Chapter | thirteen

High-Fidelity Formation Flying Simulation

I breathed enough to learn the trick,
And now, removed from air,
I simulate the breath so well,
That one, to be quite sure.

Emily Dickinson (1830–1886)

In this chapter, we will illustrate most of the previously-introduced material by performing a series of nonlinear simulations of portions of a formation flying reference mission using the `FreeFlyer`™ orbit simulator [186]. The purpose is to investigate the effects of perturbations and control techniques on a realistic satellite formation through all planned stages of operation. A commercial orbit propagator, `FreeFlyer`™, was used to propagate the orbits in an absolute frame for both satellites. Propagation included the effects of many realistic perturbations, including drag, lift, solar radiation pressure, and J_2. The propagator interacts with the controller through a `MATLAB`® interface. During each propagation step, the control algorithm is queried. If the controller is currently implementing a thrusting plan, the thrusts corresponding to the current position in the plan are converted into appropriate orbital element offsets and returned to the propagator. After each plan is completely implemented, a new plan is created.

The simulation presented in the following sections involves a three-satellite formation. The reference orbit, represented by a virtual satellite with properties similar to the average of the fleet, has a semimajor axis of 6900 km, inclination 45°, and eccentricity 0.003. Realistic perturbations (drag, J_2, solar radiation pressure, Sun/Moon effects) were included in all simulations.

Each satellite is modeled using specifications for the MELCO formation flying mission [188]. Each satellite has a mass of 900 kg and a ballistic coefficient of 0.4. The satellite thrusters are restricted to provide a maximum of 2 N of force over a 10.8-second time step.

The MELCO formation flying mission consists of two different formation shapes: (i) along-track separation formations and (ii) along-track/cross-track

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passive-aperture formations (triangular). Over the course of a mission, the formation is designed to achieve four different configurations in the following order: (1) along-track formation with 1 km (at apogee) of separation between spacecraft (represents deployment configuration); (2) passive-aperture formation with a 50-m baseline; (3) passive-aperture formation with a 500-m baseline; and (4) passive-aperture formation with a 5-km baseline.

Lawden's time-varying equations, described in Subsection 5.6.2, are used to determine the desired state for each spacecraft; however, the CW equations, described in Section 5.1, are used in solving an LP problem, to be discussed shortly. This is consistent with the observation in Ref. [26] that slightly eccentric orbits ($0.0005 < e < 0.01$) require eccentricity-invariant initial conditions, but not time-varying dynamics. Ten minutes of each orbit are reserved for observations: During this time position constraints are enforced, but no thrusting is permitted.

For each type of formation at each baseline, one or more 18-day simulations were conducted to determine the average fuel usage. A representative mission accuracy requirement is that each formation size must use an *error box* that is 10% of the baseline.

13.1 SIMULATION CONTROLLER CONFIGURATION

A *planning controller* is used in the simulations in this section. Each spacecraft designs its control individually and coordination between spacecraft is accomplished through the design of the formation desired states at the time the formation is initialized. Error box constraints of the type discussed in Ref. [26] are added at every 6 time steps to ensure that satellites achieve their performance objectives, while reducing the computational burden of imposing them at every time step. In addition, fuel inputs are permitted every 6 time steps for station-keeping (in which spacecraft are tasked to remain in formation) and every time step for formation maneuvers (those periods during which the formation is being switched from one configuration to another). Mission requirements are chosen to ensure that a ten-minute time window at apogee should be reserved during formation-keeping maneuvers in which no thruster inputs are permitted.

An always-feasible formulation was used for the simulation controller, guaranteeing that a plan is always returned, even if no feasible solution exists which satisfies the error box constraints. The always-feasible formulation is based on the *constraint-enlarging approach* presented in Ref. [26], but with a modification allowing several degrees of constraint violation. If no feasible solution is possible for the desired error box, the approach in Ref. [26] enlarges the error box as little as possible until a feasible solution is found. However, the resulting plan may end with the spacecraft outside the nominal box, which would guarantee that the next plan would use as much fuel as necessary to return the spacecraft to the error box on the first step of the plan or would also require an expansion of the nominal box. The always-feasible formulation used in this section also enlarges the error box until a feasible plan is found, but includes another soft constraint, which prefers that the planned trajectory still ends inside the nominal error box. This additional constraint is also implemented using an additional high cost-penalty variable that is used to ensure that it is only relaxed in the event that no feasible solution exists that enlarges the error box and ends inside the nominal error box. The modified always-feasible formulation allows the controller to prefer plans that make future optimizations initially feasible.

13.1.1 Parameters examined

An initial simulation of the mission described above was conducted. The results of that simulation indicated that levels of CDGPS noise expected were excessive for the station-keeping in a 50-m passive aperture formation with 5 m $\times$ 5 m $\times$ 5 m error box, the most tightly constricting phase of the mission. The controller was unable to keep the satellite constrained to its error box under those conditions and experienced errors that grew over time. In order to investigate this phenomenon and find a stable configuration for the controller that met as many mission requirements as possible, a number of variations of the basic controller setup have been implemented and tested in realistic simulations. Section 13.2 presents the results of those simulations and evaluates the effectiveness of the parameter variations in terms of effectiveness at preventing error box violations and average fuel use. The control parameters considered are as follows.

THRUSTING DURING OBSERVATIONS

Mission specifications are chosen to ensure that observations cannot occur when any spacecraft in the formation is thrusting. As a result, the basic configuration prevents thrusting during a 10 minute period at the apogee of every orbit that the formation is in the passive aperture configuration. However, the mission reference orbit is 95 minutes, so the effect of the observation thrusting prohibition is to reduce the overall control authority of any plan by more than 10%. Although this requirement is a hard constraint for the mission, it is included as a parameter in the simulation study so that its effects on formation flying mission performance can be judged.

ERROR BOXES RELAXED WHEN NOT OBSERVING

One concept for a formation flying mission could specify that a formation should be in a passive aperture during periods when the formation is taking observations and transition to a widely separated mode during periods when observations are not needed. A maneuver generation analysis indicates that there is generally no fuel advantage to formation-keeping in an along-track formation versus a passive aperture formation and that the cost of maneuvering into and out-of holding configurations is unnecessary.

Given that the tight error box requirements are derived from the needs of the distributed observation instrument, a reasonable modification of the mission requirements is to enlarge the error boxes during the 85 minutes of each orbit in which no observations are taking place and only enforce tight performance constraints when they are strictly needed. This constraint is implemented by doubling the error box size during the time steps outside the 10 minute period reserved for observations at apogee.

