Unit - 5

M.O.I. and S.F. & B.M. Diagram

Centre of Gravity:-

Centre of gravity of a body may be defined as the point at which whole weight of the body assumed to be concentrated.

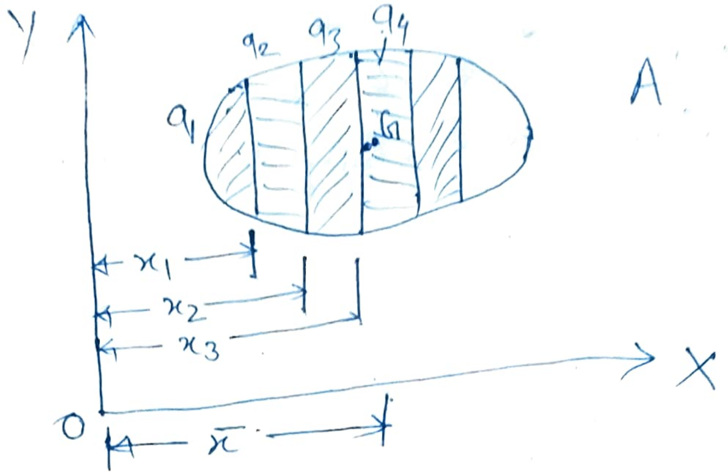
Centroid:-

The entire area of plane figures like triangle, quadrilateral, circle etc. may assumed to be concentrated at a point, which is known as centroid of the area.

centroid is used for geometrical figures like line, areas and volumes and depends only on the geometry of the body. While centre of gravity is used for physical bodies like wires, plates and solids depends upon physical properties of the body

Location of centroid of a plane Lamina:

Let A be the whole area of given figure.



It can be considered to be composed of a no. of small areas a_1, a_2, a_3, a_4 etc.

$$A = a_1 + a_2 + a_3 + a_4 + \cdots$$

Let OX and OY be the reference axes.

$x_1, x_2, x_3, x_4, \ldots$ = The distances of centroid of areas $a_1, a_2, a_3, a_4 \ldots$ from axis OY

$y_1, y_2, y_3, y_4, \ldots$ = The distances of centroid of areas $a_1, a_2, a_3, a_4 \ldots$ from axis OX respectively.

Thus moments of all small areas about axis OY

$$= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \cdots \quad \text{—(1)}$$

[∵ Moment = Area × centroid or
= Force × Perpendicular distance

Now if G be the centroid of total area A, whose distance from the axis OY is $\bar{x}$, then Moment of total area about OY axis

$$= A \cdot \bar{x} \quad \text{——— (II)}$$

Equating eq (I) & (II)

$$A \cdot \bar{x} = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + \cdots$$

$$\Rightarrow \boxed{\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + \cdots}{A}} \quad \text{——— (III)}$$

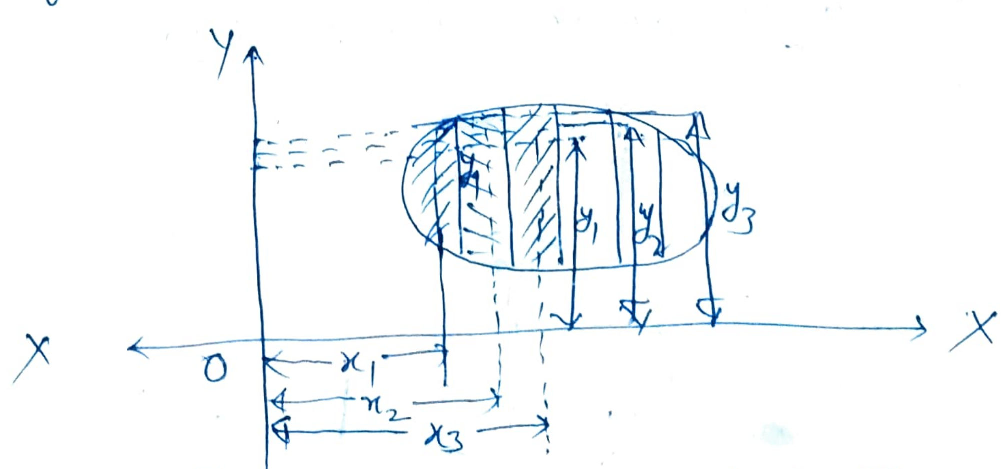
Similarly,

$$\boxed{\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4 + \cdots}{A}} \quad \text{——— (IV)}$$

Eqns (III) and (IV) give the location of centroid of plane figure.

Centre of Gravity by Method of moments:

Consider a body of mass M, whose C.G. is required to be find out.



Divide the whole body into small masses m_1, m_2, m_3, ... etc., whose centre of gravity are known. Let (x_1, y_1), (x_2, y_2), (x_3, y_3) ... etc. are the co-ordinates of C.G. of masses m_1, m_2, m_3 ... from a fixed point O.

If $\bar{x}$ and $\bar{y}$ be the co-ordinates of the C.G. of the whole body, then by the principle of moment,

$$M \cdot \bar{x} = m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots = \Sigma m \cdot x$$

$$\Rightarrow \boxed{\bar{x} = \frac{\Sigma m x}{M}}$$

Similarly,

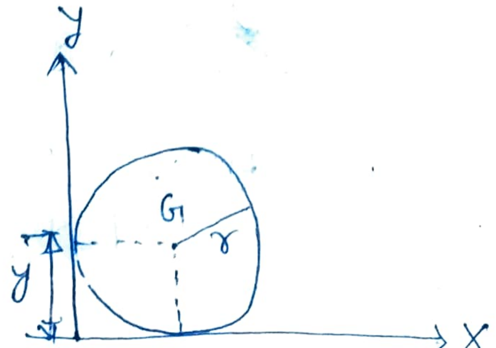
$$\boxed{\bar{y} = \frac{\Sigma m \cdot y}{M}}$$

where,

$$M = m_1 + m_2 + m_3 + \cdots$$

Centroid of Some Standard Geometrical Shapes:-

① Circle:-



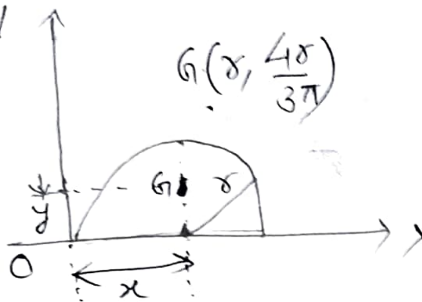
$\boxed{\text{Area} = \pi \cdot r^2}$

Centroid :- position → centre of circle

$\boxed{\bar{x} = r}$ $\boxed{\bar{y} = r}$

② semi-circle :-

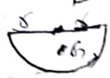
(a)



$\boxed{A = \frac{\pi r^2}{2}}$

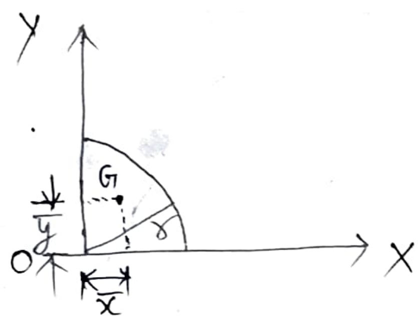
(b)  $G\left(\frac{4r}{3\pi}, r\right)$

$\boxed{\bar{x} = r}$, $\boxed{\bar{y} = \frac{4r}{3\pi}}$

(c)  $G\left(r, r - \frac{4r}{3\pi}\right)$

(d)  $G\left(r - \frac{4r}{3\pi}, r\right)$

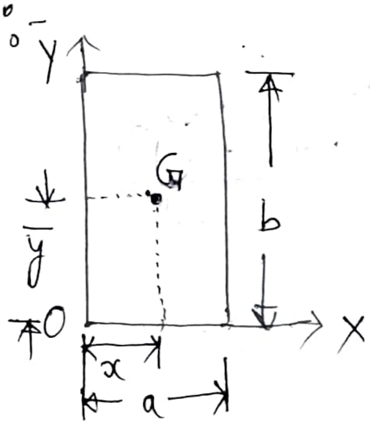
③ Quarter circle :-



$\boxed{A = \frac{\pi \cdot r^2}{4}}$

$\boxed{G\left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)}$

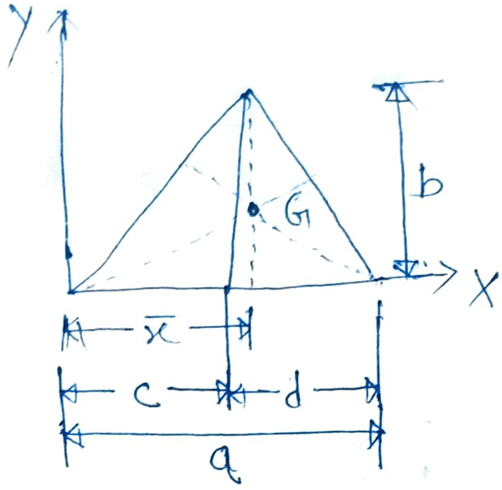
④ Rectangle :-



$\boxed{A = a \times b}$

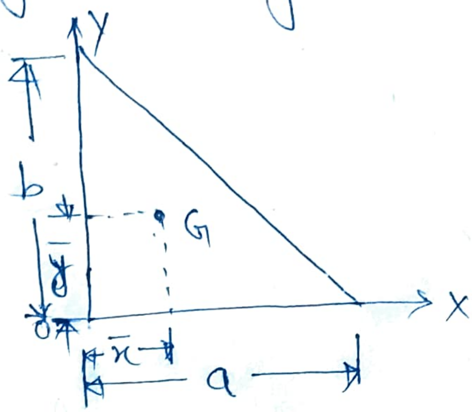
$\boxed{G\left(\frac{a}{2}, \frac{b}{2}\right)}$

(5) Triangle :-



$$\boxed{G\left(\frac{c+a}{3}, \frac{b}{3}\right)}$$

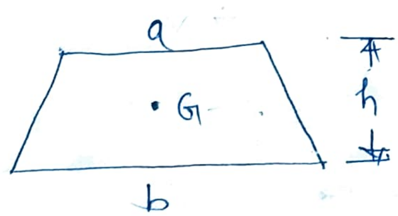
(6) Right Angle Triangle :-



$$\boxed{A = \frac{a \times b}{2}}$$

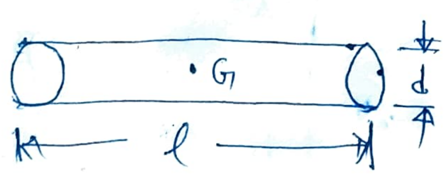
$$\boxed{G\left(\frac{a}{3}, \frac{b}{3}\right)}$$

(7) Trapezium :-



$$G\left[\frac{b}{2}, \frac{h}{2}\left(\frac{b+2a}{b+a}\right)\right]$$

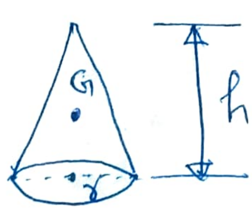
(8) Uniform Rod :-



$$\boxed{V = \frac{\pi}{4} d^2 \times l}$$

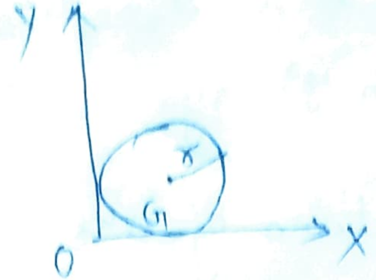
$$\boxed{G\left(\frac{l}{2}, \frac{d}{2}\right)}$$

(9) Right circular Solid cone :-



$$G\left(r, \frac{h}{4}\right)$$

(10) Sphere :-



$$\boxed{V = \frac{4}{3}\pi r^3}$$

$$\boxed{G(r, r)}$$

(11) Hemisphere :-

$$\boxed{V = \frac{2}{3}\pi r^3}$$

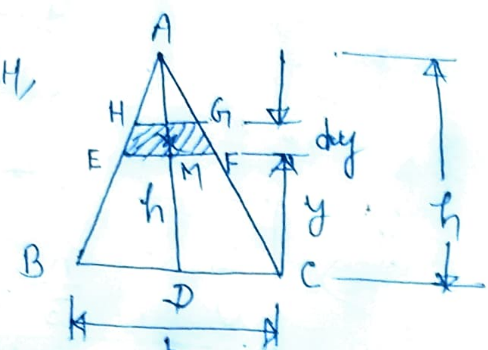
$$\boxed{G\left(r, \frac{3r}{8}\right)}$$

Prob. 1. Derive the expression for centroid of a triangular area of base (b) and height (h).

Soln. A triangle of base b and height h is shown in fig. A perpendicular DA is drawn from point D on base BC. Now consider an elementary strip of width x and height dy at a distance y from the base.

