# 基于蒙特卡罗方法模拟 Buffon 投针实验

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**摘要**

18 世纪，Buffon 提出以下问题：设我们有一个以平行且等距木纹铺成的地板，随意抛一支长度比木纹之间距离小的针，求针和其中一条木纹相交的概率。由大数定律，在试验次数足够多时，针与木纹相交的频率可以近似看作针与木纹相交的概率。Buffon 基于此提出了一种计算圆周率的方法——随机投针法。这就是 Buffon 投针问题。[1] 在本次实验中，利用计算机模拟实现 $Buffon$ 落针实验，并探究圆周率 $\pi$ 的计算精度与落针次数之间的关系。

## 1　问题背景与研究目的

法国数学家 Buffon 最早设计了投针试验。这一方法的步骤是：

1）取一张白纸，在上面画上许多条间距为 a 的平行线。

2）取一根长度为 $l(l \leq$a) 的针，随机地向画有平行直线的纸上掷 n 次，观察针与直线相交的次数，记为 m。

3）计算针与直线相交的概率.

Buffon 本人证明了，这个概率是 (其中 $\pi$ 为圆周率)：

$$p = \frac{2l}{\pi a} \tag{1}$$

由于它与 $\pi$ 有关，于是人们想到利用投针试验来估计圆周率的值。

Buffon 发现：有利的扔出与不利的扔出两者次数的比，是一个包含 $\pi$ 的表示式. 如果针的长度等于 $\frac{a}{2}$，那么扔出的概率为 $\frac{1}{\pi}$. 扔的次数越多，由此能求出越为精确的 $\pi$ 的值。

本文通过计算机编程可视化的方法，验证 Buffon 投针实验并对 Buffon 得出的落针次数和计算出的圆周率 $\pi$ 的精度之间的关系进行验证。样本数据及程序代码在附录中。

# 2 研究过程与研究方法

## 2.1 实验原理的理论证明

首先对 Buffon 的结论进行理论证明：让 $x$ 是从针的中心到最近的平行线的距离，让 $\theta$ 是针和其中一条平行线之间的锐角。$x$ 的概率密度函数为：

$$\begin{cases} \frac{2}{t} & : 0 \leq x \leq \frac{t}{2} \\ 0 & : elsewhere. \end{cases} \tag{2}$$

在这里，$x = 0$ 表示直接居中于一条线上的针，$x$=$\frac{t}{2}$ 表示在两条线之间完全居中的针。假设针头同样可能落在这个范围内的任何地方，但不能落在它之外。其概率密度函数为：

$$\begin{cases} \frac{2}{\pi} & : 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & : elsewhere \end{cases} \tag{3}$$

此外，$\theta$=0 弧度表示与标记线平行的针，$\theta$=$\frac{\pi}{2}$ 弧度表示垂直于标记线的针。此范围内的任何角度都假定为等可能事件。

两个随机变量 $theta$ 和 $x$ 是相互独立的，因此联合概率密度函数为：

$$\begin{cases} \frac{4}{t\pi} & : 0 \leq x \leq \frac{t}{2}, 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & : elsewhere. \end{cases} \tag{4}$$

则针落下时穿过一条线的概率为：

$$P = \int_{\theta=0}^{\frac{\pi}{2}} \int_{x=0}^{(l/2)\sin\theta} \frac{4}{t\pi} dx d\theta = \frac{2l}{t\pi} \tag{5}$$

