Airbrakes Control Algorithm

Marc D Nichitiu, Akash Krishna — 8 January 2026

MIT Rocket Team Simulations

1 Abstract

We wish to choose a target apogee altitude using a variable-area airbrake system. Airbrakes are deployed using a servomotor that can expose a particular fraction of the Airbrake area to the freestream, thereby allowing the drag to depend on time.

This algorithm seeks to enable a rocket having some default without-Airbrakes apogee altitude to lower this altitude to a close but particular height, to be chosen by the designer. Overall flow is as follows:

1. A linear approximation (coefficient α) of the vertical acceleration of the rocket as a function of time is characterized.
2. A non-Airbrakes apogee altitude is determined according to this approximation.
3. The difference between default apogee altitude and desired apogee altitude is calculated according to the pre-determined altitude choice. Let us denote its magnitude Δx.
4. The Airbrakes' deployed area $A(t)$ is calculated from Δx, according to the ramp shape in §2 Eq. (13).
5. Once Airbrakes are activated, they are to be extended according to $A(t)$, with a PIDF controller on position accuracy and acceleration.

2 Derivation: Energy Conservation

Consider the rocket to undergo purely vertical motion. In the case in which airbrakes are never deployed, the rocket will reach an apogee height of x_{apog}, corresponding to a gravitational potential energy of mgx_{apog}. Airbrakes deployment, with area some $A(t)$, will do work on the rocket:

$$W = \int F\ dx = \int \frac{C_D \rho A(t) \dot{x}^2}{2}\ dx \tag{1}$$

This work will contribute to some loss of altitude in the apogee, given by

$$\int \frac{C_D \rho A(t) \dot{x}^2}{2}\ dx = mg(\Delta x) \tag{2}$$

We recall that $\dot{x} = dx/dt$, and so $dx = \dot{x}dt$:

$$\int \frac{C_D \rho A(t) \dot{x}^3}{2}\ dt = mg(\Delta x) \tag{3}$$

The governing differential equation is

$$\ddot{x} = -g - B\dot{x}^2 \tag{4}$$

where

$$B = \frac{C_{D_{\text{rocket}}} \rho A_{\text{ref}}}{2m} \tag{5}$$

We approximate that acceleration varies linearly in time, of the form

$$\ddot{x} = -g + \alpha(t - t_{\text{apog}}) \tag{6}$$

We solve for $\dot{x}$:

$$\dot{x} = \sqrt{-\frac{g + \ddot{x}}{B}} \tag{7}$$

Thus,

$$\dot{x}^3 = \left(-\frac{g+\ddot{x}}{B}\right)^{3/2} \tag{8}$$

Substituting in (6) gives

$$\dot{x}^3 = \left(-\alpha\frac{t_{\text{apog}}-t}{B}\right)^{3/2} \tag{9}$$

Thus, the integral becomes

$$\frac{C_D\rho\alpha^{3/2}}{2B^{3/2}}\int A(t)(t_{\text{apog}}-t)^{3/2}\,dt = mg(\Delta x) \tag{10}$$

We move all terms to the other side:

$$\int A(t)(t_{\text{apog}}-t)^{3/2}\,dt = \frac{2mg(\Delta x)B^{3/2}}{C_D\rho\alpha^{3/2}} \tag{11}$$

Let η be the right side expression, such that

$$\int A(t)(t_{\text{apog}}-t)^{3/2}\,dt = \eta \tag{12}$$

Values of η for different altitudes are given below according to current Zephyrus statistics:

$\mathbf{\Delta x}$	$\boldsymbol{\eta}$
300 m	1.875
200 m	1.249
100 m	0.625

We are free to choose the bounds of $A(t)$'s nonzero interval, as well as the kind of function $A(t)$ is. We will choose a ramp-like function, of the form

$$A(t) = \begin{cases} 2A_0(t-t_0) & t \in \left[t_0, t_0+\dfrac{1}{2}\right] \\ A_0 & t \geq t_0 + \dfrac{1}{2} \end{cases} \tag{13}$$

The ramp slope accounts for a 1/2 second between $A(t) = 0$ and $A(t) = A_0$. Thus,

$$\frac{\eta}{A_0} = \int_{t_0}^{t_0+1/2} 2(t-t_0)(t_{\text{apog}}-t)^{3/2}\,dt + \int_{t_0+1/2}^{t_{\text{apog}}} (t_{\text{apog}}-t)^{3/2}\,dt \tag{14}$$

