Module 1

Excerpts are taken from:
Montgomery, Douglas, C. and George C. Runger. Applied Statistics and Probability for Engineers, Enhanced eText. Available from: WileyPLUS, (7th Edition). Wiley Global Education US, 2018.

Lecture 1

Setup

In[108]:=
```
<< Notation`
```

In[109]:=
```
Symbolize[ □_□ ]
Symbolize[ □_□_□ ]
```

In[111]:=
```
PopulationVariance = ResourceFunction["PopulationVariance"]
```

Out[111]= [•] PopulationVariance +

In[112]:=
```
SetOptions[DiscretePlot, PlotStyle → Thickness[.02], Frame → True];
SetOptions[Plot, PlotStyle → Thickness[.02], Frame → True];
```

Win / Loss Example

In[114]:= $P_{win} = \frac{20}{100}$ // N

Out[114]= 0.2

In[115]:= $P_{loss} = \frac{80}{100}$ // N

Out[115]= 0.8

In[116]:= $P_{win} + P_{loss}$ == 1

Out[116]= True

Rolling Dice

In[117]:= `RandomChoice[{"Heads", "Tails"}]`

Out[117]= `Tails`

In[118]:= `RandomInteger[{1, 6}]`

Out[118]= `1`

In[119]:= `RollDi := RandomInteger[{1, 6}]`

In[120]:= `RollDi`

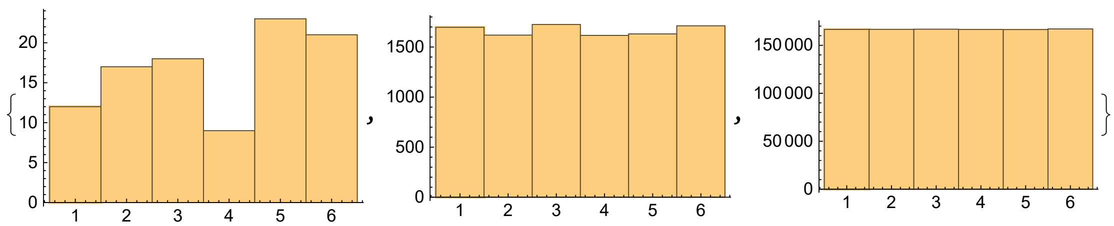
Out[120]= `1`

In[121]:= `RollDice[n_] := RandomInteger[{1, 6}, n]`

In[122]:= `rolls = RollDice[#] & /@ {100, 10000, 1000000};`

In[123]:= `Histogram /@ rolls`

Out[123]=



In[124]:= $p = \frac{1}{6};$

`diProbabilities = Association@Table[i → p, {i, 1, 6}]`

Out[125]= $\langle| 1 \to \frac{1}{6}, 2 \to \frac{1}{6}, 3 \to \frac{1}{6}, 4 \to \frac{1}{6}, 5 \to \frac{1}{6}, 6 \to \frac{1}{6} |\rangle$

In[126]:= `Total@Values@diProbabilities == 1`

Out[126]= `True`

In[127]:= `diNumbers = Range[6]`

Out[127]= `{1, 2, 3, 4, 5, 6}`

In[128]:= `diRules = Thread[x == diNumbers]`

Out[128]= `{x == 1, x == 2, x == 3, x == 4, x == 5, x == 6}`

In[129]:= $\texttt{diProbabilities = ConstantArray}\left[\frac{1}{6}, 6\right]$

Out[129]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

In[130]:=
```
pw = Piecewise[{diProbabilities, diRules}ᵀ]
```

Out[130]=
$$\begin{cases} \frac{1}{6} & x == 1 \,||\, x == 2 \,||\, x == 3 \,||\, x == 4 \,||\, x == 5 \,||\, x == 6 \\ 0 & \text{True} \end{cases}$$

In[131]:=
```
pw /. x → 1
```

Out[131]= $\frac{1}{6}$

In[132]:=
```
p = pw /. x → # & /@ {1, 2, 3}
```

Out[132]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

In[133]:=
```
Total[p]
```

Out[133]= $\frac{1}{2}$

Lecture 2

Digital Channel (Ex 3.3)

