Chapter 9 & 10 Exercises

Instructions: Complete the following exercises using Python 3. You may work with up to one other person. You may use library packages unless you are explicitly told not to. If a problem tells you to write a program/function for arbitrary input, you should always test your function and ensure it works. If you are stuck or need any help, please do not hesitate to ask the instructor or teaching assistant.

1) Go through the code for left and right Riemann sums at the beginning of Chapter 9. Make sure you understand the purpose and function of each line.

2) Write a function that computes the midpoint Riemann sum. Your function should bare close resemblance to the left and right Riemann sum functions given in Chapter 9. For an arbitrary function you pass into your Riemann sum function, plot the function and each rectangle used in the approximation.

3) Write a function that implements trapezoid rule. Also make your function graph whatever function you are integrating, and plot each trapezoid used in the approximation.

4) Go through each line in the code that implements Simpson's Rule in section 9.3. Understand the purpose and function of each line as best you can. If you can't figure out the logic behind the polynomial graphing, don't worry about it.

5) Use midpoint Riemann sums, trapezoid rule, Simpson's rule, and SciPy's quad function to find the area of the following functions for the associated range:

- $f(x) = e^{-x^{1/2}} \sin(x^3)$, $[0, 3\pi]$
- $f(x) = x^2 \ln(x)$, $[0.1, 10]$
- $f(x) = e^{-x^2}$, $[-\infty, \infty]$

For each function you integrate, what is the minimum number of rectangles/trapezoids/polynomials needed to be within 0.1% of the 'true' area given by quad? While you can brute force this part, try to write a Python script that does it for you.

6) Go through the code for the forward, backward, and central difference formulas in section (10.1) line by line. Make sure you understand the purpose and function of each line.

7) Write a function that implements the second order central difference method for computing second order derivatives, as given in equation (10.8).

8) Use the forward, backward, and central difference functions to compute the first order derivative of the following functions over the associated interval:

- $f(x) = \sin(x) + \cos(x)$, $[0, 2\pi]$
- $f(x) = \sin(x)/x$, $[1, 10]$
- $f(x) = (\sin(x) + x^2 \ln(\sqrt{x}))^{x^2}$, $[0, 5]$

For each function, plot each method on the same graph. For the first two functions, compute the derivative by hand and plot it on the graph as well. For the last function, use SciPy's derivative function to compute the derivative, and plot it on the graph too. You should find that the results from your algorithm and SciPy's are nearly the same. If there are any major discrepancies, resolve them.

9) Use the function you wrote in problem (7) to compute the second order derivatives of the following functions over the associated intervals:

- $f(x) = e^{-x^2}$, $[-5, 5]$
- $f(x) = [x \sin(\ln(x))]^{e^{-3x}}$, $[0.1, 1]$

Graph each of your solutions. Calculate the second derivative of the first function by hand and graph it on the appropriate graph. For the second function, calculate the second derivative using ScyPy's derivative, and plot it on the appropriate graph.