Teleportation as a quantum computation

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1 Introduction

Among the many exciting new applications of quantum physics in the realm of computation and information theory, I am particularly fond of quantum cryptography, quantum computing and quantum teleportation [5]. Quantum cryptography allows for the confidential transmission of classical information under the nose of an eavesdropper, regardless of her computing power or technological sophistication [2, 1, 4]. Quantum computing allows for an exponential amount of computation to take place simultaneously in a single piece of hardware [9, 7]; of particular interest is the ability of quantum computers to factorize numbers very efficiently [14], with dramatic implications for classical cryptography [6]. Quantum teleportation allows for the transmission of quantum information to a distant location despite the impossibility of measuring or broadcasting the information to be transmitted [3]. Each of these concepts had a strong overtone of science fiction when they were first introduced.

If asked to rank these ideas on a scale of technological difficulty, it is tempting to think that quantum cryptography is easiest while quantum teleportation is the most outrageous—especially when it comes to teleporting goulash [11]! This ranking is correct with respect to quantum cryptography, whose feasibility has been demonstrated by several experimental prototypes capable of reliably transmitting confidential information over distances of tens of kilometres [12, 13, 10]. The situation is less clear when it comes to comparing the technological feasibility of quantum computing with that of quantum teleportation.

On the one hand, quantum teleportation can be implemented with a quantum circuit that is much simpler than that required by any nontrivial quantum computational task: the state of an arbitrary qubit (quantum bit) can be teleported with as few as two quantum exclusive-or (controlled-not) gates. Thus, quantum teleportation is significantly easier to implement than quantum computing if we are concerned only with the complexity of the required circuitry.

On the other hand, quantum computing is meaningful even if it takes place very quickly—indeed its primary purpose is increased computational speed—and within a small region of space. Quite the opposite, the interest

of quantum teleportation would be greatly reduced if the actual teleportation had to take place immediately after the required preparation. Thus, a working demonstration of quantum teleportation is likely to be seen before the quantum factorization of even a very small integer is achieved, but quantum teleportation across significant time and space will have to await a technology that allows for the efficient long-term storage of quantum information. Nevertheless, it may be that short-distance quantum teleportation will play a role in transporting quantum information inside quantum computers. Thus we see that the fates of quantum computing and quantum teleportation are entangled!

2 Quantum teleportation

Recall that any attempt at measuring quantum information disturbs it irreversibly and yields incomplete information. This makes it impossible to transmit quantum information through a classical channel. Recall also that the purpose of quantum teleportation [3] is to circumvent this impossibility so as to allow the faithful transmission of quantum information between two parties, conventionally referred to as Alice and Bob.

In order to achieve teleportation, Alice and Bob must share prior quantum entanglement. This is usually explained in terms of Einstein-Podolsky-Rosen nonlocal quantum states [8] and Bell measurements, which makes the process seem very mysterious. The purpose of this note is to show how to achieve quantum teleportation very simply in terms of quantum computation. As interesting side product, we obtain a quantum circuit with the unusual feature that there are points in the circuit at which the quantum information can be completely disrupted by a measurement—or some types of interaction with the environment—without ill effects: the same final result is obtained whether or not measurement takes place. This is true despite that fact that the qubits affected by these measurements are entangled with the other qubits carried by the circuit, which should make these measurements even more damaging.

Of course, the uncanny power of quantum computation draws in parts on nonlocal effects inherent to quantum mechanics. The quantum teleportation circuit described in §4 is not really different in principle from the original idea [3] since it uses quantum computation to create and measure nonlocal states. Nevertheless it sheds new

^{*}Supported in part by NSERC and FCAR

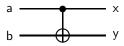
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light on teleportation, at least from a pedagogical point of view, since it makes the process completely straightforward to anyone who believes that quantum computation is a reasonable proposition. Moreover, this circuit could genuinely be used for teleportation purposes inside a quantum computer. Finally, the surprising resilience of this circuit to measurements performed while it is processing information may turn out to have relevance to quantum error correction.

3 The basic ingredients

As is often the case with quantum computation, we shall need two basic ingredients: the exclusive-or gate (also known as controlled-not), which acts on two qubits at once, and arbitrary unitary operations on single qubits. Let $|0\rangle$ and $|1\rangle$ denote basis states for single qubits and recall that pure states are given by linear combination of basis states such as $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$ where α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$.

The quantum exclusive-or (XOR), denoted as follows,



sends $|00\rangle$ to $|00\rangle$, $|01\rangle$ to $|01\rangle$, $|10\rangle$ to $|11\rangle$ and $|11\rangle$ to $|10\rangle$. In other words, provided the input states at a and b are in basis states, the output state at x is the same as the input state at a, and the output state at y is the exclusive-or of the two input states at a and b. This is also known as the controlled-not gate because the state carried by the control wire "ax" is not disturbed whereas the state carried by the controlled wire "by" is flipped if and only if the state on the control wire was $|1\rangle$. Note that the classical interpretation given above no longer holds if the input qubits are not in basis states: it is possible for the output state on the control wire (at x) to be different from its input state (at a). Moreover, the joint state of the output qubits can be entangled even if the input qubits were not, and vice versa.

