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Coupled dynamics modeling, control and simulation of a 6-DOF space-based manipulator system

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***Abstract*—Compared with the fixed-base manipulator system, kinematical and dynamical modeling and control of free-floating manipulator system (SBMS) becomes much more complicated. The primary cause is that there exists a deep dynamical coupling relationship between the floating base and the manipulator mounted on it. Firstly, in this article, generalized equation of motion of an n-DOF SBMS is built. Secondly, relevant attitudes of generalize coefficient matrixes, such as dynamical manipulability, are discussed. Then, in order to make the joints well follow a desired trajectory designed beforehand, a model-based control method is proposed aiming at a 6-DOF SBMS. Finally, a simulation test is conducted to demonstrate effectiveness of the method mentioned above.**

***Keywords*—coupled; dynamics; simulation; space manipulator; calculation**

I. INTRODUCTION

With the rapid development of aero-astronautical industry, the space robots also obtained a full development. When a space mission, which is planned to be finished by astronauts, is in baneful environment such as high radiation, large temperature difference and high vacuum, a space robot will be used to finish the work independently or semi autonomously (by receiving instructions from astronauts). Space robots are consist of various types. A special type, named space-based manipulator system (SBMS), is mainly discussed in this text. SBMS is consist of a floating base and manipulators mounted on it. SBMS is divided into three categories: the fixed-base robot system, position and attitude of whose base is controllable, the free-flying robot system, attitude of whose base is controllable, and the free-floating robot system, whose base is uncontrollable. So far, the former two systems are much more common used. But, some researchers consider that these two systems have some disadvantages such as dynamic impact phenomena, controller output saturation, disturbance of surrounding structures and fuel consumption [1]. So, in some specific cases, to avoid these negative things, researchers prefer to use the free-floating system to finish some works in space. Compared with the former two systems, dynamics and control of free-floating system becomes much more complicated. That is because it is an under-actuated system, characterized by the number of control actuators being less than the number of state variables.

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As we all know, there exists apparent dynamic coupling phenomenon between the floating base and the manipulators mounted on it. This kind of dynamic coupling may bring many negative effects. The important aspect is that complexity of dynamic modeling and control of the free-floating space manipulator system greatly increases. But, the dynamic coupling is not necessarily a bad thing. Based on the nonholonomic characteristics of the free-floating space manipulator system, Zhang etc. [2] presented a point-to-point planning approach to ensure that the manipulator end-effector can reach a desired point with the attitude of the floating base zero-disturbance. There are some other path planning methods to control the manipulators joint motion, which are benefit from the coupled dynamical properties.

Researchers have done lots of work on coupling dynamics and control of free-floating space manipulator systems. Flores-Abad etc. [3], from a global perspective, reviewed the development of space robotics and introduce the related technologies about on-orbit servicing. To better utilize those results and development, they also provide a literature review of the kinematics, dynamics, control and verification of space robotic systems for manned and unmanned on-orbit servicing missions. Sone and Nenchev mentioned the reactionless motion control problem of a free-flying space robot in practical tasks [4]. Aiming at the complex control problem of a free-flying robots, Moosavian and Papadopoulos [5] introduced a brief review of basic concepts of various algorithms about joint control. And basic issue of kinematics and dynamic modeling, trajectory planning and control strategies, cooperation of multiple arm and experimental studies were discussed. Huang etc. [6] investigated the problem of the dynamic balance control of multi-arm free-floating space robot during capturing an active object in close proximity. And a simulation were conducted and it verified the validity and efficiency of their control method. But dynamic singularity problem was not mentioned in their works. Mistry, Buchli and Schaal [7] presented an analytically correct solution by using an orthogonal decomposition to project the robot dynamics onto a reduced dimensional space and conducted a simulation test to demonstrate the feasibility and robustness of their approach. A kind of recursive implementation schemes of adaptive control for free-floating space manipulators was presented in works of Wang and Xie [8]. On the adaptive inverse dynamics for free-floating space manipulator suffering from parameter uncertainties or

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variations, Wang [9] introduce a new regressor matrix called the generalized dynamic regressor. And the simulation results demonstrate the correctness of the method mentioned in their works. Arisoy etc. [10] designed a high-order sliding mode controller for a one DOF flexible link space robotic arm with payload. And this method was demonstrated to be with a good performance on position precision. At last, external disturbances had been evaluated in real-time on an experimental setup.

