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Relative attitude dynamics and control for a satellite inspection mission

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ABSTRACT

The problem of conducting an inspection mission from a chaser satellite orbiting a target spacecraft is considered. It is assumed that both satellites follow nearly circular orbits. The relative orbital motion is described by the Hill–Clohessy–Wiltshire equation. In the case of an elliptic relative orbit, it is shown that an inspection mission is feasible when the chaser is inertially pointing, provided that the camera mounted on the chaser satellite has sufficiently large field of view. The same possibility is shown when the optical axis of the chaser's camera points in, or opposite to, the tangential direction of the local vertical local horizontal frame.

For an arbitrary relative orbit and arbitrary initial conditions, the concept of relative Euler angles is defined for this inspection mission. The expression of the desired relative angular velocity vector is derived as a function of Cartesian coordinates of the relative orbit. A quaternion feedback controller is then designed and shown to perform relative attitude control with admissible internal torques. Three different types of relative orbits are considered, namely the elliptic, Pogo and drifting relative orbits. Measurements of the relative orbital motion are assumed to be available from optical navigation.

1. Introduction

The study of relative motion between satellites has generally focused on the relative orbital dynamics for applications such as formation flying with two or more satellites. Relative attitude control has relatively received less attention. The problem under consideration is the case of an inspection mission, with a chaser satellite orbiting a target spacecraft. A number of inspection experiments were conducted on small satellite missions such as BX-1 (inspection of ShenZhou VII spaceship, see Ref. [1]), Snap-1 (inspection of Tsinghua-1, see Refs. [2,3]), Ikaros (see Ref. [4]) and Prisma (see Ref. [5]) but these were time limited experiments with little focus on relative attitude control, which was not required to ensure that the target would appear in the field of view of the chaser for limited time.

The relative attitude control of a chaser satellite orbiting a target spacecraft was considered in Ref. [6] for the case of a 2:1 elliptic relative orbit without drift and without out of plane motion. Ref. [6] did not show the attitude response, did not demonstrate convergence to zero of the torque and angular velocities and was interested in underactuated attitude control, which is not a problem under consideration here.

Relative attitude control was also considered in Ref. [7], but the focus was on a particular practical scenario for an inspection mission using optical navigation. More precisely, relative attitude control was considered during the phase that shortly precedes docking between the two satellites, when the chaser satellite is moving towards the

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target. In this case, the desired attitude of the chaser simply coincides with the current attitude of the target, which is clearly not the general case. For a general inspection scenario, the optical axis of the chaser's camera has to point towards the target while orbiting around it. Relative control was also considered in Refs. [8,9] for multiple satellites in a formation but the assumption that the desired attitude of the chaser coincides with the attitude of the target was also made in those references, where the objective was synchronization rather than inspection. In the special case of small rotations, we are also considering the energy optimal satellite attitude control subject to a fixed time [13].

In this paper, we consider different ways of conducting an inspection mission. First, we investigate the mission scenarios that only requires the chaser to be oriented to keep the target satellite within the field of view (FOV) of the chaser's instrument. The instrument is assumed, without loss of generality, to be an optical camera, which also serves, as in Ref. [10], to determine relative motion information. This type of inspection mission is shown to be feasible in two different ways:

- The natural dynamics of an inertially pointing chaser spacecraft are shown to allow for such an inspection mission to take place, assuming that the chaser's camera has a sufficiently large field of view.
- When the optical axis of chaser's camera is aligned with the tangential direction of the Local Vertical Local Horizontal (LVLH) frame, it is also possible, by adequate relative orbit design, to always keep the target within an adjustable field of view.

To address the case when pointing the camera within a relatively large field of view does not meet mission requirements, a relative attitude control problem is also considered here. The relative three axis attitude control problem is formulated for an arbitrary relative orbit (while a planar orbit was considered in [6]) that incorporates out of plane motion and can also accommodate drift and controlled relative orbits.

The attitude tracking is then demonstrated for an inclined elliptic relative orbit that includes out of plane motion (this orbit is obtained from natural dynamics), then attitude tracking is demonstrated for the Pogo orbit (also known as teardrop orbit, see Ref. [11]) that requires station keeping by applying a periodic Delta V manoeuvre and allowing the system to follow natural dynamics between those impulsive manoeuvres. The Delta V is applied when the orbit intersects itself, which is guaranteed to happen for the initial conditions provided in Ref. [11].

The paper is organized as follows. Section 2 describes the relative orbital motion, based on the Hill–Clohessy–Wiltshire (HCW) equation. Section 3 describes specific inspection mission scenarios that can be achieved within a FOV. The relative attitude control problem is formulated and solved in Section 4. Numerical simulations of 3-axis relative attitude control are given in Section 5, for three types of relative orbits (elliptic, Pogo and drifting orbits).

2. Relative orbital motion

The relative position vector between a chaser satellite and a target satellite is denoted as $\mathbf{r}_e$ and has components x,y,z in the HCW frame of the target satellite. The HCW frame is equivalent to the well-known Local Vertical Local Horizontal (LVLH) frame but with specific axes conventions. The components x, y and z of the HCW frame are respectively in the radial, tangential and out of plane directions. The relative motion in the case when the target satellite evolves on a circular orbit can be described by the Hill–Clohessy–Wiltshire (HCW) equation:

$$\ddot{x}-2n\dot{y}-3n^2x=T_x$$

$$\ddot{y}+2n\dot{x}=T_y$$

$$\ddot{z}+n^2z=T_z \tag{1}$$

where $n=2\pi/T$ represents the mean motion of the orbit, with orbital period T.

