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A.

1, $2A - B = 2\begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}2\\4\\6\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix}$

$= \begin{bmatrix}-2\\-1\\0\end{bmatrix}$

2, $||A|| = \sqrt{1^2+2^2+3^2} = \sqrt{14} \approx \boxed{3.74}$

$x = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$\vec{A} \cdot \vec{x} = |A| \cdot |x| \cos\theta$

$\rightarrow \cos\theta = \dfrac{\vec{A} \cdot \vec{x}}{|A||x|} = \dfrac{1}{\sqrt{14}\sqrt{1}} = \dfrac{1}{\sqrt{14}}$

$\rightarrow \theta = \arccos\left(\dfrac{1}{\sqrt{14}}\right) = 1.3 \text{ rad} \approx \boxed{74.498^\circ}$

3, Unit vector in direction of A: $\left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right)$

4, Direction cosines of A: $\alpha = \cos a = \dfrac{1}{\sqrt{14}}$

$\beta = \cos b = \dfrac{2}{\sqrt{14}}$

$\gamma = \cos c = \dfrac{3}{\sqrt{14}}$

5, $A \cdot B = 4 + 10 + 18 = 32$

$B \cdot A = 4 + 10 + 18 = 32$

6. $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A||B|} \approx 0.9746$

$\rightarrow \theta = \arccos(0.9746) \approx 0.2257 \text{ rad}$

$\approx \boxed{12.9°}$

7. let $\vec{v} = (x, y, z)$.

$\vec{v}$ is perpendicular to A if $\vec{v}\cdot\vec{A} = x + 2y + 3z = 0$

$\rightarrow$ any vector satisfying $x + 2y + 3z = 0$ is perpendicular to A.

$\rightarrow$ Ex: $(3, 2, -7/3)$.

8. $\cancel{A\times B = \|A\|\,\|B\| \sin(\theta)}$ $\quad A = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $B = \begin{bmatrix}4\\5\\6\end{bmatrix}$

$\cancel{= \sqrt{14}\,\sqrt{77}\,\sin(12.9°)}$

$$A\times B = \begin{bmatrix}2\times6 - 3\times5\\ 3\times4 \quad \cancel{3\times4}\ 6\\ 1\times5 - 4\times2\end{bmatrix} = \begin{bmatrix}-3\\ 6\\ -3\end{bmatrix}$$

$$B\times A = \begin{bmatrix}5\times3 - 6\times2\\ 6 - 3\times4\\ 4\times2 - 5\times1\end{bmatrix} = \begin{bmatrix}3\\ -6\\ 3\end{bmatrix}$$

9. $A\times B$ or $B\times A$ are both perpendicular to A and B

$\rightarrow$ ~~Answ~~ either $\begin{bmatrix}-3\\6\\-3\end{bmatrix}$ or $\begin{bmatrix}3\\-6\\3\end{bmatrix}$ is the answer.

(?)

11, $A^TB = [1, 2\ \ 3]\begin{bmatrix}4\\5\\6\end{bmatrix} = 4+16+18 = 32$

$AB^T = \begin{bmatrix}1\\2\\3\end{bmatrix}[4\ \ 5\ \ 6] = 32.$

B.

1, $2A - B = \begin{bmatrix}2 & 4 & 6\\8 & -4 & 6\\0 & 10 & -2\end{bmatrix} - \begin{bmatrix}1 & 2 & 1\\2 & 1 & -4\\3 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 5\\6 & -5 & 10\\-3 & 12 & -3\end{bmatrix}$

2, $AB = \begin{bmatrix}c_{11} & c_{12} & c_{13}\\c_{21} & c_{22} & c_{23}\\c_{31} & c_{32} & c_{33}\end{bmatrix}$ where $c_{ij} = \sum_{k=1}^{3} a_{ik} b_{kj}$.

$= \begin{bmatrix}14 & -2 & -4\\9 & 0 & 15\\7 & 7 & -21\end{bmatrix}$

$BA = \begin{bmatrix}9 & 3 & 8\\6 & -18 & 13\\-5 & 15 & 2\end{bmatrix}$

3, $(AB)^T = \begin{bmatrix}14 & 9 & 7\\-2 & 0 & 7\\-4 & 15 & -21\end{bmatrix}$

$\cancel{A^T}\ B^TA^T = \begin{bmatrix}1 & 2 & 3\\2 & 1 & -2\\1 & -4 & 1\end{bmatrix}\begin{bmatrix}1 & 4 & 0\\2 & -2 & 5\\3 & 3 & -1\end{bmatrix}$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix} = 1\begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} - 2\begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} + 3\begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix}$$

$$= -13 + 8 + 60$$

$$= 55$$

~~Sim~~ Using similar method, ~~tot~~ $|C| = \cancel{0.6}\,0$

5, Matrix B since dot product between each of its row vectors are $\underline{\underline{0}}$.

$\rightarrow$ form an orthogonal set

6, $$A^{-1} = \begin{bmatrix} -0.236 & 0.309 & 0.218 \\ 0.07 & -0.018 & 0.16 \\ 0.36 & -0.0909 & -0.1818 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0.166 & 0.095 & 0.214 \\ 0.333 & 0.047 & -0.14 \\ 0.166 & -0.19 & 0.07 \end{bmatrix}$$

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C.

1, $\lambda I = \lambda\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix}$

$A - \lambda I = \begin{bmatrix}1 & 2\\3 & 2\end{bmatrix} - \begin{bmatrix}\lambda & 0\\0 & \lambda\end{bmatrix} = \begin{bmatrix}1-\lambda & 2\\3 & 2-\lambda\end{bmatrix}$

$\det\left(\begin{bmatrix}1-\lambda & 2\\3 & 2-\lambda\end{bmatrix}\right) = (1-\lambda)(2-\lambda) - 6$

$= 2 - 3\lambda + \lambda^2 - 6 = \lambda^2 - 3\lambda - 4 = 0$

$\rightarrow \begin{cases}\lambda = -1\\ \lambda = 4\end{cases}$

$\rightarrow$ $\boxed{\text{eigenvalues} = [-1 \quad 4]}$

① $\lambda = -1$

$\rightarrow A - \lambda I = \begin{bmatrix}2 & 2\\3 & 3\end{bmatrix}$

$(A - \lambda I)\bar{x} = \bar{0}$

$\rightarrow \begin{bmatrix}2 & 2\\3 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\rightarrow \left[\begin{array}{cc|c}2 & 2 & 0\\3 & 3 & 0\end{array}\right] \rightarrow$ $\boxed{\text{eigenvector: } [-1, 1]}$

② $\lambda = 4$, using the same method

$\rightarrow [2/3, 1]$

3, $\begin{bmatrix} -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2/3 \\ 1 \end{bmatrix} = \frac{-2}{3} + 1 = \frac{1}{3}$.

4, Using similar method as 1., eigenvectors of B:

$\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

$\rightarrow$ dot prd. $= -1 + 1 = 0$.

5, property: the eigenvectors of B are orthogonal to each other. This is because B is a symmetric matrix (B is the same as B^T)

$\hookrightarrow$ since the eigenvalues are also different, the eigenvectors are orthogonal to each other

D.

1, $f'(x) = 2x$

$f''(x) = 2$.

2, $\frac{\partial g}{\partial x} = 2x$

$\frac{\partial g}{\partial y} = 2y$.

3, $\nabla g(x,y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$.

4, pdf $= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$