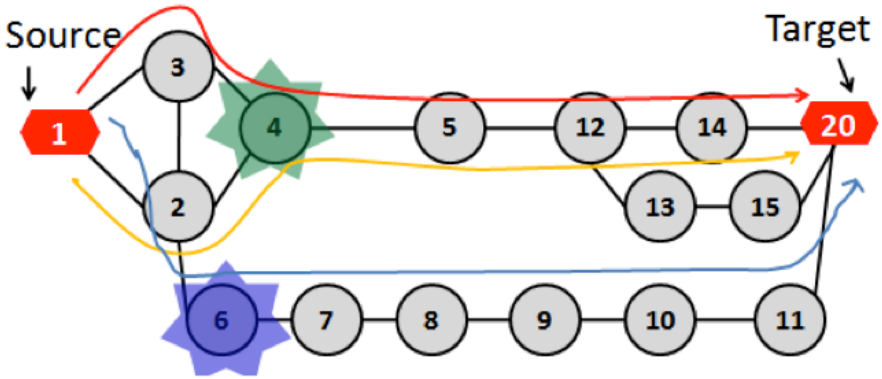
- Q: Which virus will win?
  - `virus': smartphone malware, memes, ideas
- A: if $\lambda_1 > \lambda_2$ V1 will win.
  - λ_1 and λ_2: leading eigen-values of system matrices.
- Results

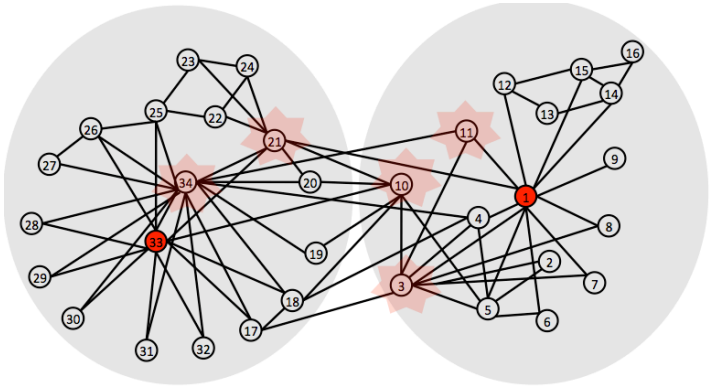


- Xuetao Wei, Nicholas Valler, B. Aditya Prakash, Iulian Neamtiu, Michalis Faloutsos, Christos Faloutsos: Competing Memes Propagation on Networks: A Network Science Perspective. IEEE Journal on Selected Areas in Communications 31(6): 1049-1060 (2013)

A9: Gateway Finder

- **Problem Definition**: Given a source (s) or a source group; and a target (t) or a target group,
  - **Q1 (Metric)**: how to measure the gateway-ness for a subset of nodes (I)?
  - **Q2 (Algorithm):** how to find a subset of k nodes with highest gateway-ness score?
- **Solutions**: Find the set whose removal causes maximal decrease of the proximity from source to target (e.g., block most paths).



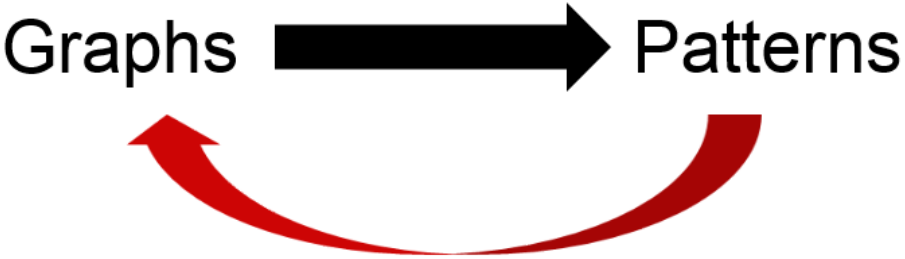


- Hanghang Tong, Spiros Papadimitriou, Christos Faloutsos, Philip S. Yu, Tina Eliassi-Rad: Gateway finder in large graphs: problem definitions and fast solutions. Inf. Retr. 15(3-4): 391-411 (2012)

Part IV: Future Trends

- N1: Learn *k* in GCO Problem
- N2: Sense-Making of GCO: How/Why?
- N3: GCO Tracking & Attribution
- N4: GCO on Multi-layered Networks
- N5: Min-Max GCO Problem
- N6: Super-Robust Network Problem
- N7: Optimal Graph Construction Problem
- N8: GCO Scalability: Challenges & Opportunities

N1: Learn *k* in GCO



Graph Connectivity Optimization (GCO) - This Lecture

Given:
(1) an initial graph
(2) a graph operation (e.g., deleting ***k*** nodes, adding ***k*** new links)
(3) a mining task

→

Find:
an 'optimal' graph

- **Q**: what is the minimum ***k***, to reduce the epidemic threshold below 1, given the strength of the virus and connectivity of the population?