In this article, a 6-DOF free-floating space manipulator system is discussed. It consisted of a floating base and a serial 6-DOF manipulator mounted on it. The coupling dynamics modeling and control issues are mainly focused on. The paper is organized as follows. Section I reviews the relevant background knowledge about the issues focused on by this paper, and proposes the target problem to be solved. Some notations and relevant backgrounds are introduced in Section II. Then, motion equation is built. Finally, relevant terms of inverse dynamics and dynamic manipulability are analyzed. In Section III, a model-based control method is provided. To demonstrate the effectiveness of this control law, a simulation test is designed shown in Section IV. Finally, Section V provides some conclusions.

II. Equation of Motion

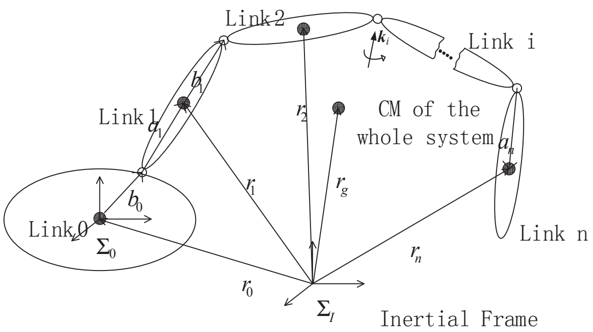

Fig. 1 Diagram of a n-DOF free-floating system

In order to show the generality of modeling process, the number of the manipulator is defined to be n. It means that the whole system can be considered as an n+1 bodies serial unrooted system. Here, we assume that each of link is rigid and connected by n independent active joints. The configuration schematic diagram is shown in Fig. 1. Here, we assume that (a) the whole system is in micro-gravity environment; (b) any external forces or torques is not exerted on the system.

The variables and relevant parameters referred in Fig. 1are defined as follows (Any vectors with its upper left corner be not marked is representation in inertial reference frame):

$\boldsymbol{q} \in \boldsymbol{R}^n$: Joint variables vector ($\boldsymbol{q} = [q_1, q_2, \dots, q_n]^{\mathrm{T}}$).

Σ_I : Inertial coordinate system.

$\boldsymbol{k}_i \in \boldsymbol{R}^3$: Unit vector in the same direction with rotational axis of the i-th joint.

$\boldsymbol{r}_i \in \boldsymbol{R}^3$: Position vector of CM (center of mass) of body i.

$\boldsymbol{r}_g \in \boldsymbol{R}^3$: Position vector of CM of the total system.

$\mathbf{a}_i, \mathbf{b}_i \in \boldsymbol{R}^3$: Distance vector between CM i to joint i and i+1.

$\boldsymbol{v}_i, \boldsymbol{\omega}_i \in \boldsymbol{R}^3$: Linear and angular velocity of the i-th body.

m_i : Mass of the i-th body.

$\boldsymbol{I}_i \in \boldsymbol{R}^{3\times 3}$: Inertia tensor of the i-th body with respect to its mass center.

A. Equation of Motion

Kinetic energy of the whole system include linear and angular energy,

$$\boldsymbol{T} = \frac{1}{2}\sum_{i=0}^{n}(\boldsymbol{\omega}_i^T \boldsymbol{I}_i \boldsymbol{\omega}_i + m_i \dot{\boldsymbol{r}}_i^T \dot{\boldsymbol{r}}_i). \tag{1}$$

Another expression can be obtained by simplifying.

$$\boldsymbol{T} = \frac{1}{2}[\boldsymbol{v}_0^T, \boldsymbol{\omega}_0^T, \dot{\boldsymbol{\phi}}^T]\begin{bmatrix} \left(\sum_{i=0}^{n} m_i\right)\boldsymbol{E} & \left(\sum_{i=0}^{n} m_i\right)\tilde{\boldsymbol{r}}_{0g}^T & \boldsymbol{J}_{Tb} \\ \left(\sum_{i=0}^{n} m_i\right)\tilde{\boldsymbol{r}}_{0g} & \boldsymbol{H}_b & \boldsymbol{H}_{bq} \\ \boldsymbol{J}_{Tb}^T & \boldsymbol{H}_{bq}^T & \boldsymbol{H}_q \end{bmatrix}\begin{bmatrix} \boldsymbol{v}_0 \\ \boldsymbol{\omega}_0 \\ \dot{\boldsymbol{\phi}} \end{bmatrix}. \tag{2}$$

