Sparse Learning for Noisy Data/Labels:

A Simple yet Effective Framework for Vision Applications

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Sparse Learning

for Noise Data Detection

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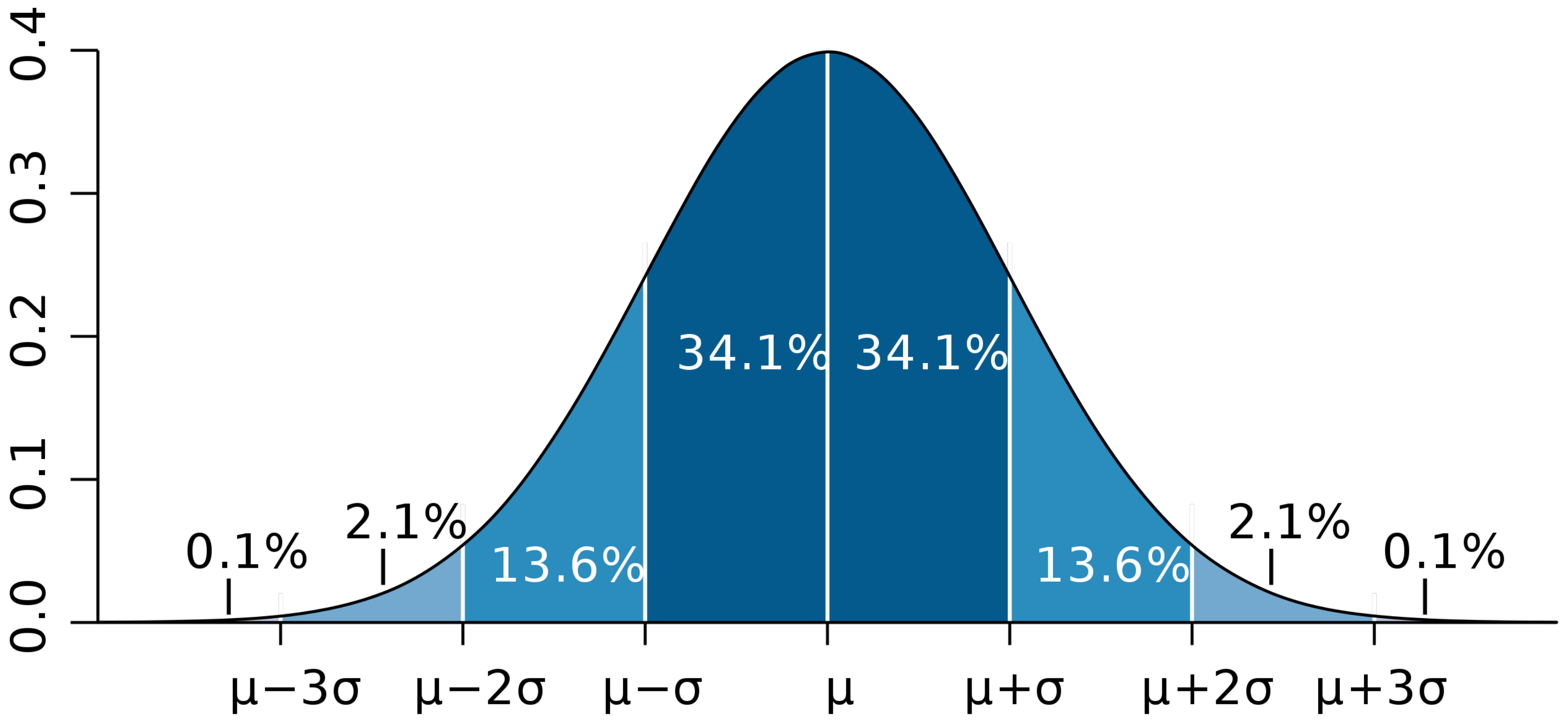
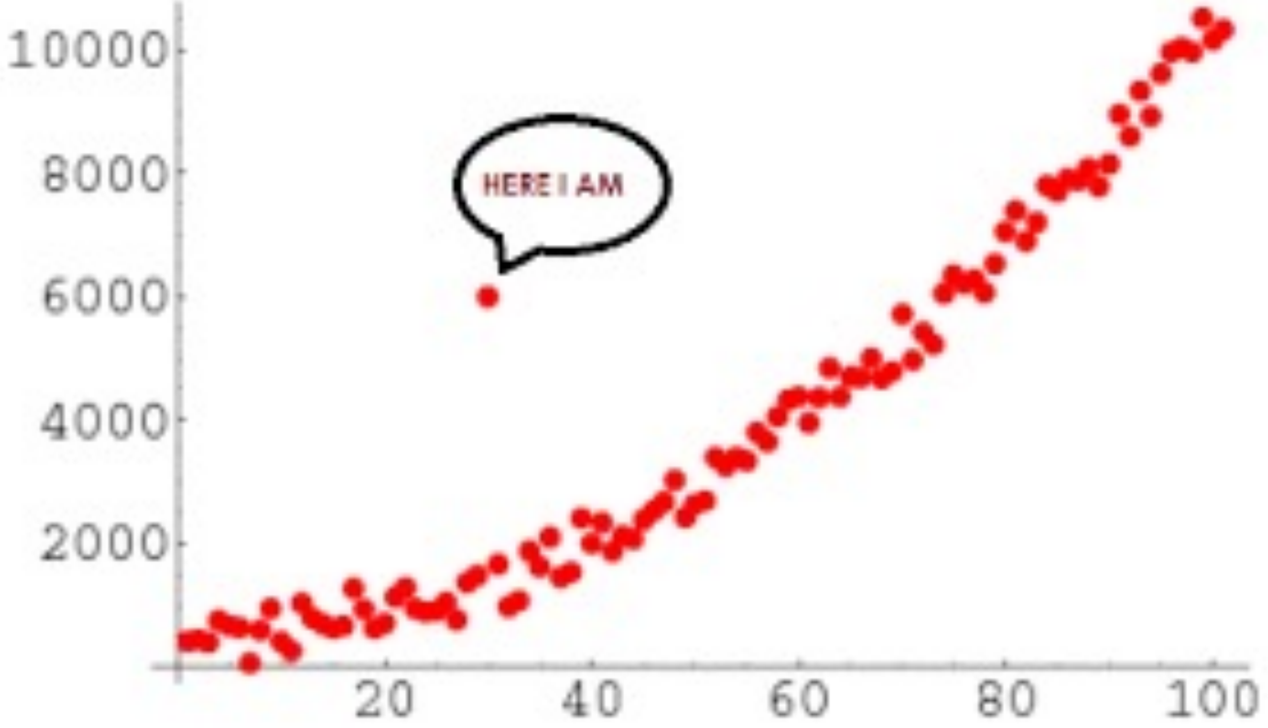
Examples of Noisy Data/Outliers



