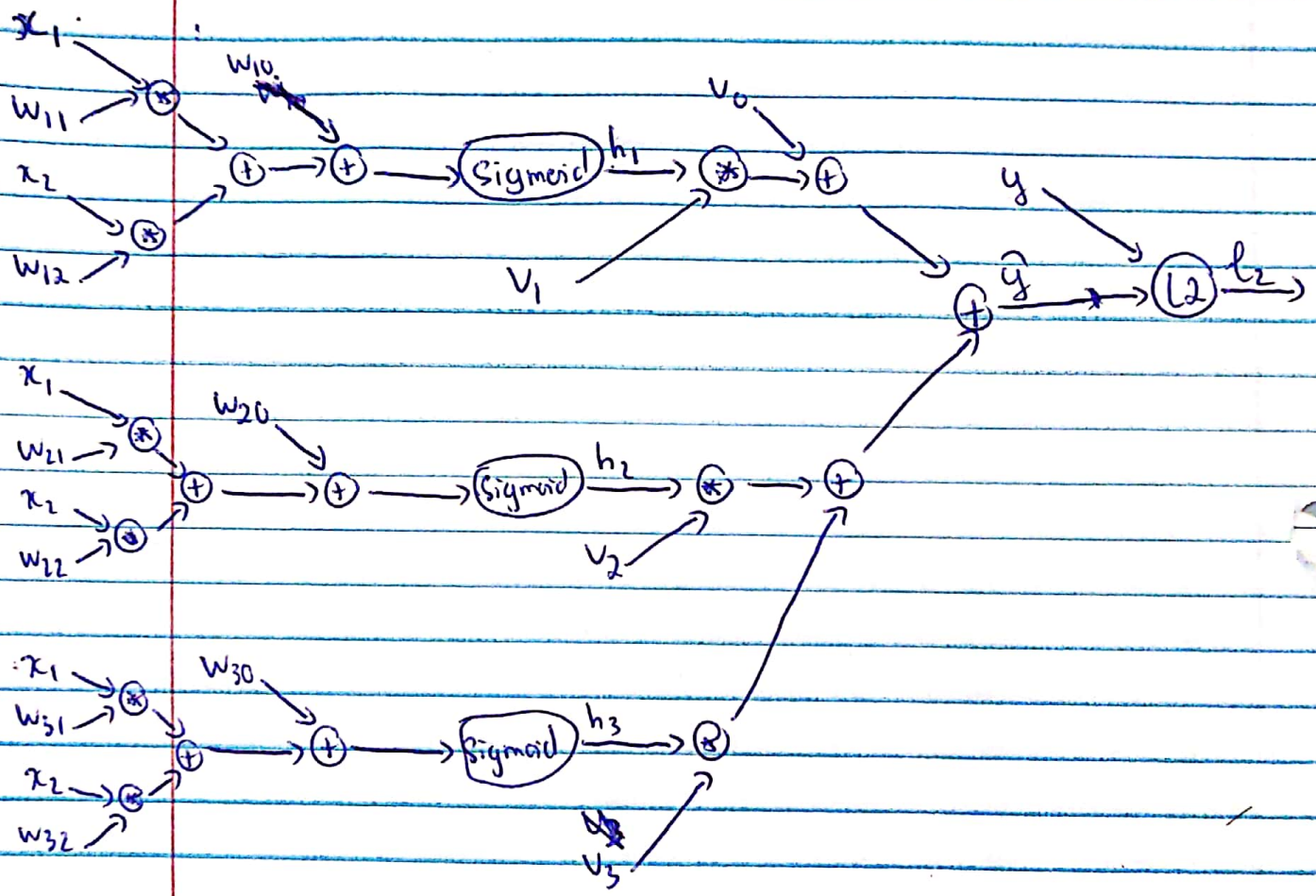
Tuan Tran - A20357888 - CS595 HW#2

1

l_1

a_1



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~~1.~~ b,

+, ~~x~~ x_1, y_1:

