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Output Feedback Control for Spacecraft with Coupled Translation and Attitude Dynamics

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Abstract— In this paper, we address a tracking control problem for a spacecraft with coupled translation and attitude motion, in the absence of translation and angular velocity measurements. We begin by describing the mutually coupled translation and attitude dynamics of the spacecraft. Next, a suitable high-pass filter is employed to estimate the spacecraft translation and angular velocities using measurements of its translational position and attitude orientation. Using a Lyapunov framework, a nonlinear output feedback control law is designed that ensures the semi-global asymptotic convergence of the spacecraft translation and attitude position tracking errors, despite the lack of translation and angular velocity feedback.

I. Introduction

Recent years have witnessed a growing interest in future space missions such as capture and removal of orbital debris [9], [11] and orbital rendezvous with maneuvering target. Such space missions rely on highly maneuverable spacecraft necessitating the development of a systematic framework for simultaneous control of translation and attitude motion of the spacecraft [8]. In contrast to the study of six-DOF rigid body dynamics and control in the realm of aircraft and underwater vehicles [2], [10], the six-DOF rigid body dynamics and control problem for spacecraft has received scant attention. Some recent exceptions include [8], [9], [11].

A typical feature of the aforementioned control designs is the requirement that the translation and angular velocity measurements of spacecraft be available for feedback. Unfortunately, this requirement is not always satisfied in practice since, e.g., a low sensor count may be desirable to keep cost/weight low. In prior research, several authors have addressed the problem of output feedback control of spacecraft attitude. Specifically, using a three-parameter representation of the spacecraft attitude, a passivity-based controller was developed in [7]. Furthermore, using the modified Rodrigues parameter representation of the spacecraft attitude, an adaptive output feedback attitude tracking controller was developed in [12]. Finally, using the four-parameter quaternion representation of the spacecraft attitude, an adaptive output feedback attitude tracking controller was developed in [4]. However, the output feedback control problem for the six-DOF coupled translation and attitude motion of spacecraft remains to be addressed.

In this paper, we address the output feedback tracking control problem for the six-DOF motion of a spacecraft using the coupled translation and attitude dynamics of a spacecraft developed in [8]. A high-pass filter is employed to generate a velocity-related signal from the translational position and attitude orientation measurements. A judicious modification of the generally recommended [5] filter is implemented to overcome the complexity arising from the mutual coupling of spacecraft translation and attitude dynamics. Using a suitable Lyaponuv function, our nonlinear output feedback control law guarantees asymptotic convergence of the translation and attitude position tracking errors, despite the lack of translation and angular velocity feedback.

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II. Mathematical Preliminaries

Throughout this paper, several reference frames are employed to characterize the translation and attitude dynamics of a spacecraft. Each reference frame used in this paper is assumed to consist of three basis vectors which are right-handed, mutually perpendicular, and of unit length. Let $\mathcal{F}$ denote a reference frame and let $\vec{i}$, $\vec{j}$, and $\vec{k}$ denote the three basis vectors of $\mathcal{F}$. Then $\vec{F} \triangleq \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix}$ denotes the vectrix of the reference frame $\mathcal{F}$ [6]. A vector $\vec{A}$ can be expressed in the reference frame $\mathcal{F}$ as $\vec{A} \triangleq a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$, where a_1, a_2, and a_3 denote the components of $\vec{A}$ along $\vec{i}$, $\vec{j}$, and $\vec{k}$, respectively. Frequently, we will assemble these components as $A \triangleq [a_1\ a_2\ a_3]^T$. Using the above vectrix formalism and the usual vector inner product, a vector $\vec{A}$ can be expressed in the reference frame $\mathcal{F}$ as $\vec{A} = A^T \vec{F} = \vec{F}^T A$.

The vectrix $\vec{F}$ has the following two properties: *i)* $\vec{F} \cdot \vec{F}^T = I_3$, where "$\cdot$" denotes the usual vector dot product and I_n denotes an n dimensional identity matrix and *ii)* $\vec{F} \times \vec{F}^T = \begin{bmatrix} \vec{0} & \vec{k} & -\vec{j} \\ -\vec{k} & \vec{0} & \vec{i} \\ \vec{j} & -\vec{i} & \vec{0} \end{bmatrix}$, where "$\times$" denotes the usual vector cross product. Thus, it follows that $A = \vec{F} \cdot \vec{A} = \vec{A} \cdot \vec{F}$. Next, let $\vec{B}$ be a vector, which is expressed in $\mathcal{F}$ by $\vec{B} \triangleq b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$, and let $B \triangleq [b_1\ b_2\ b_3]^T$. Then, it follows that $\vec{A} \cdot \vec{B} = A^T B = B^T A$ and $\vec{A} \times \vec{B} = \vec{F}^T A^\times B$, where $A^\times \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$.

Throughout this paper, various vectors will be expressed in two or more different reference frames using the rotation matrix concept. For example, consider a vector $\vec{A}$ expressed in the reference frame $\mathcal{F}_\mathrm{u}$ has components A_u, i.e., $\vec{A} = \vec{F}_\mathrm{u}^T A_\mathrm{u}$, where $\vec{F}_\mathrm{u}$ denotes the vectrix of the reference frame $\mathcal{F}_\mathrm{u}$. Similarly, consider the vector $\vec{A}$ expressed in the reference frame $\mathcal{F}_\mathrm{v}$ has components $A_{\rm v}$, i.e., $\vec{A}=\vec{F}_{\rm v}^T A_{\rm v}$, where $\vec{F}_{\rm v}$ denotes the vectrix of the reference frame $\mathcal{F}_{\rm v}$. Then, it follows that $A_{\rm v} = \vec{F}_{\rm v} \cdot \vec{F}_{\rm u}^T A_{\rm u} = C_{\rm v}^{\rm u} A_{\rm u}$, where $C_{\rm v}^{\rm u} \triangleq \vec{F}_{\rm v} \cdot \vec{F}_{\rm u}^T \in SO(3)$ denotes the rotation matrix that transforms the components of a vector expressed in $\mathcal{F}_{\rm u}$ (*viz.*, $A_{\rm u}$) to the components of the same vector expressed in $\mathcal{F}_{\rm v}$ (*viz.*, $A_{\rm v}$). Here the notation $SO(3)$ represents the set of all 3×3 rotation matrices.

Let us consider the vector $\vec{A}$ expressed in another reference frame $\mathcal{F}_{\rm w}$ has components $A_{\rm w}$, i.e., $\vec{A}=\vec{F}_{\rm w}^T A_{\rm w}$, where $\vec{F}_{\rm w}$ denotes the vectrix of the reference frame $\mathcal{F}_{\rm w}$. Then, as above, $A_{\rm u} = \vec{F}_{\rm u} \cdot \vec{F}_{\rm w}^T A_{\rm w} = C_{\rm u}^{\rm w} A_{\rm w}$ and $A_{\rm v} = \vec{F}_{\rm v} \cdot \vec{F}_{\rm w}^T A_{\rm w} = C_{\rm v}^{\rm w} A_{\rm w}$, where $C_{\rm u}^{\rm w} \triangleq \vec{F}_{\rm u} \cdot \vec{F}_{\rm w}^T \in SO(3)$ and $C_{\rm v}^{\rm w} \triangleq \vec{F}_{\rm v} \cdot \vec{F}_{\rm w}^T \in SO(3)$. Finally, it follows that $A_{\rm v} = C_{\rm v}^{\rm u} C_{\rm u}^{\rm w} A_{\rm w}$ and $C_{\rm v}^{\rm w} = C_{\rm v}^{\rm u} C_{\rm u}^{\rm w}$.

