0.1 Incremental pressure correction (IPCS):

$$\mathbf{u}^{n-\theta} = \theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}^{n} \tag{1}$$

$$\mathbf{u}^{n-\theta nl} = \theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*} \tag{2}$$

Tentative velocity step:

$$\begin{aligned} &\int_{\Omega} \mathbf{v} \frac{\mathbf{u}_{*} - \mathbf{u}_{n-1}}{\Delta t}\, d\Omega + \int_{\Omega} \mathbf{v}\left(\mathbf{u}^{n-\theta}\right) \cdot \nabla \mathbf{u}^{n-\theta nl}\, d\Omega + \int_{\Omega} \nabla \mathbf{v} \\ &\cdot \nabla \mathbf{u}^{n-\theta}\, d\Omega - \int_{\Omega} \mathbf{v}\mathbf{f}\, d\Omega + \int_{\Omega} \mathbf{v} \nabla p_{*}\, d\Omega \\ &- \int_{\Gamma} \mathbf{v} g\, d\Gamma = 0, \quad \forall \mathbf{v} \end{aligned} \tag{3}$$

Pressure equation:

$$\int_{\Omega} \nabla q \cdot \mathbf{u}_{*}\, d\Omega - \int_{\Gamma} q\left(\mathbf{u}_{n} \cdot n\right)\, d\Gamma = \Delta t \int_{\Omega} \nabla q \cdot \nabla\left(p - p_{*}\right)\, d\Omega, \quad \forall q \tag{4}$$

Corrected velocity:

$$\int_{\Omega} \mathbf{v} \cdot \frac{\mathbf{u}_{n} - \mathbf{u}_{*}}{\Delta t}\, d\Omega + \int_{\Omega} \mathbf{v} \cdot \nabla\left(p - p_{*}\right)\, d\Omega = 0, \quad \forall \mathbf{v} \tag{5}$$

0.2 Avoiding Assembly:

Tentative velocity step:

$$\begin{aligned} &\int_{\Omega} \mathbf{v} \cdot \frac{\mathbf{u}_{*} - \mathbf{u}_{n-1}}{\Delta t}\, d\Omega + \int_{\Omega} \mathbf{v} \cdot \left(\theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*}\right) \cdot \nabla\left(\theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*-1}\right)\, d\Omega \\ &+ \int_{\Omega} \nabla \mathbf{v} \cdot \nabla\left(\theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*}\right)\, d\Omega - \int_{\Omega} \mathbf{v}\mathbf{f}\, d\Omega + \int_{\Omega} \mathbf{v} \cdot \nabla p_{*}\, d\Omega \\ &- \int_{\Gamma} \mathbf{v} \cdot g\, d\Gamma = 0, \quad \forall \mathbf{v} \end{aligned} \tag{6}$$

$$\begin{aligned} &\int_{\Omega} \mathbf{v} \cdot \frac{\mathbf{u}_{*}}{\Delta t}\, d\Omega + \int_{\Omega} \mathbf{v} \cdot (1-\theta)\,\mathbf{u}_{*} \cdot \nabla\left(\theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*-1}\right)\, d\Omega + \int_{\Omega} \nabla \mathbf{v} \cdot \nabla (1-\theta)\,\mathbf{u}_{*}\, d\Omega \\ &= \int_{\Omega} \mathbf{v} \cdot \frac{\mathbf{u}_{n-1}}{\Delta t}\, d\Omega - \int_{\Omega} \mathbf{v} \cdot \theta \mathbf{u}^{n-1} \cdot \nabla\left(\theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*-1}\right)\, d\Omega - \int_{\Omega} \nabla \mathbf{v} \\ &\cdot \nabla \theta \mathbf{u}^{n-1}\, d\Omega + \int_{\Omega} \mathbf{v} \cdot \mathbf{f}\, d\Omega - \int_{\Omega} \mathbf{v} \cdot \nabla p_{*}\, d\Omega \\ &+ \int_{\Gamma} \mathbf{v} \cdot g\, d\Gamma, \quad \forall \mathbf{v} \end{aligned} \tag{7}$$

$$\begin{aligned}
&\int_\Omega \mathbf{v}\cdot\frac{\mathbf{u}_*}{\Delta t}\,d\Omega + (1-\theta)\int_\Omega \left[\mathbf{v}\cdot\mathbf{u}_*\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)+\nabla\mathbf{v}\cdot\nabla\mathbf{u}_*\right]\,d\Omega \\
&\quad=\int_\Omega \mathbf{v}\cdot\frac{\mathbf{u}_{n-1}}{\Delta t}\,d\Omega-\theta\int_\Omega\left[\mathbf{v}\cdot\mathbf{u}^{n-1}\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)+\nabla\mathbf{v}\cdot\nabla\mathbf{u}^{n-1}\right]\,d\Omega \\
&\qquad+\int_\Omega\mathbf{v}\cdot\mathbf{f}\,d\Omega-\int_\Omega\mathbf{v}\cdot\nabla p_*\,d\Omega \\
&\qquad+\int_\Gamma\mathbf{v}\cdot g\,d\Gamma,\quad\forall\mathbf{v}
\end{aligned}\tag{8}$$