由上述概率公式可推导出：

$$\pi = \frac{2l}{tP} \tag{6}$$

所以我们可以用 P 估计 $\pi$。

## 2.2 计算机实现落针实验

### 2.2.1 单次落针实验

输入扔针的次数并进行计算机模拟实验，将仿真扔针图像呈现在计算机屏幕上。

输入：扔针次数

输出：对应的 $\pi$ 估计值以及仿真扔针图，扔针图中红色为触平行线的针，黑色为未触平行线的针

参数：

- linenumber = 50 ： 间隔数，即除去桌子边缘中间应该有 49 条线
- needlewidth = 9 ： 针长，为两条平行线距离的一半

以下为计算机实现扔针次数在 500、1000、1500 时所绘图像，实验数据见附录。

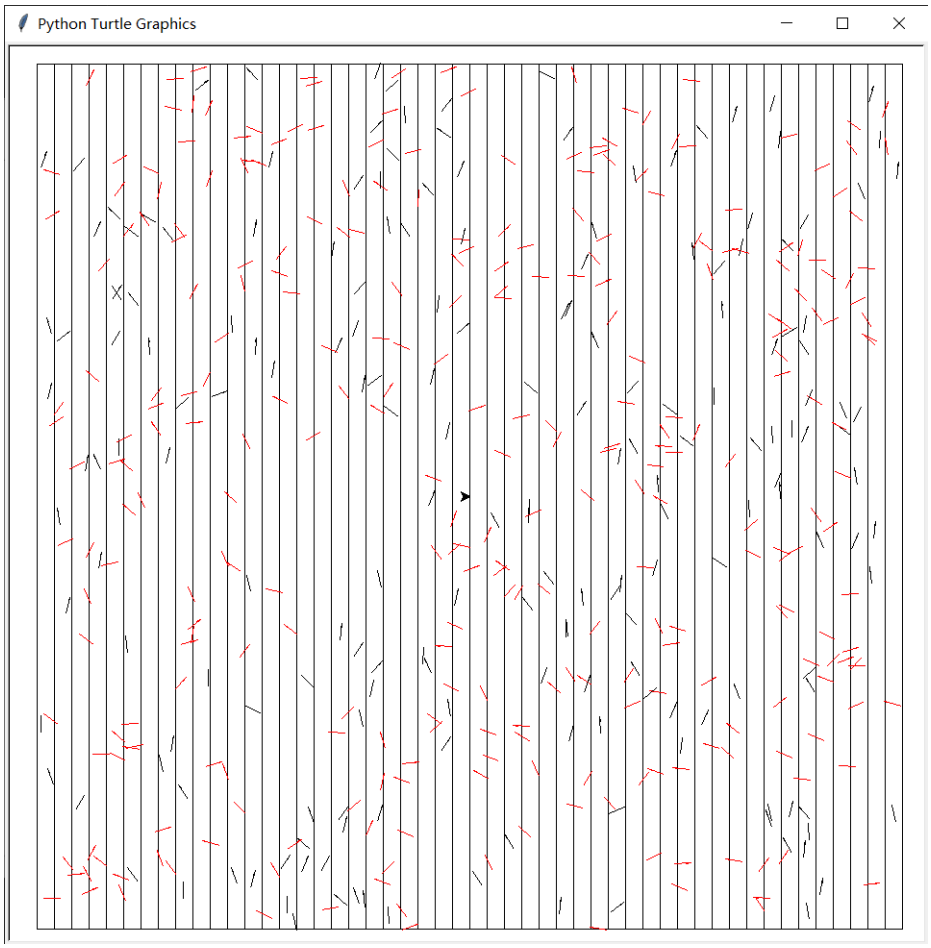


图 1: 扔针次数为 500 时的模拟图像

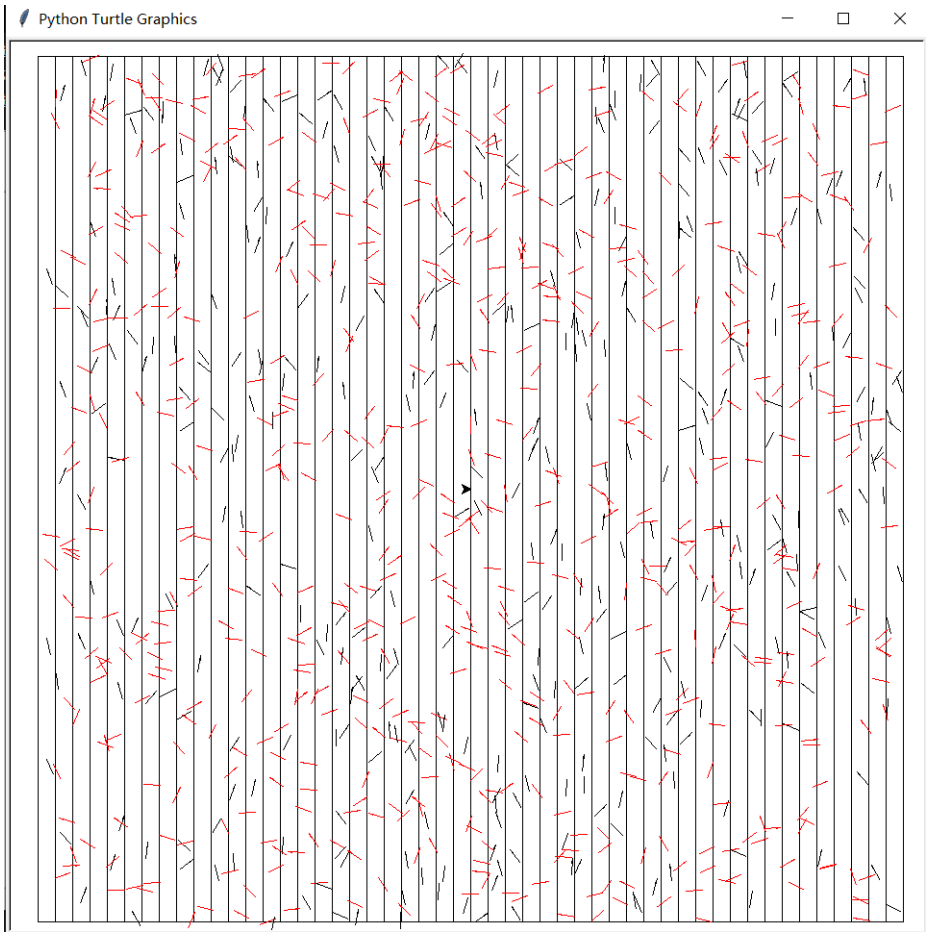


图 2: 扔针次数为 1000 时的模拟图像

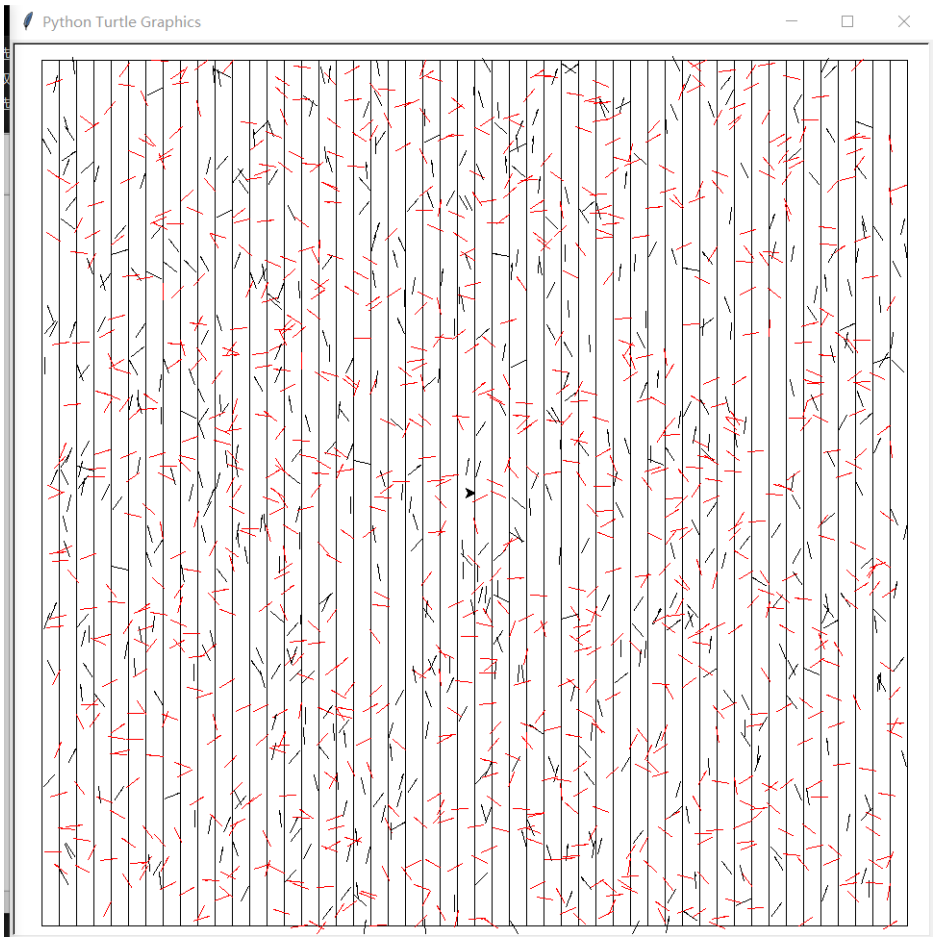


图 3: 扔针次数为 1500 时的模拟图像

### 2.2.2 多次落针实验

多次实验并输出随着扔针次数不同时不同的 $\pi$ 估计值并绘制折线图，保存仿真实验的原始数据。

对于固定的扔针次数，进行一百次重复实验，对应的 $\pi$ 估计值取重复实验的平均值。

参数：

- linenumber = 50：间隔数，即除去桌子边缘中间应该有 49 条线
- needlewidth = 9：针长，为两条平行线距离的一半

- axhy = 3.1415926：在折线图中的参考线，$\pi$ 的预期标准值
- x=[10,50,100,200,400,600,800,1000,1200,1400,1600,1800,2000,2125,2250 ,2375, 2500,2625,2750,2825,3000]：
横坐标，即扔针次数的取值
- y：对应扔针次数的估计值列表，通过计算机模拟实验得到