Where,

$$\boldsymbol{H}_b = \sum_{i=1}^{n}(\boldsymbol{I}_i + m_i \tilde{\boldsymbol{r}}_{0i}^T \tilde{\boldsymbol{r}}_{0i}) + \boldsymbol{I}_0,$$

$$\boldsymbol{H}_{bq} = \sum_{i=1}^{n}(\boldsymbol{I}_i \boldsymbol{J}_{Ri} + m_i \tilde{\boldsymbol{r}}_{0i} \boldsymbol{J}_{Ti}),$$

$$\boldsymbol{H}_q = \sum_{i=1}^{n}(\boldsymbol{J}_{Ri}^T \boldsymbol{I}_i \boldsymbol{J}_{Ri} + m_i \boldsymbol{J}_{Ti}^T \boldsymbol{J}_{Ti}),$$

$$\boldsymbol{J}_{Ti} = \left[\boldsymbol{k}_1 \times [\mathbf{a}_1 + (\boldsymbol{r}_i - \boldsymbol{r}_1)], \cdots, \boldsymbol{k}_i \times [\mathbf{a}_1 + (\boldsymbol{r}_i - \boldsymbol{r}_i)], 0, \cdots, 0\right],$$

$$\boldsymbol{J}_{Ri} = \left[\boldsymbol{k}_1, \cdots, \boldsymbol{k}_i, 0, \cdots, 0\right]$$

$$\boldsymbol{J}_{Tb} = \sum_{i=1}^{n} m_i \boldsymbol{J}_{Ti}, \ \boldsymbol{r}_{ij} = \boldsymbol{r}_j - \boldsymbol{r}_i.$$

Here, vectors $\boldsymbol{v}_0$ $\boldsymbol{\omega}_0$ and $\dot{\boldsymbol{q}}$ are defined the state variables, which are not mutually independent. Because the momentum of the total system is conserved, there are [11]:

$$\begin{bmatrix} \left(\sum_{i=0}^{n} m_i\right)\boldsymbol{E} & \left(\sum_{i=0}^{n} m_i\right)\tilde{\boldsymbol{r}}_{0g}^T \\ \left(\sum_{i=0}^{n} m_i\right)\tilde{\boldsymbol{r}}_{0g} & \boldsymbol{H}_b \end{bmatrix}\begin{bmatrix} \boldsymbol{v}_0 \\ \boldsymbol{\omega}_0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_{Tb} \\ \boldsymbol{H}_{bq} \end{bmatrix}\dot{\boldsymbol{\phi}} = 0 \tag{3}$$

From formula (3) and a Lagrange function, here we consider that potential energy is zero, the equation of motion can be expressed as a function of $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$. It is:

$$\boldsymbol{H}_q^* \ddot{\boldsymbol{\phi}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = \boldsymbol{\tau}, \tag{4}$$

where $\boldsymbol{H}_q^*$ represents generalized inertial tensor for a free-floating space manipulator system, equivalent to the one of a fixed-base system. $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ represents a velocity-dependent item.

$$\boldsymbol{H}_q^* = \boldsymbol{H}_q - \begin{bmatrix} \boldsymbol{J}_{Tb}^T & \boldsymbol{H}_{bq}^T \end{bmatrix} \begin{bmatrix} \left(\sum_{i=0}^{n} m_i\right)\boldsymbol{E} & \left(\sum_{i=0}^{n} m_i\right)\tilde{\boldsymbol{r}}_{0g}^T \\ \left(\sum_{i=0}^{n} m_i\right)\tilde{\boldsymbol{r}}_{0g} & \boldsymbol{H}_b \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{J}_{Tb} \\ \boldsymbol{H}_{bq} \end{bmatrix}.$$

Equation (4) describes the rigid multi-body dynamics for a free-floating space manipulator system, which is also known as the inverse dynamics — if the pose, velocity and acceleration is given, the required generalized joint force vector can be obtained.

To better understand the matrix coefficients in (4), other works are discussed in subsection B below.

B. Relevant terms about the inverse dynamics

The generalized inertial tensor $\boldsymbol{H}_q^*$ is a function of the pose of the floating base and the joints. Here, we mainly discuss the influence of the generalized inertial tensor brought by changes of joint variable $\boldsymbol{q}$.

The diagonal terms $(\boldsymbol{H}_q^*)_{ii}$ represents the inertia seen by joint j, and the off-diagonal terms $(\boldsymbol{H}_q^*)_{ij}$, with $i \neq j$, describe coupling component of acceleration from joint j to the generalized force on joint i.

Taking joint 2 as an example, we explore that what the change of robot configuration will lead to.Fig.2 shows the relationship between joint variable q_2 and inertia of joint 1.