$$h_1 = \text{sigmoid}(\vec{w_1} \cdot \vec{x_1})$$

$$= \frac{1}{1 + e^{-0.01 - 0.02 - 0.06}} = 0.5225$$

Similarly:

$$h_2 = \text{sigmoid}(\vec{w_2} \cdot \vec{x_1}) = 0.519989$$

$$h_3 = \text{sigmoid}(\vec{w_3} \cdot \vec{x_1}) = 0.51749$$

$$\hat{y} = \cancel{\vec{h} \cdot \vec{w_2}} = \vec{h} \cdot \vec{v} = 0.01 + 0.02 \times h_1 + 0.03 \times h_2 + 0.04 \times h_3$$

$$= 0.056749$$

$$L_2 = (\hat{y} - y_1)^2 = 63.095$$

+, x_2, y_2: Using similar formula as above:

$$h_1 = \text{sigmoid}(\vec{w_1} \cdot \vec{x_2}) = 0.529964$$

$$h_2 = 0.52498$$

$$h_3 = 0.51999$$

$$\hat{y} = 0.05715$$

$$L_2 = (\hat{y} - y_2)^2 = 119.746.$$

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<!!!!> NOTE: my $L2$ is $= (\hat{y}-y)^2$ instead of $\frac{1}{2}(\hat{y}-y)^2$

→ the answer for part c will be 2 x answer in part d since the formula on the slide is based on $L2 = \frac{1}{2}(\hat{y}-y)^2$

t, x_3, y_3:

$$h_1 = 0.52747$$

$$h_2 = 0.52248$$

$$h_3 = 0.52498$$

$$\hat{y} = 0.05722$$

$$L2 = (\hat{y} - y_3)^2 = 98.8588$$

c, t, x_1, y_1:

$$\frac{\partial L}{\partial \hat{y}} = 2(\hat{y}-y) = -15.8865$$

$$\boxed{\frac{\partial L}{\partial v}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial v} = 2(\hat{y}-y)\,\nabla_v \hat{y}$$

$$= 2(\hat{y}-y)\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

$$= [-15.8865 \quad -8.3005 \quad -8.26 \quad -8.22]$$

$$\frac{\partial L}{\partial h} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h} = 2(\hat{y}-y)\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= [-0.15886 \quad -0.3177 \quad -0.4766 \quad -0.6355]$$

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_1}$$

$$= 2(\hat{y} - y_1) \cdot v_1 h_1 (1 - h_1) x_1$$

$$= [-0.0793 \quad -0.0793 \quad -0.1585]$$

$$\frac{\partial L}{\partial w_2} = 2(\hat{y} - y_1) v_2 h_2 (1 - h_2) x_1 +$$

$$= [-0.11896 \quad -0.11896 \quad -0.23792]$$

$$\frac{\partial L}{\partial w_3} = 2(\hat{y} - y_1) v_3 h_3 (1 - h_3) x_1 .$$

$$= [-0.15867 \quad -0.15867 \quad -0.3173]$$

$+, x_2, y_2$: using similar formula and derivation as above.

$$\frac{\partial L}{\partial v} = [-21.8857 \quad -11.5986 \quad -11.4895 \quad -11.3803]$$

$$\frac{\partial L}{\partial w_1} = [-0.1090 \quad -0.1090 \quad -0.3271]$$

$$\frac{\partial L}{\partial w_2} = [-0.1637 \quad -0.1637 \quad -0.4912]$$

$$\frac{\partial L}{\partial w_3} = [-0.2185 \quad -0.2185 \quad -0.6555]$$

$+, x_3, y_3$: ~~Usin~~

$$\frac{\partial L}{\partial v} = [-19.8856 \quad -10.489 \quad -10.3899 \quad -10.4395]$$

$$\frac{\partial L}{\partial w_1} = [-0.0991 \quad -0.1983 \quad -0.1983]$$

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$$\frac{\partial L}{\partial w_2} = [-0.1488 \quad -0.2977 \quad -0.2977]$$

