



*HYPERGRAPHS: THEORY,  
APPLICATIONS AND CHALLENGES*  
**HyTAC**, September 22-25, 2020

# A Design-Methodology for Epidemic Dynamics via Time-Varying Hypergraphs

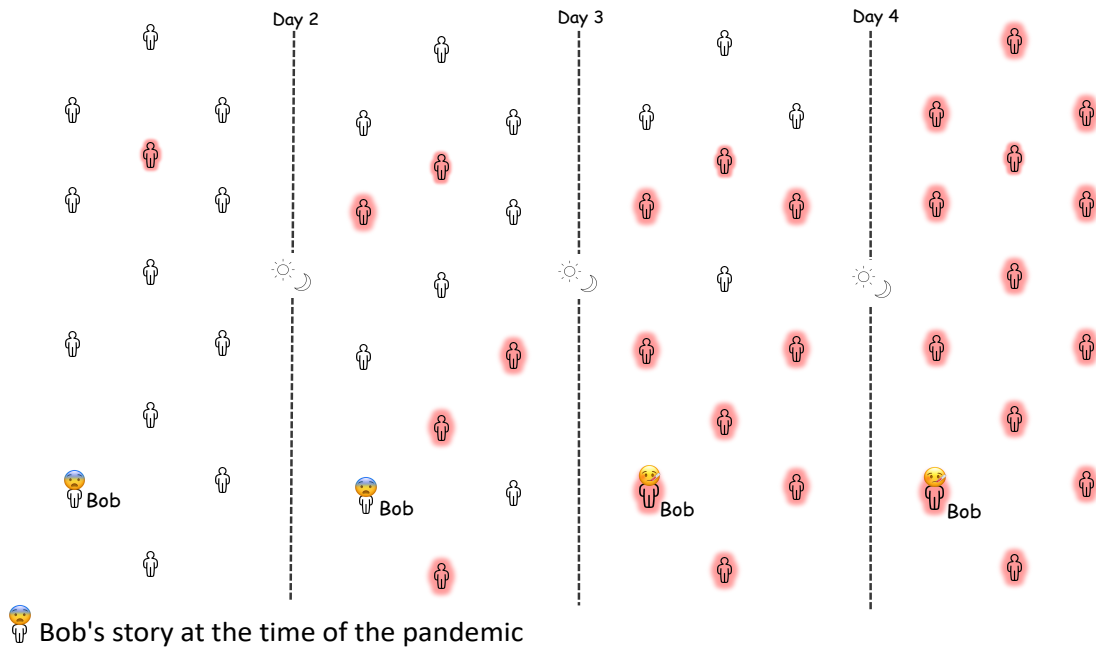
In Proceedings of the 19th International Conference on Autonomous Agents and MultiAgent Systems (pp. 61-69).


*Alessia Antelmi<sup>1</sup>, Gennaro Cordasco<sup>2</sup>, Carmine Spagnuolo<sup>1</sup>, Vittorio Scarano<sup>1</sup>*

