AUTONOMOUS AND ROBUST RENDEZVOUS GUIDANCE ON ELLIPTICAL ORBIT SUBJECT TO J_2 PERTURBATION

Emmanuel GOGIBUS [(1)], Hervé CHARBONNEL [(2)], Patrick DELPY [(3)]

[(1)] *Astrium Space Transportation, 66 route de Verneuil, 78133 LES MUREAUX Cédex, FRANCE, emmanuel.gogibus@astrium.eads.net*
[(2)] *Astrium Space Transportation, 66 route de Verneuil, 78133 LES MUREAUX Cédex, FRANCE, herve.charbonnel@astrium.eads.net*
[(3)] *Astrium Space Transportation, 66 route de Verneuil, 78133 LES MUREAUX Cédex, FRANCE, patrick.delpy@astrium.eads.net*

ABSTRACT

Until recently, the developed guidance algorithms for rendezvous were only able to deal with targets placed on circular or quasi-circular orbits. This is typically the case of the far rendezvous automatic on-board guidance of ATV (vehicle developed under ESA responsibility, with Astrium Space Transportation as prime contractor, that successfully performed an AR&D (Automated Rendezvous and Docking) in April 2008). Thanks to recent progress in the field of relative motion modelling, a guidance algorithm working on any elliptical orbit has been developed in [1].

The algorithm presented here cannot only cope with elliptical orbits but also takes into account the J_2 disturbance of the Earth gravitational field. Moreover, it is designed so as to minimize the consumption of the whole rendezvous by studying several ways to reach the rendezvous final state. The ΔV are considered instantaneous. This program was developed in the frame of the CNES study, called " Techniques de guidage autonome et robuste pour le rendez-vous entre 2 véhicules spatiaux " (Autonomous and robust guidance techniques for rendezvous between two spacecrafts), where Astrium Satellites and LAAS are prime contractors.

1. NOMENCLATURE

AR&C	=	Automated Rendezvous and Capture
AR&D	=	Automated Rendezvous and Docking
ATV	=	Automated Transfer Vehicle
CoM	=	Centre of Mass
GC	=	Guidance and Control
GNC	=	Guidance Navigation and Control
GTO	=	Geostationary Transfer Orbit
HARVD	=	Highly Autonomous RendezVous and Docking
LVLH	=	Local Vertical Local Horizontal
MCI	=	Mass, Centring and Inertia
WRT	=	With Respect To

2. INTRODUCTION

First researches on relative motion model were conducted by Clohessy and Wiltshire, in 1960, in [2]. They developed a relative motion modelling for spacecrafts placed on circular orbits. This method assesses the relative position and velocity of a chaser in a relative coordinate system whose origin is the target and whose directions are given by the local vertical and local horizontal directions. Given the initial position, velocity and true anomaly on a circular orbit, the final position and velocity at a given true anomaly are obtained by solving a linear stationary system of equations.

However, their equations are no longer valid as soon as one of the following hypotheses is not verified: the target is on circular or quasi-circular orbit, the gravitational potential is a Keplerian one and the differential of gravity can be linearized.

When the target is on an eccentric orbit, the relative motion of the chaser is described by nonlinear differential equations with periodic coefficients. The linearized equations are known as the Tschauner-Hempel equations and were developed in [3]. In that direction, another important research was driven by Carter to change the coordinate system in order to admit non-null eccentricity. The equations of motion are, however, more difficult to solve. Carter proposed a solution, in [4], that requires integrating some functions and uses the eccentric anomaly. The result is not simple to use in an engineering work and has a singularity for a circular orbit.

The solution was found by F. Ankersen and K. Yamanaka, in 2002, in [5]. They lead to a linear relative motion model removing the assumption made on the target orbit eccentricity. This model is displayed in the form of a state transition matrix. They made it possible to design a guidance algorithm able to work on elliptical orbits and keeping the simple design principles of the ATV one. Such a program has already been developed by Astrium Space Transportation, in [1], and then, used in the frame of the HARVD project (Highly Autonomous RendezVous and Docking), under ESA responsibility.

The next step in this field has been made by Gim and Alfriend, in 2003, in [6], by assuming the planet gravitational field is not any more Keplerian but includes a perturbation due to the planet oblateness and called J_2 perturbation. They developed a linear relative motion model that can, like the previous one, be presented in the form of a state transition matrix. This model is the one used by the new guidance algorithm presented here.

3. THE LINEAR RELATIVE MOTION MODEL

In this first part, one describes briefly the Gim and Alfriend relative motion model. Further information can be found in [6].