We perform a change of variables from $\tau = t_{\text{apog}} - t$ which gives $d\tau = -dt$ and $t = t_{\text{apog}} - \tau$. The integration limits become

$$t_0 \to t_{\text{apog}} - t_0 \quad ; \quad t_0 + \frac{1}{2} \to t_{\text{apog}} - t_0 - \frac{1}{2} \quad ; \quad t_{\text{apog}} \to 0 \tag{15}$$

Thus,

$$\frac{\eta}{A_0} = -2\int_{t_{\text{apog}}-t_0}^{t_{\text{apog}}-t_0-1/2} (t_{\text{apog}}-\tau-t_0)\tau^{3/2}\,d\tau - \int_{t_{\text{apog}}-t_0-1/2}^{0} \tau^{3/2}\,d\tau \tag{16}$$

$$= 2\int_{t_{\text{apog}}-t_0-1/2}^{t_{\text{apog}}-t_0} (t_{\text{apog}}-\tau-t_0)\tau^{3/2}\,d\tau + \int_{0}^{t_{\text{apog}}-t_0-1/2} \tau^{3/2}\,d\tau \tag{17}$$

We expand the expression into three integrals, and solve them one by one. The first is

$$2(t_{\text{apog}} - t_0) \int_{t_{\text{apog}}-t_0-1/2}^{t_{\text{apog}}-t_0} \tau^{3/2}\, d\tau = 2(t_{\text{apog}} - t_0) \left(\frac{\tau^{5/2}}{5/2} \right) \Bigg|_{t_{\text{apog}}-t_0-1/2}^{t_{\text{apog}}-t_0} \tag{18}$$

$$= \frac{4}{5}(t_{\text{apog}} - t_0) \left((t_{\text{apog}} - t_0)^{5/2} - (t_{\text{apog}} - t_0 - 1/2)^{5/2} \right) \tag{19}$$

The second integral evaluates to

$$-2 \int_{t_{\text{apog}}-t_0-1/2}^{t_{\text{apog}}-t_0} \tau^{5/2}\, d\tau = -2 \left(\frac{\tau^{7/2}}{7/2} \right) \Bigg|_{t_{\text{apog}}-t_0-1/2}^{t_{\text{apog}}-t_0} \tag{20}$$

$$= -\frac{4}{7} \left((t_{\text{apog}} - t_0)^{7/2} - (t_{\text{apog}} - t_0 - 1/2)^{7/2} \right) \tag{21}$$

The third integral evaluates to

$$\int_0^{t_{\text{apog}}-t_0-1/2} \tau^{3/2}\, d\tau = \left(\frac{\tau^{5/2}}{5/2} \right) \Bigg|_0^{t_{\text{apog}}-t_0-1/2} = \frac{2}{5}(t_{\text{apog}} - t_0 - 1/2)^{5/2} \tag{22}$$

Thus, A_0 evaluates to

$$\boxed{\frac{\eta}{\frac{4}{5}(t_{\text{apog}} - t_0)\left((t_{\text{apog}} - t_0)^{\frac{5}{2}} - (t_{\text{apog}} - t_0 - \frac{1}{2})^{\frac{5}{2}}\right) - \frac{4}{7}\left((t_{\text{apog}} - t_0)^{\frac{7}{2}} - (t_{\text{apog}} - t_0 - \frac{1}{2})^{\frac{7}{2}}\right) + \frac{2}{5}(t_{\text{apog}} - t_0 - \frac{1}{2})^{\frac{5}{2}}}} \tag{23}$$

The maximum area of $A(t)$ is 0.0066 m^2. A table of results is below. Note that apogee time is 31.89 seconds.

t_0	$\Delta x = 300$ m	$\Delta x = 200$ m	$\Delta x = 100$ m
15 s	63%	42%	21%
17.5 s	94%	63%	32%
19 s	124%	83%	42%
20 s	153%	102%	51%

Table 1. Values of $\frac{A_0}{A_{\max}}$ requisite for the labeled altitude difference, according to Zephyrus Test Launch Simuilations.

3 Implementation, Characterizations