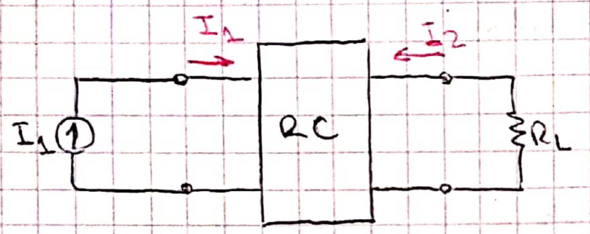
TAREA SEMANAL 12

① Ej 5 TP7



$$-\frac{I_2}{I_1} = H \cdot \frac{s^2+5s+4}{s^2+8s+12} \qquad Z_{21} = 6H$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$-\frac{I_2}{I_1} = \cancel{Z_{21}} \frac{1}{(R_L+Z_{22})} = \frac{\cancel{Z_{21}}}{6} \frac{(s+1)(s+4)}{(s+2)(s+6)}$$

$$R_L + Z_{22} = 6 \frac{s^2+8s+12}{s^2+5s+4}$$

$$Z_{22} = 6 \frac{s^2+8s+12}{s^2+5s+4} - R_L \quad \Big\} \text{ Normalizo por } \eta = R_L$$

$$Z_{22} = 6\left(\frac{s^2+8s+12}{s^2+5s+4} - \frac{1}{6}\right) = 6 \cdot \left(\frac{s^2+8s+12 - \frac{1}{6}s^2 - \frac{5}{6}s - \frac{4}{6}}{s^2+5s+4}\right)$$

$$Z_{22} = 6 \cdot \frac{\left(\frac{5}{6}s^2 + \frac{43}{6}s + \frac{34}{3}\right)}{s^2+5s+4} = \cancel{6} \cdot \frac{5}{\cancel{6}} \left(\frac{s^2 + \frac{43}{5}s + \frac{68}{5}}{s^2+5s+4}\right)$$

$$Z_{22} = 5 \cdot \frac{(s+2)(s+6{,}5)}{(s+1)(s+4)}$$

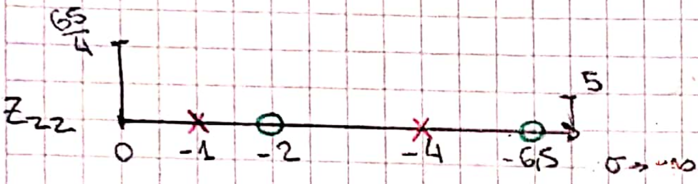
- Se toman los valores redondeados de los ceros.

↳ valores reales:
$$x_0 = \frac{-43+\sqrt{489}}{10} \wedge x_1 = \frac{-43-\sqrt{489}}{10}$$

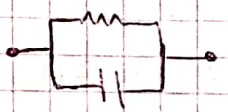
a) Síntesis Gráfica

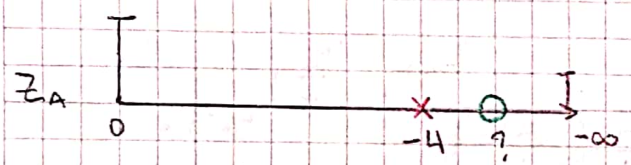
$$Z_{22} = 5\,\frac{(s+2)(s+6{,}5)}{(s+1)(s+4)} \qquad T(s) = \frac{\mathrm{den}(Z_{22})}{\left[\mathrm{den}(Z_{22})R_L + \mathrm{num}(Z_{22})\right]}$$

• Sintetizo Z_{22} ya que sus polos son los ceros de $T(s)$
↳ impedancia de salida