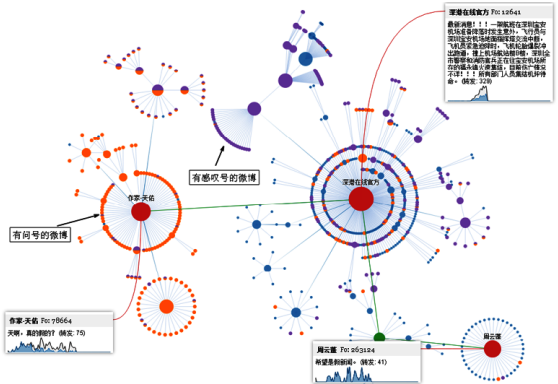
N2: Sense-Making of GCO:
What/Who → How/Why?

Given:
(1) an initial graph
(2) a graph operation (e.g., deleting ***k*** nodes, adding ***k*** new links)
(3) a mining task

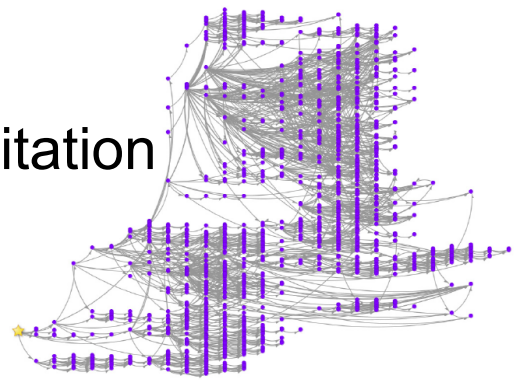
→ **Find**:
an 'optimal' graph

- **Current: A Typical GCO Instance**
  - **Given:** a social network,
  - **Find:** ***who*** or ***which links*** are the most important, in bridging different communities?
- **Next: From Who/What to How/Why**
  - **Q1**: Given an critical power-line in power-grid, explain ***why*** it is important (in maintaining the graph connectivity)
  - **Q2**: Given an influential author in scholarly network, find ***how*** s/he influences other researchers and/or fields?