以下为计算机实现多次落针实验所得数据所绘折线图，实验数据见附录。

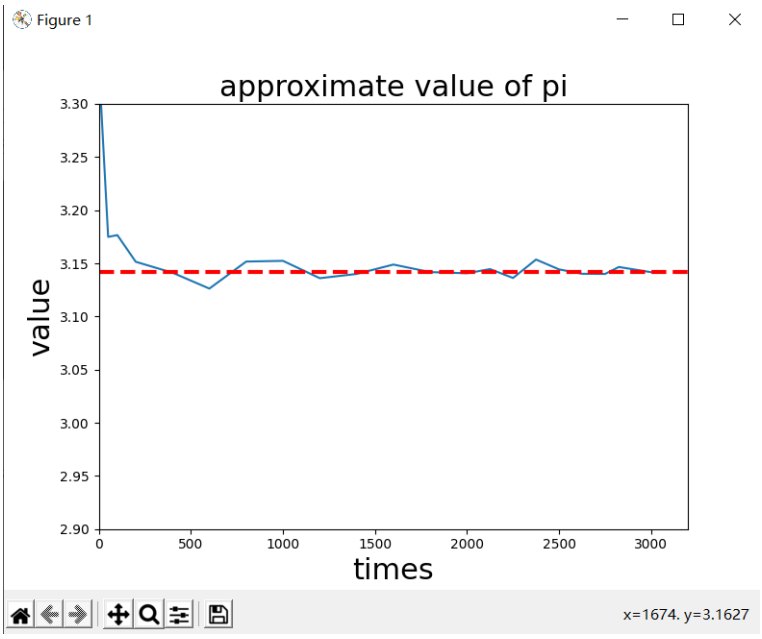


图 4: 落针次数与 $\pi$ 估计值的关系

# 3 结论

由实验结果，Buffon 投针实验可以估计 $\pi$ 的值，投针实验有效。

本实验还可以验证大数定律，由图四可知，在落针次数较少的范围内，$\pi$ 的实验估计值与真实值偏差较大。随着落针次数的增加，$\pi$ 估计值逐渐向真实值逼近，在落针次数超过 2500 次时，$\pi$ 估计值与真实值几乎一致。可推断当落针次数趋于无穷大时，$\pi$ 估计值与真实值相等。

# 附录

## 样本数据

次数 =10：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2.222222 | 5 | 6.666667 | 4 | 2.857143 | 2.857143 | 5 | 4 | 6.666667 | 3.333333 |
| 4 | 3.333333 | 3.333333 | 2.222222 | 4 | 2.222222 | 3.333333 | 2.857143 | 2.5 | 3.333333 |
| 3.333333 | 2.857143 | 3.333333 | 5 | 4 | 3.333333 | 4 | 2.5 | 2.5 | 2.5 |
| 2.222222 | 5 | 3.333333 | 2.857143 | 4 | 2.5 | 2.5 | 3.333333 | 5 | 2.5 |
| 2.857143 | 2.5 | 2.857143 | 4 | 2.857143 | 2.222222 | 2.5 | 2.857143 | 3.333333 | 2.857143 |
| 3.333333 | 2.5 | 3.333333 | 2.857143 | 2.857143 | 2.857143 | 4 | 3.333333 | 2.5 | 5 |
| 5 | 5 | 2.222222 | 5 | 4 | 2.857143 | 2.222222 | 2.857143 | 5 | 4 |
| 3.333333 | 2.857143 | 4 | 4 | 2.857143 | 2.5 | 2.857143 | 2.5 | 2.857143 | 2.857143 |
| 4 | 2.5 | 2.222222 | 3.333333 | 2.857143 | 4 | 2.857143 | 2.857143 | 2.857143 | 3.333333 |
| 2.5 | 2.857143 | 4 | 2.857143 | 2.857143 | 3.333333 | 2.5 | 2.857143 | 2.857143 | 2.5 |

次数 =50：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.125 | 2.702703 | 3.125 | 3.125 | 3.846154 | 3.703704 | 2.857143 | 3.125 | 3.030303 | 2.777778 |
| 3.225806 | 3.030303 | 2.941176 | 3.571429 | 3.703704 | 3.448276 | 3.448276 | 3.125 | 3.030303 | 2.941176 |
| 2.857143 | 3.125 | 4 | 3.125 | 2.941176 | 3.125 | 3.125 | 2.702703 | 2.857143 | 3.030303 |
| 2.857143 | 3.125 | 3.125 | 3.703704 | 3.225806 | 3.225806 | 3.571429 | 3.225806 | 5.263158 | 3.030303 |
| 3.225806 | 3.030303 | 3.030303 | 3.571429 | 2.5 | 2.857143 | 2.941176 | 3.225806 | 2.777778 | 2.941176 |
| 3.125 | 3.225806 | 3.448276 | 3.125 | 3.448276 | 3.448276 | 2.5 | 2.777778 | 2.941176 | 2.5 |
| 3.448276 | 3.225806 | 3.571429 | 3.448276 | 3.448276 | 3.030303 | 2.777778 | 3.225806 | 2.777778 | 3.125 |
| 2.941176 | 3.125 | 2.631579 | 3.846154 | 2.777778 | 3.225806 | 3.125 | 2.631579 | 2.941176 | 4 |
| 2.941176 | 3.333333 | 2.857143 | 3.448276 | 3.125 | 2.777778 | 3.571429 | 2.777778 | 3.225806 | 3.448276 |
| 3.571429 | 3.125 | 3.571429 | 3.225806 | 3.333333 | 3.703704 | 3.125 | 3.225806 | 3.448276 | 2.564103 |

次数 =100：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2.857143 | 3.125 | 3.174603 | 2.857143 | 3.225806 | 3.076923 | 3.174603 | 3.030303 | 3.125 | 2.898551 |
| 3.174603 | 2.816901 | 3.508772 | 2.941176 | 3.333333 | 3.389831 | 2.816901 | 3.448276 | 3.448276 | 3.125 |
| 3.448276 | 2.985075 | 3.389831 | 3.076923 | 2.857143 | 2.985075 | 3.030303 | 2.857143 | 3.333333 | 3.389831 |
| 3.773585 | 3.448276 | 3.225806 | 3.174603 | 3.508772 | 3.389831 | 3.703704 | 3.278689 | 2.941176 | 3.333333 |
| 3.389831 | 3.225806 | 3.278689 | 2.985075 | 3.125 | 2.857143 | 2.985075 | 3.333333 | 2.816901 | 3.333333 |
| 3.076923 | 2.941176 | 3.389831 | 2.702703 | 3.448276 | 3.125 | 3.389831 | 3.225806 | 2.857143 | 3.076923 |
| 3.125 | 3.448276 | 3.076923 | 4 | 3.703704 | 3.389831 | 3.174603 | 2.631579 | 3.278689 | 3.174603 |
| 3.174603 | 3.125 | 3.571429 | 3.174603 | 3.174603 | 3.030303 | 2.985075 | 3.508772 | 3.076923 | 2.816901 |
| 3.125 | 2.985075 | 3.030303 | 3.174603 | 3.333333 | 3.389831 | 3.333333 | 3.225806 | 2.985075 | 3.125 |
| 3.448276 | 3.125 | 3.225806 | 3.125 | 3.278689 | 3.076923 | 2.898551 | 2.666667 | 3.278689 | 2.941176 |