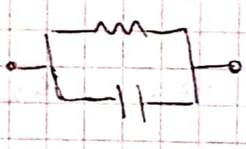


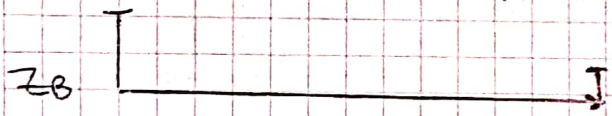
Remuevo en $\sigma = -1$



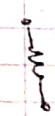


Remuevo $\sigma = -4$

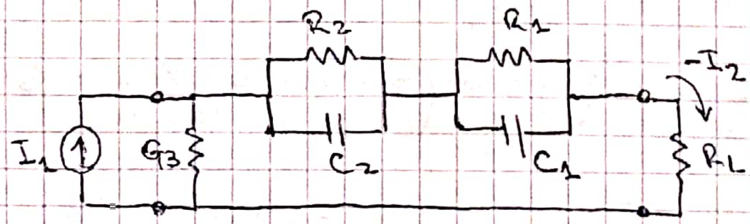




Remuevo G_∞



Red Esperada:

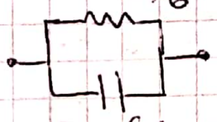


b) Síntesis analítica

R1/C1: $k_1 = \lim_{s \to -1} \cancel{(s+1)}\, 5\, \frac{(s+2)(s+6{,}5)}{\cancel{(s+1)}(s+4)} = \boxed{\frac{55}{6} = k_1}$

$$\frac{k_1}{s+1} \to s \cdot \underbrace{\frac{1}{k_1}}_{C} + \underbrace{\frac{1}{k_1}}_{G}$$

55/6



6/55

$R_1 = \frac{55}{6}$ $C_1 = \frac{6}{55}$

$\cancel{R_2/C_2:\ k_2 = \lim_{s \to -4} (s+1)\, 5\, \frac{(s+2)(s+6{,}5)}{(s+1)(s+4)} = \frac{25}{3} = k_2}$

$$Z_A = Z_{22} - \frac{55/6}{s+1} = \frac{5(s+2)(s+6{,}5)}{(s+1)(s+4)} - \frac{55/6\,(s+4)}{(s+1)(s+4)} = \cancel{5s^2 + 43s + 68 - \frac{55}{6}s - \frac{110}{3}}$$

$$Z_A = \frac{5s^2 + \frac{85}{2}s + 65 - 55/6\, s - 110/3}{} = \frac{5s^2 + \frac{100}{3}s + 85/3}{(s+1)(s+4)}$$

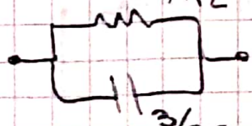
$$Z_A = \frac{\cancel{(s+1)}(s+17/3)\,5}{\cancel{(s+1)}(s+4)} = \frac{5\,(s+17/3)}{s+4}$$

• Remuevo R2/C2:

$k_2 = \lim_{s \to -4} \cancel{(s+4)}\, 5\, \frac{(s+17/3)}{\cancel{(s+4)}} = \boxed{\frac{25}{3} = k_2}$

$$\frac{k_2}{s+4} = \frac{1}{s \cdot \frac{3}{25} + \frac{12}{25}}$$

25/12

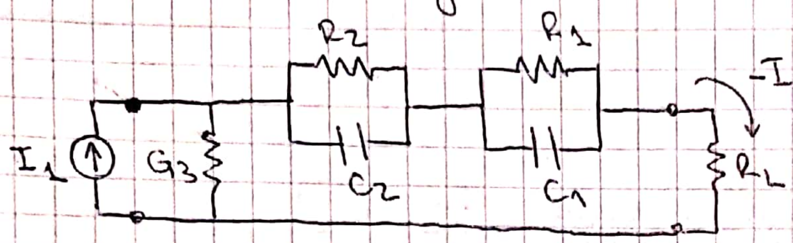


3/25

$$Z_B = Z_A - \frac{k_2}{s+4} = \frac{5s + 85/3}{s+4} - \frac{25/3}{(s+4)} = \frac{5s + 20}{s+4} = 5\,\cancel{\frac{s+4}{s+4}} = 5$$