3.1. Relative state representation

Two ways to represent the chaser state are used : the relative osculating elements and the curvilinear coordinates. The relative osculating elements δe_{osc} are the differences between the osculating elements of the two spacecrafts. The elements chosen here are $e_{osc} = (a, \theta, i, q_1, q_2, \Omega)$ where a is the semimajor axis, θ the argument of latitude, i the inclination, Ω the longitude of the ascending node and q_1 and q_2 are defined by the Eq. 1.

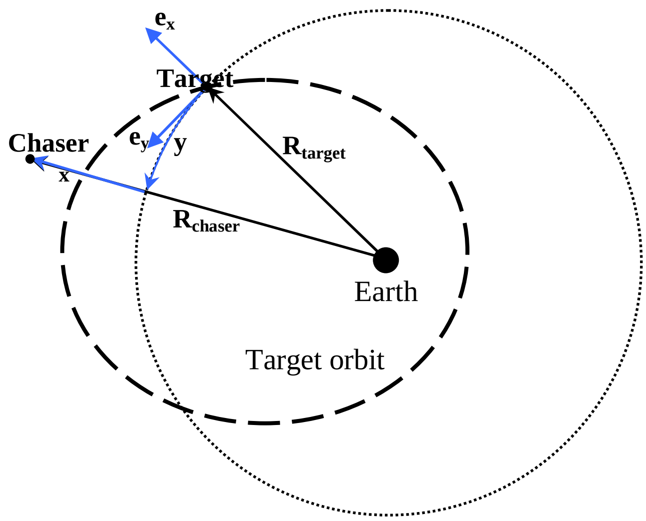

Fig. 1. The curvilinear frame

$$\begin{cases} q_1 = e\cos(\omega) \\ q_2 = e\sin(\omega) \end{cases} \qquad \textbf{(1)}$$

where e is the eccentricity and ω is the argument of perigee.

For the second way, the relative frame used here is not the usual LVLH (Local Vertical Local Horizontal) frame but a curvilinear frame, as shown in the Fig. 1. The frame position and orientation are time varying. The origin is the target CoM, x is the difference in the radii and y and z are the curvilinear distances along the imaginary circular orbit on the reference orbital plane and perpendicular to the reference orbit. The relative state is called X .

3.2. State transition matrix

One defines a first matrix Σ which is the linear transformation between the two state representations, as shown in the Eq. 2.

$$X(t) = \Sigma(t)\delta e_{osc}(t) \qquad \textbf{(2)}$$

Then, a second transformation matrix $D(t)$ is defined that allows getting the relative osculating elements from the relative mean elements, as shown in Eq. 3.

$$\delta e_{osc}(t) = D(t)\delta e_{mean}(t) \qquad \textbf{(3)}$$

The matrix $D(t)$ is computed thanks to the expressions Eq. 4-5 that provide an approximation of the osculating elements e_{osc} of a spacecraft, knowing its mean elements e_{mean} and its long and short period variations $e^{(lp)}$, $e^{(sp1)}$ and $e^{(sp2)}$.

$$e_{osc}(t) = e_{mean}(t) - J_2 R_e^2\left(e^{(lp)} + e^{(sp1)} + e^{(sp2)}\right) \qquad \textbf{(4)}$$

$$D(t) = \frac{\partial e_{osc}(t)}{\partial e_{mean}(t)} \qquad \textbf{(5)}$$

where J_2 is the J_2 disturbance constant and R_e is the Earth equatorial radius.

The core of the model is the state transition matrix $\overline{\Phi}_{\bar{e}}\left(t, t_0\right)$ that provides the chaser relative mean elements $\delta e_{mean}(t)$ at a date t, knowing the chaser relative mean elements $\delta e_{mean}(t_0)$, at a date t_0, as shown in the Eq. 6.

$$\delta e_{mean}(t) = \overline{\Phi}_{\bar{e}}(t, t_0)\delta e_{mean}(t_0) \qquad \textbf{(6)}$$

Eventually, we can get the chaser relative state $X(t)$, in the curvilinear frame, at a date t, knowing its relative state $X(t_0)$, at a previous date t_0, as shown in Eq. 7.

$$\boxed{X(t) = \Sigma(t)D(t)\overline{\Phi}_{\bar{e}}(t, t_0)D(t_0)^{-1}\Sigma(t_0)^{-1}X(t_0)} \qquad \textbf{(7)}$$