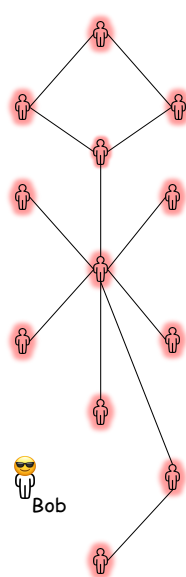
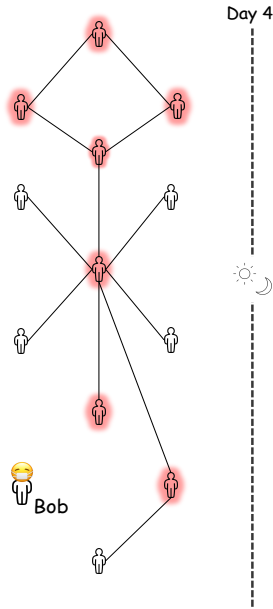
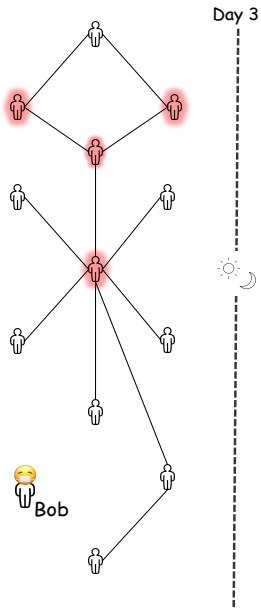
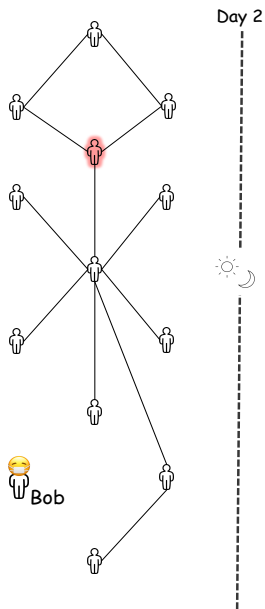
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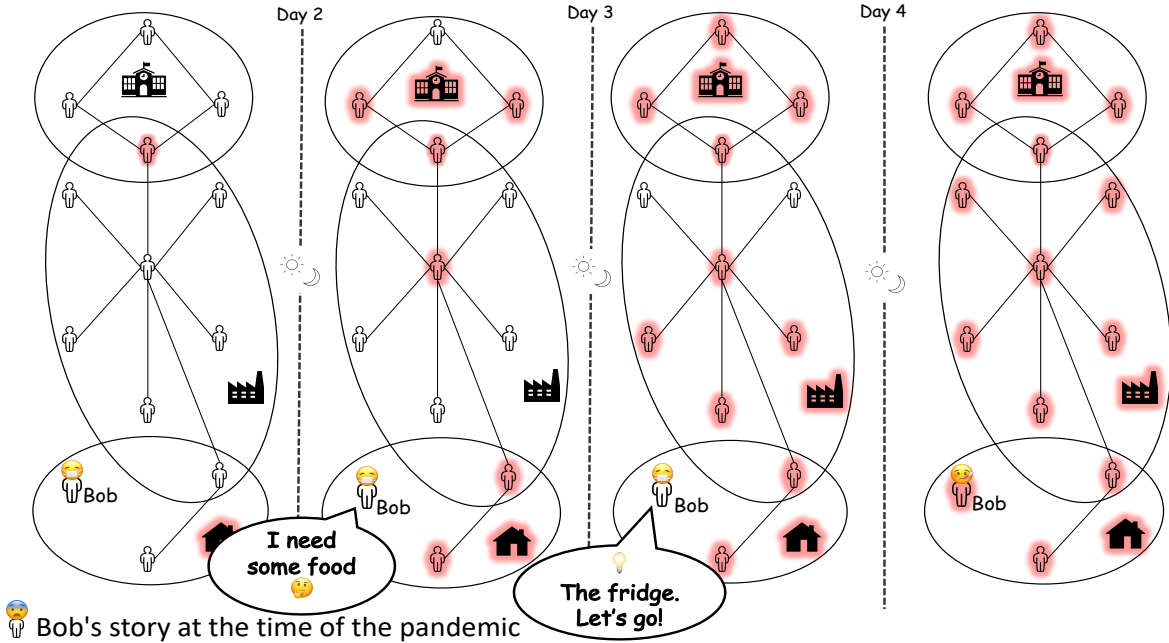
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 Bob's story at the time of the pandemic





# The potion pot of epidemic modeling



- Mathematical modeling
  - Compartmental models (SIS, SIR, ...)
- Human mobility patterns
- Human behavior
- Contagious patterns
- Pathogen properties
  - Contagiousness
  - Length of infectious period
  - Ability to persist on surfaces and environments
  - Severity

- Agent-based modeling
- Contagious pathways
  - Human-to-human
  - Human-to-environment
  - Environment-to-human
- Epidemic models on time-varying networks

## The ideas behind our contribution

We propose an innovative modeling approach to study and analyze the **propagation** of an epidemic over a set of autonomous **individuals** (agents) modeling the contagious patterns using **many-to-many** relationships by exploiting **hypergraphs**.

# Outline

1. Epidemic Dynamics via Time-Varying Hypergraphs
2. Experiments and Results
3. Evaluating Non-Pharmaceutical Interventions (ongoing research..)
4. Conclusion

In 2016, Bodó et al. highlighted two *key* properties of a real model of an epidemic outbreak.

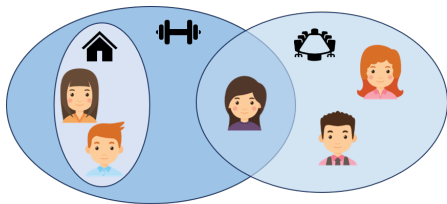
- *Community structure built-up from smaller units.* This idea is translated into practice using different contagion probabilities according to the place.
- *Infection pressure.* The probability that a susceptible individual becomes infected in a unit is not proportional to the number of infected individuals.



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# Time-Varying Hypergraphs (TVHs)

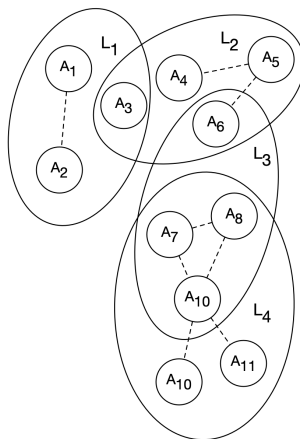


A TVH for an **epidemic diffusion** is a hypergraph  $\mathcal{H} = (V, E, \mathcal{T}, \rho)$ , where

- $V$  is the set of  $n$  vertices (*people*);
- $E$  is the set of  $m$  hyperedges (*locations*);
- $\mathcal{T}$  is the lifetime of the system;
- $\rho : V \times E \times \mathcal{T} \rightarrow \{0, ct_{v,\ell}\}$  is a function mapping whether a given vertex  $v$  has visited the location  $e$  in a given time span  $t$ . The value  $ct_{v,\ell}$  is the last check-in time of  $v$  in  $\ell$ .