$Z_B = 5 \to Y_B = 1/5$

Finalmente la red conseguida es:



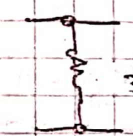
$R_1 = \frac{55}{6}$ $\quad C_1 = \frac{6}{55}$

$R_2 = \frac{25}{12}$ $\quad C_2 = \frac{3}{25}$

$G_3 = 1/5 \rightarrow R_3 = 5$

c) Para verificar planteo una cascada de dos cuadripolos con sus parametros T. Como busco $\left(\frac{-I_2}{I_1}\right)$, me va a interesar el parametro

$D = \frac{I_1}{-I_2}$ } $\frac{-I_2}{I_1} = \frac{1}{D}$ del cuadripolo resultante.

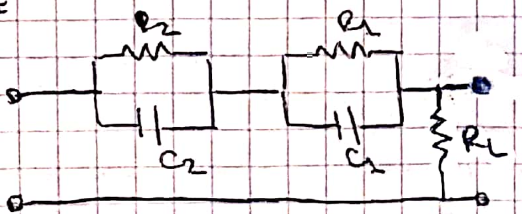
① 

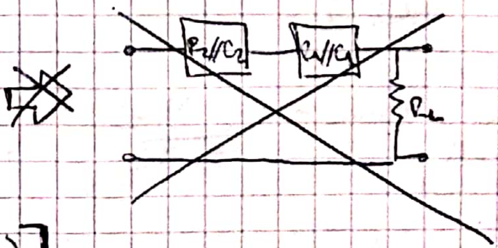
$A = \left.\frac{V_1}{V_2}\right|_{-I_2=0}$ $\quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$C = \left.\frac{I_1}{V_2}\right|_{-I_2=0}$ $\quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$

$$T_1 = \begin{pmatrix} 1 & 0 \\ 1/3 & 1 \end{pmatrix}$$

~~T2~~

② 



$$T_2 = \begin{bmatrix} 1 & (R_2//C_2 + R_1//C_1 + R_L) \\ 0 & 1 \end{bmatrix}$$

$T = T_1 \cdot T_2$ } Me interesa el parametro D tot.

$$T = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = \begin{bmatrix} (A_1A_2 + B_1D_1) & (A_1B_2 + B_1D_2) \\ (C_1A_2 + D_1C_2) & (C_1B_2 + D_1D_2) \end{bmatrix}$$

$$D_{tot} = C_1B_2 + D_1D_2 = (R_1 // C_1 + R_2 // C_2 + R_L) \cdot \frac{1}{5} + 1$$

$$D_{tot} = \left(\frac{\frac{R_1}{sC_1}}{R_1 + \frac{1}{sC_1}} + \frac{\frac{R_2}{sC_2}}{R_2 + \frac{1}{sC_2}} + R_L \right) \cdot \frac{1}{5} + 1$$

$$D_{tot} = \left(\frac{R_1}{sC_1R_1 + 1} + \frac{R_2}{sC_2R_2 + 1} + R_L \right) \frac{1}{5} + 1$$

$$D_{tot} = \left(\frac{1/C_1}{s + \frac{1}{R_1C_1}} + \frac{1/C_2}{s + \frac{1}{R_2C_2}} + R_L \right) \frac{1}{5} + 1$$

$$D_{tot} = \left(\frac{55/6}{s+1} + \frac{25/3}{s+4} + 1 \right) \frac{1}{5} + 1$$

$$D_{tot} = \frac{11/6}{s+1} + \frac{5/3}{s+4} + \underbrace{\frac{1}{5} + 1}_{6/5} = \frac{11/6\, s + 11/6 \cdot 4 + 5/3\, s + 5/3 + (s+1)(s+4)6/5}{(s+1)(s+4)}$$

