

## Ch 17

### 1. a) Aggregate method

Example Insert costs

Insert number	Cost
1	$O(1)$
2	$O(1)$
3	$O(1)$
4	$O(4) \rightarrow$ also resizes
5	$O(1)$
6	$O(1)$
7	$O(1)$
8	$O(8) \rightarrow$ also resizes
9-15	$O(1)$ each
16	$O(16) \rightarrow$ also resizes

Breaking the cost down:

Simple insertions + complex insertions = ?

$\hookrightarrow O(n)$

$\hookrightarrow O(n)$

$\hookrightarrow O(4) + O(8) + O(16) + \dots$

$$T(n) = O(n) + O(n) = O(n)$$

Now, the amortized cost per insertion =  $\frac{T(n)}{n}$

$$= \frac{O(n)}{n}$$

$$= \underline{\underline{O(1)}}$$

b) Accounting method

Operation	Amortized cost	Actual cost
Simple insertion	3	1
Resizing	Done through credit	O(1)

Now,

1st round : inserting 1, 2, 3

$$\Rightarrow 3+3+3=9$$

$$\text{Subtracting cost } 3 = 9-3=6$$

2nd round: inserting 4

$$\Rightarrow \text{Cost for resizing} = 4$$

$$\text{Bank} = 6-4=2$$

3rd round: inserting 5, 6, 7

$$\Rightarrow 2+6=8$$

4th round: inserting 8

$$\Rightarrow \text{Resize to size 8 has cost } 8$$

$$\Rightarrow \text{Bank} = 8-8=0$$

5th round: inserting 9-17

$$\Rightarrow \text{Bank} = \$18$$

6th round: inserting 18

$$\Rightarrow \text{Bank} = 18-18 = \$0$$

Amortized time per insert is  $O(1)$ .