Module 1

Excerpts are taken from:
Montgomery, Douglas, C. and George C. Runger. Applied Statistics and Probability for Engineers, Enhanced eText. Available from: WileyPLUS, (7th Edition). Wiley Global Education US, 2018.

Lecture 1

Setup

In[17]:=
```
<< Notation`
```

In[18]:=
```
Symbolize[ _ ]
Symbolize[ __ ]
```

In[20]:=
```
PopulationVariance = ResourceFunction["PopulationVariance"]
```

Out[20]=
```
[■] PopulationVariance +
```

In[21]:=
```
SetOptions[DiscretePlot, PlotStyle → Thickness[.02], Frame → True];
SetOptions[Plot, PlotStyle → Thickness[.02], Frame → True];
```

Win / Loss Example

In[23]:= $P_{win} = \frac{20}{100}$ // N

Out[23]= 0.2

In[24]:= $P_{loss} = \frac{80}{100}$ // N

Out[24]= 0.8

In[25]:= $P_{win} + P_{loss} == 1$

Out[25]= True

Rolling Dice

In[26]:=
```
RandomChoice[{"Heads", "Tails"}]
```
Out[26]=
```
Tails
```

In[27]:=
```
RandomInteger[{1, 6}]
```
Out[27]=
```
1
```

In[28]:=
```
RollDi := RandomInteger[{1, 6}]
```

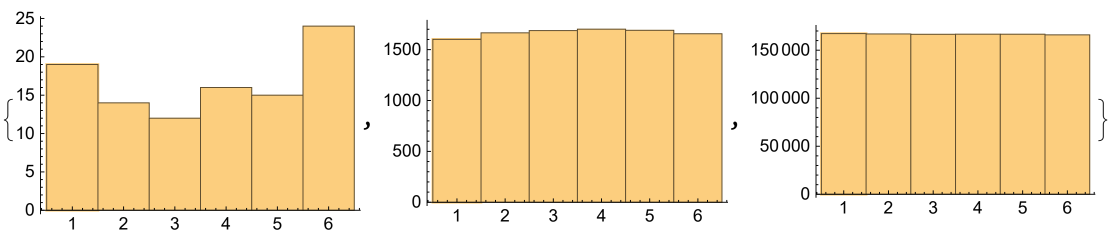
In[29]:=
```
RollDi
```
Out[29]=
```
1
```

In[30]:=
```
RollDice[n_] := RandomInteger[{1, 6}, n]
```

In[31]:=
```
rolls = RollDice[#] & /@ {100, 10000, 1000000};
```

In[32]:=
```
Histogram /@ rolls
```

Out[32]=



In[33]:=
$$p = \frac{1}{6};$$
```
diProbabilities = Association@Table[i → p, {i, 1, 6}]
```
Out[34]=
$$\langle| 1 \to \frac{1}{6}, 2 \to \frac{1}{6}, 3 \to \frac{1}{6}, 4 \to \frac{1}{6}, 5 \to \frac{1}{6}, 6 \to \frac{1}{6} |\rangle$$

In[35]:=
```
Total@Values@diProbabilities == 1
```
Out[35]=
```
True
```

In[36]:=
```
diNumbers = Range[6]
```
Out[36]=
```
{1, 2, 3, 4, 5, 6}
```

In[37]:=
```
diRules = Thread[x == diNumbers]
```
Out[37]=
```
{x == 1, x == 2, x == 3, x == 4, x == 5, x == 6}
```

In[38]:=
$$\text{diProbabilities} = \text{ConstantArray}\left[\frac{1}{6}, 6\right]$$
Out[38]=
$$\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$$

In[39]:=
```
pw = Piecewise[{diProbabilities, diRules}ᵀ]
```

Out[39]=
$$\begin{cases} \frac{1}{6} & x == 1 \,||\, x == 2 \,||\, x == 3 \,||\, x == 4 \,||\, x == 5 \,||\, x == 6 \\ 0 & \text{True} \end{cases}$$

In[40]:=
```
pw /. x → 1
```

Out[40]=
$$\frac{1}{6}$$

In[41]:=
```
p = pw /. x → # & /@ {1, 2, 3}
```

Out[41]=
$$\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$$

In[42]:=
```
Total[p]
```

Out[42]=
$$\frac{1}{2}$$

Lecture 2

Digital Channel (Ex 3.3)