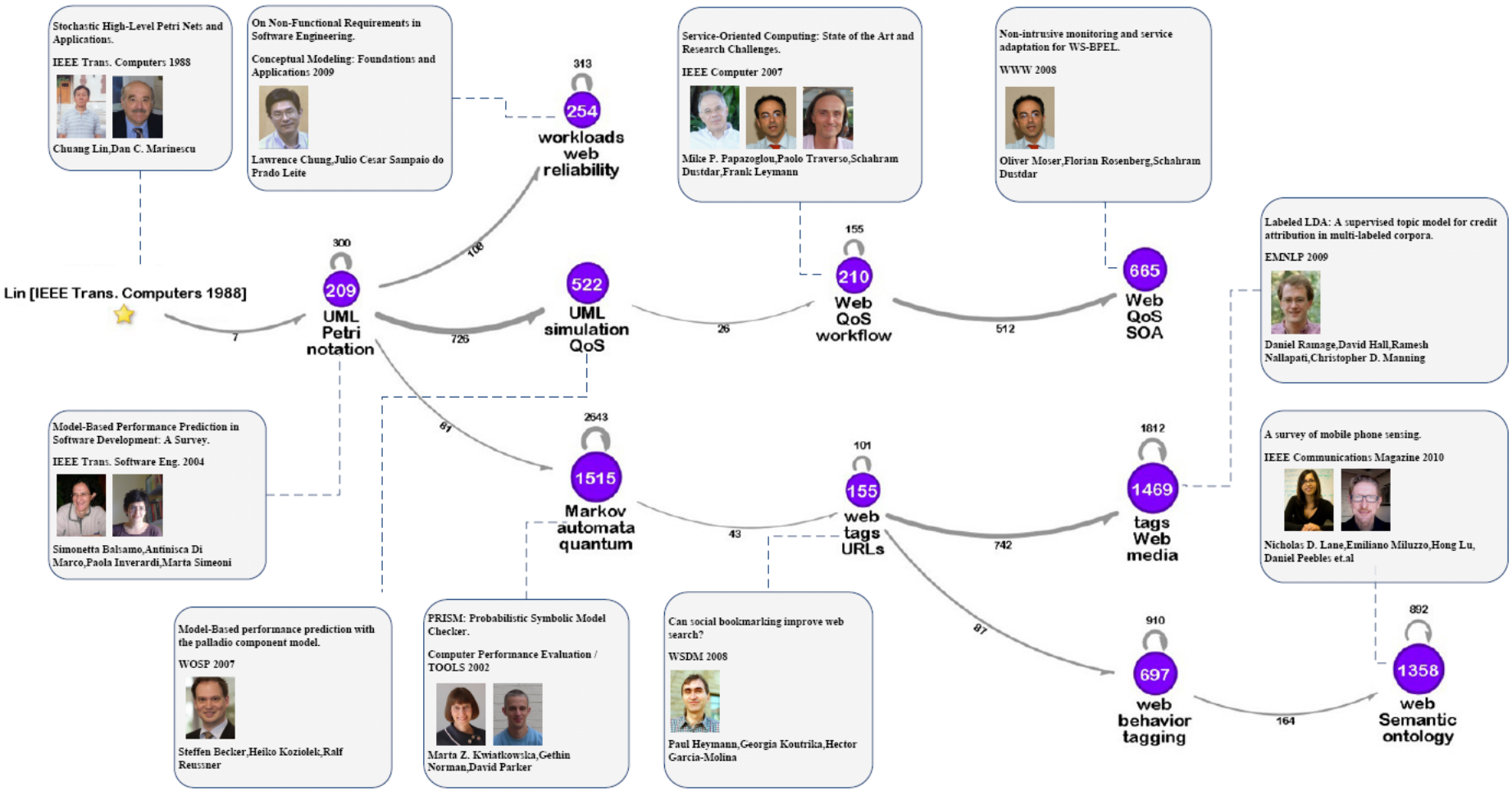
Retweeting Graph in Chinese Weibo



Reversed Citation Graph



N2: A Flow-based Summarization Solution



The influence graph of “Stochastic High-Level Petri Net and Applications”

- Lei Shi, Hanghang Tong, Jie Tang, Chuang Lin: Flow-Based Influence Graph Visual Summarization. ICDM 2014: 983-988

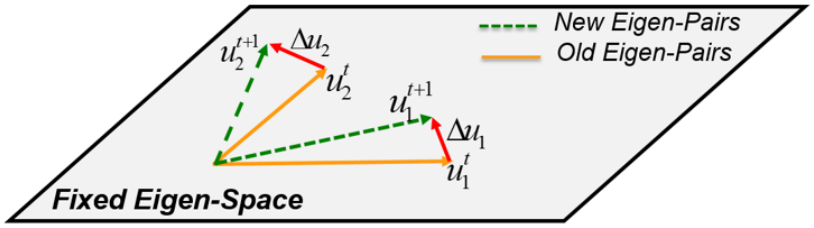
N3: GCO Tracking & Attribution

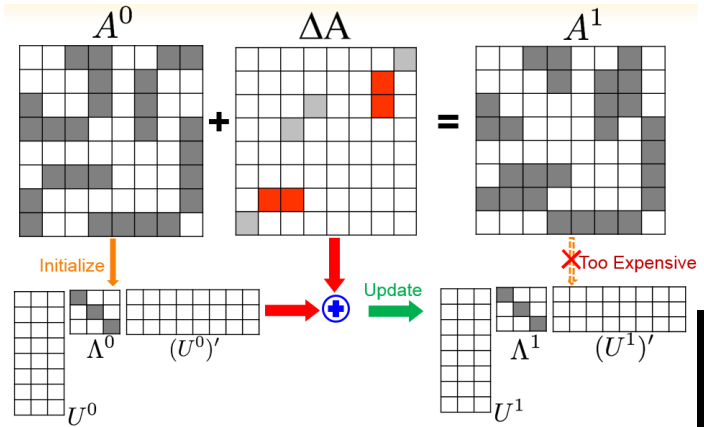
Given:
(1) an initial graph
(2) a graph operation (e.g., deleting ***k*** nodes, adding ***k*** new links)
(3) a mining task