DYNAMICALLY MOTIVATED ERROR BOX SHAPES

Because the 50 m passive aperture is the most fuel-intensive of the formation configurations, a $5 \times 10 \times 5$ meter (radial/along-track/cross-track) error box is examined as an alternative to the 10% requirement for that portion of the mission. The $5 \times 5 \times 5$ meter error box is sufficiently small that the navigation errors [38] strongly influence the closed-loop behavior. Enlarging the along-track dimension allows a slightly more natural relative elliptical motion in the radial/cross-track plane (typically a 1×2 ellipse). This observation is derived from the form of Hill's equations, in which the coefficients of the harmonic terms for the along-track axis are exactly twice the value of the coefficients of the harmonics terms for the radial axis.

TERMINAL-INVARIANCE CONDITION

A plan that is optimized to guarantee that a spacecraft remains inside an error box over some future horizon will accomplish that goal, but will not provide any guarantees for the future behavior of the spacecraft. Occasionally, a situation may arise where the spacecraft would approach its constraint boundary in the near future after the end of a plan. This situation is prevented from resulting in a constraint violation by the creation and implementation of a new plan. However, if the new plan is forced to react quickly to a potential constraint violation in the near term, the only feasible solution may require a great deal of fuel. In an effort to reduce the need for vehicles to make short term "emergency" corrections, an invariant-set terminal condition was examined. In this case, the condition guarantees that after a plan ends, the spacecraft will naturally (i.e., with no thrust inputs) remain inside its box for a full orbit and return to its state at the time the plan ended. Within the time span that the dynamics can accurately propagate the states, this terminal constraint guarantees perpetual collision avoidance in the absence of state knowledge uncertainty. This constraint is imposed using the nominal estimate of the spacecraft state, because a robust implementation is generally infeasible.

13.2 SIMULATION RESULTS

Multiple simulations were performed to study the formation flying mission and the effects of the control system parameters introduced in Section 13.1. These simulations are described in Table 13.1 and summarized in Fig. 13.1.

In Figs. 13.2–13.19, the vast majority of the trajectories remain inside the error boxes. Several instances of error box constraint violations occurred as a result of the always-feasible formulation, but were quickly corrected. Several general trends are visible: In most cases the 50 m passive aperture requires the most fuel to maintain and the 500 m passive aperture requires the least fuel to maintain. Also, enlarging the along-track dimension of the error box tended to reduce fuel use a great deal in comparable simulation configurations (i.e., Sim 1 & Sim 4 and Sim 3 & Sim 6).

Table 13.1 Fuel use results for formation flying simulations: Fuel costs for station-keeping (SK) are given in mm/s/orbit/satellite and fuel costs for maneuvers (Mvr) are given in m/s for the entire formation

	50 m PA: 5 x 5 x 5 m error box			50 m PA: 5 x 10 x 5 m error box		
	Sim 1	**Sim 2**	**Sim 3**	**Sim 4**	**Sim 5**	**Sim 6**
No thrust observation	No	Yes	Yes	No	Yes	Yes
No observe large EB	No	Yes	No	No	No	No
Mnvr. end invariance	No	No	Yes	No	No	Yes
SK 50 m	45.96	106.51	53.73	8.39	33.47	12.01
SK 500 m	0.06	0.09	1.54	0.06	0.04	1.51
SK 5 km	5.83	27.47	1.69	6.8	29.47	1.72
Mvr #1	1.06	0.56	0.65	0.71	0.57	0.57
Mvr #2	17.94	36.57	19.97	5.78	13.51	6.8
Mvr #3	51.37	65.73	48.59	39.4	48.15	35.42

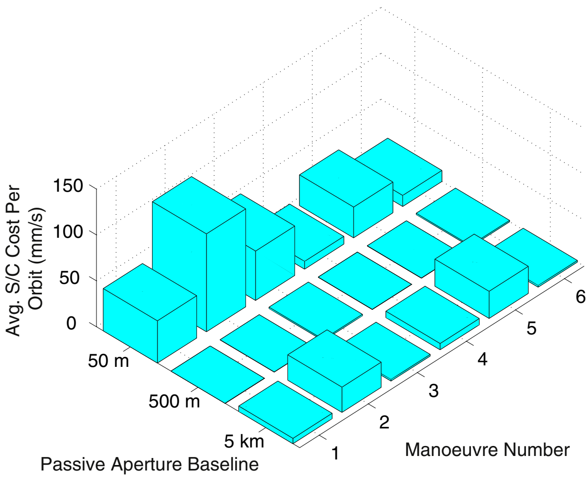

FIGURE 13.1 Summary of station-keeping results from Table 13.1.

Interestingly, the least restrictive error box (500 m for a 5 km formation baseline) did not result in the lowest fuel use. This phenomenon is likely due to the reduced likelihood of encountering constraints in a single orbit planning horizon for the larger error box. As a result, it is possible for a spacecraft to enter into a large, high-speed relative orbit that may end near an error box constraint. In those situations, future trajectory optimizations would need to take immediate corrective action to avoid constraint boundaries. This

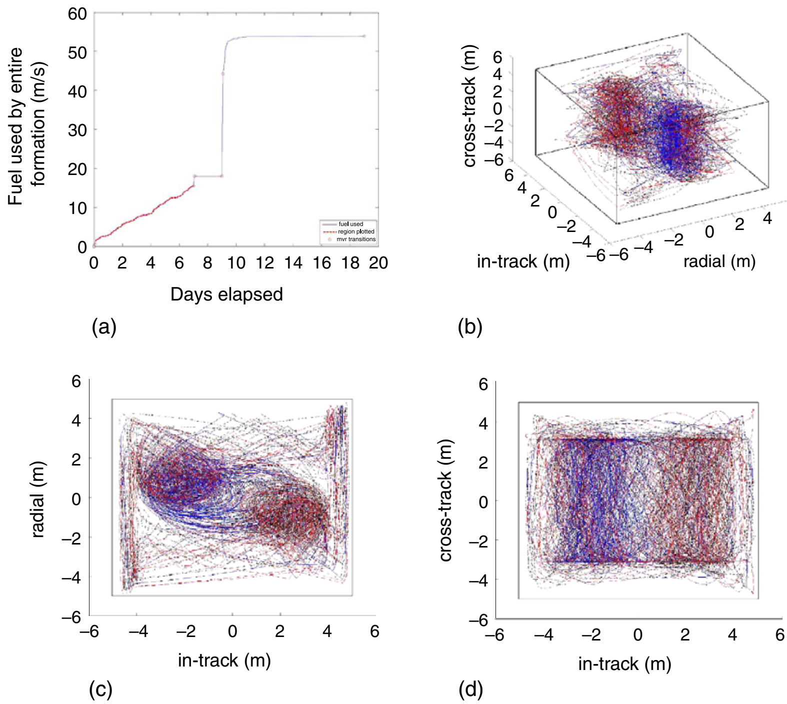

FIGURE 13.2 Simulation #1: Station-keeping in a passive aperture formation (50 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

