Engineering Electromagnetics –Experiment 4

樊青远[1]

[1]*Student ID: 11812418*

Abstract

This experiment lorentz force that is applied to a charge in an electromagnetic field and provide us a way to analyze the charge’s movement trajectory in the electromagnetic field and plot it by graph through MATLAB.

1 Background and related knowledge

Charge in electromagnetic field will be subject to the action of Lorentz force, which is expressed as:

$$\overrightarrow{\mathrm{F}} = q\overrightarrow{\mathrm{E}} + q\vec{v} \times \overrightarrow{\mathrm{B}} \tag{1}$$

Where $\vec{F}$ is vector of Lorentz force, $\vec{E}$ is the vector of electric field, $\vec{B}$is the vector of magnetic flux density, $\vec{v}$ is the vector of charge’ s velocity, q is the quantity of charge.

According to Newton’ s law of motion, charge will be accelerated under Lorentz force, hence the velocity and displacement will change. In the 3D Cartesian coordinate system, this process can be described by the following vector equations:

$$\overrightarrow{\mathrm{E}}(\mathrm{t}) = E_x(\mathrm{t})\vec{a}_x + E_y(\mathrm{t})\vec{a}_y + E_z(\mathrm{t})\vec{a}_z \tag{2}$$

$$\overrightarrow{\mathrm{B}}(\mathrm{t}) = B_x(\mathrm{t})\vec{a}_x + B_y(\mathrm{t})\vec{a}_y + B_z(\mathrm{t})\vec{a}_z \tag{3}$$

$$\begin{gathered}\overrightarrow{\mathrm{F}}(\mathrm{t}) = q\overrightarrow{\mathrm{E}}(\mathrm{t}) + q\vec{v}(\mathrm{t}) \times \overrightarrow{\mathrm{B}}(\mathrm{t}) \\ \overrightarrow{\mathrm{A}}(\mathrm{t}) = \overrightarrow{\mathrm{F}}(\mathrm{t})/m\end{gathered} \tag{4}$$

(Newton’ s second law; m is the mass of the charge; $\vec{A}$ is the acceleration vector)

$$\begin{aligned}\vec{v}(\mathrm{t}) &= \vec{v}(1) + \int_0^t \overrightarrow{\mathrm{A}}(\mathrm{t})dt \\ \vec{r}(\mathrm{t}) &= \vec{r}(1) + \int_0^t \vec{v}(\mathrm{t})dt \quad (\vec{r} \text{ is the position vector })\end{aligned} \tag{5}$$

It can be seen that this is a process that develops over time. In some cases, this process can be solved analytically by solving differential equations to get the velocity vector, position vector at every moment. This experiment is aimed to analyze this dynamic process without tedious mathematical derivation. The key point is to understand the physical essence of this dynamic process. Therefore, we discretize time and introduce small timestep Δt. And we assume at each time slot, the acceleration vector will not change. In this way, equation (5) can be rewritten as:

$$\begin{gathered}\vec{v}(\mathrm{t}+\Delta\mathrm{t}) = \vec{v}(\mathrm{t}) + \overrightarrow{\mathrm{A}}(\mathrm{t})\Delta\mathrm{t} \\ \vec{r}(\mathrm{t}+\Delta\mathrm{t}) = \vec{r}(\mathrm{t}) + \vec{v}(\mathrm{t})\Delta\mathrm{t}\end{gathered} \tag{6}$$

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DOI:

Corresponding author
Author One

Edited by
John Doe

2 Description of the task

2.1 Magnetic focusing phenomenon

In our experiment, we are required to simulate a phenomenom called ”magnetic focusing phenomenon”. For a beam of charged particles with small angle of divergence, given the same velocity component at the direction of the magnetic field B, their trajectory will have the same screw pitch. After a period, they will converge at another point. The phenomenon that the diverged charged particles focus at one point is similar to the phenomenon that lens can let the light beam focus at one point. Therefore, it is called as magnetic focusing.