# Building a TVH and epidemic model parameters

- A **check-in** specifies when an individual has visited a location.
  - $\Phi$ : time-span of the data sampling.
- **Indirect contacts** (oval shapes): touching furniture, eating contaminated food, airborne.
  - $\Delta$ : time interval within which an *indirect contact* may occur.
- **Direct interactions** (dotted lines): sneezing, whispering, shaking hands.
  - $\delta$ : time interval within which an *direct contact* may occur.



Time interval $i$		
Agent	Location	Time
A <sub>1</sub>	L <sub>1</sub>	08:00:00
A <sub>2</sub>	L <sub>1</sub>	08:00:49
A <sub>3</sub>	L <sub>1</sub>	08:30:00
A <sub>3</sub>	L <sub>2</sub>	09:00:00
A <sub>4</sub>	L <sub>2</sub>	09:30:00
A <sub>5</sub>	L <sub>2</sub>	09:30:40
A <sub>6</sub>	L <sub>2</sub>	09:30:45
A <sub>6</sub>	L <sub>3</sub>	10:00:00
A <sub>7</sub>	L <sub>3</sub>	09:00:00
A <sub>8</sub>	L <sub>3</sub>	09:00:10
A <sub>9</sub>	L <sub>3</sub>	09:00:15
A <sub>7</sub>	L <sub>4</sub>	09:30:00
A <sub>8</sub>	L <sub>4</sub>	09:40:00
A <sub>9</sub>	L <sub>4</sub>	10:00:00
A <sub>10</sub>	L <sub>4</sub>	10:00:10
A <sub>11</sub>	L <sub>4</sub>	10:00:20

*Suppose we want to analyze the spreading of an epidemic over a population of agents through a compartmental model, such as SIS.*

- ① Given a set of check-ins, build a TVH from it.
- ② Define model parameters regulating the contagion:
  - the probabilities of a *direct* and an *indirect* contagion;
  - the probabilities of a spontaneous recovery;
  - the infection pressure of indirect propagation.
- ③ For each time interval in  $|\mathcal{T}|$ , simulate the epidemic spreading over a population of agents according to a **diffusion algorithm**.

- $\Gamma_t$  and  $N_t$  define the neighborhood functions of an agent  $a \in V$  in a given simulation time  $t$ . Specifically,

$$\Gamma_t(a) = \{\ell \in E : \omega(a, \ell, t) = 1\}$$

is the set of locations visited by  $a$  during the interval  $t$ .

- $N_t(a)$  is the set of neighbors of  $a$  during the simulation time  $t$ , which corresponds to the agents that visited at least one of the locations visited by  $a$ . Formally,

$$N_t(a) = \bigcup_{\ell \in \Gamma_t(a)} V_t(\ell),$$

where  $V_t(\ell)$  denotes the set of agents that visited the location  $\ell$  during the interval  $t$ .

- $\Upsilon(a, \ell)$  is a time function which provides the last check-in time of the agent  $a$  in the venue  $\ell$ . In other words, it returns the weight of  $a$  in  $\ell$  in the hypergraph  $\mathcal{H}$ .
- $T_t(a)$  and  $T_t(\ell)$  denote the infection state of an agent or a location at a given time  $t$ , respectively.
- $X_t(a, b)$  is a direct contact function. Given two agents  $a$  and  $b$ , it returns 1 if they have a direct contact in the time span  $t$ , 0 otherwise. Formally,

$$X_t(a, b) = \begin{cases} 1, & \text{if } \exists \ell \in \Gamma_t(a) \cap \Gamma_t(b) \text{ AND } |\Upsilon(a, \ell) - \Upsilon(b, \ell)| < \delta \\ 0, & \text{otherwise.} \end{cases}$$

Each iteration of the diffusion algorithm consists of three contagion phases:

- ① *Agent-to-Environment* → Infection of environment by agents;
  - Evaluated on the number of infected agents that have visited a non-contaminated location.
- ② *Agent-to-Agent* → Direct propagation to agents;
  - Evaluated on number of infected neighbors for each non-infected agent.
- ③ *Environment-to-Agent* → Indirect propagation to agents;
  - Evaluated on number of infected locations visited for each non-infected agent.

# (1) Agent-to-Environment

For all non contaminated locations, (i.e.,  $\ell \in E : T(\ell) = 0$ ), we compute the number of infected agents that have visited that location:

$$I^e(\ell) = \sum_{a \in V(\ell)} T(a).$$

This value is then used to update the contagiousness level of  $\ell$  as expressed by the following:

$$T(\ell) = \begin{cases} 1, & \text{infected according to the value } f^e(I^e(\ell)) \\ 0, & \text{not infected,} \end{cases}$$

where  $f^e()$  is a non-linear function:

$$f^e(x) = \begin{cases} x, & \text{if } 0 \leq x \leq c \\ c, & \text{if } x > c, \end{cases}$$

where  $c$  is a constant value given as parameter.