4. THE GUIDANCE ALGORITHM

4.1. Objectives

The usual task of the guidance algorithms for rendezvous is to compute the two boosts of the transfer that binds two consecutive way points of the rendezvous trajectory. This trajectory has been designed and optimized on ground, in order to minimize the overall consumption while meeting the mission requirements on time schedule, safety, etc. It manages only one transfer at a time. The linear motion model is used at this step. It provides us the 2 ΔV needed to the chaser to get from a first state (position/velocity) to another in a given duration. This is simply done by solving the linear equation Eq. 7, where 6 variables are known (the 3 position coordinates of $X(t_0)$ and the 3 position coordinates of $X(t)$) and 6 variables are unknown (the 3 velocity coordinates of $X(t_0)$ and the 3 velocity coordinates of $X(t)$). If the linear model is not precise enough, a more accurate model can be used, afterwards, so as to correct the computed ΔV and be sure they will drive the chaser at the right position, at the right date.

When the chaser trajectory during the rendezvous is not to follow a specific corridor or to respect predefined way points, one can give more autonomy to the guidance program. This can be done by building an algorithm that seeks itself the best trajectory and boosts to be performed so as to drive the chaser from its state at the beginning of the rendezvous to the state aimed at the end of the rendezvous in a given duration. The optimality is evaluated in terms of consumption. This is the objective of the guidance program presented here.

However, studying a rendezvous problem without a priori implies that many variables are to be taken into account and optimized:

- The number of ΔV to be performed
- The dates where these ΔV are applied
- The 3 components of each ΔV

The computations duration would be high and their convergence would not be guaranteed. Several simplifications have to be considered here.

4.2. Simplifications

One restrains the study to a rendezvous trajectory implying at most 3 ΔV. This means that the rendezvous is to be performed with one or two transfers. In this last case, the two transfers would be consecutive: the second ΔV of the first transfer would be added to the first ΔV of the second transfer and this sum would form only one ΔV.

The last ΔV is forced to be performed at the end of the rendezvous, in order to be sure the chaser will have the aimed velocity when reaching the aimed position.

The ΔV computation of the transfers boosts thanks to the linear equation Eq. 7 requires that the initial position, the final position and the duration of the transfers are known. This fact has consequences on the way the transfers are computed and optimized.

In the case of a rendezvous in one transfer, i.e. with 2 ΔV, its characteristics are fully determined by the choice of the date where the first ΔV is performed. Indeed, its state at the beginning of the transfer only depends on its state at the guidance call and on the drift duration before this transfer since no ΔV is performed before this date. Its state at the end of the transfer is an input to the program. And the transfer duration is easily deduced from the rendezvous duration and the date of the first ΔV. This kind of rendezvous is shown in Fig. 2.

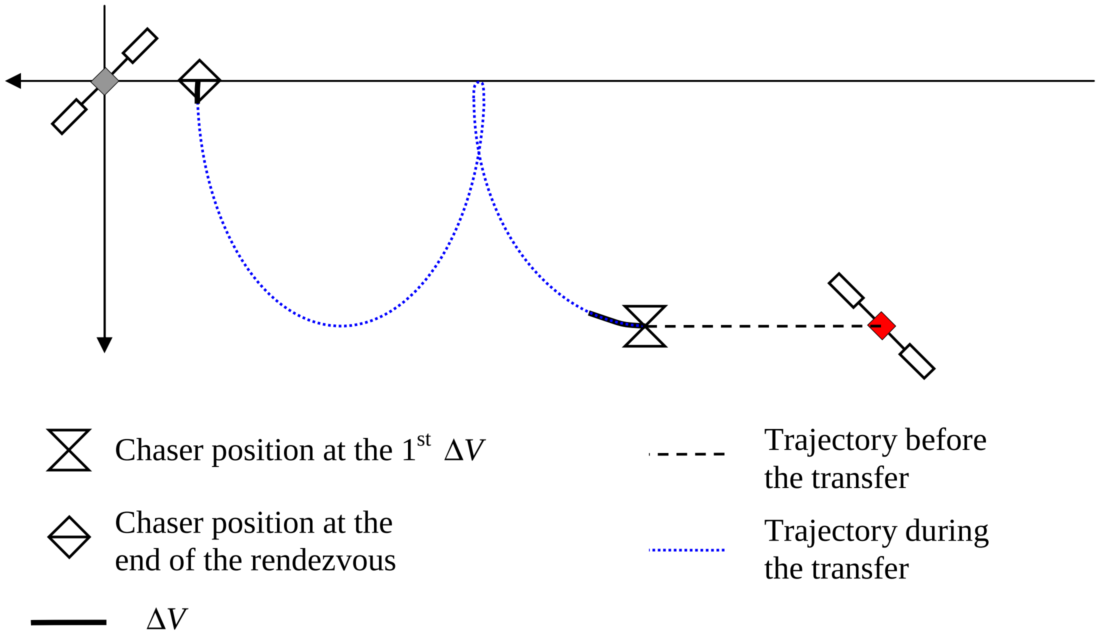

Fig. 2. Rendezvous in 2 boosts (in the LVLH frame)