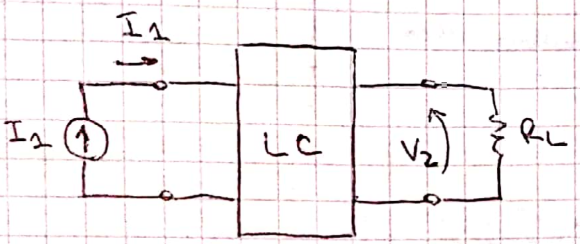
$$D_t = \frac{6}{5} \frac{s^2 + 19/2\, s + 69/5}{(s+1)(s+4)} = \frac{6}{5} \cdot \frac{s^2 + 95/12\, s + 23/2}{(s+1)(s+4)}$$

$$T(s) = \frac{1}{D_{tot}} = \frac{5}{6} \frac{(s+1)(s+4)}{\underbrace{(s+1{,}91)(s+6)}_{\text{aproximados}}} \Big\} \text{ Se verifica}$$

$$\downarrow$$

$$\boxed{H = \frac{5}{6}}$$

② TRANSIMPEDANCIA



$$T(s) = \frac{V_2}{I_1} = \frac{k(s^2+9)}{s^3+2s^2+2s+1}$$

$$-I_2 R_L = V_2 \rightarrow I_2 = -\frac{V_2}{R_L}$$

$$Z: \quad V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow V_2 = Z_{21} I_1 + Z_{22}\left(-\frac{V_2}{R_L}\right)$$

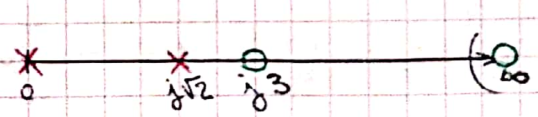
$$V_2\left(1 + \frac{Z_{22}}{R_L}\right) = Z_{21} I_1$$

$$\frac{V_2}{I_1} = \frac{Z_{21}}{1 + \frac{Z_{22}}{R_L}} = \frac{\overset{\nearrow \text{par}}{k(s^2+9)}}{s^3+2s^2+2s+1} \Big\} \rightarrow \text{saco f.c. parte impar}$$

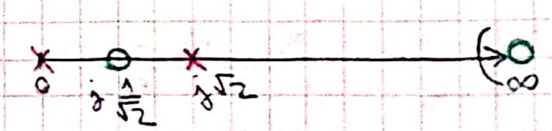
$$T(s) = \frac{Z_{21}}{1 + \frac{Z_{22}}{R_L}} = k \cdot \frac{\overbrace{\frac{s^2+9}{s^3+2s}}^{Z_{21}}}{1 + \underbrace{\frac{2s^2+1}{s^3+2s}}_{Z_{22}}}$$

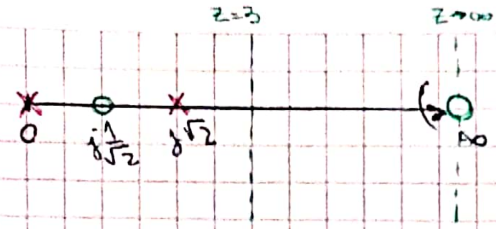
$\hookrightarrow R_L = 1$

$$Z_{21} = \frac{s^2+3^2}{(s^2+2)s}$$

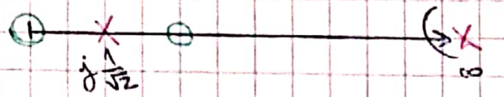


$$Z_{22} = \frac{2(s^2+1/2)}{s(s^2+2)}$$



Z_{22} 

$$Z_{22} = 2\,\frac{(s^2+1/2)}{s(s^2+2)}$$

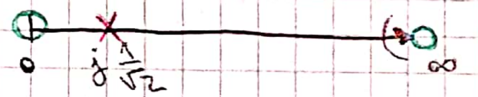
Y_{22} 

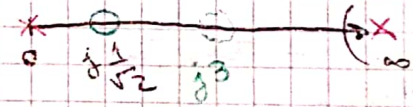
} remuevo k_∞  $k_\infty = 1/2$

$$k_\infty = \lim_{s\to\infty} \frac{1}{s}\cdot Y_{22} = \lim_{s\to\infty} \frac{1}{s}\,\frac{s(s^2+2)}{2(s^2+1/2)} = \boxed{\frac{1}{2} = k_\infty}$$