III. Spacecraft Dynamic Modeling

In this section, we review a nonlinear model characterizing the translation and attitude dynamics of a spacecraft [8]. The spacecraft is modeled as a rigid body with actuators that provide body-fixed forces and torques about three mutually perpendicular axes that define a body-fixed reference frame $\mathcal{F}_{\rm b}$ located at the mass center of the spacecraft as shown in Figure 1. In this paper, we fully account for the mutual coupling between the translation and attitude motion of the spacecraft. For a given desired translation and attitude motion trajectory, we develop the translation and attitude error dynamics of the spacecraft. Finally, we state our control objective for the translation and attitude motion of the spacecraft.

A. Spacecraft Translation Motion Dynamics

Let $\mathcal{F}_{\rm i}$ be an inertial reference frame fixed at the center of the earth and let $\vec{A}$ be an arbitrary vector measured with respect to the origin of $\mathcal{F}_{\rm i}$. Then, in this paper, $\overset{\bullet}{\vec{A}}(t)$ denotes the time derivative of $\vec{A}(t)$ measured in $\mathcal{F}_{\rm i}$. Using this notation, the translation motion dynamics of the spacecraft is given by [6]

$$\overset{\bullet}{\vec{R}} = \vec{V}, \qquad m\,\overset{\bullet}{\vec{V}} = \vec{f}_{\rm e} + \vec{f}_{\rm d} - \vec{f}, \tag{1}$$

where m denotes the mass of the spacecraft, $\vec{R}(t)$ and $\vec{V}(t)$ denote the position and velocity of its mass center, $\vec{f}_{\rm e}(t)$ denotes the inverse-square gravitational force that leads to an elliptical orbit [3], [6], $\vec{f}_{\rm d}(t)$ denotes the attitude-dependent disturbance force that causes the spacecraft trajectory to deviate from an ellipse [6], and $\vec{f}(t)$ denotes the external control force. The inverse-square gravitational force and the attitude-dependent disturbance force are characterized as $\vec{f}_{\rm e} = -\frac{\mu m}{||\vec{R}||^3}\vec{R}$ and $\vec{f}_{\rm d} = -\frac{3\mu}{2||\vec{R}||^4}\left[\left\{{\rm tr}(J)\,\vec{I} + 2\,\vec{J}\right\}\cdot\vec{Z} - 5\left(\vec{Z}\cdot\vec{J}\cdot\vec{Z}\right)\vec{Z}\right]$, respectively, where $\mu \triangleq MG$ with M being the mass of the earth and G being the universal gravitational constant, J is the constant, positive-definite, symmetric inertia matrix of the spacecraft expressed in $\mathcal{F}_{\rm b}$, $\vec{J} \triangleq \vec{F}_{\rm b}^T J\, \vec{F}_{\rm b}$ denotes the central inertia dyadic of the spacecraft [6], $\vec{I}$ denotes the dyadic of a 3×3 identity matrix, $\vec{Z} \triangleq \frac{\vec{R}}{||\vec{R}||}$ with $\|\vec{R}\| \triangleq \sqrt{\vec{R}\cdot\vec{R}}$, and ${\rm tr}\,(\cdot)$ denotes the trace of a matrix.

In this paper, the desired translation motion dynamics of the spacecraft is assumed to track an elliptical orbit given by

$$\overset{\bullet}{\vec{R}}_{\rm d} = \vec{V}_{\rm d}, \qquad \overset{\bullet}{\vec{V}}_{\rm d} + \frac{\mu}{||\vec{R}_{\rm d}||^3}\vec{R}_{\rm d} = 0, \tag{2}$$

where $\vec{R}_{\rm d}(t)$ and $\vec{V}_{\rm d}(t)$ denote the desired position and velocity of the spacecraft mass center.

Next, let $\vec{\omega}_{\rm b}(t)$ denote the angular velocity of $\mathcal{F}_{\rm b}$ relative to $\mathcal{F}_{\rm i}$. In this paper, $\overset{\circ}{\vec{A}}(t)$ denotes the time derivative of an arbitrary vector $\vec{A}$ measured in $\mathcal{F}_{\rm b}$. Using this notation, the time derivative of vector $\vec{A}$ measured in $\mathcal{F}_{\rm i}$ is given by

$$\overset{\bullet}{\vec{A}} = \overset{\circ}{\vec{A}} + \vec{\omega}_{\rm b} \times \vec{A}. \tag{3}$$

Now we develop the translation motion error dynamics of the spacecraft. Before proceeding, for convenience, we introduce the notation

$$\vec{e}_{\rm R} \triangleq \vec{R}_{\rm d} - \vec{R}, \qquad \vec{e}_{\rm V} \triangleq \vec{V}_{\rm d} - \vec{V}. \tag{4}$$

Computing the time derivative on both sides of the two equations in (4) measured in $\mathcal{F}_{\rm i}$ and using (3), we obtain

$$\overset{\bullet}{\vec{e}}_{\rm R} = \overset{\circ}{\vec{R}}_{\rm d} - \overset{\circ}{\vec{R}} + \vec{\omega}_{\rm b} \times \vec{e}_{\rm R}, \quad \overset{\bullet}{\vec{e}}_{\rm V} = \overset{\circ}{\vec{V}}_{\rm d} - \overset{\circ}{\vec{V}} + \vec{\omega}_{\rm b} \times \vec{e}_{\rm V}. \tag{5}$$

Note that from the first equation of (4), $\overset{\circ}{\vec{e}}_{\rm R} = \overset{\circ}{\vec{R}}_{\rm d} - \overset{\circ}{\vec{R}}$ and $\overset{\bullet}{\vec{e}}_{\rm R} = \overset{\bullet}{\vec{R}}_{\rm d} - \overset{\bullet}{\vec{R}}$. Similarly, from the second equation of (4), $\overset{\circ}{\vec{e}}_{\rm V} = \overset{\circ}{\vec{V}}_{\rm d} - \overset{\circ}{\vec{V}}$ and $\overset{\bullet}{\vec{e}}_{\rm V} = \overset{\bullet}{\vec{V}}_{\rm d} - \overset{\bullet}{\vec{V}}$. Combining (1), (2), (4), and (5), we obtain the translation motion error dynamics of the spacecraft given by

$$\overset{\circ}{\vec{e}}_{\rm R} = \vec{e}_{\rm V} - \vec{\omega}_{\rm b} \times \vec{e}_{\rm R}, \tag{6}$$

$$\overset{\circ}{\vec{e}}_{\rm V} = -\vec{\omega}_{\rm b} \times \vec{e}_{\rm V} - \frac{\mu\,\vec{R}_{\rm d}}{||\vec{R}_{\rm d}||^3} - \frac{1}{m}\left(\vec{f}_{\rm e} + \vec{f}_{\rm d}\right) + \frac{1}{m}\vec{f}. \tag{7}$$