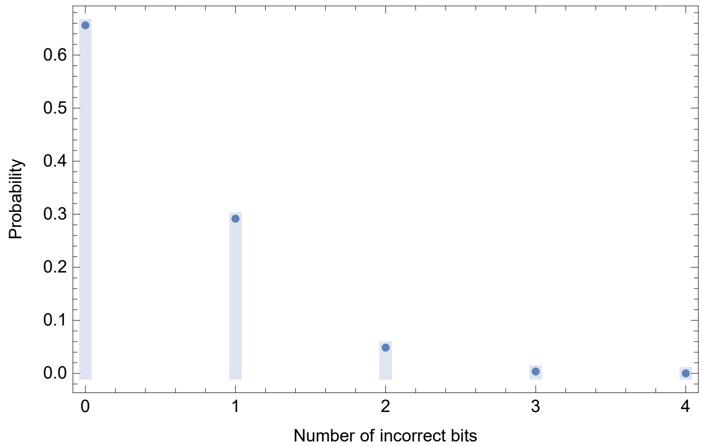
There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors that is presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$\begin{array}{ll} P(X=0)=0.6561 & P(X=1)=0.2916 \\ P(X=2)=0.0486 & P(X=3)=0.0036 \\ P(X=4)=0.0001 & \end{array}$$

In[134]:=
$$\text{digitalChannel} = \begin{cases} 0.6561 & x == 0 \\ 0.2916 & x == 1 \\ 0.0486 & x == 2 \\ 0.0036 & x == 3 \\ 0.0001 & x == 4 \end{cases};$$