Area of elementary strip EFGH,

$$da = x \cdot dy \quad \text{——— (i)}$$



Now from geometry of fig. (ΔAEF & ΔABC), we have,

$$\frac{EF}{BC} = \frac{AM}{AD} \quad \text{or} \quad EF = BC \times \frac{AM}{AD}.$$

$$\text{or} \quad x = b \times \left(\frac{h-y}{h}\right) \quad \text{——— (ii)}$$

Substituting value of x in eqn (i), we get,

$$da = \frac{b}{h}(h-y)\cdot dy \qquad \text{(iii)}$$

Now location of centroid of the triangle will be given by,

$$\bar{y} = \frac{\int y\, da}{A} \qquad \left[\because A = \frac{b\cdot h}{2}\right]$$

$$\because \bar{y} = \frac{a_1 y_1 + a_2 y_2 + \cdots}{a_1 + a_2} = \frac{\Sigma ay}{A} = \frac{\int y\, da}{A}$$

$$= \frac{\int_0^h y\cdot \frac{b}{h}(h-y)\cdot dy}{\frac{b\cdot h}{2}}$$

$$= \frac{2}{h^2}\int_0^h (hy - y^2)\, dy$$

$$= \frac{2}{h^2}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$= \frac{2}{h^2}\left[\frac{h\cdot h^2}{2} - \frac{h^3}{3} - 0 + 0\right]$$

$$= \frac{2}{h^2}\left[\frac{h^3}{2} - \frac{h^3}{3}\right]$$

$$= \frac{\cancel{2}^{1}}{\cancel{h^2}} \times \frac{h^{\cancel{3}}}{\cancel{6}_3}$$

$$\Rightarrow \boxed{\bar{y} = \frac{h}{3}}$$

Thus the centroid of a triangle lies at a distance $h/3$ from its base.

Prob2. Determine the centroid of a 100mm X 150mm X 30mm T-section.

Solution:

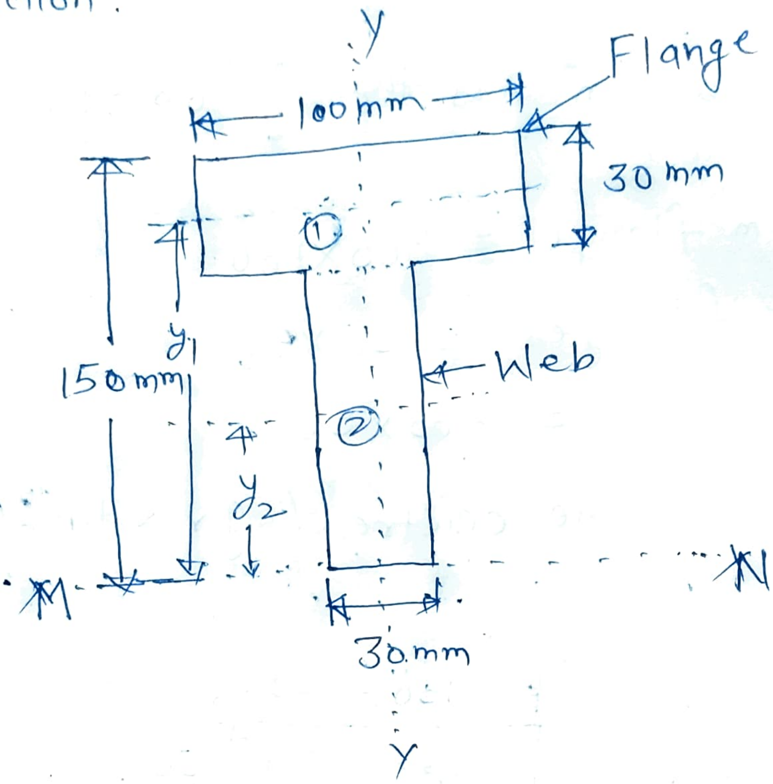


Fig. T-Section

Section is symmetrical about Y-Y axis;

$a_1 = 100\,mm \times 30\,mm = 3000\,mm^2$

$y_1 = 150 - \frac{30}{2} = 135\,mm$

$a_2 = (150 - 30) \times 30 = 3600\,mm^2$

$y_2 = \frac{150 - 30}{2} = 60\,mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$= \frac{3000 \times 135 + 3600 \times 60}{3000 + 3600}$

$\boxed{\bar{y} = 94.09\,mm}$ Ans.

Q.3 Determine the centroid of a 120mm x 250mm x 40mm T-section.

Q.4. An I-section has the following dimensions in mm units:

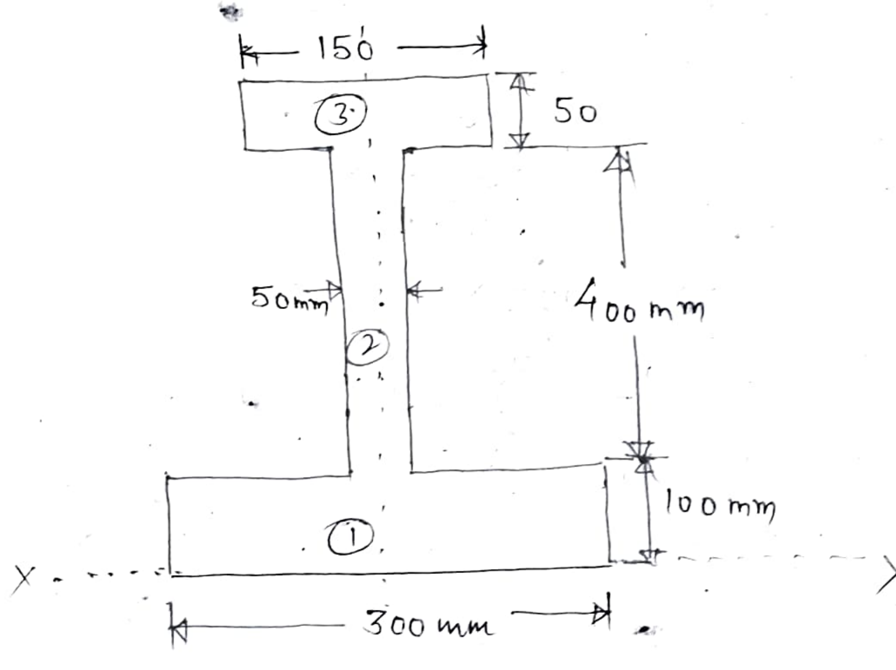
Bottom flange = 300 X 100

Top flange = 150 X 50

Web = 400 X 50

Determine the centre of gravity of given section.

Sol^n



Given I-section is symmetrical about y-y axis.

i) Bottom flange:

$a_1 = 300 \times 100 = 30000 \text{ mm}^2$

$y_1 = \frac{100}{2} = 50 \text{mm}$

ii) Web

$a_2 = 400 \times 50 = 20000 \text{ mm}^2$

$y_2 = 100 + \frac{400}{2} = 300 \text{ mm}$

(ii) Top Flange

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

$$y_3 = 100 + 400 + \frac{50}{2} = 525 \text{ mm}$$

Now centroid of given I-section;

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{30000 \times 50 + 20000 \times 300 + 7500 \times 525}{30000 + 20,000 + 7500}$$

$$\boxed{\bar{y} = 198.91 \text{ mm}}$$ Ans.

Q. 5. An I-section has the following dimensions in mm units:

Bottom flange = 400×120

Top flange = 200×60

Web = 500×80

Q. 6. Find the C.G or centroid of an inverted T-section with flange 60mm × 10mm and web 50mm × 10mm.

Soln. The inverted T-section is shown in figure. Section is symmetrical about Y-Y axis, its C.G. will lie on this axis.

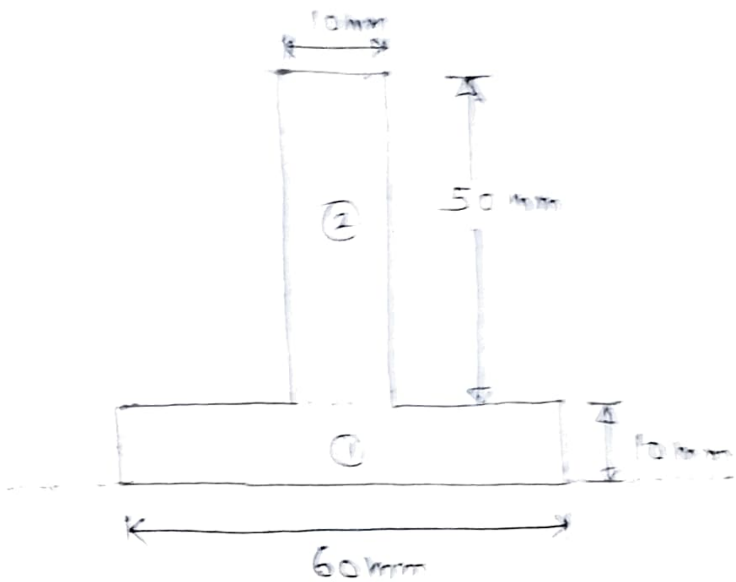


Fig. Inverted T- section

(i) Flange

$a_1 = 60 \times 10 = 600 \, mm^2$

$y_1 = \frac{10}{2} = 5 \, mm$

(ii) Web

$a_2 = 50 \times 10 = 500 \, mm$

$y_2 = 10 + \frac{50}{2} = 35 \, mm$

Distance of C.G. from bottom of the T-section is given by

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{600 \times 5 + 500 \times 35}{600 + 500}$$

$$\boxed{\bar{y} = 18.636 \, mm}$$

Q.7. Find the centroid (C.G.) of an inverted T-section with flange 160 mm X 18 mm and web 150 mm X 18 mm

Q. 8. A channel section 300 mm X 100 mm is 20 mm thick. Find the centroid of the section.

Solution.

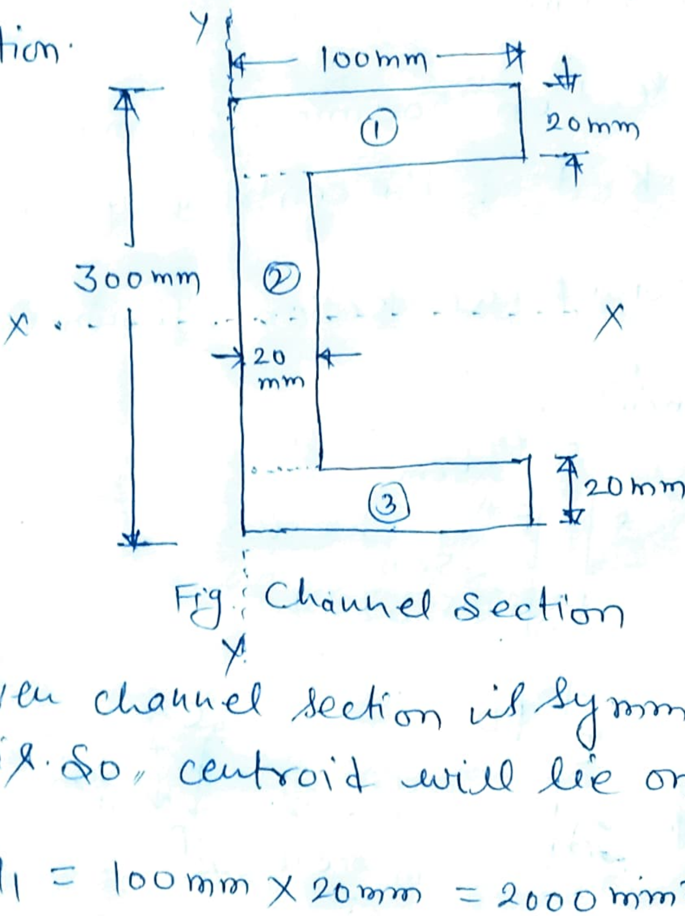


Fig: Channel Section

Given channel section is symmetrical about X-X axis. So, centroid will lie on this axis.

