1 Incremental pressure correction (IPCS):

$$\mathbf{u}^{n-\theta} = \theta\mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}^{n} \tag{1}$$

$$\mathbf{u}^{n-\theta nl} = \theta\mathbf{u}^{n-1} + (1-\theta)\,\mathbf{u}_{*} \tag{2}$$

Tentative velocity step:

$$\begin{aligned}&\int_{\Omega}\mathbf{v}\frac{\mathbf{u}_{*}-\mathbf{u}_{n-1}}{\Delta t}\;d\Omega + \int_{\Omega}\mathbf{v}\left(\mathbf{u}^{n-\theta}\right)\cdot\nabla\mathbf{u}^{n-\theta nl}\;d\Omega + \int_{\Omega}\nabla\mathbf{v}\\ &\cdot\nabla\mathbf{u}^{n-\theta}\;d\Omega - \int_{\Omega}\mathbf{v}\mathbf{f}\;d\Omega + \int_{\Omega}\mathbf{v}\nabla p_{*}\;d\Omega\\ &-\int_{\Gamma}\mathbf{v}g\;d\Gamma = 0, \quad \forall\mathbf{v}\end{aligned} \tag{3}$$

Pressure equation:

$$\int_{\Omega}\nabla q\cdot\mathbf{u}_{*}\;d\Omega - \int_{\Gamma}q\left(\mathbf{u}_{n}\cdot n\right)\;d\Gamma = \Delta t\int_{\Omega}\nabla q\cdot\nabla\left(p-p_{*}\right)\;d\Omega, \quad \forall q \tag{4}$$

Corrected velocity:

$$\int_{\Omega}\mathbf{v}\cdot\frac{\mathbf{u}_{n}-\mathbf{u}_{*}}{\Delta t}\;d\Omega + \int_{\Omega}\mathbf{v}\cdot\nabla\left(p-p_{*}\right)\;d\Omega = 0, \quad \forall\mathbf{v} \tag{5}$$

1.1 Avoiding Assembly:

Tentative velocity step:

$$\begin{aligned}&\int_{\Omega}\mathbf{v}\cdot\frac{\mathbf{u}_{*}-\mathbf{u}_{n-1}}{\Delta t}\;d\Omega + \int_{\Omega}\mathbf{v}\cdot\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*}\right)\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\;d\Omega\\ &+\int_{\Omega}\nabla\mathbf{v}\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*}\right)\;d\Omega - \int_{\Omega}\mathbf{v}\mathbf{f}\;d\Omega + \int_{\Omega}\mathbf{v}\cdot\nabla p_{*}\;d\Omega\\ &-\int_{\Gamma}\mathbf{v}\cdot g\;d\Gamma = 0, \quad \forall\mathbf{v}\end{aligned} \tag{6}$$

$$\begin{aligned}&\int_{\Omega}\mathbf{v}\cdot\frac{\mathbf{u}_{*}}{\Delta t}\;d\Omega + \int_{\Omega}\mathbf{v}\cdot(1-\theta)\,\mathbf{u}_{*}\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\;d\Omega + \int_{\Omega}\nabla\mathbf{v}\cdot\nabla(1-\theta)\,\mathbf{u}_{*}\;d\Omega\\ &=\int_{\Omega}\mathbf{v}\cdot\frac{\mathbf{u}_{n-1}}{\Delta t}\;d\Omega - \int_{\Omega}\mathbf{v}\cdot\theta\mathbf{u}^{n-1}\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\;d\Omega - \int_{\Omega}\nabla\mathbf{v}\\ &\cdot\nabla\theta\mathbf{u}^{n-1}\;d\Omega + \int_{\Omega}\mathbf{v}\cdot\mathbf{f}\;d\Omega - \int_{\Omega}\mathbf{v}\cdot\nabla p_{*}\;d\Omega\\ &+\int_{\Gamma}\mathbf{v}\cdot g\;d\Gamma, \quad \forall\mathbf{v}\end{aligned} \tag{7}$$