In addition to the quantum exclusive-or, we shall need two single-qubit rotations L and R, and two single-qubit conditional phase-shifts S and T. Rotation L sends $|0\rangle$ to $(|0\rangle+|1\rangle)/\sqrt{2}$ and $|1\rangle$ to $(-|0\rangle+|1\rangle)/\sqrt{2}$, whereas R sends $|0\rangle$ to $(|0\rangle-|1\rangle)/\sqrt{2}$ and $|1\rangle$ to $(|0\rangle+|1\rangle)/\sqrt{2}$. Note that LR|\psi = RL|\psi > |\psi > |

$$\mathsf{L} = \frac{1}{\sqrt{2}} \left(\begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array} \right) \quad \mathsf{R} = \frac{1}{\sqrt{2}} \left(\begin{array}{cc} 1 & 1 \\ -1 & 1 \end{array} \right)$$

$$\mathsf{S} = \left(\begin{array}{cc} i & 0 \\ 0 & 1 \end{array} \right) \quad \text{and} \quad \mathsf{T} = \left(\begin{array}{cc} -1 & 0 \\ 0 & -i \end{array} \right)$$

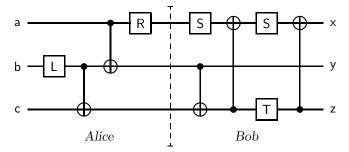
if $\alpha|0\rangle + \beta|1\rangle$ is represented by vector $\binom{\alpha}{\beta}$. Similarly the quantum exclusive-or operation is given by matrix

$$\mathsf{XOR} = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

if $\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ is represented by the transpose of vector $(\alpha, \beta, \gamma, \delta)$.

4 The teleportation circuit

Consider the following quantum circuit. Please disregard the dashed line for the moment.



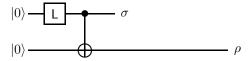
Let $|\psi\rangle$ be an arbitrary one-qubit state. Consider what happens if you feed $|\psi00\rangle$ in this circuit, that is if you set upper input a to $|\psi\rangle$ and both other inputs b and c to $|0\rangle$. It is a straightforward exercise to verify that state $|\psi\rangle$ will be transferred to the lower output z, whereas both other outputs x and y will come out in state $|\phi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. In other words the output will be $|\phi\phi\psi\rangle$. If the two upper outputs are measured in the standard basis $(|0\rangle$ versus $|1\rangle$), two random classical bits will be obtained in addition to quantum state $|\psi\rangle$ on the lower output.

Now, let us consider the state of the system at the dashed line. A simple calculation shows that all three qubits are entangled. We should therefore be especially careful not to disturb the system at that point. Nevertheless, let us measure the two upper qubits, leaving the lower qubit undisturbed. This measurement results in two purely random classical bits u and v, bearing no correlation whatsoever with the original state $|\psi\rangle$. Let us now turn u and v back into quantum bits and reinject $|u\rangle$ and $|v\rangle$ in the circuit immediately after the dashed line.

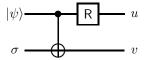
Needless to say that the quantum state carried at the dashed line has been completely disrupted by this measurement-and-resend process. We would therefore expect this disturbance to play havoc with the final output of the circuit. Not at all! In the end, the state carried at xyz is $|uv\psi\rangle$. In other words, $|\psi\rangle$ is still obtained at z and the other two qubits, if measured, are purely random provided we forget the measurement outcomes at the dashed line. Another way of seeing this phenomenon is that the

outcome of the circuit will not be altered if the state of the upper two qubits leaks to the environment (in the standard basis) at the dashed line.

To turn this circuit into a quantum teleportation device, we need the ability to store qubits. Assume Alice prepares two qubits in state $|0\rangle$ and pushes them through the first two gates of the circuit.



She keeps the upper qubit σ in quantum memory and gives the other, ρ , to Bob. [We do not denote these qubits by kets because they are not individual pure states: together they are in state $\Phi^+ = (|00\rangle + |11\rangle)/\sqrt{2}$.] At some later time, Alice receives a mystery qubit in unknown state $|\psi\rangle$. In order to teleport this qubit to Bob, she releases σ from her quantum memory and pushes it together with the mystery qubit through the next two gates of the circuit. She measures both output wires to turn them into classical bits u and v.



To complete teleportation, Alice has to communicate uand v to Bob by way of a classical communication channel. Upon reception of the signal, Bob creates quantum states $|u\rangle$ and $|v\rangle$ from the classical information received from Alice, he releases the qubit ρ he had kept in quantum memory, and he pushes all three qubits into his part of the circuit (on the right of the dashed line). Finally Bob may wish to measure the two upper qubit at x and y to make sure that he gets u and v; otherwise something went wrong in the teleportation apparatus. At this point, teleportation is complete as Bob's output z is in state $|\psi\rangle$. Note that this process works equally well if Alice's mystery qubit is not in a pure state. In particular, Alice can teleport to Bob entanglement with an arbitrary auxiliary system, possibly outside both Alice's and Bob's laboratories.

In practice, Bob need not use the quantum circuit shown right of the dashed line at all. Instead, he may choose classically one of 4 possible rotations to apply to the qubit he had kept in quantum memory, depending on the 2 classical bits he receives from Alice. (This would be more in tune with the original teleportation proposal [3].) This explains the earlier claim that quantum teleportation can be achieved at the cost of only two quantum exclusive-ors: those of Alice. Nevertheless, the unitary version of Bob's process given here may be more appealing than choosing classically among 4 courses of action if teleportation is used inside a quantum computer.

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