There are two conditions for magnetic focusing:

1. The charged particles have similar initial velocity v;
2. The angle between v and B is sufficiently small so that each particle will do helical motion.

2.2 Parameters of the experiment

16 charges, they have equal mass $m = 0.02\mathrm{kg}$ and each carries $q = 0.016\mathrm{C}$. The initial velocities are the same: $\vec{r}(1) = 0$ (at the origin of the coordinate). Electric field: $\overrightarrow{\mathrm{E}} = 0$ magnetic flux density: $\overrightarrow{\mathrm{B}} = 8\vec{a}_z\mathrm{Wb/m^2}$. These 16 charges' initial velocities along z-axis are equal: $v_z(1) = 10\mathrm{m/s}$. Their initial velocities along x-axis and y-axis can be expressed as: $v_x = 0.1\sin(\mathrm{k}\pi/8)\mathrm{m/s}$, $v_y(0) = 0.1\cos(\mathrm{k}\pi/8)\mathrm{m/s}$, where $\mathrm{k} = 0, 1, 2, \cdots\cdots, 15$

3 Experiment content

3.1 Define the Parameters

First, we define the relevant parameters and generate the time matrix, velocity matrix for every particle.
We assign 3 to t in order to get a clearer plot.

```
clear all; %clean the env;

m=0.02; %Set the mass;
q=1.6e-2; %Set the quantity of charge;

dt=0.001;  %Set the timestep to be 0.001s;
t=0:dt:3; %Construct the array of time;
vx = zeros(16,length(t));
vy = vx;
vz = vx; %Construct the velocity vector;
for c2 = 1:16
    vx(c2,1)=0.1*sin((c2-1)*pi/8); %assign the initial velocity of for every particle
    vy(c2,1)=0.1*cos((c2-1)*pi/8);
end
vz(:,1) = (zeros(16,1)+10);
```

3.2 Define the field and vector

Next, we define the electric, magnetic fields and the acceleration vector.

```
%%
rx=linspace(0,0,length(t));ry=rx;rz=rx; %Set the position vector;
Ex=0; Ey=0; Ez=0;  %Set the electric field vector;
Bx=0; By=0; Bz=8; %Set the magnetic flux density vector;
Fx=linspace(0,0,length(t));Fy=Fx;Fz=Fx; %Construct the force vector;
```

```
ax=linspace(0,0,length(t));ay=ax;az=ax; %Construct the acceleration vector;

vx_rst=vx;
vy_rst=vy;
vz_rst=vz;
```

3.3 Calculate the track of the particle

After that, we use two FOR loop to calculate the track of the particle. We used the idea of differentiation to calculate the position of the particle second by second.

```
figure
hold on;

for c1 = 1:16
vx = vx_rst;
vy = vy_rst;
vz = vz_rst;
    for c2=1:(length(t)-1) % Calculate each position point

        Fx(c2)=q*Ex+q*(vy(c1,c2)*Bz-vz(c1,c2)*By);% Calculate the acted force at position c2
        Fy(c2)=q*Ey+q*(vz(c1,c2)*Bx-vx(c1,c2)*Bz);% Calculate the acted force at position c2
        Fz(c2)=q*Ez+q*(vx(c1,c2)*By-vy(c1,c2)*Bx);% Calculate the acted force at position c2

        ax(c2)=Fx(c2)/m;% Calculate the acceleration at position c2
        ay(c2)=Fy(c2)/m;% Calculate the acceleration at position c2
        az(c2)=Fz(c2)/m;% Calculate the acceleration at position c2

        vx(c1,c2+1)=vx(c1,c2)+ax(c2)*dt;% Calculate the velocity at position c2+1
        vy(c1,c2+1)=vy(c1,c2)+ay(c2)*dt;% Calculate the velocity at position c2+1
        vz(c1,c2+1)=vz(c1,c2)+az(c2)*dt;% Calculate the velocity at position c2+1

        rx(c2+1)=rx(c2)+vx(c1,c2)*dt;% Calculate the position at point c2+1
        ry(c2+1)=ry(c2)+vy(c1,c2)*dt;% Calculate the position at point c2+1
        rz(c2+1)=rz(c2)+vz(c1,c2)*dt;% Calculate the position at point c2+1

    end
 plot3(rx,ry,rz);
end
```

