Module 1

Excerpts are taken from:
Montgomery, Douglas, C. and George C. Runger. Applied Statistics and Probability for Engineers, Enhanced eText. Available from: WileyPLUS, (7th Edition). Wiley Global Education US, 2018.

Lecture 1

Setup

In[1]:=
```
<< Notation`
```

In[2]:=
```
Symbolize[ _ ]
Symbolize[ __ ]
```

In[4]:=
```
PopulationVariance = ResourceFunction["PopulationVariance"]
```

Out[4]=
```
[•] PopulationVariance +
```

In[5]:=
```
SetOptions[DiscretePlot, PlotStyle → Thickness[.02], Frame → True];
SetOptions[Plot, PlotStyle → Thickness[.02], Frame → True];
```

Win / Loss Example

In[7]:= $P_{win} = \frac{20}{100}$ // N

Out[7]= 0.2

In[8]:= $P_{loss} = \frac{80}{100}$ // N

Out[8]= 0.8

In[9]:= $P_{win} + P_{loss} == 1$

Out[9]= True

Rolling Dice

In[10]:=
```
RandomChoice[{"Heads", "Tails"}]
```
Out[10]= Heads

In[11]:=
```
RandomInteger[{1, 6}]
```
Out[11]= 4

In[12]:=
```
RollDi := RandomInteger[{1, 6}]
```

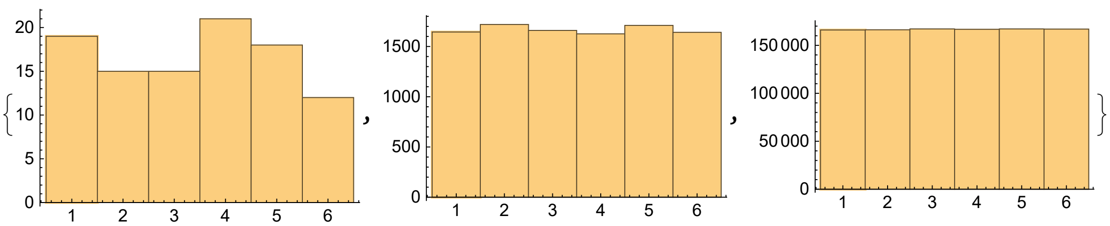
In[13]:=
```
RollDi
```
Out[13]= 4

In[14]:=
```
RollDice[n_] := RandomInteger[{1, 6}, n]
```

In[15]:=
```
rolls = RollDice[#] & /@ {100, 10000, 1000000};
```

In[16]:=
```
Histogram /@ rolls
```

Out[16]=



In[17]:=
```
p = 1/6;
diProbabilities = Association@Table[i → p, {i, 1, 6}]
```
Out[18]= $\langle| 1 \to \frac{1}{6}, 2 \to \frac{1}{6}, 3 \to \frac{1}{6}, 4 \to \frac{1}{6}, 5 \to \frac{1}{6}, 6 \to \frac{1}{6} |\rangle$

In[19]:=
```
Total@Values@diProbabilities == 1
```
Out[19]= True

In[20]:=
```
diNumbers = Range[6]
```
Out[20]= {1, 2, 3, 4, 5, 6}

In[21]:=
```
diRules = Thread[x == diNumbers]
```
Out[21]= {x == 1, x == 2, x == 3, x == 4, x == 5, x == 6}

In[22]:=
```
diProbabilities = ConstantArray[1/6, 6]
```
Out[22]= $\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$

In[23]:=
```
pw = Piecewise[{diProbabilities, diRules}ᵀ]
```

Out[23]=
$$\begin{cases} \frac{1}{6} & x == 1 \,||\, x == 2 \,||\, x == 3 \,||\, x == 4 \,||\, x == 5 \,||\, x == 6 \\ 0 & \text{True} \end{cases}$$