In the case of a rendezvous in two transfers, i.e. with 3 ΔV, its characteristics are fully determined by the choice of the dates where the first and the second ΔV are performed and the position common to both transfers. This position is where the chaser is situated at the end of the first transfer and at the beginning of the second transfer. It is called intermediate position. Indeed, its state at the beginning of the 1st transfer only depends on its state at the guidance call and on the drift duration before this transfer. Its state at the end of the 1st transfer and at the beginning of the 2nd transfer is the intermediary position. Its state at the end of the 2nd transfer is an input to the program. The durations of the transfers are computed thanks to the rendezvous duration and the dates of the 1st and the 2nd ΔV. This kind of rendezvous is shown in Fig. 3.

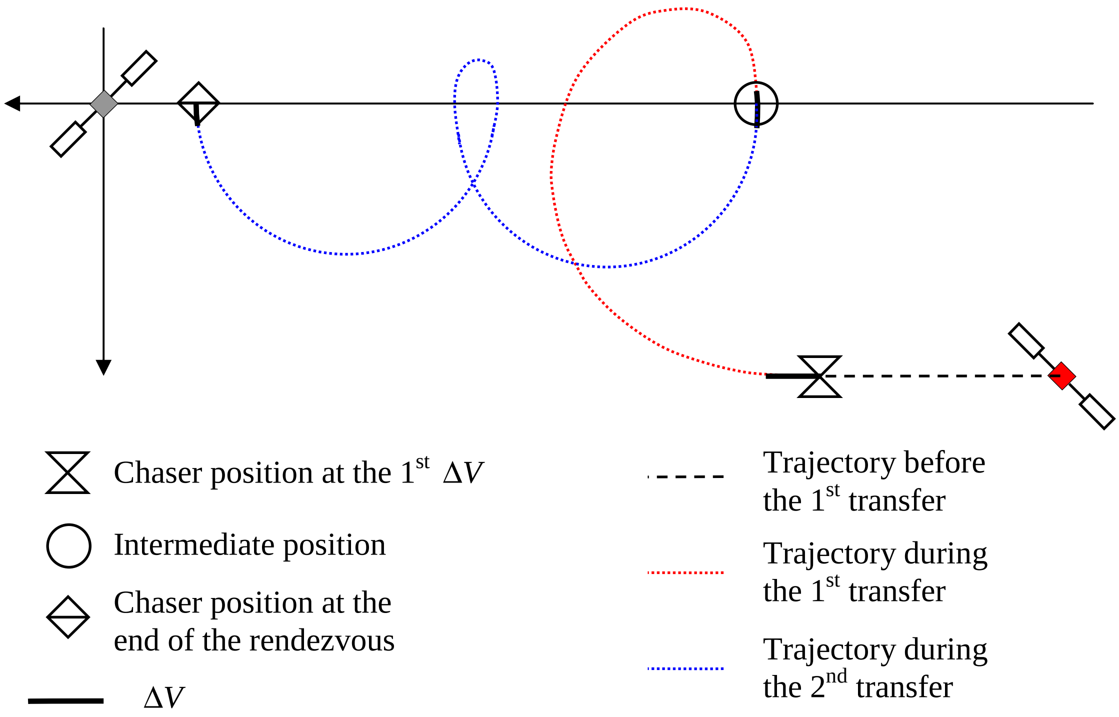

Fig. 3. Rendezvous in 3 boosts (in the LVLH frame)

Therefore, when the guidance program is called, it studies 2 types of rendezvous, one in 2 ΔV and the other in 3 ΔV. For each of them, it finds the best set of variables that allows performing the rendezvous while minimizing the overall consumption. At the end, the program chooses the type of rendezvous that requires the lowest consumption. We call "manoeuvres plan", the set constituted by the dates of the ΔV and their 3 components. This plan is the output of the program.

