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CS 584 - Summer 2020

A,

1, $2A - B = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

2, $\|A\| = \cancel{1+2+3=6}\ \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{1+2^2+3^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$$

3, $\vec{A} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

4, $\alpha_x, \alpha_y, \alpha_z$ = ~~cos~~ direction cosine $y = A$. along x, y, and z axis

$$\alpha_x = \frac{1}{\sqrt{14}}, \alpha_y = \frac{2}{\sqrt{14}}, \alpha_z = \frac{3}{\sqrt{14}}$$

5, $A \cdot B = B \cdot A = 4 + 10 + 18 = 32$

6, Angle between ~~2x~~ A and B.

$$\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|} = \frac{32}{\sqrt{14} \cdot \sqrt{77}}$$

$\alpha \approx 0.2257$ rad $\approx 12.93°$

7, $\vec{A_p} = (x_p, y_p, z_p)$

$\vec{A_p} \perp \vec{A} \rightarrow x_p + 2y_p + 3z_p = 0$

let $x_p = 1$, $y_p = -1$

$\rightarrow z_p = \frac{1}{3} \rightarrow \vec{A_p} = (1, -1, \frac{1}{3})$

8, $A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$

$= -3\hat{i} + 6\hat{j} + -3\hat{k}$

$B \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 3\hat{i} - 6\hat{j} + 3\hat{k}.$

9, $A \times B$ → results in a vector ⊥ to both A and B.

$\rightarrow \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$

11, $A^T B =$

$A B^T =$

B,

1, $2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 8 & -1 \end{bmatrix}$

2, $AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$

$BA = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$

3,

$$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4, $\det A = |A| = \det \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix}$

$$= 1 \cdot \begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} - 2 \begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} + 3 \begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix}$$

$$= -13 + 8 + \cancel{20}\,60 = 55$$

$$\det C = |C| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix}$$

$$= 9 - 12 \cancel{+} -3 = -6.$$

~~Q.6~~

~~6,~~ D,

1, $f'(x) = 2x$ 2, $\frac{\partial g}{\partial x} = 2x$, $\frac{\partial g}{\partial y} = 2y$

$f''(x) = 2.$

3, $\nabla g(x,y) = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

4, pdf $= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

C,

1, $A\vec{v} = \lambda\vec{v}$ $\forall$ non zero $\vec{v}$ if and only if $\det(\lambda I - A) = 0$

$\to$ ~~det~~ $\det(\lambda I - A) = 0$

$\to \det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\right) = 0$

$\to \det\left(\begin{bmatrix} \lambda-1 & -2 \\ -3 & \lambda-2 \end{bmatrix}\right) = 0$

$\to \begin{vmatrix} \lambda-1 & -2 \\ -3 & \lambda-2 \end{vmatrix} = 0$

$\to (\lambda-1)(\lambda-2) - 6 = 0$

$\to \lambda^2 - 3\lambda + 2 - 6 = 0$

$\to \lambda^2 - 3\lambda - 4 = 0$

$\to (\lambda+1)(\lambda-4) = 0$

$\to \begin{cases} \lambda = -1 \\ \lambda = 4 \end{cases}$

$\to$ let $\lambda = -1$

$\to \begin{bmatrix} \lambda-1 & -2 \\ -3 & \lambda-2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix} = C$

$\to C\vec{x} = \vec{0} \to \begin{bmatrix} -2 & -2 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$