## (2) Agent-to-Agent

For all non infected agents (i.e.,  $a \in V : T(a) = 0$ ), the total number of infected neighbors is computed. Formally,

$$I^d(a) = \sum_{b \in N(a)} T(b)X(a, b).$$

This value is then used to update the infection state of  $a$ , as

$$T(a) = \begin{cases} 1, & \text{infected according to the value } I^d(a) \\ 0, & \text{not infected.} \end{cases}$$

### (3) *Environment-to-Agent.*

For all non infected agents, (i.e.,  $a \in V : T(a) = 0$ ), we compute the number of infected locations visited. Formally,

$$I^i(a) = \sum_{\ell \in \Gamma(a)} T(\ell).$$

This value is then used to update the infection state of  $a$ , as

$$T(a) = \begin{cases} 1, & \text{infected according to the value } I^i(a) \\ 0, & \text{not infected.} \end{cases}$$

# Diffusion Algorithm Parameters

Parameter	Description
$\beta_d$	Probability that an agent $a_i$ is infected by another agent $a_j$ via a direct-contact in <i>Agent-to-Agent</i>
$\beta_i$	Probability that an agent $a$ is infected via an indirect-contact due to a location $\ell$ in <i>Environment-to-Agent</i>
$\beta_e$	Probability that a location $\ell$ is infected by an agent in <i>Agent-to-Environment</i>
$\gamma_a$	Probability that an agent $a$ spontaneously recovers
$\gamma_e$	Probability that a location $\ell$ is sanitized
$c$	Number of contact in <i>Agent-to-Environment</i>

---

$r$  ▷ a random number generator  $\in [0, 1]$ .

**for**  $t \in \mathcal{T}$  **do**

$\mathcal{H} \leftarrow \xi(t)$

**for**  $\ell \in E$  **do** ▷ Agent-to-Environment.

**if**  $T_t(\ell) == 0$  **then**

**if**  $r_{next} < 1 - e^{-\beta_e f^e(I^e(\ell))}$  **then**

$T_{t+1}(\ell) = 1$

**else if**  $r_{next} < 1 - e^{-\gamma_e}$  **then**

$T_{t+1}(\ell) = 0$

**for**  $a \in V$  **do** ▷ Agent-to-Agent.

**if**  $T_t(a) == 0$  **then**

**if**  $r_{next} < 1 - e^{-\beta_d I^d(\ell)}$  **then**

$T_{t+1}(a) = 1$

**for**  $a \in V$  **do** ▷ Environment-to-Agent.

**if**  $T_t(a) == 0$  **then**

**if**  $r_{next} < 1 - e^{-\beta_i I^i(\ell)}$  **then**

$T_{t+1}(a) = 1$

**else if**  $r_{next} < 1 - e^{-\gamma_a}$  **then**

$T_{t+1}(a) = 0$

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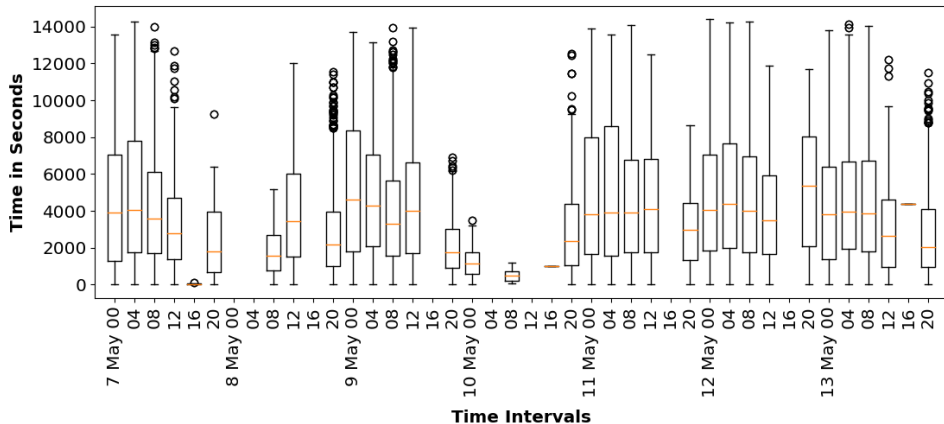
# The Foursquare Data set



- Foursquare social network data [YANG2015+].
- Tokyo, from 12 April 2012 to 16 February 2013.
- 573,703 check-ins, 2,293 users, 61,858 locations.
- Most crowded month: May, 2012.

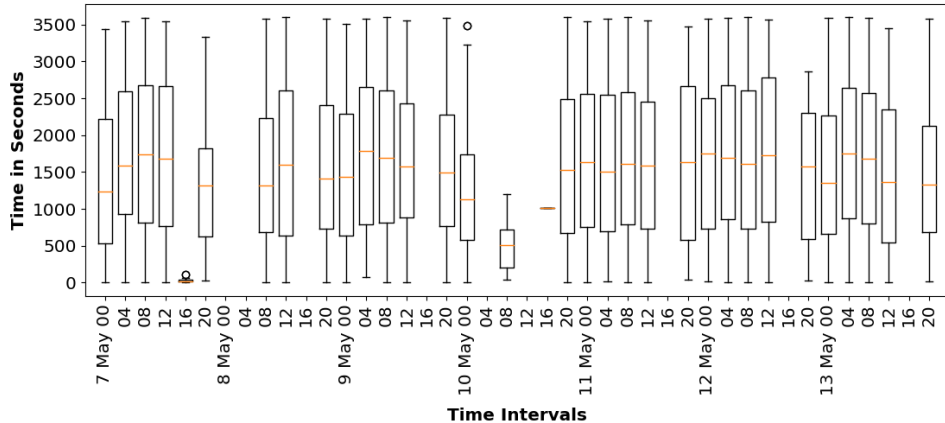
# The Foursquare Data set

Time difference distribution of check-ins within the same place in 7 days and  $\Delta = 4$  hours.