4.3. Robustness

To ensure the algorithm robustness, improvements are brought. The performed ΔV are not only dependent on the guidance program computations, but also on navigation information and propulsion system accuracies. As a consequence, it is necessary to recall the guidance program during the rendezvous in such a way that it can update the manoeuvres plan. The variables needed to compute the plan (dates of the ΔV and intermediary position if needed) are kept from the first call to the guidance program. Only the ΔV components are updated.
Since, during a transfer, the precision on the position at the end of the transfer must be ensured, it is also necessary to perform correction boosts. A correction boost is also performed, when the chaser is close to the final aimed position of the rendezvous so as to ensure the final accuracy in position. The guidance program must be able to compute these boosts and perform them when necessary.

5. TESTS

To assess the guidance program performances for different kinds of rendezvous, several tests are performed with targets placed on low or high orbits, on circular or elliptical orbits, for short or long rendezvous durations and for close or far initial relative states. One presents here 2 of these tests.

The tests are driven on a simulator that takes into account the Earth Keplerian gravitational field, the J_2 disturbance and the Earth atmosphere drag. The target and the chaser characteristics, for the first and the second test, are described in the Table 1.

	Target	**Chaser**
Mass (kg)	2000	2000
Aerodynamic Reference Surface (m^2)	0.6	3
Aerodynamic coefficient	2	2

Table 1 Spacecrafts characteristics

5.1. Tests on a circular orbit

The first test is a rendezvous with a target placed on a low circular orbit. The initial states of the two spacecrafts are described in the Table 2 and in the Table 3.

The rendezvous duration is 12 orbits (68448 s), it begins at the date 100 s and the final aimed state is at -100 m on the X axis of the LVLH frame, with a nil velocity. The trajectory is placed in Appendix, in Fig. 4.

	Target
Semimajor axis (km)	6900
Eccentricity	0
Inclination (°)	70
Longitude of the ascending node (°)	45
Argument of perigee (°)	0
True anomaly (°)	180

Table 2 Target initial conditions (circular)

	Chaser
X Position in LVLH (m)	-30000.0
Y Position in LVLH (m)	5000.0
Z Position in LVLH (m)	5000.0
X Velocity in LVLH (m/s)	-11.8872
Y Velocity in LVLH (m/s)	-1.7334
Z Velocity in LVLH (m/s)	3.7696

Table 3 Chaser initial conditions (circular)

The guidance program drives the chaser with an accuracy better than 2 m, in position, and better than 5.10^{-3} m/s, in velocity. At the first step, it computes the manoeuvres plan described in the Table 4. The rendezvous eventually required 3 correction boosts of 1.5 m/s plus the last correction boost (6.10^{-3} m/s) before the end of the rendezvous.

ΔV number	1	2
ΔVx (LVLH) (m/s)	0.5310	0.1866
ΔVy (LVLH) (m/s)	5.7051	0.2423
ΔVz (LVLH) (m/s)	-5.8641	-3.4667
Date (s)	1185.5	68548.0

Table 4 Manoeuvres plan for the circular rendezvous

5.2. Tests on an elliptical orbit

The second test is a rendezvous with a target placed on an elliptical orbit. The initial states of both spacecrafts are described in the Table 5 and in the Table 6.

The rendezvous duration is 3.4 orbits (41728.2 s), it begins at the date 100 s and the final aimed state is at -100 m on the X axis of the LVLH frame, with a nil velocity. The trajectory is placed in Appendix, in Fig. 5.

	Target
Semimajor axis (km)	11500
Eccentricity	0.4
Inclination (°)	80
Longitude of the ascending node (°)	60
Argument of perigee (°)	100
True anomaly (°)	20

Table 5 Target initial conditions (elliptical)

	Chaser
X Position in LVLH (m)	-30000.0
Y Position in LVLH (m)	5000.0
Z Position in LVLH (m)	5000.0
X Velocity in LVLH (m/s)	-13.4497
Y Velocity in LVLH (m/s)	-15.6038
Z Velocity in LVLH (m/s)	12.5705

Table 6 Chaser initial conditions (elliptical)

The guidance program drives the chaser with an accuracy better than 2 m, in position, and better than 1.10^{-3} m/s, in velocity. At the first step, it computes the manoeuvres plan described in the Table 7. The rendezvous eventually required 3 correction boosts of 1.5 m/s plus the last correction boost (8.10^{-3} m/s) before the end of the rendezvous.

ΔV number	1	2	3
ΔVx (LVLH) (m/s)	-1.4167	-0.0675	0.0773
ΔVy (LVLH) (m/s)	-6.6762	4.0523	0.0004
ΔVz (LVLH) (m/s)	1.6307	-1.5641	0.3643
Date (s)	7518.0	13659.6	41828.2

Table 7 Manoeuvres plan for the elliptical rendezvous

5.3. Tests on ATV rendezvous

The third test is the ATV rendezvous. The target is placed on a low circular orbit. The characteristics of both spacecrafts are described in the Table 8. The initial states of both spacecrafts are described in the Table 9 and in the Table 10.

