Istituto Nazionale di RIcerca Metrologica

Phase tracking and intensity fading due to polarization change in optical fibre link

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Contents

1 Noise sensitivity of the algorithm

1.1 Theoretical background

Two kinds of random process (at least) must be taken into account in order to perform simulations with a good reliability:

- *intrinsic noise:* this is actually the metrological information we want to reveal. A clock stability as well as the optical link performance is measured in terms of noise power spectral density. In particular, considering that these measurements are based on the phase difference between two oscillators, we are interested in the phase noise power spectral density (PSD) $S_\phi(f)\ \left(\frac{\mathrm{rad}^2}{\mathrm{Hz}}\right)$. This quantity is usually modeled using a power-law function $S_\phi(f) = b_0 f^0 + b_1 f^{-1} + b_2 f^{-2} + b_3 f^{-3} + \ldots$ which corresponds, in the typical log-log plot, to a sum of lines with different slope.
  Recalling that $S_\phi(f)$ is intended to be one-side PSD [1], namely:
  $$S(f) = \frac{2}{T}\left|\hat{X}(f)\right|^2 \tag{1.1}$$
  where T is the measurement time and $\left|\hat{X}(f)\right|^2$ is the Fourier Transform in the frequency domain f.
  Moving to the digital domain, the one-side PSD becomes:
  $$S(\mathrm{F}) = \frac{2}{N\cdot f_s}\left|\hat{X}(F)\right|^2 \tag{1.2}$$
  where N is the number of samples and f_s is the sampling frequency, while F is the digital (discrete) frequency.
  However for a preliminary analysis, a simpler model $S_\phi(f) = b_0 f^0 + b_2 f^{-2}$ will be considered, whose terms are called respectively *white noise* and *random walk (noise)*. In this case, only two meaningful quantities must be known to fully describe the function: the b_0 factor for the white noise and the $b_2(1)^{-2} = b_2$ product i.e. the noise value at $f = 1$ for the random walk term.

- *detection noise:* this is due to the opto-electronic read-out system. In spite of the several phenomena involved for a complete noise description (shot noise, flickering,...) we choose a simplified model for a preliminary analysis as above. This takes into account the only contribution of the white noise. In particular, the four channels are considered uncorrelated i.e. each one is characterized by a single white noise random process.

- *quantization noise:* this phenomenon arises from the ADC process. A correspondence between the time and frequency domains is quite complex to be derived, but the frequency spectrum can be assumed flat (i.e. quantization noise becomes white) for the whiteness condition [2]:
  $$f_s < 9.2\frac{\sigma_x}{q}BW = 9.2\frac{1}{\sqrt{12}}BW \approx 2.7BW \tag{1.3}$$
  The condition can thus be satisfied for an observed signal with frequencies components which extend up to near the nyquist frequency.

In Digital Signal Process, random processes such as white noise and random walk are modeled with a sequence of data arising from random data generation. In the former case every value of the sequence is obtained with the formula $A_{\mathrm{rw}}\cdot\mathcal{N}(0,1)$ where $\mathcal{N}(0,1)$ is a value randomly extracted from a Normal Distribution with zero mean and unitary variance while the values of a random walk sequence are $A_{\mathrm{rn}}\cdot\mathscr{C}(\mathcal{N}(0,1))$ where $\mathscr{C}(\mathcal{N}(0,1))$ is the cumulative sum of the white noise n-length sequence up to the considered n-th value. The magnitudes of the amplitudes A_{wn} and A_{rn} are determined respectively by the chosen values for b_0 and b_2 as described in Tab.(1.1):

[1] The PSD of a real function is an even function. Furthermore the one-side PSD is what is displayed by a spectrum analyzer instrument.

[2] Eq (20.54) of http://oldweb.mit.bme.hu/books/quantization/spectrum.pdf

	Amplitude	PSD	Applied to
random walk	$\sqrt{\frac{2\pi^2 b_2}{f_s}}$ (rad)	$b_2(1)^{-2} = b_2$ (rad^2/Hz)	Simul. params
white noise	$\sqrt{\frac{b_0 f_s}{2}}$ (rad)	b_0 (rad^2/Hz)	Simul. electr. channels
quant noise			Simul. electr. channels