(i) $a_1 = 100\,mm \times 20\,mm = 2000\,mm^2$

$x_1 = \frac{100}{2} = 50\,mm$

(ii) $a_2 = (300 - 20 - 20) \times 20 = 260 \times 20 = 5200\,mm^2$

$x_2 = \frac{20}{2} = 10\,mm.$

(iii) $a_3 = 100 \times 20 = 2000\,mm^2$

$x_3 = \frac{100}{2} = 50\,mm$

Distance of C.G. from Y-Y axis,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

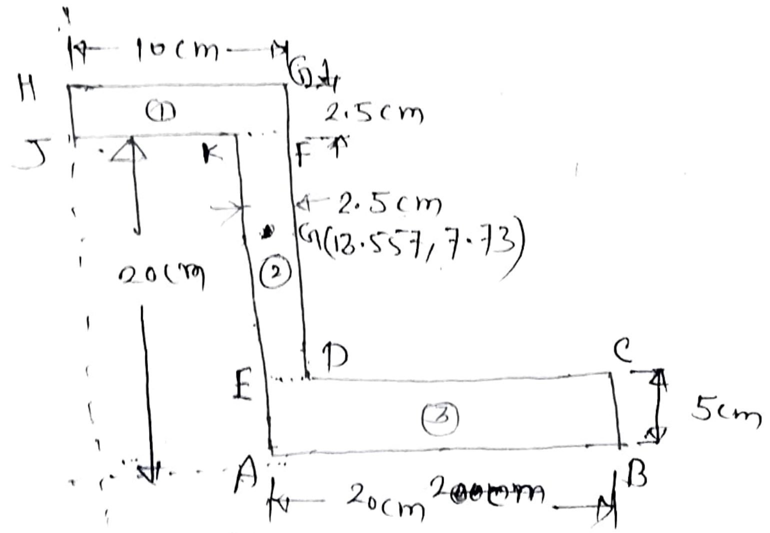
$$= \frac{2000 \times 50 + 5200 \times 10 + 2000 \times 50}{2000 + 5200 + 2000}$$

$= 27.39$ mm

Q. 9. A channel section 500 mm × 200 mm is 30 mm thick. Find the centroid of the section.

Q. 10. Find the C.G. of the section shown in fig.

Sol^n



(i) Rectangle HGFJ.

$a_1 = 10 \times 2.5 = 25 \text{ cm}^2$

$x_1 = \frac{10}{2} = 5 \text{ cm}$

$y_1 = 20 + \frac{2.5}{2} = 21.25 \text{ cm}$

(ii) Rectangle KFDE

$a_2 = (20-5) \times 2.5 = 37.5 cm^2$

$x_2 = (10-2.5) + \frac{2.5}{2} = 8.75 cm$

$y_2 = 5 + \left(\frac{20-5}{2}\right) = 12.5 cm$

(iii) Rectangle ABCE

$a_3 = 20 \times 5 = 100 cm^2$

$x_3 = (10-2.5) + \frac{20}{2} = 17.5 cm$

$y_3 = \frac{5}{2} = 2.5 cm$

$$\therefore \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{25 \times 5 + 37.5 \times 8.75 + 100 \times 17.5}{25 + 37.5 + 100}$$

$\boxed{\bar{x} = 13.557 cm}$ Ans.

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{25 \times 21.5 + 37.5 \times 12.5 + 100 \times 2.5}{25 + 37.5 + 100}$$

$\boxed{\bar{y} = 7.73 cm}$

Q. 11. Detn the C.G. of the plane lamina shown in fig.

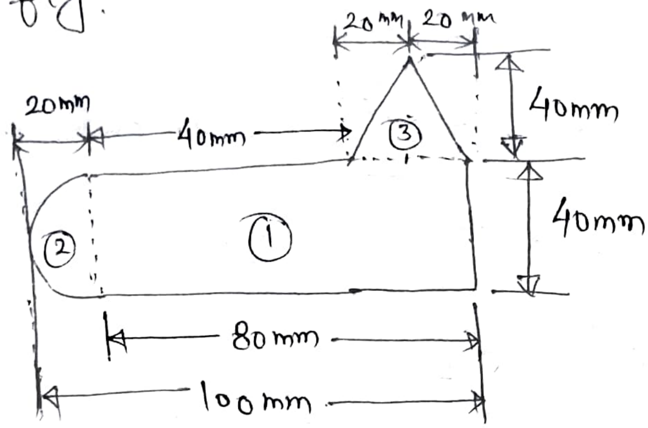


Fig.

Soln. Given lamina can be divided in to three parts viz. ① rectangle, ② semi-circle and ③ triangle as shown in figure

Let left edge of the circular portion and bottom face of the rectangular portion be the axes of reference.

① Rectangular portion

$$a_1 = 80 \times 40 = 3200 \text{ mm}^2$$

$$x_1 = 20 + \frac{80}{2} = 60 \text{ mm}$$

$$y_1 = \frac{40}{2} = 20 \text{ mm}$$

(ii) Semi-circular portion:

$$a_2 = \frac{\pi r^2}{2} = \frac{\pi \times 20^2}{2} = 628.31 \text{ mm}^2$$

$$x_2 = r - \frac{4r}{3\pi} = 20 - \frac{4 \times 20}{3 \times \pi} = 11.51 \text{ mm}$$

$$y_2 = r = 20 \text{ mm}$$

(ii) Triangular portion

$a_3 = \frac{1}{2} \times 40 \times 40 = 800 mm^2$

$x_3 = 20 + 40 + 20 = 80$ mm

$y_3 = 40 + \frac{40}{3} = 53.33$ mm

Now distance between C.G. of the section and left edge of circular section;

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{3200 \times 60 + 628.31 \times 11.51 + 800 \times 80}{3200 + 628.31 + 800}$$

$$\boxed{\bar{x} = 56.87 mm}$$

Now distance between C.G. of the section and bottom face of the rectangular portion,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{3200 \times 20 + 628.31 \times 20 + 800 \times 53.33}{3200 + 628.31 + 800}$$

$$\boxed{\bar{y} = 25.76 mm}$$ Ans

Moment of Inertia

Moment of a force about any axis is the product of the force (F) and perpendicular distance (x) between the axis and line of action of force i.e.

$$\boxed{\text{Moment} = F.x}$$

If this moment is again multiplied by the same perpendicular distance (x), the product so obtained called moment of the moment of a force or second moment of force or 'Moment of Inertia'.

If instead of force, area of a plane figure or mass of body is taken into consideration, then second moment is known as moment of inertia of plane area or mass moment of inertia respectively.

Moment of Inertia of plane Area:

Moment of inertia of a plane area about a given axis is the sum of the products of the elementary areas into which the given area can be sub-divided and the squares of the distance of the centroids of these elementary areas from that axis.

Consider a plane area whose moment of inertia is required to be found out. Let this area be splitted in to elementary areas $a_1, a_2, a_3, \ldots$ If $r_1, r_2, r_3, \ldots$ are the distances of these elementary areas from the axis about which moment of Inertia have to be determined, then M.O.I. of the area will be given by,

$$\boxed{I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots = \Sigma a r^2}$$

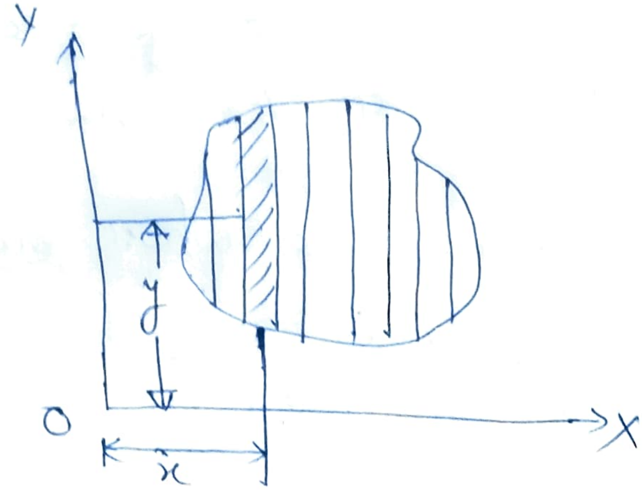
Units of M.O.I are m^4, cm^4, mm^4 etc.

Determination of Moment of Inertia :-

1. Method of Integration
2. ~~Routh~~ Routh Rule

1. Method of Integration :-

Consider a plane area as shown in fig where moment of inertia about X-X-axis and Y-Y-axis have to be determined.
Let the whole area has been divided into a no. of strips. consider an elementary strip shown by shaded in fig.



Let dA = Area of elementary strip

x = Distance of C.G. of the strip on XX-axis

y = Distance of C.G. of the strip on YY-axis

Moment of inertia of the strip about XX-axis

$$= dA \cdot y^2 \quad \text{—— (i)}$$

∴ Now M.O.I. of the whole area can be obtained by integrating the eqn (i),

$$\boxed{I_{xx} = \int dA \cdot y^2 = \Sigma dA \cdot y^2} \quad \text{—— (ii)}$$

Similarly $$\boxed{I_{yy} = \int dA \cdot x^2 = \Sigma dA \cdot x^2} \quad \text{—— (iii)}$$

(2) Routh's Rule :-

Routh's Rule is used to find out the MOI. of a body which is symmetrical about three mutually perpendicular axes. According to this rule M.O.I. of a body about any one axis passing through its C.G. is given by,

(a) For square or rectangular lamina

$$\boxed{I = \frac{A \times S}{3}}$$

(b) For circular or ~~lam~~ elliptical lamina

$$\boxed{I = \frac{A \times S}{4}}$$

where, A = Area of the lamina

S = Sum of the squares of the two semi-axes, other than the axis about which MOI is required to be foundout.

Parallel Axis Theorem :-

"M.O.I. of an area about an axis is equal to the sum of (i) moment of inertia about an axis passing through the centroid and parallel to the given axis (ii) product of area and square of the distance between the two parallel axis."

Thus If I_G is the MOI of an area about an axis passing thorough the centroid of given area and 'h' is the distance between centroid (or C.G.) and axis AB about which M.O.I. has to be determined then,

$$\boxed{I_{AB} = I_G + A \cdot h^2}$$

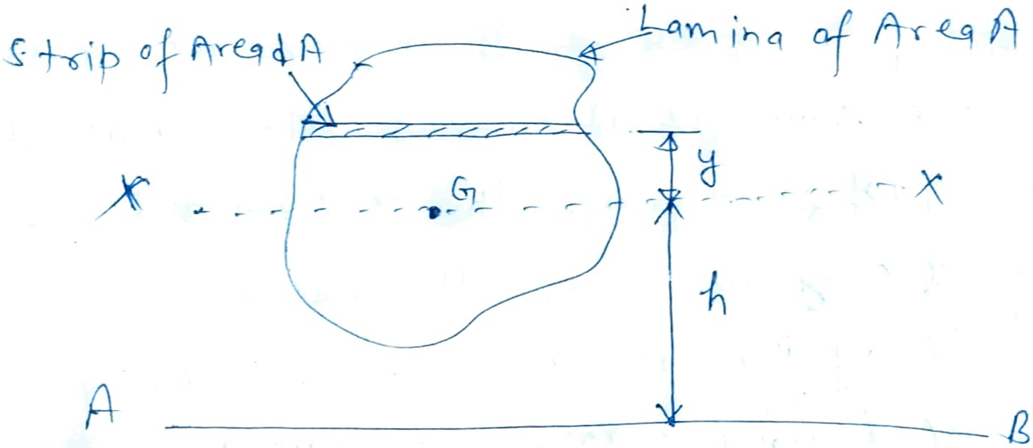
Where, I_{AB} = M.O.I. about an axis AB

I_G = M.O.I. about its C.G.

A = Area of given lamina

h = Distance between C.G. of the lamina and the axis AB

Proof:- Let a plane lamina of area A. Let XX be the axis in the plane of lamina and passing through its C.G.. AB is an axis at a distance h in the plane of given lamina and parallel to XX-axis.



Consider an elementary strip of lamina at a distan y from the XX-axis. Let the area of the strip is dA.

Now moment of inertia of elementary about XX-axis $= dA \cdot y^2$ —— (I)

Thus moment of Inertia of total area A about XX-axis

$$I_{XX} \text{ or } I_G = \Sigma dA \cdot y^2 \quad \text{—— (II)}$$

Now MOI of total area about AB,

$$I_{AB} = \Sigma dA \cdot (h+y)^2 = \Sigma dA \cdot (h^2 + y^2 + 2h \cdot y)$$

or $I_{AB} = \Sigma dA \cdot h^2 + \Sigma dA \cdot y^2 + \Sigma 2hy \cdot dA$ —— (III)

$\because \Sigma dA = A$ and from eq (II) I_{XX} or $I_G = \Sigma dA \cdot y^2$

Substituting these values in above eqn (III)

$$I_{AB} = A \cdot h^2 + I_G + 2h \cdot \Sigma dA \cdot y \quad \text{—— (IV)}$$

$\because \Sigma dA \cdot y$ represents the MOI about XX axis, but MOI is equal to the product of total area and distance of C.G. of total area from XX ~~axis~~, axis. In this case distance of the C.G. of the total area from XX-axis is zero. $\therefore \Sigma dA \cdot y = 0$

$$I_{AB} = A \cdot h^2 + I_G + 0$$

$$\boxed{I_{AB} = I_G + A \cdot h^2}$$

Proved

Perpendicular Axis Theorem :-

"Moment of Inertia of a plane lamina about an axis perpendicular to the lamina and passing through its centroid is equal to the sum of the moment of inertias of the lamina about two mutually perpendicular axes passing through the centroid and in the plane of the lamina."

