Module 1

Lecture 1

Setup

In[∘]:=
```
<< Notation`
```

In[∘]:=
```
Symbolize[ _ ]
Symbolize[ __ ]
```

In[∘]:=
```
PopulationVariance = ResourceFunction["PopulationVariance"]
```

Out[∘]=
```
[•] PopulationVariance +
```

In[∘]:=
```
SetOptions[DiscretePlot, PlotStyle → Thickness[.02], Frame → True];
SetOptions[Plot, PlotStyle → Thickness[.02], Frame → True];
```

Win / Loss Example

In[∘]:= $P_{win} = \frac{20}{100}$ // N

Out[∘]= 0.2

In[∘]:= $P_{loss} = \frac{80}{100}$ // N

Out[∘]= 0.8

In[∘]:= $P_{win} + P_{loss}$ == 1

Out[∘]= True

Rolling Dice

In[∘]:=
```
RandomChoice[{"Heads", "Tails"}]
```

Out[∘]= Heads

In[ ]:=
```
RandomInteger[{1, 6}]
```
Out[ ]= 5

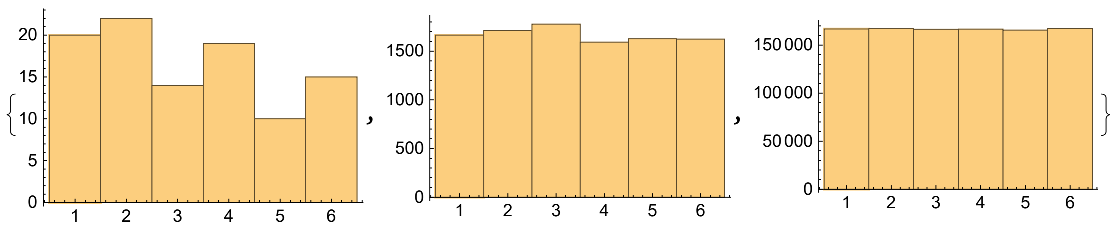
In[ ]:=
```
RollDi := RandomInteger[{1, 6}]
```

In[ ]:=
```
RollDi
```
Out[ ]= 4

In[ ]:=
```
RollDice[n_] := RandomInteger[{1, 6}, n]
```

In[ ]:=
```
rolls = RollDice[#] & /@ {100, 10000, 1000000};
```

In[ ]:=
```
Histogram /@ rolls
```

Out[ ]=


In[ ]:=
```
p = 1/6;
diProbabilities = Association@Table[i → p, {i, 1, 6}]
```
Out[ ]= $\langle| 1 \to \frac{1}{6}, 2 \to \frac{1}{6}, 3 \to \frac{1}{6}, 4 \to \frac{1}{6}, 5 \to \frac{1}{6}, 6 \to \frac{1}{6} |\rangle$

In[ ]:=
```
Total@Values@diProbabilities == 1
```
Out[ ]= True

In[ ]:=
```
diNumbers = Range[6]
```
Out[ ]= {1, 2, 3, 4, 5, 6}

In[ ]:=
```
diRules = Thread[x == diNumbers]
```
Out[ ]= {x == 1, x == 2, x == 3, x == 4, x == 5, x == 6}

In[ ]:=
```
diProbabilities = ConstantArray[1/6, 6]
```
Out[ ]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

In[ ]:=
```
pw = Piecewise[{diProbabilities, diRules}ᵀ]
```
Out[ ]= $\begin{cases} \frac{1}{6} & x == 1 \,||\, x == 2 \,||\, x == 3 \,||\, x == 4 \,||\, x == 5 \,||\, x == 6 \\ 0 & \text{True} \end{cases}$

```
pw /. x → 1
```

Out[∘]= $\frac{1}{6}$

```
In[∘]:= p = pw /. x → # & /@ {1, 2, 3}
```

Out[∘]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

```
In[∘]:= Total[p]
```

Out[∘]= $\frac{1}{2}$

Lecture 2

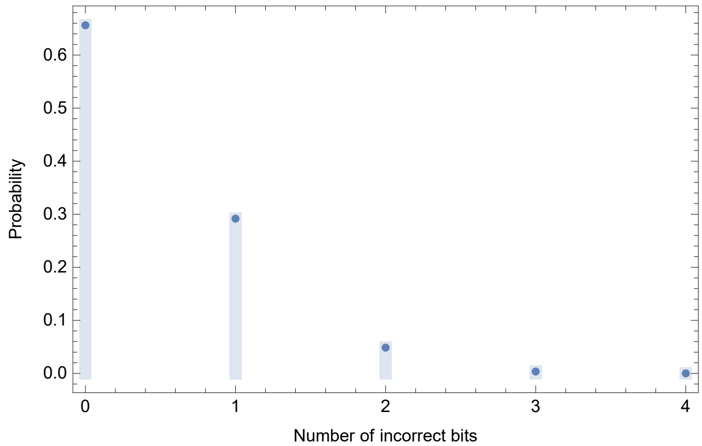
Digital Channel (Ex 3.3)