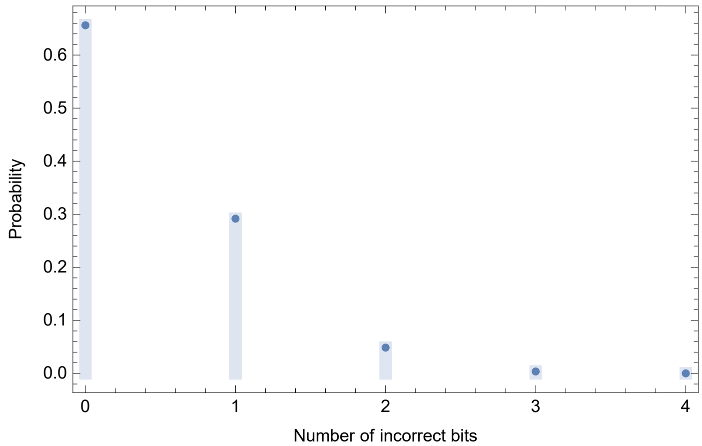
There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are $\{0, 1, 2, 3, 4\}$. Based on a model for the errors that is presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$\begin{aligned} P(X=0) &= 0.6561 \qquad & P(X=1) &= 0.2916 \\ P(X=2) &= 0.0486 \qquad & P(X=3) &= 0.0036 \\ P(X=4) &= 0.0001 \end{aligned}$$

In[43]:=
$$\text{digitalChannel} = \begin{cases} 0.6561 & x == 0 \\ 0.2916 & x == 1 \\ 0.0486 & x == 2 \\ 0.0036 & x == 3 \\ 0.0001 & x == 4 \end{cases};$$

In[44]:=
```
DiscretePlot[digitalChannel, {x, 0, 4},
 FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[44]=



Lecture 3

Digital Channel (Ex 3.5)

In Example 3.3, we might be interested in the probability that three or fewer bits are in error. This question can be expressed as $P(X \leq 3)$.

The event that $\{X \leq 3\}$ is the union of the events $\{X = 0\}$, $\{X = 1\}$, $\{X = 2\}$, and $\{X = 3\}$. Clearly, these three events are mutually exclusive. Therefore,

$$\begin{aligned} P(X \leq 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= 0.6561 + 0.2916 + 0.0486 + 0.0036 = 0.9999 \end{aligned}$$

Ways to access values from Piecewise

In[45]:=
```
values = digitalChannel⟦1, ;; , 1⟧
```

Out[45]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

In[46]:=
```
values = digitalChannel /. x → # & /@ Range[0, 4]
```

Out[46]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

Cumulative Sum

In[47]:=
```
Accumulate@values
```

Out[47]=
```
{0.6561, 0.9477, 0.9963, 0.9999, 1.}
```

P(X ≤ 3)

In[48]:=
```
Total[values[[ ;; 4]]]
```

Out[48]=
```
0.9999
```

Lecture 4

Digital Channel (Ex 3.7)

In Example 3.3, there is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X = 0) = 0.6561 \; P(X = 2) = 0.0486 \; P(X = 4) = 0.0001$$
$$P(X = 1) = 0.2916 \; P(X = 3) = 0.0036$$

Discrete Distribution

In[49]:=
```
dist = ProbabilityDistribution[digitalChannel, {x, 0, 4, 1}]
```

Out[49]=
```
ProbabilityDistribution[ { 0.6561  x == 0
                           0.2916  x == 1
                           0.0486  x == 2
                           0.0036  x == 3
                           0.0001  x == 4
                           0       True     , {x, 0, 4, 1}]
```

Expectation Value (several methods)

In[50]:=
```
x[i_] := i
```

In[51]:=
```
f[i_] := digitalChannel /. x → i
```

In[52]:=
```
μ = 0 f[0] + 1 f[1] + 2 f[2] + 3 f[3] + 4 f[4]
```

Out[52]=
```
0.4
```

In[53]:=
```
μ = Range[0, 4].values
```
Out[53]=
```
0.4
```

The mean of a distribution gives the expectation value.

In[54]:=
```
μ = Mean[dist]
```
Out[54]=
```
0.4
```

Standard Deviation

The variance can be computed manually using a sum.

In[55]:= $V = \sum_{i=0}^{4} f[x[i]]\ (x[i] - \mu)^2$

Out[55]=
```
0.36
```

Note that this is variance of a distribution, which considers weights appropriately.

In[56]:=
```
Variance@dist
```
Out[56]=
```
0.36
```

In[57]:= $\sigma = \sqrt{V}$

Out[57]=
```
0.6
```

In[58]:= $\text{Around}\left[\mu, \sqrt{V}\right]$

Out[58]=
```
0.4 ± 0.6
```

NiCd Battery (3.3.6)

3.3.6 In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states. Assume the following proportions of the states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

In[59]:=
```
battery = { 0.17  x == 0
            0.35  x == 2
            0.33  x == 3
            0.15  x == 4 ;
```

a. Determine the cumulative distribution function of the nickel charge.

In[60]:=
```
dist = ProbabilityDistribution[battery, {x, 0, 4, 1}];
```

In[61]:=
```
cdf = CDF[dist]
```

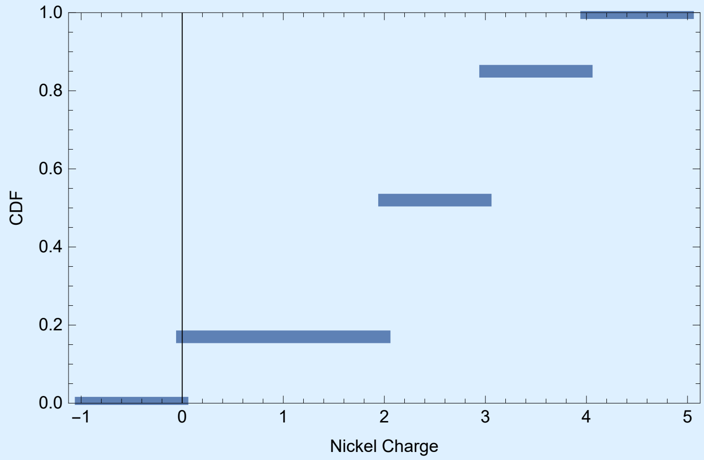
Out[61]=
```
Function[x, { 0.    x < 0
              0.17  0 ≤ x < 2
              0.52  2 ≤ x < 3 , Listable]
              0.85  3 ≤ x < 4
              1.    True
```

In[62]:=
```
cdf[#] & /@ Range[0, 4]
```

Out[62]=
```
{0.17, 0.17, 0.52, 0.85, 1.}
```