In[135]:=
```
DiscretePlot[digitalChannel, {x, 0, 4},
 FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[135]=



Lecture 3

Digital Channel (Ex 3.5)

In Example 3.3, we might be interested in the probability that three or fewer bits are in error. This question can be expressed as $P(X \le 3)$.

The event that $\{X \le 3\}$ is the union of the events $\{X = 0\}$, $\{X = 1\}$, $\{X = 2\}$, and $\{X = 3\}$. Clearly, these three events are mutually exclusive. Therefore,

$$\begin{aligned} P(X \le 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= 0.6561 + 0.2916 + 0.0486 + 0.0036 = 0.9999 \end{aligned}$$

Ways to access values from Piecewise

In[136]:=
```
values = digitalChannel[[1, ;; , 1]]
```

Out[136]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

In[137]:=
```
values = digitalChannel /. x → # & /@ Range[0, 4]
```

Out[137]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

Cumulative Sum

In[138]:=
```
Accumulate@values
```

Out[138]=
```
{0.6561, 0.9477, 0.9963, 0.9999, 1.}
```

P(X ≤ 3)

In[139]:=
```
Total[values[[ ;; 4]]]
```

Out[139]=
```
0.9999
```

Lecture 4

Digital Channel (Ex 3.7)

In Example 3.3, there is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X=0)=0.6561 \; P(X=2)=0.0486 \; P(X=4)=0.0001$$
$$P(X=1)=0.2916 \; P(X=3)=0.0036$$

Discrete Distribution

In[140]:=
```
dist = ProbabilityDistribution[digitalChannel, {x, 0, 4, 1}]
```

Out[140]=
```
ProbabilityDistribution[ { 0.6561  x == 0
                           0.2916  x == 1
                           0.0486  x == 2
                           0.0036  x == 3
                           0.0001  x == 4
                           0       True   , {x, 0, 4, 1}]
```

Expectation Value (several methods)

In[141]:=
```
x[i_] := i
```

In[142]:=
```
f[i_] := digitalChannel /. x → i
```

In[143]:=
```
μ = 0 f[0] + 1 f[1] + 2 f[2] + 3 f[3] + 4 f[4]
```

Out[143]=
```
0.4
```

In[144]:=
```
μ = Range[0, 4].values
```

Out[144]=
```
0.4
```

The mean of a distribution gives the expectation value.

In[145]:=
```
μ = Mean[dist]
```

Out[145]=
```
0.4
```

Standard Deviation

The variance can be computed manually using a sum.

In[146]:= $V = \sum_{i=0}^{4} f[x[i]]\ (x[i] - \mu)^2$

Out[146]= 0.36

Note that this is variance of a distribution, which considers weights appropriately.

In[147]:=
```
Variance@dist
```

Out[147]= 0.36

In[148]:= $\sigma = \sqrt{V}$

Out[148]= 0.6

In[149]:= $\text{Around}\left[\mu, \sqrt{V}\right]$

Out[149]=
```
0.4 ± 0.6
```

NiCd Battery (3.3.6)

3.3.6 In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states. Assume the following proportions of the states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

In[150]:=
```
battery = ⎧ 0.17  x == 0
          ⎨ 0.35  x == 2
          ⎨ 0.33  x == 3 ;
          ⎩ 0.15  x == 4
```

a. Determine the cumulative distribution function of the nickel charge.

In[151]:=
```
dist = ProbabilityDistribution[battery, {x, 0, 4, 1}];
```

In[152]:=
```
cdf = CDF[dist]
```

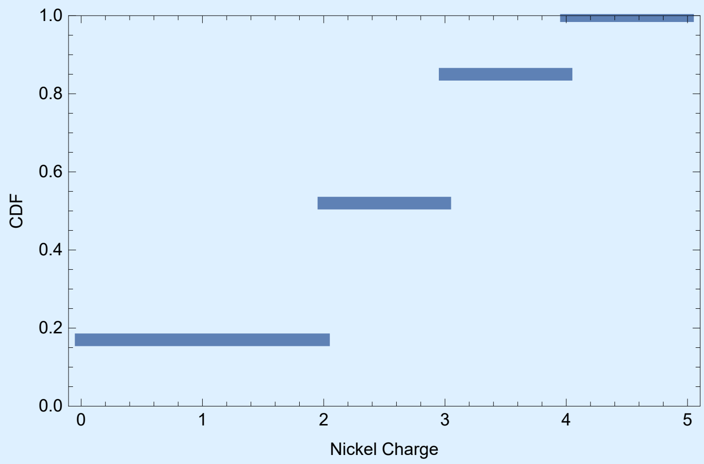
Out[152]=
```
Function[x, ⎧ 0.    x < 0
            ⎨ 0.17  0 ≤ x < 2
            ⎨ 0.52  2 ≤ x < 3 , Listable]
            ⎨ 0.85  3 ≤ x < 4
            ⎩ 1.    True
```

In[153]:=
```
cdf[#] & /@ Range[0, 4]
```

Out[153]=
```
{0.17, 0.17, 0.52, 0.85, 1.}
```

In[154]:=
```
Plot[cdf[x], {x, 0, 5}, PlotRange → {0, 1}, FrameLabel → {"Nickel Charge", "CDF"}]
```

Out[154]=



b. Determine the mean and variance of the nickel charge.

In[155]:=
```
μ = Mean@dist
```

Out[155]= 2.29

In[156]:=
```
V = Variance@dist;
σ = √V
```

Out[157]= 1.23527

In[158]:=
```
charge = Around[μ, σ]
```

Out[158]= 2.3 ± 1.2

Symbolic Mean and Variance

In[159]:=
$$\text{dummyValues} = \begin{cases} p_0 & x == \text{"}a_0\text{"} \\ p_1 & x == \text{"}a_1\text{"} \\ p_2 & x == \text{"}a_2\text{"} \\ p_3 & x == \text{"}a_3\text{"} \\ p_4 & x == \text{"}a_4\text{"} \end{cases};$$

Symbolize: Warning: The box structure attempting to be symbolized has a similar or identical symbol already defined, possibly overriding previously symbolized box structure.

Symbolize: Warning: The box structure attempting to be symbolized has a similar or identical symbol already defined, possibly overriding previously symbolized box structure.

Symbolize: Warning: The box structure attempting to be symbolized has a similar or identical symbol already defined, possibly overriding previously symbolized box structure.

General: Further output of Symbolize::bsymbexs will be suppressed during this calculation.

In[160]:=
```
x[i_] := i
```

In[161]:=
```
f[i_] := dummyValues /. x → i
```

f[i] acts as a lookup function for the discrete probability.

In[162]:=
```
{f["a₀"], f["a₁"], f["a₂"], f["a₃"], f["a₄"]}
```

Out[162]= $\{p_0, p_1, p_2, p_3, p_4\}$

In[163]:=
```
f[5]
```

Out[163]= 0

In[164]:= $V = \sum_{i=0}^{4} f[x[i]]\,(x[i] - \mu)^2$

Out[164]= 0.

Code Graveyard

Exam Scores

In[165]:=
```
scores = <|"50-60" → 20, "61-80" → 30, "81-100" → 50|>
```

Out[165]=
```
<|50-60 → 20, 61-80 → 30, 81-100 → 50|>
```

In[166]:=
```
values = Values@scores;
total = Total@values;
values / total // N
```

Out[168]= {0.2, 0.3, 0.5}

In[169]:=
```
Total[values / total] == 1
```

Out[169]= True

Piecewise Function

In[170]:= $\text{scores} = \begin{cases} \frac{20}{60-50} & (x \geq 50) \,\&\&\, (x \leq 60) \\ \frac{30}{80-61} & (x \geq 61) \,\&\&\, (x \leq 80) \\ \frac{50}{100-81} & (x \geq 81) \,\&\&\, (x \leq 100) \end{cases};$

Integration of First Group

In[171]:= $\frac{\int_{50}^{60} scores\, dx}{\int_{0}^{100} scores\, dx}$ // N

Out[171]= 0.2

Integration of All Groups

In[172]:= MapThread$\left[\frac{\int_{\#1}^{\#2} scores\, dx}{\int_{0}^{100} scores\, dx}\right.$ &, {{50, 61, 81}, {60, 80, 100}}$\left.\right]$ // N

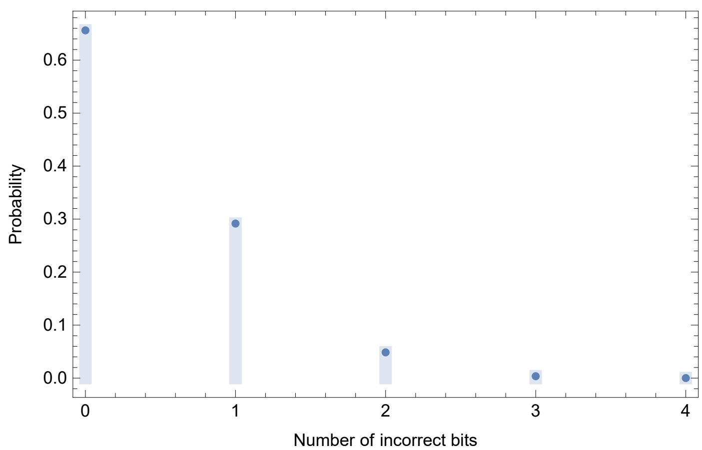
Out[172]= {0.2, 0.3, 0.5}

Probability Distribution