➡ **Find**:
an 'optimal' graph

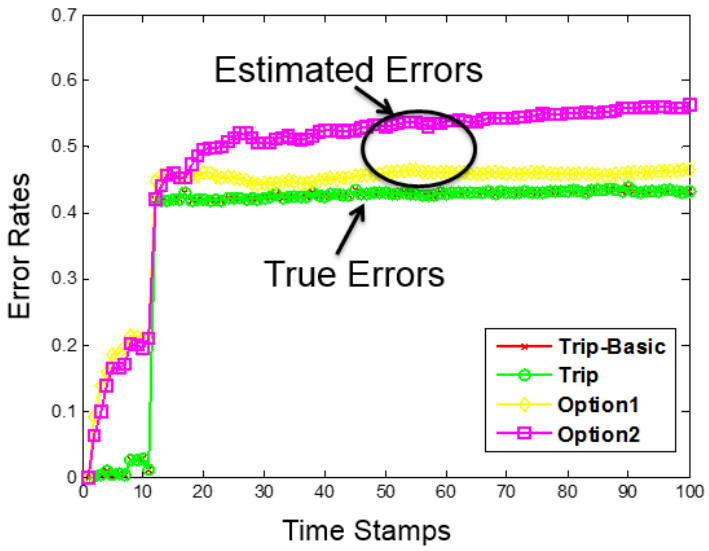
- **Observations**
  - **#1:** Graphs are changing over time
  - **#2:** Many graph connectivity measures can be expressed as an ***eigen-function*** of the adjacency matrix

- **Solutions: Tracking eigen-function**





- **Results**



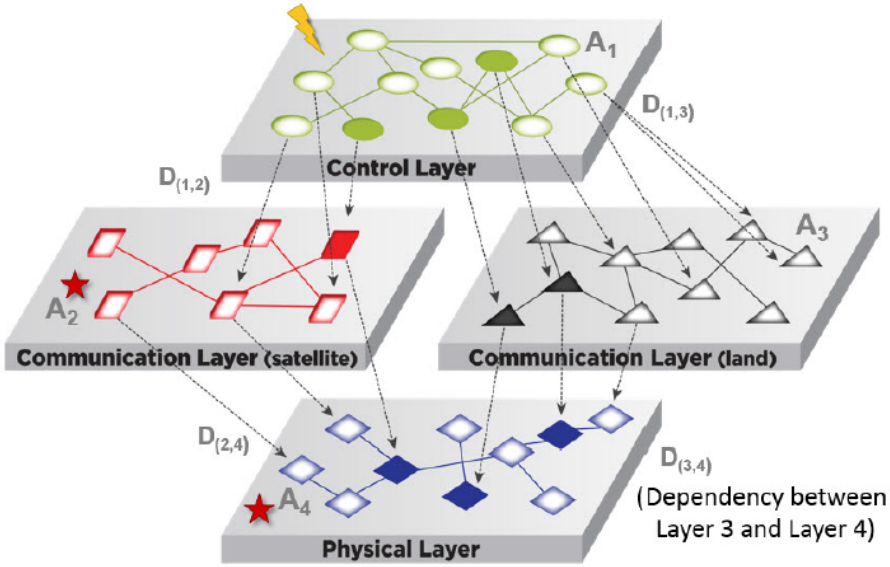
- C. Chen and H. Tong: "Fast Eigen-Functions Tracking on Dynamic Graphs". SDM 2015

N4: GCO on Multi-layered Networks

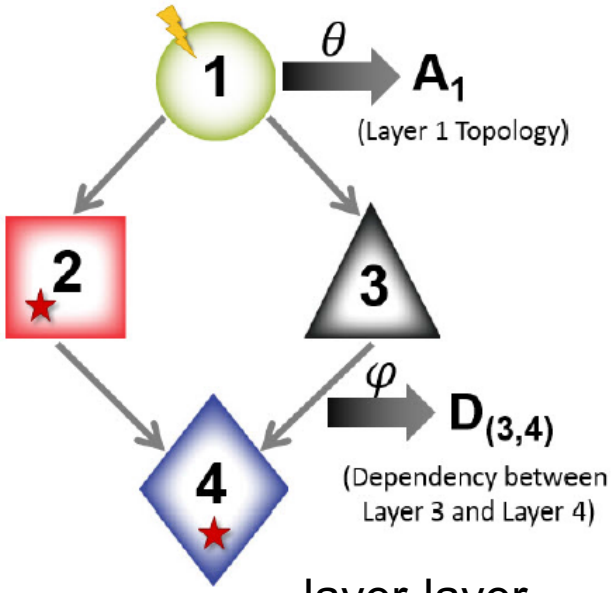
Graph Connectivity Optimization (GCO) - This Tutorial

Given:
(1) an initial graph
(2) a graph operation (e.g., deleting ***k*** nodes, adding ***k*** new links)
(3) a mining task

→ **Find**: an 'optimal' graph



A four-layered network



layer-layer dependency network

- **A Multi-layered Network Model (Mulan)**
  - A Quintuple: $\Gamma =< \mathbf{G}, \mathcal{A}, \mathcal{D}, \theta, \varphi >$
- **Q:** How to find an optimal node set in the ***control layer***, to minimize the connectivity of the ***target layer(s)***?

- C. Chen, J. He, N. Bliss and H. Tong: "On the Connectivity of Multi-layered Networks: Models, Measures and Optimal Control" ICDM 2015.