Now using the framework of Section II, various vectors of interest can be expressed in the spacecraft body-fixed reference frame $\mathcal{F}_{\rm b}$ as follows

$$\left[R\ \dot{R}\ R_{\rm d}\ \dot{R}_{\rm d}\ V\ V_{\rm d}\ e_{\rm R}\ \dot{e}_{\rm R}\ e_{\rm V}\ \dot{e}_{\rm V}\ \omega_{\rm b}\ f\ f_{\rm e}\ f_{\rm d}\right] \triangleq \vec{F}_{\rm b} \cdot \left[\vec{R}\ \overset{\circ}{\vec{R}}\ \vec{R}_{\rm d}\ \overset{\circ}{\vec{R}}_{\rm d}\ \vec{V}\ \vec{V}_{\rm d}\ \vec{e}_{\rm R}\ \overset{\circ}{\vec{e}}_{\rm R}\ \vec{e}_{\rm V}\ \overset{\circ}{\vec{e}}_{\rm V}\ \vec{\omega}_{\rm b}\ \vec{f}\ \vec{f}_{\rm e}\ \vec{f}_{\rm d}\right], \tag{8}$$

where $R(t), \dot{R}(t), R_{\rm d}(t), \dot{R}_{\rm d}(t), V(t), V_{\rm d}(t), e_{\rm R}(t), \dot{e}_{\rm R}(t), e_{\rm V}(t), \dot{e}_{\rm V}(t), \omega_{\rm b}(t), f(t), f_{\rm e}(t), f_{\rm d}(t) \in \mathbb{R}^3$ and $\vec{F}_{\rm b}$ denotes the vectrix of the reference frame $\mathcal{F}_{\rm b}$. Next, an application of the vectrix formalism of Section II on (6) and (7) yields

$$\dot{e}_{\rm R} = e_{\rm V} - \omega_{\rm b}^{\times} e_{\rm R}, \tag{9}$$

$$\dot{e}_{\rm V} = -\omega_{\rm b}^{\times} e_{\rm V} - \frac{\mu}{||R_{\rm d}||^3} R_{\rm d} + \frac{\mu}{||R||^3} R + \frac{3\mu}{2m||R||^4}$$

$$\cdot \left\{ \mathrm{tr}\,(J)\, I_3 + 2J - \frac{5R^T JR}{||R||^2} I_3 \right\} \frac{R}{||R||} + \frac{1}{m} f. \quad (10)$$

Remark 3.1: As in [8], the control design framework of Section IV can handle arbitrary time-varying desired trajectories $\vec{R}_{\rm d}(t)$ under the assumption that $\vec{R}_{\rm d}(t)$ and its first two time derivatives are all bounded functions of time.

B. Spacecraft Attitude Dynamics

The attitude dynamics of the spacecraft is given by

$$\overset{\bullet}{\vec{h}} \;=\; \vec{\tau}_{\rm g} + \vec{\tau}, \quad (11)$$

where $\vec{h}(t)$ given by $\vec{h}{=}\vec{J}\cdot\vec{\omega}_{\rm b}$ denotes the angular momentum of the spacecraft about its mass center, $\vec{\tau}_{\rm g}(t)$ given by $\vec{\tau}_{\rm g}{=}\frac{3\mu}{||\vec{R}||^3}\vec{Z}\times\vec{J}\cdot\vec{Z}$ denotes the gravity gradient torque [6], and $\vec{\tau}(t)$ denotes the control torque.

Using (3) we obtain $\overset{\bullet}{\vec{h}} = \overset{\circ}{\vec{h}} + \vec{\omega}_{\rm b}\times\vec{h}$. Once again, using the framework of Section II, various vectors of interest can be expressed in the spacecraft body-fixed reference frame $\mathcal{F}_{\rm b}$ as

$$\begin{bmatrix} h & \dot{\omega}_{\rm b} & \tau_{\rm g} & \tau \end{bmatrix} \;\triangleq\; \vec{F}_{\rm b}\cdot \begin{bmatrix} \vec{h} & \overset{\circ}{\vec{\omega}}_{\rm b} & \vec{\tau}_{\rm g} & \vec{\tau} \end{bmatrix}, \quad (12)$$

where $h(t), \dot{\omega}_{\rm b}(t), \tau_{\rm g}(t), \tau(t) \in \mathbb{R}^3$. Next, an application of the vectrix formalism of Section II on (11) yields

$$J\dot{\omega}_{\rm b} \;=\; -\omega_{\rm b}^{\times} J\omega_{\rm b} + \tau_{\rm g} + \tau. \quad (13)$$

Now we use the nonsingular, four-parameter quaternion representation to relate the time derivative of the spacecraft angular orientation to the angular velocity $\omega_{\rm b}$ as follows [6]

$$\begin{bmatrix} \dot{\varepsilon}_{\rm b}^T & \dot{\zeta}_{\rm b}^T \end{bmatrix}^T \;=\; E(\varepsilon_{\rm b}, \zeta_{\rm b})\omega_{\rm b}, \quad (14)$$

where $E(\varepsilon_{\rm b}, \zeta_{\rm b}) \triangleq \frac{1}{2}\begin{bmatrix} \varepsilon_{\rm b}^{\times} + \zeta_{\rm b} I_3 \\ -\varepsilon_{\rm b}^T \end{bmatrix}$ and $[\varepsilon_{\rm b}(t),\, \zeta_{\rm b}(t)] \in \mathbb{R}^3 \times\mathbb{R}$ represents the quaternion, which characterizes the attitude of $\mathcal{F}_{\rm b}$ with respect to $\mathcal{F}_{\rm i}$. By construction, the quaternion $[\varepsilon_{\rm b},\, \zeta_{\rm b}]$ must satisfy the unit norm constraint $\varepsilon_{\rm b}^T\varepsilon_{\rm b} + \zeta_{\rm b}^2 = 1$. Following [6], the rotation matrix $C_{\rm b}^{\rm i} \in SO(3)$ that brings the inertial frame $\mathcal{F}_{\rm i}$ onto the spacecraft body-fixed reference frame $\mathcal{F}_{\rm b}$ is given as $C_{\rm b}^{\rm i} = C(\varepsilon_{\rm b}, \zeta_{\rm b}) \triangleq \left(\zeta_{\rm b}^2 - \varepsilon_{\rm b}^T\varepsilon_{\rm b}\right) I_3 + 2\varepsilon_{\rm b}\varepsilon_{\rm b}^T - 2\zeta_{\rm b}\varepsilon_{\rm b}^{\times}$. The dynamic and kinematic equations of (13) and (14) represent the attitude dynamics of the spacecraft.