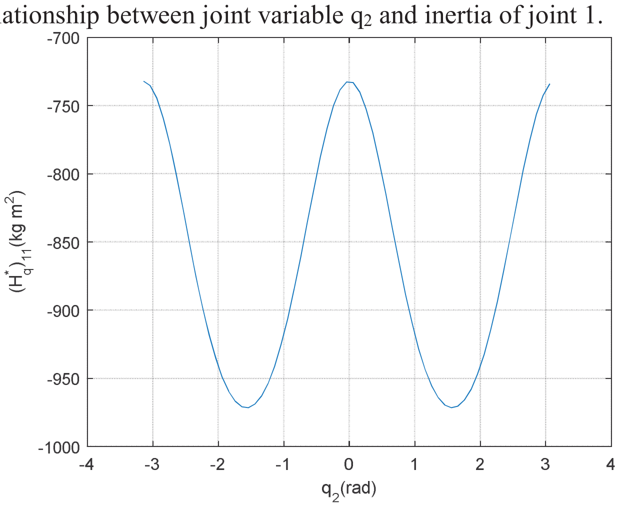

Fig. 2. Inertia of joint 1 caused by joint 2

Taking joint 2 and 3 as another example, we explore that what the change of robot configuration will lead to. Fig.3 and Fig. 4 respectively describe the coupling inertia from joint 2 or 3 to 1 caused by joint 2 and 3.

As for the velocity-dependent item $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$, which is a function of joint position and velocity, a special input of $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ are given, and then solve it.

When **q** = [0 0 0 0 0 0] and qd = 0.5*[1 1 1 1 1 1], the velocity-dependent item is

C_coriolis = [0.75 0 -0.375 0.75 0 -0.375]'.

C. Dynamic Manipulability

Here, we discussed a dynamic measure of manipulability for a free-floating space manipulator system, named as dynamic manipulability, which is to consider how well the manipulator is able to acceleration in different Cartesian directions. Following a similar approach with the one mentioned in fixed-base system, we investigate the dynamic manipulability of a special joint robot configuration.

