## Recursive Sequences

General Form:  $t_n = t_1 r^{n-1}$ 

Common Ratio:  $r = \frac{t_i}{t_{i-1}}$  i > 1

1. For each of the following sequences, write the first 5 terms:

(a) 
$$a_1 = 4$$
 and  $a_n = n + a_{n-1} + 6$ 

(d) 
$$a_1 = 5$$
 and  $a_n = n^2 - a_{n-1}$ 

(b) 
$$a_1 = 0$$
 and  $a_n = a_{n-1} - n^2$ 

(e) 
$$a_1 = 4$$
,  $a_2 = 2$  and  $a_n = a_{n-1} - a_{n-2}$ 

(c) 
$$a_1 = 2$$
 and  $a_n = (a_{n-1})^2 + 2$ 

(f) 
$$a_1 = 1$$
,  $a_2 = 3$  and  $a_n = a_{n-1} \times a_{n-2}$ 

2. For each of the following sequences:

i write as an explicit rule

ii write as a recursive rule

iii find the  $43^{rd}$  term

(a) 
$$9, 1, -7, -15, \dots$$

(c) 
$$\frac{\pi}{4}, \pi, \frac{7\pi}{4}, \frac{5\pi}{2}, \dots$$

(e) Geometric: 
$$a_1 = \sqrt[3]{4}, r = \sqrt[3]{2}$$

(b) 
$$40, -20, 10, -5, \dots$$

(d) Geometric: 
$$a_1 = 2, r = 10$$

(f) Arithmetic: 
$$a_1 = 14, d = \frac{1}{2}$$

- 3. Write the equation of the general term of an arithmetic sequence  $t_n = t_1 + (n-1)d$  as a recursive rule.
- 4. Write the equation of the general term of a geometric sequence  $t_n = t_1 r^{n-1}$  as a recursive rule.

- 5. You have just bought a new swimming pool and need to add chlorine to the water. You add 750mL of chlorine the first week and 350mL every week thereafter. Each week 40% of the chlorine in the pool evaporates.
  - (a) Write a recursive rule for the amount of chlorine in the pool each week. How much chlorine is in the pool at the beginning of the sixth week?
  - (b) What happens to the amount of chlorine after an extended period of time?
- 6. Give an example of a sequence in which each term after the third term is a function of the three terms preceding it. Write a recursive rule for the sequence and find the first 8 terms.
- 7. You can define a sequence using a piece-wise rule. The following is an example of a piece-wise defined sequence:

$$a_n = \begin{cases} 7 & \text{if } n = 1\\ \frac{a_{n-1}}{2} & \text{if } a_{n-1} \text{ is even}\\ 3a_{n-1} + 1 & \text{if } a_{n-1} \text{ is odd} \end{cases}$$

- (a) Write the first ten terms of the sequence.
- (b) Choose three different values for  $a_1$  (other than  $a_1 = 7$ ). For each value of  $a_1$ , find the first ten terms of the sequence. What conclusions can you make about the behavior of this sequence?

- 8. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s? (2019 AMC 10B Problems/Problem 25)(try to solve using a recursive rule)
  - (A) 55 (B) 60 (C) 65 (D) 70 (E) 75