次数 =200：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.478261 | 3.225806 | 3.053435 | 3.2 | 3.333333 | 3.125 | 3.149606 | 3.278689 | 3.076923 | 3.225806 |
| 3.100775 | 3.174603 | 3.252033 | 3.305785 | 3.252033 | 2.898551 | 2.985075 | 3.007519 | 3.030303 | 3.278689 |
| 2.962963 | 3.149606 | 3.278689 | 3.389831 | 3.174603 | 3.305785 | 2.898551 | 3.100775 | 3.361345 | 2.919708 |
| 3.149606 | 3.305785 | 2.941176 | 3.508772 | 2.797203 | 3.100775 | 3.225806 | 2.985075 | 3.278689 | 3.278689 |
| 3.252033 | 3.361345 | 3.225806 | 3.305785 | 2.962963 | 3.571429 | 3.252033 | 3.100775 | 2.857143 | 3.125 |
| 2.857143 | 3.305785 | 3.100775 | 3.305785 | 3.278689 | 3.100775 | 3.305785 | 2.836879 | 3.225806 | 3.149606 |
| 2.962963 | 3.053435 | 2.962963 | 3.007519 | 3.149606 | 2.816901 | 3.149606 | 2.857143 | 3.2 | 3.305785 |
| 3.125 | 3.149606 | 3.539823 | 3.252033 | 3.007519 | 3.278689 | 3.125 | 3.100775 | 3.225806 | 3.030303 |
| 3.053435 | 3.076923 | 3.2 | 3.174603 | 3.076923 | 3.174603 | 3.007519 | 3.125 | 3.076923 | 3.053435 |
| 3.100775 | 3.2 | 3.076923 | 3.007519 | 3.333333 | 3.571429 | 3.125 | 3.174603 | 3.225806 | 2.985075 |

次数 =400：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.149606 | 3.053435 | 3.030303 | 3.076923 | 3.448276 | 3.076923 | 3.162055 | 3.174603 | 3.389831 | 3.162055 |
| 3.389831 | 2.973978 | 3.088803 | 3.292181 | 3.030303 | 2.909091 | 3.018868 | 3.187251 | 3.125 | 2.996255 |
| 3.212851 | 2.985075 | 3.225806 | 3.149606 | 3.265306 | 3.053435 | 3.088803 | 3.053435 | 3.125 | 3.252033 |
| 3.162055 | 3.212851 | 3.137255 | 3.252033 | 3.389831 | 3.088803 | 3.125 | 3.225806 | 3.187251 | 3.278689 |
| 3.238866 | 3.212851 | 2.996255 | 3.225806 | 3.053435 | 3.187251 | 3.041825 | 3.065134 | 3.34728 | 3.065134 |
| 3.065134 | 2.846975 | 3.187251 | 3.212851 | 3.100775 | 3.174603 | 3.041825 | 3.187251 | 3.11284 | 3.137255 |
| 3.149606 | 3.174603 | 2.930403 | 3.053435 | 3.100775 | 3.433476 | 3.125 | 3.076923 | 3.149606 | 3.088803 |
| 3.252033 | 3.125 | 3.319502 | 3.041825 | 3.252033 | 3.278689 | 3.174603 | 3.137255 | 2.919708 | 3.076923 |
| 3.125 | 3.007519 | 3.2 | 3.225806 | 3.252033 | 3.041825 | 3.11284 | 3.100775 | 3.076923 | 3.11284 |
| 3.018868 | 3.053435 | 3.319502 | 2.973978 | 3.162055 | 3.389831 | 3.252033 | 3.030303 | 2.919708 | 3.174603 |

次数 =600：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.092784 | 3.166227 | 3.045685 | 3.287671 | 3.2 | 3.26087 | 3.174603 | 2.977667 | 3.133159 | 3.116883 |
| 3.069054 | 3.061224 | 3.166227 | 3.243243 | 3.084833 | 3.141361 | 3.174603 | 3.092784 | 3.208556 | 3.217158 |
| 3.02267 | 3.174603 | 3.149606 | 3.234501 | 3.133159 | 3.076923 | 2.962963 | 3.252033 | 3.183024 | 3.157895 |
| 3.287671 | 3.015075 | 3.191489 | 3.037975 | 3.108808 | 3.234501 | 3.084833 | 3.125 | 2.977667 | 2.955665 |
| 3.076923 | 3.084833 | 3.125 | 3.183024 | 3.076923 | 3.02267 | 3.183024 | 2.955665 | 3.166227 | 3.389831 |
| 3.061224 | 3.174603 | 3.26087 | 3.166227 | 3.314917 | 3.030303 | 3.133159 | 3.225806 | 3.269755 | 2.985075 |
| 2.962963 | 3.069054 | 3.069054 | 3.208556 | 3.278689 | 3.149606 | 3.133159 | 3.269755 | 3.069054 | 3.149606 |
| 3.045685 | 3.174603 | 3.2 | 3.125 | 3.108808 | 3.015075 | 3.045685 | 3.208556 | 2.977667 | 3.100775 |
| 3.174603 | 3.149606 | 3.076923 | 2.985075 | 3.053435 | 3.116883 | 2.850356 | 3.174603 | 3.2 | 2.992519 |
| 3.061224 | 3.269755 | 3.166227 | 3.076923 | 3.157895 | 3.030303 | 3.030303 | 3.166227 | 3.174603 | 3.191489 |

次数 =800：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.125 | 3.030303 | 3.018868 | 3.018868 | 3.082852 | 3.292181 | 3.168317 | 3.076923 | 3.013183 | 3.094778 |
| 3.11284 | 3.094778 | 3.118908 | 3.174603 | 3.024575 | 3.013183 | 2.973978 | 3.174603 | 3.065134 | 3.174603 |
| 3.125 | 3.168317 | 3.082852 | 3.162055 | 3.206413 | 2.985075 | 3.024575 | 3.225806 | 3.225806 | 3.312629 |
| 3.149606 | 3.187251 | 3.065134 | 3.265306 | 3.143418 | 3.053435 | 3.418803 | 3.168317 | 3.155819 | 3.155819 |
| 3.065134 | 3.212851 | 3.047619 | 3.252033 | 3.137255 | 3.258656 | 3.137255 | 2.973978 | 3.238866 | 3.312629 |
| 3.053435 | 3.082852 | 3.149606 | 3.193613 | 3.212851 | 3.252033 | 3.285421 | 3.024575 | 3.137255 | 3.312629 |
| 3.100775 | 3.118908 | 3.155819 | 3.047619 | 3.206413 | 3.2 | 3.059273 | 3.2 | 3.245436 | 3.2 |
| 3.245436 | 3.252033 | 3.131115 | 3.265306 | 3.180915 | 3.174603 | 3.076923 | 3.238866 | 3.206413 | 3.265306 |
| 3.071017 | 3.155819 | 3.082852 | 3.155819 | 3.368421 | 3.137255 | 3.206413 | 3.180915 | 3.137255 | 3.11284 |
| 3.168317 | 3.354298 | 3.143418 | 3.088803 | 3.100775 | 3.312629 | 3.118908 | 3.11284 | 3.036053 | 3.082852 |

