2.1 $\hat{M}\psi_n(x) = \frac{d^2}{dx^2} \cdot \sqrt{2}\sin(\pi n x)$

$= \frac{d}{dx} \cdot \sqrt{2}\pi n \cos(\pi n x)$

$= -\sqrt{2}\pi^2 n^2 \sin \pi n x$

$= \lambda_n \psi_n(x)$

2.2 if $n = m$:

$\int_0^1 \psi_n^2(x)\,dx = 2\int_0^1 \sin^2(\pi n x)\,dx$

$= \frac{2}{\pi n}\int_0^1 \sin^2(\pi n x)\,d(\pi n x)$

$= \frac{2}{\pi n}\int_0^{\pi n} \sin^2 t\,dt$

$= \frac{1}{\pi n}\int_0^{\pi n} (1-\cos 2t)\,dt$

$= 1$

if $n \pm m$:

$\int_0^1 \psi_n(x)\,\psi_m(x)\,dx = 2\int_0^1 \sin(\pi n x)\sin(\pi m x)$

$= \int_0^1 \cos((n-m)\pi x) - \cos((n+m)\pi x)\,dx$

$= \frac{1}{(m-n)\pi}\sin((m-n)\pi) - \frac{1}{(m+n)\pi}\sin((m+n)\pi)$

$= 0$

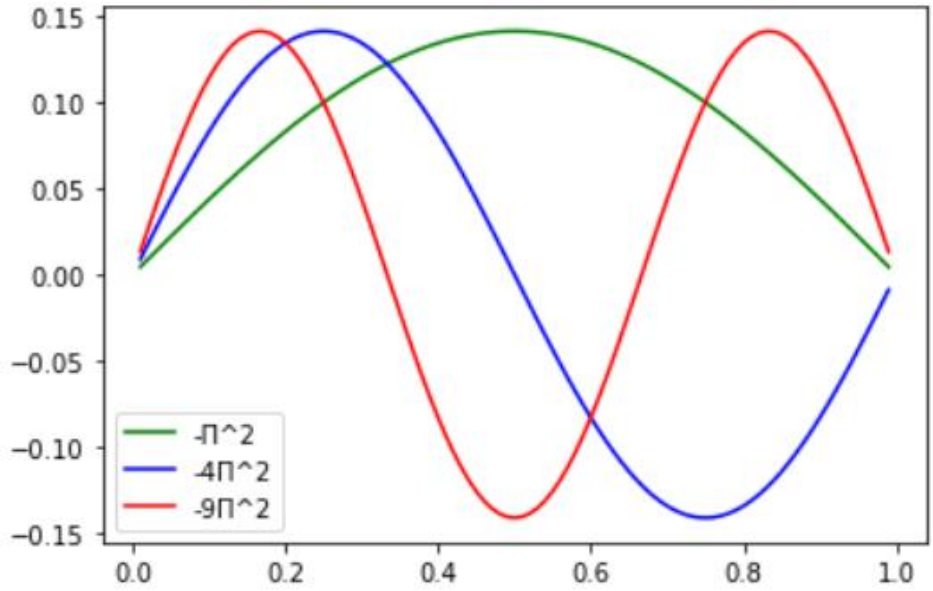
2.3

True

2.4

```
The result is :

[ -9.86879269 -39.46543143 -88.76070794]
```



3.1

```