There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are $\{0, 1, 2, 3, 4\}$. Based on a model for the errors that is presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X=0)=0.6561 \qquad P(X=1)=0.2916$$
$$P(X=2)=0.0486 \qquad P(X=3)=0.0036$$
$$P(X=4)=0.0001$$

```
In[10]:= digitalChannel = {
  0.6561  x == 0
  0.2916  x == 1
  0.0486  x == 2 ;
  0.0036  x == 3
  0.0001  x == 4
```

In[ ]:=
```
DiscretePlot[digitalChannel, {x, 0, 4},
 FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[ ]=


Lecture 3

Digital Channel (Ex 3.5)

In Example 3.3, we might be interested in the probability that three or fewer bits are in error. This question can be expressed as $P(X \leq 3)$.

The event that $\{X \leq 3\}$ is the union of the events $\{X = 0\}$, $\{X = 1\}$, $\{X = 2\}$, and $\{X = 3\}$. Clearly, these three events are mutually exclusive. Therefore,

$$\begin{aligned} P(X \leq 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= 0.6561 + 0.2916 + 0.0486 + 0.0036 = 0.9999 \end{aligned}$$

Ways to access values from Piecewise

In[ ]:=
```
values = digitalChannel[[1, ;; , 1]]
```

Out[ ]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

In[ ]:=
```
values = digitalChannel /. x → # & /@ Range[0, 4]
```

Out[ ]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

Cumulative Sum

In[ ]:=
```
Accumulate@values
```

Out[ ]=
```
{0.6561, 0.9477, 0.9963, 0.9999, 1.}
```

Lecture 4

Digital Channel (Ex 3.7)

In Example 3.3, there is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X=0)=0.6561 \; P(X=2)=0.0486 \; P(X=4)=0.0001$$
$$P(X=1)=0.2916 \; P(X=3)=0.0036$$

Expectation Value (several methods)

```
In[•]:= x[i_] := i
```

```
In[•]:= f[i_] := digitalChannel /. x → i
```

```
In[•]:= 0 f[0] + 1 f[1] + 2 f[2] + 3 f[3] + 4 f[4]
Out[•]= 0.4
```

```
In[•]:= Range[0, 4].values
Out[•]= 0.4
```

The mean of a distribution gives the expectation value.