$$\frac{\partial L}{\partial w_3} = [-0.1984 \quad -0.3967 \quad -0.3967]$$

~~d,~~

~~$\Delta v =$~~

↳ For all examples: (Summing over 3 examples)

$$\frac{\partial L}{\partial v} = [-57.6577 \quad -30.3882 \quad -30.1402 \quad -30.0481]$$

$$\frac{\partial L}{\partial w_1} = [-0.2874 \quad -0.3866 \quad -0.6839]$$

$$\frac{\partial L}{\partial w_2} = [-0.4315 \quad -0.5804 \quad -1.0268]$$

$$\frac{\partial L}{\partial w_3} = [-0.5755 \quad -0.7738 \quad -1.3696]$$

d, $\Delta v = \sum_{i=1}^{m} (\hat{y}^i - y^i) h^i = [-28.829 \quad -15.194 \quad -15.070 \quad -15.0204]$

$$\Delta w_1 = \sum_{i=1}^{3} (\hat{y}^i - y^i) v_1 h_1^i (1 - h_1^i) x^i$$

$$= [-0.1437 \quad -0.1932 \quad -0.34195]$$

$$\Delta w_2 = \sum_{i=1}^{3} (\hat{y}^i - y^i) v_2 h_2^i (1 - h_2^i) x^i$$

$$= [-0.2157 \quad -0.2902 \quad -0.5134]$$

$$\Delta w_3 = \sum_{i=1}^{3} (\hat{y}^i - y^i) v_3 h_3^i (1 - h_3^i) x^i$$

$$= [-0.2877 \quad -0.386985 \quad -0.6848]$$

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→ The answer is the <u>same</u>, as noted, it's just that I use $L2 = (\hat{y}-y)^2 \rightarrow \frac{\partial L}{\partial \hat{y}} = 2(\hat{y}-y)$ where the ~~form~~ formula used $L2 = \frac{1}{2}(\hat{y}-y)^2 \rightarrow \frac{\partial L}{\partial \hat{y}} = (\hat{y}-y)$

→ my answer (part c) is 2 × the answer ~~in part d~~ from the formula (part d).

2,

a, $$\nabla f(x,y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2(2x+3y)2 \\ 2(2x+3y)3 \end{bmatrix}$$

$$= \begin{bmatrix} 8x + 12y \\ 12x + 18y \end{bmatrix}$$

b, $$DF(x,y) = \begin{bmatrix} \frac{\partial(x^2+2y)}{\partial x} & \frac{\partial(x^2+2y)}{\partial y} \\ \frac{\partial(3x+4y^2)}{\partial x} & \frac{\partial(3x+4y^2)}{\partial x} \end{bmatrix}$$

$$= \begin{bmatrix} 2x & 2 \\ 3 & 8y \end{bmatrix} \rightarrow \boxed{DF(1,2) = \begin{bmatrix} 2 & 2 \\ 3 & 16 \end{bmatrix}}$$

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c, ~~Without~~

+, $F(G(x)) = \begin{bmatrix} x^2 + 2x^2 \\ 3x + 4x^4 \end{bmatrix} = \begin{bmatrix} 3x^2 \\ 3x+4x^4 \end{bmatrix}$

$\rightarrow D(F \circ G)(x) = DF(G(x)) = \begin{bmatrix} \frac{\partial(3x^2)}{\partial x} \\ \frac{\partial(3x+4x^4)}{\partial x} \end{bmatrix}$

$= \begin{bmatrix} 6x \\ 3+16x^3 \end{bmatrix}$

$\rightarrow D(F \circ G)(2) = \begin{bmatrix} 12 \\ 131 \end{bmatrix}$

+, $DF(x,y) = \begin{bmatrix} 2x & 2 \\ 3 & 8y \end{bmatrix}$

$\rightarrow DF(G(x)) = \begin{bmatrix} 2x & 2 \\ 3 & 8x^2 \end{bmatrix} \rightarrow DF(G(2)) = \begin{bmatrix} 4 & 2 \\ 3 & 32 \end{bmatrix}$