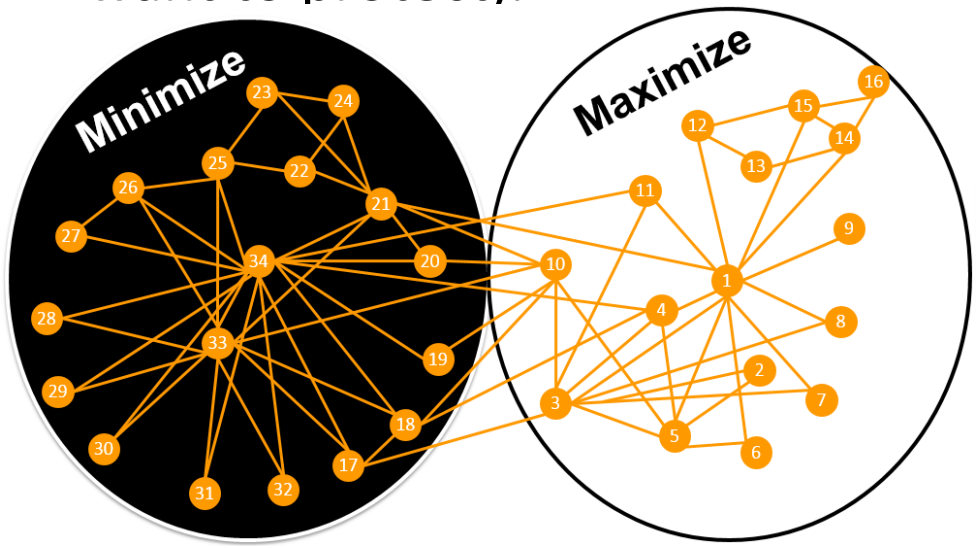
N5: Min-Max GCO Problem (Angels & Demons)

Graph Connectivity Optimization (GCO) - This Tutorial

Given:
(1) an initial graph
(2) a graph operation (e.g., deleting ***k*** nodes, adding ***k*** new links)
(3) a mining task

➡ **Find**:
an 'optimal' graph

- **Given:** two inter-connected networks (or two inter-connected components within the same network);
- **Find:** the optimal graph operation, that
  - ***minimizes*** the connectivity of the (adversarial) network, and
  - ***maximizes*** the connectivity of the other network (the one we want to protect).



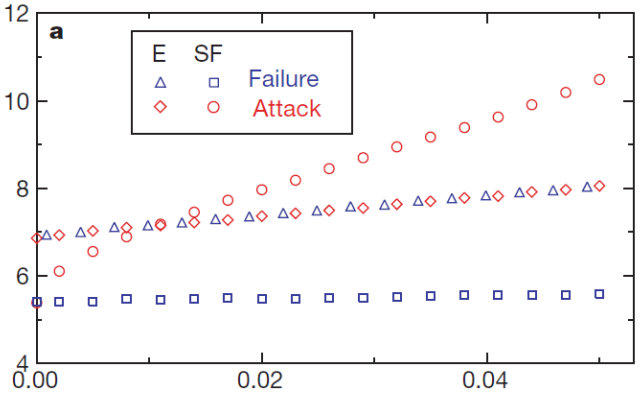
Graph Connectivity Optimization (GCO) - This Tutorial

Given:
(1) an initial graph
(2) a graph operation (e.g., deleting ***k*** nodes, adding ***k*** new links)
(3) a mining task

➡ **Find**:
an 'optimal' graph

N6: Super-Robust Network

- **Observations** (Nature 2000):
  - **Scale-free Networks** (e.g., power-law): resilient to random failure, but vulnerable to targeted attack
  - **Exponential Networks** (e.g., ER, Small-World model): resilient to targeted attacks.



- X: fraction of removed nodes
- Y: diameter of the residual network
- E: ER model; SF: scale-free
- Blue: (random) failure
- Red: (intentional) attack

- **Q1:** How to design a robust network that is resilient to both failure and attacks?
- **Q2:** If we know the type of attack (e.g., HDA, or even based on GCO algorithms), How to tailor the GCO-defending algorithms (e.g., knowing your enemies)?