次数 =1000：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.189793 | 3.11042 | 3.236246 | 3.12989 | 3.067485 | 3.139717 | 3.189793 | 3.09119 | 3.08642 | 3.11042 |
| 3.220612 | 3.125 | 3.194888 | 3.174603 | 3.246753 | 3.220612 | 3.115265 | 3.134796 | 3.210273 | 3.11042 |
| 3.12989 | 3.149606 | 3.115265 | 3.10559 | 3.215434 | 3.2 | 3.053435 | 3.159558 | 2.985075 | 3.04878 |
| 3.338898 | 3.252033 | 3.169572 | 3.174603 | 3.10559 | 3.144654 | 3.09119 | 3.120125 | 3.154574 | 3.095975 |
| 3.174603 | 3.10559 | 3.154574 | 3.08642 | 3.194888 | 3.081664 | 3.062787 | 3.125 | 3.205128 | 3.205128 |
| 3.246753 | 3.10559 | 3.215434 | 3.2 | 3.053435 | 3.284072 | 3.115265 | 3.164557 | 3.174603 | 3.095975 |
| 3.120125 | 3.215434 | 3.120125 | 3.10559 | 3.231018 | 3.149606 | 3.134796 | 3.189793 | 3.267974 | 3.17965 |
| 3.389831 | 3.241491 | 3.149606 | 3.115265 | 3.144654 | 3.10559 | 3.081664 | 3.072197 | 3.072197 | 3.154574 |
| 3.11042 | 3.267974 | 3.053435 | 3.062787 | 3.174603 | 3.139717 | 3.139717 | 3.262643 | 3.174603 | 3.12989 |
| 3.100775 | 3.164557 | 3.273322 | 3.08642 | 3.159558 | 3.072197 | 3.225806 | 3.241491 | 3.04414 | 3.144654 |

次数 =1200：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.065134 | 3.133159 | 3.178808 | 3.274216 | 3.221477 | 3.137255 | 3.065134 | 3.191489 | 3.049555 | 3.092784 |
| 3.030303 | 3.129074 | 3.145478 | 3.11284 | 3.080873 | 3.178808 | 3.11284 | 3.191489 | 3.234501 | 3.129074 |
| 3.116883 | 3.187251 | 3.049555 | 3.17041 | 3.088803 | 3.092784 | 3.11284 | 3.153745 | 3.296703 | 3.100775 |
| 3.137255 | 3.204272 | 3.157895 | 3.145478 | 3.076923 | 3.212851 | 3.108808 | 3.204272 | 3.088803 | 3.178808 |
| 2.977667 | 3.088803 | 3.204272 | 3.053435 | 3.153745 | 3.129074 | 3.088803 | 3.11284 | 3.166227 | 3.174603 |
| 3.283174 | 3.157895 | 3.195739 | 3.02267 | 3.11284 | 3.100775 | 3.069054 | 3.084833 | 3.080873 | 3.217158 |
| 3.092784 | 3.049555 | 3.125 | 3.137255 | 3.069054 | 3.072983 | 3.11284 | 3.080873 | 3.238866 | 3.080873 |
| 3.204272 | 3.096774 | 3.225806 | 3.069054 | 3.221477 | 3.076923 | 3.234501 | 3.037975 | 3.287671 | 3.120936 |
| 3.274216 | 3.096774 | 3.174603 | 3.225806 | 3.096774 | 3.26087 | 3.204272 | 3.162055 | 3.183024 | 3.153745 |
| 3.057325 | 3.034134 | 3.141361 | 3.108808 | 3.096774 | 3.034134 | 3.096774 | 3.061224 | 3.212851 | 3.178808 |

次数 =1400：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.100775 | 3.131991 | 3.222094 | 3.063457 | 3.259604 | 3.001072 | 3.076923 | 3.013994 | 3.040174 | 3.146067 |
| 3.142536 | 3.111111 | 3.203661 | 3.222094 | 3.192702 | 3.060109 | 3.139013 | 3.104213 | 3.100775 | 3.156708 |
| 3.203661 | 3.271028 | 3.11804 | 3.093923 | 3.090508 | 3.229527 | 3.160271 | 3.063457 | 3.192702 | 3.107658 |
| 3.097345 | 3.063457 | 3.013994 | 3.080308 | 3.178207 | 3.178207 | 3.104213 | 3.2 | 3.189066 | 3.192702 |
| 3.128492 | 3.203661 | 3.149606 | 3.104213 | 3.214696 | 3.0837 | 3.070175 | 3.167421 | 3.142536 | 3.163842 |
| 3.181818 | 3.020496 | 3.149606 | 3.156708 | 3.185438 | 3.073546 | 3.125 | 3.171008 | 3.196347 | 3.036876 |
| 3.240741 | 3.100775 | 3.114572 | 3.146067 | 3.043478 | 3.171008 | 3.278689 | 3.135498 | 3.167421 | 3.097345 |
| 3.153153 | 3.100775 | 3.114572 | 3.167421 | 3.290247 | 3.189066 | 3.244496 | 3.036876 | 3.156708 | 3.222094 |
| 3.2 | 3.2 | 3.053435 | 3.255814 | 3.153153 | 3.244496 | 3.185438 | 3.080308 | 3.160271 | 3.004292 |
| 3.185438 | 3.104213 | 3.121516 | 3.111111 | 3.185438 | 3.056769 | 3.139013 | 3.056769 | 3.207331 | 3.0837 |

次数 =1600：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.121951 | 3.162055 | 3.165183 | 3.059273 | 3.149606 | 3.18408 | 3.050524 | 3.252033 | 3.082852 | 3.193613 |
| 3.193613 | 3.152709 | 3.206413 | 3.155819 | 3.137255 | 3.094778 | 3.131115 | 3.168317 | 3.106796 | 3.125 |
| 3.265306 | 3.094778 | 3.212851 | 3.094778 | 3.177756 | 3.068073 | 3.180915 | 3.001876 | 3.121951 | 3.125 |
| 3.203203 | 3.11284 | 3.149606 | 3.248731 | 3.118908 | 3.079885 | 3.140334 | 3.271984 | 3.11284 | 3.245436 |
| 3.106796 | 3.180915 | 3.076923 | 3.056351 | 3.149606 | 3.143418 | 3.225806 | 3.238866 | 3.056351 | 3.088803 |
| 3.268641 | 3.155819 | 3.118908 | 3.203203 | 3.177756 | 3.100775 | 3.152709 | 3.165183 | 3.106796 | 3.103783 |
| 3.059273 | 3.155819 | 3.158934 | 3.174603 | 3.225806 | 3.177756 | 3.21608 | 3.174603 | 3.149606 | 3.115871 |
| 3.162055 | 3.212851 | 3.079885 | 3.068073 | 3.088803 | 3.097773 | 3.106796 | 3.152709 | 3.076923 | 3.158934 |
| 3.152709 | 3.088803 | 3.196803 | 3.11284 | 3.071017 | 3.109815 | 3.235592 | 3.146509 | 3.203203 | 3.248731 |
| 3.158934 | 3.252033 | 3.155819 | 3.245436 | 3.295572 | 3.100775 | 3.068073 | 3.149606 | 3.134182 | 3.149606 |