3.4 Plot the graph

Last, we plot and overlay the image.

```
grid;
title({'Charge’ s movement trajectory in the electromagnetic field';'(single particle) 樊青远 11812418'}
xlabel('X Axis', 'fontsize', 12);% Label x axis
ylabel('Y Axis', 'fontsize', 12);% Label y axis
zlabel('Z Axis', 'fontsize', 12);% Label z axis
pbaspect([1 1 1]);
```

4 Graphs

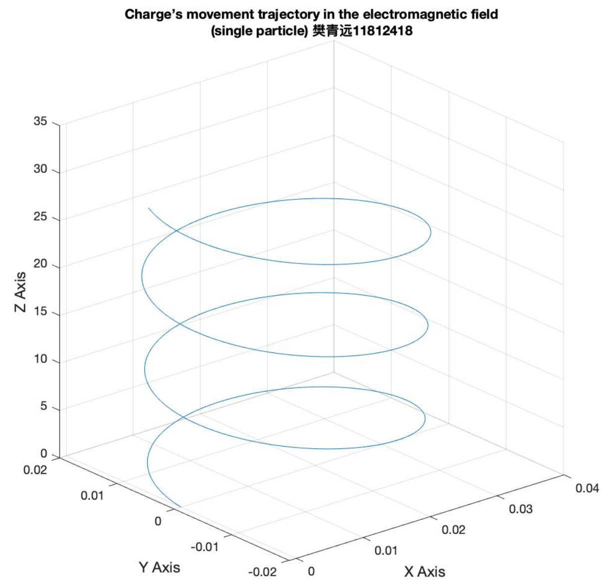

Figure 1. The track of single particle

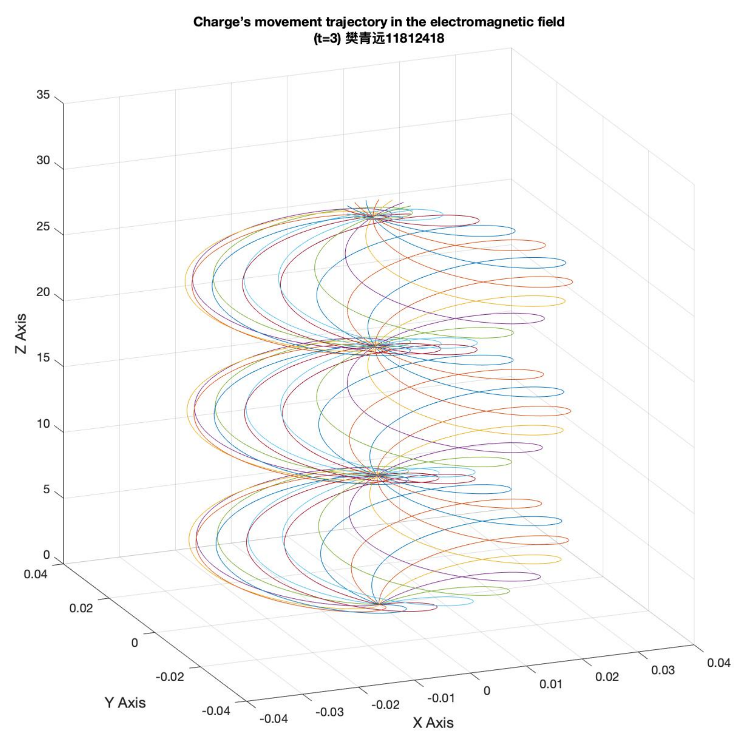

Figure 2. The track of 16 particles (t=3)