For now, the thrust is assumed to be zero: $T_x=T_y=T_z=0$. Note that the out of plane z motion is decoupled from the in plane motion in the (x,y) plane.

3. Inspection mission scenarios within a field of view

Before considering the relative attitude control problem, where the chaser is controlled to point precisely in the direction of the target satellite, we first explore possible mission scenarios allowing for an inspection mission to take place, not by pointing precisely in the desired direction but by having a camera with a sufficient field of view to always keep the target under inspection. Two scenarios are considered:

- Inspection mission using the natural dynamics of an inertially pointing chaser satellite;
- Inspection mission where the optical axis of chaser's camera is controlled to be aligned with the tangential direction of the HCW frame.

3.1. Exploiting natural dynamics for an inspection mission

We seek to express the relative position vector $\mathbf{r}_e$ in the Earth Centred Inertial (ECI) frame, where the $\vec{i}$ unit vector points in the direction of the point of Aries, the $\vec{k}$ unit vector points in the direction of the north pole and the $\vec{j}$ unit vector completes an orthonormal set. For simplicity, we assume that the orbit lies in the $\vec{j}$, $\vec{k}$ plane. It is also more generally possible to define unit vectors in the orbit plane, perpendicular to the momentum vector.

We consider here the special case when the initial conditions of the HCW equations are adjusted to make the relative orbit (in plane) consist of a 2:1 ellipse in the Local Vertical Local Horizontal (LVLH) frame. This is obtained from the following initial conditions:

$$y_0=\frac{2}{n}\dot{x}_0$$

$$\dot{y}_0=-2nx_0 \tag{2}$$

Note that the relative orbit has period T, the same as the circular target orbit.

The relative position vector of the Hill–Clohessy–Wiltshire equation can be seen as the result of introducing a relative eccentricity about the circular orbit of the target satellite. It was shown in Ref. [12] that a variation of the eccentricity ε away from the circular orbit leads to the following variation of the relative position vector in the ECI coordinate frame:

$$r_{ej} = -\frac{l^2}{\mu}(1+\sin^2 v)\delta\varepsilon\vec{j}$$

$$r_{ek} = -\frac{l^2}{\mu}(\cos v \sin v)\delta\varepsilon\vec{k} \tag{3}$$

where l denotes the magnitude of the momentum vector, μ is the earth's gravitation parameter and v is a relative true anomaly angle.

From the above equations, the relative position vector describes a circle in the ECI frame, which after rescaling to get a radius of 1, can be found to be centered about the point (0,1.5) in the (j,k) plane. The relative position vector (scaled as explained) is shown in Fig. 1, in which we can consider the target satellite to be at the origin, while the chaser forms the circle in the ECI frame.

It is assumed that the chaser spacecraft points inertially and such that the optical axis of the instrument points opposite to the $\vec{k}$ inertial direction. This is feasible for an inertially pointing satellite because the rotation matrix from body to camera frame is generally constant. It can then be shown by calculating the maximum ratio between the components r_{ej},r_{ek} in Eq. (3) and from Fig. 1 that the chaser can observe the target as long as it is equipped with an instrument having a minimum field of view (FOV) of

$$FOV_{min} = 2 \times \arctan(\tfrac{1}{3}) = 2 \times 18.435 \simeq 36.87^\circ$$

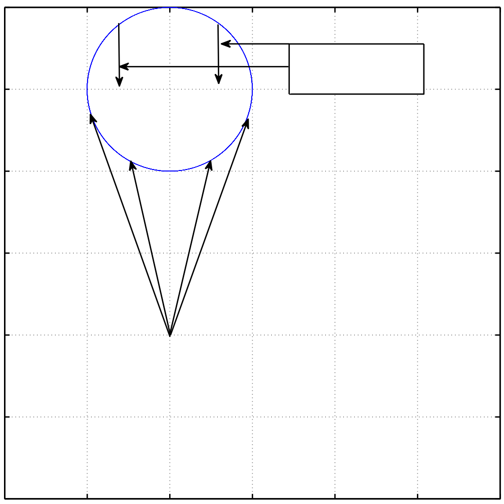

Fig. 1. Relative positions in ECI frame.

In other words, an inspection mission can theoretically be conducted with a chaser spacecraft in inertial pointing mode, provided that the inspector satellite is equipped with a camera having the aforementioned minimum FOV.

The angular off-pointing between the instrument on the inspector and it's target will be guaranteed to never exceed half of that maximum FOV angle as shown in Fig. 2. Note that the chaser has to be initially pointing in the right inertial direction towards the target. However, at all later times, no control effort is required to keep the target within the field of view.

3.2. *Inspection mission with fixed orientation in the LVLH frame*

In this subsection, we consider the conventional scenario when the attitude of the chaser satellite is controlled in the LVLH frame, which would in particular be the case for normal operations in a typical nadir pointing mode.

Assuming the elliptic relative orbit obtained from Eq. (2), it would not be possible to conduct an inspection mission in the inertial pointing mode from any point on the orbit, since even with a theoretical instrument having 180° FOV, the chaser would only keep the target under inspection whenever the inspector is further from earth than the target, which only happens during half the relative orbit T.

A better relative orbit design is obtained in this case by adjusting the initial conditions to shift the center the ellipse to a fixed point on the y-axis and away from the origin. In other words, the chaser would orbit around a point with constant phase offset with respect to the target. This solution is feasible by carefully adjusting the initial relative position and velocity vectors.