$$
\begin{aligned}
&\int_\Omega \mathbf{v}\cdot\frac{\mathbf{u}_*}{\Delta t}\,d\Omega + (1-\theta)\int_\Omega \left[\mathbf{v}\cdot\mathbf{u}_*\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)+\nabla\mathbf{v}\cdot\nabla\mathbf{u}_*\right]\,d\Omega\\
&\quad=\int_\Omega \mathbf{v}\cdot\frac{\mathbf{u}_{n-1}}{\Delta t}\,d\Omega-\theta\int_\Omega\left[\mathbf{v}\cdot\mathbf{u}^{n-1}\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)+\nabla\mathbf{v}\cdot\nabla\mathbf{u}^{n-1}\right]\,d\Omega\\
&\qquad+\int_\Omega \mathbf{v}\cdot\mathbf{f}\,d\Omega-\int_\Omega\mathbf{v}\cdot\nabla p_*\,d\Omega\\
&\qquad+\int_\Gamma \mathbf{v}\cdot g\,d\Gamma,\quad\forall\mathbf{v}
\end{aligned}
\tag{8}
$$

$$
\mathbf{u}_* = \phi_i\mathbf{U}_* \tag{9}
$$

$$
\mathbf{u}^{n-1} = \phi_i\mathbf{U}^{n-1} \tag{10}
$$

$$
\mathbf{v} = \phi_j \tag{11}
$$

$$
\begin{aligned}
&\int_\Omega \phi_j\cdot\frac{\phi_i\mathbf{U}_*}{\Delta t}\,d\Omega+(1-\theta)\int_\Omega\left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}_*+\nabla\phi_j\cdot\nabla\phi_i\mathbf{U}_*\right]\,d\Omega\\
&\quad=\int_\Omega\phi_j\cdot\frac{\phi_i\mathbf{U}^{n-1}}{\Delta t}\,d\Omega-\theta\int_\Omega\left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}^{n-1}+\nabla\phi_j\cdot\nabla\phi_i\mathbf{U}^{n-1}\right]\,d\Omega\\
&\qquad+\int_\Omega\phi_j\cdot\mathbf{f}\,d\Omega-\int_\Omega\phi_j\cdot\nabla p_*\,d\Omega\\
&\qquad+\int_\Gamma\phi_j\cdot g\,d\Gamma,\quad\forall\phi_j
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&\int_\Omega\frac{\phi_j\cdot\phi_i}{\Delta t}\mathbf{U}_*\,d\Omega+(1-\theta)\int_\Omega\left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i+\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}_*\,d\Omega\\
&\quad=\int_\Omega\frac{\phi_j\cdot\phi_i}{\Delta t}\mathbf{U}^{n-1}\,d\Omega-\theta\int_\Omega\left[\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i+\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}^{n-1}\,d\Omega\\
&\qquad+\int_\Omega\phi_j\cdot\mathbf{f}\,d\Omega-\int_\Omega\phi_j\cdot\nabla p_*\,d\Omega\\
&\qquad+\int_\Gamma\phi_j\cdot g\,d\Gamma,\quad\forall\phi_j
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&\int_\Omega\left[\frac{\phi_j\cdot\phi_i}{\Delta t}+(1-\theta)\,\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}_*\,d\Omega+\int_\Omega(1-\theta)\,\phi_j\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}_*\,d\Omega\\
&\quad=\int_\Omega\left[\frac{\phi_j\cdot\phi_i}{\Delta t}-\theta\,\nabla\phi_j\cdot\nabla\phi_i\right]\mathbf{U}^{n-1}\,d\Omega-\int_\Omega\theta\,\phi_j\\
&\qquad\cdot\nabla\left(\theta\mathbf{u}^{n-1}+(1-\theta)\,\mathbf{u}_{*-1}\right)\phi_i\mathbf{U}^{n-1}\,d\Omega+\int_\Omega\phi_j\cdot\mathbf{f}\,d\Omega-\int_\Omega\phi_j\cdot\nabla p_*\,d\Omega\\
&\qquad+\int_\Gamma\phi_j\cdot g\,d\Gamma,\quad\forall\phi_j
\end{aligned}
\tag{14}
$$

$$M = \phi_j \cdot \phi_i \tag{15}$$

$$K = \nabla \phi_j \cdot \nabla \phi_i \tag{16}$$

$$A = \phi_j \cdot \nabla \left(\theta \mathbf{u}^{n-1} + (1-\theta)\, \mathbf{u}_{*-1} \right) \phi_i \tag{17}$$

