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CS325-401
Project Group 2
Project 3: Linear Programming

Problem 1

Part A: Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

Objective Function:

Minimum cost:

$10W_{11} + 15W_{12} + 11W_{21} + 8W_{22} + 13W_{31} + 8W_{32} + 9W_{33} + 14W_{42} + 8W_{43} + 5R_{11} + 6R_{12} + 7R_{13} + 10R_{14} + 12R_{23} + 8R_{24} + 10R_{25} + 14R_{26} + 14R_{34} + 12R_{35} + 12R_{36} + 6R_{37}$

Constraints:

$W_{11} + W_{12} < 150$
$W_{21} + W_{22} < 450$
$W_{31} + W_{32} + W_{33} < 250$
$W_{42} + W_{43} < 150$
$W_{11} + W_{21} + W_{31} - R_{11} - R_{12} - R_{13} - R_{14} > 0$
$W_{12} + W_{22} + W_{32} + W_{42} - R_{23} - R_{24} - R_{25} - R_{26} > 0$
$W_{33} + W_{43} - R_{34} - R_{35} - R_{36} - R_{37} > 0$
$R_{11} > 100$
$R_{12} > 150$
$R_{13} + R_{23} > 100$
$R_{14} + R_{24} + R_{34} > 200$
$R_{25} + R_{35} > 200$
$R_{26} + R_{36} > 150$
$R_{37} > 100$
$W_{11} > 0$
$W_{12} > 0$
$W_{13} = 0$
$W_{21} > 0$
$W_{22} > 0$
$W_{23} = 0$
$W_{31} > 0$
$W_{32} > 0$
$W_{33} > 0$
$W_{41} = 0$
$W_{42} > 0$
$W_{43} > 0$
$R_{11} > 0$

$R_{12} > 0$
$R_{13} > 0$
$R_{14} > 0$
$R_{15} = 0$
$R_{16} = 0$
$R_{17} = 0$
$R_{21} = 0$
$R_{22} = 0$
$R_{23} > 0$
$R_{24} > 0$
$R_{25} > 0$
$R_{26} > 0$
$R_{27} = 0$
$R_{31} = 0$
$R_{32} = 0$
$R_{33} = 0$
$R_{34} > 0$
$R_{35} > 0$
$R_{36} > 0$
$R_{37} > 0$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

Using Lindo: 1.A

```
MIN 10w11 + 15w12 + 11w21 + 8w22 + 13w31 + 8w32 + 9w33 + 14w42 + 8w43 + 5R11 + 6R12 + 7R13
+ 10R14 + 12R23 + 8R24 + 10R25 + 14R26 + 14R34 + 12R35 + 12R36 + 6R37

ST

w11 + w12 < 150
w21 + w22 < 450
w31 + w32 + w33 < 250
w42 + w43 < 150
w11 + w21 + w31 - R11 - R12 - R13- R14 > 0
w12 + w22 + w32 + w42 - R23 - R24 - R25 - R26 > 0
w33 + w43 - R34 - R35 - R36 - R37 > 0
R11 > 100
R12 > 150
R13 + R23 > 100
R14 + R24 + R34 > 200
R25 + R35 > 200
R26 + R36 > 150
R37 > 100
w11 > 0
w12 > 0
w13 = 0
w21 > 0
w22 > 0
w23 = 0
```

```
w31 > 0
w32 > 0
w33 > 0
w41 = 0
w42 > 0
w43 > 0
R11 > 0
R12 > 0
R13 > 0
R14 > 0
R15 = 0
R16 = 0
R17 = 0
R21 = 0
R22 = 0
R23 > 0
R24 > 0
R25 > 0
R26 > 0
R27 = 0
R31 = 0
R32 = 0
R33 = 0
R34 > 0
R35 > 0
R36 > 0
R37 > 0

END
```

iii. What are the optimal shipping routes and minimum cost.
The optimal solution is $17100.00

	W1	W2	W3
P1	150	0	
P2	200	250	
P3	0	150	100
P4		0	150
Totals	350	400	250

	R1	R2	R3	R4	R5	R6	R7

W1	100	150	100	0			
W2			0	200	200	0	
W3				0	0	150	100
Totals	100	150	100	200	200	150	100

Above are shown the routes and numbers for 1A.

Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model?
Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?

It is not feasible since the supply from warehouse 3 does not meet the demand for their retailers. The supply is 400, but the demand is 450 thus violating a constraint and giving an infeasible solution.

```
MIN 10W11 + 11W21 + 13W31 + 9W33 + 8W43 + 5R11 + 6R12 + 7R13 + 10R14 + 14R34 +
12R35 + 12R36 + 6R37
ST

W11 = 150
W21 = 450
W31 + W33 < 250
W43 = 150

R11 = 100
R12 = 150
R13 = 100
R14 + R34 > 200
R35 = 200
R36 = 150
R37 = 100

W11 + W21 + W31 - R11 - R12 - R13 - R14 > 0
W33 + W43 - R34 - R35 - R36 - R37 > 0
W11 > 0
W21 > 0
W31 > 0
W33 > 0
W43 > 0

R11 > 0
R12 > 0
R13 > 0
R14 > 0
```

```
R34 > 0
R35 > 0
R36 > 0
R37 > 0

END
```

Part C: Instead of closing Warehouse 2 management has to decide to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so, what is the optimal solution when warehouse 2 is limited to 100 refrigerators?

Yes, it is feasible, and it decreases costs slightly to $18300.

```
MIN 10w11 + 15w12 + 11w21 + 8w22 + 13w31 + 8w32 + 9w33 + 14w42 + 8w43 + 5R11 +
6R12 + 7R13 + 10R14 + 12R23 + 8R24 + 10R25 + 14R26 + 14R34 + 12R35 + 12R36 +
6R37

ST

      w11 + w12 < 150
      w21 + w22 < 450
      w31 + w32 + w33 < 250
      w42 + w43 < 150
      w11 + w21 + w31 - R11 - R12 - R13- R14 > 0
      w12 + w22 + w32 + w42 - R23 - R24 - R25 - R26 > 0
      w33 + w43 - R34 - R35 - R36 - R37 > 0
      w12 + w22 + w32 + w42 = 100
      R11 > 100
      R12 > 150
      R13 + R23 > 100
      R14 + R24 + R34 > 200
      R25 + R35 > 200
      R26 + R36 > 150
      R37 > 100
      w11 > 0
      w12 > 0
      w13 = 0
      w21 > 0
      w22 > 0
      w23 = 0
      w31 > 0
      w32 > 0
      w33 > 0
      w41 = 0
      w42 > 0
      w43 > 0
      R11 > 0
      R12 > 0
      R13 > 0
```

```
      R14 > 0
      R15 = 0
      R16 = 0
      R17 = 0
      R21 = 0
      R22 = 0
      R23 > 0
      R24 > 0
      R25 > 0
      R26 > 0
      R27 = 0
      R31 = 0
      R32 = 0
      R33 = 0
      R34 > 0
      R35 > 0
      R36 > 0
      R37 > 0
END
```

	W1	**W2**	**W3**
P1	150		
P2	350	100	
P3			250
P4			150
Totals	500	100	400

	R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**
W1	100	150	100	150			
W2			0	50	50	0	
W3				0	150	150	100
Totals	100	150	100	200	200	150	100

Part D: Formulate a generalized linear programming model for the transshipment problem. Give the objective function and constraints as mathematical formulas.

Min sum of transportation $\sum_{i=1}^{m}\sum_{j=1}^{n} P_i W_j + \sum_{j=1}^{n}\sum_{k=1}^{q} W_j R_k$ where P is plant, W is warehouse and R is retailer.

