

The Attitude Control Problem

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Abstract—A general framework for the analysis of the attitude tracking control problem for a rigid body is presented in this paper. In contrast to the approach that feedback linearizes the attitude dynamics to a double integrator form with respect to some minimal representation of the orientation, a large family of globally stable control laws are obtained by using the globally nonsingular unit quaternion representation in a Lyapunov function candidate whose form is motivated by the consideration of total energy of the rigid body. The controllers share the common structure of a proportional-derivative feedback plus some feedforward which can be zero (the model-independent case), the Coriolis torque compensation, or an adaptive compensation. These controller structures are compared in terms of the requirement on the *a priori* model information, guaranteed transient performance, and robustness. The global stability of the Luh-Walker-Paul robot end-effector controller is also analyzed in this framework.

I. INTRODUCTION

THE orientation control of a rigid body has important applications from pointing and slewing of aircraft, helicopter, spacecraft, and satellites [1]–[8], to the orientation control of a rigid object held by a single or multiple robot arms [9]–[19]. In the case of robot arm control, the arms can be viewed as actuators maneuvering the attitude of the held object where the control loop is closed around the tip force (which acts to move the held object), and the joint torques are then selected to effect the desired tip force profile in a feedforward manner.

The Euler equation which describes the evolution of the orientation on the attitude configuration space, the rotation group $SO(3)$, is usually viewed as too complex to work with directly. For this reason, an additional compensation is sometimes done to place the attitude dynamics in a double integrator form with respect to some minimal 3-parameter representation of attitude [1], [7], [10]–[13]. Such an approach therefore introduces an additional level of complexity to the control law. Furthermore, this approach can never result in a control law which is globally stable on $SO(3)$, as there are

no globally nonsingular 3-parameter representations of the rotation group [6], [8]. When working with three-parameter representations of attitude, it is consequently necessary to avoid singularities of attitude representation. This complicates path planning and can seriously constrain admissible attitudes. This limit on the allowable motions is for purely mathematical reasons which have nothing to do with the physically admissible motions.

The research reported in this paper results from an investigation into the proper attitude error measures needed to have globally stable control on the rotation group when controlling the Euler equation, and into the proper formulation of, and tools for, the study of such control. The work reported here is motivated in part by the early work of [1] on spacecraft attitude control, which focused on appropriate attitude representations and adopted a Lyapunov approach, and by the recent developments in energy-like Lyapunov functions appropriate for control analysis of systems whose behavior arises from a Lagrangian analysis [20]. Our work is also closely related to that in [3]. In fact, the results given in this paper can be viewed as a natural extension of the work in [1], [3].

The unit quaternion, which uses the least possible number of parameters (four) to represent orientation globally, is chosen as the representation for the attitude error. Unit quaternions have been previously used in the robotics context in path planning [22] and, recently, their ability to represent attitude in a singularity-free way has gained appreciation in robot kinematics analysis [23], [24]. For spacecraft control analysis, unit quaternions have been applied in [1], [3]. The use of the energy Lyapunov function is not a new concept—it has appeared in the context of the stabilization of mechanical systems [25], [26], attitude control [2]–[5] and, more recently, robotics [20], [27]–[30]. With the novel use of a product term, first introduced in [20], this framework allows for robustness analysis with respect to parameter error, signal noise, and external disturbance (such as friction), robustness enhancement by using sliding-mode type of modification [31] and adaptive control [30]. The generality of this framework is demonstrated by the wide range of stabilizing control laws that have been obtained: from model-independent, proportional derivative (PD) tracking control, to model-dependent tracking control, and finally to adaptive control. The trade-off of controller complexity, achievable performance, and required *a priori* model information between these control laws can be rigorously quantified.

Similar considerations, but with a slightly different error measure, result in a second class of control laws which are “almost” globally asymptotically stable and which are exponentially stable in a neighborhood. This class of control laws

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contains the attitude control law first proposed in [9] as a special case. In [9], only a heuristic stability proof is given, which is based on the linearization about the nominal trajectory, so the work reported here gives the first rigorous analysis and justification for the attitude control law given in [9]. In [16], the generalization of the control law in [9] is claimed. However, this is not strictly true since [16] uses a different attitude error and a nonlinear feedback gain. The generalization here involves the original form of the controller as stated in [9]. The lack of global stability of the control law in [9] is also pointed out in [21].

Similar lines of inquiry to those in this paper have appeared in [3]–[5], [16]–[19]. A comparison between our approach and this related body of work is summarized below.

1) The stability of a large number of control laws is shown in this paper, including model independent control, model based tracking control, and adaptive control. Most of these control laws are new; the setpoint control laws (from a constant angular velocity to another constant angular velocity) in [3], and the setpoint control (zero desired angular velocity) in [4], [5] are special cases of the more general control laws developed here.

2) A cross term we added in the Lyapunov function is critical for the derivation of the new results in this paper. This technique has been used in robot control [20] but not in the attitude control literature. The energy motivated Lyapunov function has also been used in [3]–[5], [19], but without the cross term. An energy Lyapunov function is used in [17] for the angular velocity observer analysis, but a feedback linearizing feedforward compensation is used for control. Similarly, an energy Lyapunov analysis is used in [16] with feedback linearization. A complicated Lyapunov analysis is used in [18] to produce a globally asymptotically stable control law that is more complex than the ones presented here.

3) Global stability of the controllers in this paper is shown based on the globally nonsingular attitude parameterization provided by unit quaternions together with a Lyapunov analysis. Global stability for a special type of setpoint control is shown in [3]. Globally stable tracking control with feedback linearization is shown in [16], [17] (the latter also incorporated an angular velocity observer). Global stability for setpoint control is achieved in [4], [5], [18]. The global stability in [19] requires sufficiently high gain due to the use of the three-parameter representation of Gibb's vector. As noted in [3], no globally asymptotically stable control law that is also continuous on $SO(3)$ exists. Indeed, the controllers mentioned above are only continuous on either $S(3)$ or R^3 .

4) As pointed out in [3], unit quaternion feedback control also produces an unstable equilibrium (which also corresponds to the same attitude). This may lead to an undesirable situation where the rigid body reaches the desired attitude and then, with any arbitrarily small perturbation, turns 360° about some axis. We have derived a sufficient condition to avoid this situation.

5) Global stability (but only local asymptotic stability) of the control law in [9] is shown here. This control law is computationally efficient and performs almost identically to

the globally asymptotically stable control law for small errors.

6) In contrast to previous work, our development is completely coordinate-free; the actual implementation can be done in any coordinate frame depending on convenience.

The insights gained from this inquiry have proven fruitful in other contexts. The idea that the error measure should correspond to the topology of the error space, and that the energy terms in the Lyapunov functions should also conform to this topology, has lead to a new class of control laws appropriate for control of all-revolute joint manipulators whose joint space is the N -torus, T^N [31]. The energy Lyapunov technique itself has proven to be valuable for investigating control issues pertaining to constrained mechanical systems [33], [15], [32].

The rest of this paper is organized as follows. Section II provides the background of the attitude control problem: the representation issue on $SO(3)$, different attitude error measures, and the differential equations governing their evolution (kinematic and dynamic equations). Section III presents a family of stable tracking control laws based on the unit quaternion error measure. Three controller structures, model-independent, model-dependent, and parameter adaptive control structures, are analyzed in detail and their relative merits discussed. The global stability of control laws based on the vector quaternion error measure is also shown. Some simulation results are given in Section IV.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Notations

A basis-free vectorial perspective will be adopted throughout this paper. Let V be a normed linear vector space with dimension 3 and be endowed with an inner product, called the dot product. Bold face lowercase letters, e.g., v , are used to denote vectors, and bold face uppercase letters, e.g., L , are used to denote linear transforms (or linear operators) that act on such vectors.

Given a vector v , the crossproduct operation $v \times$ is a linear operator, and can be represented in a coordinate frame by an antisymmetric matrix \tilde{v}

$$\tilde{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \quad (2.1)$$

where (v_1, v_2, v_3) are the components of v in the given coordinate frame.

There are three coordinate frames of interest, denoted by “ o ” for the inertial frame, “ s ” for the body frame, and “ d ” for the desired body frame, respectively. For the coordinate frame representation of a vector, a leading superscript indicates the frame of reference. For the angular velocity ω , the subscripts ab in ω_{ab} means the angular velocity of the a -frame with respect to the b -frame. For the attitude matrix, the subscript frame is transformed to the superscript frame. For time derivatives of vectors, the frame with respect to which the derivative is taken is denoted by a trailing superscript of dt .