$$\begin{aligned} &\int_\Omega \left[\frac{M}{\Delta t} + (1-\theta)\, K \right] \mathbf{U}_*\, d\Omega + \int_\Omega (1-\theta)\, A \mathbf{U}_*\, d\Omega \\ &= \int_\Omega \left[\frac{M}{\Delta t} - \theta\, K \right] \mathbf{U}^{n-1}\, d\Omega - \int_\Omega \theta\, A \mathbf{U}^{n-1}\, d\Omega + \int_\Omega \phi_j \cdot \mathbf{f}\, d\Omega - \int_\Omega \phi_j \cdot \nabla p_*\, d\Omega \\ &\quad + \int_\Gamma \phi_j \cdot g\, d\Gamma, \quad \forall \phi_j \end{aligned} \tag{18}$$

no forcing and full dirichlet conditions

$$\int_\Omega \left[\frac{M}{\Delta t} + (1-\theta)\, K + (1-\theta)\, A \right] \mathbf{U}_*\, d\Omega = \int_\Omega \left[\frac{M}{\Delta t} - \theta\, K - \theta\, A \right] \mathbf{U}^{n-1}\, d\Omega - \int_\Omega \phi_j \cdot \nabla p_*\, d\Omega, \ \ \forall \phi_j \tag{19}$$

Pressure equation:

$$\int_\Omega \nabla q \cdot \mathbf{u}_*\, d\Omega - \int_\Gamma q \left(\mathbf{u}_n \cdot n \right)\, d\Gamma = \Delta t \int_\Omega \nabla q \cdot \nabla \left(p - p_* \right)\, d\Omega, \quad \forall q \tag{20}$$

$$\int_\Omega \nabla q \cdot \nabla p\, d\Omega = \int_\Omega \nabla q \cdot \nabla p_*\, d\Omega + \int_\Omega \frac{1}{\Delta t} \nabla q \cdot \mathbf{u}_*\, d\Omega\, d\Gamma, \quad \forall q \tag{21}$$

Corrected velocity:

$$\int_\Omega \mathbf{v} \cdot \frac{\mathbf{u}_n - \mathbf{u}_*}{\Delta t}\, d\Omega + \int_\Omega \mathbf{v} \cdot \nabla \left(p - p_* \right)\, d\Omega = 0, \quad \forall \mathbf{v} \tag{22}$$

$$\int_\Omega \mathbf{v} \cdot \mathbf{u}_n\, d\Omega = \int_\Omega \mathbf{v} \cdot \mathbf{u}_*\, d\Omega - \Delta t \int_\Omega \mathbf{v} \cdot \nabla \left(p - p_* \right)\, d\Omega, \quad \forall \mathbf{v} \tag{23}$$

2 Advection of sediment:

$$\frac{\partial \left(c^n - c^{n-1} \right)}{\Delta t} + \nabla \cdot \left(\mathbf{u}^{n-\theta} c^{n-\theta} \right) + \nabla \cdot \left(\mathbf{k} u_{sink} c^{n-\theta} \right) - \nabla \cdot \left(\nu_T \nabla c^{n-\theta} \right) = s \tag{24}$$

$$\begin{aligned} &\int_\Omega \psi \frac{\partial \left(c^n - c^{n-1} \right)}{\Delta t}\, d\Omega + \int_\Omega \psi \nabla \cdot \left(\mathbf{u}^{n-\theta} c^{n-\theta} \right)\, d\Omega + \int_\Omega \psi \nabla \\ &\cdot \left(\mathbf{k} u_{sink} c^{n-\theta} \right)\, d\Omega - \int_\Omega \psi \nabla \cdot \left(\nu_T \nabla c^{n-\theta} \right)\, d\Omega = \int_\Omega \psi s\, d\Omega \end{aligned} \tag{25}$$

$$
\begin{aligned}
&\int_\Omega \psi \frac{\partial \left(c^n - c^{n-1}\right)}{\Delta t} \, d\Omega + \int_\Omega \psi \nabla \cdot \left(\mathbf{u}^{n-\theta} c^{n-\theta}\right) \, d\Omega - \\
&\quad \int_\Omega \nabla \psi \cdot \left(\mathbf{k} u_{sink} c^{n-\theta}\right) \, d\Omega + \int_\Omega \nabla \psi \cdot \left(\nu_T \nabla c^{n-\theta}\right) \, d\Omega = \int_\Omega \psi s \, d\Omega \\
&\qquad - \int_{\Gamma_{deposit}} \psi \cdot \left(\mathbf{n} \cdot \mathbf{k} u_{sink} c^{n-\theta}\right) \, d\Gamma + \int_{\Gamma_N} \psi \cdot g \, d\Gamma
\end{aligned}
\tag{26}
$$