If I_{XX} and I_{yy} be the moment of inertia of a plane lamina about two mutually perpendicular axes XX and YY in the plane of lamina, then the MOI of the lamina about axis-ZZ perpendicular to the lamina and passing through the intersection of XX and YY-axes is given by,

$$\boxed{I_{ZZ} = I_{xx} + I_{yy}}$$

Proof: Let a lamina of area A. Let OX and OY be the two mutually perpendicular axes lying in the plane of the lamina and OZ be an axis normal to the plane of lamina and passing through O.

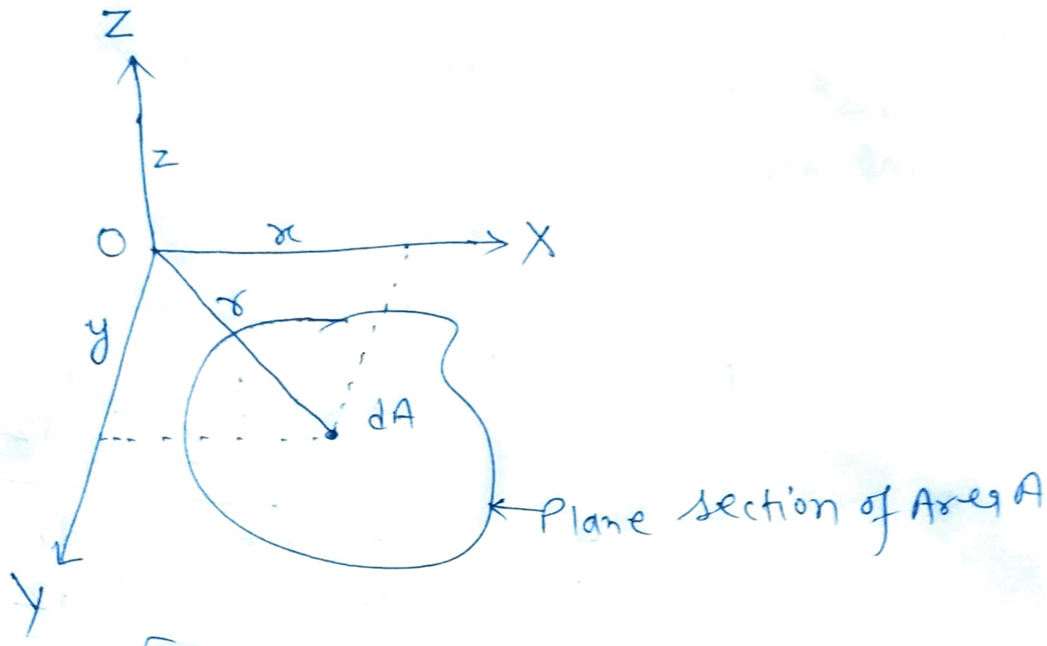


Fig.

Consider an elementary component of lamina having area dA shown in above fig.

Let

x = Distance of dA from axis OY

y = Distance of dA from axis OX

r = Distance of dA from axis OZ

From geometry of fig., we have

$$r^2 = x^2 + y^2 \qquad \text{—— (I)}$$

Now M.O.I. of dA about OX-axis $= dA \cdot y^2$

Thus, M.O.I. of total area A about OX-axis

$$I_{xx} = \Sigma\, dA \cdot y^2 \qquad \text{—— (II)}$$

Similarly, MOI of total Area A about OY-axis

$$I_{yy} = \Sigma\, dA \cdot x^2 \qquad \text{—— (III)}$$

Now MOI of total area A about OZ axis

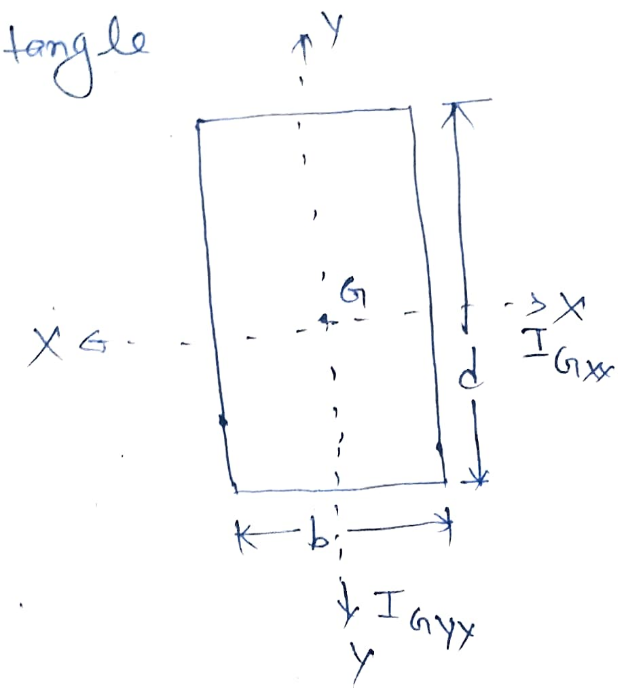
$$I_{zz} = \Sigma\, dA \cdot r^2 = \Sigma\, dA\,(x^2 + y^2) = \Sigma\, dA \cdot x^2 + \Sigma\, dA \cdot y^2$$

$$I_{zz} = I_{yy} + I_{xx}$$

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$ Proved

Moment of Inertia of Some Standard Laminas about centroidal axis –

① Rectangle

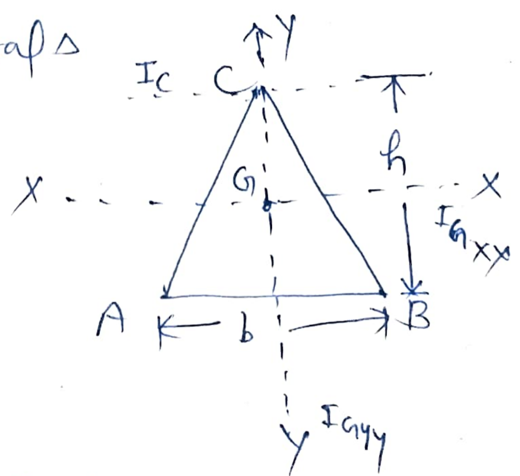


$$\boxed{I_{Gxx} = \frac{b \cdot d^3}{12}}$$

$$\boxed{I_{Gyy} = \frac{d b^3}{12}}$$

② Triangle

ⓐ Equilateral △



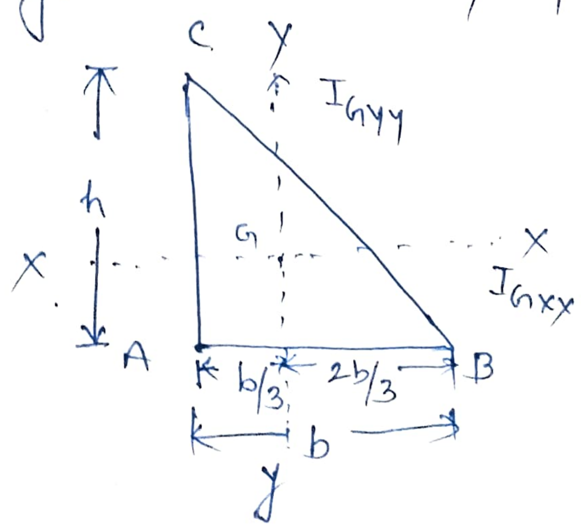
$$I_{Gxx} = \frac{b \cdot h^3}{36}$$

$$I_{Gyy} = \frac{h \cdot b^3}{36}$$

$$I_{AB} = \frac{b \cdot h^3}{12}$$

$$I_C = \frac{b \cdot h^3}{4}$$

ⓑ Right Ang △



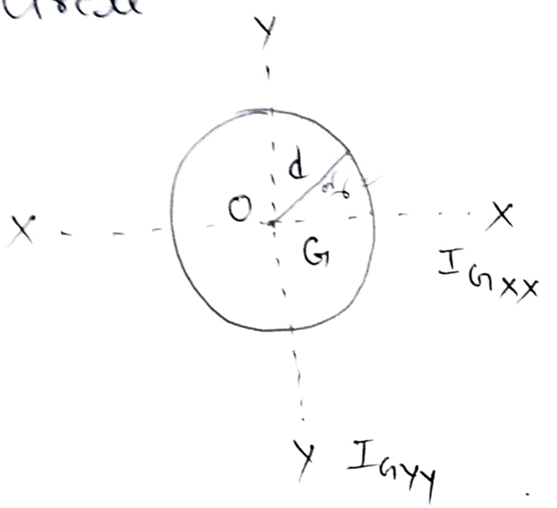
$$I_{Gxx} = \frac{b \cdot h^3}{36}$$

$$I_{Gyy} = \frac{h \cdot b^3}{36}$$

$$I_{AB} = \frac{b \cdot h^3}{12}$$

$$I_C = \frac{b \cdot h^3}{4}$$

③ Circle

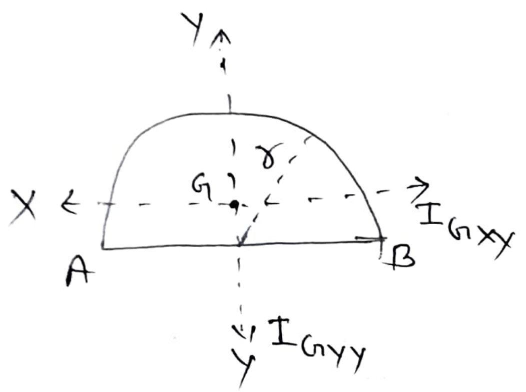


$$I_{Gxx} = \frac{\pi}{64} d^4 \text{ or } \frac{\pi}{64} r^4$$

$$I_{Gyy} = \frac{\pi}{64} d^4 \text{ or } \frac{\pi}{64} r^4$$

d = r = radius

④ Semi-circle :

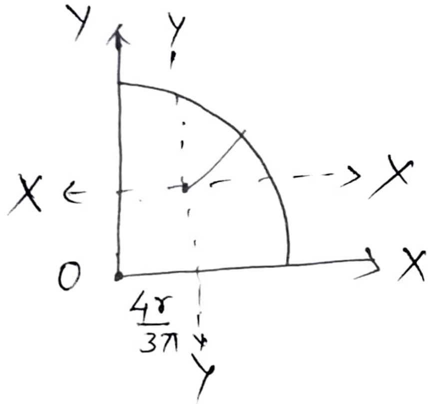


$$I_{Gxx} = 0.11 r^4$$

$$I_{Gyy} = 0.393 r^4$$

$$I_{AB} = 0.393 r^4$$

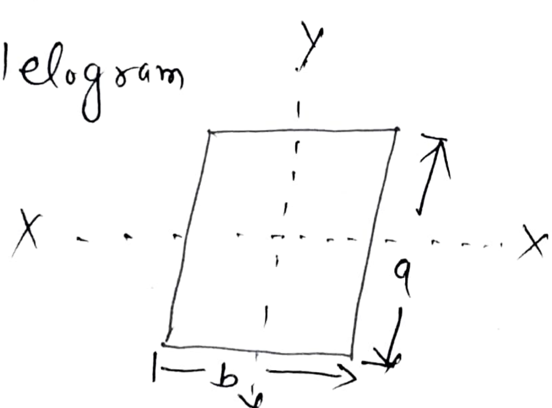
⑤



$$I_{Gxx} = 0.055 r^4 = I_{Gyy}$$

$$I_{ox} = \frac{\pi}{256} d^4 = I_{oy}$$

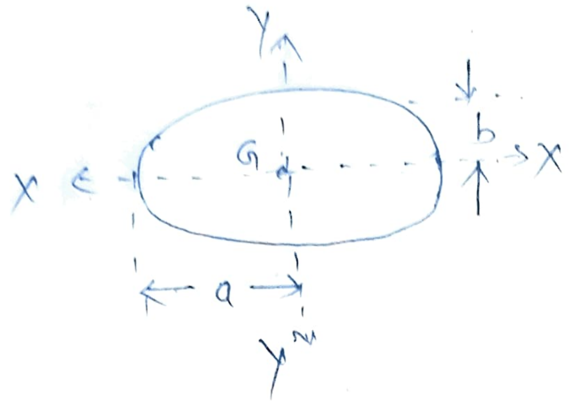
⑥ Parallelogram



$$I_{Gxx} = \frac{b \cdot a^3}{12}$$

$$I_{Gyy} = \frac{a \cdot b^3}{12}$$

(i) Elliptical

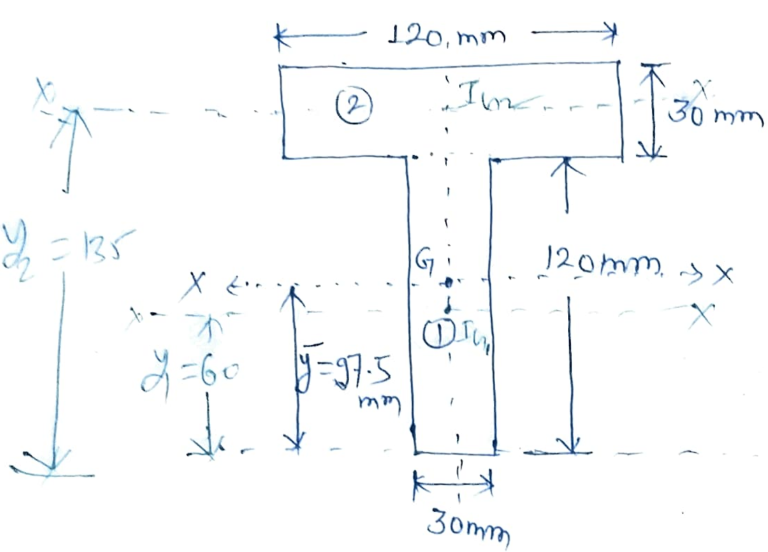


$$I_{Gxx} = \frac{\pi a . b^3}{4}$$

$$I_{Gyy} = \frac{\pi b . a^3}{4}$$

Q.1. Determine the moment of Inertia of a T-section having flange and web both 120 mm X 30 mm about XX and YY- axes passing through the C.G. of the section.