B. Representation of Attitude

A coordinate frame in V is given by a set of orthonormal basis vectors that obey the right hand rule, say, $\{e_{o1}, e_{o2}, e_{o3}\}$, and can be considered as a linear operator, $E_o: \mathbb{R}^3 \rightarrow V$ (a row of three vectors) defined by

$$E_o = [e_{o1}, e_{o2}, e_{o3}]. \quad (2.2)$$

The attitude operator for a rigid body is defined as a linear transformation taking an inertial reference frame, E_o , to a fixed body frame, E_s . This transform is of the form

$${}^oA_s = E_s E_o^* \quad (2.3)$$

where $*$ denotes the adjoint of a linear transform. It follows from the definition that ${}^sA_o = {}^oA_s^*$. The orthonormality of the basis vectors implies that oA_s is orthogonal, i.e., ${}^oA_s^* {}^oA_s = \mathcal{I}$ (\mathcal{I} is the identity operator). The right-hand convention of the basis vectors imply that the product of the eigenvalues of oA_s is equal to $+1$. Linear transforms having these properties form the manifold $SO(3)$. There are three eigenvalues for oA_s : $\{1, e^{j\phi}, e^{-j\phi}\}$. Call the eigenvector corresponding to the eigenvalue 1, k . By the Euler theorem, oA_s is equivalent to a rotation transformation about k over an angle ϕ .

The coordinate frame representation of oA_s in either E_o or E_s is the same and is known as the attitude matrix (equivalently, orientation matrix, direction cosine matrix, or rotation matrix) and is given by

$${}^oA_s = E_o^* E_s. \quad (2.4)$$

The columns of oA_s , are the basis vectors in E_s represented in the E_o frame. The rows of oA_s are the basis vectors in E_o represented in the E_s frame. There are 9 parameters in the attitude matrix, subject to 6 constraints imposed by the orthogonality. For manipulation, analysis, and implementation reasons, frequently it is simpler to use other representations. The minimal number of parameters needed to represent oA_s is 3 with no constraint. Some common minimal representations are: Euler angles, Gibbs vector, unit equivalent axis/angle, and vector quaternion. Minimal representations are only *locally* one-to-one and onto mappings of the attitude matrix, and there are always singular orientations (i.e., the Jacobian that maps differential changes in the representation to differential changes in the attitude matrix is singular for some orientations). The minimal number of parameters that can globally represent attitude without singularities is 4, with one constraint equation. The unit quaternion (or the Euler parameters) is a popular nonsingular four-parameter representation due to their desirable computational properties [8], [22].

The unit quaternion $q_A = \begin{bmatrix} q_{oA} \\ q_A \end{bmatrix}$ can be defined for the attitude operator, oA_s , as

$$q_{oA} = \cos \frac{\phi}{2} \quad q_A = \sin \frac{\phi}{2} k. \quad (2.5)$$

q_{oA} is called the scalar part of q_A and q_A is called the vector part of q_A . Clearly, $|q_{oA}|^2 + \|q_A\|^2 = 1$. oA_s can

be related to q_A through the Rodriguez formula [8]:

$${}^oA_s = 1 + 2q_A \times (q_A \times) + 2q_{oA}q_A \times. \quad (2.6)$$

Algorithms for computing q_A from oA_s can be found in [35]. In general, $\pm q_A$ both represent oA_s and this sign ambiguity must be resolved consistently during implementation. Many important properties of the quaternion representation are given in [1], [3].

C. Kinematic Equation of Attitude

The evolution of the attitude operator in time is governed by the rigid body kinematics equation. This equation, as well as the kinematic equations for other representations for the attitude operator, are summarized below (for a representation-dependent derivation of these equations for a specified coordinate frame, see, for example, [34], [8], [3]).

1) Attitude Operator:

$$\frac{d{}^oA_s}{dt^o} = \omega_{so} \times {}^oA_s \quad (2.7)$$

2) Unit Quaternion:

$$\begin{aligned} \frac{dq_{oA}}{dt} &= -\frac{1}{2} \omega_{so} \cdot q_A = -\frac{1}{2} q_A \cdot \omega_{so} \\ \frac{dq_A}{dt^o} &= \frac{1}{2} \omega_{so} q_{oA} + \frac{1}{2} \omega_{so} \times q_A = \frac{1}{2} (q_{oA} \mathcal{I} + q_A \times) \omega_{so}. \end{aligned} \quad (2.8)$$

As noted earlier, the unit quaternion, although globally nonsingular, contains a sign ambiguity in that (q_{oA}, q_A) and $(-q_{oA}, -q_A)$ represent the same attitude. In many quaternion extraction algorithms [35], the sign of q_{oA} is arbitrarily chosen positive. This approach is not used here, instead, the sign ambiguity is resolved by choosing the one that satisfies the associated kinematic differential equation. In implementation, this would probably imply keeping some immediate past values of the quaternion.

3) Minimal Representations:

$$J_{p_o}(p) \frac{dp}{dt^o} = \omega_{so} \quad (2.9)$$

where p is any minimal representation and J_{p_o} is the Jacobian that transforms the rate of change of p in E_o to the angular velocity. There always exists at least a configuration p such that J_{p_o} is singular, at which point, the representation is no longer valid and a different minimal representation that is not singular at the point may be used.

D. Representation of Attitude Error

Suppose a desired coordinate frame E_d is given. The control objective is to drive the body frame E_s toward E_d . Hence, we define the frame error as

$$\Delta E \triangleq E_s - E_d. \quad (2.10)$$

Note that ΔE is no longer a frame (since $\Delta E^* \Delta E \neq I$ in general). We now form an attitude error operator by post-multiplying ΔE by the adjoint of a reference frame. If the

reference frame is E_o , then the attitude error operator is

$$\Delta A_o = (E_s - E_d)E_o^* = {}^oA_s - {}^oA_d \quad (2.11)$$

if the reference frame is E_s , then the attitude error operator is

$$\Delta A_s = (E_s - E_d)E_s^* = \mathcal{J} - E_dE_s^* = \mathcal{J} - {}^sA_d. \quad (2.12)$$

Clearly, when either attitude error operator is zero, E_s coincides with E_d as required. We will mainly consider ΔA_s . Note that the second term in (2.12), $E_dE_s^* = {}^sA_d$, is an attitude operator (which rotates E_s to E_d). We call sA_d the relative attitude. Represented in the E_s frame, sA_d is

$$S \triangleq ({}^sA_d)_s = E_s^* {}^sA_d E_s = E_s^* E_d = {}^sA_d \quad (2.13)$$

and in the E_o frame

$$R \triangleq ({}^sA_d)_o = E_o^* {}^sA_d E_o = E_o^* E_d E_s^* E_o = {}^oA_d {}^sA_o. \quad (2.14)$$

The S representation has been used in [1], [3]–[5] and R representation used in [9] (without the quaternion kinematic equation). Here, we have shown that they are just different coordinate representations for the same linear operator sA_d .

Note that in both (2.11) and (2.12), the attitude error operator appears as the difference between two attitude operators. Motivated by this observation, in general, a measure of the attitude error will be taken as the norm of the difference between some representation of the each of the attitude operators. Some examples motivated by the ΔA_o case are

- o1. induced norm of ${}^oA_s - {}^oA_d$;
- o2. norm of $q_{oA_s} - q_{oA_d}$ on $\mathbb{R} \times \mathbb{V}$;
- o3. norm of $q_{oA_s} - q_{oA_d}$ on \mathbb{V} , using the unit quaternion representation;
- o4. norm of $p_{oA_s} - p_{oA_d}$ on \mathbb{V} , for any 3-parameter representation p_{oA_s} .

Some examples motivated by the ΔA_s case are

- s1. induced norm of $\mathcal{J} - {}^sA_d$;
- s2. norm of $q_s - q_{sA_d}$ on $\mathbb{R} \times \mathbb{V}$;
- s3. norm of $q_s - q_{sA_d}$ on \mathbb{V} , using the unit quaternion representation for sA_d .
- s4. norm of $p_s - p_{sA_d}$ on \mathbb{V} , for any 3-parameter representation p_{sA_d} .

Error measures motivated by ΔA_o are avoided in this paper since they are either unsuitable for our stability analysis (cases o1 and o3) or do not avoid the singularity of representation problem (case o4). A notable exception is case o2 above, which was used in [16] to derive a globally stable control law which, interestingly, can also be obtained by using case s2 of the ΔA_s error measures. In [7], case o4 is used and p is chosen to the vector part of the quaternion. A special maneuver (2-stage detumbling) is necessary to avoid representation singularities. In contrast to case s4 of the ΔA_s error measures, singularities of both oA_s and oA_d need to be avoided rather than just the singularities of a single attitude operator in $SO(3)$, i.e., sA_d . For these reasons, we will use principal error measures based on ΔA_s . In particular, 2-norms of cases s2–s3 are considered in detail and they provide a rich class of stable control algorithms; this

error measure is also used in [3]–[5]. Case s4 is used in [1]. In case s2, either sign in $q_{oA_s} = \pm 1$ can be used; without loss of generality, only the +1 case is considered. Some of these error measures are related, for example, the 2-norm of the case s3 error is the same as the Frobenius norm of the case s1 error. In cases s3 and s4, $q_s = p_s = \mathbf{0}$, hence, the attitude error is simply given by the norm of q or p .