2.1 SU Stabilisation:

$$
S = \int_\Omega \frac{\nu h}{||\mathbf{u}^{n-\theta}||} \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi\right) \left(\mathbf{u}^{n-\theta} \cdot \nabla c^{n-\theta}\right) \, d\Omega \tag{27}
$$

$$
\begin{aligned}
&\int_\Omega \psi \frac{\partial \left(c^n - c^{n-1}\right)}{\Delta t} \, d\Omega + \int_\Omega \psi \nabla \cdot \left(\mathbf{u}^{n-\theta} c^{n-\theta}\right) \, d\Omega - \\
&\quad \int_\Omega \nabla \psi \cdot \left(\mathbf{k} u_{sink} c^{n-\theta}\right) \, d\Omega + \int_\Omega \nabla \psi \cdot \left(\nu_T \nabla c^{n-\theta}\right) \, d\Omega \\
&\quad + S = \int_\Omega \psi s \, d\Omega - \int_{\Gamma_{deposit}} \psi \cdot \left(\mathbf{n} \cdot \mathbf{k} u_{sink} c^{n-\theta}\right) \, d\Gamma + \int_{\Gamma_N} \psi \cdot g \, d\Gamma
\end{aligned}
\tag{28}
$$

2.2 Avoiding Assembly:

$$
\begin{aligned}
&\int_\Omega \psi \frac{\left(c^n - c^{n-1}\right)}{\Delta t} \, d\Omega + \int_\Omega \psi \nabla \cdot \left(\mathbf{u}^{n-\theta} \left(\theta c^{n-1} + (1-\theta) c^n\right)\right) \, d\Omega - \\
&\quad \int_\Omega \nabla \psi \cdot \left(\mathbf{k} u_{sink} \left(\theta c^{n-1} + (1-\theta) c^n\right)\right) \, d\Omega + \int_\Omega \nabla \psi \cdot \left(\nu_T \nabla \left(\theta c^{n-1} + (1-\theta) c^n\right)\right) \, d\Omega \\
&\quad + S \left(\theta c^{n-1} + (1-\theta) c^n\right) = \int_\Omega \psi s \, d\Omega - \int_{\Gamma_{deposit}} \psi \cdot \left(\mathbf{n} \cdot \mathbf{k} u_{sink} \left(\theta c^{n-1} + (1-\theta) c^n\right)\right) \, d\Gamma + \int_{\Gamma_N} \psi \cdot g \, d\Gamma
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&\int_\Omega \psi \frac{c^n}{\Delta t} \, d\Omega + \int_\Omega (1-\theta) \psi \nabla \cdot \left(\mathbf{u}^{n-\theta} c^n\right) \, d\Omega - \int_\Omega (1-\theta) \nabla \psi \cdot \left(\mathbf{k} u_{sink} c^n\right) \, d\Omega + \\
&\quad \int_\Omega (1-\theta) \nabla \psi \cdot \left(\nu_T \nabla c^n\right) \, d\Omega + \int_{\Gamma_{deposit}} (1-\theta) \psi \cdot \left(\mathbf{n} \cdot \mathbf{k} u_{sink} c^n\right) \, d\Gamma + S \left((1-\theta) c^n\right) = \\
&\qquad \int_\Omega \psi \frac{c^{n-1}}{\Delta t} \, d\Omega - \int_\Omega \theta \psi \nabla \cdot \left(\mathbf{u}^{n-\theta} c^{n-1}\right) \, d\Omega + \int_\Omega \theta \nabla \psi \cdot \left(\mathbf{k} u_{sink} c^{n-1}\right) \, d\Omega + \\
&\qquad \int_\Omega \theta \nabla \psi \cdot \left(\nu_T \nabla c^{n-1}\right) \, d\Omega - \int_{\Gamma_{deposit}} \theta \psi \cdot \left(\mathbf{n} \cdot \mathbf{k} u_{sink} c^{n-1}\right) \, d\Gamma + \\