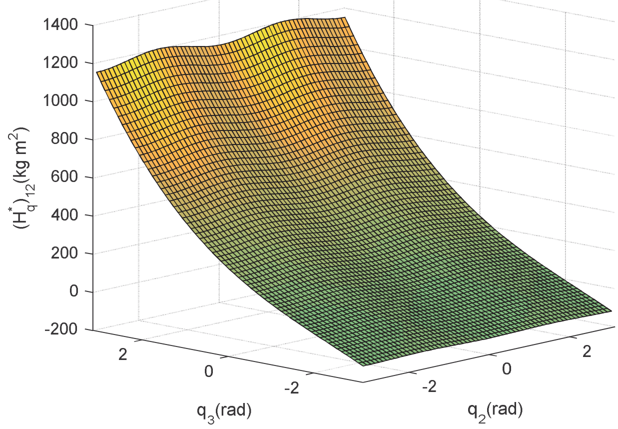

Fig. 3 coupling inertia from joint 2 to joint 1 cuased by joint 2 and 3

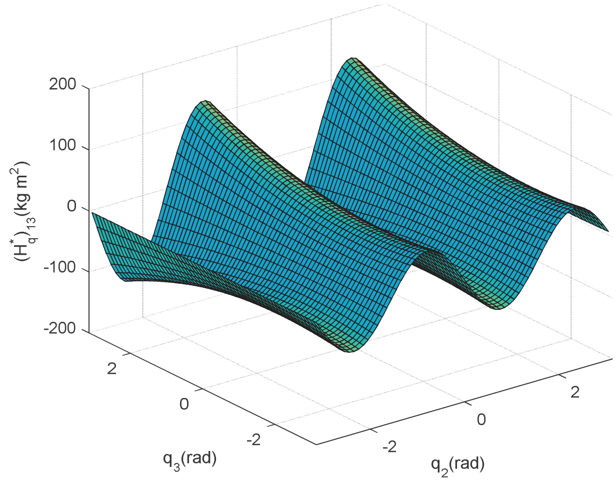

Fig. 4 coupling inertia from joint 3 to joint 1 cuased by joint 2 and 3

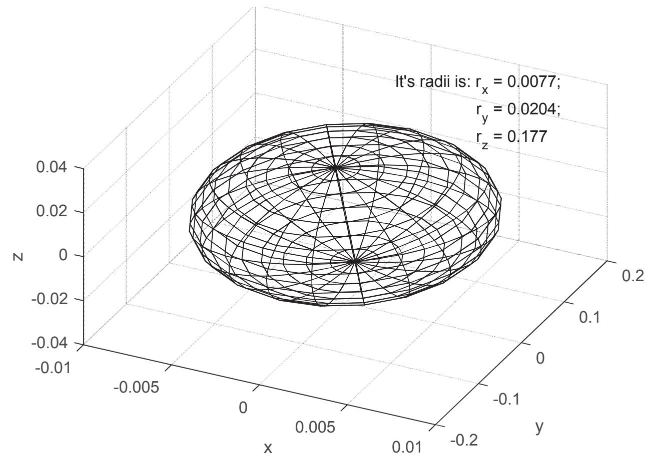

Fig. 5. Spatial acceleratioin ellipsoid for a 6-DOF SBMS

Now, assume $\boldsymbol{v}_0$, $\boldsymbol{\omega}_0$ and $\dot{\boldsymbol{q}}$ to be zero and set generalized joint forces is with unit form, namely,

$$\boldsymbol{\tau}^T\boldsymbol{\tau} = 1\,. \tag{5}$$

The forward kinematics in functional form can be written as $\boldsymbol{v} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}$, considering $\dot{\boldsymbol{q}}$==0 , we have,

$$\dot{\boldsymbol{v}} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}}\,, \tag{6}$$

where $\boldsymbol{J}$ represents the manipulator Jacobian which describe the relationship between velocity of robot's end-effector and that of joints.

Combining (4), (5) and (6), there are

$$\dot{\boldsymbol{v}}^T\left[\boldsymbol{J}\left(\boldsymbol{H}_q^*\right)^{-1}\right]\left[\boldsymbol{J}\left(\boldsymbol{H}_q^*\right)^{-1}\right]^T\dot{\boldsymbol{v}} = 1 \tag{7}$$

This is an equation of a hyper-ellipsoid in Cartesian acceleration space (6×6 dimensions).

Consider just the translational acceleration, and define q = [0 pi/2 0 0 pi/2 0]. A 3-dimensional ellipsoid is plotted with its radii are [0.0077 0.0204 0. 177] in Fig. 5.

III. PATH FOLLOWING CONTROL

In fact, a desired trajectory in joint space is obtained from a path-planning process. Then each of joints will be controlled to follow well this trajectory. In this section, we discuss an approach, computed torque control method, to joint control for a 6-DOF SBMS. Fig. 6 shows a schematic of it.

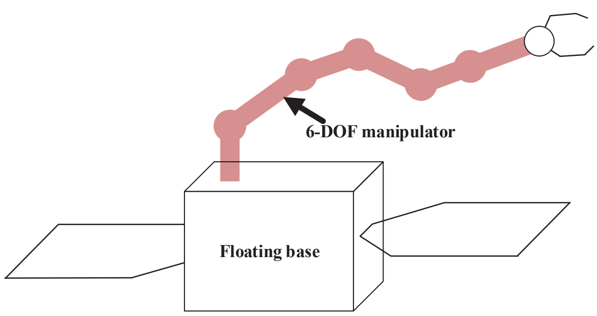

Fig. 6. Skematic of a 6-DOF SBMS

TABLE 1. GEOMETRICAL AND INERTIAL PARAMETERS OF A SYSTEM

No.	Mass(kg)	Length(m)		Inertia tensor(kg.m²)		
				Ixx	Iyy	Izz
0	100	-	-	10	10	10
		$\boldsymbol{b}_0$	0.5			
1	10	$\boldsymbol{a}_1$	0.5	1	1	0.1
		$\boldsymbol{b}_1$	0.5			
2	10	$\boldsymbol{a}_2$	0.5	1	1	0.1
		$\boldsymbol{b}_2$	0.5			
3	10	$\boldsymbol{a}_3$	0.5	1	0.1	1
		$\boldsymbol{b}_3$	0.5			
4	10	$\boldsymbol{a}_4$	0.5	1	1	0.1
		$\boldsymbol{b}_4$	0.5			
5	10	$\boldsymbol{a}_5$	0.5	1	0.1	1
		$\boldsymbol{b}_5$	0.5			
6	10	$\boldsymbol{a}_6$	0.5	1	0.1	1
		$\boldsymbol{b}_6$	0.5			

To propose a path following control method of joint trajectories and demonstrate its efficiency, a simulation is conducted. Geometrical and inertia parameters of a 6-DOF SBMS focused in this section are shown in Table 1.