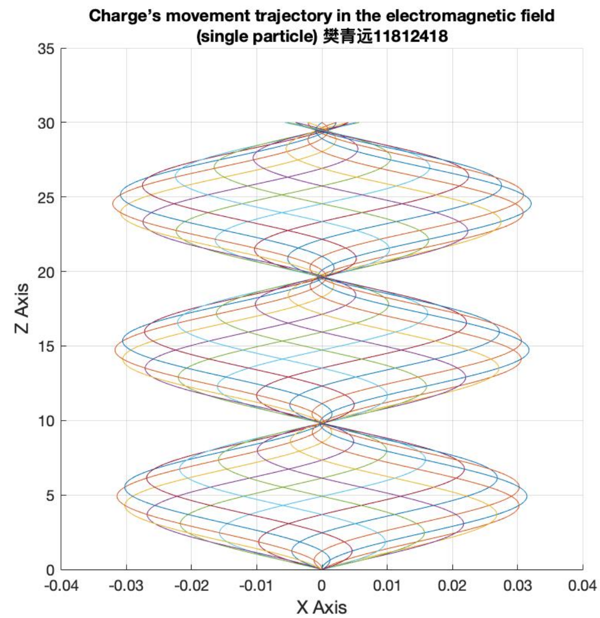

Figure 3. The track of 16 particles (t=3, view from XZ plane)

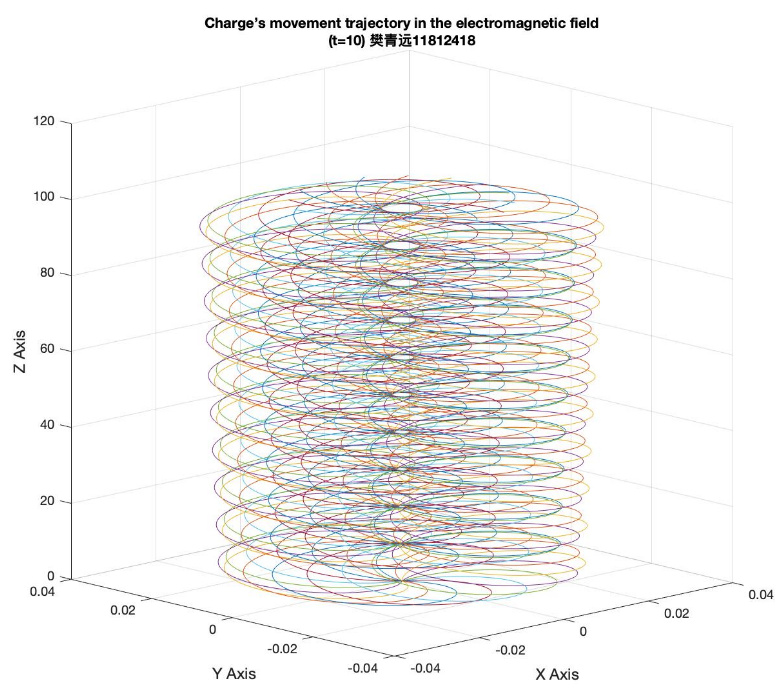

Figure 4. The track of 16 particles (t=10)

5 Discussion

5.1 Effect of different dt on results

We have noticed that different dt could relsult to plot with great difference, here are some examples:

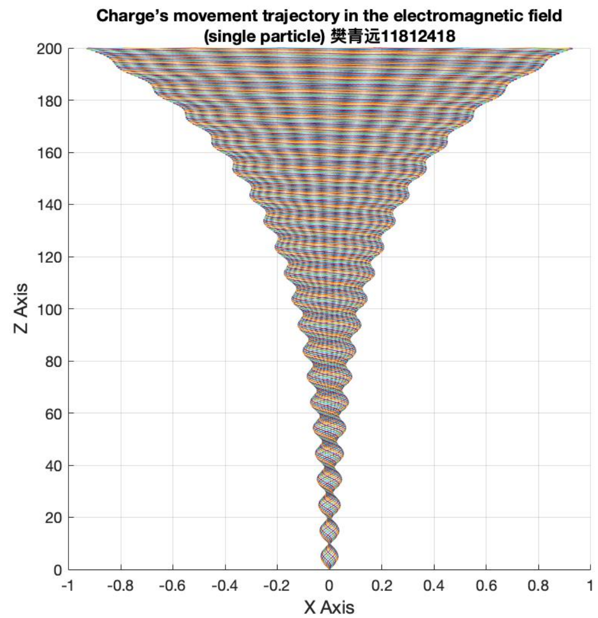

Figure 5. The track of 16 particles (t=20, **dt=0.01**)