$$Y_A = Y_{22} - k_\infty\cdot s = \frac{s(s^2+2)}{2(s^2+1/2)} - \frac{k_\infty\, s(s^2+1/2)\cdot 2}{2(s^2+1/2)} = \frac{\cancel{s^3} + \overset{3/2}{\cancel{2s}} - \cancel{s^3} - s\,1/2}{2(s^2+1/2)}$$

$$Y_A = \frac{3/2}{2}\,\frac{s}{(s^2+1/2)} \rightarrow Z_A = \frac{4}{3}\,\frac{(s^2+1/2)}{s}$$



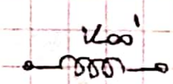


- Debo remover k_∞ parcialmente para llevar el cero a $j3$.

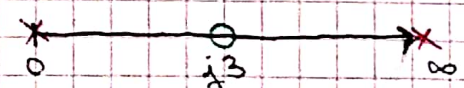
$$Z_B\Big|_{s=j3} = Z_A\Big|_{s=j3} - k_\infty' s\Big|_{s=j3} = 0$$

$$\frac{4}{3}\,\frac{(j3)^2+1/2}{j3} = k_\infty'\, j3$$

$$\frac{4}{3}\cdot(-9+1/2) = -9\,k_\infty'$$

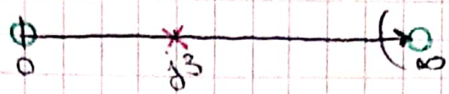
$$-\frac{34}{3}\cdot\left(-\frac{1}{9}\right) = k_\infty' \rightarrow \boxed{k_\infty' = \frac{34}{27}}$$ 

$$Z_B = \frac{4}{3}\,\frac{s^2+1/2}{s} - \frac{34}{27}\,\frac{s^2}{s} = \frac{2/27\, s^2 + 2/3}{s} = \frac{2}{27}\cdot\frac{(s^2+9)}{s}$$

Z_B: 

} Ahora invierto y remuevo en $j3$

$$Y_B = \frac{27}{2}\,\frac{s}{(s^2+9)}$$



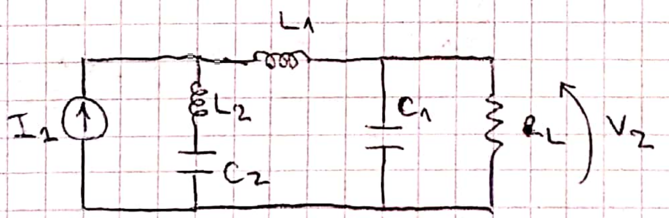
$$2k_1 = \lim_{s^2 \to -9} \frac{\cancel{(s^2+9)}}{\cancel{s}}\,\frac{27}{2}\,\frac{\cancel{s}}{\cancel{(s^2+9)}} = \boxed{\frac{27}{2} = 2k_1}$$

$$\cancel{\frac{2k_1 s}{s^2+9}}$$

$$\frac{2k_1 s}{s^2+9} = \frac{1}{\frac{s^{\cancel{2}}}{\frac{27}{2}\cancel{s}} + \frac{9}{\frac{27}{2}\cdot s}} = \frac{1}{s\cdot\underbrace{\frac{2}{27}}_{L} + \frac{1}{\underbrace{\frac{3}{2}}_{C}s}}$$

$$Y_C = Y_B - \frac{2k_1 s}{s^2+9} = \frac{27}{2}\,\frac{s}{s^2+9} - \frac{27}{2}\,\frac{s}{s^2+9} = \boxed{0}$$ //

Red final:



$L_2 = 2/27$ $\quad C_2 = 3/2$ $\quad L_1 = 34/27$ $\quad C_1 = 1/2$ $\quad R_L = 1$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ ‖ $B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

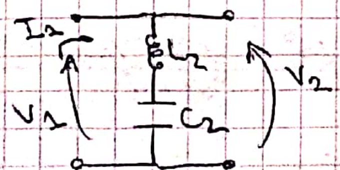
$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ $\quad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$

Verificación: $\frac{V_2}{I_1} = \frac{1}{C_{tot}}$ } parámetro C total

$T = T_1 \cdot T_2 \cdot T_3$

T_1:

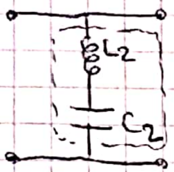
$$T_1 = \frac{1}{sC_2 + \frac{1}{sL_2}}$$



Verificación: $T(s) = \frac{V_2}{I_1} = \frac{1}{C_{tot}}$

$T = T_1 \cdot T_2 \cdot T_3$

T_1:



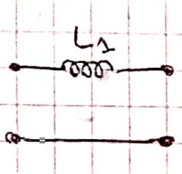
$Z_B = \frac{(s^2+9)}{s} \cdot \frac{2}{27}$

$$T_1 = \begin{pmatrix} 1 & 0 \\ 1/Z_B & 1 \end{pmatrix}$$

$A' \ldots \quad B' \ldots$

$C' = \frac{1}{Z_B} \qquad D' = \frac{sL_1}{Z_B} + 1$

$T_2 =$



$$T_2 = \begin{pmatrix} 1 & sL_1 \\ 0 & 1 \end{pmatrix}$$

$T_3 =$

$$T_3 = \begin{pmatrix} 1 & 0 \\ \underbrace{\frac{1}{R_L} + sC_1}_{\frac{1}{Z_1}} & 1 \end{pmatrix}$$

$C_1 = 1/2 \rightsquigarrow \frac{1}{1} + s\frac{1}{2} = \frac{2+s}{2}$

$$C_{tot} = C' + D' \cdot \frac{s+2}{2} = \frac{27}{2}\frac{s}{(s^2+9)} + \frac{sL_1 + Z_B}{Z_B} \cdot \frac{s+2}{2}$$

$$C_{tot} = \frac{27}{2}\frac{s}{(s^2+9)} + \frac{s \cdot \cancel{\frac{34}{27}} + \left[\frac{(s^2+9)}{s}\frac{2}{27}\right]}{(s^2+9)} \cdot s\,\frac{27}{2} \cdot \frac{(s+2)}{2}$$

$$C_{tot} = \frac{27}{2}\frac{s}{(s^2+9)} + \frac{s^2\,\overset{17}{\cancel{34}} + s^2 + 9}{(s^2+9)} \cdot \frac{1}{\cancel{s}}\frac{\cancel{2}}{\cancel{27}} \cdot \cancel{s}\,\frac{\cancel{27}}{\cancel{2}} \cdot \frac{(s+2)}{2}$$

$$C_{tot} = \frac{27\,s + (s^2 \cdot \overset{18}{\cancel{36}} + 9)\cdot(s+2)}{2\,(s^2+9)} = \frac{27\,s + \overset{18}{\cancel{36}}s^3 + \overset{36}{\cancel{72}}s^2 + 9s + 18}{2\,(s^2+9)}$$

$$C_{tot} = \frac{\overset{18}{\cancel{36}}s^3 + \overset{36}{\cancel{72}}s^2 + 36\,s + 18}{2\,(s^2+9)} = \frac{18}{2} \cdot \frac{(s^3 + 2s^2 + 2s + 1)}{(s^2+9)}$$

$$T(s) = \frac{1}{C_{tot}} = \frac{1}{9} \cdot \frac{(s^2+9)}{(s^3+2s^2+2s+1)} \quad \boxed{K = 1/9}$$