&\qquad \int_{\Gamma_N} \psi \cdot g \, d\Gamma + \int_\Omega \psi s \, d\Omega - S \left(\theta c^{n-1}\right)
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&\int_\Omega \frac{\psi_i \psi_j}{\Delta t} C^n \, d\Omega + \int_\Omega (1-\theta)\, \psi_i \nabla \cdot \left(\mathbf{u}^{n-\theta} \psi_j\right) C^n \, d\Omega - \int_\Omega (1-\theta)\, \nabla \psi_i \cdot (\mathbf{k} u_{sink} \psi_j)\, C^n \, d\Omega + \\
&\int_\Omega (1-\theta)\, \nabla \psi_i \cdot (\nu_T \nabla \psi_j)\, C^n \, d\Omega + \int_{\Gamma_{deposit}} (1-\theta)\, \psi_i \cdot (\mathbf{n} \cdot \mathbf{k} u_{sink} \psi_j)\, C^n \, d\Gamma \\
&+ \int_\Omega (1-\theta) \frac{\nu h}{||\mathbf{u}^{n-\theta}||} \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi_i\right) \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi_j\right) C^n \, d\Omega = \\
&\int_\Omega \frac{\psi_i \psi_j}{\Delta t} C^{n-1} \, d\Omega - \int_\Omega \theta \psi_i \nabla \cdot \left(\mathbf{u}^{n-\theta} \psi_j\right) C^{n-1} \, d\Omega + \int_\Omega \theta \nabla \psi_i \cdot (\mathbf{k} u_{sink} \psi_j)\, C^{n-1} \, d\Omega + \\
&\int_\Omega \theta \nabla \psi_i \cdot (\nu_T \nabla \psi_j)\, C^{n-1} \, d\Omega - \int_{\Gamma_{deposit}} \theta \psi_i \cdot (\mathbf{n} \cdot \mathbf{k} u_{sink} \psi_j)\, C^{n-1} \, d\Gamma + \\
&\int_{\Gamma_N} \psi_i \cdot g \, d\Gamma + \int_\Omega \psi_i s \, d\Omega - \int_\Omega \theta \frac{\nu h}{||\mathbf{u}^{n-\theta}||} \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi_i\right) \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi_j\right) C^{n-1} \, d\Omega
\end{aligned} \tag{31}
$$

$$
M = \psi_i \psi_j \tag{32}
$$

$$
D = \nabla \psi_i \cdot (\nu_T \nabla \psi_j) \tag{33}
$$

$$
A = \psi_i \nabla \cdot \left(\mathbf{u}^{n-\theta} \psi_j\right) \tag{34}
$$

$$
A_{sink} = \nabla \psi_i \cdot (\mathbf{k} u_{sink} \psi_j) \tag{35}
$$

$$
\tilde{A}_{sink} = \psi_i \cdot (\mathbf{n} \cdot \mathbf{k} u_{sink} \psi_j) \tag{36}
$$

$$
S = \frac{\nu h}{||\mathbf{u}^{n-\theta}||} \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi_i\right) \left(\mathbf{u}^{n-\theta} \cdot \nabla \psi_j\right) \tag{37}
$$

$$
\begin{aligned}
&\int_\Omega \left[\frac{M}{\Delta t} + (1-\theta)\, A - (1-\theta)\, A_{sink} + (1-\theta)\, D + (1-\theta)\, S\right] C^n \, d\Omega + \int_{\Gamma_{deposit}} (1-\theta)\, \tilde{A}_{sink} C^n \, d\Gamma = \\
&\int_\Omega \left[\frac{M}{\Delta t} - \theta A + \theta A_{sink} - \theta D - \theta S\right] C^{n-1} \, d\Omega - \int_{\Gamma_{deposit}} \theta \tilde{A}_{sink} C^{n-1} \, d\Gamma + \\
&\int_{\Gamma_N} \psi_i \cdot g \, d\Gamma + \int_\Omega \psi_i s \, d\Omega
\end{aligned} \tag{38}
$$