Table 1.1: Amplitude-PSD relation for random processes

The starting point for our noise sensitivity analysis is the choice of proper values for b_0 and b_2.
Fig(1.1) shows a typical optical fiber link phase noise spectrum for an installed long distance optical link ($\approx 10^2$ km) while Fig(1.2) presents spectrum obtained via the recovery algorithm in a lab setup, where the total length is limited for testing reasons to just few meters.
Both show the same "f^{-2} shape" for all the low frequency range and the "flat shape" for the higher one. Furthermore the share the same ratio between the phase noise at 1 Hz (b_2 value) and the beginning of the white noise plateau which is around $10^7 - 10^8$. We will therefore model the noise sources of Tab(1.1) assuming the above power ratio, and we will chose the absolute value for b_2 from Fig(1.2). However the spectrum in Fig(1.2) does not represents a "pure" random walk spectrum in fact the phase signal is also formed with an important contribute of low a frequency component (<1 Hz). This component overlaps in the frequency spectrum the random walk spectrum. Therefore a more reliable value for b_2 to be adopted in the simulations is around two order of magnitude less than the one obtained in Fig(1.2): 1 (rad^2/Hz) $\rightarrow$ 0.01 (rad^2/Hz). (Furthermore i cannot build a random walk process from the plot of Fig(1.1) if we do not know the sampling frequency)

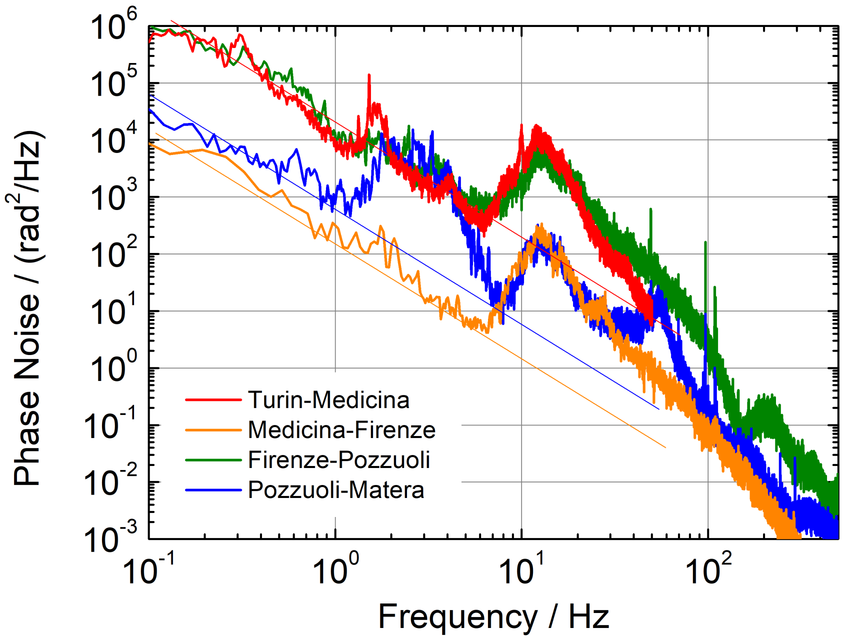

Figure 1.1: Phase PSD for a long distance link in a real frequency dissemination system

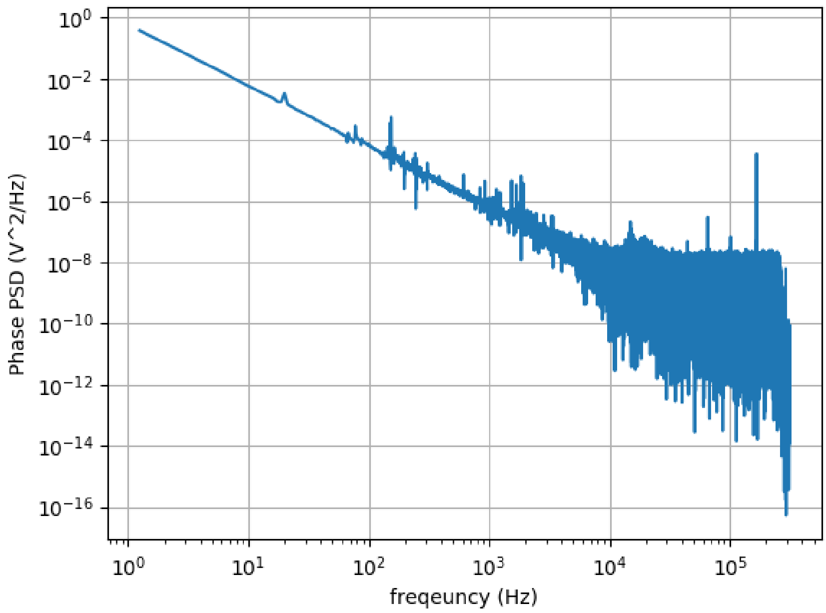

Figure 1.2: Phase PSD for a long short link in lab environment. (downsampling=2, lowfilter=100kHz)