	Target	Chaser
Mass (kg)	204321	20000
Aerodynamic Reference Surface (m^2)	1261	3
Aerodynamic coefficient	2.35	2.2

Table 8 Spacecrafts characteristics for the ATV rendezvous

The rendezvous duration is 1 orbit (5560 s), it begins at the date 10 s and the final aimed state is at -250 m on the X axis of the LVLH frame, with a nil velocity. The trajectory is placed in Appendix, in Fig. 6.

	Target
Semimajor axis (km)	6783
Eccentricity	0.008
Inclination (°)	51.6
Longitude of the ascending node (°)	-1.14
Argument of perigee (°)	1.0
True anomaly (°)	0.0

Table 9 Target initial conditions (ISS)

	Chaser
X Position in LVLH (m)	-15000.0
Y Position in LVLH (m)	0.0
Z Position in LVLH (m)	5200.0
X Velocity in LVLH (m/s)	9.0
Y Velocity in LVLH (m/s)	0.0
Z Velocity in LVLH (m/s)	0.0

Table 10 Chaser initial conditions (ATV)

The guidance program drives the chaser with an accuracy better than 3 m, in position, and better than 1.10^{-2} m/s, in velocity. At the first step, it computes the manoeuvres plan described in the Table 11. The rendezvous eventually required 1 correction boosts of 1.5 m/s plus the last correction boost ($3.7.10^{-3}$ m/s) before the end of the rendezvous.

ΔV number	1	2
ΔVx (LVLH) (m/s)	2.2177	0.6717
ΔVy (LVLH) (m/s)	-0.0224	-0.0139
ΔVz (LVLH) (m/s)	-0.1930	-4.0921
Date (s)	1735.4	5570.0

Table 11 Manoeuvres plan for the elliptical rendezvous

The sum of the ΔV performed for this rendezvous is 8.15 m/s. Knowing that the ATV rendezvous mission analysis solution requires 8 m/s, the guidance program found a good solution, while performing 4 boosts when the ATV mission analysis used 8 boosts (2 " 2-boosts transfers " with 2 corrections each).

6. CONCLUSIONS

The presented guidance program fulfils its objectives: driving the chaser to the right position at the right date, while minimizing the overall consumption of the rendezvous. It works on any elliptical orbit, takes into account the J_2 disturbance of the Earth gravitational field and is also robust to the Earth atmosphere drag. Moreover, it is able to compute correction boosts so as to keep the chaser on the initial computed trajectory.

This guidance program can still be improved by taking into account the atmospheric drag when computing the boosts. However, no developed linear relative motion model is, for the moment, able to do it. Another improvement could consist in computing spread boosts instead of impulse ΔV . A function can also be added so as to ensure the chaser trajectory respects some safety constraints wrt. the target, e.g. an avoidance area.

7. REFERENCES

[1] Silva N., Ganet M., Gogibus E., Delpy P. and Augros P., *Autonomous rendezvous : Guidance and Control in elliptical orbit*, AIAA, Guidance, Navigation and Control Conference and Exhibit (20 - 23/08/2007)

[2] Clohessy W. H. and Wiltshire R. S., *Terminal Guidance System for Satellite Rendezvous*, Journal of Aerospace and Science, Vol. 27, No. 9, Sept. 1960, pp. 653-658, 674

[3] Tschauner J. and Hempel P., *Optimale Beschleunigungsprogramme für das Rendezvous-Manöver*, Astronautica Acta, Vol. 10, No. 296, 1964, pp. 339-343

[4] Carter T. E., *State Transition Matrices for Terminal Rendezvous Studies: Brief Survey and New Example*, Journal of Guidance, Control and Dynamics, Vol. 21, No. 1, 1998, pp. 148-155

[5] Yamanaka K. and Ankersen F., *New State Transition Matrix for Relative Motion on an Arbitrary Elliptical Orbit*, Journal of Guidance, Control and Dynamics, Vol. 25, No. 1, Jan.-Feb. 2002, pp. 60-66

[6] Gim D.-W. and Alfriend K. T., *State transition matrix of relative motion for the perturbed non-circular reference orbit*, Journal of Guidance, Control and Dynamics, Vol. 26, No. 6, Nov-Dec 2003, pp 956-971