In this case, it is possible to always keep the target under inspection with a camera pointing, positively or negatively, in the tangential direction and having sufficient FOV. This relative orbit can be designed, as shown in

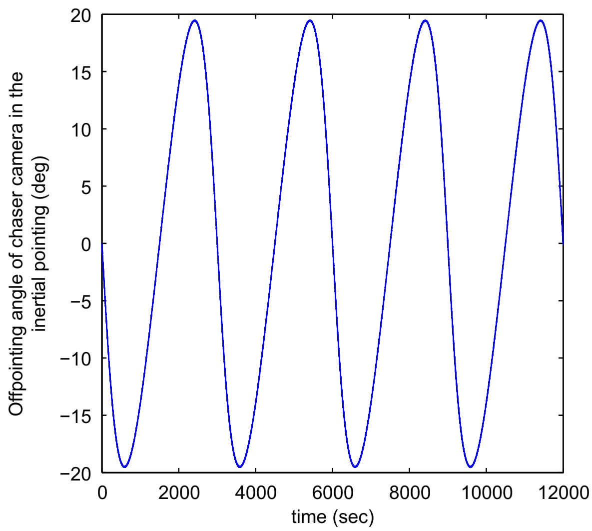

Fig. 2. Time history of the off-pointing angle.

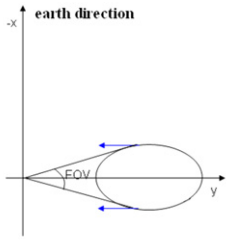

Fig. 3. Inspection mission possibility in a nadir pointing case.

Fig. 3. The minimum required FOV is then given by:

$$FOV_{min} = 2 \times \arctan\left(\left|\frac{x_0}{y_0}\right|\right)$$

where $\dot{y}_0$ is directly obtained from Eq. (2) and the orbit is shifted as shown in Fig. 3 by setting $\dot{x}_0 = 0$, with y_0 nonzero. The chaser should then point in the tangential $+y$-axis direction of the HCW frame if $y_0 < 0$ or in the $-y$ inverse tangential direction of the HCW frame if $y_0 > 0$. The latter scenario is considered in Fig. 3.

The optical axis of the camera is often designed to be a principal axis of the satellite. With the aforementioned y-axis shifted 2:1 ellipse, the required camera orientation for in orbit inspection can in particular be obtained from a nadir pointing satellite with a camera mounted such that the optical axis is aligned with the X (or roll) spacecraft body axis. An advantage of nadir pointing is the possibility or real time imaging of the target during a pass over a ground station. It is also feasible with a $+90^\circ$ pitch orientation of the satellite if the camera is mounted to align the optical axis with the Z (or yaw) body axis of the satellite.

Another possibility for in-orbit inspection from a nadir pointing chaser is to consider an adjustable Pogo or teardrop orbit (similar to the one shown in Fig. 6). The optical axis would again have to be aligned with the yaw body axis of the satellite. The difference from Fig. 6 is that the teardrop should be designed in the half plane where $y > 0$. This is always possible, because a new Pogo orbit can always be obtained from an existing one by symmetry about the y-axis of the HCW frame. This type of orbit is however less attractive in a nadir pointing mode than the shifted 2:1 ellipse because of the need for relative orbit maintenance.

An inspection mission is therefore conceptually feasible by equipping the inspector with a suitable instrument, whether the chaser is inertially pointing or has a fixed orientation in the LVLH frame.

4. Relative attitude control

In Section 3, the use of natural dynamics or nadir pointing was shown to be possible options for an inspection mission when the chaser is required to keep the target within a relatively large field of view. Also, the relative orbital motion was assumed in both cases to be planar. Relative or target tracking attitude control would be necessary not only for nonplanar orbits but also in the case of small satellites with restrictions on the size of the camera and on the camera field of view.

In this section, we formulate a three axis relative (or target tracking) attitude control problem for an arbitrary relative orbit, expressed in the HCW frame. In the following, we take the results of Ref. [6] further by allowing for out of plane motion, which is likely to be required in an inspection mission to observe a target satellite from various relative inclinations. In fact, we allow for any relative orbit, with or without drift and with or without orbit control, as long as the closed loop orbit can be expressed in the HCW frame.

4.1. Coordinates of relative attitude

Prior to formulating the relative attitude control problem, the concept of relative attitude has to be clearly defined. To this end, spherical coordinates are used to represent the relative position of the chaser satellite with respect to the target spacecraft. Fig. 4 is used to illustrate the desired relative attitude in the HCW frame. In this figure, the Chaser satellite is located at the point P, while the target spacecraft is at the origin denoted O. The HCW frame has unit vectors $X_{TLO}, Y_{TLO}, Z_{TLO}$, where the subscript TLO stands for the Target's Local Orbit.

A sequence of two rotations ϕ,θ is sufficient to represent the relative position of the chaser satellite with respect to the target. These angles can be calculated from the relative position vector in HCW frame:

$$\theta = \arctan\left(\frac{y}{x}\right) \tag{4}$$

$$\phi = \arcsin\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \tag{5}$$

It is then straightforward to obtain expressions for the time derivatives of θ and ϕ.