1.2 Step 1: Random Walk affecting the parameters

As stated in the previous chapter, we add a random walk noise on all the three parameters δ,θ,ϕ as shown. Fig.(1.3) shows the result for a simple case where the optical fiber parameters (δ,θ) are simply set to sinusoidal functions while ϕ is set to zero (therefore, it becomes a "pure" random walk signal). The correspondent PSD spectrum of ϕ is shown in Fig.(1.4)

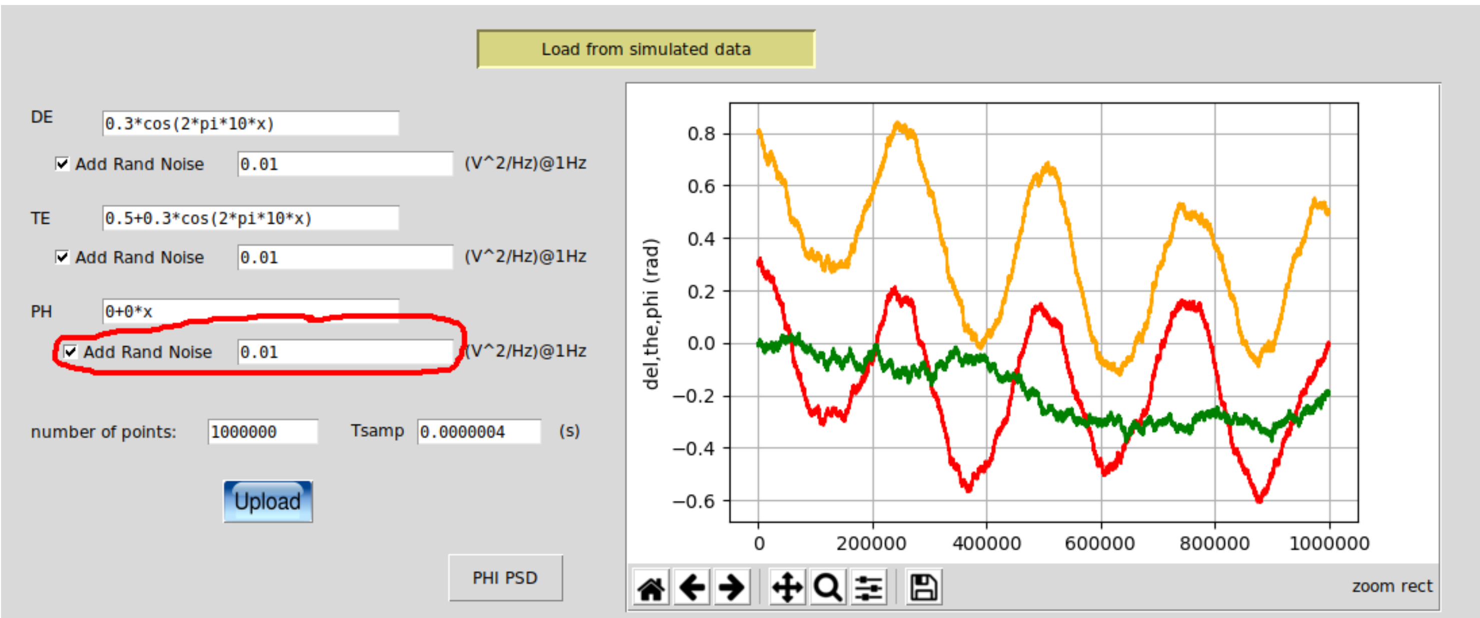

Figure 1.3: Optical fiber parameters (δ,θ) and metrological phase (ϕ) chosen for the building of a simulated data set. Red circle shows the application of additional random walk noise