8. APPENDIX

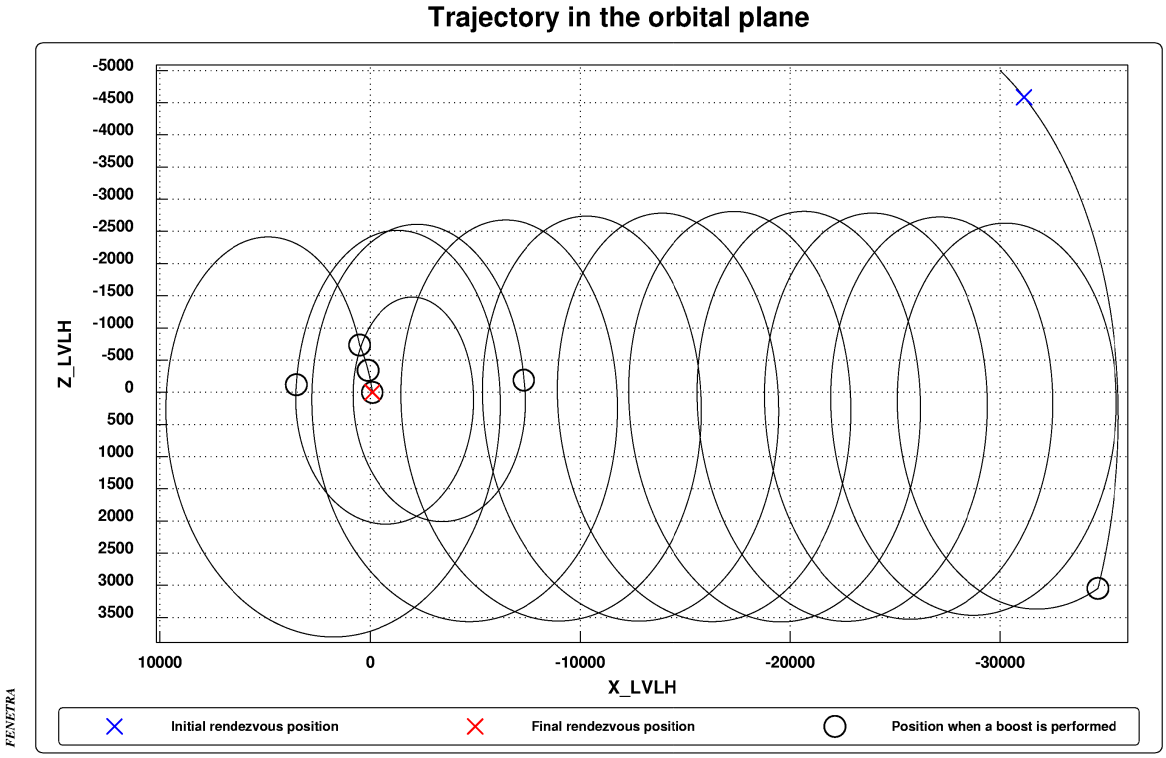

Fig. 4. In plane trajectory for the circular rendezvous

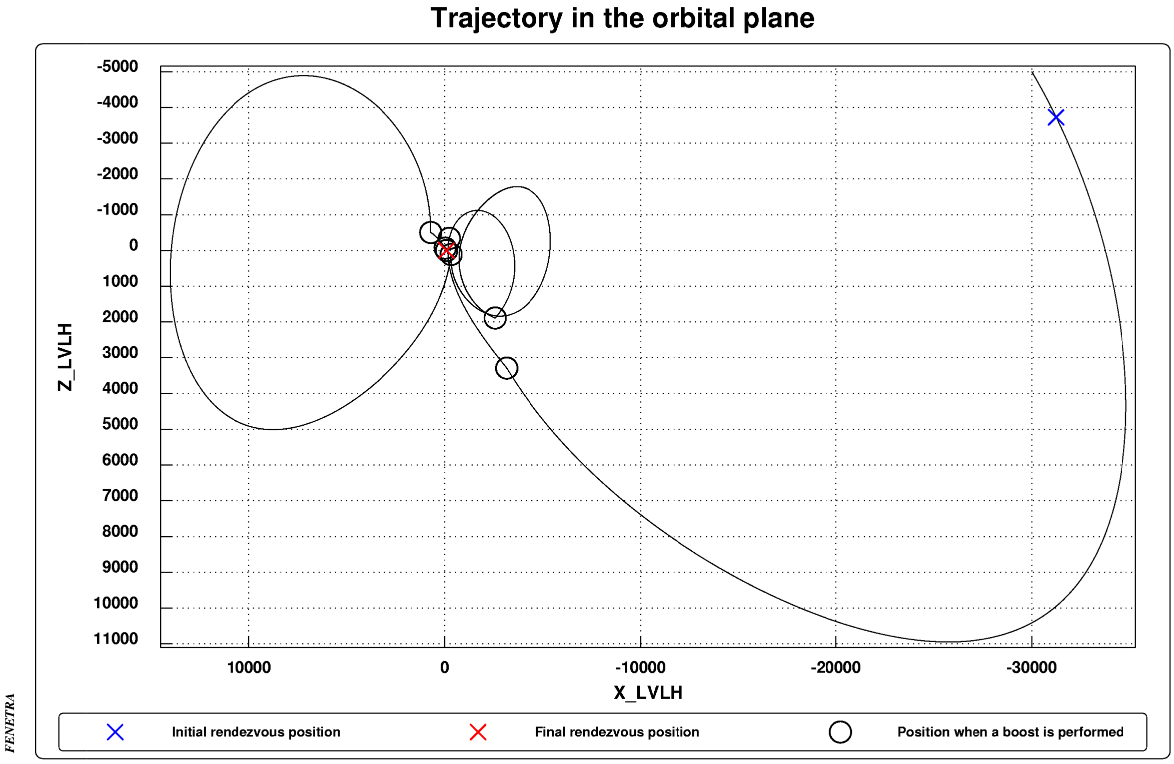