hypothesis is supported by significant reductions in fuel use for the 5 km passive aperture between Sim 1 & Sim 3 and Sim 4 & Sim 6, even with fewer thrusting times available in Sim 3 and Sim 6 due to constraints against thrusting during observations. In both cases, the only difference between the simulations is the inclusion of the terminal invariance constraint and a restriction against thrusting during observations. The restriction against thrusting tends to increase the fuel use, as in the Sim 4 & Sim 5 pair. In Sim 4, thrusting is permitted at all times in the orbit, whereas in Sim 5 10% of the orbit is reserved for observations. In the 50 m and 5 km configurations, the 10% restriction increased fuel use by more than an order of magnitude for those simulations. It is likely that the difference between the fuel use numbers for the 500 m configuration is insignificant due to random elements in the simulations.

The addition of the terminal invariance constraint in Sim 6 lowers the fuel consumption for the 5 km configuration, but raises it for the 50 m and 500 m baselines compared to Sim 4. At the 5 km level, the reduction is greater than a factor of 3. Figures 13.13 and 13.19 show the error box motion for all three satellites in the formation for station-keeping in the 5 km formation for Sims 4 and 6, respectively. While Fig. 13.13 shows a considerable amount of motion throughout the entire error box, Fig. 13.19 shows that the spacecraft is actively

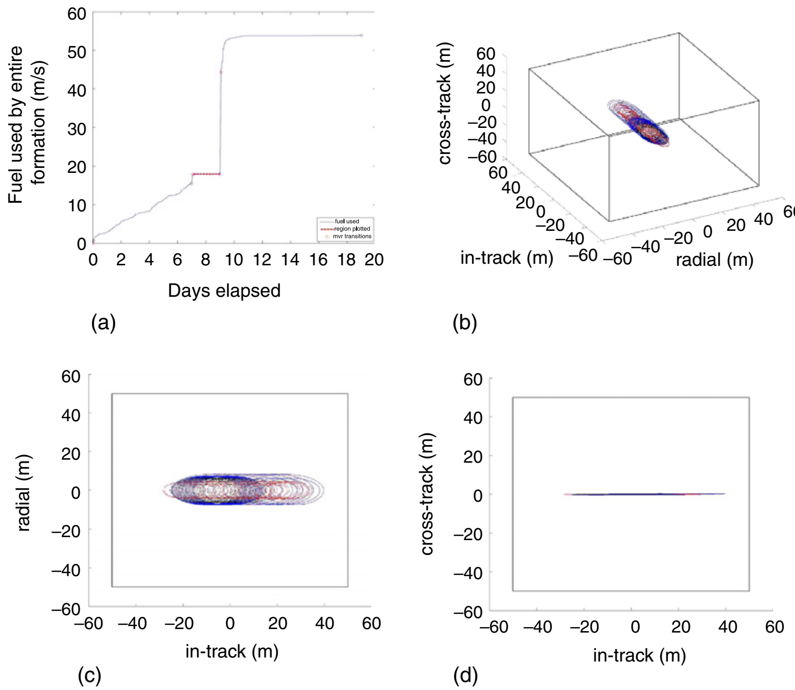

FIGURE 13.3 Simulation #1: Station-keeping in a passive aperture formation (500 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

controlled in the center of the error box. These results are counterintuitive for several reasons: (i) the addition of constraints to an optimization would typically be associated with increased fuel cost; and (ii) expending fuel to keep a spacecraft in a small box would usually be more expensive than allowing the same spacecraft to stay in a large box. The fact that the fuel costs were reduced is not in conflict with the fact that individual optimizations should have increased cost; the fuel costs measured are the steady-state closed-loop levels of consumption, as opposed to expected maneuver costs. Also, note that the fuel use in Sim 4 remained high for a period after the maneuver to the 5 km configuration before settling into a steady state, whereas the Sim 5 fuel use settled almost immediately. This is likely because the terminal invariance condition forces the trajectories to enter closed ellipses. In Sim 4, where no invariance condition was specified, the spacecraft did not start in a closed ellipse, but did eventually enter one and did not exit for the duration of the simulation. This is because even without requiring a closed-ellipse, the optimization recognizes that there is no need to expend fuel to change a trajectory, which will result in no constraint violations. An almost identical pattern is visible between Sim 1 (which does not impose terminal invariance) and Sim 3 (which uses terminal invariance).

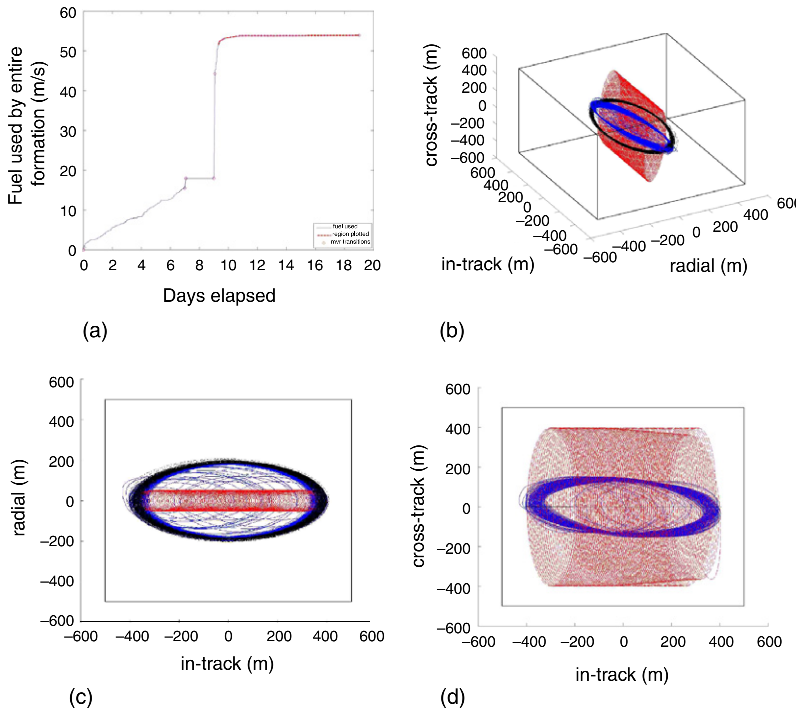

FIGURE 13.4 Simulation #1: Station-keeping in a passive aperture formation (5000 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