The desired pointing orientation of the chaser with respect to the target is also shown on Fig. 4 and depends

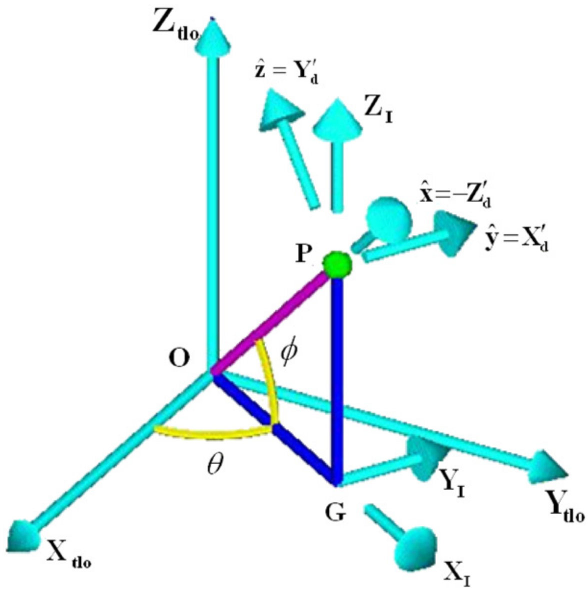

Fig. 4. Relative positions in ECI frame.

on the angles θ,ϕ. The notation $\hat{x},\hat{y},\hat{z}$ on the diagram stands for the unit vectors following the sequence of rotations θ,ϕ of the HCW frame. These are not to be confused with the italicized x,y,z coordinates of the relative orbit. In Fig. 4, the unit vectors X'_d,Y'_d,Z'_d are the principal body axes of the chaser's camera frame. For simplicity, we consider in this section that the constant rotation matrix from the chaser's body frame to the chaser's camera frame is the identity matrix. This implies that the camera axis is one of the body axes of the chaser, which is chosen to be in the Z-body axis direction of the satellite. This assumption is not too restrictive because the camera axis is often chosen to coincide with a body axis. The unit vectors X'_d,Y'_d,Z'_d are chosen to respectively represent the desired relative roll, pitch and yaw (or camera) axes of the chaser satellite. The relative orientation of the body axes X, Y and Z of the satellite with respect to the (X'_d,Y'_d,Z'_d) frame represents the relative attitude to be controlled to zero.

More generally, the camera may have an orientation with respect to the chaser, but this orientation would still have to be fixed. The roll and pitch axes are chosen in an analogous way to the planar relative orbit case and to form an orthonormal reference frame. The (X'_d,Y'_d,Z'_d) reference frame was also chosen to make the chaser's camera point towards the target when roll and pitch angles are both zero.

The relative attitude is the orientation of the body axes of the chaser with respect to the chaser's desired frame X'_d,Y'_d,Z'_d. Different attitude parameterizations are possible for this relative rotation but quaternion modeling is used here to allow for a singularity free representation of the attitude.

The relative angular velocity of the chaser's desired frame with respect to the target satellite's local orbit is a particularly important quantity. Expressed in the $(\hat{x},\hat{y},\hat{z})$ frame obtained from the sequence of two rotations θ,ϕ, the desired angular velocity is given by:

$$\omega_{TLO}^{des} = \dot{\theta}(\sin(\phi)\hat{x}+\cos(\phi)\hat{z})-\dot{\phi}\hat{y} \tag{6}$$

To express the angular velocity using the usual attitude convention where unit vectors (X'_d,Y'_d,Z'_d) define the relative roll, pitch and yaw axes respectively, a further transformation is required: $X'_d=\hat{y}$, $Y'_d=-\hat{z}$, $Z'_d=-\hat{x}$. The analytic expressions of $\dot{\phi}$, $\dot{\theta}$ are easily derived from Eqs. (4) and (5). Note that with $\phi=0$, we get $\dot{\theta}\hat{z}=-\dot{\theta}Y'_d$, which is the planar relative angular velocity of Ref. [6]. It is taken negatively with the attitude convention because the Y-axis of its LVLH frame is in the direction of the orbit anti-normal.

4.2. Relative attitude control by quaternion feedback

We assume that the target satellite is uncontrolled. Euler's dynamic equation of motion for the target satellite is then given by:

$$\mathbf{I}_T\dot{\boldsymbol{\omega}}_I^T = -\boldsymbol{\omega}_I^T \times (\mathbf{I}_T\boldsymbol{\omega}_I^T) \tag{7}$$

where:

- $\mathbf{I}_T$ denotes the moment of inertia matrix of the target satellite
- ω_I^T represents the target satellite's angular velocity with respect to the inertial frame, expressed in the target body frame.

Euler's dynamic equation of motion for the chaser satellite, which is controlled on all three axes, is given by:

$$\mathbf{I_C}\dot{\boldsymbol{\omega}}_I^C = -\boldsymbol{\omega}_I^C \times (\mathbf{I}_C\boldsymbol{\omega}_I^C+\mathbf{h})+\mathbf{N}_c \tag{8}$$

where:

- $\mathbf{I}_C$ denotes the moment of inertia matrix of the target satellite;
- ω_I^C represents the Chaser satellite's angular velocity with respect to the inertial frame, expressed in the chaser's body frame;
- $\mathbf{N}_c$ represents the control torque vector of the satellite, which is generated by reaction wheels on all three axes;
- $\mathbf{h}$ represents the angular momentum vector of the reaction wheels.

We assume that the relative position coordinates x,y,z are measured and that the measurements are refined with an Extended Kalman Filtering (EKF). However, the implementation of this relative attitude control in conjunction with the EKF falls beyond the scope of this paper.