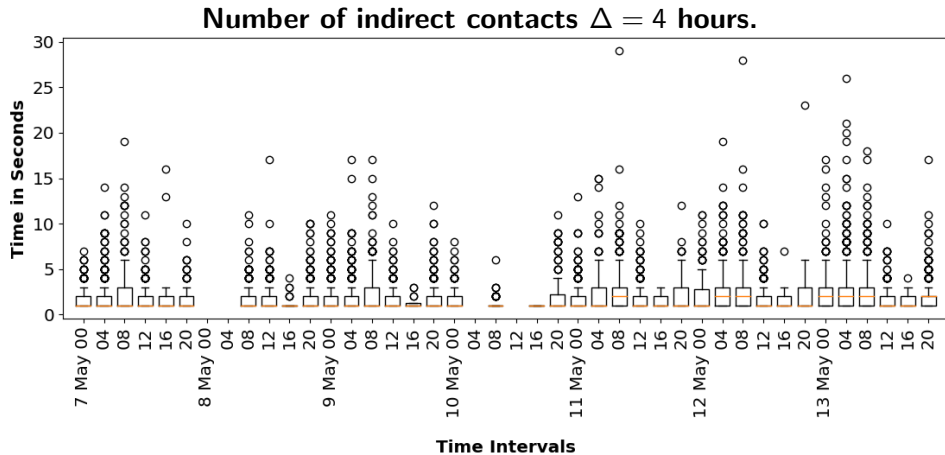
# The Foursquare Data set

Number of direct contacts  $\Delta = 4$  and  $\delta = 1$  hours.

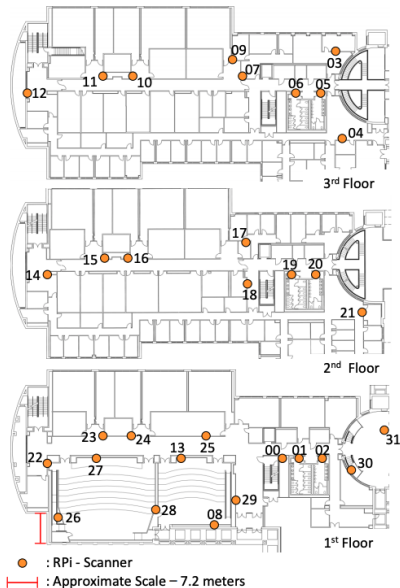




# The Foursquare Data set

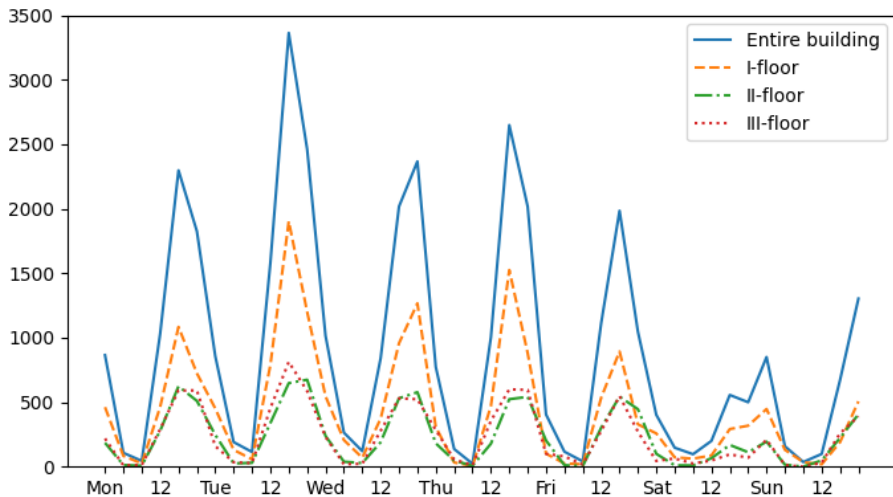


# The BLEBeacon Data set

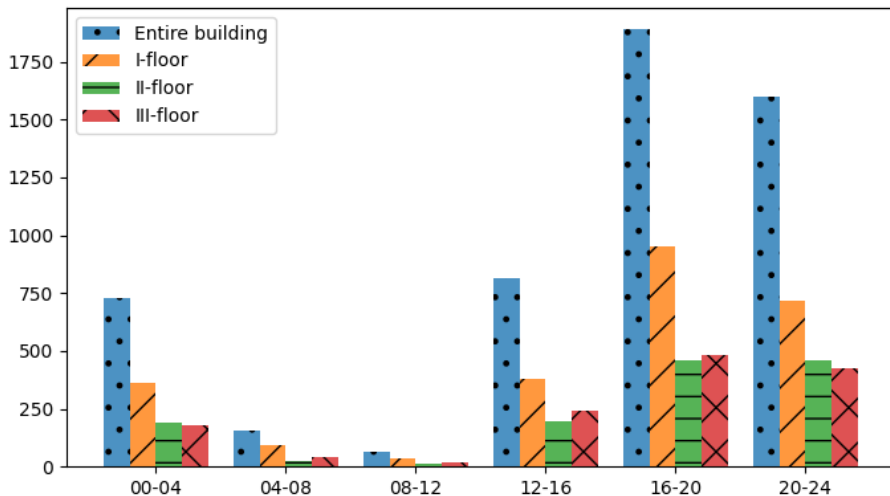


- BLEBeacon data [Sikeridis2018+].
- A collection of Bluetooth Low Energy (BLE) advertisement packets/traces generated from BLE beacons carried by people following their daily routine inside a university building.
- From 15 September 2016 to 17 October 2016.
- 46 users, 32 locations.