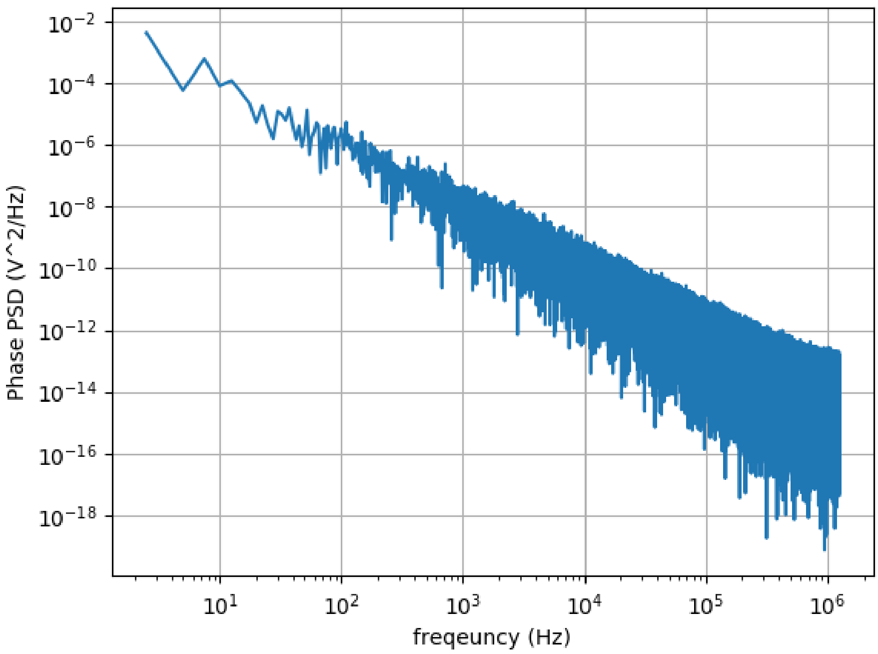

Figure 1.4: Phase PSD of the simulated data for ϕ: b_2 (i.e. PSD @ 1Hz) corresponds to the chosen value in Fig.(1.3). (In this case, the value must be extrapolated due to the low resolution of the psd calculation. A resolution of 1Hz would require 2500000 data for a F_s=2500000 Hz)

1.3 Step 2: Electronic Noise on the DPOH channels

At this step, the simulator produces the four lists of the Optical Hybrid channels (Fig.1.5). We then add to the channels respectively four independent white noise, following the assumed criterion $\frac{b_2}{b_0} \approx 10^8$ obtaining the data set shown in Fig(1.6)