E. Kinematic Equation for the Attitude Error

The evolution of sA_d in time can be obtained via direct differentiation in time with respect to the E_s frame

$$\frac{d{}^sA_d}{dt^s} = \frac{dE_d}{dt^s} E_s^* = \omega_{ds} \times E_d E_s^* = \omega_{ds} \times {}^sA_d. \quad (2.15)$$

Define the angular velocity error as $\Delta \omega \triangleq \omega_{so} - \omega_{do}$. By the additive property of the angular velocity, $\omega_{ds} = \omega_{do} - \omega_{so} = -\Delta \omega$. Equation (2.15) is of the same form as (2.7) except (o, s) in (2.7) is replaced by (s, d) . Let $q = \begin{bmatrix} q_o \\ q \end{bmatrix}$ be the unit quaternion representation for sA_d . Then the kinematic equation q can be obtained from (2.8) with (o, s) replaced by (s, d)

$$\begin{aligned} \frac{dq_o}{dt} &= -\frac{1}{2} \omega_{ds} \cdot q \\ \frac{dq}{dt^s} &= \frac{1}{2} q_o \omega_{ds} + \frac{1}{2} \omega_{ds} \times q. \end{aligned} \quad (2.16)$$

All the time derivatives can also be taken with respect to the E_o frame, resulting in the following kinematic equations:

$$\begin{aligned} \frac{d{}^sA_d}{dt^o} &= \omega_{so} \times {}^sA_d - {}^sA_d \omega_{so} \times + \frac{d{}^sA_d}{dt^s} \\ &= \omega_{so} \times {}^sA_d - {}^sA_d \omega_{so} \times + \omega_{ds} \times {}^sA_d \\ &= \omega_{do} \times {}^sA_d - {}^sA_d \omega_{do} \times \\ &\quad \times \frac{dq}{dt^o} = \omega_{so} \times q + \frac{dq}{dt^s} \\ &= \frac{1}{2} q_o \omega_{ds} - \frac{1}{2} \omega_{ds} \times q + \omega_{do} \times q. \end{aligned} \quad (2.17)$$

F. Dynamic Equation for the Attitude

Dynamics of the rotation of a rigid body is given by Euler's equation [6], [8]

$$\begin{aligned} \frac{dI\omega_{so}}{dt^o} &= I \frac{d\omega_{so}}{dt^s} + \omega_{so} \times I\omega_{so} \\ &= I \frac{d\omega_{so}}{dt^o} + \omega_{so} \times I\omega_{so} = \tau \end{aligned} \quad (2.18)$$

where I is the rigid body inertia, and the facts that $dI/dt^s = 0$ (due to the rigid body assumption) and that the time derivative of ω_{so} is the same when taken in either E_o or E_s .

If I is equal to the identity operator multiplied by a scalar, the Coriolis term $\omega_{so} \times I\omega_{so}$ vanishes. The dynamic equation then becomes

$$\dot{\omega}_{so} = v \quad (2.19)$$

for an effective control $v = I^{-1}\tau$. In general, this equation

can be obtained for a general I by using the so-called feedback linearization control (as in [9], [14])

$$\tau = Iv + \omega_{so} \times I\omega_{so}. \quad (2.20)$$

This method was used in [1], [9] where the control problem effectively reduces to the consideration of (2.19). The stability results to be presented in the next section—which hold for arbitrary I —hold for the exact linearization case also, by taking I to be the identity. The linearization procedure can be taken one step further to a minimal representation with [recall (2.9)]

$$v = J_p w + \frac{dJ_p}{dt^o} \frac{dp}{dt^o} \quad (2.21)$$

where w is now the effective control. Provided that J_p is nonsingular, the dynamics becomes a decoupled double integrator

$$\frac{d^2 p}{dt^{o^2}} = w. \quad (2.22)$$

This approach was used by [7], [10]–[12] (among many others) in the robotics literature.

G. Problem Statement

Let E_s and E_d be the actual and the desired coordinate frames of the rigid body and ω_{ds} their relative angular velocities. This paper considers the following problem:

Find a feedback control law $\tau = f(E_s, E_d, \omega_{so}, \omega_{do})$,

such that asymptotically $E_s \rightarrow E_d$ and $\omega_{ds} \rightarrow 0$.

Perfect and instantaneous measurement of E_s and ω_{ds} are assumed. The desired quantities E_d , ω_{do} , and $d\omega_{do}/dt^o$ (angular velocity and acceleration of E_d relative to the inertial frame E_o) are also assumed instantaneously available. $\omega_{do}(t)$ and $d\omega_{do}(t)/dt^o$ are assumed uniformly bounded in t .

III. GLOBALLY STABLE ATTITUDE TRACKING CONTROL LAWS

A. Introduction

We will show the *global* asymptotic stability of the zero equilibrium of the attitude trajectory error system for a family of control laws with the following general controller structure:

$$\tau = \text{proportional and derivative feedback} \\ + \text{feedforward compensation.} \quad (3.1)$$

The proportional feedback is in terms of the vector quaternion of the relative attitude, and the derivative feedback is in terms of the angular velocity error ω_{ds} . Together, their role is to ensure stability, correct for tracking error and reject disturbances. The feedforward term, on the other hand, is used to enhance the tracking performance (in terms of the maximum tracking error) by compensating for the plant dynamics. Three different types of feedforward compensation are considered

- 1) no compensation (a model independent control law);

- 2) full model based compensation with known parameters;
- 3) compensation with parameter adaptation (a parameter independent control law).

We will show that global asymptotic stability is achieved in each of these cases, but there is the classical trade-off between the achievable tracking performance and the amount of a prior available model information.

The main tool for the stability analysis is the Lyapunov direct method with a judicious choice of the Lyapunov function candidate. This choice is motivated by the consideration of the total energy in the system; it consists of three terms: the kinetic energy error (kinetic energy with the angular velocity error substituted for the angular velocity), an artificial potential energy and a product term of angular momentum, and the position error. The product term is chosen small enough so that the Lyapunov function candidate is positive definite. The purpose of this term is to establish local exponential convergence and to facilitate the generalization to adaptive control. Without the product term, stability can still be shown by using the invariance principle [36], but neither the local exponential rate of convergence nor the generalization to adaptive control are possible. Motivation of the product term is based on the work in [20].

In order to establish a global result, the globally nonsingular unit quaternion is used to parameterize $SO(3)$. We will consider a potential energy based on the 2-norm of the unit quaternion of the relative attitude sA_d resulting in control laws that are globally asymptotically stabilizing. For comparison, we also consider an artificial potential energy based on the vector part of the quaternion of sA_d ; the resulting control laws are globally stable but only almost globally asymptotically stable. They also have the drawback of turning off at error angle of 180° . The control law in [9] is a special case of this class of control laws.

The following notation is used for various bounds needed in the stability proofs:

$$\gamma_I \triangleq \|I\| \\ \gamma_d \triangleq \sup_{t \geq 0} \|\omega_{do}(t)\|.$$

Euclidean vector norm is used throughout this paper. The norm of a tensor is defined in the sense of the induced norm. All of the control laws to be presented below will be written in the vectorial form. Either the body frame or the inertial frame can be used for the implementation.

B. Model-Independent Control Law

By using the unit quaternion error measure, the PD controller is shown in this section to be globally stabilizing for a class of desired trajectories. This result is a generalization of the setpoint control laws in [3] (with zero final desired angular velocity) and [4], [5]; in fact, if the desired trajectory is a step change in attitude, then we obtain those results in [3]–[5].

Theorem 1: Consider the following control law:

$$\tau = k_p q - k_v \Delta \omega \quad (3.2)$$

where k_p, k_v are positive scalar constants. Let

$$\rho_1(t) \triangleq \left\| \frac{d\omega_{do}(t)}{dt^o} \right\| + \|\omega_{do}(t)\|^2. \quad (3.3)$$

If $\rho \in L_2[0, \infty) \cap L_\infty[0, \infty)$, then $q(t)$ and $\Delta\omega(t) \rightarrow 0$ as $t \rightarrow \infty$.

Proof: Consider the following scalar function:

$$V = (k_p + ck_v)((q_o - 1)^2 + q \cdot q) + \frac{1}{2} \Delta\omega \cdot I \Delta\omega - cq \cdot I \Delta\omega. \quad (3.4)$$

V can be bounded below by

$$V \geq \begin{bmatrix} \|q\| \\ \|\Delta\omega\| \end{bmatrix}^T P_c \begin{bmatrix} \|q\| \\ \|\Delta\omega\| \end{bmatrix} \quad (3.5)$$

where

$$P_c = \frac{1}{2} \begin{bmatrix} 2(k_p + ck_v) & c\gamma_I \\ c\gamma_I & \mu_I \end{bmatrix} \quad (3.6)$$

μ_I is defined as $\inf_{\|v\|=1} v \cdot I v$, which is positive since the inertia is positive definite. For c sufficiently small, P_c is positive definite. Now, compute the time derivative of V along the solution trajectory:

$$\begin{aligned} \dot{V} = & -(k_p + ck_v)q \cdot \Delta\omega + \Delta\omega \cdot \left(\tau + \omega_{do} \times I \omega_{do} - I \frac{d\omega_{do}}{dt^o} \right) \\ & - c \left(\frac{1}{2} \Delta\omega \times q - \frac{1}{2} q_o \Delta\omega + \omega_{do} \times q \right) \cdot I \Delta\omega \\ & - cq \cdot \left(\tau - \omega_{so} \times I \omega_{so} - I \frac{d\omega_{do}}{dt^o} \right) \\ & - cq \cdot (\omega_{so} \times I - I \omega_{so} \times) \Delta\omega. \end{aligned}$$