Constraints:

$\sum W_j R_k \geq d_k$ for each retailer where d is the demand

$\sum P_i W_j \leq s_i$ for each plant where s is the supply

s_i - d_k ≥ 0 (Demand cannot be greater than supply)

$P_i \geq 0, W_j \geq 0, R_k \geq 0$

Transportation model for Lindo

Objective Function: Take the minimum sum of transportation from plant-1 + plant-2 + ... + plant-n to Warehouse 1 + plant-1 + plant-2 + ...+ plant-n to Warehouse 2 + ... + plant-1 + plant-2 + ... + plant-n to Warehouse n + retail-1 from Warehouse-1 +...+ retail-1 from Warehouse-n +...+ retail-n from Warehouse-1 +...+ retail-n from Warehouse-n

Constraints:
limit by sum of plant-1 to Warehouse-1 + ... + plant-1 to Warehouse-n output limit if possible to ship from p -> w
limit by sum of plant-2 to Warehouse-1 + ... + plant-2 to Warehouse-n output limit if possible to ship from p -> w
limit by sum of plant-n to Warehouse-1 + ... + plant-n to Warehouse-n output limit if possible to ship from p -> w

limit by sum of plant-1 to Warehouse-1 + ... + plant-n to Warehouse-1 - retailer-1 from Warehouse-1 +...+ retailer-n from Warehouse-1 > 0
limit by sum of plant-1 to Warehouse-n + ... + plant-n to Warehouse-n - retailer-1 from Warehouse-n +...+ retailer-n from Warehouse-n > 0

limit by sum of warehouse-1 to retail-1 + ... + warehouse-n to retail-1 if possible to ship >= demand for retail-1
limit by sum of warehouse-1 to retail-2 + ... + warehouse-n to retail-2 if possible to ship >= demand for retail-2
limit by sum of warehouse-1 to retail-n + ... + warehouse-n to retail-n if possible to ship >= demand for retail-n

if possible to ship from plant to warehouse
limit by plant-1 to warehouse-1 >= 0
limit by plant-1 to warehouse-2 >= 0
limit by plant-1 to warehouse-n >= 0

limit by plant-2 to warehouse-1 >= 0
limit by plant-2 to warehouse-2 >= 0
limit by plant-2 to warehouse-n >= 0

limit by plant-n to warehouse-1 >= 0
limit by plant-n to warehouse-2 >= 0
limit by plant-n to warehouse-n >= 0

if possible to ship from warehouse to retail
limit by warehouse-1 to retail-1 >= 0
limit by warehouse-1 to retail-2 >= 0
limit by warehouse-1 to retail-n >= 0

limit by warehouse-2 to retail-1 >= 0
limit by warehouse-2 to retail-2 >= 0
limit by warehouse-2 to retail-n >= 0

limit by warehouse-n to retail-1 >= 0
limit by warehouse-n to retail-2 >= 0
limit by warehouse-n to retail-n >= 0

Problem 2

<u>Part A:</u> Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

i. Formulate the problem as a linear program with an objective function and all constraints.

X_i = Salad ingredient

<u>Objective Function:</u>

Min calories:

