

Kernels:

A kernel, convolution matrix, or mask is a small matrix. It is used for blurring, sharpening, embossing, edge detection, and more. This is accomplished by doing a convolution between a kernel and an image.

Image Convolution:

Convolution is the process of adding each element of the image to its local neighbors, weighted by the kernel. This is related to a form of mathematical convolution. The matrix operation being performed—convolution—is not traditional matrix multiplication, despite being similarly denoted by $*$.

$$\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \right) [2,2] = (i \cdot 1) + (h \cdot 2) + (g \cdot 3) + (f \cdot 4) + (e \cdot 5) + (d \cdot 6) + (c \cdot 7) + (b \cdot 8) + (a \cdot 9).$$

Pseudocode:

```
for each image row in input image:
    for each pixel in image row:

        set accumulator to zero

        for each kernel row in kernel:
            for each element in kernel row:

                if element position corresponding* to pixel position then
                    multiply element value corresponding* to pixel value
                    add result to accumulator
                endif

        set output image pixel to accumulator
```

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} * \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} = \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} x_{(m-i)(n-j)} y_{(1+i)(1+j)}$$

OR

$$O(i, j) = \sum_{k=1}^m \sum_{l=1}^n I(i+k-1, j+l-1) K(k, l)$$