Chapter 11 Exercises

Instructions: Complete the following exercises using Python 3. You may work with up to one other person. You may use library packages unless you are explicitly told not to. If a problem tells you to write a program/function for arbitrary input, you should always test your function and ensure it works. If you are stuck or need any help, please do not hesitate to ask the instructor or teaching assistant.

1) Write a function that implements the Forward Euler method. Further, make your function graph the solution to the ODE as well. Use this function to solve and graph $dy/dt = y\sin(t)$, $y(0) = 1$ over the interval $[0, 2]$. Solve this ODE by hand and plot the solution on the same graph. Resolve any discrepancies.

2) Go through line by line the function that implements the Euler-Cromer method in section (11.2). Make sure you understand the purpose and function of each line.

3) Go through line by line the function that implements the Verlet method in section (11.3). Make sure you understand the purpose and function of each line.

4) Write a function that implements the velocity-Verlet method given in section (11.4). Have your function also plot the velocity and position of the system on the same graph.

5) In section (11.2), there was a plot generated titled "Energy in Euler-Cromer Method." Recreate that plot yourself.

6) Recreate the energy plot in problem (5), but for the velocity-Verlet method.

7) Use the Forward Euler and Euler-Cromer methods to solve the following ODE over the associated interval and initial conditions:

- $m\ddot{y} = -mg$, $y(0) = 5$, $v(0) = 0$, $[0, 15]$

Graph the velocity and position over the interval for both methods. On one graph, display the results of the Forward Euler method only. On a second graph, display the results of the Euler-Cromer method only. On a third graph, plot the position of the Forward-Euler method and the Euler-Cromer method. On a fourth graph, do the same with the velocities.

8) Use the velocity-Verlet method to solve the following ODE over the associated interval and initial conditions:

- $\ddot{y} = -1/y^2$, $y(0) = 1$, $v(0) = 0$. $[1, 2]$

Solve the same problem, but using ODEint. Plot the position from velocity-Verlet and ODEint on one graph. On a second graph, plot the velocities from both methods. Commend on the plots–how similar are they? What happens if you extend the interval?