From (4), the computed torque controller is designed as

$$\boldsymbol{\tau} = \boldsymbol{H}_q^*\left\{\ddot{q}_d + \boldsymbol{K}_v\left(\dot{q}_d - \dot{q}\right) + \boldsymbol{K}_p\left(q_d - q\right)\right\} + \boldsymbol{C}(q,\dot{q})\dot{q}\,, \tag{8}$$

where $\boldsymbol{K}_{\mathrm{p}}$ and $\boldsymbol{K}_{\mathrm{v}}$ are control gains about position and velocity respectively. The block diagram of a model-based control method is shown in Fig. 7.

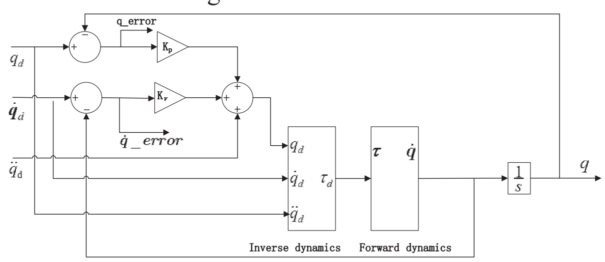

Fig. 7. Control block diagram based on a model of the 6-DOF SBMS

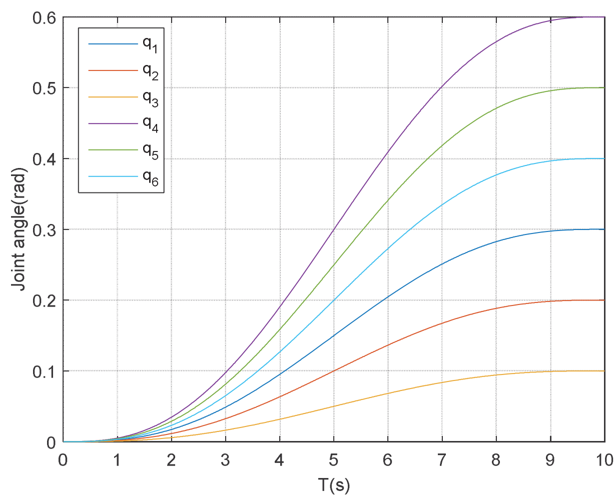

Fig. 8. Desired joint trajectories to be followed

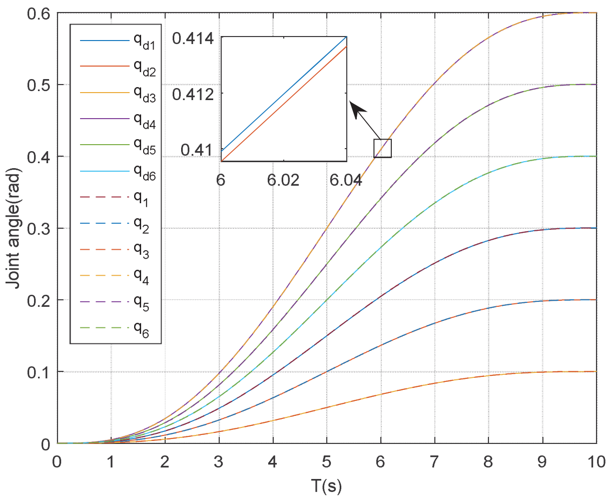

Fig. 9. The comparison chart between desired and actual joint trajectories

Next, a desired joint trajectories drawn in figure 8 is given. It represents that each joint arrives its desired position from the initial position, i.e. from $\mathbf{q}_0$ = [0 0 0 0 0 0] to $\mathbf{q}_d$ = [0.3 0.2 0.1 0.6 0.5 0.4].Take it as inputs of control, and relevant outputs, which is actual path followed by the joints, are obtained after some control loops.

Fig.9 describes a comparison chart between the desired and actual joint trajectories. The results indicate that the control method mentioned in Fig.7 gains a good control effect.

IV. Conclusion

The conclusions of this study are as follows:

(1) Without loss of generality, dynamic model of an n-DOF SBMS without any external forces/torques exerted on is built. The inverse dynamics and relevant attitude of the generalized coefficient matrix are discussed. Also, concept of dynamical manipulability for SBMS is presented. Furthermore, some examples are given to make it clear.

(2) A model-based control approach, computed torque control method is proposed for a 6-DOF SBMS.

(3) To demonstrate the efficiency of this method, a simulation test is conducted, and the results illustrate that it has a good control effect.

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