Terminal invariance raises the fuel cost between Sims 1 & 3 and Sims 4 & 6 for the 50 m and 500 m configurations. At the 50 m level, this effect is complicated by the fact that the increase is small and Sims 3 and 6 both also restrict thrusting at apogee. Sims 1 and 4 were found to not be feasible with thrust restrictions. Sim 3 was feasible most of the time, but Fig. 13.8 shows that there were a number of instances in which error box violations occurred. It should be noted that Sim 3 was the only one of the simulations that used the original MELCO mission control specifications and succeeded in remaining stable. All of the other simulations used modifications that enlarged the error boxes at some or all times. Sim 6 used a $5\times10\times5$ meter error box for the 50 m configuration and did not require any error box violations. At the 500 m level, the addition of the invariance constraint causes an almost two orders of magnitude fuel consumption increase. It appears that this is because almost no fuel is used without invariance as a constraint, but the resulting trajectories are naturally invariant (see Figs. 13.3, 13.6, 13.12 and 13.15) with slight semimajor axis mismatches, which cause the ellipses to travel inside the error box. The trajectories with invariance (see Figs. 13.18 and 13.19) take on similar shapes but expend fuel to cancel any real or perceived drift introduced through navigation errors with each successive optimization.

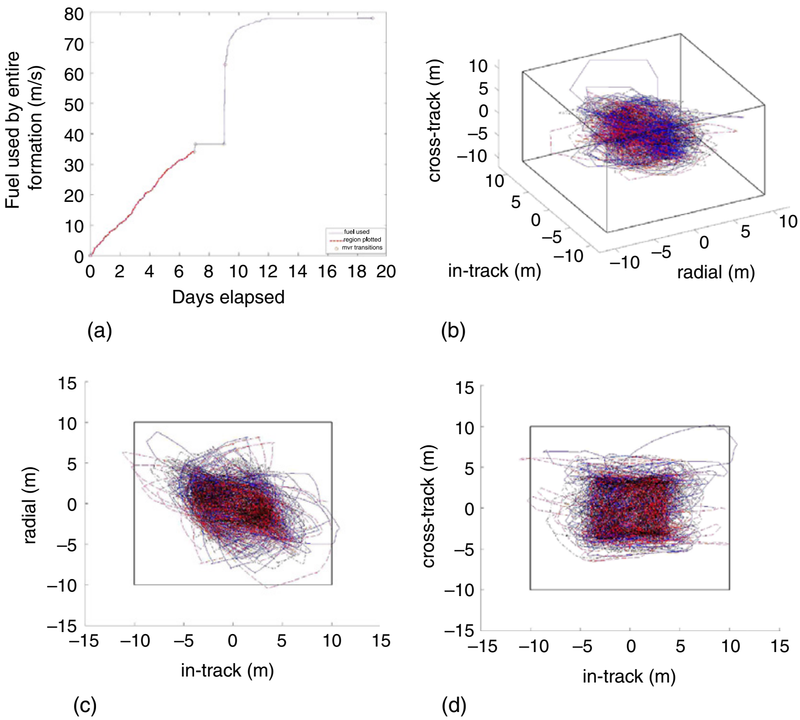

FIGURE 13.5 Simulation #2: Station-keeping in a passive aperture formation (50 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

Overall, it appears that the most consistent combination of successful constraint satisfaction and low fuel use came from the simulations using the invariance constraints (Sims 3 and 6). These simulations used the observation thrusting restriction and the specified error box size for Sim 3 and a slightly larger error box for the 50 m configuration in Sim 6. Thus, the 500 m and 5 km configurations were the same in Sims 3 and 6 and, as would be expected, they have nearly identical fuel use. The only difference is at the 50 m level, where a 5 m increase in the along-track error box size decreases the fuel use by more than a factor of 4.

The fuel numbers for the maneuvers between formation types are included for completeness, but cannot be used to draw conclusions because of the stochastic nature of the simulations. Many additional simulations would need to be run and averages examined. This fact does not reduce the validity of the conclusions regarding the fuel use for station-keeping, because in those situations the fuel use tends to reach a steady state, as is evidenced by the linear (fixed slope) rates of fuel use in the Δv plots in this section.

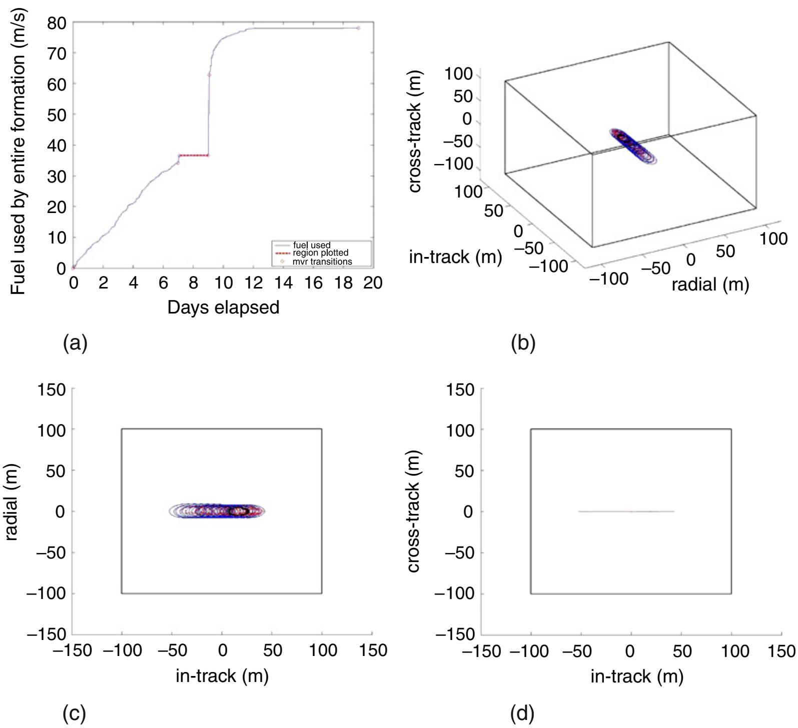

FIGURE 13.6 Simulation #2: Station-keeping in a passive aperture formation (500 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

