Deductive Reasoning & Logical Connectives (10-Minute YouTube Script)

[Opening – 30s]

Before we start, you can download a free PDF with today's problems and solutions using the link below.

Don't panic, just watch. Just Watch Math.

Hi everyone, this is Just Watch Math.

Today we're starting one of the most important foundations of proof-based math:

Deductive Reasoning and Logical Connectives.

This is Section 1.1 from *How to Prove It* by Daniel Velleman.

If you plan to study proofs, Putnam problems, or math competitions,

this section is non-negotiable.

Let's start.

[Part 1: What is Deductive Reasoning – 2 min]

A deductive argument has two parts:

premises and a conclusion.

Premises are the given assumptions.

The conclusion is what must logically follow.

The key word is MUST.

If all premises are true and the logic is valid, the conclusion is guaranteed to be true.

Validity is about structure, not truth.

An argument can be valid even if everything in it is false.

Example:

“All cats are dogs. Fluffy is a cat. Therefore Fluffy is a dog.”

The logic is valid.

The premises are nonsense, but the structure works.

Here’s the biggest exam tip:

To prove an argument is invalid,

you only need ONE counterexample.

One situation where all premises are true,

but the conclusion is false.

That’s it.

[Part 2: Logical Connectives – 3 min]

We build logical statements using three symbols.

OR is written as $P \lor Q$.

AND is written as $P \land Q$.

NOT is written as $\neg P$.

Important:

In math, OR is inclusive.

$P \vee Q$ is true if P is true, Q is true, or both.

AND is stricter.

$P \wedge Q$ is true only when both are true.

NOT flips the truth value.

True becomes false, false becomes true.

Memory trick:

Think of OR like addition.

At least one nonzero makes it work.

Think of AND like multiplication.

One zero kills everything.

Common trap:

$\neg P \wedge Q$ does NOT mean $\neg(P \wedge Q)$.

The NOT only applies to what comes right after it.

[Part 3: Translating English to Logic – 3 min]

This is the number one skill in this section.

You are not translating words.

You are translating meaning.

Three-step method:

First, identify the basic statements.

Second, assign symbols.

Third, connect them correctly.

Example:

"Neither Alice nor Bob is happy."

This means Alice is not happy AND Bob is not happy.

So the correct translation is $\neg A \land \neg B$.

Another example:

"John is tall but not heavy."

"But" means AND.

So we get $T \land \neg H$.

Watch out:

English "or" is often misleading.

In math, OR is always inclusive.

[Part 4: Well-Formed Formulas – 1.5 min]

Not every string of symbols is legal.

AND and OR go BETWEEN two statements.

NOT goes BEFORE one statement.

Parentheses matter.

Just like algebra.

$\neg P \lor Q \land R$ is ambiguous.

You must add parentheses.

Exams love this.

[Part 5: Key Argument Form – 2 min]

The most important argument form here is

Disjunctive Syllogism.

P or Q.

Not Q.

Therefore P.

This is pure elimination.

Multiple choice tests use this constantly.

But be careful:

P or Q.

P.

Therefore not Q.

This is INVALID.

Because OR allows both to be true.

Always remember:

Structure decides validity, not meaning.

[Closing – 30s]

This section looks simple,

but it powers everything that comes later:

proofs, contrapositive, contradiction, De Morgan’s laws.

If you master this now,

the rest becomes much easier.

Download the PDF below and try the practice problems.

Don’t panic, just watch. Just Watch Math.