~~Df~~ $DG(x) = \begin{bmatrix} \frac{\partial(x)}{\partial x} \\ \frac{\partial(x^2)}{\partial x} \end{bmatrix} = \begin{bmatrix} 1 \\ 2x \end{bmatrix} \rightarrow DG(2) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$\rightarrow D(F \circ G)(2) = \begin{bmatrix} 4 & 2 \\ 3 & 32 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 12 \\ 131 \end{bmatrix}$

d,

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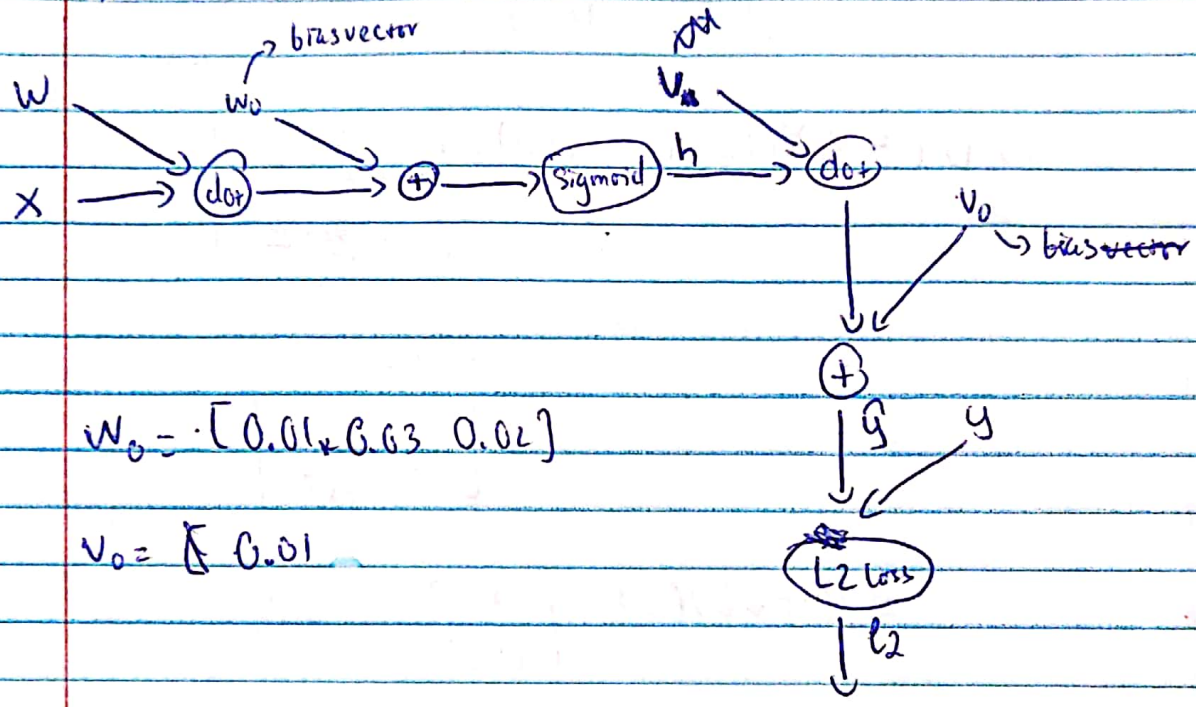