Now substitute in the control law (3.2), then

$$\begin{aligned} \dot{V} = & -ck_p \|q\|^2 - k_v \|\Delta\omega\|^2 + (\Delta\omega - cq) \cdot \left(-\omega_{do} \times I \omega_{do} - I \frac{d\omega_{do}}{dt^o} \right) \\ & - c \Delta\omega \cdot I \left(\frac{1}{2} \Delta\omega \times q - \frac{1}{2} q_o \Delta\omega + \omega_{do} \times q \right) \\ & - cq \cdot (\Delta\omega \times I \Delta\omega + (\omega_{do} \times I - I \omega_{do} \times) \Delta\omega) \\ \leq & -x^T Q_c x + w^T x \\ \leq & -\lambda \|x\|^2 + \rho(t) \|x\| \end{aligned} \quad (3.7)$$

where

$$x \triangleq \begin{bmatrix} \|q\| \\ \|\Delta\omega\| \end{bmatrix} \quad (3.8)$$

$$Q_c \triangleq \begin{bmatrix} ck_p & \frac{3}{2} \gamma_I \gamma_d c \\ \frac{3}{2} \gamma_I \gamma_d c & k_v - 2c\gamma_I \end{bmatrix} \quad (3.9)$$

$$w \triangleq \gamma_I \left(\left\| \frac{d\omega_{do}}{dt^o} \right\| + \|\omega_{do}\|^2 \right) \begin{bmatrix} 1 \\ c \end{bmatrix} \quad (3.10)$$

$$\rho \triangleq \sqrt{1 + c^2} \gamma_I \left(\left\| \frac{d\omega_{do}}{dt^o} \right\| + \|\omega_{do}\|^2 \right) \quad (3.11)$$

and $\lambda \triangleq \lambda_{\min}(Q_c)$ denotes the minimum eigenvalue of Q_c . For c sufficiently small, Q_c is positive definite (i.e., $\lambda > 0$). Now, integrate both sides of (3.7), then we have

$$V_t - V_0 \leq -\lambda \int_0^t \|x(s)\|^2 ds + \int_0^t \rho(s) \|x(s)\| ds$$

which can be further written as

$$\lambda \int_0^t \|x(s)\|^2 ds - \int_0^t \rho(s) \|x(s)\| ds \leq V_0. \quad (3.12)$$

By moving the second term on the left-hand side of (3.12) to the right-hand side and applying Schwarz inequality, we have (the assumption $\rho_1 \in L_2$ is used here)

$$\lambda \|x\|_{L_2}^2 \leq V_0 + \|\rho\|_{L_2} \|x\|_{L_2}.$$

After some more algebra (completing the square involving $\|x\|_{L_2}$), we obtain a bound on the $\|x\|_{L_2}$

$$\|x\|_{L_2} \leq \left[\frac{1}{\lambda} \left(V_0 + \frac{\|\rho\|_{L_2}^2}{4\lambda} \right) \right]^{1/2} + \frac{\|\rho\|_{L_2}}{2\lambda}. \quad (3.13)$$

Hence, $x \in L_2[0, \infty)$. Substitute (3.13) into (3.12), it follows that V along the solution trajectory is uniformly bounded in t . From the kinematic and dynamic equations, \dot{x} is also uniformly bounded, and, therefore, x is uniformly continuous. It follows by Barbalat's theorem [37], that $x(t) \rightarrow 0$ as $t \rightarrow \infty$. The above proof hinges on choosing c sufficiently small so that P_c and Q_c are both positive definite. Since c is not implemented in the control law, it is a free parameter that can be chosen small enough to satisfy the required conditions. For given gains, k_p and k_v , there is a permissible range of c , which in turn determines the convergence rate. ■

When the initial condition is known and the desired trajectory is planned so that there is no initial tracking error, i.e., $V_0 = 0$, we have the following uniform upperbound for V_t :

$$V_t \leq \|\rho\|_{L_2} \|x\|_{L_2}. \quad (3.14)$$

From (3.13), this bound shrinks with large λ , which is in turn determined by the PD gains, k_p and k_v . Thus, as expected, higher gains imply better tracking performance. Practically, there are limits to the level of gains, determined by, for example, sampling, actuator saturation, noise etc. Therefore, a nonzero transient tracking error may be incurred even with $V_0 = 0$.

The control law (3.2) do not depend on any model information, in contrast to the feedback linearization approach in (2.20) where a nonlinear compensation term of the form $\omega \times I \omega$ always needs to be present. However, the achievable performance (as measured by the maximum tracking error) for a given set of gains depends on the body inertia.

This theorem is most useful for the slewing types of

operation, where the problem is to move from the initial attitude to a goal attitude with certain desired transient response. In that case, the L_2 requirement of $\rho_1(t)$ can be trivially satisfied.

For the setpoint control case, $\omega_{do}(t) \equiv 0$, $d\omega_{do}(t)/dt \equiv 0$, which is also considered in [3]–[5], $\rho_1 \in L_2$ is trivially satisfied. Therefore, the zero equilibrium is globally asymptotically stable. Furthermore, $V_t \leq V_o$ and

$$\tau \leq \max(k_p, k_v) \|x\| \leq \max(k_p, k_v) \sqrt{\frac{V_o}{\lambda_{\min}(P_c)}}. \quad (3.15)$$

Since k_p and k_v only need to be positive, τ can be made arbitrarily small, uniformly in t , as pointed out in [3].

C. Model-Dependent Control Law

The lack of model information implies a nonzero transient tracking error in the simple PD control law in the previous section. In many instances, high feedback gains are not possible due to practical constraints, yet good tracking performance is still demanded. The result below shows that this is possible by incorporating the structure of the model in the control law. Note that the restriction placed on the class of desired trajectories in Theorem 1 is removed here. This result is a generalization of the control law in [3], where the desired angular velocity is a constant vector.

Theorem 2: Consider the following control laws:

$$\tau = k_p q - k_v \Delta\omega + I \frac{d\omega_{do}}{dt} + z_1 \times I z_2 \quad (3.16)$$

where z_1, z_2 can be either ω_{so} or ω_{do} , and k_p, k_v are positive scalar constants. If

$$k_v > a_1 \gamma_I \gamma_d \quad (3.17)$$

with a_1 a constant depending on z_1 and z_2 as follows:

z_1	z_2	a_1
ω_{do}	ω_{do}	0
ω_{do}	ω_{so}	1
ω_{so}	ω_{do}	0
ω_{so}	ω_{so}	1

then $q(t) \rightarrow 0$ and $\Delta\omega(t) \rightarrow 0$ as $t \rightarrow \infty$.

If $q_o(t) \rightarrow +1$, as $t \rightarrow \infty$, then the convergence is of exponential rate. A sufficient condition for this to happen is

$$\frac{1}{2} \Delta\omega(0) \cdot I \Delta\omega(0) < 2k_p(1 + q_o(0)). \quad (3.18)$$

Proof:

1) The proof is similar to the proof of Theorem 1. Consider the Lyapunov function candidate in (3.4). Take the derivative along the solution as before, and substitute in the control law (3.16), then we have

$$\dot{V} \leq -x^T Q_c x \quad (3.19)$$

where Q_c is now

$$Q_c = \begin{bmatrix} ck_p & \frac{1}{2} a_2 \gamma_d \gamma_I c \\ \frac{1}{2} a_2 \gamma_d \gamma_I c & k_v - a_1 \gamma_I \gamma_d - a_3 \gamma_I c \end{bmatrix} \quad (3.20)$$

where

z_1	z_2	a_2	a_3
ω_{do}	ω_{do}	3	2
ω_{do}	ω_{so}	0	1
ω_{so}	ω_{do}	1	1
ω_{so}	ω_{so}	1	2

Hence, if k_v satisfies (3.17), then there exists a range of c sufficiently small so that Q_c is positive definite. Again by using the Barbalat's theorem, it follows that $x(t) \rightarrow 0$ as $t \rightarrow \infty$ as stated.

2) It remains to show the exponential rate of convergence.