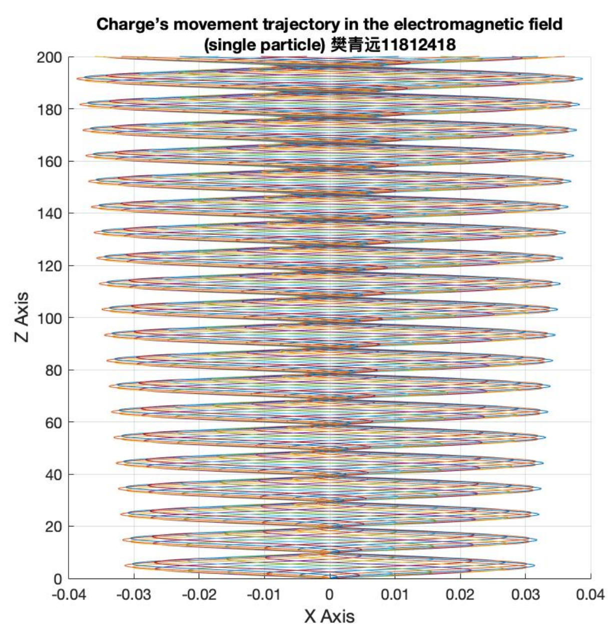

Figure 6. The track of 16 particles (t=20, **dt=0.001**)

Comparing to the two graph, we found that the increase of the gyration radius of the second particle is significantly slower than the first particle.

Observing from the track of single prticle from the following figure, we could finally get a reasonable explanation: we we set a relative larger dt, the particle may have turned serval laps on its orbit during one dt. At that time, we still consider that the particle has only turned one lap. Whats more, the discontinuous brought by a greater dt also make the increase of the orbit radius much faster than the real condition.

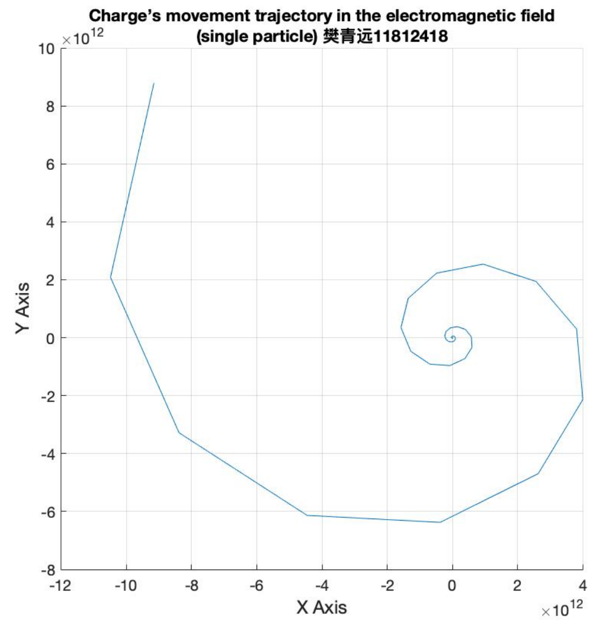

Figure 7. The track of one particle (t=20, **dt=0.1**, viewed from xy plane)