$w_0 = [0.01 \quad 0.03 \quad 0.02]$

$v_0 = 0.01$

$t_1 \quad X_1 = [1 \quad 2]$

$$\cancel{h = \frac{1}{1+e^{-(w_0 + X_0 W)}}}$$

$$X_1 W = [1 \quad 2] \begin{bmatrix} 0.02 & 0.01 & 0.03 \\ 0.03 & 0.02 & 0.01 \end{bmatrix}$$

$$h = \frac{1}{1+e^{-(w_0 + X_1 W)}} = [\cancel{0.52996} \quad \cancel{0.52498} \quad \cancel{0.51999} \quad 0.52248 \quad 0.519989 \quad 0.51749]$$

$$v_0 + h v = v_0 + h \cdot \begin{bmatrix} 0.02 \\ 0.03 \\ 0.04 \end{bmatrix} = \hat{y} = 0.05675$$

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$\cancel{\rightarrow \hat{y} = hv =}$

$$L_2 = (\hat{y} - y)^2 = 63.095.$$

+, $X_2 = [1 \;\; 1 \;\; 3]$, using similar formula:

$$h = \frac{1}{1+e^{-X_2 w}} = [0.52996 \;\; 0.52498 \;\; 0.51999]$$

$$\hat{y} = hv + v_0 = 0.05715; L_2 = 119.746$$

+, $X_3 = [1 \;\; 2 \;\; 2]$

$$h = \frac{1}{1+e^{-X_3 w}} = [0.52747 \;\; 0.52248 \;\; 0.52498]$$

$$\hat{y} = hv + v_0 = 0.05722$$

$$L_2 = (\hat{y} - y_3)^2 = 98.8588.$$

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Back prop: Assume we feed all 3 examples in as a batch.

$$\hat{y} = \begin{bmatrix} \hat{y}^1 \\ \hat{y}^2 \\ \hat{y}^3 \end{bmatrix} \; ; \; y = \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix} \rightarrow \boxed{\text{each row is an instance}}.$$

$$h = \begin{bmatrix} h_1^1 & h_2^1 & h_3^1 \\ h_1^2 & h_2^2 & h_3^2 \\ h_1^3 & h_2^3 & h_3^3 \end{bmatrix} \; ; \; v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\frac{\partial L_2}{\partial \hat{y}} = \begin{bmatrix} \frac{\partial L_2}{\partial \hat{y}^1} \\ \frac{\partial L_2}{\partial \hat{y}^2} \\ \frac{\partial L_2}{\partial \hat{y}^3} \end{bmatrix} = \begin{bmatrix} 2(\hat{y}^1 - y^1) \\ 2(\hat{y}^2 - y^2) \\ 2(\hat{y}^3 - y^3) \end{bmatrix} = \begin{bmatrix} -15.8865 \\ -21.8857 \\ -19.88555 \end{bmatrix}$$

bias

$$\boxed{\frac{\partial L_2}{\partial v_0}} = \sum_{i=1}^{3} \frac{\partial L_2}{\partial \hat{y}^i} = -57.65776.$$

$$\boxed{\frac{\partial L_2}{\partial v}} = \text{Sum}\left(\frac{\partial L_2}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial v}\right) = \cancel{\frac{\partial L_2}{\partial \hat{y}}} \cdot \cancel{h} \; \text{Sum}\left(\frac{\partial L_2}{\partial \hat{y}} \cdot h\right)$$

Sum across 3 examples

$$= \text{Sum}\left(\frac{\partial L_2}{\partial \hat{y}} \begin{bmatrix} h^1 \\ h^2 \\ h^3 \end{bmatrix}\right) = \text{Sum}\left(\frac{\partial L_2}{\partial \hat{y}} \begin{bmatrix} h_1^1 & h_2^1 & h_3^1 \\ h_1^2 & h_2^2 & h_3^2 \\ h_1^3 & h_2^3 & h_3^3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -30.3882 & -30.1402 & -30.04098 \end{bmatrix}$$

$$\frac{\partial L_2}{\partial h} = \frac{\partial L_2}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h} = \frac{\partial L_2}{\partial \hat{y}} \cdot \begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$$

$$= \begin{bmatrix} 2(\hat{y}^1 - y^1)v_1 & \cdots & 2(\hat{y}^1 - y^1)v_3 \\ \vdots & & \\ 2(\hat{y}^3 - y^3)v_1 & \cdots & 2(\hat{y}^3 - y^3)v_3 \end{bmatrix}$$