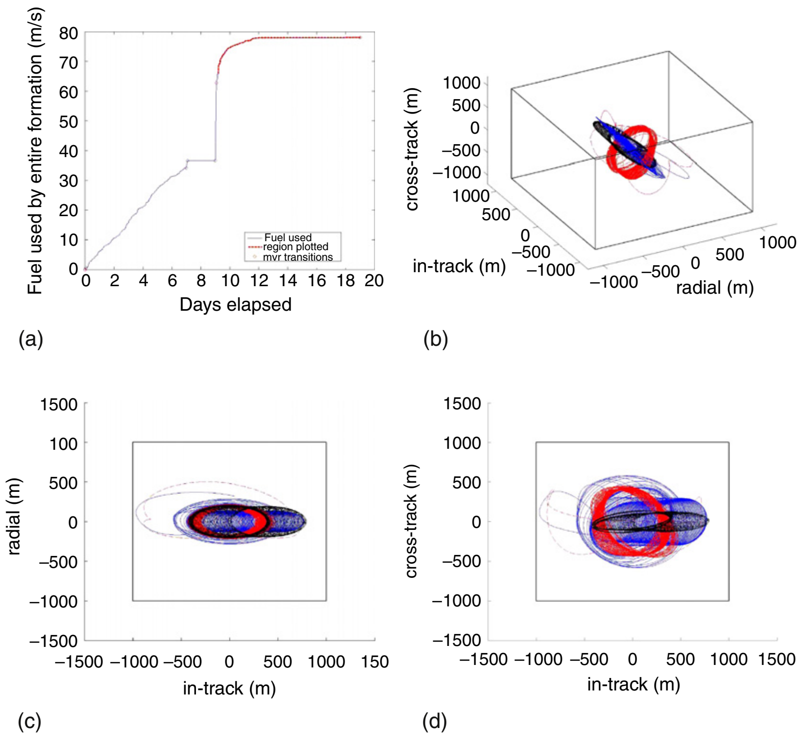

FIGURE 13.7 Simulation #2: Station-keeping in a passive aperture formation (5000 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

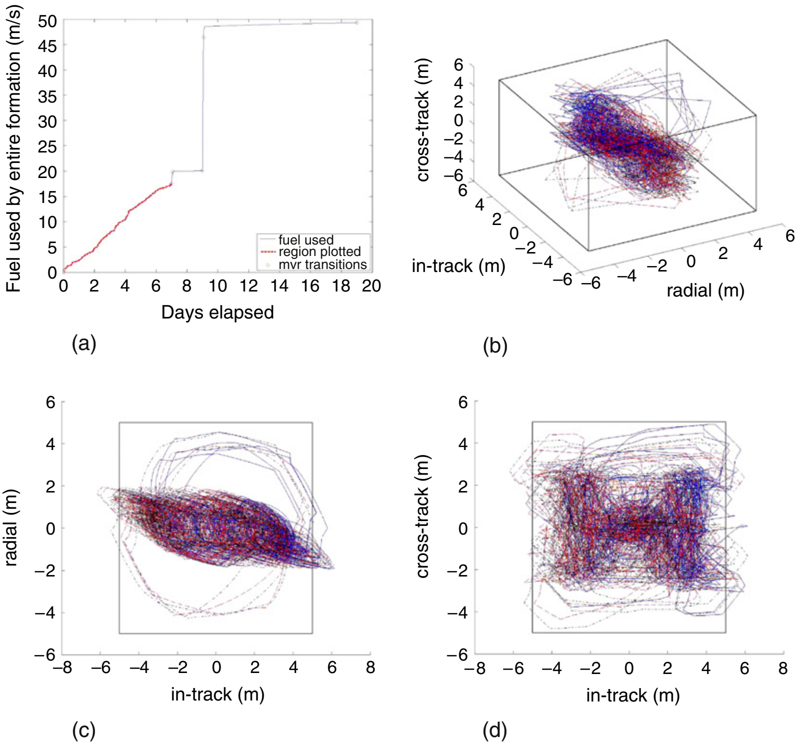

FIGURE 13.8 Simulation #3: Station-keeping in a passive aperture formation (50 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

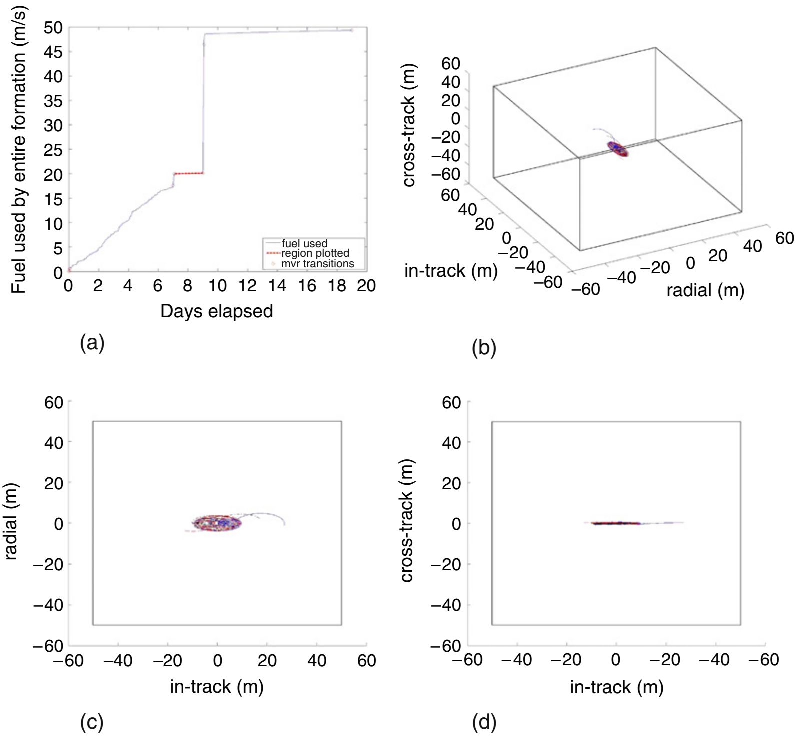

FIGURE 13.9 Simulation #3: Station-keeping in a passive aperture formation (500 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

