Practice Problems: Exam Level

> *Work through each problem, then check the Answer Key at the end.*

Section A: Standard Exam Problems

1. Analyze the logical form: "We'll have either a reading assignment or homework, but we won't have both homework and a test."

2. Let S = "Steve is happy" and G = "George is happy." Translate to English: $(S \vee G) \wedge (\neg S \vee \neg G)$

3. Analyze: (a) "Alice and Bob are not both in the room." (b) "Alice and Bob are both not in the room." (c) "Neither Alice nor Bob is in the room." Which of (a)(b)(c) are equivalent?

4. Translate: "If either the main course is beef or the vegetable is corn, then the dessert won't be cake." (Hint: just use $\vee$, $\wedge$, $\neg$ for now; treat "if...then" as a single unit for Section 1.5)

5. Let T = "Taxes go up," D = "Deficit goes up." What do these formulas mean in English? (a) $T \vee D$ (b) $\neg(T \wedge D) \wedge \neg(\neg T \wedge \neg D)$ (c) $(T \wedge \neg D) \vee (D \wedge \neg T)$

6. Determine validity: "The main course is beef or fish. The vegetable is peas or corn. We won't have both fish and corn. Therefore, we won't have both beef and peas."

Section B: Competition-Style Problems 🏆

7. [Knights & Knaves] On an island, Knights always tell truth; Knaves always lie. You meet X and Y.

X says: "At least one of us is a Knave."

Let P = "X is a Knight," Q = "Y is a Knight." (a) Write X's statement using $\neg, \vee$. (b) Determine what X and Y are.

8. [Liar Puzzle] Three people A, B, C each make one claim:

A: "Either B or C is lying." B: "A is telling the truth." C: "B is lying."

Find all consistent truth assignments (who tells truth, who lies).

9. [Number Theory + Logic] Let $D_2(n)$ = "n is divisible by 2" and $D_3(n)$ = "n is divisible by 3."

(a) Write "n is divisible by 6" using D_2, D_3, and $\wedge$.

(b) Write "n is not divisible by 6" using D_2, D_3, and $\neg$, $\vee$.

(c) Is this valid? Premise: $\neg D_2(n) \vee \neg D_3(n)$. Premise: $D_3(n)$. Conclusion: $\neg D_2(n)$.

10. [Chain Argument] Determine validity:

$P \vee Q$, $\neg P \vee R$, $\neg R$. Conclusion: Q.

11. [AMC-Style Logic] Five suspects A,B,C,D,E. Exactly one committed the crime. Given:

(i) If A is guilty, then B was at the scene.

(ii) B was not at the scene.

(iii) Either C or D is guilty, or A is guilty.

(iv) D is not guilty.

Using only $\vee$, $\wedge$, $\neg$ and the argument forms from 1.1, determine who is guilty.