When $\|q\| \rightarrow 0$, q_o tends to either $+1$ or -1 . Suppose $q_o \rightarrow +1$. Then there exists some finite time T such that $q_o(t) \geq 0$ for all $t \geq T$. Since $|q_o| \leq 1$, for $t \geq T$

$$\|q\|^2 = 1 - q_o^2 \geq 1 - q_o \geq (1 - q_o)^2.$$

Therefore

$$\begin{aligned} \|q\|^2 &= \frac{1}{2} \|q\|^2 + \frac{1}{2} \|q\|^2 \\ &\geq \frac{1}{2} (\|q\|^2 + (1 - q_o)^2). \end{aligned}$$

Then for all $t \geq T$, there exists $\lambda > 0$ such that

$$\dot{V} \leq -\lambda V.$$

Hence, $\|q\|$ and $\Delta\omega \rightarrow 0$ exponentially. If $q_o \rightarrow -1$, no such conclusion can be drawn. In that case

$$V \rightarrow 4(k_p + ck_v).$$

If

$$V(0) < 4(k_p + ck_v) \quad (3.21)$$

then this situation ($q_p \rightarrow -1$) cannot occur for V is nonincreasing. Since c is not implemented and can be chosen arbitrarily small, (3.21) can be replaced by

$$\begin{aligned} \frac{1}{2} \Delta\omega(0) \cdot I \Delta\omega(0) &< k_p (4 - (1 - q_o(0))^2 - \|q(0)\|^2) \\ &= 2k_p(1 + q_o(0)). \end{aligned} \quad (3.22)$$

Hence, (3.18) is a condition of exponential convergence of $V(t)$ to zero. ■

The desired torque term, $d\omega_{do}/dt$, in the feedforward compensation can be replaced by $d\omega_{do}/dt^s$, with the same stability result except that the constant a_1 in (3.17) is re-

placed by

$$\begin{array}{ccc} z_1 & z_2 & a_1 \\ \omega_{do} & \omega_{do} & 1 \\ \omega_{do} & \omega_{so} & 0 \\ \omega_{so} & \omega_{do} & 1 \\ \omega_{so} & \omega_{so} & 0. \end{array}$$

The model-independent control law in Theorem 1 does not require any model information and therefore can be applied to the attitude control of any rigid body. However, when high performance is sought without the expense of very high feedback gains, model-dependent control laws are helpful. From the proof, $V_t \leq V_o$ for all $t \geq 0$. Therefore, if the initial tracking error is zero ($V_o = 0$), V_t will remain zero (for the model independent case, the bound of V_t is given by (3.14) and (3.13), which is nonzero). The nonlinear compensation term has a similar form as the crossproduct term in the model dynamics; indeed, in one case, this term is the output of the inverted plant driven by ω_{do} . In the next section, we will discuss the adaptive form the model-dependent tracking control laws with the inertia replaced by its estimate.

In Theorem 2, q_o may converge to either $+1$ or -1 . We will show later in Section III-E that $q_o = -1$ corresponds to an unstable equilibrium (this is also pointed out in [3]). The case that $q_o \rightarrow -1$ is clearly undesirable, as any small perturbation will cause a rotation of 360° to $q_o = +1$. This situation is avoided if condition (3.18) is satisfied by choosing k_p large enough or $\Delta\omega(0) = 0$ (for the case considered in [3], $\Delta\omega(0) \neq 0$).

The exponential convergence condition (3.18) also implies robustness with respect to the inertia in the control law. For the setpoint control case, this result reduces to the robustness result in [5]. Suppose the inertia I in (3.16) is perturbed from the true inertia, i.e., $I = I_{true} + \Delta I$, and $\|\Delta I\| = \eta_I$. Then the Lyapunov derivative of V can be bounded by

$$\dot{V} \leq -\lambda V + \frac{\rho \eta_I}{\lambda p} \sqrt{V} \quad (3.23)$$

where ρ is defined as in Theorem 1, and $\lambda p \triangleq \lambda_{\min}(P_c)$. Note that ρ is only assumed to be uniformly bounded but *not* necessarily in $L_2[0, \infty)$ (the L_2 assumption is only needed for the model independent control). Let W be the positive root of $W^2 = V$. Then

$$\dot{W} \leq -\frac{\lambda}{2} W + \frac{\rho \eta_I}{2 \lambda p} \quad (3.24)$$

which gives the bound

$$W(t) \leq e^{-(\lambda/2)t} W(0) + \frac{\rho \eta_I}{\lambda \lambda p} (1 - e^{-(\lambda/2)t}). \quad (3.25)$$

Hence, the system remains Lagrange stable with the ultimate bound linear in the size of the inertia error.

Comparison between Theorem 2 with the results in [16] reveals the relative strength and weakness of the two approaches. Both approaches are based on using a globally nonsingular parameterization, the unit quaternion. The atti-

tude error measure in the stability analysis in [16] uses the S error of (2.13), and the generalization to the R case of (2.14) is not obvious. Here both the R and S types of error measure are treated in the same framework. The controller in [16] allows positive definite matrix PD feedback gains, in contrast to our scalar gain requirement (however, this can be ameliorated by a more complex control law; see Section III-E later). This may be important if the response time is different in different directions (due to perhaps highly different inertia parameters). Explicit cancellation of the nonlinear terms is required in [16]. This is a consequence of the choice of unweighted 2-norm of $\Delta^s \omega$ in the Lyapunov function. By using the energy-type Lyapunov function proposed here, similar results to Theorem 2 may be obtained.

D. Adaptive Control

In the model dependent control law (3.16), if the inertia matrix is unknown or poorly known, we can replace it by an estimate and update the estimate by a simple gradient scheme. The result below shows global asymptotic stability in this adaptive scheme. We adopt the following notations: Given a symmetric 3×3 matrix W , define

$$v(W) = [W_{11}, W_{12}, W_{13}, W_{22}, W_{23}, W_{33}]^T. \quad (3.26)$$

For arbitrary vectors, a, b , define h from the relation

$$a^T W b = h^T(a, b) v(W). \quad (3.27)$$

Theorem 3: In Theorem 2, suppose I is replaced by the estimate \hat{I} , and ${}^s I$, the body frame representation of \hat{I} is updated by

$$\frac{dv({}^s \hat{I})}{dt} = -K_I \left[h \left(\Delta^s \omega - c q_s, {}^s \left(\frac{d\omega_{do}}{dt^o} \right) \right) - h({}^s \tilde{z}_1 (\Delta^s \omega - c^s q), {}^s z_2) \right] \quad (3.28)$$

for any $K_I > 0$, and ${}^s z_1, {}^s z_2$ can be either ${}^s \omega_{so}$ or ${}^s \omega_{do}$. If c is chosen sufficiently small, and k_v is chosen sufficiently large in the sense of (3.17), then $q(t)$ and $\Delta\omega(t) \rightarrow 0$ as $t \rightarrow \infty$.

Proof: Define

$$\Delta^s I \triangleq {}^s \hat{I} - {}^s I. \quad (3.29)$$

Introduce a new Lyapunov function candidate, V_1 , as the sum of the previous Lyapunov function candidate (used in Theorem 2) and an estimation error term

$$V_1 = V + \Delta V$$

where V is given by (3.4) and ΔV is given by

$$\Delta V = \frac{1}{2} v({}^s \hat{I})^T K_I^{-1} v({}^s \hat{I}).$$

Because of the estimated inertia instead of the true inertia in the control law, \dot{V} now contains the following additional term:

$$(\Delta\omega - c q) \cdot \left(z_1 \times \Delta \hat{I} z_2 + \Delta \hat{I} \frac{d\omega_{do}}{dt^o} \right)$$

which, in the body frame, can be written as

$$v^T(\Delta^s I) \left[h \left(\Delta^s \omega - c^s q, {}^s \left(\frac{d\omega_{do}}{dt} \right) \right) - h \left({}^s \tilde{z}_1 (\Delta^s \omega - c^s q), {}^s z_2 \right) \right].$$

The derivative of ΔV is

$$v^T(\Delta^s I) K_I^{-1} \frac{dv({}^s \hat{I})}{dt}.$$

With the chosen adaptation law (3.28), these two extra terms cancel with each other exactly. It follows then

$$\dot{V}_1 \leq -x^T Q_c x. \quad (3.30)$$

As in Theorem 2, for c sufficiently small (but independent of the initial condition), Q_c is positive definite, and the Barbalat's theorem can be used to show that $x(t) \rightarrow 0$ as $t \rightarrow \infty$ as stated. ■

We have exploited the fact that I in the body frame is symmetric and, therefore, only 6 parameters are updated. In general, ${}^s I - {}^s \hat{I}$ does not converge to zero (except under a sufficient excitation condition), and ${}^s \hat{I}$ need not be positive definite as does ${}^s I$.

In the proof for the aforementioned theorem, the parameter c needs to be sufficiently small to guarantee that P_c (in (3.6)), and Q_c in (3.9) are positive definite. In contrast to the model independent control laws (3.2) and the model based control laws (3.16), c is implemented in the control law for the adaptive case, and it should be chosen small enough to ensure stability. In contrast to the robot control case [20], [30], the allowable size for c is not dependent on the initial condition, so the stability result here is a global one for c chosen as a small enough constant.

E. Discussion

In Theorems 1 through 3, we have obtained a large class of control laws with the basic structure as in (3.1) and we have proved their *global* asymptotic stability in the closed-loop. This global result is achieved by combining a globally nonsingular parameterization (quaternion) with a global stability analysis tool (Lyapunov's direct method). In this section, we will discuss some additional properties of this class of controllers.

The choice of the Lyapunov function candidate (3.4) is clearly the foundation of all our results. Some important features of this choice should be noted.

1) The position error is measured by the Euclidean norm of the unit quaternion of the relative attitude ${}^s A_d$. This representation is globally nonsingular.

2) The velocity error is a quadratic form weighted by the inertia matrix (the Riemannian metric). It is like the kinetic energy of the rigid body except that the angular velocity is replaced by the angular velocity error. In the setpoint control case ($\omega_{do} = 0$), this term is simply the kinetic energy. This choice of velocity error avoids the need to explicitly compensate for the nonlinear Coriolis term (as required in [16]).