```
In[173]:= dist = ProbabilityDistribution[digitalChannel, {x, 0, 4, 1}];
          pdf = Simplify@PDF[dist, x];

In[175]:= DiscretePlot[pdf, {x, 0, 4}, FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[175]=



In[176]:=
$$\text{dummyValues} = \begin{cases} p_0 & x == a_0 \\ p_1 & x == a_1 \\ p_2 & x == a_2 \\ p_3 & x == a_3 \\ p_4 & x == a_4 \end{cases};$$

Symbolize: Warning: The box structure attempting to be symbolized has a similar or identical symbol already defined, possibly overriding previously symbolized box structure.

Symbolize: Warning: The box structure attempting to be symbolized has a similar or identical symbol already defined, possibly overriding previously symbolized box structure.

Symbolize: Warning: The box structure attempting to be symbolized has a similar or identical symbol already defined, possibly overriding previously symbolized box structure.

General: Further output of Symbolize::bsymbexs will be suppressed during this calculation.

In[177]:= $\$\text{Assumptions} = a_0 \neq a_1 \neq a_2 \neq a_3 \neq a_4$

Out[177]= $a_0 \neq a_1 \neq a_2 \neq a_3 \neq a_4$

In[178]:=
```
x[i_] := i
```

In[179]:=
```
f[i_] := dummyValues /. x → i
```

f[i] acts as a lookup function for the discrete probability.

In[180]:= $\{f[a_0], f[a_1], f[a_2], f[a_3], f[a_4]\}$ // FullSimplify

Out[180]= $\{p_0, p_1, p_2, p_3, p_4\}$

In[181]:= $V = \sum_{i=0}^{4} f[x[i]]\,(x[i] - \mu)^2$

Out[181]=
$$5.2441 \left(\begin{cases} p_0 & 0 == a_0 \\ p_1 & 0 == a_1 \\ p_2 & 0 == a_2 \\ p_3 & 0 == a_3 \\ p_4 & 0 == a_4 \\ 0 & \text{True} \end{cases}\right) + 1.6641 \left(\begin{cases} p_0 & 1 == a_0 \\ p_1 & 1 == a_1 \\ p_2 & 1 == a_2 \\ p_3 & 1 == a_3 \\ p_4 & 1 == a_4 \\ 0 & \text{True} \end{cases}\right) +$$
$$0.0841 \left(\begin{cases} p_0 & 2 == a_0 \\ p_1 & 2 == a_1 \\ p_2 & 2 == a_2 \\ p_3 & 2 == a_3 \\ p_4 & 2 == a_4 \\ 0 & \text{True} \end{cases}\right) + 0.5041 \left(\begin{cases} p_0 & 3 == a_0 \\ p_1 & 3 == a_1 \\ p_2 & 3 == a_2 \\ p_3 & 3 == a_3 \\ p_4 & 3 == a_4 \\ 0 & \text{True} \end{cases}\right) + 2.9241 \left(\begin{cases} p_0 & 4 == a_0 \\ p_1 & 4 == a_1 \\ p_2 & 4 == a_2 \\ p_3 & 4 == a_3 \\ p_4 & 4 == a_4 \\ 0 & \text{True} \end{cases}\right)$$

Print Notebook

Assumes that Mathematica notebook ends with .nb extension. Make sure the .pdf file is not open on the computer.

In[182]:=
```
Export[StringDrop[NotebookFileName[], -2] <> "pdf", EvaluationNotebook[]]
```

Out[•]=
```
C:\Users\sterg\Documents\GitHub\sparks-baird\mete-3070\mathematica\module-1.pdf
```