- Albert, Réka, Hawoong Jeong, and Albert-László Barabási. "Error and attack tolerance of complex networks." nature 406.6794 (2000): 378-382.
- István A. Kovács & Albert-László Barabási: Network science: Destruction perfected. Nature 524, 38–39, 2015

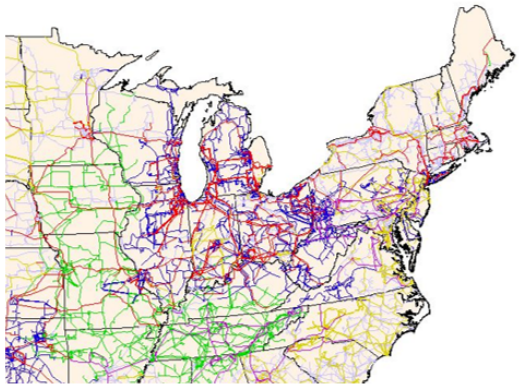
Graph Connectivity Optimization (GCO) - This Tutorial

Given:
(1) an initial graph
(2) a graph operation
(3) a mining task

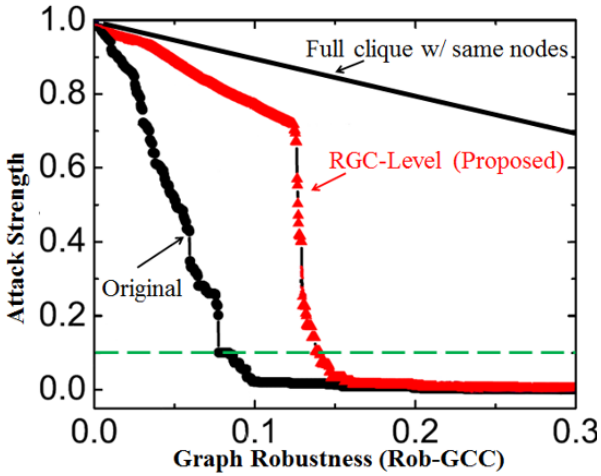
Find:
an 'optimal' graph

N7: Optimal Graph Construction

- **Q: What if the initial graph does not exist?**
- **Robust Network Construction again intentional attacks (e.g., HDA)**
  - **Given:** (1) the number of nodes *n* of the graph, and (2) its desired degree vector *d* (i.e., node capacity);
  - **Output:** a graph *A* with (1) *n* nodes, (2) the maximal robustness, (3) deg(*A*) = *d*
- **An Effective Heuristic**
  - **H1:** Avoid disassortative mix by degree
  - **H2:** Large loop coverage

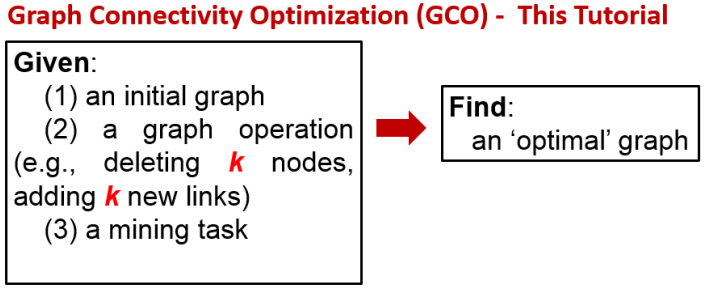


US Power Grid



- Hui Wang, Wanyun Cui, Yanghua Xiao, Hanghang Tong: Robust network construction against intentional attacks. BigComp 2015: 279-286

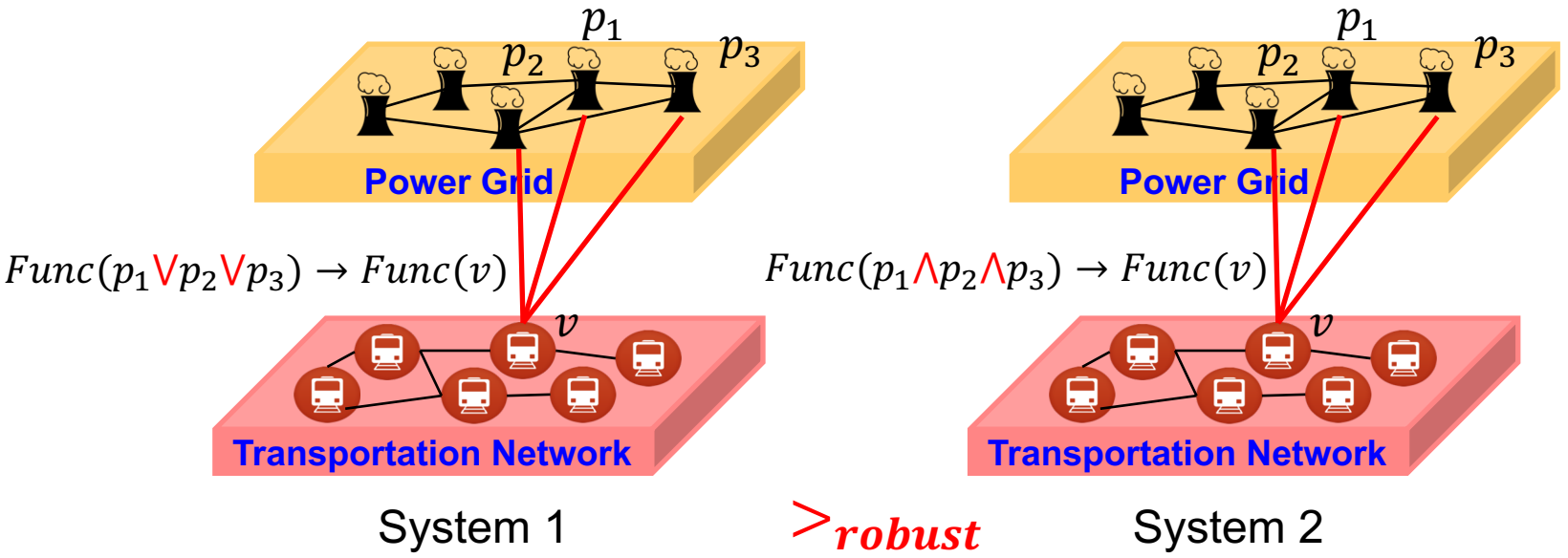
N8: GCO Scalability: Challenges & Opportunities