Next, we characterize the desired attitude of the spacecraft using a desired, spacecraft body-fixed reference frame $\mathcal{F}_{\rm d}$. Let $\vec{\omega}_{\rm d}(t)$ denote the desired angular velocity of $\mathcal{F}_{\rm d}$ with respect to $\mathcal{F}_{\rm i}$ and let $\overset{\ominus}{\vec{\omega}}_{\rm d}(t)$ denote the time derivative of $\vec{\omega}_{\rm d}$ measured in $\mathcal{F}_{\rm d}$. Using the framework of Section II, we express $\vec{\omega}_{\rm d}$ and $\overset{\ominus}{\vec{\omega}}_{\rm d}$ in the desired, spacecraft body-fixed reference frame $\mathcal{F}_{\rm d}$ as follows

$$\begin{bmatrix} \omega_{\rm d} & \dot{\omega}_{\rm d} \end{bmatrix} \;\triangleq\; \vec{F}_{\rm d}\cdot \begin{bmatrix} \vec{\omega}_{\rm d} & \overset{\ominus}{\vec{\omega}}_{\rm d} \end{bmatrix}, \quad \omega_{\rm d}(t), \dot{\omega}_{\rm d}(t) \in \mathbb{R}^3, (15)$$

where $\vec{F}_{\rm d}$ denotes the vectrix of the reference frame $\mathcal{F}_{\rm d}$. The angular orientation of the desired, spacecraft body-fixed reference frame $\mathcal{F}_{\rm d}$ with respect to the inertial frame $\mathcal{F}_{\rm i}$ is characterized by the desired unit quaternion $[\varepsilon_{\rm d}(t),\, \zeta_{\rm d}(t)] \in \mathbb{R}^3 \times \mathbb{R}$, whose kinematics are governed by

$$\begin{bmatrix} \dot{\varepsilon}_{\rm d}^T & \dot{\zeta}_{\rm d}^T \end{bmatrix}^T \;=\; E(\varepsilon_{\rm d}, \zeta_{\rm d})\omega_{\rm d}. \quad (16)$$

The rotation matrix $C_{\rm d}^{\rm i} \in SO(3)$ that brings the reference frame $\mathcal{F}_{\rm i}$ onto the reference frame $\mathcal{F}_{\rm d}$ is given by $C_{\rm d}^{\rm i} = C(\varepsilon_{\rm d}, \zeta_{\rm d})$. In addition, using (16), it follows that $\omega_{\rm d} = 2(\zeta_{\rm d}\dot{\varepsilon}_{\rm d} - \dot{\zeta}_{\rm d}\varepsilon_{\rm d}) - 2\varepsilon_{\rm d}^{\times}\dot{\varepsilon}_{\rm d}$ and $\dot{\omega}_{\rm d} = 2(\zeta_{\rm d}\ddot{\varepsilon}_{\rm d} - \ddot{\zeta}_{\rm d}\varepsilon_{\rm d}) - 2\varepsilon_{\rm d}^{\times}\ddot{\varepsilon}_{\rm d}$. In this paper, we assume that $\varepsilon_{\rm d}$, $\zeta_{\rm d}$, and their first two time derivatives are all bounded functions of time, which yields boundedness of $\omega_{\rm d}$ and $\dot{\omega}_{\rm d}$ given above.

Now we develop the attitude error dynamics of the spacecraft. Let $[e_\varepsilon(t),\, e_\zeta(t)] \in \mathbb{R}^3 \times \mathbb{R}$ denote the quaternion characterizing the mismatch between the actual orientation of the spacecraft $\mathcal{F}_{\rm b}$ and desired orientation of the spacecraft $\mathcal{F}_{\rm d}$. By construction, the error quaternion $[e_\varepsilon,\, e_\zeta]$ must satisfy the unit norm constraint $e_\varepsilon^T e_\varepsilon + e_\zeta^2 = 1$. In addition, $[e_\varepsilon,\, e_\zeta]$ can be characterized using $[\varepsilon_{\rm b},\, \zeta_{\rm b}]$ and $[\varepsilon_{\rm d},\, \zeta_{\rm d}]$ as follows $e_\varepsilon = \zeta_{\rm d}\varepsilon_{\rm b} - \zeta_{\rm b}\varepsilon_{\rm d} + \varepsilon_{\rm b}^{\times}\varepsilon_{\rm d}$ and $e_\zeta = \zeta_{\rm d}\zeta_{\rm b} + \varepsilon_{\rm d}^T\varepsilon_{\rm b}$ [1], [4], [6]. The corresponding rotation matrix $C_{\rm b}^{\rm d} \in SO(3)$ that brings the desired, spacecraft body-fixed reference frame $\mathcal{F}_{\rm d}$ onto the spacecraft body-fixed frame $\mathcal{F}_{\rm b}$ is given by

$$C_{\rm b}^{\rm d} \;=\; C(e_\varepsilon, e_\zeta) = C_{\rm b}^{\rm i} {C_{\rm d}^{\rm i}}^T. \quad (17)$$

Next, let $\vec{\omega}(t)$ denote the angular velocity of $\mathcal{F}_{\rm b}$ with respect to $\mathcal{F}_{\rm d}$. Then, it follows that

$$\vec{\omega} \;=\; \vec{\omega}_{\rm b} - \vec{\omega}_{\rm d}\,. \quad (18)$$

Now using the framework of Section II, we express $\vec{\omega}$ in the spacecraft body-fixed reference frame $\mathcal{F}_{\rm b}$ as

$$\omega \;\triangleq\; \vec{F}_{\rm b}\cdot\vec{\omega}, \quad \omega(t) \in \mathbb{R}^3. \quad (19)$$

Using $\omega_{\rm b} = \vec{F}_{\rm b}\cdot\vec{\omega}_{\rm b}$ from (8), $\omega_{\rm d} = \vec{F}_{\rm d}\cdot\vec{\omega}_{\rm d}$ from (15), and (19), Eq. (18) yields

$$\omega \;=\; \omega_{\rm b} - C_{\rm b}^{\rm d}\omega_{\rm d}. \quad (20)$$

Note that by applying (3) to $\vec{\omega}_{\rm b}$ and $\vec{\omega}$ we get $\overset{\bullet}{\vec{\omega}}_{\rm b}{=}\overset{\circ}{\vec{\omega}}_{\rm b}$ and $\overset{\bullet}{\vec{\omega}}{=}\overset{\circ}{\vec{\omega}} + \vec{\omega}_{\rm b}\times\vec{\omega}$, respectively. Similarly, it can be shown that $\overset{\bullet}{\vec{\omega}}_{\rm d}{=}\overset{\ominus}{\vec{\omega}}_{\rm d}$. Now computing the time derivative of (18) measured in $\mathcal{F}_{\rm i}$ and performing simple manipulations, it can be shown that the time derivative of $\vec{\omega}$ measured in $\mathcal{F}_{\rm b}$ is given by $\overset{\circ}{\vec{\omega}}{=}\overset{\circ}{\vec{\omega}}_{\rm b} - \vec{\omega}_{\rm b}\times\vec{\omega} - \overset{\ominus}{\vec{\omega}}_{\rm d}$.