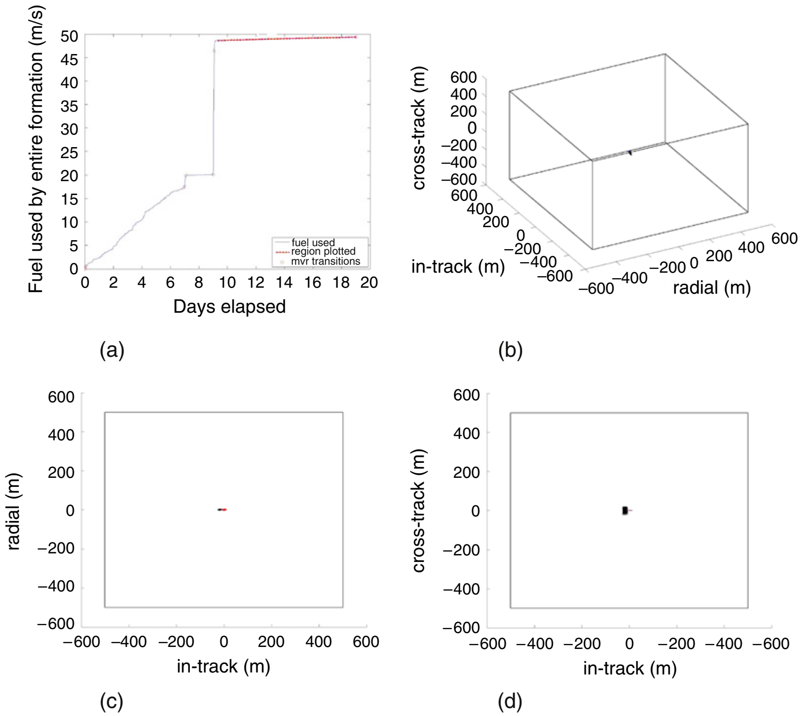

FIGURE 13.10 Simulation #3: Station-keeping in a passive aperture formation (5000 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

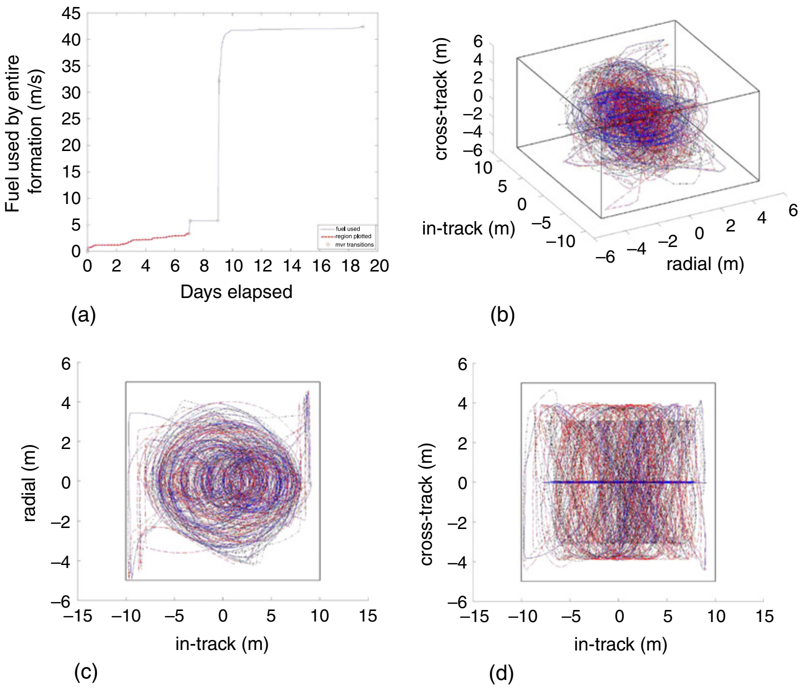

FIGURE 13.11 Simulation #4: Station-keeping in a passive aperture formation (50 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

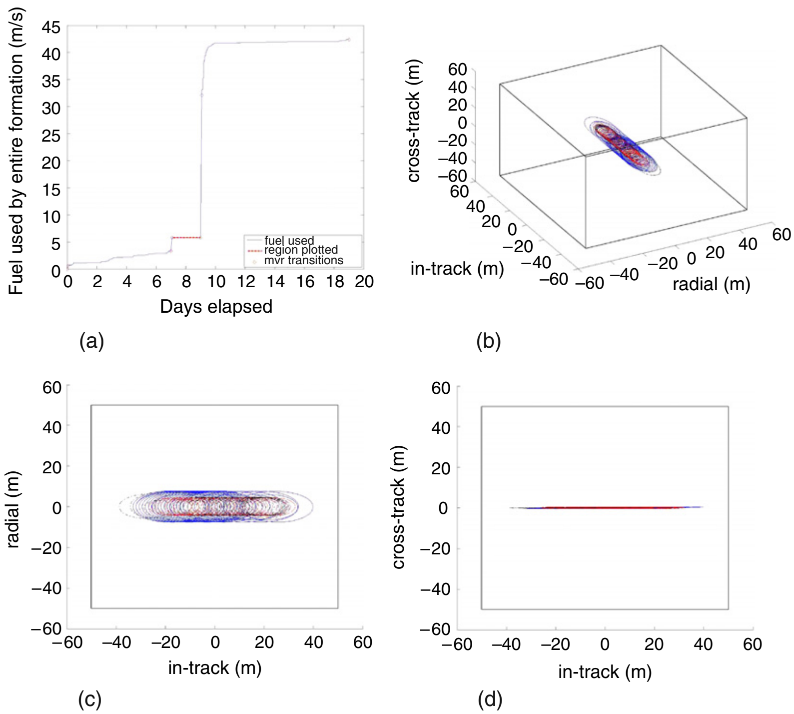

FIGURE 13.12 Simulation #4: Station-keeping in a passive aperture formation (500 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

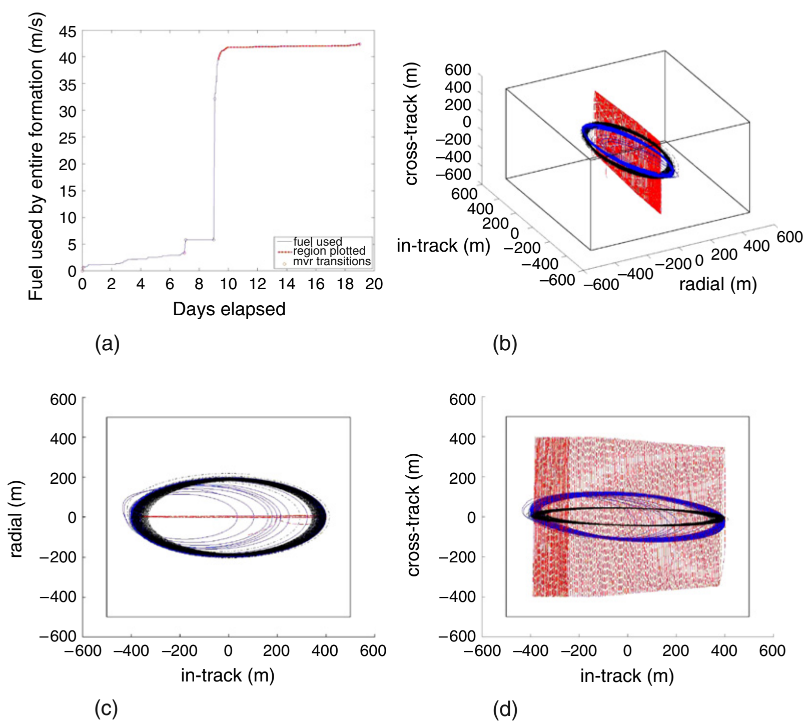

FIGURE 13.13 Simulation #4: Station-keeping in a passive aperture formation (5000 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