次数 =1800：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.007519 | 3.106126 | 3.116883 | 3.114187 | 3.211418 | 3.119584 | 3.076923 | 3.135889 | 3.090129 | 3.174603 |
| 3.157895 | 3.133159 | 3.183024 | 3.030303 | 3.155127 | 3.056027 | 3.06644 | 3.02267 | 3.146853 | 3.108808 |
| 3.249097 | 3.103448 | 3.095443 | 3.103448 | 3.146853 | 3.202847 | 3.116883 | 3.141361 | 3.119584 | 3.185841 |
| 3.108808 | 3.211418 | 3.171806 | 3.053435 | 3.163445 | 3.266788 | 3.197158 | 3.160667 | 3.222919 | 3.272727 |
| 3.177405 | 3.079555 | 3.2 | 3.194321 | 3.141361 | 3.092784 | 3.087479 | 3.111495 | 3.010033 | 3.231598 |
| 3.246168 | 3.217158 | 3.152364 | 3.127715 | 3.114187 | 3.108808 | 3.157895 | 3.205699 | 3.23741 | 3.116883 |
| 3.197158 | 3.166227 | 3.225806 | 3.133159 | 3.141361 | 3.185841 | 3.180212 | 3.108808 | 3.135889 | 3.177405 |
| 3.061224 | 3.119584 | 3.116883 | 3.188663 | 3.079555 | 3.12229 | 3.146853 | 3.125 | 3.045685 | 3.082192 |
| 3.135889 | 3.152364 | 3.155127 | 3.138622 | 3.2287 | 3.174603 | 3.191489 | 3.141361 | 3.06644 | 3.135889 |
| 3.166227 | 3.152364 | 3.202847 | 3.183024 | 3.084833 | 3.043111 | 3.127715 | 3.144105 | 3.114187 | 3.180212 |

次数 =2000：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.197442 | 3.149606 | 3.074558 | 3.117693 | 3.189793 | 3.095975 | 3.149606 | 3.187251 | 3.10559 | 3.162055 |
| 3.225806 | 3.098373 | 3.058104 | 3.067485 | 3.12989 | 3.069839 | 3.149606 | 3.11284 | 3.197442 | 3.223207 |
| 3.246753 | 3.174603 | 3.065134 | 3.142184 | 3.205128 | 3.095975 | 3.157064 | 3.164557 | 3.164557 | 3.223207 |
| 3.093581 | 3.134796 | 3.134796 | 3.147128 | 3.018868 | 3.152088 | 3.172086 | 3.100775 | 3.095975 | 3.157064 |
| 3.132341 | 3.125 | 3.051106 | 3.144654 | 3.108003 | 3.069839 | 3.207698 | 3.147128 | 3.187251 | 3.139717 |
| 3.238866 | 3.103181 | 3.24412 | 3.08404 | 3.172086 | 3.2 | 3.149606 | 3.202562 | 3.159558 | 3.039514 |
| 3.04878 | 3.139717 | 3.194888 | 3.212851 | 3.003003 | 3.053435 | 3.046458 | 3.060444 | 3.149606 | 3.11284 |
| 3.154574 | 3.262643 | 3.088803 | 3.194888 | 3.223207 | 3.08404 | 3.11284 | 3.164557 | 3.174603 | 3.120125 |
| 3.051106 | 3.162055 | 3.12989 | 3.142184 | 3.103181 | 3.18218 | 3.154574 | 3.144654 | 3.127443 | 3.164557 |
| 3.132341 | 3.197442 | 3.137255 | 3.12989 | 3.164557 | 3.088803 | 3.210273 | 3.238866 | 3.11284 | 3.177125 |

次数 =2125：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.141168 | 3.145818 | 3.122704 | 3.138848 | 3.193088 | 3.136531 | 3.234399 | 3.141168 | 3.118122 | 3.185907 |
| 3.143491 | 3.15516 | 3.111274 | 3.176383 | 3.127299 | 3.07971 | 3.125 | 3.190691 | 3.104456 | 3.131909 |
| 3.190691 | 3.207547 | 3.136531 | 3.176383 | 3.08418 | 3.185907 | 3.181138 | 3.141168 | 3.162202 | 3.141168 |
| 3.064167 | 3.08642 | 3.064167 | 3.111274 | 3.075253 | 3.171642 | 3.115836 | 3.176383 | 3.10219 | 3.134218 |
| 3.185907 | 3.131909 | 3.159851 | 3.066378 | 3.113553 | 3.113553 | 3.104456 | 3.08642 | 3.190691 | 3.159851 |
| 3.111274 | 3.264209 | 3.231939 | 3.229483 | 3.08418 | 3.136531 | 3.202713 | 3.166915 | 3.136531 | 3.131909 |
| 3.222138 | 3.143491 | 3.20997 | 3.159851 | 3.070809 | 3.136531 | 3.120411 | 3.136531 | 3.106725 | 3.08642 |
| 3.068592 | 3.053161 | 3.118122 | 3.202713 | 3.195489 | 3.164557 | 3.169277 | 3.095412 | 2.999294 | 3.185907 |
| 3.106725 | 3.178758 | 3.136531 | 3.183521 | 3.093159 | 3.157504 | 3.141168 | 3.148148 | 3.07303 | 3.07303 |
| 3.181138 | 3.178758 | 3.193088 | 3.141168 | 3.21726 | 3.249235 | 3.229483 | 3.120411 | 3.143491 | 3.249235 |

次数 =2250：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.042596 | 3.153469 | 3.114187 | 3.109883 | 3.200569 | 3.214286 | 3.046716 | 3.114187 | 3.15568 | 3.214286 |
| 3.099174 | 3.146853 | 3.112033 | 3.10559 | 3.171247 | 3.207413 | 3.149055 | 3.162333 | 3.122831 | 3.116343 |
| 3.225806 | 3.04878 | 3.225806 | 3.164557 | 3.101309 | 3.216583 | 3.153469 | 3.069577 | 3.107735 | 3.140265 |
| 3.084304 | 3.173484 | 3.221188 | 3.189227 | 3.040541 | 3.196023 | 3.107735 | 3.180212 | 3.177966 | 3.133705 |
| 3.186969 | 3.092784 | 3.193754 | 3.101309 | 3.109883 | 3.2097 | 3.114187 | 3.151261 | 3.223496 | 3.080082 |
| 3.103448 | 3.153469 | 3.166784 | 3.15568 | 3.082192 | 3.127172 | 3.135889 | 3.125 | 3.2097 | 3.092784 |
| 3.191489 | 3.144654 | 3.171247 | 3.207413 | 3.092784 | 3.118503 | 3.118503 | 3.177966 | 3.120666 | 3.138075 |
| 3.061224 | 3.004005 | 3.173484 | 3.092784 | 3.112033 | 3.171247 | 3.129346 | 3.135889 | 3.077975 | 3.15568 |
| 3.196023 | 3.090659 | 3.133705 | 3.153469 | 3.107735 | 3.065395 | 3.092784 | 3.129346 | 3.069577 | 3.173484 |
| 3.175723 | 3.109883 | 3.059143 | 3.173484 | 3.131524 | 3.180212 | 3.131524 | 3.059143 | 3.090659 | 3.173484 |

次数 =2375：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.100522 | 3.173013 | 3.170895 | 3.060567 | 3.135314 | 3.179384 | 3.198653 | 3.205128 | 3.168779 | 3.260124 |
| 3.078419 | 3.156146 | 3.110675 | 3.164557 | 3.112713 | 3.187919 | 3.094463 | 3.220339 | 3.066495 | 3.14778 |
| 3.173013 | 3.211629 | 3.118844 | 3.19006 | 3.202967 | 3.168779 | 3.141534 | 3.16245 | 3.120894 | 3.145695 |
| 3.110675 | 3.200809 | 3.248974 | 3.09648 | 3.15405 | 3.14778 | 3.235695 | 3.164557 | 3.181514 | 3.112713 |
| 3.143614 | 3.231293 | 3.141534 | 3.196501 | 3.194351 | 3.151958 | 3.14778 | 3.122945 | 3.143614 | 3.173013 |
| 3.192204 | 3.116798 | 3.100522 | 3.120894 | 3.173013 | 3.149867 | 3.084416 | 3.092448 | 3.160346 | 3.170895 |
| 3.116798 | 3.145695 | 3.013959 | 3.160346 | 3.145695 | 3.160346 | 3.166667 | 3.15405 | 3.207292 | 3.179384 |
| 3.173013 | 3.060567 | 3.066495 | 3.143614 | 3.104575 | 3.264605 | 3.218157 | 3.215978 | 3.0985 | 3.181514 |
| 3.177258 | 3.135314 | 3.127057 | 3.151958 | 3.177258 | 3.179384 | 3.127057 | 3.287197 | 3.078419 | 3.151958 |
| 3.307799 | 3.090436 | 3.141534 | 3.127057 | 3.151958 | 3.143614 | 3.108639 | 3.129117 | 3.149867 | 3.149867 |