Soln.



Given T-section can be splitted into two rectangle (1) and (2) shown in fig.

(1) Rectangle

$$a_1 = 120 \times 30 = 3600 \text{ mm}^2$$

$$y_1 = \frac{120}{2} = 60 \text{ mm}$$

(ii) Rectangle (2)

$a_2 = 120 \times 30 = 3600 \text{ mm}^2$

$y_2 = 120 + \frac{30}{2} = 135 \text{ mm}$

C.G. of T section will lie at,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{3600 \times 60 + 3600 \times 135}{3600 + 3600}$$

$$\boxed{\bar{y} = 97.5 \text{ mm}}$$

M.I. of the Rectangle (1) about an axis passing through its C.G. and parallel to XX-axis,

$$I_{G_1} = \frac{b \cdot d^3}{12} = \frac{30 \times 120^3}{12} = 4320000 \text{ mm}^4$$

Distance between C.G. of Rectangle (1) and XX-axis,

$$h_1 = \bar{y} - y_1 = 97.5 - 60 = 37.5 \text{ mm}$$

M.I. of Rectangle (1) about XX-axis (By Parallel axis theorem)

$$I_{XX_1} = I_{G_1} + a_1 h_1^2.$$

$$= 4320000 + 3600 \times (37.5)^2$$

$$I_{XX_1} = 9382500 \text{ mm}^4$$

Now M.I. of Rectangle (2) about an axis passing through its C.G. and parallel to XX-axis

$$I_{G_2} = \frac{120 \times 30^3}{12} = 270000 \text{ mm}^4$$

Distance between C.G. of Rectangle (2) and XX-axis,

$$h_2 = 135 - 97.5 = 37.5 \text{ mm}$$

M.I. of Rectangle (2) about XX-axis, (By parallel axis theorem)

$$I_{XX_2} = I_{G_2} + a_2 h_2^2$$

$$= 270000 + 3600 \times (37.5)^2$$

$$I_{XX_2} = 5332500 \text{ mm}^4$$

Thus M.I. of whole section about XX-axis,

$$I_{XX} = I_{XX_1} + I_{XX_2}$$

$$= 9382500 + 5332500$$

$$= 14715000$$

$$\boxed{I_{XX} = 14715 \times 10^3 \text{ mm}^4}$$

Ans.

Now, M.I. about YY-axis :-

M.I. of Rectangle (1) about YY-axis,

$$I_{YY_1} = \frac{120 \times 30^3}{12} = 270000 \text{ mm}^4$$

M.I. of Rectangle (2) about YY-axis,

$$I_{YY_2} = \frac{30 \times 120^3}{12} = 4320000 \text{ mm}^4$$

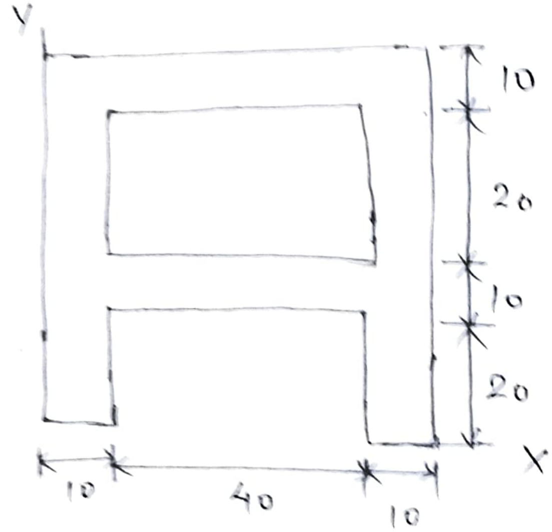
∴ M.I. of whole section about YY axis

$$I_{YY} = I_{YY_1} + I_{YY_2}$$

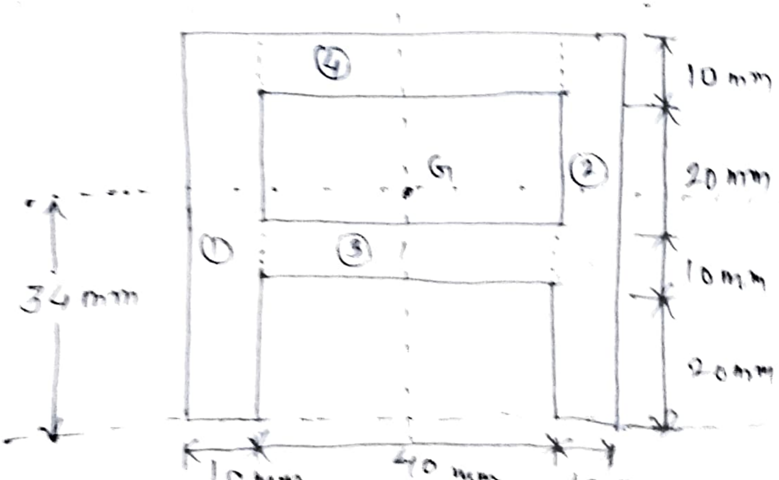
$$= 270000 + 4320000$$

$$\boxed{I_{YY} = 4590 \times 10^3 \ mm^4}$$ Ans.

Q.2. Detⁿ the M.I. of the section shown in fig. about an axis passing through its centroid and parallel to the base. All dimensions are in mm.



Solⁿ: Given section is symmetrical about YY-axis, hence centroid will be on this axis.



We can divide the given section into 4 parts as shown in fig.,

Let the bottom face of the section be axis of reference

(i) Rectangle (1)

$$a_1 = 60 \times 10 = 600 \, mm^2$$

$$y_1 = \frac{60}{2} = 30 \, mm$$

(ii) Rectangle (2)

$$a_2 = 60 \times 10 = 600 \, mm^2$$

$$y_2 = 30 \, mm$$

(iii) Rectangle (3),

$$a_3 = 40 \times 10 = 400 \, mm^2$$

$$y_3 = 20 + \frac{10}{2} = 25 \, mm$$

(iv) Rectangle (4)

$$a_4 = 40 \times 10 = 400 \, mm^2$$

$$y_4 = 50 + \frac{10}{2} = 55 \, mm$$

∴ Distance between centroid of section and bottom face of the section,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4}{a_1 + a_2 + a_3 + a_4}$$

$$= \frac{600 \times 30 + 600 \times 30 + 400 \times 25 + 400 \times 55}{600 + 600 + 400 + 400}$$

$$\boxed{\bar{y} = 34 \, mm}$$

Now M.I. of Rectangle ① w.r.t. an axis passing through its centroid and parallel to XX-axis,

$$I_{G_1} = \frac{10 \times 60^3}{12} = 180000 \text{ mm}^4$$

Distance between centroid of rectangle ① and XX-axis,

$$h_1 = 34 - 30 = 4 \text{ mm}$$

∴ M.I. of rectangle ① about XX-axis, (By parallel axis theorem)

$$I_{XX_1} = I_{G_1} + a_1 h_1^2$$

$$I_{XX_1} = 180000 + 600 \times 4^2 = 189600 \text{ mm}^4$$

As rectangles ① and ② are identical, so

$$I_{XX_2} = 189600 \text{ mm}^4$$

Now for rectangle (3),

$$I_{G_3} = \frac{40 \times 10^3}{12} = 3333.3 \text{ mm}^4$$

$$h_3 = 34 - 25 = 9 \text{ mm}$$

$$\therefore I_{XX_3} = I_{G_3} + a_3 h_3^2 = 3333.3 + 400 \times 9^2$$

$$I_{XX_3} = 35733.3 \text{ mm}^4$$

Similarly for rectangle (4)

$$I_{G_4} = \frac{40 \times 10^3}{12} = 3333.3 \text{ mm}^4$$

$$h_4 = 55 - 34 = 21 \text{ mm}$$

$\therefore I_{xx_4} = I_{G_4} + a_4 h_4^2$

$= 3333.3 + 400 \times 21^2$

$I_{xx_4} = 179733.3 \text{ mm}^4$

So, the total M.I. of the whole section about XX-axis,

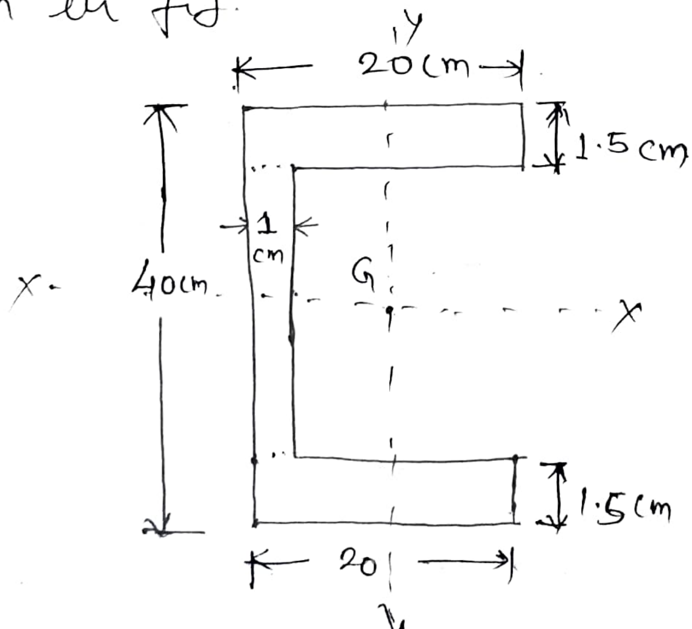
$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} + I_{xx_4}$

$= 189600 + 189600 + 35733.3 + 179733.3$

$I_{xx} = 594666.6 \text{ mm}^4$

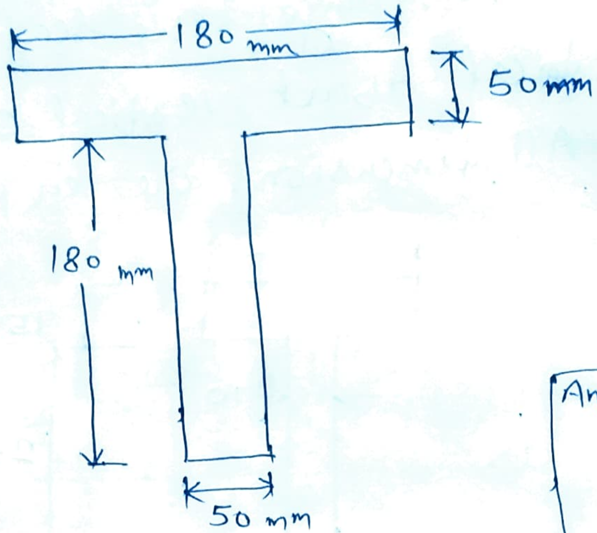
$\boxed{I_{xx} = 59.47 \times 10^4 \text{ mm}^4}$ Ans.

Q.3. Deth the moment of Inertia about centroidal axis-XX of the channel section shown in fig.