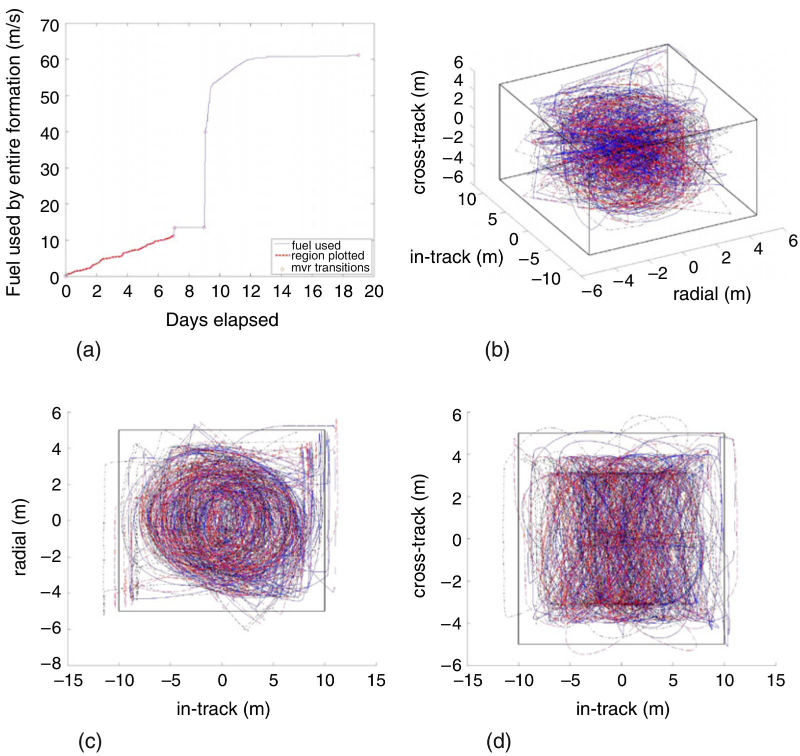

FIGURE 13.14 Simulation #5: Station-keeping in a passive aperture formation (50 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

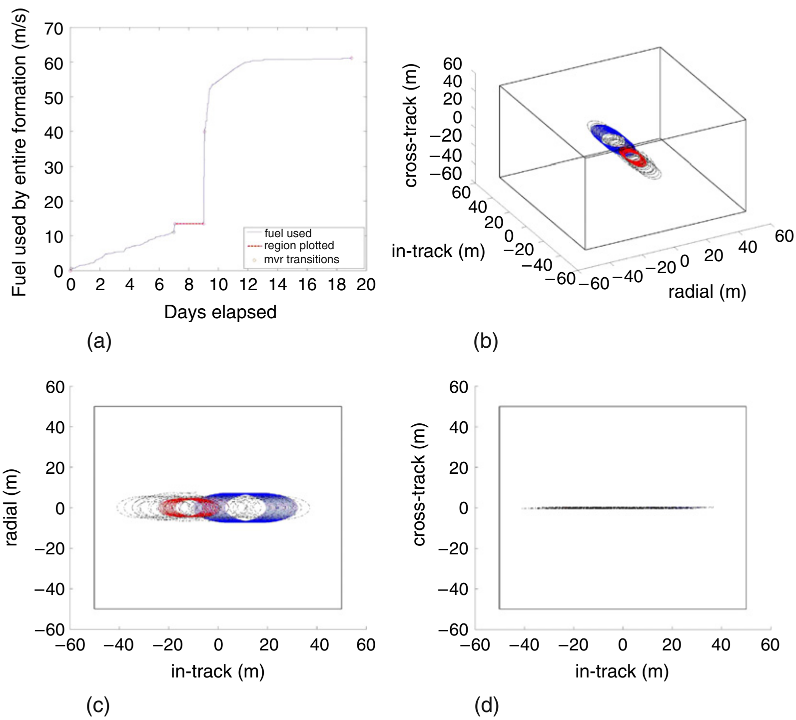

FIGURE 13.15 Simulation #5: Station-keeping in a passive aperture formation (500 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

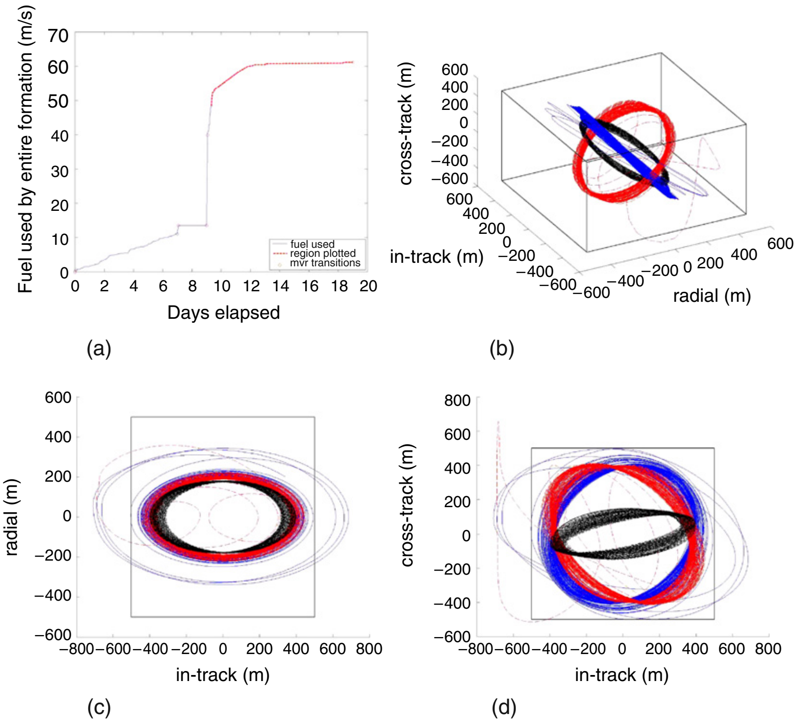

FIGURE 13.16 Simulation #5: Station-keeping in a passive aperture formation (5000 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

