Module 1

Excerpts taken from:
[1] Montgomery, Douglas, C. and George C. Runger. Applied Statistics and Probability for Engineers, Enhanced eText. Available from: WileyPLUS, (7th Edition). Wiley Global Education US, 2018.

Lecture 1

Setup

In[ ]:=
```
<< Notation`
```

In[ ]:=
```
Symbolize[ _ ]
Symbolize[ __ ]
```

In[ ]:=
```
PopulationVariance = ResourceFunction["PopulationVariance"]
```

Out[ ]= PopulationVariance

In[ ]:=
```
SetOptions[DiscretePlot, PlotStyle → Thickness[.02], Frame → True];
SetOptions[Plot, PlotStyle → Thickness[.02], Frame → True];
```

Win / Loss Example

In[ ]:= $P_{win} = \frac{20}{100}$ // N

Out[ ]= 0.2

In[ ]:= $P_{loss} = \frac{80}{100}$ // N

Out[ ]= 0.8

In[ ]:= $P_{win} + P_{loss} == 1$

Out[ ]= True

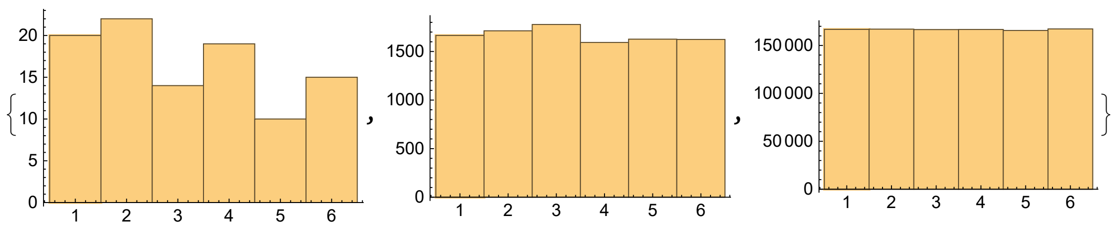
Rolling Dice

In[∘]:=
```
RandomChoice[{"Heads", "Tails"}]
```
Out[∘]= Heads

In[∘]:=
```
RandomInteger[{1, 6}]
```
Out[∘]= 5

In[∘]:=
```
RollDi := RandomInteger[{1, 6}]
```
In[∘]:=
```
RollDi
```
Out[∘]= 4

In[∘]:=
```
RollDice[n_] := RandomInteger[{1, 6}, n]
```
In[∘]:=
```
rolls = RollDice[#] & /@ {100, 10 000, 1 000 000};
```
In[∘]:=
```
Histogram /@ rolls
```
Out[∘]=



In[∘]:=
```
p = 1/6;
diProbabilities = Association@Table[i → p, {i, 1, 6}]
```
Out[∘]= $\langle|1 \to \frac{1}{6}, 2 \to \frac{1}{6}, 3 \to \frac{1}{6}, 4 \to \frac{1}{6}, 5 \to \frac{1}{6}, 6 \to \frac{1}{6}|\rangle$

In[∘]:=
```
Total@Values@diProbabilities == 1
```
Out[∘]= True

In[∘]:=
```
diNumbers = Range[6]
```
Out[∘]= {1, 2, 3, 4, 5, 6}

In[∘]:=
```
diRules = Thread[x == diNumbers]
```
Out[∘]= {x == 1, x == 2, x == 3, x == 4, x == 5, x == 6}

In[∘]:=
```
diProbabilities = ConstantArray[1/6, 6]
```
Out[∘]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

```
In[•]:= pw = Piecewise[{diProbabilities, diRules}ᵀ]
```

$$\text{Out[•]=} \begin{cases} \frac{1}{6} & x == 1 \,||\, x == 2 \,||\, x == 3 \,||\, x == 4 \,||\, x == 5 \,||\, x == 6 \\ 0 & \text{True} \end{cases}$$

```
pw /. x → 1
```

$$\text{Out[•]=} \frac{1}{6}$$

```
In[•]:= p = pw /. x → # & /@ {1, 2, 3}
```

$$\text{Out[•]=} \left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$$

```
In[•]:= Total[p]
```

$$\text{Out[•]=} \frac{1}{2}$$

Lecture 2

Digital Channel (Ex 3.3)

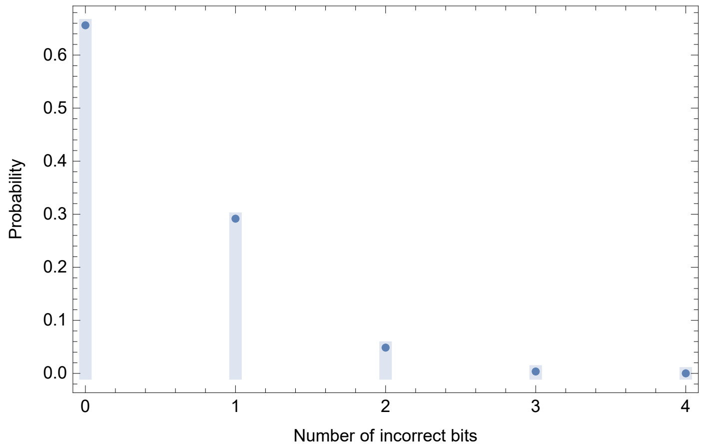
There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are $\{0, 1, 2, 3, 4\}$. Based on a model for the errors that is presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X=0)=0.6561 \qquad P(X=1)=0.2916$$
$$P(X=2)=0.0486 \qquad P(X=3)=0.0036$$
$$P(X=4)=0.0001$$