3) There is a product term between position and velocity errors weighted by a small constant c . This term produces a

$-c\|q\|^2$ term in \dot{V} , which combines with the $-\|\Delta\omega\|^2$ term to form the $-\|x\|^2$ term. This allows us to show exponential stability in model independent and model-dependent control laws. In the latter case, we can also assert robustness with respect to the inertia matrix. Our development of the adaptive control also depends crucially on the $-\|x\|^2$ term in \dot{V} . In contrast to the the stability proof without the cross term, the invariance principle is no longer needed. The constant c needs to be sufficiently small in order for the stability property to hold (both for V to be positive definite and \dot{V} negative definite in x). In the nonadaptive cases, c is not implemented in the control law, therefore, it can be freely chosen to be sufficiently small for the analysis purpose. In the adaptive case, c needs to be chosen small enough to guarantee global asymptotic stability.

Instead of using the Riemannian metric for the angular velocity error, the Euclidean metric (the unweighted two-norm) can also be used to show global stability for the feedback linearizing control laws as discussed in (2.20).

We have used $q - q_d$ to form the attitude tracking error, where $q_d = [1 \ 0 \ 0 \ 0]^T$. But, as noted in Section II-E, the scalar quaternion part of q_d (the first element) can be either ± 1 . If q_{d_o} is set to -1 , the same stability analysis goes through in Theorems 1 through 3, provided the sign of the proportional (vector quaternion) feedback is changed to -1 and the sign of the cross term in V (cf. (3.4)) is changed to $+1$. The two different signs in the proportional feedback correspond to two opposite directions of rotation about the equivalent axis to reduce the error angle to zero. Unless the initial error angle is exactly π , clearly one of the rotations is over the shorter span. Which sign should one choose to effect a smaller range of rotation? To answer this, consider the initial error angle, angular velocity, and desired angular velocity to be very small so that local linearization (about $q_o = +1$, $q = 0$, $\omega = 0$, $\omega_d = 0$) holds. In that case, the linearized kinematic and dynamic equations are:

$$\begin{aligned} \Delta\omega &= -2\dot{q} \\ -2I\dot{q} &= \tau + I\dot{\omega}_{do} \end{aligned}$$

where the “ $\dot{\cdot}$ ” is the derivative in either the inertial or body frame. By using any one of the controllers in Theorems 1 through 3, the closed-loop dynamics is determined by $2I\dot{q} + 2k_v q + k_p q$ which is exponentially stable. This procedure is also used in [3]. If the opposite sign in the proportional feedback is used, the linearized system is *unstable*, and the rigid body will turn completely around to reduce the error angle to zero. Of course, linearization can again be performed when tracking error has eventually become small, but now the linearization is about $q_o = -1$, and for that linearized system, negative proportional feedback produces an exponentially stable closed-loop system. The choice of positive proportional feedback may not *always* be the best choice. If the initial kinetic energy kicks the rigid body in the wrong direction of rotation, and it is so large that the error angle increases beyond π at some t , then negative proportional feedback may yield a faster convergence.

Another issue caused by the equivalent representation of q_d by $[\pm 1, 0, 0, 0]^T$ is that, in general, we only know that q

converges to zero by q_o may converge to either ± 1 . This is due to the fact that the closed-loop equilibria consist of

$$\mathbf{q} = \begin{bmatrix} \pm 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \Delta\omega = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

We now show that the equilibrium corresponding to $q_o = -1$ is unstable, i.e., there exists an arbitrarily small perturbation that will cause divergence from the equilibrium (this is also proved in [3]). Suppose that $q_o(t) \rightarrow -1$ as $t \rightarrow \infty$. Then V converges to a steady-state value of

$$V_{ss} = 4(k_p + ck_v) > 0.$$

Suppose q_o is perturbed to $q_o = -1 + \epsilon$ where ϵ is a small positive constant. Then, the perturbed V becomes (to the first order of ϵ)

$$V = 2(k_p + ck_v)(2 - \epsilon) = V_{ss} - 2(k_p + ck_v)\epsilon < V_{ss}.$$

But V is strictly decreasing as long as $\mathbf{q} \neq 0$ (cf. (3.19)). This implies that V will never return to V_{ss} . From Theorem 2, asymptotic stability is still assured, therefore q_o must converge to $+1$, and the convergence is of exponential rate. As noted in Section III-C, if k_p is sufficiently large or $\Delta\omega(0) = 0$, then q_o always converges to $+1$.

One drawback of the class of control laws presented so far is that the gains must be chosen as scalars. One can try to replace the unweighted two-norm of $\mathbf{q} - \mathbf{q}_d$ by a weighted two-norm of the form:

$$\begin{bmatrix} q_o - 1 \\ \mathbf{q} \end{bmatrix}^* \begin{bmatrix} k_p & 0 \\ 0 & \mathbf{K}_p \end{bmatrix} \begin{bmatrix} q_o - 1 \\ \mathbf{q} \end{bmatrix}.$$

After a little algebra, one can show that the derivative of this term along the solution is

$$\begin{aligned} \Delta\omega \cdot (-k_p \mathbf{q} - (\mathbf{K}_p - k_p \mathcal{J}) q_o \mathbf{q} + \mathbf{q} \times \mathbf{K}_p \mathbf{q}) \\ + 2\mathbf{q} \cdot \omega_{do} \times \mathbf{q} \quad \text{if } \mathbf{K}_p \text{ is constant in the "o" frame,} \\ \Delta\omega \cdot (-k_p \mathbf{q} - (\mathbf{K}_p - k_p \mathcal{J}) q_o \mathbf{q} - \mathbf{q} \times \mathbf{K}_p \mathbf{q}) \\ \text{if } \mathbf{K}_p \text{ is constant in the "s" frame.} \end{aligned}$$

The same stability results as in Theorems 1 through 3 can be therefore obtained if $k_p \mathbf{q}$ in the control laws are replaced by

$$k_p \mathbf{q} + (\mathbf{K}_p - k_p \mathcal{J}) q_o \mathbf{q} \mp \mathbf{q} \times \mathbf{K}_p \mathbf{q} \quad (3.31)$$

where the minus sign corresponds to a constant \mathbf{K}_p in the inertial frame and the plus sign to the constant \mathbf{K}_p in the body frame. The scalar gain case is recovered if \mathbf{K}_p is chosen to be a constant times the identity. A direct generalization by replacing the scalar gain by a positive definite matrix in the vector quaternion feedback is still under investigation. A related result was shown in [5] that \mathbf{K}_p can also be replaced by $(k_{p1} \mathbf{I} + k_{p2} \mathcal{J})^{-1}$ where k_{p1} and k_{p2} are positive scalars.

Now consider the case that k_v is replaced by an arbitrary positive definite matrix \mathbf{K}_v . If the ck_v term is removed in the Lyapunov function candidate in (3.4), then the only additional term introduced in V can be bounded by $c \|\mathbf{K}_v\| \|\Delta\omega\| \|\mathbf{q}\|$. Since c can be chosen arbitrarily small in the nonadaptive case and will be chosen sufficiently small in the adaptive case, this term does not affect the stability conclusion. Hence, in general, one can replace the scalar k_v in the control laws by any positive definite matrix.

Depending on the choices of \mathbf{z}_1 and \mathbf{z}_2 , (3.16) consists of four controllers. For the case of $\mathbf{z}_1 = \mathbf{z}_2 = \omega_{do}$, for feedforward compensation only depends on the desired trajectory; if it is preplanned, the feedforward term can be precomputed and then played back at run time. The trade-off between the memory space and real-time computational load is a useful one if a very high sampling rate is required (for example, for high bandwidth control).

For the model-based control law (3.16), the feedforward compensation is of the same structure as the plant dynamics. Other partial compensation schemes can also be used, without affecting the closed-loop asymptotic stability. For example, if only $I(d\omega_{do}/dt^o)$ is included, then we have a cross between the model independent case and the full model based case. With the same Lyapunov analysis, it is straightforward to show that perfect tracking is no longer possible when the initial tracking error is zero, but the maximum tracking error will be smaller than the model independent case, since the constant ρ (cf. (3.11)) is reduced in size.

It may appear that the control laws presented here create a continuous, globally asymptotically stable vector field on $SO(3) \times \mathbb{R}^3$. This is not possible, as pointed out in [3], [18]. Indeed, their implementation would require memory since the sign ambiguity in \mathbf{q} cannot be resolved from the attitude kinematic equation (2.7); the quaternion kinematic equation (2.8) must be used. This means that the control laws do not generate a vector field on $SO(3) \times \mathbb{R}^3$. However, on $S(3) \times \mathbb{R}^3$ ($S(3)$ is the unit sphere in \mathbb{R}^4 , where the quaternion lies), then we do have a globally asymptotically stable vector field in the closed-loop, i.e., consider (2.8) as the kinematics equation instead of (2.7).

It was shown in [17] that a stable observer for ω can be constructed based on the attitude information alone. Though the feedback linearizing control was used in [17], its observer can be combined with the results here to yield a large class of globally asymptotically stable control laws with only the attitude measurements.

When there is less than three independent actuators, the technique in this paper is no longer applicable. The difficulty lies in part in that the linearized system is not controllable, and indeed, no smooth locally asymptotically stable feedback law exists. This problem has been discussed extensively in [39], [40], [41]. The Lyapunov approach can still be useful (though not as useful as the three-actuator case) in this case. For example, it can be shown that stable rotation to any attitude can be achieved about the axis where actuation is available. Then any arbitrary attitude can be attained by connecting together no more than five such rotations. The detail of this result will be communicated in the future.