Given the angular velocity of the chaser with respect to the inertial frame, which is estimated from available measurements, it is possible to extract the angular velocity of the chaser satellite with respect to the desired frame:

$$\boldsymbol{\omega}_{des}^C = \boldsymbol{\omega}_I^C - \mathbf{A}_{des}^C\boldsymbol{\omega}_I^{des} \tag{9}$$

This quantity will have to be controlled to zero. The angular velocity of the desired frame with respect to the inertial frame is then given by:

$$\boldsymbol{\omega}_I^{des} = \boldsymbol{\omega}_{TLO}^{des} - \mathbf{A}_{TLO}^{des}\mathbf{A}_I^{TLO}\boldsymbol{\omega}_{TLO}^I \tag{10}$$

In Eq. (10), the desired angular velocity of the chaser with respect to the TLO frame is known as a function of x,y and z, while the rotation matrix from the TLO frame to

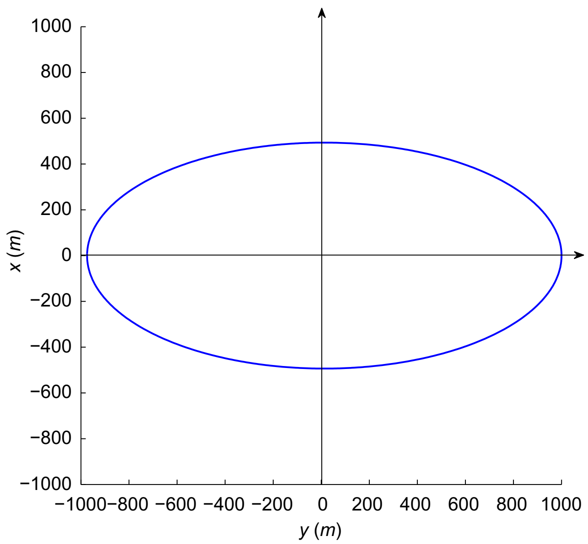

Fig. 5. Elliptic relative orbit.

the chaser desired frame is given by:

$$\mathbf{A}_{TLO}^{des} = \begin{pmatrix} \cos\phi\cos\theta & -\cos\phi\sin\theta & -\sin\phi \\ \sin\theta & \cos\theta & 0 \\ \sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{pmatrix} \tag{11}$$

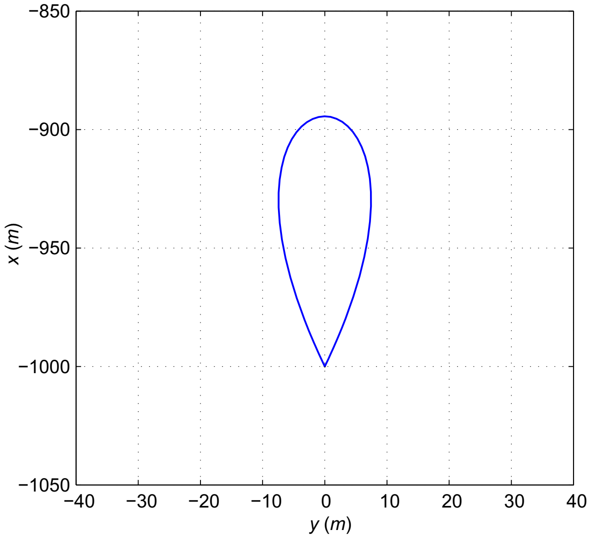

Fig. 6. Pogo or teardrop relative orbit.

The angular velocity of the inertial frame with respect to the TLO frame is simply given by $[0,n,0]^T$ and $\mathbf{A}_I^{TLO}$ is a pitch rotation matrix, where the pitch angle is obtained by integration over time of the constant angular velocity $\boldsymbol{\omega}_I^{TLO}$.

The relative attitude of the spacecraft has to be controlled to zero. Using the quaternion representation of attitude kinematics, the desired quaternion is $[0,0,0,1]^T$. The relative attitude quaternion kinematics are given by:

$$\dot{\mathbf{q}}_{des}^{C} = -\tfrac{1}{2}\,\boldsymbol{\omega}_{des}^{C} \times \mathbf{q}_{des}^{C} + \tfrac{1}{2} q_{4des}^{C}\boldsymbol{\omega}_{des}^{C}$$

$$\dot{q}_{4des}^{C} = -\tfrac{1}{2}\boldsymbol{\omega}_{des}^{C\,T}\mathbf{q}_{des}^{C} \tag{12}$$

where $\mathbf{q}_{des}^{C}$ denotes the vector part of the quaternion error and q_{4des}^{C} is the scalar real part of the quaternion error.

From Eqs. (12) and (9), both the quaternion and angular velocity errors are recovered. The chaser satellite can now be controlled to the desired orientation to inspect the target by the following feedback control law:

$$\mathbf{N}_c = -k_p\mathbf{q}_{des}^{C} - k_d\boldsymbol{\omega}_{des}^{C} \tag{13}$$

As in Ref. [7], a Lyapunov stability proof (for $k_p > 0, k_d > 0$) was deemed unnecessary here because this controller is known to work when the desired reference is inertial or local orbital and the fundamental problem here is the same, the only difference being the reference to be