Expressing $\overset{\circ}{\vec{\omega}}$ in the spacecraft body-fixed reference frame $\mathcal{F}_{\rm b}$ as $\dot{\omega} \triangleq \vec{F}_{\rm b}\cdot\overset{\circ}{\vec{\omega}}$, $\dot{\omega}(t) \in \mathbb{R}^3$, and using (12), (15), (19), and (20), we can now express $\overset{\circ}{\vec{\omega}}$ in the spacecraft body-fixed reference frame $\mathcal{F}_{\rm b}$. Finally, multiplying the resultant expression by J on both sides, we obtain

$$J\dot{\omega} \;=\; J\dot{\omega}_{\rm b} + J\omega^{\times}C_{\rm b}^{\rm d}\omega_{\rm d} - JC_{\rm b}^{\rm d}\dot{\omega}_{\rm d}. \quad (21)$$

We now use (13), (20), and (21) to obtain the following open-loop attitude tracking error dynamics of the spacecraft

$$J\dot{\omega} = -\left(\omega + C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}}\right)^{\times} J\left(\omega + C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}}\right) + J\left(\omega^{\times} C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}} - C_{\mathrm{b}}^{\mathrm{d}}\dot{\omega}_{\mathrm{d}}\right) + \frac{3\mu}{||R||^5} R^{\times} J R + \tau. \quad (22)$$

In addition, the open-loop attitude tracking error kinematics is given by [1], [4], [6]

$$\begin{bmatrix}\dot{e}_{\varepsilon}^T & \dot{e}_{\zeta}^T\end{bmatrix}^T = E(e_{\varepsilon}, e_{\zeta})\omega. \quad (23)$$

Note that the attitude dynamics of (22) can be rearranged to yield

$$J\dot{\omega} = -\omega^{\times} J\omega + \tau_{\mathrm{un}} + \tau_{\mathrm{kn}} + \tau, \quad (24)$$

where $\tau_{\mathrm{un}}(t), \tau_{\mathrm{kn}}(t) \in \mathbb{R}^3$ are defined as

$$\tau_{\mathrm{un}} \triangleq \left((JC_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}})^{\times} - (C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}})^{\times} J - J(C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}})^{\times}\right)\omega, \quad (25)$$

$$\tau_{\mathrm{kn}} \triangleq -(C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}})^{\times} J(C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}}) - JC_{\mathrm{b}}^{\mathrm{d}}\dot{\omega}_{\mathrm{d}} + \frac{3\mu}{\|R\|^5} R^{\times} J R. \quad (26)$$

Remark 3.2: The definition of τ_{un} in (25) depends on ω, which is not measured. Thus, τ_{un} can not be used in the control design. On the other hand, the definition of τ_{kn} in (26) depends on the desired attitude trajectory and the translational position R, signals that are known/measured. Thus, τ_{kn} can be used in the control design.

C. Control Objectives

In this paper, the control objective for the translation motion dynamics of the spacecraft requires that the mass center of the spacecraft track the desired translation motion trajectory, i.e., $\vec{R}(t) \to \vec{R}_{\mathrm{d}}(t)$ as $t \to \infty$. In addition, it is required that $\dot{\vec{R}}(t) \to \dot{\vec{R}}_{\mathrm{d}}(t)$ as $t \to \infty$. Using (1), (2), (4), and (8), the spacecraft translation motion tracking control objective can be stated as follows

$$\lim_{t\to\infty} e_{\mathrm{R}}(t),\, e_{\mathrm{V}}(t) = 0. \quad (27)$$

The control objective for the attitude dynamics of the spacecraft requires that the actual attitude of the spacecraft track the desired attitude trajectory, i.e., the rotation matrix $C_{\mathrm{b}}^{\mathrm{i}}$ must coincide with the rotation matrix $C_{\mathrm{d}}^{\mathrm{i}}$ in steady-state. Using (17), this control objective can be equivalently characterized as $\lim_{t\to\infty} C_{\mathrm{b}}^{\mathrm{d}} = I_3$. Furthermore, it is required that $\vec{\omega}_{\mathrm{b}}(t) \to \vec{\omega}_{\mathrm{d}}(t)$ as $t \to \infty$. With the aid of the unit norm constraint of $(e_{\varepsilon}, e_{\zeta})$ and using (17)–(19), the spacecraft attitude tracking control objective can be equivalently stated as follows

$$\lim_{t\to\infty} e_{\varepsilon}(t),\, \omega(t) = 0. \quad (28)$$

The control objectives of (27) and (28) are to be met under the constraint of no direct velocity feedback (i.e., V and ω_{b} are not measured).

IV. Spacecraft Output Feedback Control Design

In this section, we develop an output feedback controller based on the system dynamics of (9), (10), (23), and (24) such that the tracking error variables e_{R}, e_{V}, e_{ε}, and ω exhibit asymptotic stability. Before we proceed with the control design, for notational convenience, we introduce two matrices $T(t), P(t) \in \mathbb{R}^{3\times 3}$ defined as

$$T \triangleq \frac{1}{2}\begin{bmatrix} e_{\zeta} & -e_{\varepsilon_3} & e_{\varepsilon_2} \\ e_{\varepsilon_3} & e_{\zeta} & -e_{\varepsilon_1} \\ -e_{\varepsilon_2} & e_{\varepsilon_1} & e_{\zeta}\end{bmatrix},\ P \triangleq T^{-1}, \quad (29)$$

where $e_{\varepsilon_1}(t)$, $e_{\varepsilon_2}(t), e_{\varepsilon_3}(t) \in \mathbb{R}$ are the components of e_{ε}. Using (29), $\dot{e}_{\varepsilon}$ of (23) can be written in a compact form as follows

$$\dot{e}_{\varepsilon} = T\omega, \quad \Rightarrow \quad P\dot{e}_{\varepsilon} = \omega. \quad (30)$$

Next, we define the position and velocity tracking error variables $r_0(t), v_0(t) \in \mathbb{R}^6$ as follows

$$r_0 \triangleq \begin{bmatrix} e_{\mathrm{R}}^T & e_{\varepsilon}^T\end{bmatrix}^T, \quad v_0 \triangleq \begin{bmatrix} e_{\mathrm{V}}^T & \dot{e}_{\varepsilon}^T\end{bmatrix}^T. \quad (31)$$

Now differentiating r_0 in (31) with respect to time and using (9) and (31), we obtain

$$\dot{r}_0 = \begin{bmatrix}\dot{e}_{\mathrm{R}}^T & \dot{e}_{\varepsilon}^T\end{bmatrix}^T = v_0 - \Omega, \quad (32)$$

where $\Omega(t) \in \mathbb{R}^6$ is defined as $\Omega \triangleq \begin{bmatrix}(\omega_{\mathrm{b}}^{\times} e_{\mathrm{R}})^T & 0_{1\times 3}\end{bmatrix}^T$.