```
Plot[cdf[x], {x, -1, 5}, PlotRange → {0, 1}, FrameLabel → {"Nickel Charge", "CDF"}]
```

Out[63]=



b. Determine the mean and variance of the nickel charge.

```
In[64]:= μ = Mean@dist
Out[64]= 2.29

In[65]:= V = Variance@dist;
         σ = √V
Out[66]= 1.23527

In[67]:= charge = Around[μ, σ]
Out[67]= 2.3 ± 1.2
```

Symbolic Mean and Variance

In[68]:=

$$\text{dummyValues} = \begin{cases} p_0 & x == \text{"}a_0\text{"} \\ p_1 & x == \text{"}a_1\text{"} \\ p_2 & x == \text{"}a_2\text{"} \\ p_3 & x == \text{"}a_3\text{"} \\ p_4 & x == \text{"}a_4\text{"} \end{cases};$$

```
In[69]:= x[i_] := i

In[70]:= f[i_] := dummyValues /. x → i
```

f[i] acts as a lookup function for the discrete probability.

In[71]:= $\{f[\text{"}a_0\text{"}], f[\text{"}a_1\text{"}], f[\text{"}a_2\text{"}], f[\text{"}a_3\text{"}], f[\text{"}a_4\text{"}]\}$

Out[71]= $\{p_0, p_1, p_2, p_3, p_4\}$

In[72]:=
```
f[5]
```
Out[72]= 0

In[73]:=
$$V = \sum_{i=0}^{4} f[x[i]]\,(x[i] - \mu)^2$$

Out[73]= 0.

Code Graveyard

Exam Scores

In[74]:=
```
scores = <|"50-60" → 20, "61-80" → 30, "81-100" → 50|>
```
Out[74]= <|50-60 → 20, 61-80 → 30, 81-100 → 50|>

In[75]:=
```
values = Values@scores;
total = Total@values;
values / total // N
```
Out[77]= {0.2, 0.3, 0.5}

In[78]:=
```
Total[values / total] == 1
```
Out[78]= True

Piecewise Function

In[79]:=
$$\text{scores} = \begin{cases} \frac{20}{60-50} & (x \geq 50)\ \&\&\ (x \leq 60) \\ \frac{30}{80-61} & (x \geq 61)\ \&\&\ (x \leq 80) \\ \frac{50}{100-81} & (x \geq 81)\ \&\&\ (x \leq 100) \end{cases};$$

Integration of First Group

In[80]:=
$$\frac{\int_{50}^{60} \text{scores}\, dx}{\int_{0}^{100} \text{scores}\, dx} \text{ // N}$$

Out[80]= 0.2

Integration of All Groups

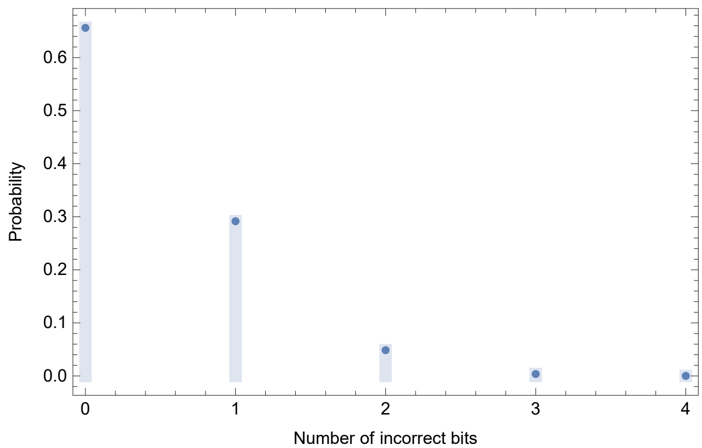
In[81]:=
$$\text{MapThread}\left[\frac{\int_{\#1}^{\#2}\text{scores}\,dx}{\int_{0}^{100}\text{scores}\,dx}\ \&,\ \{\{50, 61, 81\}, \{60, 80, 100\}\}\right]\ //\ N$$

Out[81]= {0.2, 0.3, 0.5}

Probability Distribution

In[82]:=
```
dist = ProbabilityDistribution[digitalChannel, {x, 0, 4, 1}];
pdf = Simplify@PDF[dist, x];
```

In[84]:=
```
DiscretePlot[pdf, {x, 0, 4}, FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[84]=