F. Controller Based on the Vector Quaternion Error Measure

The attitude error is zero when $q = 0$. Hence, in the artificial potential energy in the Lyapunov function candidate (3.4), the unit quaternion of the relative attitude can be replaced by the vector quaternion of the relative attitude. This gives rise to a slightly modified control law. This section will analyze the closed-loop stability for this case. We will only discuss the model-dependent tracking case, but the model independent and the parameter adaptive cases can be analyzed similarly.

The theorem below states the global stability property of this class of control laws. We will also show that the controller in [9] is a special case.

Theorem 4: Consider the following control laws

$$\tau = k_p q_o q + k_v \Delta \omega + \frac{dI\omega_{do}}{dt} + z_1 \times I z_2 \quad (3.32)$$

where z_1, z_2 , can be either ω_{so} or ω_{do} . If k_v satisfies the inequality (3.17), then $q_o(t)q(t)$ and $\Delta \omega(t) \rightarrow 0$ as $t \rightarrow \infty$. The equilibria that correspond to $(q_o = 0, \Delta \omega = 0)$ are all unstable.

If the initial condition satisfies

$$|q_o(0)|^2 > \frac{\Delta \omega(0) \cdot I \Delta \omega(0)}{k_p} \quad (3.33)$$

then $q(t)$ and $\Delta \omega(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$.

Proof: The proof is based on a slightly modified Lyapunov function candidate:

$$V = (k_p + ck_v)q \cdot q + \frac{1}{2}\Delta \omega \cdot I \Delta \omega - cq_o q \cdot I \Delta \omega. \quad (3.34)$$

The derivative of V along the solution is the same as (3.7), except in each of the expressions, c in the second and third terms is replaced by cq_o , and there are two additional terms

$$-(k_p + ck_v)(q_o - 1)q \cdot \Delta \omega - \frac{c}{2}\Delta \omega \cdot qq \cdot I \Delta \omega.$$

Define

$$z \triangleq \begin{bmatrix} |q_o| \|q\| \\ \|\Delta \omega\| \end{bmatrix}.$$

Then with the control law (3.32), \dot{V} can be bounded above by (in the same way as in the proof of Theorem 2):

$$\dot{V} \leq -z^T Q_c z \quad (3.35)$$

where Q_c is given by (3.20), except the 2-2 block contains an additional $-1/2\gamma_c c$ term. For c sufficiently small, Q_c is positive definite. Therefore, the asymptotic convergence of z to zero follows from the Barbalat's theorem.

When $z = 0$, either $q_o = 0$ or $\|q\| = 0$, with the latter being the desired equilibrium. If the former case is true, then $V \rightarrow 1/2(k_p + ck_v)$. Suppose q_o is now perturbed to $q_o = \epsilon$, where ϵ is an arbitrary constant. Then, the perturbed V

becomes

$$V = \frac{1}{2}(k_p + ck_v)(1 - \epsilon^2) < \frac{1}{2}(k_p + ck_v).$$

Since V is strictly decreasing, the trajectory will never return to any of the equilibria corresponding to $q_o = 0$, in other words, $q(t) \rightarrow 0$, as $t \rightarrow \infty$. Hence, the equilibria corresponding to $(q_o = 0, \Delta \omega = 0)$ are all unstable.

If the initial condition satisfies

$$\frac{1}{2}(k_p + ck_v) > V_0. \quad (3.36)$$

The above argument also shows that q converges to zero, otherwise, V would increase at some t , violating (3.35). Since c can be chosen arbitrarily small, it can be set to zero in (3.36), which then becomes (3.33). If (3.33) holds, q_o is uniformly bounded below. Then, $\|z\|^2$ can be bounded below by V , implying an exponential rate of convergence. ■

The control law (3.32) is globally stabilizing and the velocity tracking error goes to zero asymptotically. The closed-loop equilibria consist of the following two sets:

$$\begin{aligned} \omega_{so} = 0 \quad q_o = 1 \quad q = 0 \quad \text{and} \\ \omega_{so} = 0 \quad q_o = 0 \quad \|q\| = 1. \end{aligned}$$

The first equilibrium is the desired one, corresponding to zero tracking error. The rest are π away from the desired attitude about some axis. We have shown that the undesirable equilibria are all unstable, i.e., there exists an arbitrarily small perturbation which will cause divergence from these equilibria. Hence, the closed-loop stability is only slightly weaker than before, in that the global asymptotic stability is weakened to *almost* global asymptotic stability. Furthermore, in most tracking control applications, the initial kinetic energy error is zero (by choosing the initial desired velocity to be the actual initial velocity). Then (3.33) is trivially satisfied, implying exponential convergence to the desired equilibrium.

Even though global asymptotic stability is also achieved (almost), the performance in general is poorer for large error angles. To see this, write the proportional feedback term in (3.16) and (3.32) as

$$-\frac{k_p}{2} \sin \frac{\phi}{2} \cdot k \quad \text{for (3.16)} \quad (3.37)$$

$$-\frac{k_p}{2} \sin \phi \cdot k \quad \text{for (3.32)} \quad (3.38)$$

where ϕ and k are the equivalent angle and axis that represent the attitude error. For (3.32), the error feedback decreases with ϕ when ϕ is greater than $\pi/2$. In the extreme case of $\phi = \pi$, the proportional term is turned off completely. Simulation has also confirmed poor performance for large error angles. This deficiency is clearly avoided in the vector quaternion feedback case. However, when the error is small, this control law performs equally well as (3.16) since $\sin \phi \approx \sin \phi/2$ (or $q_o \approx 1$).

By using the relationship between the direction cosine and quaternion representations of R , (3.32) can be stated in an

equivalent form in terms of R :

$${}^o\tau = -\frac{k_p}{4}F(R) - k_v\Delta^o\omega + {}^oI^o\left(\frac{d\omega_{do}}{dt^o}\right) + {}^o z_1 {}^oI^o z_2 \quad (3.39)$$

where $F: SO(3) \rightarrow \mathbb{R}^3$ is a linear function defined as

$$F(A) = \frac{1}{2} \begin{bmatrix} A_{32} - A_{23} \\ A_{13} - A_{31} \\ A_{21} - A_{12} \end{bmatrix}. \quad (3.40)$$

If oA_s and oA_d are written as ${}^oA_s = [n, o, a]$ and ${}^oA_d = [n_d, o_d, a_d]$, then [9]

$$F(R) = n \times n_d + o \times o_d + a \times a_d. \quad (3.41)$$

This simplifies the computation of the attitude error feedback term in ${}^o\tau$ considerably. The stability analysis for this controller can also be carried out directly with R itself by replacing the desired potential energy terms in the Lyapunov functions with the Frobenius norm of $I - R$. The ease in computing R is a strong plus for the control law (3.39), since no quaternion extraction algorithm is needed to form the error feedback signal. The formula (3.41) does not hold for the S error, however.

The computation of q in the control laws in Theorems 1 through 3 requires the use of a quaternion extraction algorithm at sample rate. Since the sign ambiguity in quaternion is resolved by requiring the kinematic equation (2.8) to hold (rather than simply choosing the positive q_o , as in most of the existing algorithms), these control laws incur a greater computational cost than that in the control laws in Theorem 4 which, by using (3.41), can be more efficiently implemented.

The control law in (3.32) is of a very similar form to the resolved acceleration control in [9]. Indeed, if the inertial matrix is the identity, the controllers are exactly the same. Hence, Theorem 4 also serves as a rigorous justification of the resolved acceleration control law given in [9]. This proof is superior than the one given in [16] in which the error measure is required to be S (instead of R as in [9]); as stated in the previous remark, error feedback involving R has a more desirable computational property) and the feedback gain in nonlinear.

IV. EXAMPLE

A simple example is given, which is motivated by the one given in [7], to illustrate some of the results presented in this paper. The moment of inertia matrix in the body frame is given by

$${}^sI = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 0.63 & 0.0 \\ 0.0 & 0.0 & 0.85 \end{bmatrix}. \quad (4.1)$$

The (2, 2) element is chosen to be 0.63 instead of the 0.93 used in [7].

A choice of the attitude representation needs to be made for the simulation. If the nine-parameter attitude matrix is used, it is difficult to ensure that the six constraints imposed by the orthogonality are satisfied throughout the simulation.

If a minimal representation is used, there will be the singularity issue. We chose to use the unit quaternion kinematic equation (2.8). The unit norm constraint is enforced by updating \dot{q}_o through

$$\dot{q}_o = -\frac{q \cdot \dot{q}}{q_o}. \quad (4.2)$$

This method breaks down near $q_o = 0$, so when $|q_o|$ is small, it is updated by (2.8). In this way, the violation of the unit norm constraint due to the integration error can be kept small.

We will first compare the performance of point-to-point tracking between the model independent, model dependent, and adaptive controllers. Then the model dependent vector quaternion feedback control law (3.16) is compared to the vector and scalar quaternion product feedback control (3.32) for both tracking a periodic trajectory and setpoint control.