$$\text{In[10]:=} \; \texttt{digitalChannel} = \begin{cases} 0.6561 & x == 0 \\ 0.2916 & x == 1 \\ 0.0486 & x == 2 \\ 0.0036 & x == 3 \\ 0.0001 & x == 4 \end{cases};$$

```
In[•]:= DiscretePlot[digitalChannel, {x, 0, 4},
         FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[•]=



Lecture 3

Digital Channel (Ex 3.5)

In Example 3.3, we might be interested in the probability that three or fewer bits are in error. This question can be expressed as $P(X \leq 3)$.

The event that $\{X \leq 3\}$ is the union of the events $\{X = 0\}$, $\{X = 1\}$, $\{X = 2\}$, and $\{X = 3\}$. Clearly, these three events are mutually exclusive. Therefore,

$$
\begin{aligned}
P(X \leq 3) &= P(X=0)+P(X=1)+P(X=2)+P(X=3) \\
&= 0.6561+0.2916+0.0486+0.0036=0.9999
\end{aligned}
$$

Ways to access values from Piecewise

```
In[•]:= values = digitalChannel⟦1, ;; , 1⟧
Out[•]= {0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

```
In[•]:= values = digitalChannel /. x → # & /@ Range[0, 4]
Out[•]= {0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

Cumulative Sum

```
In[•]:= Accumulate@values
Out[•]= {0.6561, 0.9477, 0.9963, 0.9999, 1.}
```

Lecture 4

Digital Channel (Ex 3.7)

In Example 3.3, there is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X=0)=0.6561\ P(X=2)=0.0486\ P(X=4)=0.0001$$
$$P(X=1)=0.2916\ P(X=3)=0.0036$$

Expectation Value (several methods)

In[∘]:=
```
x[i_] := i
```

In[∘]:=
```
f[i_] := digitalChannel /. x → i
```

In[∘]:=
```
0 f[0] + 1 f[1] + 2 f[2] + 3 f[3] + 4 f[4]
```

Out[∘]=
```
0.4
```

In[∘]:=
```
Range[0, 4].values
```

Out[∘]=
```
0.4
```

The mean of a distribution gives the expectation value.

In[∘]:=
```
μ = Mean[dist]
```

Out[∘]=
```
0.4
```

Standard Deviation

The variance can be computed manually using a sum.

In[∘]:= $V = \sum_{i=0}^{4} f[x[i]]\ (x[i] - \mu)^2$

Out[∘]=
```
0.36
```

Note that this is variance of a distribution, which considers weights appropriately.

In[•]:=
```
Variance@dist
```
Out[•]= 0.36

In[•]:=
$$\sigma = \sqrt{V}$$
Out[•]= 0.6

In[•]:=
$$\text{Around}\left[\mu, \sqrt{V}\right]$$
Out[•]= 0.4 ± 0.6

NiCd Battery (3.3.6)

3.3.6 In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states. Assume the following proportions of the states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

In[•]:=
$$\text{battery} = \begin{cases} 0.17 & x == 0 \\ 0.35 & x == 2 \\ 0.33 & x == 3 \\ 0.15 & x == 4 \end{cases};$$

a. Determine the cumulative distribution function of the nickel charge.

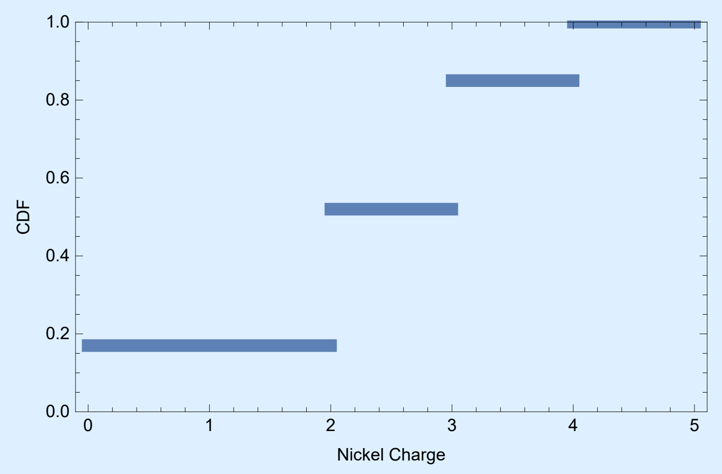
In[•]:=
```
dist = ProbabilityDistribution[battery, {x, 0, 4, 1}];
```

In[•]:=
```
cdf = CDF[dist]
```
Out[•]=
$$\text{Function}\left[\{x\}, \begin{cases} 0. & x < 0 \\ 0.17 & 0 \le x < 2 \\ 0.52 & 2 \le x < 3 \\ 0.85 & 3 \le x < 4 \\ 1. & \text{True} \end{cases}, \text{Listable}\right]$$

In[•]:=
```
cdf[#] & /@ Range[0, 4]
```
Out[•]= {0.17, 0.17, 0.52, 0.85, 1.}

In[∘]:=
```
Plot[cdf[x], {x, 0, 5}, PlotRange → {0, 1}, FrameLabel → {"Nickel Charge", "CDF"}]
```

Out[∘]=



b. Determine the mean and variance of the nickel charge.

In[∘]:=
```
μ = Mean@dist
```

Out[∘]=
```
2.29
```

In[∘]:=
```
V = Variance@dist;
σ = √V
```

Out[∘]=
```
1.23527
```

In[∘]:=
```
charge = Around[μ, σ]
```

Out[∘]=
```
2.3 ±1.2
```

Code Graveyard

Print Notebook

Assumes that Mathematica notebook ends with .nb extension. Make sure the .pdf file is not open on the computer.

In[14]:=
```
Export[StringDrop[NotebookFileName[], -2] <> "pdf", EvaluationNotebook[]]
```

Out[13]=
```
C:\Users\sterg\Documents\GitHub\sparks-baird\mete-3070\mathematica\module-1.pdf
```