Chapter 5 & 6 Exercises

Instructions: Complete the following exercises using Python 3. You may work with up to one other person. You may use library packages unless you are explicitly told not to. If a problem tells you to write a program/function for arbitrary input, you should always test your function and ensure it works. If you are stuck or need any help, please do not hesitate to ask the instructor or teaching assistant.

1) For each line in the Gaussian elimination code (beginning on page 35), understand the purpose of each line/block of code and how that line/block accomplishes that purpose. In other words, make sure you understand the code. While not required, you may find it helpful to run the code yourself and add additional print statements.

2)Repeat question (1), but for the LU decomposition code and backwards substitution code, beginning on pages 38 and 39 respectively.

3) Write an algorithm that implements forward substitution to solve equation (5.13) for some arbitrary matrix $\mathbb{L}$ and some vector $\mathbf{b}$. The output should be a set of new vectors, $\mathbf{z}$.

4) Write a function that calculates the trace of a $n \times n$ matrix for any $n > 0$.

5) Compute the determinates and inverses of the following matrices (you may use library functions):

$$\begin{bmatrix} 4 & 5 \\ 8 & 10 \end{bmatrix}, \begin{bmatrix} 8 & 2 \\ 16 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 6 & 11 \\ 8 & 13 & 4 \\ 1 & 3 & 7 \end{bmatrix}$$

6) Solve the following two systems of equations using both Gaussian Elimination and LU decomposition:

$$\begin{cases} 5x + 2y = 6 \\ 7y + 4x = 2 \end{cases}, \quad \begin{cases} 14x + 12y + 6z = 5 \\ 9x + 2y + 11z = 14 \\ x - y - z = 10 \end{cases}$$

7) Plot equations (6.1) and (6.2) to replicate the figure on page 42.

8) Go through the Bisection function on page 45 line by line. Make sure you understand

the purpose of each line, and how it accomplishes that purpose.

9) Write a function that implements Newton's method of finding roots (given by equation (6.4) on page 44).

10) Use both Newton's method and the Bisection method to find the zeros of the following functions:

$$x^3 - 2x + 4 = 5 \tag{1}$$
$$\cos(x) = x^3 \tag{2}$$
$$\ln(x) = x^{-4}e^x \tag{3}$$

For each function, compare Newton's method and Bisection method. How many iterations does each method take to find a zero?

11) Use fsolve() to find zeros of the following (systems of) equations:

$$x + 2\cos(x) = 0 \tag{4}$$
$$\begin{cases} xy - x = 3 \\ x\cos(x) = 2 \end{cases} \tag{5}$$