$21X_1 + 16X_2 + 40X_3 + 41X_4 + 585X_5 + 120X_6 + 164X_7 + 884X_8$

<u>Constraints:</u>

$0.85X_1 + 1.62X_2 + 2.86X_3 + 0.93X_4 + 23.4X_5 + 16X_6 + 9X_7 > 15$

$0.33X_1 + 0.20X_2 + 0.39X_3 + 0.24X_4 + 48.7X_5 + 5.00X_6 + 2.6X_7 + 100X_8 > 2$

$0.33X_1 + 0.20X_2 + 0.39X_3 + 0.24X_4 + 48.7X_5 + 5.00X_6 + 2.6X_7 + 100X_8 < 8$

$4.64X_1 + 2.37X_2 + 3.63X_3 + 9.58X_4 + 15.00X_5 + 3.00X_6 + 27.0X_7 > 4$

$9.00X_1 + 28.00X_2 + 65.00X_3 + 69.00X_4 + 3.80X_5 + 120.00X_6 + 78X_7 < 200$

$0.6X_2 + 0.6X_3 - 0.4X_1 - 0.4X_4 - 0.4X_5 - 0.4X_6 - 0.4X_7 - 0.4X_8 > 0$

$X_i > 0$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The minimum calories for the salad meeting those constraints are 114.7541.

```
MIN 21X1 + 16X2 + 40X3 + 41X4 + 585X5 + 120X6 + 164X7 + 884X8
ST

0.85X1 + 1.62X2 + 2.86X3 + 0.93X4 + 23.4X5 + 16X6 + 9X7 > 15
0.33X1 + 0.20X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 > 2
0.33X1 + 0.20X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 < 8
4.64X1 + 2.37X2 + 3.63X3 + 9.58X4 + 15.00X5 + 3.00X6 + 27.0X7 > 4
9X1 + 28X2 + 65X3 + 69X4 + 3.8X5 + 120X6 + 78X7 < 200

0.6X2 + 0.6X3 - 0.4X1 - 0.4X4 - 0.4X5 - 0.4X6 - 0.4X7 - 0.4X8 > 0

X1 > 0
X2 > 0
X3 > 0
X4 > 0
X5 > 0
X6 > 0
X7 > 0
X8 > 0

END
```

iii. What is the cost of the low calorie salad?

The cost is $2.33 for the low calorie salad.

Part B: Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

X_i = salad ingredients

Objective Function:

min cost to make salad:

$1X_1 + 0.75X_2 + 0.50X_3 + 0.50X_4 + 0.45X_5 + 2.15X_6 + 0.95X_7 + 2.00X_8$

Constraints:

$0.85X_1 + 1.62X_2 + 2.86X_3 + 0.93X_4 + 23.4X_5 + 16X_6 + 9X_7 > 15$

$0.33X_1 + 0.20X_2 + 0.39X_3 + 0.24X_4 + 48.7X_5 + 5.00X_6 + 2.6X_7 + 100X_8 > 2$

$0.33X_1 + 0.20X_2 + 0.39X_3 + 0.24X_4 + 48.7X_5 + 5.00X_6 + 2.6X_7 + 100X_8 > 8$

$4.64X_1 + 2.37X_2 + 3.63X_3 + 9.58X_4 + 15.00X_5 + 3.00X_6 + 27.0X_7 > 4$

$9.00X_1 + 28.00X_2 + 65.00X_3 + 69.00X_4 + 3.80X_5 + 120.00X_6\ 78X_7 < 200$

$0.6X_2 + 0.6X_3 - 0.4X_1 - 0.4X_4 - 0.4X_5 - 0.4X_6 - 0.4X_7 - 0.4X_8 > 0$
$X_i > 0$

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The optimal solution is $1.55 for a low cost salad.

```
MIN 1X1 + 0.75X2 + 0.50X3 + 0.50X4 + 0.45X5 + 2.15X6 + 0.95X7 + 2.00X8
ST
0.85X1 + 1.62X2 + 2.86X3 + 0.93X4 + 23.4X5 + 16X6 + 9X7 > 15
0.33X1 + 0.20X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 > 2
0.33X1 + 0.20X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 < 8
4.64X1 + 2.37X2 + 3.63X3 + 9.58X4 + 15.00X5 + 3.00X6 + 27.0X7 > 4
9X1 + 28X2 + 65X3 + 69X4 + 3.8X5 + 120X6 + 78X7 < 200

0.6X2 + 0.6X3 - 0.4X1 - 0.4X4 - 0.4X5 - 0.4X6 - 0.4X7 - 0.4X8 > 0

X1 > 0
X2 > 0
X3 > 0
X4 > 0
X5 > 0
X6 > 0
X7 > 0
X8 > 0

END
```

iii. How many calories are in the low cost salad?
There are 278.49 calories in the low cost salad.