A. Velocity Filter Design

To account for the lack of spacecraft translation and angular velocity measurements *viz.*, V and ω_{b}, or equivalently the velocity tracking errors *viz.*, e_{V} and ω, a filtered velocity error signal $e_f(t) \in \mathbb{R}^6$ is produced using a filter. The filter is constructed as shown below

$$e_f = -kr_0 + p, \quad (33)$$

where $k > 0$ is a positive, constant filter gain, $p(t) \in \mathbb{R}^6$ is a pseudo-velocity tracking error generated using

$$\dot{p} = -(k+1)p + k^2 r_0 + \Gamma r_0 + \Delta,\ p(0) = kr_0(0), \quad (34)$$

where $\Gamma(t) \in \mathbb{R}^{6\times 6}$ and $\Delta(t) \in \mathbb{R}^6$ are defined as $\Gamma \triangleq \mathrm{diag}\{k_0 I_3, \frac{k_1}{(1-e_{\varepsilon}^T e_{\varepsilon})^2} I_3\}$ and $\Delta \triangleq \left[(k\left[P\left(e_{f_2} + e_{\varepsilon}\right) - C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}}\right]^{\times} e_{\mathrm{R}})^T \quad 0_{1\times 3}\right]^T$, respectively, $k_0 > 0$ is a constant, $k_1 > 1$ is a constant, and $e_{f_2}(t)$ is obtained by decomposing e_f as $e_f = \begin{bmatrix} e_{f_1}^T & e_{f_2}^T\end{bmatrix}^T$ with $e_{f_1}(t), e_{f_2}(t) \in \mathbb{R}^3$.

To assist in the development of the filtered velocity error signal e_f dynamics, we introduce an auxiliary tracking error variable $\eta(t) \in \mathbb{R}^6$ as follows

$$\eta \triangleq v_0 + e_f + r_0. \quad (35)$$

Note that using (35) in (32) produces

$$\dot{r}_0 = \eta - e_f - r_0 - \Omega. \quad (36)$$

Next, we decompose η as $\eta = \begin{bmatrix}\eta_1^T\ \eta_2^T\end{bmatrix}^T$, where $\eta_1(t), \eta_2(t), \in \mathbb{R}^3$. Using this decomposition, (35) yields

$$\eta_1 = e_{\mathrm{V}} + e_{f_1} + e_{\mathrm{R}}, \qquad \eta_2 = \dot{e}_{\varepsilon} + e_{f_2} + e_{\varepsilon}. \quad (37)$$

In addition, solving for ω_{b} in (20) and substituting for ω from (30) in the resulting equation, we obtain

$$\omega_{\mathrm{b}} = P(\eta_2 - e_{f_2} - e_{\varepsilon}) + C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}}, \quad (38)$$

where (37) has been used. Next, we substitute (38) into the definition of Ω and rearrange terms to decompose Ω as

$$\Omega = \Omega_1 + \Omega_2, \quad \Omega_1(t), \Omega_2(t) \in \mathbb{R}^6, \quad (39)$$

where $\Omega_1 \triangleq [((P\eta_2)^{\times} e_{\mathrm{R}})^T\ 0_{1\times 3}]^T$ and $\Omega_2 \triangleq \left[((-P\left(e_{f_2} + e_{\varepsilon}\right) + C_{\mathrm{b}}^{\mathrm{d}}\omega_{\mathrm{d}})^{\times} e_{\mathrm{R}})^T\ 0_{1\times 3}\right]^T$.

To obtain the closed-loop dynamics of e_f, we take the time derivative of (33), which yields

$$\dot{e}_f = -k\eta - e_f + \Gamma r_0 + k\Omega_1, \quad (40)$$

where (33), (34), (36), and $\Delta = -k\Omega_2$ have been used.

B. Open-Loop Auxiliary Tracking Error Dynamics

We begin by differentiating η of (35) with respect to time and substituting the time derivative of (30) and (31) to produce

$$\dot{\eta} = \begin{bmatrix} \dot{e}_{\mathrm{V}} \\ \dot{T}\omega + T\dot{\omega} \end{bmatrix} + \dot{e}_f + v_0 - \Omega, \quad (41)$$

where (32) has been used. Next, we multiply $M \in \mathbb{R}^{6\times 6}$ defined as $M \triangleq \mathrm{diag}\{mI_3, P^TJP\}$ on both sides of (41) to yield

$$M\dot{\eta} = \begin{bmatrix} m\dot{e}_{\mathrm{V}} \\ P^TJP\dot{T}\omega + P^TJPT\dot{\omega} \end{bmatrix} + M\left(\dot{e}_f + v_0 - \Omega\right). \quad (42)$$

Using (24) and (29), $P^TJPT\dot{\omega}$ in (42) is expressed as $P^TJPT\dot{\omega} = P^T(J\omega)^\times\omega + P^T(\tau_{\mathrm{un}} + \tau_{\mathrm{kn}} + \tau)$. Next, we substitute for ω from (30) into $P^TJP\dot{T}\omega + P^TJPT\dot{\omega}$ to obtain

$$\begin{aligned} P^TJP\dot{T}\omega + P^TJPT\dot{\omega} &= (P^TJP\dot{T}P + P^T(JP\dot{e}_\varepsilon)^\times P)\dot{e}_\varepsilon \\ &\quad + P^T\left(\tau_{\mathrm{un}} + \tau_{\mathrm{kn}} + \tau\right). \quad (43) \end{aligned}$$

To simplify notation, we define an inertia-like matrix $J^\star(t) \in \mathbb{R}^{3\times 3}$ as $J^\star(e_\varepsilon, e_\zeta) \triangleq P^TJP$ and a coriolis-like matrix $C^\star(t) \in \mathbb{R}^{3\times 3}$ as $C^\star(e_\varepsilon, e_\zeta, \dot{e}_\varepsilon, \dot{e}_\zeta) \triangleq J^\star\dot{T}P + P^T(JP\dot{e}_\varepsilon)^\times P$. Using these definitions, (43) is given by

$$\begin{aligned} P^TJP\dot{T}\omega + P^TJPT\dot{\omega} &= C^\star\eta_2 - C^\star\left(e_{f_2} + e_\varepsilon\right) \\ &\quad + P^T\left(\tau_{\mathrm{un}} + \tau_{\mathrm{kn}} + \tau\right), \quad (44) \end{aligned}$$

where (37) has been used.

Next, to simplify $\dot{e}_f + v_0 - \Omega$ term in (42), we use (39), (40), and v_0 from (35) to produce

$$\dot{e}_f + v_0 - \Omega = -\bar{k}\eta - 2e_f + \bar{\Gamma}r_0 + \bar{k}\Omega_1 - \Omega_2, \quad (45)$$

where $\bar{k} \triangleq k - 1$ and $\bar{\Gamma} \triangleq \Gamma - I_6$.