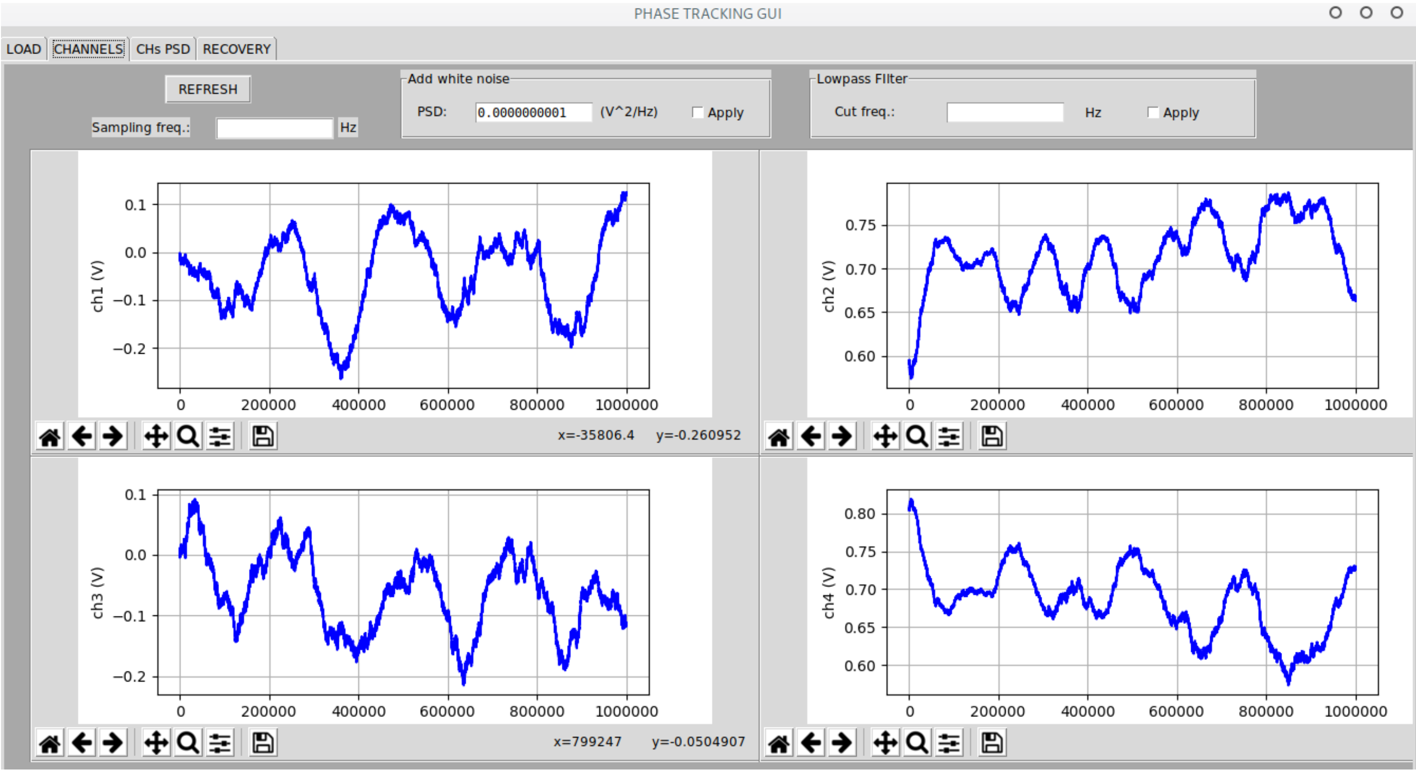

Figure 1.5: DPOH channels for set of simulated data.