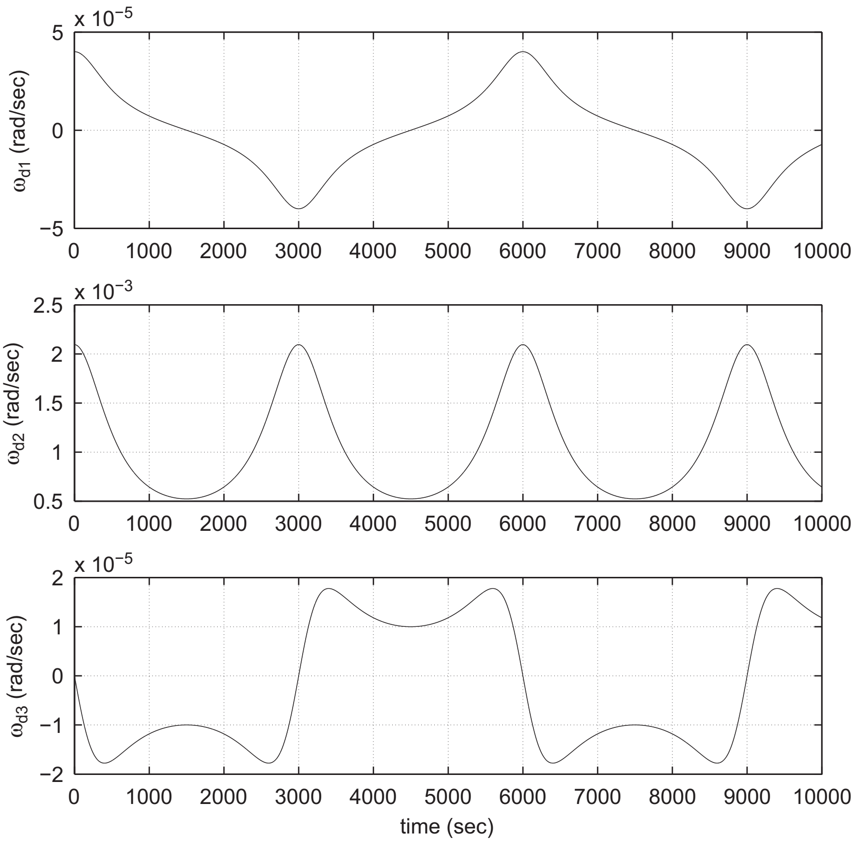

Fig. 7. Desired angular velocity of the chaser on an inclined elliptic relative orbit.

tracked. The numerical simulation section will demonstrate the acceptable efficiency of this controller for an inspection mission, for which the time scales of attitude control are small in comparison to the orbital period.

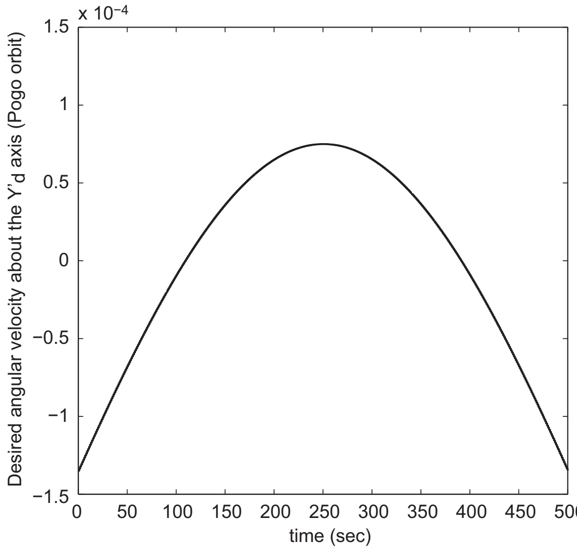

Fig. 8. Desired angular velocity of the chaser on a Pogo (teardrop) relative orbit during a Pogo period.

5. Numerical simulations

The parameters of a 50 kg microsatellite equipped with three reaction wheels are considered to model the inspector satellite. The moments of inertia of the principal axes are

$$I_1 = 7, \quad I_2 = 7, \; I_3 = 5 \; (\mathrm{kg\,m^2})$$

A realistic level for the maximum torque of the wheels is 10 m Nm. Considering a simple rest to rest maneuver for illustration purposes that the initial angular velocity of the chaser is

$$\omega^c_{d1}(0) = 0, \quad \omega^c_{d2}(0) = 0, \; \omega^c_{d3}(0) = 0 \; (\mathrm{rad\,s^{-1}})$$

The initial quaternion errors are

$$q^c_{d1}(0) = 0.1, \quad q^c_{d2}(0) = 0.1, \quad q^c_{d3}(0) = 0.1, \; q^c_{d4}(0) = 0.9849$$

A low earth circular orbit at 540 km altitude was assumed for the target spacecraft, which corresponds to an orbital period of approximately 100 min.

The relative orbit of the chaser with respect to the target satellite's local orbit is shown in Fig. 5 for the case of a 2:1 ellipse and in Fig. 6 for a Pogo or teardrop orbit.

The inclined 2:1 ellipse is obtained with $x_0 = 500$ m, $y_0 = 0$ m and the initial velocities are obtained from Eq. (2). The initial out of plane velocity is $\dot{z}_0 = -0.02$ m/s. The Pogo orbit is initialized by setting $x_0 = -1$ km, $y_0 = 0$ km, a Pogo period of 500 s and following the procedure used in Ref. [11].