Ans. $\boxed{I_{xx} = 6130.84 \text{ cm}^4}$

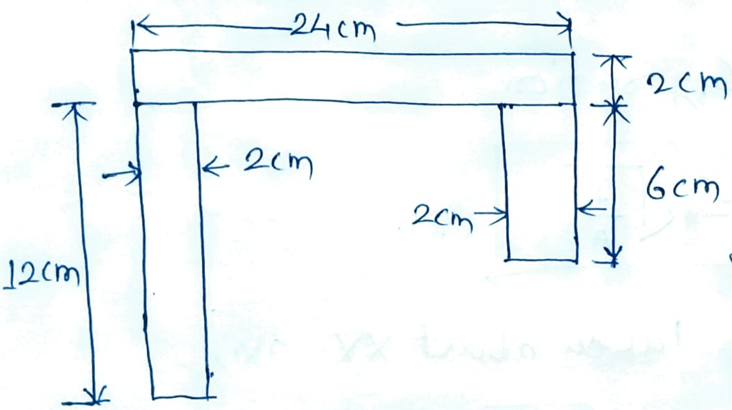
Q.4. Find the Moment of Inertia of a T-section as shown in fig. All dimensions are in mm.



Ans. $I_{xx} = 85.6875 \times 10^6$ mm^4

$I_{yy} = 26.175 \times 10^6$ mm^4

Q.5. Find the moment of Inertia of the section shown in fig. about XX and YY axes passing through the centroid.

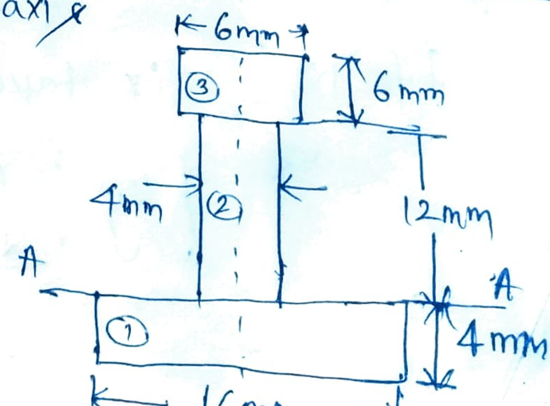


Ans,

$I_{xx} = 1152.54$ cm^4

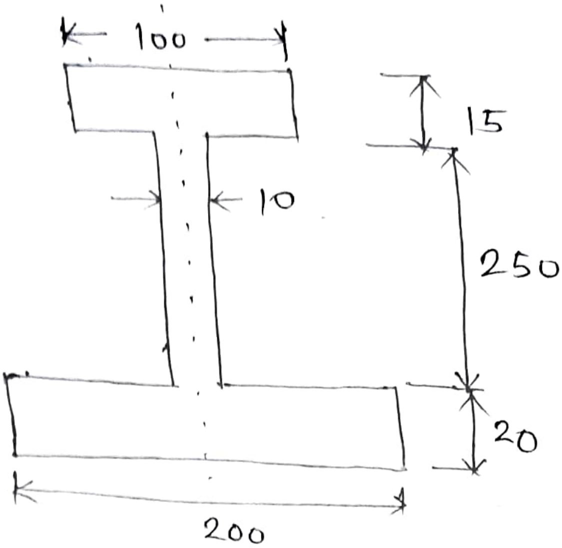
$I_{yy} = 6464.5$ cm^4

Q.6 Find the M.I. about AA-axis for the cross-section shown in fig.



Ans. $I_{AA} = 10853.33$ mm^4.

7. Find the moment of Inertia of the section shown in fig. about centroidal XX-axis parallel to flange. The section is symmetrical about vertical centroidal axis. All dimensions are in mm.



Ans. $\boxed{I_{XX} = 90876236.6\ mm^4}$

Radius of Gyration:

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

If M.I. is taken about XX-axis,

$$K_x = \sqrt{\frac{I_{XX}}{A}}$$

If M.I. is taken about YY-axis,

$$\boxed{K_y = \sqrt{\frac{I_{yy}}{A}}}$$

Mass Moment of Inertia of Solid Bodies:

Total Mass moment of Inertia of the body about axis AA will be,

$$\boxed{I_{AA} = \int dM r^2}$$

eg Mass moment of Inertia of a right circular Cone

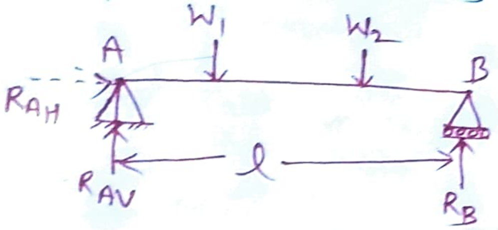
$$\boxed{I = \frac{3m}{20}(r^2 + 4 \cdot h^2)}$$

Unit-5

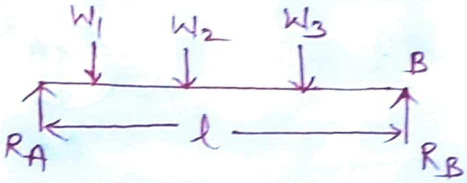
Beam:- "Beam is a rigid horizontal part of any structure" or "A member which bends under external loads applied transversely to the axis of the member is known as beam." The internal forces that appear in beams are bending moment and shear forces.

Types of Beam :- ① Simply supported Beam ② Cantilever beam ③ Overhanging beam ④ Fixed beam ⑤ Continuous beam

① **Simply supported beam:-** A beam supported by a hinge or roller on smooth surface at the ends having one span is known as simply supported beam.



or



② **Cantilever Beam:-** A beam, fixed at one end while free at the other, is known as cantilever beam. It can not rotate or move transversely.

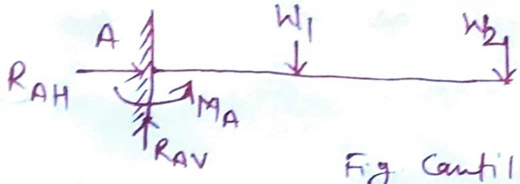


Fig Cantilever beam

③ **Overhanging Beam:-** In simply suported beam, one or both ends may overhang i.e. project beyond the supports. Such beams are called overhanging beam.

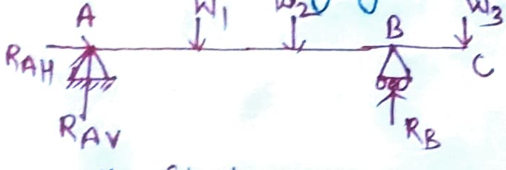


Fig Single overhanging beam

or

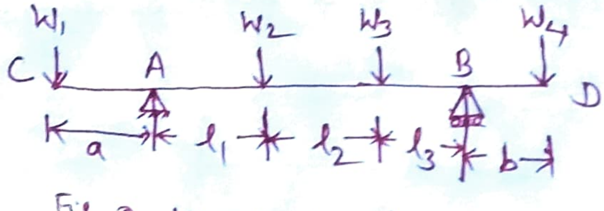


Fig Double overhanging

④ **Fixed Beam:-** A fixed beam is one whose both ends are rigidly fixed into its supporting wall or columns.

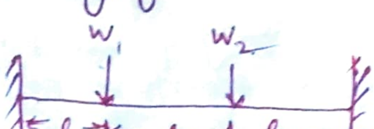


Fig. Fixed Beam

(5) Continuous Beam :- A continuous beam is one which has more than two supports eg. bridge

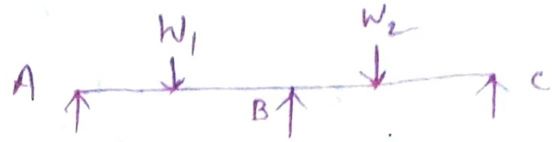
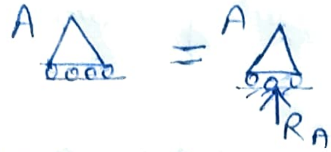
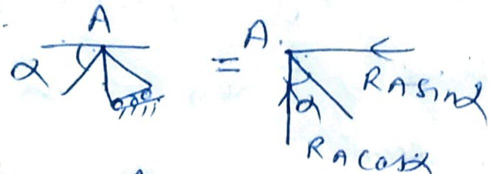


Fig. continuous beam

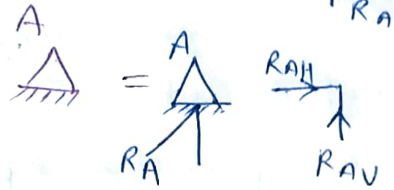
Types of supports :-

(1) Roller support :- 

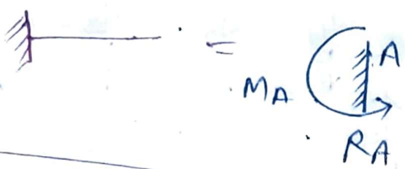
Reaction
Unknowns
= 1 Unknown = 1 force

(2) Inclined Roller Support :- 

= 2 (R_{AH}, R_{AV})

(3) Hinge or pin support :- 

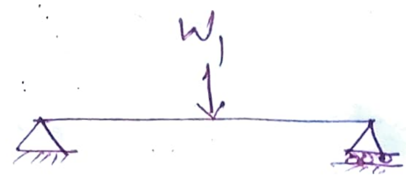
= 2 (R_{AH}, R_{AV})

(4) Fixed :- 

= 3 (R_{AH}, R_{AV}, M_A)

Types of Loads :-

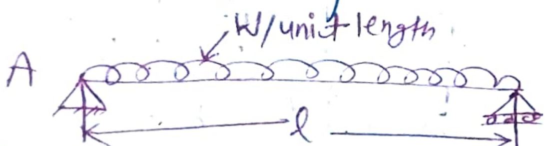
(1) Point or Concentrated Load :- If a load is acting on a beam over a very small length compared a span is called point load.



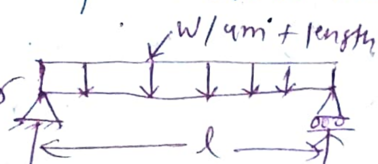
eg. Load occurring when a person stands on the beam.

(2) Uniformly Distributed Load (UDL) :- In this load, the intensity remains constant over a considerable length. Its unit is N/m

eg. Self weight of a beam of uniform cross-section



or



(3) Uniformly Varying Load (UVL) :-
When intensity of load is varying linearly over a considerable length, it is known as UVL.
eg. Water pressure acting on the side wall of water tank.

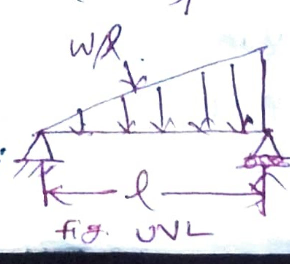


Fig. UVL

Shear force:- "The shear force at any cross-section of a beam may be defined as the" Algebraic sum of all unbalanced vertical forces to the right or left of the section."

Shear force produces shearing stress. ~~Let~~ ~~fs~~

Torsion eqn is -

$$\boxed{\frac{T}{J} = \frac{f_s}{r}},$$

where,

T = Torque , f_s = shear stress produced

J = Polar M.O.I. , r = Radius of component

Bending Moment:- The internal moment is known as Bending moment. Bending moment at the cross-section of a beam is defined as the 'algebraic sum of the unbalanced moments of all the forces, to the right or left of the section."

When a body is subjected to vertical forces, it will produce bending moment due to which bending stress are produced. Bending eqn is given as:

$$\boxed{\frac{M}{I} = \frac{f_b}{y}}$$

where, f_b = Bending stress

M = Bending Moment

I = M.I. of area of the cross section of Beam

y = Distance b/w neutral axis and extreme fibre of beam

Sign Convention:-

(1) For Shear Force:-

(a) For Right Portion of Beam:- Right Downward = +Ve

Right Upward = −Ve

(b) For Left Portion of Beam:- Left downward = −Ve

Left Upward = +Ve

(2) For Bending Moment:-

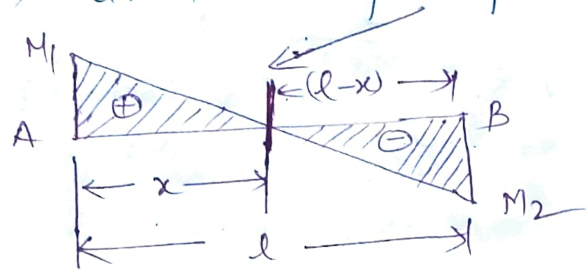
⊕ Ve

Sagging Moment

⊖ Ve

Hogging Moment

Point of Contraflexure:- It is the point where bending moment changes its sign. This point can be located by equating the bending moment equation to zero. This point occurs in the portion where bending moment changes its sign i.e. from +ve to −ve or vice-versa. This point is also known as point of inversion or point of inflexion.



Q.1. Draw the shear force and bending moment diagrams for the beam loaded as shown in fig. Clearly mark the position of maximum BM and determine its value.