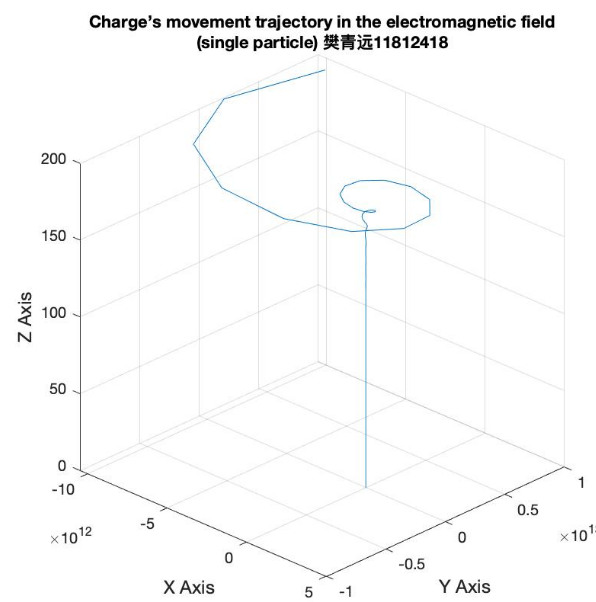

Figure 8. The track of one particle (t=20, **dt=0.1**,)

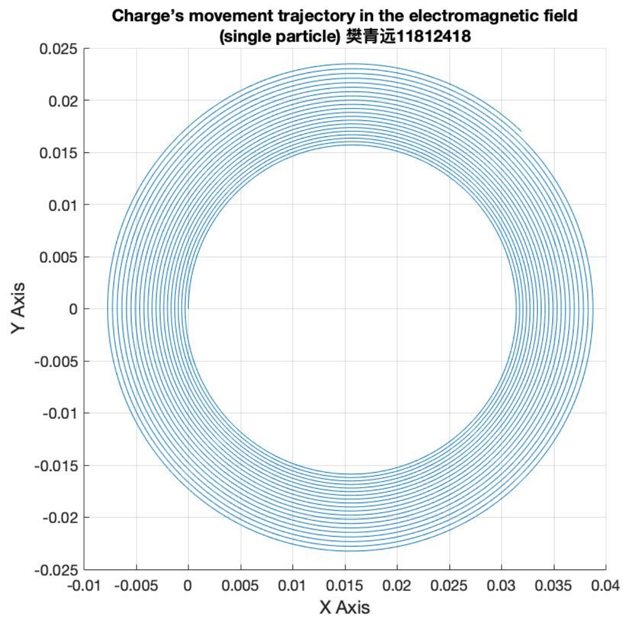

Figure 9. The track of one particle (t=20, **dt=0.001**, viewed from xy plane)

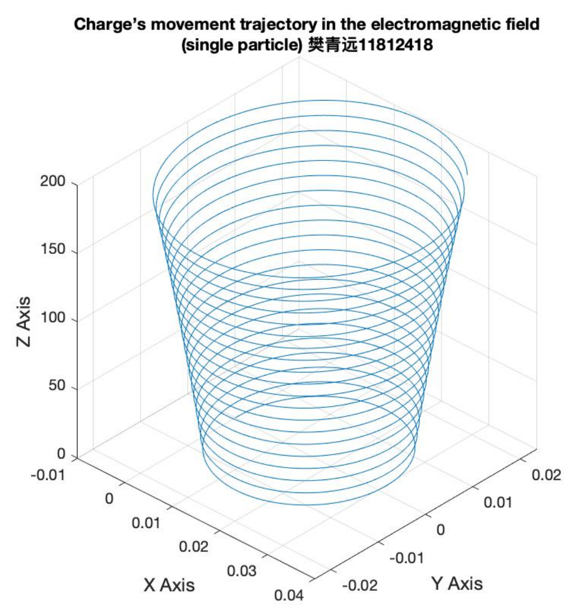

Figure 10. The track of one particle (t=20, **dt=0.001**)

6 Analyze and Inspiration

During the experiment, we observe and deepen our understanding of the particle movement in the magnetic field.

We understand the principle magnetic focusing, which has great potential to be used in infrastructure like magnetic lenses and other devices.

Lastly, we have also notice that the accuracy of the parameters could significant impact the result of the experiment.