🔑 Answer Key and Explanations

Section A Answers

1. $(R \vee H) \wedge \neg(H \wedge T)$

R=reading, H=homework, T=test. "Either R or H" = $R \vee H$. "Not both H and T" = $\neg(H \wedge T)$. "But" = $\wedge$.

2. "Either Steve or George is happy, but they're not both happy."

$(S \vee G)$ says at least one is happy. $(\neg S \vee \neg G)$ says at least one is NOT happy. Together: exactly one is happy. This is the exclusive or!

3. (a) $\neg(A \wedge B)$ (b) $\neg A \wedge \neg B$ (c) $\neg A \wedge \neg B$. (b) and (c) are equivalent.

(a) says it's not the case that BOTH are in the room (one might be). (b) says Alice is not in AND Bob is not in. (c) "neither...nor" = same as (b). Note: (a) is NOT the same as (b)/(c)! $\neg(A \wedge B) \neq \neg A \wedge \neg B$. (This is De Morgan's law — (a) actually equals $\neg A \vee \neg B$.)

4. Use letters B=beef, C=corn, K=cake. Formula: $(B \vee C)$ — for now, write the "if...then" structure as: if $(B \vee C)$ then $\neg K$.

Full formal version (Section 1.5): $(B \vee C) \to \neg K$. For now, identifying the components is enough: antecedent = $B \vee C$, consequent = $\neg K$.

5. (a) "Taxes or deficit will go up (or both)." (b) "Exactly one of taxes/deficit will go up." (c) Same as (b).

(b) $\neg(T \wedge D)$ = not both; $\neg(\neg T \wedge \neg D)$ = not neither. Together = exactly one. (c) $(T \wedge \neg D) \vee (D \wedge \neg T)$ = taxes without deficit, OR deficit without taxes = also exactly one. So (b) and (c) are equivalent!

6. INVALID.

B=beef, F=fish, P=peas, C=corn. Premises: $B \vee F$, $P \vee C$, $\neg(F \wedge C)$. Conclusion: $\neg(B \wedge P)$. Counterexample: B=T, F=T, P=T, C=F. Check: $B \vee F = T$✓, $P \vee C = T$✓, $\neg(F \wedge C) = \neg(T \wedge F) = T$✓. But $\neg(B \wedge P) = \neg(T \wedge T) = F$✗. Premises all true, conclusion false → invalid.

Section B Answers

7. X is a Knight, Y is a Knave.

(a) X's statement: "At least one of us is a Knave" = $\neg P \vee \neg Q$. (b) Case 1: X is Knight (P=T). Statement is true: $\neg T \vee \neg Q = F \vee \neg Q = \neg Q$. So Q=F, Y is Knave. ✓ Case 2: X is Knave (P=F). Statement is false: $\neg P \vee \neg Q$ must be F. Need $\neg P$=F AND $\neg Q$=F, so P=T and Q=T. But P=F — contradiction! ✗ Only Case 1 works.

8. A tells truth, B tells truth, C lies. (P=T, Q=T, R=F)

From C: if C tells truth (R=T) then Q=F; if C lies (R=F) then Q=T. Case R=F, Q=T: B tells truth, so P=T (B says A tells truth). A tells truth, so $\neg Q \vee \neg R = F \vee T = T$. ✓ Consistent! Case R=T, Q=F: B lies, so P=F. A lies, so $\neg Q \vee \neg R$ must be false, meaning Q=T. But Q=F — contradiction! ✗

9. (a) $D_2(n) \wedge D_3(n)$ (b) $\neg D_2(n) \vee \neg D_3(n)$ (c) VALID.

(a) Divisible by 6 = divisible by both 2 and 3. (b) Negate (a): $\neg(D_2 \wedge D_3) = \neg D_2 \vee \neg D_3$ (De Morgan). (c) From $\neg D_2 \vee \neg D_3$ and D_3, since D_3 is true, $\neg D_3$ is false, so $\neg D_2$ must be true (disjunctive syllogism). Valid!

10. VALID. Conclusion Q is correct.

From $\neg P \vee R$ and $\neg R$: disjunctive syllogism gives $\neg P$. From $P \vee Q$ and $\neg P$: disjunctive syllogism gives Q. Two-step chain!

11. C is guilty.

Step 1: From (i) “if A guilty then B at scene” and (ii) “B not at scene” (¬B_scene), contrapositive reasoning (or: treat as P∨Q form — “A not guilty OR B at scene” + ¬B_scene) gives: A is not guilty. Step 2: From (iii) C∨D∨A and ¬A: C∨D. Step 3: From C∨D and (iv) ¬D: disjunctive syllogism gives C. Answer: C is guilty. This chains three argument steps — classic competition technique!