In[24]:=
```
pw /. x → 1
```

Out[24]=
$$\frac{1}{6}$$

In[25]:=
```
p = pw /. x → # & /@ {1, 2, 3}
```

Out[25]=
$$\left\{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\}$$

In[26]:=
```
Total[p]
```

Out[26]=
$$\frac{1}{2}$$

Lecture 2

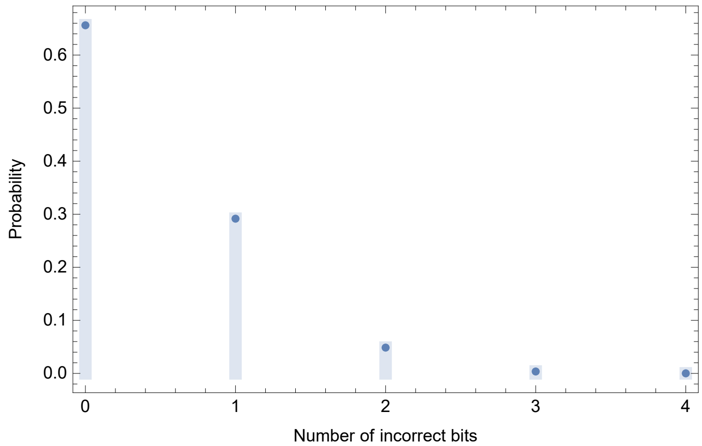
Digital Channel (Ex 3.3)

There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors that is presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$\begin{array}{ll} P(X=0)=0.6561 & P(X=1)=0.2916 \\ P(X=2)=0.0486 & P(X=3)=0.0036 \\ P(X=4)=0.0001 & \end{array}$$

In[27]:=
$$\text{digitalChannel} = \begin{cases} 0.6561 & x == 0 \\ 0.2916 & x == 1 \\ 0.0486 & x == 2 \\ 0.0036 & x == 3 \\ 0.0001 & x == 4 \end{cases};$$

In[28]:=
```
DiscretePlot[digitalChannel, {x, 0, 4},
 FrameLabel → {"Number of incorrect bits", "Probability"}]
```

Out[28]=


Lecture 3

Digital Channel (Ex 3.5)

In Example 3.3, we might be interested in the probability that three or fewer bits are in error. This question can be expressed as $P(X \leq 3)$.

The event that $\{X \leq 3\}$ is the union of the events $\{X = 0\}$, $\{X = 1\}$, $\{X = 2\}$, and $\{X = 3\}$. Clearly, these three events are mutually exclusive. Therefore,

$$\begin{aligned} P(X \leq 3) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ &= 0.6561 + 0.2916 + 0.0486 + 0.0036 = 0.9999 \end{aligned}$$

Ways to access values from Piecewise

In[29]:=
```
values = digitalChannel[[1, ;; , 1]]
```

Out[29]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

In[30]:=
```
values = digitalChannel /. x → # & /@ Range[0, 4]
```

Out[30]=
```
{0.6561, 0.2916, 0.0486, 0.0036, 0.0001}
```

Cumulative Sum

In[31]:=
```
Accumulate@values
```

Out[31]=
```
{0.6561, 0.9477, 0.9963, 0.9999, 1.}
```

P(X ≤ 3)

In[32]:=
```
Total[values[[ ;; 4]]]
```

Out[32]=
```
0.9999
```

Lecture 4

Digital Channel (Ex 3.7)

In Example 3.3, there is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Based on a model for the errors presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$$P(X=0)=0.6561 \; P(X=2)=0.0486 \; P(X=4)=0.0001$$
$$P(X=1)=0.2916 \; P(X=3)=0.0036$$

Discrete Distribution

In[33]:=
```
dist = ProbabilityDistribution[digitalChannel, {x, 0, 4, 1}]
```

Out[33]=
```
ProbabilityDistribution[ { 0.6561  x == 0
                           0.2916  x == 1
                           0.0486  x == 2
                           0.0036  x == 3
                           0.0001  x == 4
                           0       True   , {x, 0, 4, 1}]
```

Expectation Value (several methods)

In[34]:=
```
x[i_] := i
```

In[35]:=
```
f[i_] := digitalChannel /. x → i
```

In[36]:=
```
μ = 0 f[0] + 1 f[1] + 2 f[2] + 3 f[3] + 4 f[4]
```

Out[36]=
```
0.4
```

In[37]:=
```
μ = Range[0, 4].values
```
Out[37]=
```
0.4
```

The mean of a distribution gives the expectation value.