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 2.23939409e+00 2.17416334e+00 2.06789065e+00 2.06118874e+00
 2.02593012e+00 1.98634931e+00 1.96619070e+00 1.93862002e+00
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 1.37644974e+00 1.34822142e+00 1.32540873e+00 1.30171470e+00
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1.17163227e-03 8.32607758e-04 2.62305623e-04 1.65859499e-04]

3.2

K=2

K=4

K=8

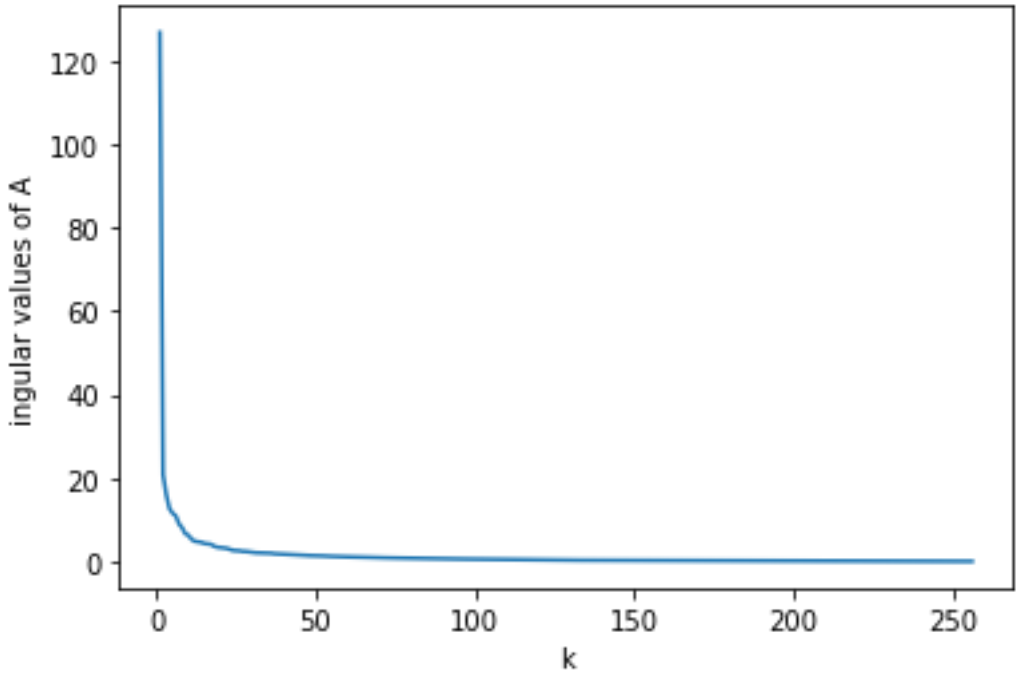
k=16

K=32

K=64

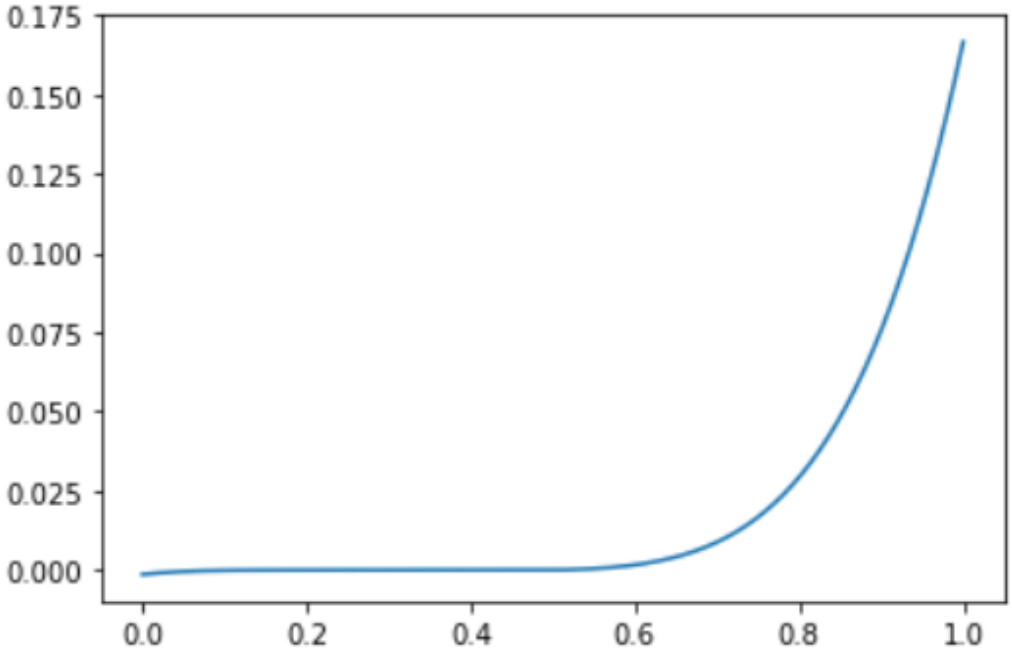
	
K=128	K=256

3.3



4.1

```
[0.5        0.33333333 0.25       0.2        0.16666667]
```



There are five unique roots.

4.2

```
The root for Newton method is  0.16666666666666893
```

4.3

```
The root for Secant method is: 0.49999999850055993
```

It is not same as the root for Newton method.