Fig. 5. In plane trajectory for the elliptical rendezvous

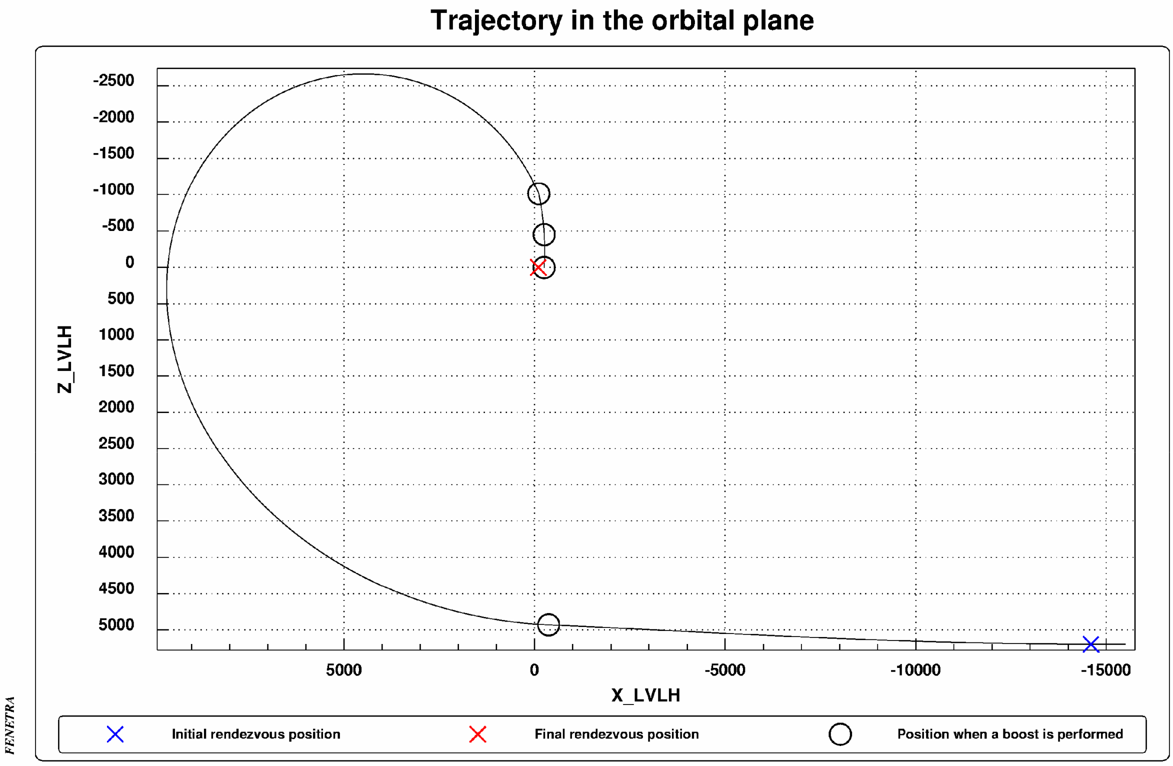

Fig. 6. In plane trajectory for the ATV rendezvous