次数 =2500：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.136763 | 3.188776 | 3.073141 | 3.17662 | 3.227889 | 3.092146 | 3.082614 | 3.115265 | 3.078818 | 3.19081 |
| 3.095975 | 3.250975 | 3.142678 | 3.10752 | 3.186743 | 3.092146 | 3.097893 | 3.075031 | 3.126954 | 3.213368 |
| 3.037667 | 3.146633 | 3.103662 | 3.10752 | 3.192848 | 3.164557 | 3.15856 | 3.162555 | 3.125 | 3.152585 |
| 3.119152 | 3.152585 | 3.128911 | 3.184713 | 3.170577 | 3.17864 | 3.136763 | 3.117207 | 3.138732 | 3.125 |
| 3.128911 | 3.113325 | 3.099814 | 3.209243 | 3.182686 | 3.184713 | 3.109453 | 3.152585 | 3.184713 | 3.15856 |
| 3.205128 | 3.221649 | 3.134796 | 3.138732 | 3.194888 | 3.215434 | 3.160556 | 3.075031 | 3.162555 | 3.095975 |
| 3.10752 | 3.236246 | 3.152585 | 3.119152 | 3.17662 | 3.134796 | 3.084516 | 3.148615 | 3.142678 | 3.138732 |
| 3.203075 | 3.138732 | 3.144654 | 3.136763 | 3.111388 | 3.205128 | 3.148615 | 3.08642 | 3.168568 | 3.184713 |
| 3.150599 | 3.227889 | 3.188776 | 3.174603 | 3.13087 | 3.121099 | 3.172589 | 3.069368 | 3.121099 | 3.103662 |
| 3.103662 | 3.205128 | 3.088326 | 3.186743 | 3.138732 | 3.061849 | 3.162555 | 3.188776 | 3.061849 | 3.121099 |

次数 =2625：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.115727 | 3.153153 | 3.128725 | 3.187614 | 3.104672 | 3.155048 | 3.158845 | 3.143713 | 3.13059 | 3.158845 |
| 3.191489 | 3.047011 | 3.082795 | 3.101004 | 3.108348 | 3.160747 | 3.123141 | 3.132458 | 3.11943 | 3.121284 |
| 3.143713 | 3.084606 | 3.08642 | 3.093695 | 3.102837 | 3.166466 | 3.079179 | 3.112033 | 3.141831 | 3.099174 |
| 3.160747 | 3.11943 | 3.158845 | 3.18568 | 3.207086 | 3.134328 | 3.179891 | 3.115727 | 3.183748 | 3.17029 |
| 3.106509 | 3.139952 | 3.102837 | 3.166466 | 3.082795 | 3.071972 | 3.099174 | 3.17029 | 3.106509 | 3.132458 |
| 3.177966 | 3.199269 | 3.125 | 3.132458 | 3.211009 | 3.191489 | 3.145596 | 3.091873 | 3.181818 | 3.126861 |
| 3.166466 | 3.138075 | 3.132458 | 3.18568 | 3.136201 | 3.136201 | 3.14937 | 3.18955 | 3.176044 | 3.123141 |
| 3.090053 | 3.18568 | 3.101004 | 3.18955 | 3.115727 | 3.132458 | 3.17029 | 3.155048 | 3.164557 | 3.115727 |
| 3.195374 | 3.209046 | 3.160747 | 3.134328 | 3.104672 | 3.205128 | 3.095519 | 3.230769 | 3.177966 | 3.166466 |
| 3.155048 | 3.11019 | 3.101004 | 3.121284 | 3.162651 | 3.123141 | 3.102837 | 3.13059 | 3.115727 | 3.151261 |

次数 =2750：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.15729 | 3.125 | 3.08642 | 3.102087 | 3.109101 | 3.173687 | 3.069196 | 3.132118 | 3.119682 | 3.155479 |
| 3.205128 | 3.192107 | 3.218256 | 3.08642 | 3.062361 | 3.159104 | 3.102087 | 3.077784 | 3.159104 | 3.244838 |
| 3.114383 | 3.15729 | 3.142857 | 3.164557 | 3.074343 | 3.201397 | 3.107345 | 3.162737 | 3.031974 | 3.203262 |
| 3.220141 | 3.188406 | 3.141062 | 3.119682 | 3.181029 | 3.15729 | 3.171857 | 3.114383 | 3.168203 | 3.162737 |
| 3.216374 | 3.133903 | 3.203262 | 3.151862 | 3.098592 | 3.076063 | 3.18287 | 3.170029 | 3.190255 | 3.100338 |
| 3.08642 | 3.128555 | 3.119682 | 3.125 | 3.117914 | 3.126777 | 3.079507 | 3.142857 | 3.141062 | 3.241014 |
| 3.168203 | 3.11086 | 3.119682 | 3.166379 | 3.193961 | 3.121453 | 3.125 | 3.076063 | 3.141062 | 3.067485 |
| 3.109101 | 3.159104 | 3.205128 | 3.126777 | 3.089888 | 3.177354 | 3.079507 | 3.096847 | 3.119682 | 3.126777 |
| 3.173687 | 3.210741 | 3.130336 | 3.195816 | 3.144654 | 3.137479 | 3.04878 | 3.093363 | 3.168203 | 3.16092 |
| 3.139269 | 3.10559 | 3.159104 | 3.166379 | 3.117914 | 3.15367 | 3.159104 | 3.123225 | 3.069196 | 3.210741 |

次数 =2825：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.079019 | 3.106102 | 3.161724 | 3.16704 | 3.226728 | 3.202948 | 3.109521 | 3.16704 | 3.144129 | 3.181306 |
| 3.118102 | 3.137146 | 3.163494 | 3.16704 | 3.130194 | 3.206583 | 3.170595 | 3.195701 | 3.087432 | 3.16704 |
| 3.075667 | 3.149387 | 3.184893 | 3.14588 | 3.138889 | 3.16704 | 3.068984 | 3.210227 | 3.138889 | 3.149387 |
| 3.144129 | 3.156425 | 3.168817 | 3.152902 | 3.152902 | 3.116382 | 3.224886 | 3.128461 | 3.158189 | 3.119823 |
| 3.128461 | 3.118102 | 3.082379 | 3.100988 | 3.144129 | 3.161724 | 3.14238 | 3.131929 | 3.106102 | 3.243398 |
| 3.140634 | 3.016551 | 3.186689 | 3.14588 | 3.232265 | 3.097588 | 3.195701 | 3.163494 | 3.138889 | 3.055706 |
| 3.156425 | 3.092501 | 3.168817 | 3.181306 | 3.16704 | 3.084061 | 3.130194 | 3.159955 | 3.130194 | 3.245261 |
| 3.140634 | 3.168817 | 3.158189 | 3.19209 | 3.193895 | 3.119823 | 3.107811 | 3.179516 | 3.031116 | 3.170595 |
| 3.109521 | 3.111233 | 3.128461 | 3.102691 | 3.202948 | 3.210227 | 3.092501 | 3.152902 | 3.137146 | 3.19209 |
| 3.19209 | 3.119823 | 3.158189 | 3.111233 | 3.188488 | 3.068984 | 3.170595 | 3.206583 | 3.055706 | 3.193895 |