Part C: Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However, if she can advertise that the salad has under 250 calories then she may be able to sell more.
i. Suggest some possible ways that she select a combination of ingredients that is "near optimal" for both objectives. This is a type of multi-objective optimization.

To optimize for both low calorie and low cost, add a constraint for calories (i.e. >100 and < 250). From Part B, the low cost salad was $1.55. For low calorie and low cost, the ingredient cost cannot go over $2.00 since that would result in a profit of less than $3.00 if she sold the salad for $5.00. From B, the low cost salad was $1.55. We could set the range of < 2.00 and >1.55.

ii. What combination of ingredient would you suggest and what is the associated cost and calorie.

X_i = salad ingredient
<u>Objective Function:</u>
min calories of low cost salad:

$21X_1 + 16X_2 + 40X_3 + 41X_4 + 585X_5 + 120X_6 + 164X_7 + 884X_8$

Constraints:

$0.85X_1 + 1.62X_2 + 2.86X_3 + 0.93X_4 + 23.4X_5 + 16X_6 + 9X_7 > 15$

$0.33X_1 + 0.20X_2 + 0.39X_3 + 0.24X_4 + 48.7X_5 + 5.00X_6 + 2.6X_7 + 100X_8 > 2$

$0.33X_1 + 0.20X_2 + 0.39X_3 + 0.24X_4 + 48.7X_5 + 5.00X_6 + 2.6X_7 + 100X_8 > 8$

$4.64X_1 + 2.37X_2 + 3.63X_3 + 9.58X_4 + 15.00X_5 + 3.00X_6 + 27.0X_7 > 4$

$9.00X_1 + 28.00X_2 + 65.00X_3 + 69.00X_4 + 3.80X_5 + 120.00X_6\ 78X_7 < 200$

$0.6X_2 + 0.6X_3 - 0.4X_1 - 0.4X_4 - 0.4X_5 - 0.4X_6 - 0.4X_7 - 0.4X_8 > 0$

$1X_1 + 0.75X_2 + 0.50X_3 + 0.50X_4 + 0.45X_5 + 2.15X_6 + 0.95X_7 + 2.00X_8 > 1.55$

$1X_1 + 0.75X_2 + 0.50X_3 + 0.50X_4 + 0.45X_5 + 2.15X_6 + 0.95X_7 + 2.00X_8 < 2.00$

$21X_1 + 16X_2 + 40X_3 + 41X_4 + 585X_5 + 120X_6 + 164X_7 + 884X_8 > 100$

$21X_1 + 16X_2 + 40X_3 + 41X_4 + 585X_5 + 120X_6 + 164X_7 + 884X_8 < 250$

$X_i > 0$

The salad is 134.76 calories and costs $2.00

iii. Note: There is not one “right” answer. Discuss how you derived your solution.

Lindo code here:

```
MIN 21X1 + 16X2 + 40X3 + 41X4 + 585X5 + 120X6 + 164X7 + 884X8
ST
0.85X1 + 1.62X2 + 2.86X3 + 0.93X4 + 23.4X5 + 16X6 + 9X7 > 15
0.33X1 + 0.20X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 > 2
0.33X1 + 0.20X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 < 8
4.64X1 + 2.37X2 + 3.63X3 + 9.58X4 + 15.00X5 + 3.00X6 + 27.0X7 > 4
9X1 + 28X2 + 65X3 + 69X4 + 3.8X5 + 120X6 + 78X7 < 200

0.6X2 + 0.6X3 - 0.4X1 - 0.4X4 - 0.4X5 - 0.4X6 - 0.4X7 - 0.4X8 > 0
1X1 + 0.75X2 + 0.50X3 + 0.50X4 + 0.45X5 + 2.15X6 + 0.95X7 + 2.00X8 > 1.55
1X1 + 0.75X2 + 0.50X3 + 0.50X4 + 0.45X5 + 2.15X6 + 0.95X7 + 2.00X8 < 2.00
21X1 + 16X2 + 40X3 + 41X4 + 585X5 + 120X6 + 164X7 + 884X8 > 100
21X1 + 16X2 + 40X3 + 41X4 + 585X5 + 120X6 + 164X7 + 884X8 < 250

X1 > 0
X2 > 0
X3 > 0
X4 > 0
X5 > 0
X6 > 0
X7 > 0
X8 > 0

END
```