Finally, using (10), (44), and (45), the open-loop dynamics η of (42) yields

$$M\dot{\eta} = \Lambda - \bar{k}M\eta + N + M(\bar{\Gamma}r_0 - 2e_f - \Omega_2) + \chi + \bar{k}M\Omega_1 + u, \quad (46)$$

where $\Lambda(t)$, $N(t)$, $\chi(t)$, $u(t) \in \mathbb{R}^6$ are defined as $\Lambda \triangleq \left[0_{1\times 3}\ (C^\star\eta_2)^T\right]^T$, $N \triangleq \left[-\frac{m\mu}{\|R_{\mathrm{d}}\|^3}R_{\mathrm{d}}^T + \frac{m\mu}{\|R\|^3}R^T + \left(\frac{3\mu}{2\|R\|^4}\right)\frac{R^T}{\|R\|}\left\{\mathrm{tr}\left(J\right)I_3 + 2J - \frac{5R^TJR}{\|R\|^2}I_3\right\}\ (P^T\tau_{\mathrm{kn}})^T\right]^T$, $\chi \triangleq [(-m\omega_{\mathrm{b}}^\times e_{\mathrm{V}})^T\ \ (P^T\tau_{\mathrm{un}} - C^\star(e_{f_2} + e_\varepsilon))^T]^T$, and $u \triangleq \left[f^T\ (P^T\tau)^T\right]^T$.

Remark 4.1: The inertia- and coriolis-like matrices of $J^\star$ and $C^\star$ satisfy the skew-symmetric property of $z^T\left(\frac{1}{2}\dot{J}^\star + C^\star\right)z = 0$, $\forall z \in \mathbb{R}^3$. See [4] for details.

C. Stability Analysis

To facilitate the following stability analysis, we introduce several variables. We define an auxiliary error variable $y(t) \in \mathbb{R}^6$ as $y \triangleq \left[\eta_1^T\ (P\eta_2)^T\right]^T = \left[y_1^T\ y_2^T\right]^T$, where $y_1(t), y_2(t) \in \mathbb{R}^3$. Next, we define a combined error variable $x(t) \in \mathbb{R}^{18}$ as $x \triangleq \left[\begin{array}{cccc} e_{\mathrm{R}}^T & e_f^T & \frac{\sqrt{k_1}e_\varepsilon^T}{\sqrt{1-e_\varepsilon^Te_\varepsilon}} & y^T \end{array}\right]^T$, and a constant matrix $\hat{M} \in \mathbb{R}^{6\times 6}$ as $\hat{M} \triangleq \mathrm{diag}\{mI_3, J\}$. In addition, we define $\chi_1(t) \in \mathbb{R}^6$ and $\chi_2(t) \in \mathbb{R}^3$ as

$$\chi_1 \triangleq \begin{bmatrix} I_3 & 0_{3\times 3} \\ 0_{3\times 3} & (P^T)^{-1} \end{bmatrix}\chi, \chi_2 \triangleq \left[\left(ke_f^T + \bar{k}\eta^TM\right)\begin{bmatrix} (P\eta_2)^\times \\ 0_{3\times 3} \end{bmatrix}\right]^T. \quad (47)$$

Finally, we define positive constants, λ_1, λ_2, $k_{e_{\mathrm{R}}}$, $\hat{k}$, k_y, and $k_{\max}$ as $\lambda_1 \triangleq \frac{1}{2}\min\{k_0, 1, m, \lambda_{\min}\{J\}\}$, $\lambda_2 \triangleq \frac{1}{2}\max\{k_0, 1, m, \lambda_{\max}\{J\}\}$, $k_{e_{\mathrm{R}}} \triangleq k_0 - 1$, $\hat{k} \triangleq \bar{k}\lambda_{\min}\{\hat{M}\}$, $k_y \triangleq \hat{k} - 1$, and $k_{\max} \triangleq \max\left\{k_{e_{\mathrm{R}}}, k_y\right\}$, respectively, where $\lambda_{\min}\{X\}$ and $\lambda_{\max}\{X\}$ represent the minimum and maximum eigenvalue, respectively, of a matrix X.

Theorem 4.1: The output feedback control law $u(t)$ given by

$$u = ke_f - N - M(\bar{\Gamma}r_0 - 2e_f - \Omega_2) - \begin{bmatrix} k_0e_{\mathrm{R}} \\ \frac{k_1e_\varepsilon}{\left(1-e_\varepsilon^Te_\varepsilon\right)^2} \end{bmatrix}, \quad (48)$$

ensures semi-global asymptotic convergence of the spacecraft translational position and velocity tracking errors and the spacecraft attitude position and velocity tracking errors, delineated by $\lim_{t\to\infty} e_{\mathrm{R}}(t), e_{\mathrm{V}}(t), e_\varepsilon(t), \omega(t) = 0$, if the initial condition of e_ζ is selected such that $\|e_\zeta(0)\| \neq 0$ and k, k_0 are selected such that $k_{\max} > \rho_1^2\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\|x(0)\|\right) + \rho_2^2\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\|x(0)\|\right)$, where $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are nondecreasing functions.

Proof. We begin by substituting (48) into (46) to obtain the closed-loop dynamics for η

$$M\dot{\eta} = \Lambda - \bar{k}M\eta + \chi + \bar{k}M\Omega_1 + ke_f - \begin{bmatrix} k_0e_{\mathrm{R}} \\ \frac{k_1e_\varepsilon}{\left(1-e_\varepsilon^Te_\varepsilon\right)^2} \end{bmatrix}. \quad (49)$$

Next, we define a positive-definite, candidate Lyapunov function as

$$V \triangleq \frac{1}{2}k_0e_{\mathrm{R}}^Te_{\mathrm{R}} + \frac{1}{2}e_f^Te_f + \frac{1}{2}\frac{k_1e_\varepsilon^Te_\varepsilon}{1-e_\varepsilon^Te_\varepsilon} + \frac{1}{2}y^T\hat{M}y. \quad (50)$$

Applying Rayleigh-Ritz's theorem on (50) results in

$$\lambda_1\|x\|^2 \leq V \leq \lambda_2\|x\|^2. \quad (51)$$

Next, we compute the time derivative of (50) to produce

$$\dot{V} = k_0e_{\mathrm{R}}^T\dot{e}_{\mathrm{R}} + e_f^T\dot{e}_f + \frac{k_1e_\varepsilon^T\dot{e}_\varepsilon}{\left(1-e_\varepsilon^Te_\varepsilon\right)^2} + y^T\hat{M}\dot{y}. \quad (52)$$

Note that $y^T\hat{M}\dot{y}$ on the right hand side of (52) is simplified as follows

$$y^T\hat{M}\dot{y} = \eta^TM\dot{\eta} + \frac{1}{2}\eta_2^T\dot{J}^\star\eta_2, \quad (53)$$

where we used the time derivative of $J^\star$ defined earlier. Evaluating (52) along the trajectories of (36), (40), (49), and (53), we get

$$\begin{aligned} \dot{V} &= -k_0e_{\mathrm{R}}^Te_{\mathrm{R}} - e_f^Te_f - \frac{k_1e_\varepsilon^Te_\varepsilon}{\left(1-e_\varepsilon^Te_\varepsilon\right)^2} - \bar{k}y^T\hat{M}y \\ &\quad + y^T\chi_1 + e_{\mathrm{R}}^T\chi_2, \quad (54) \end{aligned}$$

where the definitions of (47) and the skew-symmetry property in Remark 4.1 have been used. Based on (47), it can be shown that χ_1 and χ_2 satisfy $\|\chi_1\| \leq$