```
In[•]:= μ = Mean[dist]
Out[•]= 0.4
```

Standard Deviation

The variance can be computed manually using a sum.

In[•]:= $V = \sum_{i=0}^{4} f[x[i]]\,(x[i] - \mu)^2$

Out[•]= 0.36

Note that this is variance of a distribution, which considers weights appropriately.

In[∘]:=
```
Variance@dist
```
Out[∘]= 0.36

In[∘]:=
$$\sigma = \sqrt{V}$$
Out[∘]= 0.6

In[∘]:=
$$\text{Around}\left[\mu, \sqrt{V}\right]$$
Out[∘]= 0.4 ± 0.6

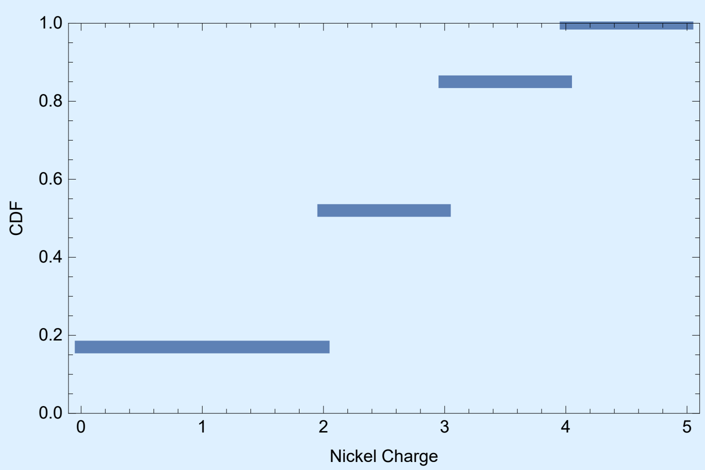
NiCd Battery (3.3.6)

In[∘]:=
$$\text{battery} = \begin{cases} 0.17 & x == 0 \\ 0.35 & x == 2 \\ 0.33 & x == 3 \\ 0.15 & x == 4 \end{cases};$$

In[∘]:=
```
dist = ProbabilityDistribution[battery, {x, 0, 4, 1}];
```

In[∘]:=
```
cdf = CDF[dist]
```
Out[∘]=
$$\text{Function}\left[\{x\}, \begin{cases} 0. & x < 0 \\ 0.17 & 0 \le x < 2 \\ 0.52 & 2 \le x < 3 \\ 0.85 & 3 \le x < 4 \\ 1. & \text{True} \end{cases}, \text{Listable}\right]$$

In[∘]:=
```
cdf[#] & /@ Range[0, 4]
```
Out[∘]= {0.17, 0.17, 0.52, 0.85, 1.}

In[∘]:=
```
Plot[cdf[x], {x, 0, 5}, PlotRange → {0, 1}, FrameLabel → {"Nickel Charge", "CDF"}]
```
Out[∘]=


In[ ]:=
```
μ = Mean@dist
```
Out[ ]=
```
2.29
```

In[ ]:=
```
V = Variance@dist;
σ = √V
```
Out[ ]=
```
1.23527
```

In[ ]:=
```
charge = Around[μ, σ]
```
Out[ ]=
```
2.3 ± 1.2
```

Code Graveyard

Exam Scores

In[ ]:=
```
scores = <|"50-60" → 20, "61-80" → 30, "81-100" → 50|>
```
Out[ ]=
```
<|50-60 → 20, 61-80 → 30, 81-100 → 50|>
```

In[ ]:=
```
values = Values@scores;
total = Total@values;
values / total // N
```
Out[ ]=
```
{0.2, 0.3, 0.5}
```

In[ ]:=
```
Total[values / total] == 1
```
Out[ ]=
```
True
```

Piecewise Function

In[ ]:=

$$\text{scores} = \begin{cases} \frac{20}{60-50} & (x \geq 50) \,\&\&\, (x \leq 60) \\ \frac{30}{80-61} & (x \geq 61) \,\&\&\, (x \leq 80) \\ \frac{50}{100-81} & (x \geq 81) \,\&\&\, (x \leq 100) \end{cases};$$

Integration of First Group

Out[ ]=
```
0.2
```

Integration of All Groups

In[∘]:=

```
MapThread[(∫_#1^#2 scores dx)/(∫_0^100 scores dx) &, {{50, 61, 81}, {60, 80, 100}}] // N
```

Probability Distribution

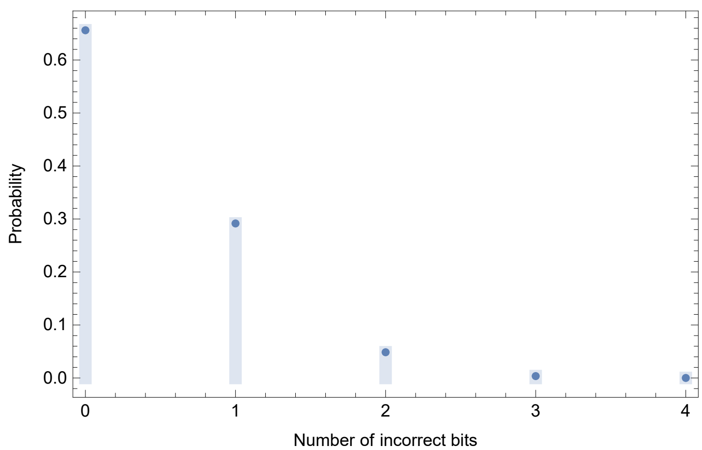
In[∘]:=

```
dist = ProbabilityDistribution[digitalChannel, {x, 0, 4, 1}];
pdf = Simplify@PDF[dist, x];
```

In[∘]:=

```
DiscretePlot[pdf, {x, 0, 4}, FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[∘]=



Print Notebook

Assumes that Mathematica notebook ends with .nb extension

In[13]:=

```
Export[StringDrop[NotebookFileName[], -2] <> "pdf", EvaluationNotebook[]]
```

Export: Cannot open C:\Users\sterg\Documents\GitHub\sparks-baird\mete-3070\mathematica\module-1.pdf.

Out[12]= $Failed