This solution was derived using the above objective function and constraints. The rationale for choosing calories between 100-250 was that a quality salad most likely must be > 100 calories to be considered nutritional at all and < 250 since Veronica wanted to advertise a salad under 250 calories. The price point from part B was $1.55

and this was excluding caloric intake; therefore, \$1.55 was the lower bound. Veronica also wanted to list the salad for \$5.00 with a net profit of \$3.00, so the salad price itself could not go over \$2.00. Therefore, the upper bound of the constraint was set to \$2.00.

Problem 3

a) What are the lengths of the shortest paths from vertex a to all other vertices.

A to A: 0
A to B: 2
A to C: 3
A to D: 8
A to E: 9
A to F: 6
A to G: 8
A to H: 9
A to I: 8
A to J: 10
A to K: 14
A to L: 15
A to M: 17

Using Lindo:

```
max b + c + d + e + f + g + h + i + j + k + l + m
ST
a = 0
b - a <= 2
c - a <= 3
d - a <= 8
h - a <= 9
a - b <= 4
c - b <= 5
e - b <= 7
f - b <= 4
d - c <= 10
b - c <= 5
g - c <= 9
i - c <= 11
f - c <= 4
a - d <= 8
g - d <= 2
j - d <= 5
f - d <= 1
h - e <= 5
c - e <= 4
i - e <= 10
i - f <= 2
g - f <= 2
d - g <= 2
j - g <= 8
k - g <= 12
```

```
i - h <= 5
k - h <= 10
a - i <= 20
k - i <= 6
j - i <= 2
m - i <= 12
i - j <= 2
k - j <= 4
l - j <= 5
h - k <= 10
m - k <= 10
m - l <=2
END
```

b) If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a).

This causes an error. Vertex Z is listed as unbound, and the optimal value it produces is 0.9999990E+08

```
max b + c + d + e + f + g + h + i + j + k + l + m + z
ST
      a = 0
      b - a <= 2
      c - a <= 3
      d - a <= 8
      h - a <= 9
      a - b <= 4
      c - b <= 5
      e - b <= 7
      f - b <= 4
      d - c <= 10
      b - c <= 5
      g - c <= 9
      i - c <= 11
      f - c <= 4
      a - d <= 8
      g - d <= 2
      j - d <= 5
      f - d <= 1
      h - e <= 5
      c - e <= 4
      i - e <= 10
      i - f <= 2
      g - f <= 2
      d - g <= 2
      j - g <= 8
      k - g <= 12
      i - h <= 5
      k - h <= 10
```

```
      a - i <= 20
      k - i <= 6
      j - i <= 2
      m - i <= 12
      i - j <= 2
      k - j <= 4
      l - j <= 5
      h - k <= 10
      m - k <= 10
      m - l <= 2
      m - z <= 5
END
```

c) What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?

By using the sum of the shortest paths as the optimal, you can get the results from the variable values listed in the results.

The optimal value was 145.