In[38]:=
```
μ = Mean[dist]
```
Out[38]=
```
0.4
```

Standard Deviation

The variance can be computed manually using a sum.

In[39]:= $V = \sum_{i=0}^{4} f[x[i]]\ (x[i] - \mu)^2$

Out[39]=
```
0.36
```

Note that this is variance of a distribution, which considers weights appropriately.

In[40]:=
```
Variance@dist
```
Out[40]=
```
0.36
```

In[41]:= $\sigma = \sqrt{V}$

Out[41]=
```
0.6
```

In[42]:= $\text{Around}\left[\mu, \sqrt{V}\right]$

Out[42]=
```
0.4 ± 0.6
```

NiCd Battery (3.3.6)

3.3.6 In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states. Assume the following proportions of the states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

In[43]:=
```
battery = ⎧ 0.17  x == 0
          ⎨ 0.35  x == 2
          ⎨ 0.33  x == 3 ;
          ⎩ 0.15  x == 4
```

a. Determine the cumulative distribution function of the nickel charge.

In[44]:=
```
dist = ProbabilityDistribution[battery, {x, 0, 4, 1}];
```

In[45]:=
```
cdf = CDF[dist]
```

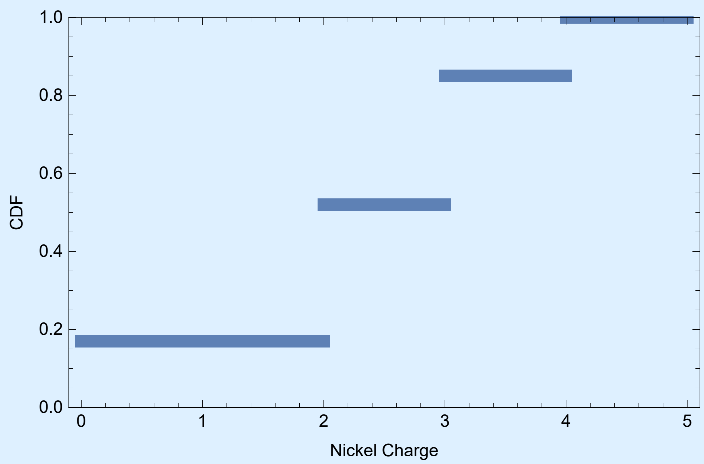
Out[45]=
```
Function[x, ⎧ 0.    x < 0
            ⎨ 0.17  0 ≤ x < 2
            ⎨ 0.52  2 ≤ x < 3 , Listable]
            ⎨ 0.85  3 ≤ x < 4
            ⎩ 1.    True
```

In[46]:=
```
cdf[#] & /@ Range[0, 4]
```

Out[46]=
```
{0.17, 0.17, 0.52, 0.85, 1.}
```

In[47]:=
```
Plot[cdf[x], {x, 0, 5}, PlotRange → {0, 1}, FrameLabel → {"Nickel Charge", "CDF"}]
```

Out[47]=



b. Determine the mean and variance of the nickel charge.

In[48]:=
```
μ = Mean@dist
```

Out[48]= 2.29

In[49]:=
```
V = Variance@dist;
σ = √V
```

Out[50]= 1.23527

In[51]:=
```
charge = Around[μ, σ]
```

Out[51]= 2.3 ± 1.2

Code Graveyard

Print Notebook

Assumes that Mathematica notebook ends with .nb extension. Make sure the .pdf file is not open on the computer.

In[63]:=
```
Export[StringDrop[NotebookFileName[], -2] <> "pdf", EvaluationNotebook[]]
```

Out[∘]= C:\Users\sterg\Documents\GitHub\sparks-baird\mete-3070\mathematica\module-1.pdf