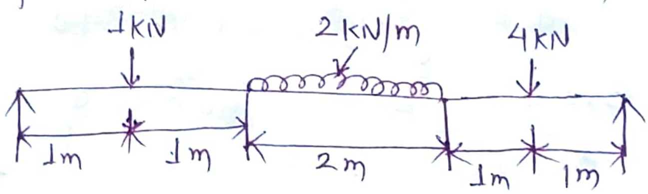


Fig. ①

Soln:- Step-I:- Calculation of Support Reaction:-

Let R_A and R_B are reactions at A and B respectively. Now consider the equilibrium of beam and apply conditions of equilibrium.

$\Sigma V = 0$

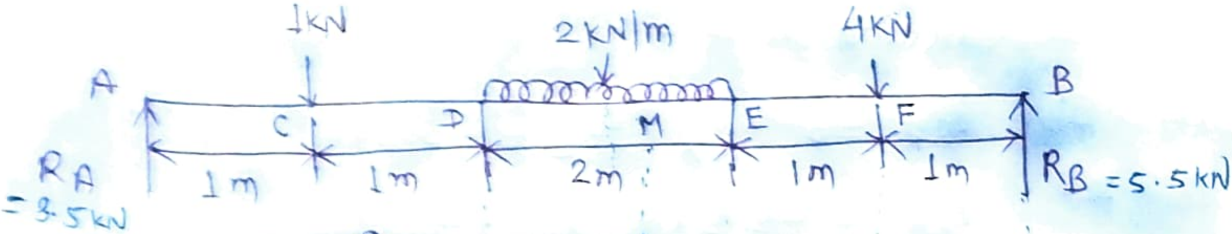
$R_A + R_B - 1 - 2\times2 - 4 = 0$

$\Rightarrow R_A + R_B = 9$ ——— ①

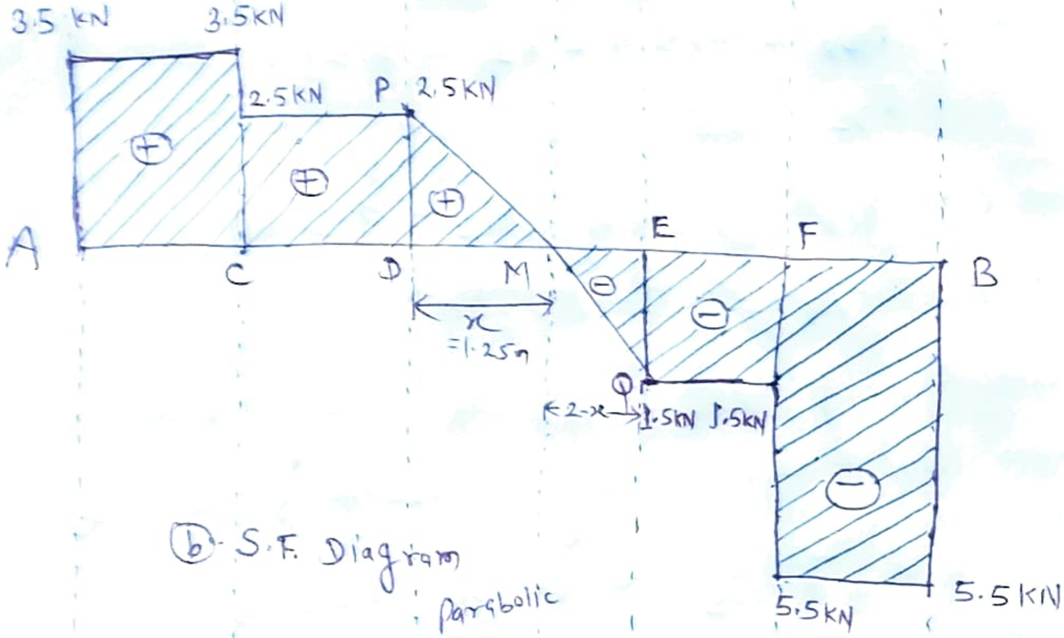
Taking moment about A, $\Sigma M_A = 0$

$R_B \times 6 - 1\times1 - 2\times2\times3 - 4\times5 = 0 \Leftrightarrow \boxed{R_B = 5.5\,KN}$

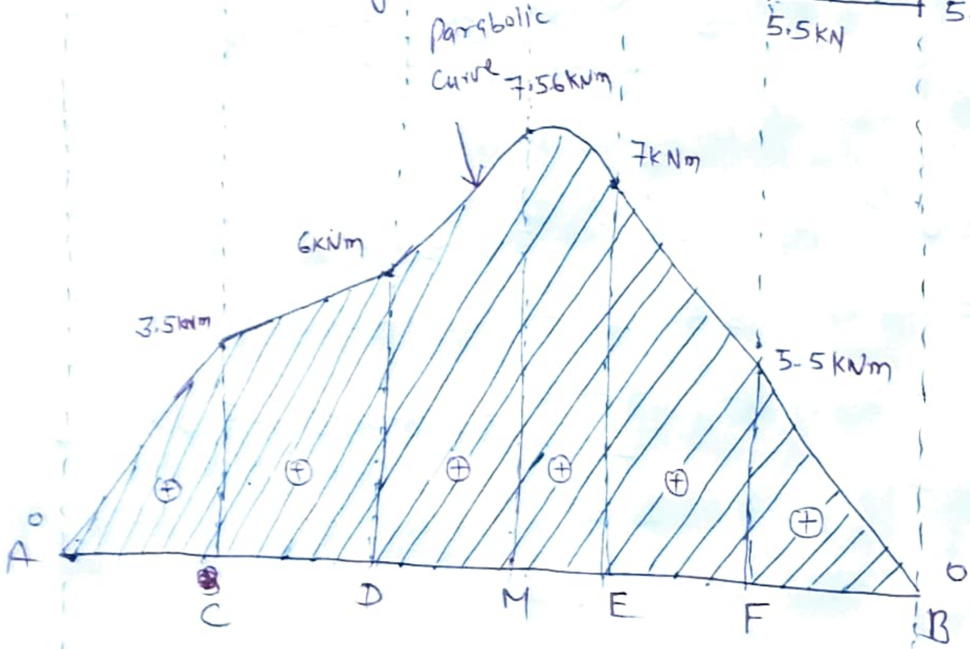
from eqn ① $R_A + 5.5 = 9 \Leftrightarrow \boxed{R_A = 3.5\,KN}$



(a) Load Diagram



(b) S.F. Diagram



(c) B.M. Diagram

Scale: 1kN = 1cm
2kNm = 1cm

Fig 1 (a), (b), (c)

Step-II :- Calculation of S.F.:-

Point B: $F_{BR} = 0$, $F_{BL} = -5.5\,kN$

Point F: $F_{FR} = -5.5\,kN$, $F_{FL} = -5.5 + 4 = -1.5\,kN$

Point E: $F_{ER} = -1.5\,kN$, $F_{EL} = -1.5\,kN$

Point D: $F_{DR} = -1.5 + 2 \times 2 = 2.5\,kN$

$F_{DL} = 3.5 - 1 = 2.5\,kN$

Point C: $F_{CR} = 2.5\,kN$, $F_{CL} = 3.5\,kN$

Point A: $F_{AR} = 3.5\,kN$, $F_{AL} = 0$

Calculation of position of Max. Bending Moment:-

From two similar triangles (ΔDPM and ΔMEQ)

$$\frac{x}{2-x} = \frac{2.5}{1.5} \Leftrightarrow \boxed{x = 1.25\ m}$$

Step-III

Calculation of B.M:-

$M_B = 0$, $M_F = 5.5 \times 1 = 5.5\,kNm$

$M_E = -(4 \times 1) + 5.5 \times 2 = 7\,kN\text{-}m$

$M_M = -\left(2 \times 0.75 \times \frac{0.75}{2}\right) - 4 \times 1.75 + 5.5 \times 2.75 = 7.56\,kNm$

$M_D = -1 \times 1 + 3.5 \times 2 = 6\,kNm$

$M_C = 3.5 \times 1 = 3.5\,kNm$

$M_A = 0$

Q.2. Draw the shear force and bending moment diagrams for the beam loaded as shown in fig

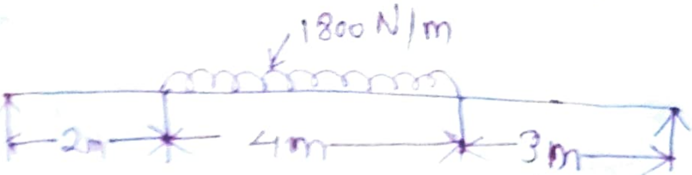


Fig ②

Sol^n:-

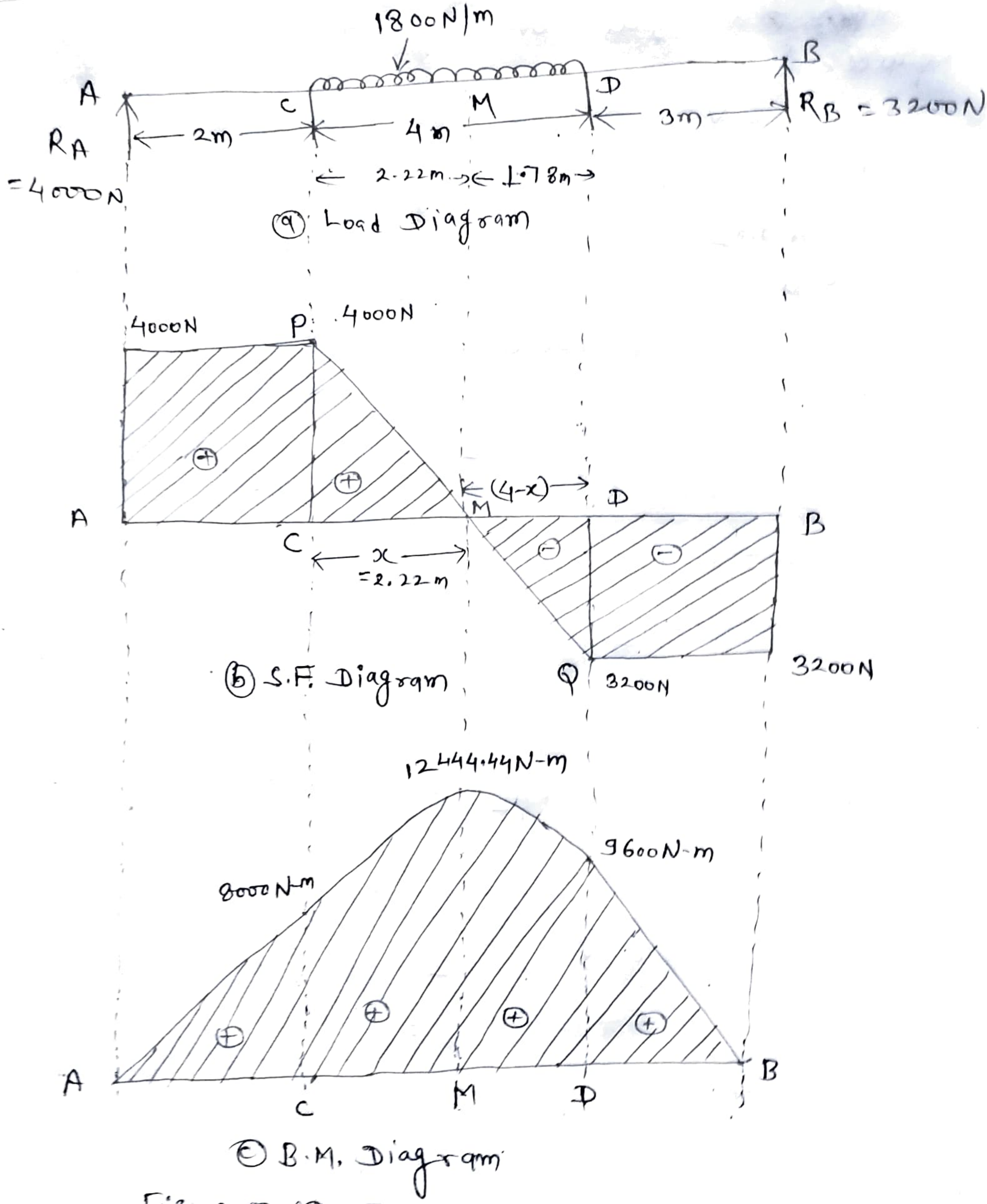


Fig. 2.(a), (b), (c)

(i) Calculation of Support Reaction :-

Consider the equilibrium of beam. Apply condition of equilibrium.

$$\Sigma V = 0$$

$$R_A + R_B - 1800 \times 4 = 0$$

$$\Rightarrow R_A + R_B = 7200 \qquad \text{--- (1)}$$

$$\Sigma M = 0$$

Taking moment about 'A',

$$R_B \times 9 - 1800 \times 4 \times \left(2 + \frac{4}{2}\right) = 0$$

$$\Rightarrow \boxed{R_B = 3200\,N}$$

$$\therefore \quad R_A + 3200 = 7200$$

$$\Rightarrow \boxed{R_A = 4000\,N}$$

(ii) S.F. Calculation :-

At B, $F_{BR} = 0$, $F_{BL} = -3200\,N$

At D, $F_{DR} = -3200\,N$, $F_{DL} = 3200\,N$.