In[85]:=
$$\text{dummyValues} = \begin{cases} p_0 & x == a_0 \\ p_1 & x == a_1 \\ p_2 & x == a_2 \\ p_3 & x == a_3 \\ p_4 & x == a_4 \end{cases};$$

In[86]:= $\$\text{Assumptions} = a_0 \neq a_1 \neq a_2 \neq a_3 \neq a_4$

Out[86]= $a_0 \neq a_1 \neq a_2 \neq a_3 \neq a_4$

In[87]:=
```
x[i_] := i
```

In[88]:=
```
f[i_] := dummyValues /. x → i
```

f[i] acts as a lookup function for the discrete probability.

In[89]:= $\{f[a_0], f[a_1], f[a_2], f[a_3], f[a_4]\}$ // FullSimplify

Out[89]= $\{p_0, p_1, p_2, p_3, p_4\}$

In[90]:=
```
V = Sum[f[x[i]] (x[i] - μ)^2, {i, 0, 4}]
```
$$V = \sum_{i=0}^{4} f[x[i]]\,(x[i]-\mu)^2$$

Out[90]=

$$5.2441 \left(\begin{cases} p_0 & 0 == a_0 \\ p_1 & 0 == a_1 \\ p_2 & 0 == a_2 \\ p_3 & 0 == a_3 \\ p_4 & 0 == a_4 \\ 0 & \text{True} \end{cases} \right) + 1.6641 \left(\begin{cases} p_0 & 1 == a_0 \\ p_1 & 1 == a_1 \\ p_2 & 1 == a_2 \\ p_3 & 1 == a_3 \\ p_4 & 1 == a_4 \\ 0 & \text{True} \end{cases} \right) +$$

$$0.0841 \left(\begin{cases} p_0 & 2 == a_0 \\ p_1 & 2 == a_1 \\ p_2 & 2 == a_2 \\ p_3 & 2 == a_3 \\ p_4 & 2 == a_4 \\ 0 & \text{True} \end{cases} \right) + 0.5041 \left(\begin{cases} p_0 & 3 == a_0 \\ p_1 & 3 == a_1 \\ p_2 & 3 == a_2 \\ p_3 & 3 == a_3 \\ p_4 & 3 == a_4 \\ 0 & \text{True} \end{cases} \right) + 2.9241 \left(\begin{cases} p_0 & 4 == a_0 \\ p_1 & 4 == a_1 \\ p_2 & 4 == a_2 \\ p_3 & 4 == a_3 \\ p_4 & 4 == a_4 \\ 0 & \text{True} \end{cases} \right)$$

Print Notebook

Assumes that Mathematica notebook ends with .nb extension. Make sure the .pdf file is not open on the computer.

In[91]:=
```
Export[StringDrop[NotebookFileName[], -2] <> "pdf", EvaluationNotebook[]]
```

Out[◦]=
```
C:\Users\sterg\Documents\GitHub\sparks-baird\mete-3070\mathematica\module-1.pdf
```