次数 =3000：

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.181336 | 3.159558 | 3.08642 | 3.157895 | 3.203417 | 3.184713 | 3.09119 | 3.120125 | 3.080082 | 3.213712 |
| 3.113648 | 3.107198 | 3.159558 | 3.115265 | 3.220612 | 3.157895 | 3.208556 | 3.131524 | 3.183024 | 3.095975 |
| 3.094379 | 3.217158 | 3.166227 | 3.19659 | 3.156234 | 3.131524 | 3.11042 | 3.128259 | 3.059663 | 3.120125 |
| 3.094379 | 3.116883 | 3.118503 | 3.156234 | 3.120125 | 3.149606 | 3.069054 | 3.107198 | 3.128259 | 3.07377 |
| 3.112033 | 3.2 | 3.125 | 3.172924 | 3.070624 | 3.118503 | 3.139717 | 3.125 | 3.205128 | 3.172924 |
| 3.162889 | 3.139717 | 3.154574 | 3.084833 | 3.19659 | 3.17965 | 3.172924 | 3.070624 | 3.181336 | 3.12989 |
| 3.151261 | 3.167899 | 3.12989 | 3.151261 | 3.205128 | 3.201708 | 3.213712 | 3.151261 | 3.075346 | 3.184713 |
| 3.149606 | 3.167899 | 3.12989 | 3.151261 | 3.134796 | 3.211991 | 3.118503 | 3.059663 | 3.194888 | 3.05499 |
| 3.134796 | 3.08642 | 3.115265 | 3.146303 | 3.194888 | 3.166227 | 3.144654 | 3.174603 | 3.161222 | 3.147954 |
| 3.11042 | 3.108808 | 3.08642 | 3.12989 | 3.123373 | 3.174603 | 3.161222 | 3.201708 | 3.089598 | 3.103983 |

## 程序代码

### 单次落针实验

```
#try to paint the needle event
import turtle as t
import random

```

```
def paint(x,y,angle,nw):
    #first get to know whether touch the line
    #if: paint in red else:paint in black
    touch=0
    t.up()
    t.color("black")
    t.goto(x,y)
    t.left(angle)
    t.forward(nw/2)
    pos1=t.pos()
    t.backward(nw*2)
    t.right(angle)
    pos2=t.pos()
    linex=-450
    if pos1[0]<-450 or pos1[0]>450 or pos2[0]<-450 or pos2[0]>450 :
        return 1
    for j in range(1,50):
        linex=linex+nw*2
        if pos1[0]<=linex and pos2[0] >=linex or pos1[0]>=linex and pos2[0]<=linex:
            t.color("red")
            touch=1
    t.goto(x,y)
    t.down()
    t.left(angle)
    t.forward(nw/2)
    t.backward(nw*2)
    t.right(angle)
    return touch

t.Screen()
t.speed(10)
t.tracer(False)
#borders and lines
```

```
38 t.up()
39 t.goto(-450,450)
40 t.down()
41 t.goto(450,450)
42 t.goto(450,-450)
43 t.goto(-450,-450)
44 t.goto(-450,450)
45 linenumber=50
46 #represent for the space nums between left and right border
47 width=900/linenumber
48 needlewidth=width/2
49 x=-450
50 y=0
51 angle=0
52 count=0
53  result =0
54 trytime=int(input("please input the times of throwing: "))
55 for i in range(linenumber-1):
56     x=x+width
57     t.up()
58     t.goto(x,450)
59     t.down()
60     t.goto(x,-450)
61
62 for i in range(1,trytime):
63     x=random.uniform(-450,450)
64     y=random.uniform(-450,450)
65     angle= random.uniform(0,180)
66      result =paint(x,y,angle,needlewidth)
67      if result ==1:
68          count=count+1
69      if result ==-1:
70          i=i-1
```

```
print("the approximate value is: ")
print(trytime*2/count)
#end the painting

#-- do add more times'trying and show it in a chart
```

**多次落针实验**

```
#the output file of needle event
import random
import turtle as t
import matplotlib.pyplot as plt
import csv
from matplotlib import gridspec
def throw(x):
    #throw x times
    appv=0
    #get 10 to count for the average value
    #each for one time for throwing
    count=0
    for j in range(x):
        t.up()
        t.color("white")
        locatex=random.uniform(-450,450)
        locatey=random.uniform(-450,450)
        angle=random.uniform(0,180)
        t.goto(locatex,locatey)
        t.left(angle)
        t.forward(4.5)
        pos1=t.pos()
        t.backward(18)
        t.right(angle)
        pos2=t.pos()
```

```
            linex=-450
            if pos1[0]<-450 or pos1[0]>450 or pos2[0]<-450 or pos2[0]>450:
                count=count+1
                continue
            for k in range(1,50):
                linex=linex+18
                if pos1[0]<=linex and pos2[0] >=linex or
                    pos1[0]>=linex and pos2[0]<=linex:
                     count=count+1
                     break
        appv=appv+x*2/count
        print(round(appv,4))
        return appv

#t.Screen()
t.speed(10)
t.tracer(False)

x=[10,50,100,200,400,600,800,1000,1200,1400,1600,1800,2000,2125,2250]
x.append(2375)
x.append(2500)
x.append(2625)
x.append(2750)
x.append(2825)
x.append(3000)
#each try 100 times in the csv and get the average values;
y=[]
total=0
row=[1,1]
f=open('output.csv','w',encoding='utf-8',newline='')
csv_writer=csv.writer(f)
csv_writer.writerow(["times","value"])
for i in x:
```

```
59      row[0]=i
60      total=0
61      for j in range(1,101):
62          row[1]=throw(i)
63          csv_writer.writerow(row)
64          total=total+row[1]
65      total=total/100
66      y.append(total)
67      print(x,end='')
68      print("times, ",end='')
69      print("the average is :",end='')
70      print(y)
71  t.bye()
72  f.close()
73  plt.plot(x,y)
74  plt.title("approximate value of pi", fontsize=22)
75  plt.xlabel("times", fontsize=22)
76  plt.ylabel("value", fontsize=22)
77  plt.axis([0,3200,2.9,3.3])
78  plt.tick_params(axis='both',which='major', labelsize=10)
79  plt.axhline(y=3.1415926,c='red',ls='--',lw=3)
80  plt.show()
81  #plt.scatter(x,y,s=20)
```

# 参考文献

[1] Buffon's needle problem. https://en.wikipedia.org/wiki/Buffon