

# Algorithm 5: Intersect

Comp175: Introduction to Computer Graphics – Fall 201

Due: Tuesday November 15th at 11:59pm

Your Name:

Paul Nixon

Your CS Login:

Sophia Panuthos  
spanuthos10  
nixon

## 1 Instructions

Complete this assignment by yourself without any help from anyone or anything except a current Comp175 TA, the lecture notes, official textbook, and the professor. Standard rules for notation apply. Show all work; write your answers on this sheet. This assignment is worth 10% of your final grade for Intersect.

## 2 Generating rays

For this assignment, you need to shoot a ray from the eye point through the center of each pixel.

[1 point] Given a pixel with screen-space coordinates  $x$  and  $y$ , and the width and height of the screen  $x_{max}$  and  $y_{max}$ , what is the corresponding point,  $p_{film}$ , on the normalized film plane? Assume that this is taking place after all of the perspective viewing transformations have been applied except for the unhooking transformation. Use the far clip plane as your film plane, and remember that a pixel  $y$ -value of 0 corresponds to the top of the screen.

$$p_{film} = \begin{matrix} -1 + 2 \frac{x}{x_{max}}, \\ 1 - 2 \frac{y}{y_{max}}, \\ -1 \end{matrix}$$

[1 point] In Camera, you transformed a point from world space to screen space by using the normalizing transformation. In Intersect and Ray, you need to transform  $p_{film}$  on the normalized film plane into an untransformed world-space point,  $p_{world}$ , by performing the viewing transformation. Using only the components of the viewing transformation (and/or their inverses) write the equation for  $M_{film-to-world}$ , the composite transformation matrix that transforms  $p_{film}$  to  $p_{world}$ , such that  $p_{world} = M_{film-to-world} * p_{film}$ .

$M_{film-to-world} =$

$$\begin{bmatrix} u_x & v_x & w_x & eye_x \\ u_y & v_y & w_y & eye_y \\ u_z & v_z & w_z & eye_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[1 point] Given your eye-point  $p_{eye}$  and the world-space point on the normalized film plane  $p_{world}$  give the equation for the world-space ray you want to shoot into the scene. Specify your ray in the format  $p + t\vec{d}$ , where  $p$  is a point and  $\vec{d}$  is a normalized vector.

$$r(t) = p_{eye} + t \frac{M_{film-to-world} * (p_{film} - p_{eye})}{\|M_{film-to-world} * (p_{film} - p_{eye})\|}$$

$$x^2 + z^2 = \left(\frac{y - 0.5}{2}\right)^2$$

### 3 Cone-Ray Intersection

*Special Instructions: For this problem, show all your work and circle, box, or bold your final answers.*

Write out both of the cone-ray intersect equations in terms of  $t$ . There are two equations: one for the body of the cone, and one for the bottom cap. For your cone, use the same dimensions that you did in Shapes. Use the definition of a ray used in question 1, i.e.  $p + t\vec{d}$ . To get you started you might want to define an intersection point as  $(x, y, z) = (p_x + \vec{d}_x t, p_y + \vec{d}_y t, p_z + \vec{d}_z t)$ , where  $p$  is the eyepoint, and  $\vec{d}$  is the direction of the ray we are shooting. Looking over the the derivation of the implicit equations for the cylinder in the Raytracing lecture might prove to be useful.

Recall that the equation of a circle on the 2D XZ coordinate plane is  $x^2 + z^2 = r^2$ . Think of our canonical unit cone as an infinite number of "differential" circles in the XZ plane stacked on top of one another in the Y direction; the bottommost circle has a radius of 1/2 and the topmost circle has a radius of 0. Then the equation of the unit cone is  $x^2 + z^2 = k(y)^2$ , where  $k$  linearly interpolates the radius of the differential circle from 1/2 at the base to 0 at the top.

The intersection points you compute are possible intersection points and need to be examined further (such as the  $-0.5 \leq y \leq 0.5$  restriction for the body of the cylinder in the lecture notes). However for this problem you are NOT required to list these restrictions.

Note that in your program you will need to find intersection points by finding a value for  $t$ . If you do not find an explicit formula for  $t$  (ie.  $t = \text{some value(s)}$ ) for both the cone and the cap then you will have a very hard time writing the program.

Finally, the equations you write should not use vectors but should be functions of the individual components of the vectors. By reducing your equations after deriving them, you eliminate unnecessary computation and thereby optimize your code before you even write it!

$$\left(\frac{p_y + t d_y - 0.5}{2}\right)^2 = \left(\frac{p_x}{2} + \frac{t d_x}{2} - 0.25\right)^2$$

[2 points] Cone Body

$$x^2 + z^2 - \left(\frac{y - 0.5}{2}\right)^2 = 0 = f(x, y, z)$$

$$f(x, y, z) = p + t d = 0$$

$$(p_x + t d_x)^2 + (p_z + t d_z)^2 - \left(\frac{p_y + t d_y - 0.5}{2}\right)^2 = 0$$

$$p_x^2 + 2 p_x t d_x + t^2 d_x^2 + p_z^2 + 2 p_z t d_z + t^2 d_z^2 - \frac{1}{4} p_y^2 - \frac{1}{2} p_y t d_y + \frac{1}{4} t^2 d_y^2 - \frac{1}{4} = 0$$

$$\text{we } -b \pm \sqrt{b^2 - 4ac}$$

$$a = d_x^2 + d_z^2 - \frac{1}{4} d_y^2 \quad b = 2 p_x d_x + 2 p_z d_z - \frac{1}{2} p_y d_y$$

$$c = p_x^2 + p_z^2 - \frac{1}{4} p_y^2 - \frac{1}{4}$$

[2 points] Cone Cap

$$x^2 + z^2 \leq (0.5)^2, y = -0.5$$

$$x^2 + z^2 - 0.25 \leq 0 = f(x, y, z)$$

$$(p_x + t d_x)^2 + (p_z + t d_z)^2 - 0.25 \leq 0$$

$$p_x^2 + 2 t d_x + t^2 d_x^2 + p_z^2 + 2 t d_z + t^2 d_z^2 - 0.25 \leq 0$$

Check if this is true

after solving

$$p_y + t d_y = -0.5$$

$$t = \frac{-1}{d_y} (p_y + 0.5)$$

## 4 Illuminating Samples

[2 points] When you are attempting to illuminate a transformed object, you will need to know that object's normal vector in world-space. Assume you know the normal vector in object-space,  $\vec{N}_{object}$ . Give an equation for the normal vector in world-space,  $\vec{N}_{world}$ , using the object's modeling transformation  $M$  and  $\vec{N}_{object}$ .

$$M^* \vec{N}_{object}$$

[1 point] In the lighting equation, what does  $\vec{N} \cdot \vec{L}$  represent, i.e. what is equivalent to it? What is its purpose?

how directly the light rays are hitting the surface, so angled light is less bright.

## 5 How to Submit

Hand in a PDF version of your solutions using the following command:

```
provide comp175 a4-alg
```