# The BLEBeacon Data set: Weekly check-ins



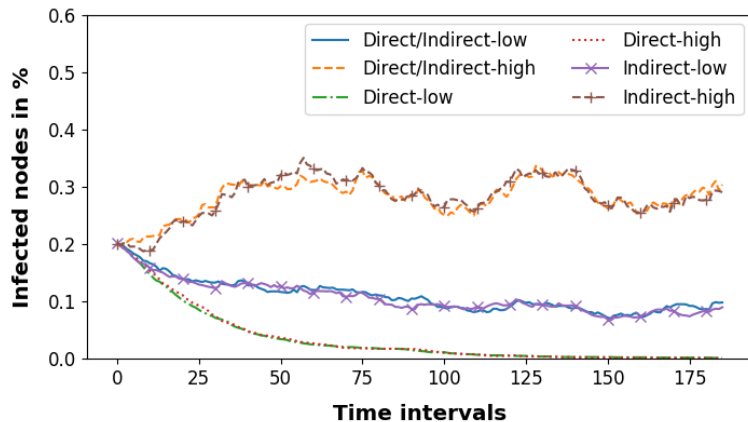
# The BLEBeacon Data set: Daily check-ins



- We experimented the SIS model on the contact-network built upon the Foursquare and BLEBeacon data sets.
- Experimental scenarios:
  - 1 **Direct vs Indirect**, testing the model expressiveness in distinguishing direct and indirect contagion pathways.
  - 2 **Time proprieties of contacts**, effect of time varying intervals length when direct or indirect contacts happen.

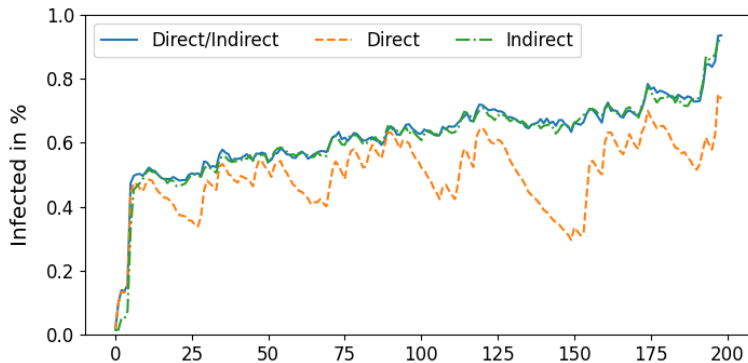
- **Goal.** Testing the model expressiveness in distinguishing direct and indirect contagion pathways;
- $\Delta = 4$  hours,  $\delta = 1$  minutes;
- 80% of the agents *susceptible*, the remaining 20% to *infected*;
- Two parameters configurations has been investigated:
  - *Low*:  $\beta_d = 0.2$ ,  $\beta_i = 0.1$ ,  $\beta_e = 0.06$ ,  $\gamma_e = 0.06$ ,  $\gamma_a = 0.1$ , and  $c = 5$ ;
  - *High*:  $\beta_d = 0.8$ ,  $\beta_i = 0.4$ ,  $\beta_e = 0.26$ ,  $\gamma_e = 0.06$ ,  $\gamma_a = 0.1$ , and  $c = 5$
  - Fixed  $\beta_d$ 
    - $\beta_i = \frac{\beta_d}{2}$ ;
    - $\beta_e = \frac{\beta_d}{4}$ .

# Direct vs Indirect Contagions: Foursquare



Indirect contacts are crucial in spreading the epidemic. They should be investigated when studying epidemic diffusion processes.

# Direct vs Indirect Contagions: BLEBeacon

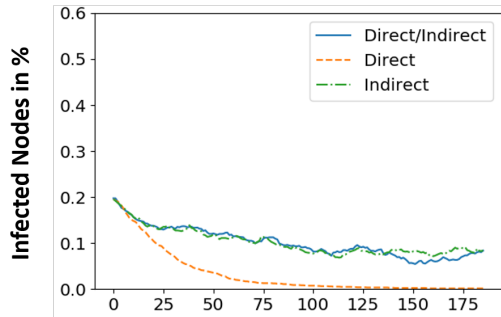


- 1 randomly infected at time  $t = 0$ .
- $\beta_d = 0.56$ ,  $\beta_i = 0.29$ ,  $\beta_e = 0.29$ ,  $\gamma_e = 0.017$ ,  $\gamma_a = 0.034$ , and  $c = 5$ .