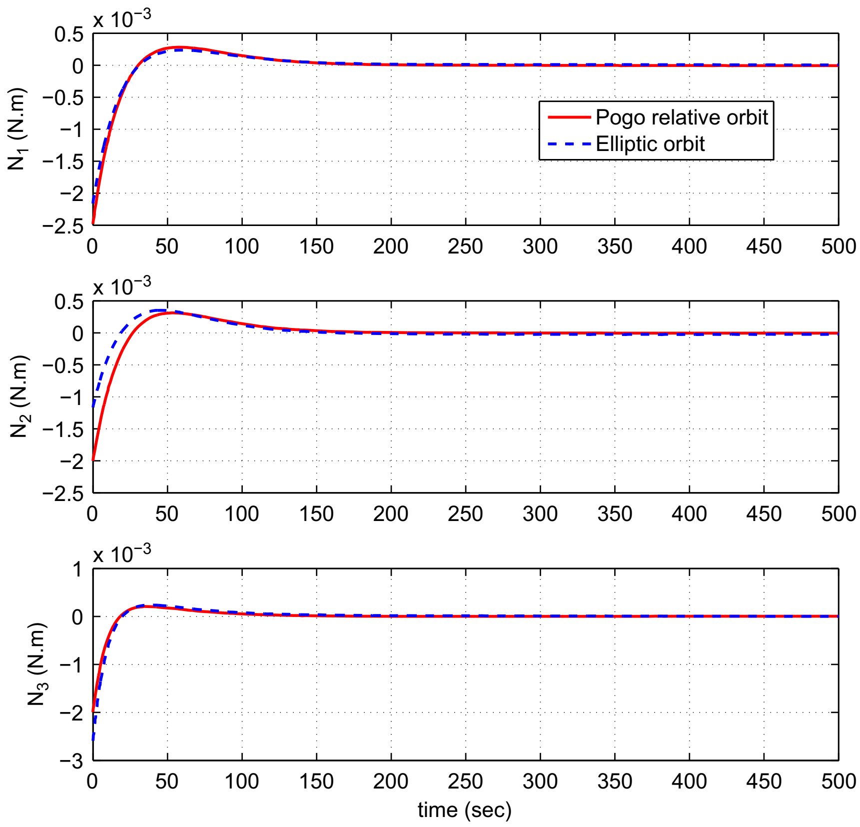

Fig. 9. Control torque vector on the chaser satellite.

The desired angular velocity of the chaser satellite with respect to the target satellite's local orbit is shown on Fig. 7 for an inclined elliptic orbit (with out of plane motion) and Fig. 8 for a Pogo orbit. The attitude convention whereby the out of place motion is in the Y-axis was used to refer this angular velocity. In the case of the elliptic orbit (Fig. 7), the second component stands for the 'planar' angular velocity and has period $T/2$ with an average equal to $-n$, where n is the mean motion and T is the corresponding orbital period. The other two components have period T and zero average and are required to achieve tracking in the presence of out of plane relative motion. In the case of the Pogo orbit (Fig. 8), out of plane motion was not included in the simulation.

Fig. 9 shows that the relative attitude control application is achievable by an admissible and smooth torque profile, for both elliptic and Pogo relative orbits. The torque profiles are similar, although the elliptic orbit requires less torque initially on the Y-axis and less overall integrated torque.

Fig. 10 shows the relative attitude control response for the elliptic and Pogo relative orbits. The settling time at $\pm 0.5^\circ$ on all three axes is 128.2 s for the elliptic orbit against 169.7 s with a Pogo orbit.

The attitude tracking is more precise for an elliptic relative orbit, despite an overall lower torque expenditure. However, in both cases the attitude is tracked within 2–3 min with admissible torque, without exceeding the assumed torque saturation level on all three axes.

Attitude tracking is also possible within a certain range along a drifting orbit. In this case, the desired angular velocity is shown on Fig. 12 to consist of damped oscillations. The orbit was initialized to create a drift along the y-axis of the HCW frame as shown in Fig. 11.

Among the three scenarios considered here (elliptic orbit, Pogo orbit and drift), the first two scenarios are the

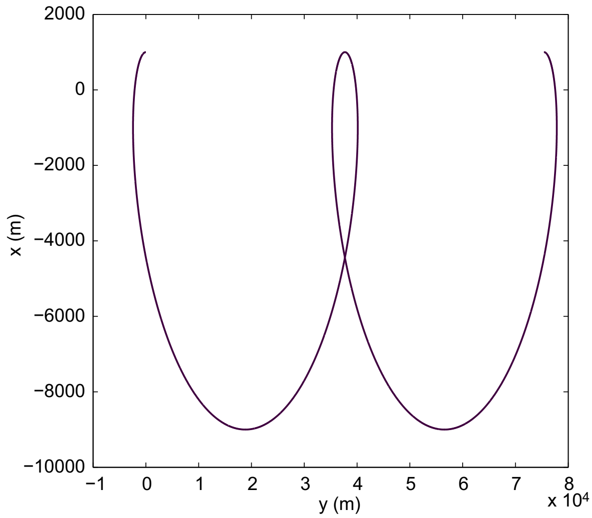

Fig. 11. Tangentially drifting relative orbit.