- **Challenges: How to Scale-up & Speed-up**
  - E1: O(m) or better on a single machine
  - E2: Parallelism (implementation, decouple, analysis)
- **Opportunities**:
  - Solving GCO problems **trivially** by scale?
  - **Conjecture**: when the initial graph is big enough, (1) adding any new links will make little improvement, and (2) the graph becomes impossible to demolish with any limited budget.
  - Is this true? If so, where is the tipping point?

N9. Connectivity Measures: Beyond SUBLINE

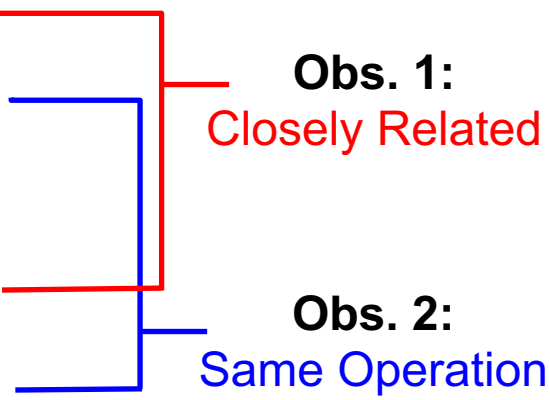
- Obs.
  - Different dependency types may greatly affect the robustness of multi-layered networks



- **Q. How to model and measure the connectivity in such complex multi-layered networks?**

N10. Connectivity Optimization vs. Adversarial Attack

- Connectivity Optimization
  - Goal: optimize network connectivity
  - Method: manipulate network structure
- Adversarial Attack
  - Goal: attack network learning models
  - Method: manipulate network structure



- **Q1. Is it possible to model the network learning tasks from the connectivity perspective?**
- **Q2. Is it possible to attack the learning results through connectivity optimization?**

Reference

- Hanghang Tong, B. Aditya Prakash, Tina Eliassi-Rad, Michalis Faloutsos, Christos Faloutsos: Gelling, and melting, large graphs by edge manipulation. CIKM 2012: 245-254

- Hui Wang, Wanyun Cui, Yanghua Xiao, Hanghang Tong: Robust network construction against intentional attacks. BigComp 2015: 279-286

- Lei Shi, Hanghang Tong, Jie Tang, Chuang Lin: Flow-Based Influence Graph Visual Summarization. ICDM 2014: 983-988

- B. Aditya Prakash, Lada Adamic, Theodore Iwashnya, Hanghang Tong and Christos Faloutsos: Fractional Immunization on Networks. SDM 2013

- Hau Chan, Leman Akoglu, Hanghang Tong: Make It or Break It: Manipulating Robustness in Large Networks. SDM 2014: 325-333

Reference

- Hanghang Tong, Spiros Papadimitriou, Christos Faloutsos, Philip S. Yu, Tina Eliassi-Rad: Gateway finder in large graphs: problem definitions and fast solutions. Inf. Retr. 15(3-4): 391-411 (2012)

- Hanghang Tong, Jingrui He, Zhen Wen, Ravi Konuru, Ching-Yung Lin: Diversified ranking on large graphs: an optimization viewpoint. KDD 2011: 1028-1036

- Nicholas Valler, B. Aditya Prakash, Hanghang Tong, Michalis Faloutsos, Christos Faloutsos: Epidemic Spread in Mobile Ad Hoc Networks: Determining the Tipping Point. Networking (1) 2011: 266-280