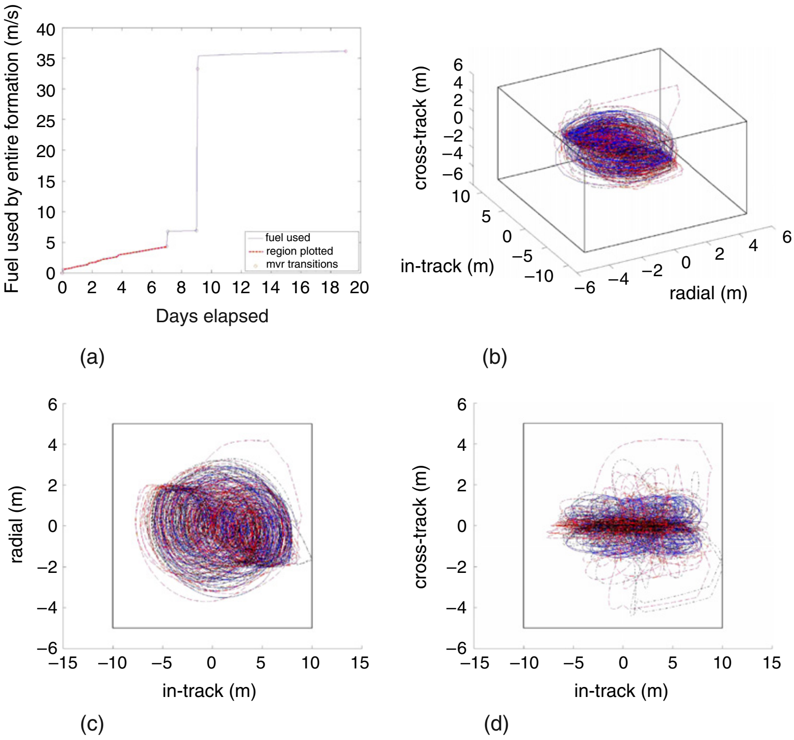

FIGURE 13.17 Simulation #6: Station-keeping in a passive aperture formation (50 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

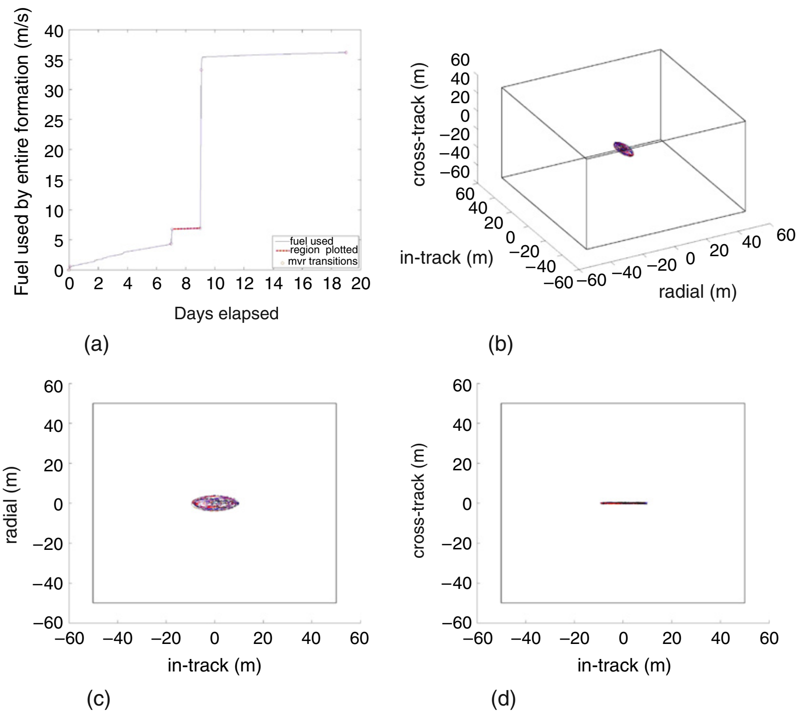

FIGURE 13.18 Simulation #6: Station-keeping in a passive aperture formation (500 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

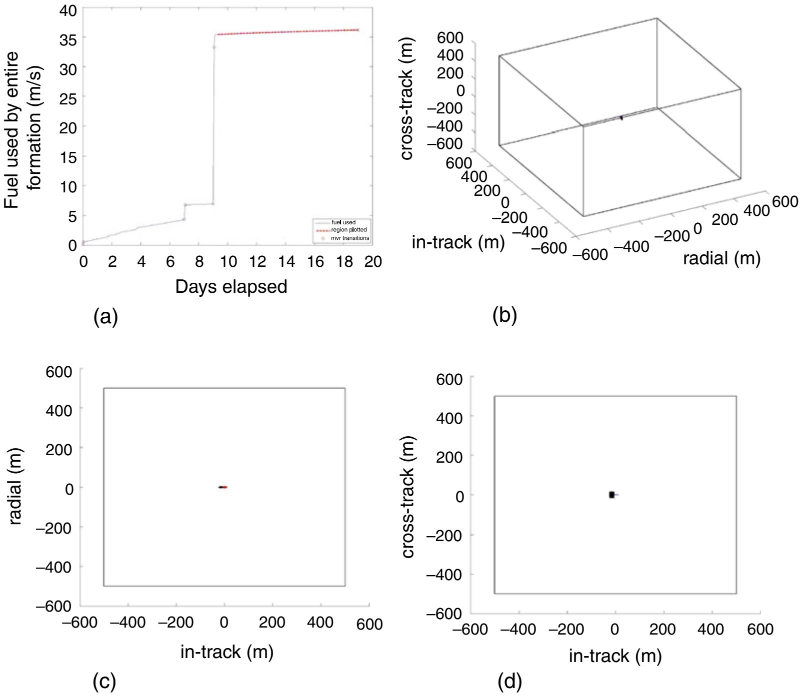

FIGURE 13.19 Simulation #6: Station-keeping in a passive aperture formation (5000 m). (a) Fuel use. (b) 3D LVLH trajectory. (c) LVLH trajectory: Along-track/radial. (d) LVLH trajectory: Along-track/cross-track.

SUMMARY

Multiple high-fidelity simulations of a reference formation flying mission were performed successfully. These simulations demonstrated, with the most rigorous tools available, that a realistic LEO reference mission can be feasibly controlled and made to meet all of its performance constraints. The simulations indicated that the fuel cost of reserving passive observation time can be significant. Also, it appears that the fewest constraint violations and most consistently low fuel usage occur when a terminal-invariance constraint is added to the control formulation. It was demonstrated that small increases in error box size can result in large fuel savings.