A to M: 17

B to M: 15

C to M: 15

D to M: 12

E to M: 19

F to M: 11

G to M: 14

H to M: 14

I to M: 9

J to M: 7

K to M: 10

L to M: 2

M to M: 0

```
MAX a + b + c + d + e + f + g + h + i + j + k + l
ST
      m = 0
      a - b <= 2
      a - c <= 3
      a - d <= 8
      a - h <= 9
      b - a <= 4
      b - c <= 5
      b - e <= 7
      b - f <= 4
      c - d <= 10
      c - b <= 5
      c - g <= 9
      c - i <= 11
      c - f <= 4
```

```
        d - a <= 8
        d - g <= 2
        d - j <= 5
        d - f <= 1
        e - h <= 5
        e - c <= 4
        e - i <= 10
        f - i <= 2
        f - g <= 2
        g - d <= 2
        g - j <= 8
        g - k <= 12
        h - i <= 5
        h - k <= 10
        i - a <= 20
        i - k <= 6
        i - j <= 2
        i - m <= 12
        j - i <= 2
        j - k <= 4
        j - l <= 5
        k - h <= 10
        k - m <= 10
        l - m <= 2

END
```

d) Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).

The values are listed in the chart below. The row of letter is the starting vertex and the column of letters is the destination. NP represents no path.

Shortest Paths Including a Pass Through Vertex I

	A	B	C	D	E	F	G	H	I	J	K	L	M
A	28	26	26	23	30	22	25	25	20	22	35	NP	NP
B	30	28	28	25	32	24	27	27	22	24	37	NP	NP
C	31	29	29	26	33	25	28	28	23	25	38	NP	NP
D	36	34	34	31	38	30	33	33	28	30	43	NP	NP
E	37	35	35	32	39	31	34	34	29	31	44	NP	NP
F	34	32	32	29	36	28	31	31	26	28	41	NP	NP

G	36	34	34	31	38	30	33	33	28	30	43	NP	NP
H	24	22	22	19	26	18	21	21	16	18	31	NP	NP
I	8	6	6	3	10	2	5	5	0	2	15	NP	NP
J	10	8	8	5	12	4	7	7	2	4	17	NP	NP
K	14	12	12	9	16	8	11	11	6	8	21	NP	NP
L	15	13	13	10	17	9	12	12	7	9	22	NP	NP
M	17	15	15	12	19	11	14	14	9	11	24	NP	NP

Lindo code:

```
MAX 12i -a - b - c - d - e - f - g - h - j - k - l - m
ST
      a - b <= 2
      a - c <= 3
      a - d <= 8
      a - h <= 9
      b - a <= 4
      b - c <= 5
      b - e <= 7
      b - f <= 4
      c - d <= 10
      c - b <= 5
      c - g <= 9
      c - i <= 11
      c - f <= 4
      d - a <= 8
      d - g <= 2
      d - j <= 5
      d - f <= 1
      e - h <= 5
      e - c <= 4
      e - i <= 10
      f - i <= 2
      f - g <= 2
      g - d <= 2
      g - j <= 8
      g - k <= 12
      h - i <= 5
      h - k <= 10
      i - a <= 20
      i - k <= 6
      i - j <= 2
      i - m <= 12
      j - i <= 2
      j - k <= 4
      j - l <= 5
      k - h <= 10
      k - m <= 10
```

```
      l - m <= 2

END
```

```
MAX a + b + c + d + e + f + g + h + j + k - 10i
ST
      a - b <= 2
      a - c <= 3
      a - d <= 8
      a - h <= 9
      b - a <= 4
      b - c <= 5
      b - e <= 7
      b - f <= 4
      c - d <= 10
      c - b <= 5
      c - g <= 9
      c - i <= 11
      c - f <= 4
      d - a <= 8
      d - g <= 2
      d - j <= 5
      d - f <= 1
      e - h <= 5
      e - c <= 4
      e - i <= 10
      f - i <= 2
      f - g <= 2
      g - d <= 2
      g - j <= 8
      g - k <= 12
      h - i <= 5
      h - k <= 10
      i - a <= 20
      i - k <= 6
      i - j <= 2
      i - m <= 12
      j - i <= 2
      j - k <= 4
      j - l <= 5
      k - h <= 10
      k - m <= 10
      l - m <= 2

END
```