In terms of the equivalent axis and angle representation, the initial attitude sA_o for all cases is given as:

$$\begin{aligned} \text{equivalent axis} \quad k &= [0.4896, 0.2032, 0.8480]^T \\ \text{equivalent angles} \quad \phi &= 2.4648 \text{ rad } (141.22^\circ). \end{aligned}$$

The initial angular velocity is zero.

A. Comparison of Point-to-Point Tracking Performance

Suppose the desired trajectory is given by a rotation along the equivalent axis k of the initial attitude towards the asymptotic desired attitude $I_{3 \times 3}$. The desired rotational angle given by

$$\begin{aligned} \phi_d(t) &= \phi_f - (\phi_f - \phi_i)e^{-\alpha t^2} \\ {}^o\omega_{do}(t) &= \dot{\phi}_d(t) \cdot k = 2\alpha(\phi_f - \phi_i)te^{-\alpha t^2} \cdot k \\ {}^o\ddot{\omega}_{do}(t) &= \ddot{\phi}_d(t) \cdot k = 2\alpha(\phi_f - \phi_i)(1 - 2\alpha t^2)e^{-\alpha t^2} \cdot k \end{aligned}$$

where $\phi_f = 0$ and ϕ_i and k are the initial angle and axis.

We compare the performance of three tracking control laws: the model independent control (3.2), model dependent control (3.16), and adaptive control (3.28) (with $z_1 = z_2 = \omega_{do}$ for the latter two cases). The gains are selected to be $k_p = 4$ and $k_v = 8$. In the adaptive case, the adaptation gain is selected to be $K_f = 100$, and the initial inertia parameters (6 of them) are all chosen zero. The plot of the scalar part of the quaternion of the desired case versus the three controlled cases is shown in Fig. 1. The actual trajectory closely tracks the desired trajectory in all three cases. The tracking error (in terms of the scalar quaternion) is shown in Fig. 2. Perfect tracking is achieved, as expected, in the model dependent case. A maximum tracking error of $q_o = 0.012$ is incurred for the model independent case. This error is reduced by about 30% in the adaptive case. It has been noticed in simulation that this error can be further reduced by using higher adaptation gains.

The estimated inertia is initially set to zero. The eigenvalues converge to $(-0.0072, -0.0655, 0.2365)$, while the eigenvalues of the true inertia is $(0.63, 0.85, 1.00)$. As mentioned earlier, the convergence of the tracking error to zero

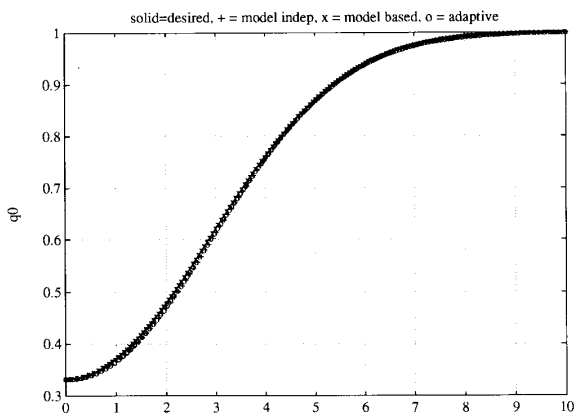


Fig. 1. Tracking comparison.

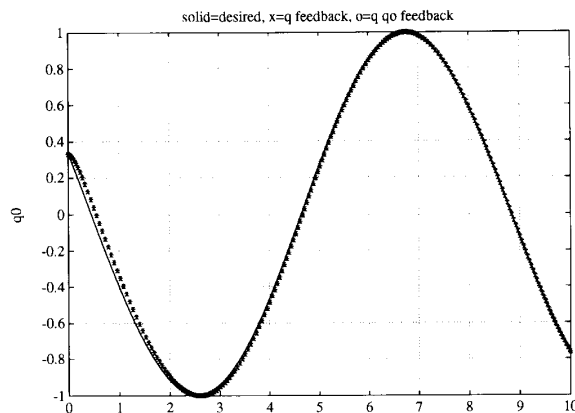


Fig. 3. Tracking comparison.

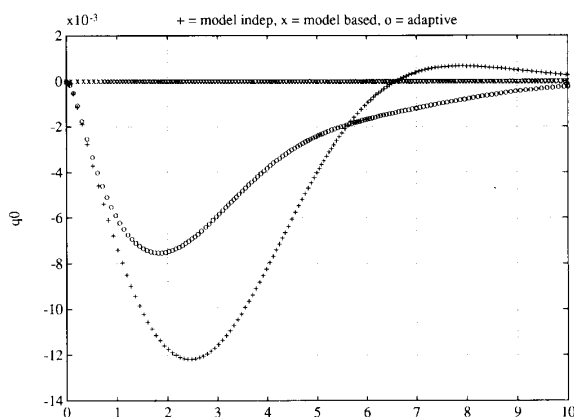


Fig. 2. Tracking error.

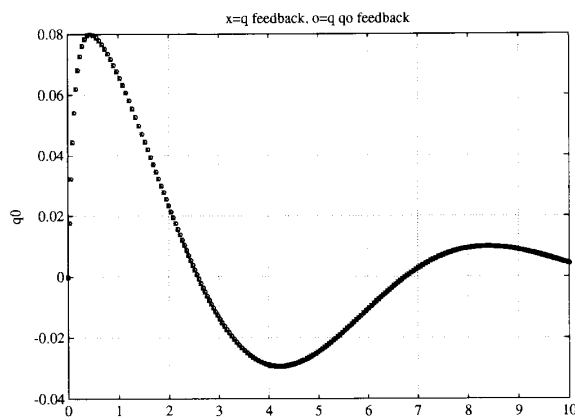


Fig. 4. Tracking error.

does not imply nor require the convergence of the parameter estimation error. If an accurate inertia estimate is desired, a sufficient excitation condition must be satisfied by the desired trajectory.

B. Comparison Between Control Laws in (3.16) and (3.32)

The control laws (3.16) and (3.32) are compared in this section. We first consider the tracking of a constant spin (with angular velocity 1.5 rad/s) about the initial equivalent axis. By choosing the initial desired attitude to be the true initial attitude, the initial attitude error is zero. However, there is an initial velocity error since the rigid body is initially at rest. Since the attitude tracking error remains small for all t (due to the small size of the initial error state), the two controllers are virtually indistinguishable; see Figs. 3 and 4. This demonstrates the usefulness of the control law (3.32) in the tracking context despite its sluggishness for large error angles. Coupled with its computational efficiency for the R type relative attitude, this may well be the control law of choice for the tracking application.

For the setpoint control over large angular range, the responses become quite different. As shown in Fig. 5, the response in the case of control law (3.32) is much slower than that of the quaternion feedback. This is due to the fact

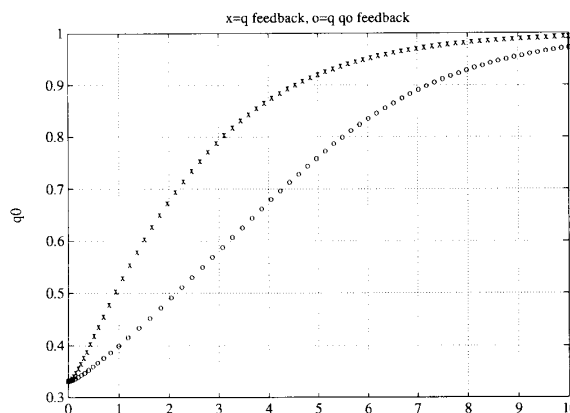


Fig. 5. Set point comparison.

that the proportional feedback term in (3.32) is the product of q_o and q and q_o in this case is initially small (around 0.33).

V. CONCLUSION

A general analytic framework based on the coordinate independent vectorial algebra for the stability analysis of a large family of globally stable tracking control laws is pre-

sented. The attitude error is represented as the difference between the actual body frame and the desired body frame with respect to the inertial frame. For the stability analysis, two measures of the attitude error are used: the Euclidean norm of the difference between the unit quaternions of the relative attitude sA_d and of the desired frame \mathcal{J} , and the Euclidean norm of the vector quaternion of sA_d . Based on an energy motivated Lyapunov function using each of these error measures, a family of control laws is constructed; and due to the global nature of the unit quaternion representation, the global stability of the closed-loop system can be shown. The results given in this paper can be viewed as a natural extension of the work begun in [1] and significantly advanced in [3].

The controller structure is of the form of proportional and derivative feedback and a feedforward compensation. The proportional term is either the vector quaternion feedback (for the unit quaternion error measure) or the vector quaternion and scalar quaternion product (for the vector quaternion error measure). In the first case, the zero error equilibrium is globally asymptotically stable. In the second case, there are two sets of equilibria, 180° apart about some axis. Only the desired one (zero error) is asymptotically stable and all the others are unstable. The control law in [9] is a special case of this class. The feedforward may be zero, a nonlinear compensation similar to the inverted plant, or the nonlinear compensation with adaptation for the rigid body inertia. The strength of the stability result, required model information, real-time computation load, maximum tracking error are some of the trade-offs between the different cases of feedforward compensation.

Current research in this area involves the generalization of the approach here to the attitude control of a rigid body with flexible appendages.

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