$$\mathbf{u}_*=\phi_i\mathbf{U}_*\tag{9}$$

$$\mathbf{u}^{n-1}=\phi_i\mathbf{U}^{n-1}\tag{10}$$

$$\mathbf{v}=\phi_j\tag{11}$$

$$\begin{aligned}
&\int_\Omega \phi_j\cdot\frac{\phi_i\mathbf{U}_*}{\Delta t}\,d\Omega + (1-\theta)\int_\Omega \left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}_*+\nabla\phi_j\cdot\nabla\phi_i\mathbf{U}_*\right]\,d\Omega \\
&\quad=\int_\Omega \phi_j\cdot\frac{\phi_i\mathbf{U}^{n-1}}{\Delta t}\,d\Omega-\theta\int_\Omega\left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}^{n-1}+\nabla\phi_j\cdot\nabla\phi_i\mathbf{U}^{n-1}\right]\,d\Omega \\
&\qquad+\int_\Omega\phi_j\cdot\mathbf{f}\,d\Omega-\int_\Omega\phi_j\cdot\nabla p_*\,d\Omega \\
&\qquad+\int_\Gamma\phi_j\cdot g\,d\Gamma,\quad\forall\phi_j
\end{aligned}\tag{12}$$

$$\begin{aligned}
&\int_\Omega \frac{\phi_j\cdot\phi_i}{\Delta t}\mathbf{U}_*\,d\Omega + (1-\theta)\int_\Omega \left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i+\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}_*\,d\Omega \\
&\quad=\int_\Omega \frac{\phi_j\cdot\phi_i}{\Delta t}\mathbf{U}^{n-1}\,d\Omega-\theta\int_\Omega\left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i+\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}^{n-1}\,d\Omega \\
&\qquad+\int_\Omega\phi_j\cdot\mathbf{f}\,d\Omega-\int_\Omega\phi_j\cdot\nabla p_*\,d\Omega \\
&\qquad+\int_\Gamma\phi_j\cdot g\,d\Gamma,\quad\forall\phi_j
\end{aligned}\tag{13}$$

$$\begin{aligned}
&\int_\Omega\left[\frac{\phi_j\cdot\phi_i}{\Delta t}+(1-\theta)\,\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}_*\,d\Omega+\int_\Omega(1-\theta)\,\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}_*\,d\Omega \\
&\quad=\int_\Omega\left[\frac{\phi_j\cdot\phi_i}{\Delta t}-\theta\,\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}^{n-1}\,d\Omega-\int_\Omega\theta\,\phi_j \\
&\qquad\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}^{n-1}\,d\Omega+\int_\Omega\phi_j\cdot\mathbf{f}\,d\Omega-\int_\Omega\phi_j\cdot\nabla p_*\,d\Omega \\
&\qquad+\int_\Gamma\phi_j\cdot g\,d\Gamma,\quad\forall\phi_j
\end{aligned}\tag{14}$$

$$M = \phi_j \cdot \phi_i \tag{15}$$

$$K = \nabla\phi_j \cdot \nabla\phi_i \tag{16}$$

$$A = \phi_j \cdot \nabla \left(\theta \mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*-1}\right) \phi_i \tag{17}$$

$$\begin{aligned} &\int_\Omega \left[\frac{M}{\Delta t} + (1-\theta)\,K\right] \mathbf{U}_*\, d\Omega + \int_\Omega (1-\theta)\, A\mathbf{U}_*\, d\Omega \\ &\quad = \int_\Omega \left[\frac{M}{\Delta t} - \theta\, K\right] \mathbf{U}^{n-1}\, d\Omega - \int_\Omega \theta\, A\mathbf{U}^{n-1}\, d\Omega + \int_\Omega \phi_j \cdot \mathbf{f}\, d\Omega - \int_\Omega \phi_j \cdot \nabla p_*\, d\Omega \\ &\qquad + \int_\Gamma \phi_j \cdot g\, d\Gamma, \quad \forall \phi_j \end{aligned} \tag{18}$$

no forcing and full dirichlet conditions

$$\int_\Omega \left[\frac{M}{\Delta t} + (1-\theta)\,K + (1-\theta)\,A\right] \mathbf{U}_*\, d\Omega = \int_\Omega \left[\frac{M}{\Delta t} - \theta\, K - \theta\, A\right] \mathbf{U}^{n-1}\, d\Omega - \int_\Omega \phi_j \cdot \nabla p_*\, d\Omega, \quad \forall \phi_j \tag{19}$$