- Dashun Wang, Zhen Wen, Hanghang Tong, Ching-Yung Lin, Chaoming Song, Albert-László Barabási: Information spreading in context. WWW 2011: 735-744

- Hanghang Tong, B. Aditya Prakash, Charalampos E. Tsourakakis, Tina Eliassi-Rad, Christos Faloutsos, Duen Horng Chau: On the Vulnerability of Large Graphs. ICDM 2010: 1091-1096

Reference

- Yao Zhang and B. Aditya Prakash: Scalable Vaccine Distribution in Large Graphs given Uncertain Data. ICDM 2014

  Code available at: http://people.cs.vt.edu/badityap/CODE/UDAV.zip

- L. Le, T. Eliassi-Rad and H. Tong: MET: A Fast Algorithm for Minimizing Propagation in Large Graphs with Small Eigen-Gaps. SDM 2015

- István A. Kovács & Albert-László Barabási: Network science: Destruction perfected. Nature 524, 38–39, 2015

- Rinaldi, Steven M., James P. Peerenboom, and Terrence K. Kelly. "Identifying, understanding, and analyzing critical infrastructure interdependencies." Control Systems, IEEE 21.6 (2001): 11-25.

- Nguyen, Duy T., Yilin Shen, and My T. Thai. "Detecting critical nodes in interdependent power networks for vulnerability assessment." Smart Grid, IEEE Transactions on 4.1 (2013): 151-159.

Reference

- Xuetao Wei, Nicholas Valler, B. Aditya Prakash, Iulian Neamtiu, Michalis Faloutsos, Christos Faloutsos: Competing Memes Propagation on Networks: A Network Science Perspective. IEEE Journal on SAC 31(6): 1049-1060 (2013)

- Liangyue Li, Hanghang Tong, Nan Cao, Kate Ehrlich, Yu-Ru Lin, Norbou Buchler:Replacing the Irreplaceable: Fast Algorithms for Team Member Recommendation. WWW 2015: 636-646

- B. Aditya Prakash, Hanghang Tong, Nicholas Valler, Michalis Faloutsos, Christos Faloutsos: Virus Propagation on Time-Varying Networks: Theory and Immunization Algorithms. ECML/PKDD (3) 2010: 99-114

Reference

- C. Chen, H. Tong, B. Prakash, C. Tsourakakis, T. Eliassi-Rad, C. Faloutsos, D. Chau: Node Immunization on Large Graphs: Theory and Algorithms. IEEE TKDE 2015

- Chen Chen, Hanghang Tong, Lei Xie, Lei Ying, Qing He, "Cross-Dependency Inference in Multi-layered Networks: A Collaborative Filtering Perspective", ACM Transactions on Knowledge Discovery from Data, Special Issue of "Bests of KDD 2016" 2017

- Chen Chen, Hanghang Tong, B Aditya Prakash, Tina Eliassi-Rad, Michalis Faloutsos, Christos Faloutsos, "Eigen-Optimization on Large Graphs by Edge Manipulation", ACM Transactions on Knowledge Discovery from Data 2016

- Chen Chen, Hanghang Tong, "On the Eigen-Functions of Dynamic Graphs: Fast Tracking and Attribution Algorithms", SAM Special Issue of "Best of SDM 2015"

- Chen Chen, Ruiyue Peng, Lei Ying, Hanghang Tong. "Network Connectivity Optimization: Fundamental Limits and Effective Algorithms", Proceedings of KDD 2018

Reference

- Chen Chen*, Jundong Li*, Hanghang Tong, Huan Liu. ”Multi-Layered Network Embedding”, Proceedings of SDM, 2018

- Qiao Liu, Chen Chen, Annie Gao, Hanghang Tong, Lei Xie, “VariFunNet, an integrated multiscale modeling framework to study the effects of rare non-coding variants in genome-wide association studies: Applied to Alzheimer's disease”, Proceedings of BIBM, 2017

- Chen Chen, Hanghang Tong, Lei Xie, Lei Ying, Qing He, “FASCINATE: Fast Cross-Layer Dependency Inference on Multi-layered Networks”, Proceedings of SIGKDD, 2016 (***Bests of KDD’16***)

- Chen Chen, Jingrui He, Nadya Bliss, Hanghang Tong, “On the Connectivity of Multi-layered Networks: Models, Measures and Optimal Control”, Proceedings of IEEE ICDM, 2015

- Chen Chen, Hanghang Tong, “Fast Eigen-Functions Tracking on Dynamic Graphs”, Proceedings of SDM, 2015 (***Bests of SDM’15***)