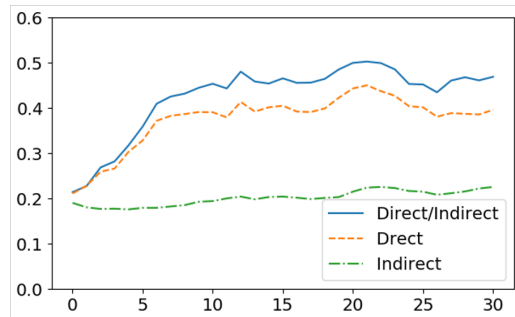


# Contacts time analysis: Foursquare

$\Delta = 4$  hours  
 $\delta = 1$  minute



$\Delta = 24$  hours  
 $\delta = 60$  minutes



Time Intervals

First and last of the 16 configurations  
in the paper.

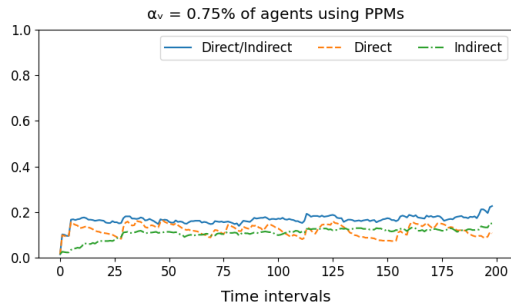
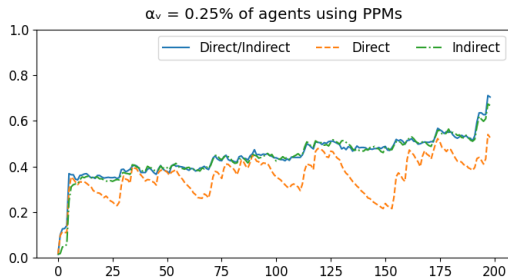


Changing the direct and indirect contagious  
contact-time dramatically shift the epidemic  
spreading pathways.

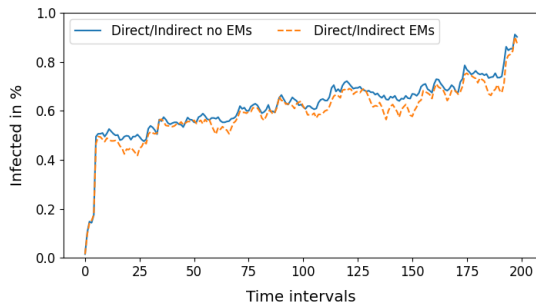
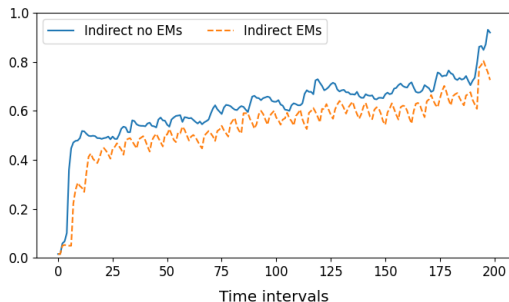
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- *WHO World Health Organization. 2019. Non-pharmaceutical public health measures for mitigating the risk and impact of epidemic and pandemic influenza.*
  - ① Personal protective measures (PPMs).
  - ② Environmental measures (EMs).
  - ③ Social distancing measures (SDMs):
    - Isolation;
    - Quarantine;
    - Avoiding crowding;
    - Contact Tracing.
- Experiments on NPIs

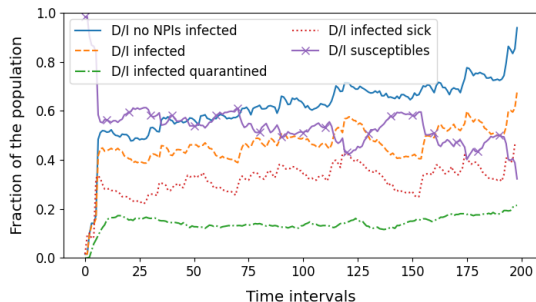
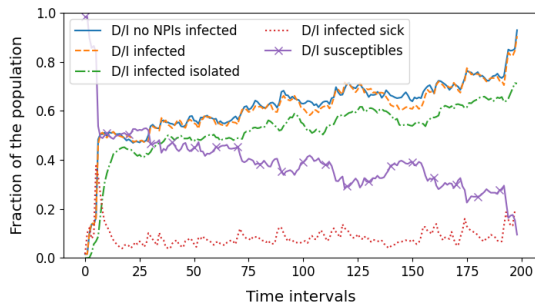


- The transmission probabilities for both direct/indirect are reduced.
- Varying the  $\alpha\%$  of agents that adopt the PPM.