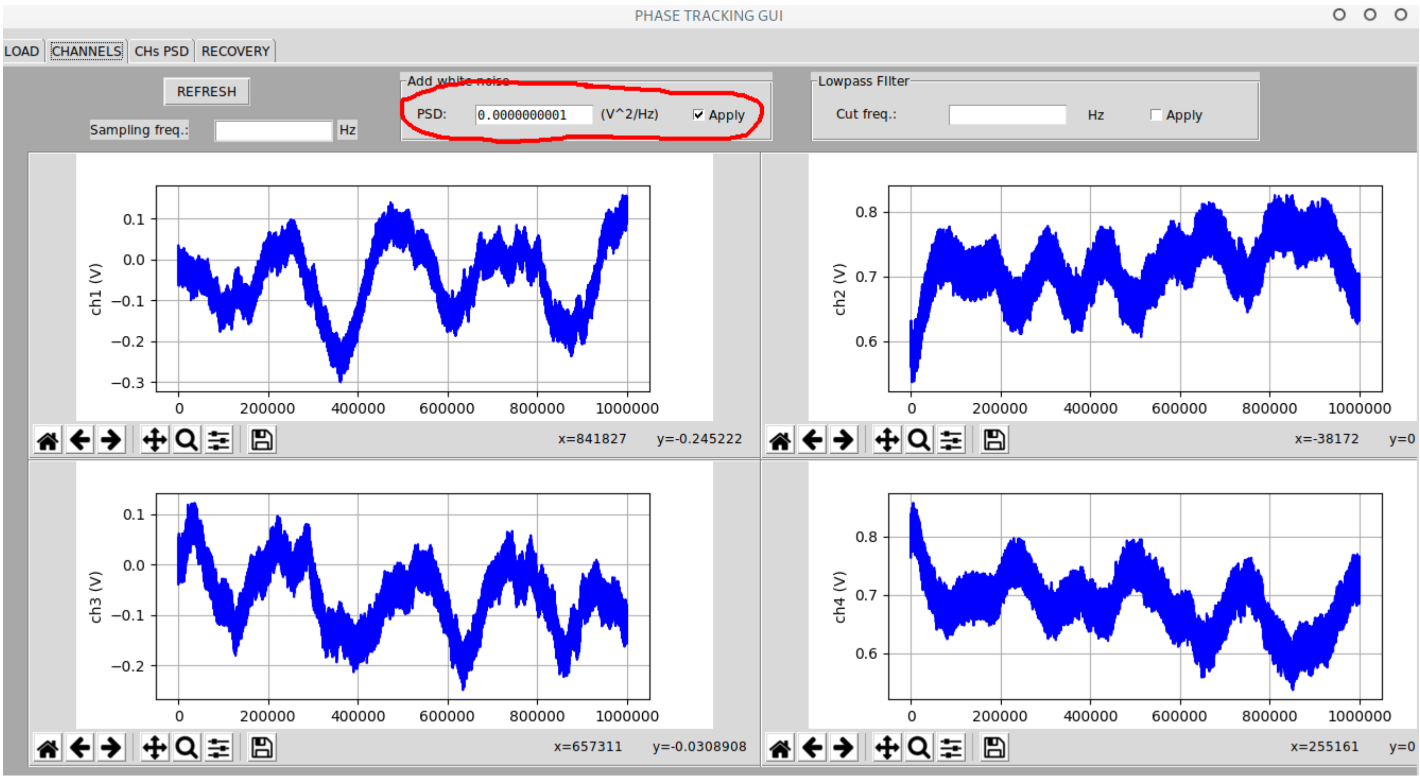

Figure 1.6: DPOH channels for set of simulated data + white noise 10^{-9} (rad^2/Hz).

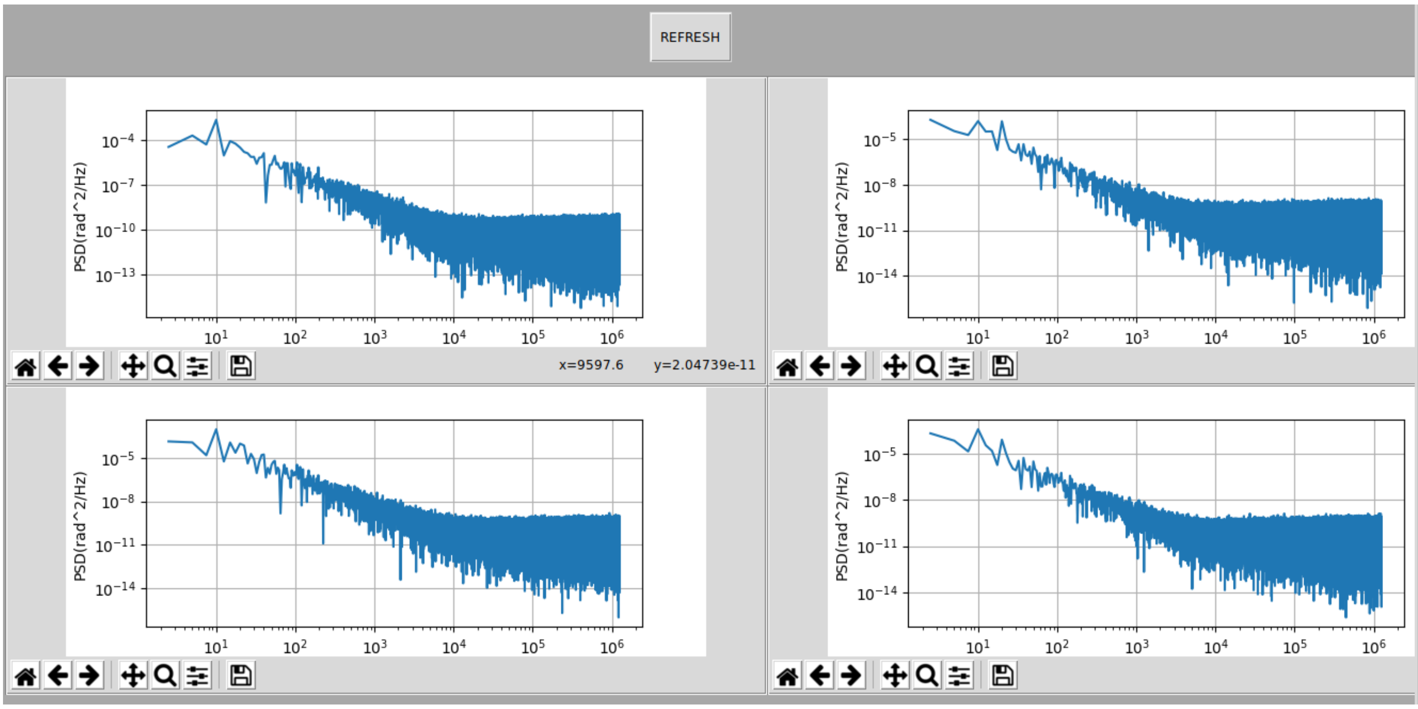

Figure 1.7: ....