$\rho_1(\|x\|)\|x\|$ and $\|\chi_2\| \le \rho_2(\|x\|)\|x\|$, respectively, using which, (54) can be upper bounded as follows

$$\begin{aligned}\dot V \le & -k_0\|e_{\mathrm{R}}\|^2 - \|e_f\|^2 - \frac{k_1\|e_\varepsilon\|^2}{(1-e_\varepsilon^T e_\varepsilon)^2} - \hat k\|y\|^2 \\ & +\rho_1(\|x\|)\|x\|\|y\| + \rho_2(\|x\|)\|x\|\|e_{\mathrm{R}}\|, \end{aligned} \tag{55}$$

where the definition of $\hat k$ has been used.

Then, using the definition of $k_{e_{\mathrm{R}}}$ and k_y yields the following expression for (55)

$$\begin{aligned}\dot V \ \le\ & -\|x\|^2 - \underbrace{k_y\|y\|^2 + \rho_1(\|x\|)\|x\|\|y\|}_{D_1} \\ & \underbrace{-k_{e_{\mathrm{R}}}\|e_{\mathrm{R}}\|^2 + \rho_2(\|x\|)\|x\|\|e_{\mathrm{R}}\|}_{D_2}, \end{aligned} \tag{56}$$

where the definition of x has been used. Bounding D_1 and D_2 in (56) by completion of squares produces $\dot V \le -\left(1-\frac{\rho_1^2(\|x\|)}{4k_y}-\frac{\rho_2^2(\|x\|)}{4k_{e_{\mathrm{R}}}}\right)\|x\|^2$. Note that if $k_{\max}$ is chosen such that $k_{\max} \ge \frac{\rho_1^2(\|x\|)+\rho_2^2(\|x\|)}{4}$, then $\dot V$ is negative semidefinite, i.e.,

$$\dot V \ \le\ -\beta\|x\|^2, \tag{57}$$

where β is some positive constant. Utilizing (51) yields a sufficient condition for (57) as follows

$$\dot V \le -\beta\|x\|^2, \text{ if } k_{\max} > \frac{1}{4}\left[\rho_1^2\left(\sqrt{\frac{V(t)}{\lambda_1}}\right)+\rho_2^2\left(\sqrt{\frac{V(t)}{\lambda_1}}\right)\right]. \tag{58}$$

Since $V(x(t)) \ge 0$ and $\dot V(x(t)) \le 0$, we conclude that

$$0 \ \le\ V(x(t)) \le V(x(0)) < \infty. \tag{59}$$

Using (51) and (59) yields a sufficient condition for (58) given by $\dot V \le -\beta\|x\|^2$, $k_{\max} > \frac14\rho_1^2\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\|x(0)\|\right)$ $+\frac14\rho_2^2\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\|x(0)\|\right)$.

From (59), we know that $V \in \mathcal{L}_\infty$, thus $e_{\mathrm{R}}, e_f, \frac{e_\varepsilon}{\sqrt{1-e_\varepsilon^T e_\varepsilon}}, \eta \in \mathcal{L}_\infty$. Since $\frac{e_\varepsilon}{\sqrt{1-e_\varepsilon^T e_\varepsilon}} \in \mathcal{L}_\infty$, we can conclude that $\|e_\varepsilon(t)\| < 1$ for all time. Next, use of the unit norm constraint $e_\varepsilon^T e_\varepsilon + e_\zeta^2 = 1$ with $\|e_\varepsilon(t)\| < 1$, $\forall t \ge 0$, reveals that $e_\zeta(t) \ne 0$, $\forall t \ge 0$. In addition, it follows from (29) that $\det(T) = e_\zeta$, using which, we conclude that T is invertible for all time. Since $e_{\mathrm{R}}, e_f, e_\varepsilon, \eta \in \mathcal{L}_\infty$, it follows from (35) that $v_0 \in \mathcal{L}_\infty$; hence, due to the bound of R_{d}, $\dot R_{\mathrm{d}}$, ε_{d}, and ω_{d}, we can use (4), (30), (38), (40), and (49) to conclude that $R, e_{\mathrm{V}}, V, \omega_{\mathrm{b}}, \omega, \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{e_\varepsilon}{\sqrt{1-e_\varepsilon^T e_\varepsilon}}\right), \dot e_f, \dot\eta \in \mathcal{L}_\infty$. Similar signal chasing arguments can now be employed to show that all other signals in the closed-loop system remain bounded.

Using (57) and (59), it can be easily shown that e_{R}, $\frac{e_\varepsilon}{\sqrt{1-e_\varepsilon^T e_\varepsilon}}$, e_f, $\eta \in \mathcal{L}_2$. Since we have already shown that e_{R}, $\frac{e_\varepsilon}{\sqrt{1-e_\varepsilon^T e_\varepsilon}}$, e_f, $\eta \in \mathcal{L}_\infty$, we can utilize Barbalat's Lemma to conclude that $\lim_{t\to\infty} e_{\mathrm{R}}(t)$, $\frac{e_\varepsilon(t)}{\sqrt{1-e_\varepsilon(t)^T e_\varepsilon(t)}}$, $e_f(t)$, $\eta(t) = 0$, which is then used to show that $\lim_{t\to\infty} e_\varepsilon(t) = 0$. Finally, using (30), (31), and (35) it is immediate that $\lim_{t\to\infty} e_{\mathrm{V}}, \omega = 0$. Thus, the result of Theorem 4.1 follows. $\square$

V. Conclusion

In this paper, we addressed an output feedback tracking control problem for a spacecraft with coupled translation and attitude motion when only translational position and attitude orientation measurements are available. A Lyapunov based tracking controller was designed with guaranteed semi-global, asymptotic stability for the position and velocity tracking errors. This control design methodology required only translation and attitude position measurements while estimating translation and angular velocity errors through a high pass filter.

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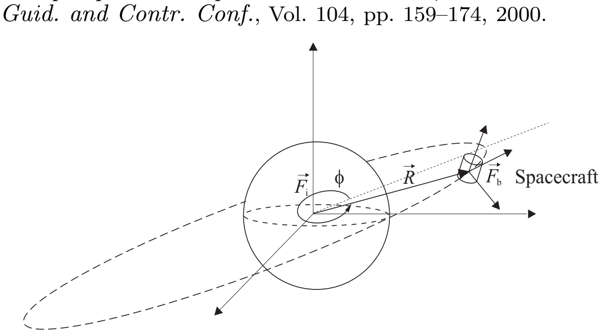

Fig. 1. Schematic representation of the spacecraft system