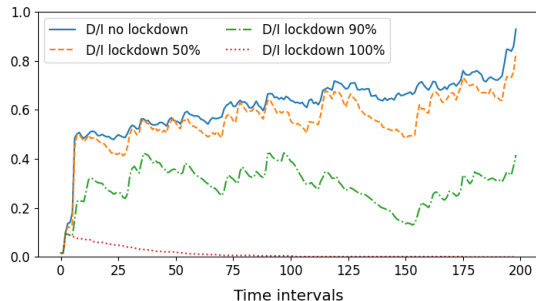
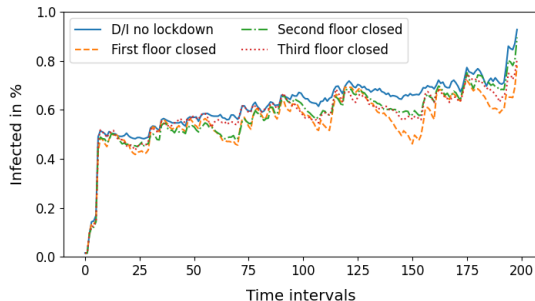
- Each location is cleaned (the infection is removed) at the end of the most crowded intervals: 12:00-16:00 — 16:00-20:00 — 20:00-24:00.

# NPIs - SDMs: Isolation and quarantine



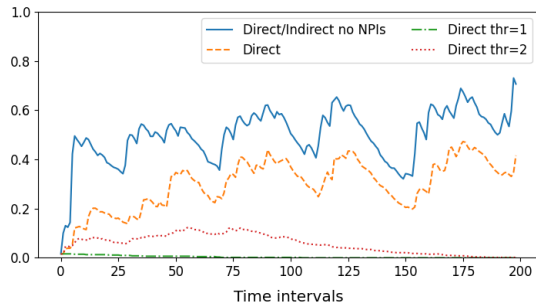
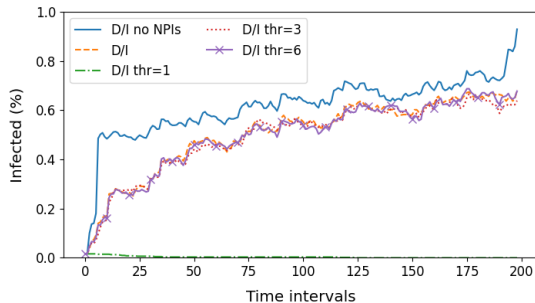
- **Isolation** (left): at the beginning of each step, an agent put itself in isolation with a probability  $\beta_{isolation}$ , proportional to the number of the infected meets, if it is infected.
- **Quarantine** (right): at the beginning of each step, an agent put itself in quarantine with probabilities  $\beta_{q-direct}$ , proportional to the number of the infected meets, and  $\beta_{i-direct}$ , proportional to the number of the infected location visited.

# NPIs - SDMs: Location closure (lockdown)



- **By type** (left): all location of a certain type is closed.
- **Most crowded** (right): a % of the most crowded location is closed.

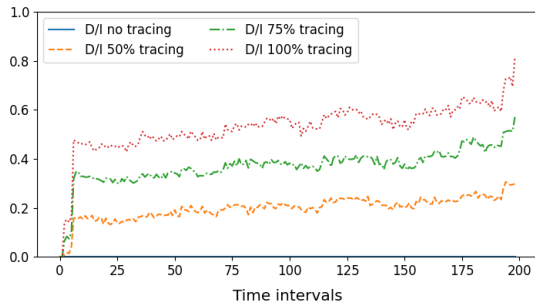
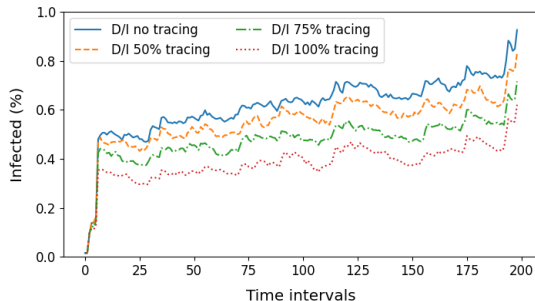
# NPIs - SDMs: Avoiding crowding



- For each day the building capacity is reduced of 50%.
- **Special Number (thr)**: only a limited number of agents (1 – 3 – 6) are allowed to stay in a location together.



# NPIs - SDMs: Contact tracing



- **Quarantine approach:** at the beginning of each step, an agent put itself in quarantine with probability  $\beta_{tracing}$ , proportional to the number of the infected reported by a tracing application.
- Only a  $\alpha\%$  of the agents use the tracing application.
- Percentage of quarantined individuals on the right.

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- We formally defined the **Time Varying Hypergraphs** for modeling contact-networks.
- We provided a design-methodology for **enhancing the accuracy** in epidemiological study combining:
  - Agent-based Modeling and Hypergraphs.
- We showed how hypergraphs allow us to **distinguish** different contagious pathways among the contact-network: *direct* and *indirect*.
- We experimented the SIS model on Foursquare users-mobility data, revealing:



- 1 the **importance of indirect** propagation in epidemic contact-network;
- 2 the **consequence of modeling contact-time** in epidemic simulation for both direct and indirect contacts.

# What's next

- Study epidemic control strategies (NPIs, ongoing research).
- Analyze immunization and quarantine techniques on agents or environment:
  - by developing dedicated immunization strategies for hypergraphs.
- Testing our methodology on other classical epidemic models.
- Experiments on more real-world datasets.
  - Unfortunately, now we should find data about human mobility patterns during the *COVID-19* pandemic, generated by pandemic control applications already adopted in many countries.

# Time for questions



Thank You  
For Your Attention!

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