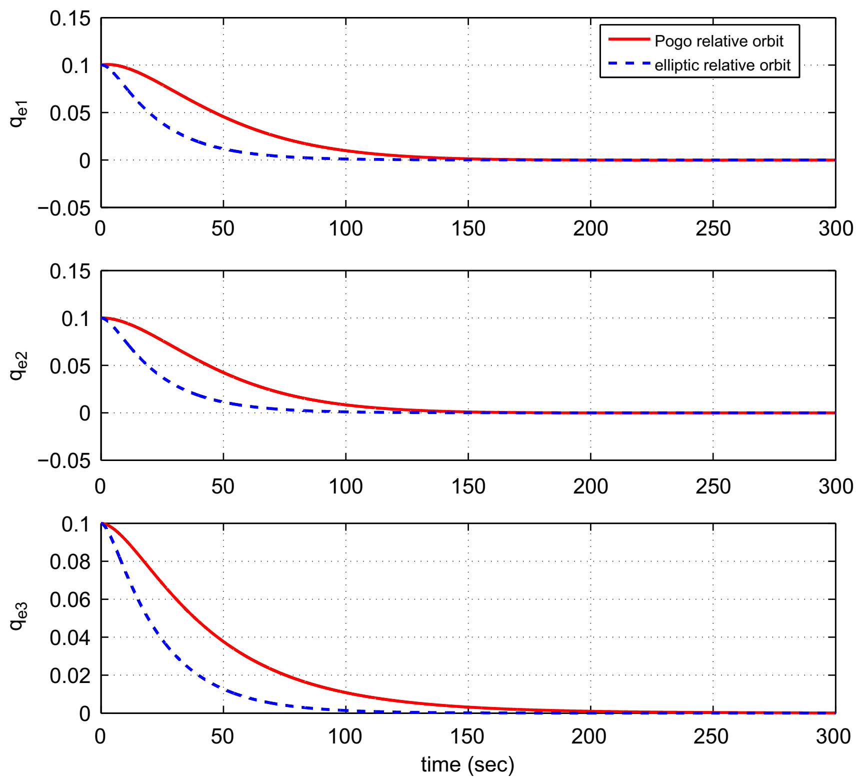

Fig. 10. Quaternion error from the tracking of the elliptic and Pogo orbits.

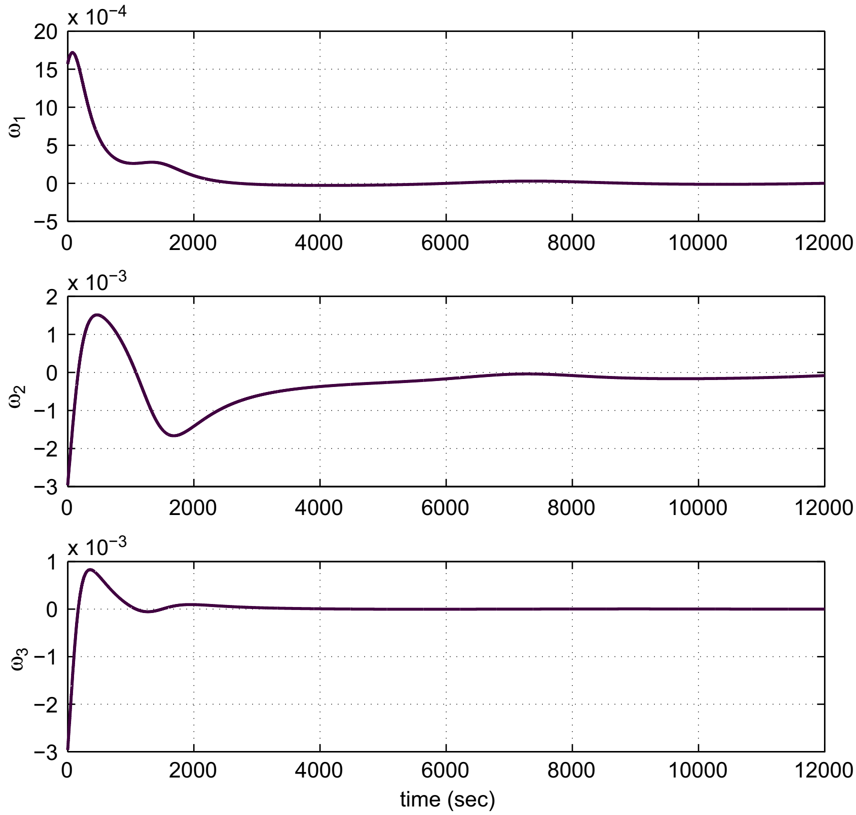

Fig. 12. Desired angular velocity of the chaser on a tangentially drifting relative orbit.

most advantageous because a drifting orbit can only be convenient for a limited time (a few orbits) and the optical navigation equipment will become less precise outside that range. An inspection mission on all three types of relative orbits has however been shown to be feasible with admissible torque.

The 2:1 ellipse is a good reference orbit to track for the in plane motion but the desired angular velocity should be computed to allow for out of plane motion to observe the target satellite from different relative inclinations. This orbit has the advantage of not theoretically requiring any orbit station keeping but there is a constraint that the period of the relative orbit has to be T (orbital period of the target satellite) and that the orbit has a ratio of 2:1 between semi-major and semi-minor axes of the ellipse. In practice, this type of orbit would require techniques to achieve relatively inexpensive station keeping in the presence of external disturbances, which are not included in this paper.

The Pogo orbit has the advantage that the size and shape of the teardrop can be specified as shown in Ref. [11]. Also, it makes it possible to keep one of the two satellites closer to earth at all times, which can be advantageous for certain inspection missions, such as those requiring real time imaging of the target when the chaser passes over a ground station with earth in the background, because the required pointing would be close to nadir pointing. The main disadvantage of the approach is that significant fuel consumption is required to generate the periodic velocity increments needed.

6. Conclusion

Different approaches to conducting an inspection mission with a chaser satellite orbiting a target spacecraft have been considered. An inspection mission which consists of keeping the target spacecraft within a FOV of the chaser's instrument was shown to be feasible in two ways: with the natural dynamics of an inertially pointing inspector satellite or with a fixed controlled orientation of the inspector satellite in the LVLH frame.

In the case of an inspection mission requiring finer relative pointing, the problem of relative attitude control of a chaser satellite orbiting a target in the Hill–Clohessy–Wiltshire frame was formulated in the case of an arbitrary relative orbit. The concept of relative attitude has been defined for such an inspection mission scenario. Relative attitude control has been shown to be feasible with a good settling time and admissible torque expenditure.

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