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Has $\boxed{\frac{\partial L_2}{\partial w_0}} = \text{Sum}\left(\frac{\partial L_2}{\partial h} \cdot \frac{\partial h}{\partial w_0}\right) = \text{Sum}\left(\frac{\partial L_2}{\partial h} \cdot h(1-h)\right)$

$$= [-0.287435 \quad +0.4315 \quad -0.5755].$$

$$\boxed{\frac{\partial L_2}{\partial w}} = \cancel{\frac{\partial L_2}{\partial h} \cdot \frac{\partial h}{\partial w}} \; \text{Sum}\left(\frac{\partial L_2}{\partial h} \cdot \frac{\partial h}{\partial w}\right) \rightarrow \text{sum over 3 examples}$$

$$= \text{Sum}\left(\begin{bmatrix} 2(\hat{y}^1 - y^1)v_1 h_1^1(1-h_1^1)x^1 & & 2(\hat{y}^1 - y^1)v_3 h_3^1(1-h_3^1)x^1 \\ 2(\hat{y}^2 - y^2)v_1 h_1^2(1-h_1^2)x^2 & \cdots & \vdots \\ 2(\hat{y}^3 - y^3)v_1 h_1^3(1-h_1^3)x^3 & & 2(\hat{y}^3 - y^3)v_3 h_3^3(1-h_3^3)x^3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -0.3866 & -0.5804 & -0.7739 \\ -0.6839 & -1.0268 & -1.3696 \end{bmatrix}$$

$\hookrightarrow \frac{\partial L_2}{\partial w_1}$

$\hookrightarrow$ $\boxed{\text{Answers matched ~~part~~ 1c and 1d}}$

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$$z_j = \frac{1}{1+e^{-xw_j}} \rightarrow \frac{\partial z_j}{\partial w_j} = \cancel{z_j(1-}\, z_j(1-z_j)x.$$

3,

$$L = \frac{1}{2}\sum_{i=1}^{m}(\hat{y}^i - y^i)^2 \; ; \; \hat{y} = \frac{1}{1+e^{-zv}} \rightarrow \boxed{\begin{aligned} \frac{\partial \hat{y}}{\partial v} &= \hat{y}(1-\hat{y})\, z \\ \frac{\partial \hat{y}}{\partial z_j} &= \hat{y}(1-\hat{y})\, v_j \end{aligned}}$$

$$\frac{\partial L}{\partial \hat{y}} = \sum_{i=1}^{m}(\hat{y}^i - y^i)$$

$$\frac{\partial L}{\partial v} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial v} = \underbrace{\sum_{i=1}^{m}(\hat{y}^i - y^i)}\,\underbrace{\hat{y}^i(1-\hat{y}^i)}\, z^i$$

$$\frac{\partial L}{\partial z_j} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_j} = \underbrace{\sum_{i=1}^{m}(\hat{y}^i - y^i)}\,\underbrace{\hat{y}^i(1-\hat{y}^i)}\, v_j$$

$$\frac{\partial L}{\partial w_j} = \frac{\partial L}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_j} = \underbrace{\sum_{i=1}^{m}(\hat{y}^i - y^i)\hat{y}^i(1-\hat{y}^i)v_j}\,\underbrace{z_j^i(1-z_j^i)x^i}$$

↳ Update: $v \leftarrow v \; \epsilon_1 \sum_{i=1}^{m}(\hat{y}^i - y^i)\hat{y}^i(1-\hat{y}^i)\, z^i$

$$w_j \leftarrow w_j - \epsilon_1 \sum_{i=1}^{m}(\hat{y} - y^i)\hat{y}^i(1-\hat{y}^i)v_j\, z_j^i(1-z_j^i)x^i.$$

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