At C, $F_{CR} = 4000\,N$, $F_{CL} = 4000\,N$

At A, $F_{AR} = 4000\,N$, $F_{AL} = 0$

Let M is point where shear force changes its sign i.e. Max B.M. point. So to find position of M, consider similar triangles, ΔPMC and ΔMDQ.

$$\frac{x}{4-x} = \frac{4000}{3200}$$

$$\Rightarrow \boxed{x = 2.22\,m}$$

(iii) Calculation of Bending Moment:-

$M_B = 0$

$M_D = 3200 \times 3 = 9600$ N-m

$M_M = -(1800 \times 1.78 \times \frac{1.78}{2}) + 3200 \times 4.78$

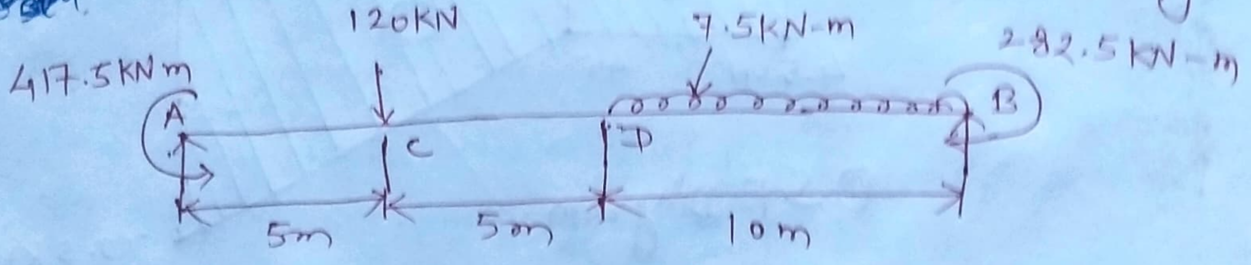
$\Rightarrow M_M = 12444.44$ N-m

$M_C = +4000 \times 2 = 8000$ N-m

$M_A = 0$

Q.3. A beam of span 20m, hinged at both ends is loaded as shown in fig. Draw the SFD and BMD. Also determine point of contraflexure, if any.

Soln.



Soln Calculation of Support Reactions:-

Consider equilibrium of beam and apply

$\Sigma V = 0$

$R_A + R_B - 120 - 7.5 \times 10 = 0$

$\Rightarrow R_A + R_B = 195$ ——— (1)

$\Sigma M = 0$

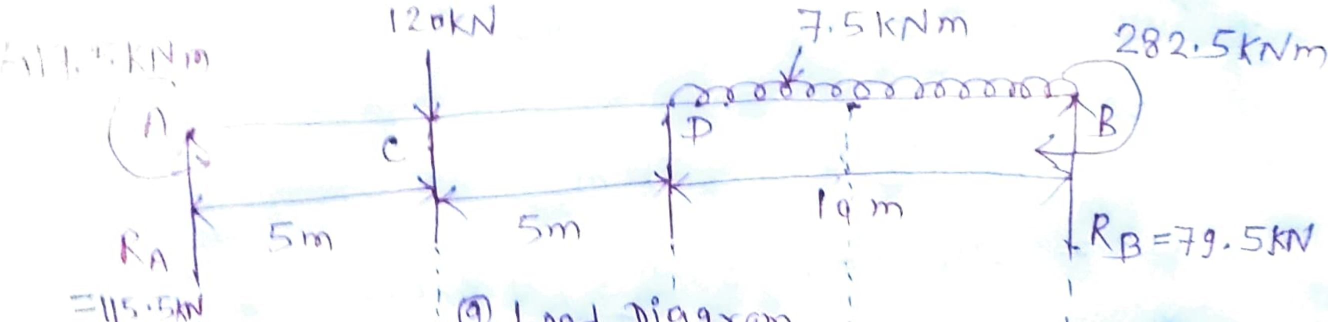
Taking moment about A,

~~$R_B \times 20 - 120 \times 5 - (7.5 \times 10) \times 15$~~

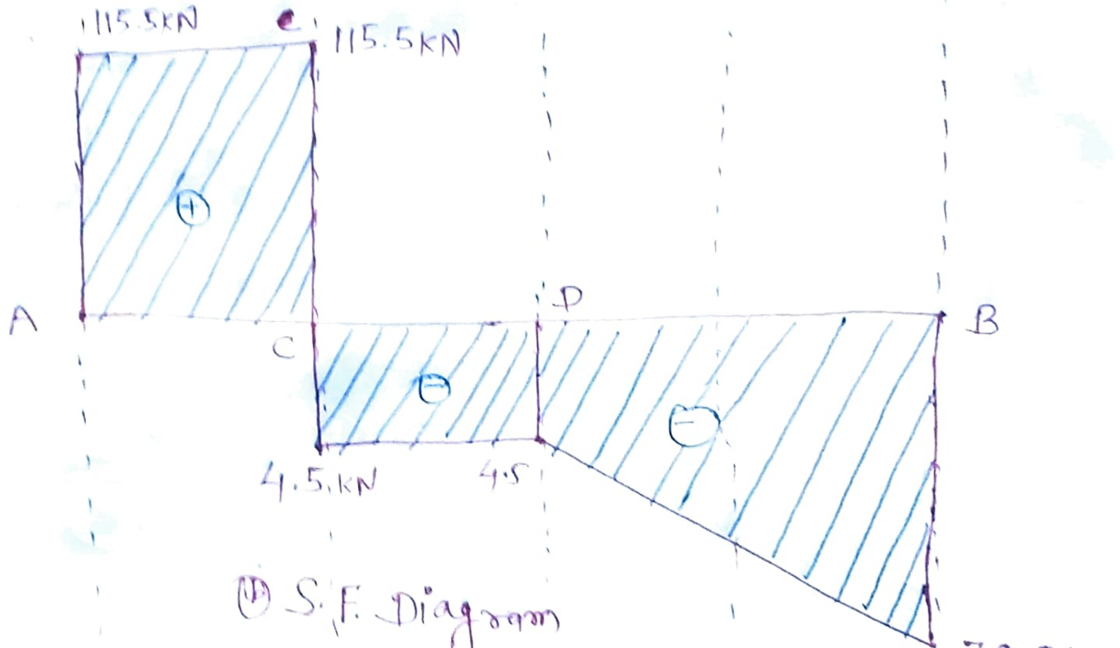
$120 \times 5 + (7.5 \times 10) \times 15 - R_B \times 20 - 417.5 + 282.5 = 0$

$\Rightarrow \boxed{R_B = 79.5 \text{ KN}}$

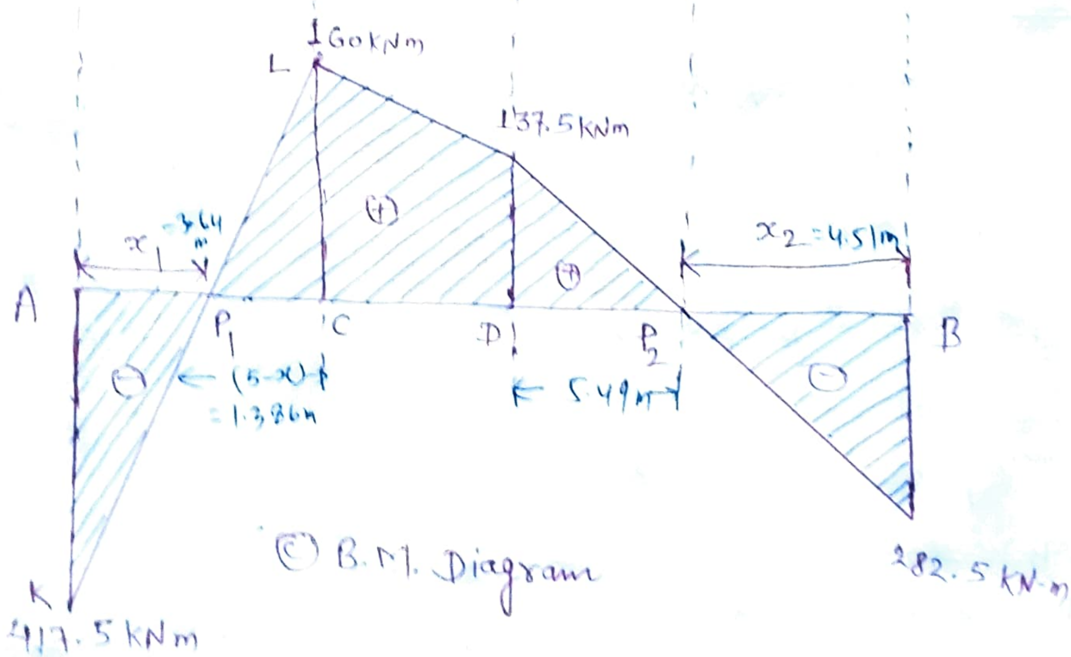
$\therefore R_A + 79.5 = 195 \Rightarrow \boxed{R_A = 115.5 \text{ KN}}$ (from eqn (1))



(a) Load Diagram



(b) S.F. Diagram



(c) B.M. Diagram

Calculation of Shear Forces:-

At B, $F_{BR} = 0$, $F_{BL} = -79.5\ kN$

At D, $F_{DR} = -79.5 + 7.5 \times 10 = -4.5\ kN$

$F_{DL} = -4.5\ kN$

At C, $F_{CR} = -4.5\ kN$, $F_{CL} = 120 - 4.5 = 115.5\ kN$

At A, $F_{AR} = 115.5\ kN$, $F_{AL} = 0$

Calculation of B.M.:-

$M_B = -282.5\ kNm$

$M_D = -(7.5 \times 10 \times 5) + 79.5 \times 10 - 282.5 = 137.5\ kNm$

$M_C = -417.5 + 115.5 \times 5 = 160\ kNm$

$M_A = -417.5\ kNm$

Point of Contraflexure:-

Let x_1 and x_2 be the distances of P_1 and P_2 from extreme left and right respectively. Now to find x_1, from similar triangles ~~ΔAKP_1~~ ΔAKP_1 and P_1LC,

$$\frac{x_1}{417.5} = \frac{5-x}{160} \Rightarrow \boxed{x_1 = 3.614\ m}$$ Ans.

Now to find x_2:-

$M_{x_2} = 0$

$\Rightarrow -282.5 + 79.5 \times x_2 - 7.5 \times x_2 \times \frac{x_2}{2} = 0$

$\Rightarrow 3.75 x_2^2 - 79.5 x_2 + 282.5 = 0$ —— (11)

$\Rightarrow x_2 = 16.68\ m$ (It is not possible)

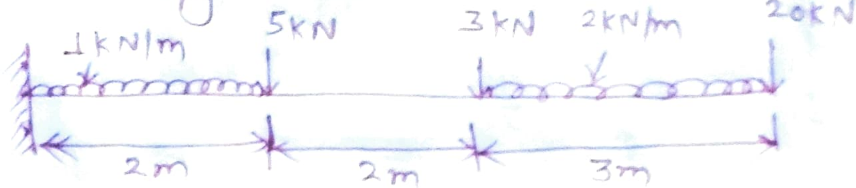
$\Rightarrow \boxed{x_2 = 4.51\ m}$ Ans

or By Shri Dharacharya Sutra,

$$x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-79.5) \pm \sqrt{(-79.5)^2 - 4 \times 3.75 \times 282.5}}{2 \times 3.75}$$

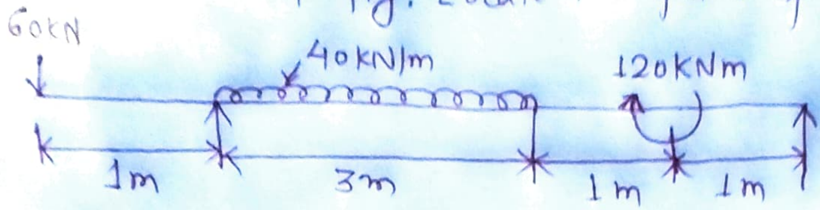
$$\Rightarrow x_2 = \frac{79.5 \pm 45.63}{7.5} = 16.68\ m \text{ or } 4.51\ m$$ Ans

Draw S.F. and B.M. Diagram for the Cantilever beam loaded as shown in fig



Soln.

Q.5 Draw shear force and bending moment diagrams for the beam shown in fig. Locate the point of Contraflexure, if any



Sol^n.