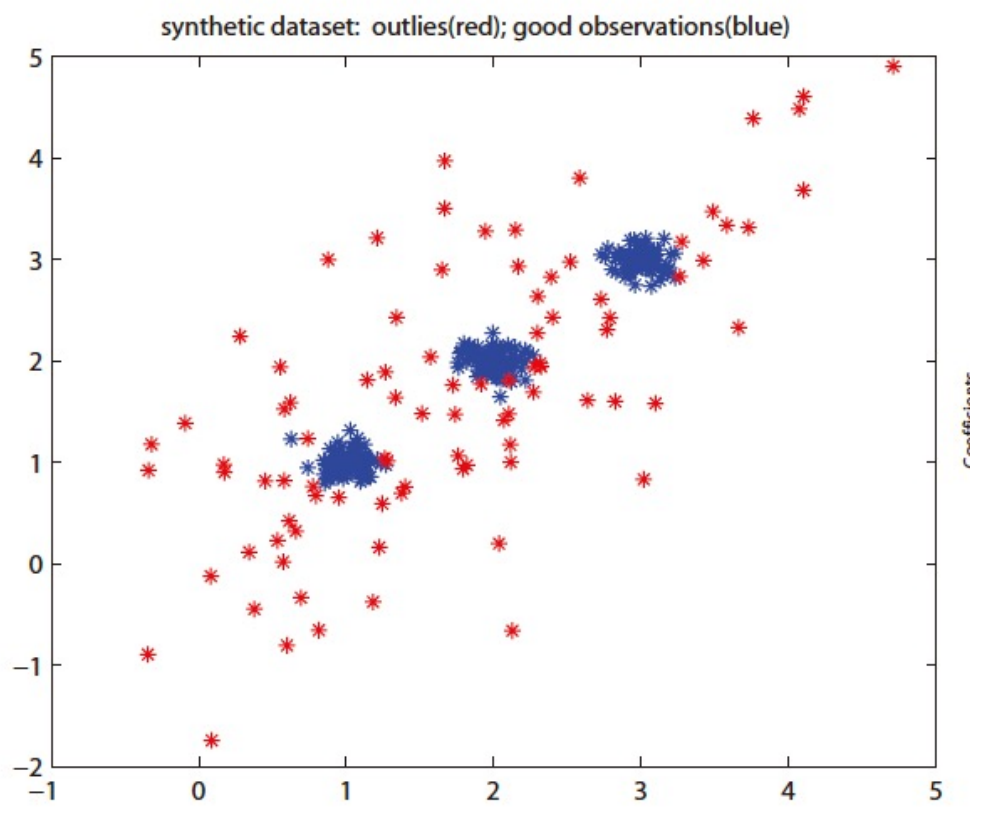


Outliers are the **irregular** data compared with the majority of the dataset.

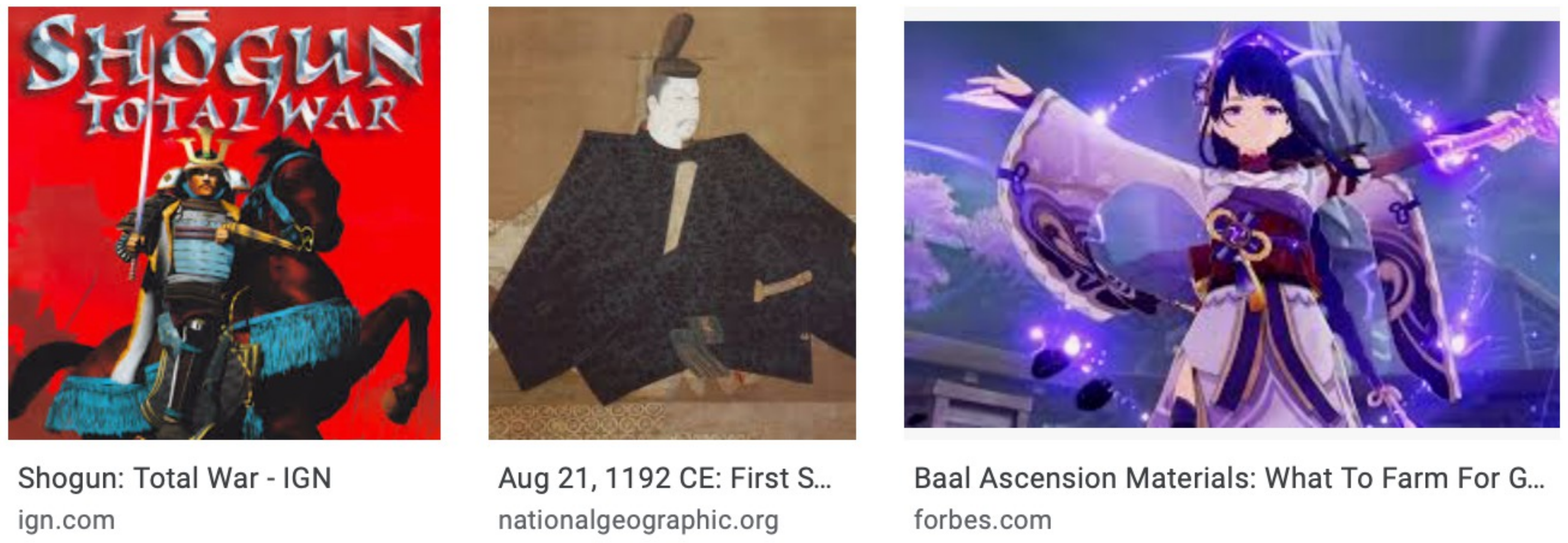
Figures from
[1] towardsdatascience.com/this-article-is-about-identifying-outliers-through-funnel-plots-using-the-microsoft-power-bi-d7ad16ac9ccc
[2] en.wikipedia.org/wiki/Outlier#/media/File:Standard_deviation_diagram_micro.svg
[3] medium.com/analytics-vidhya/its-all-about-outliers-cbe172aa1309

Noisy Data in Label Space

- Random Corruptions



- Noisy search engine results

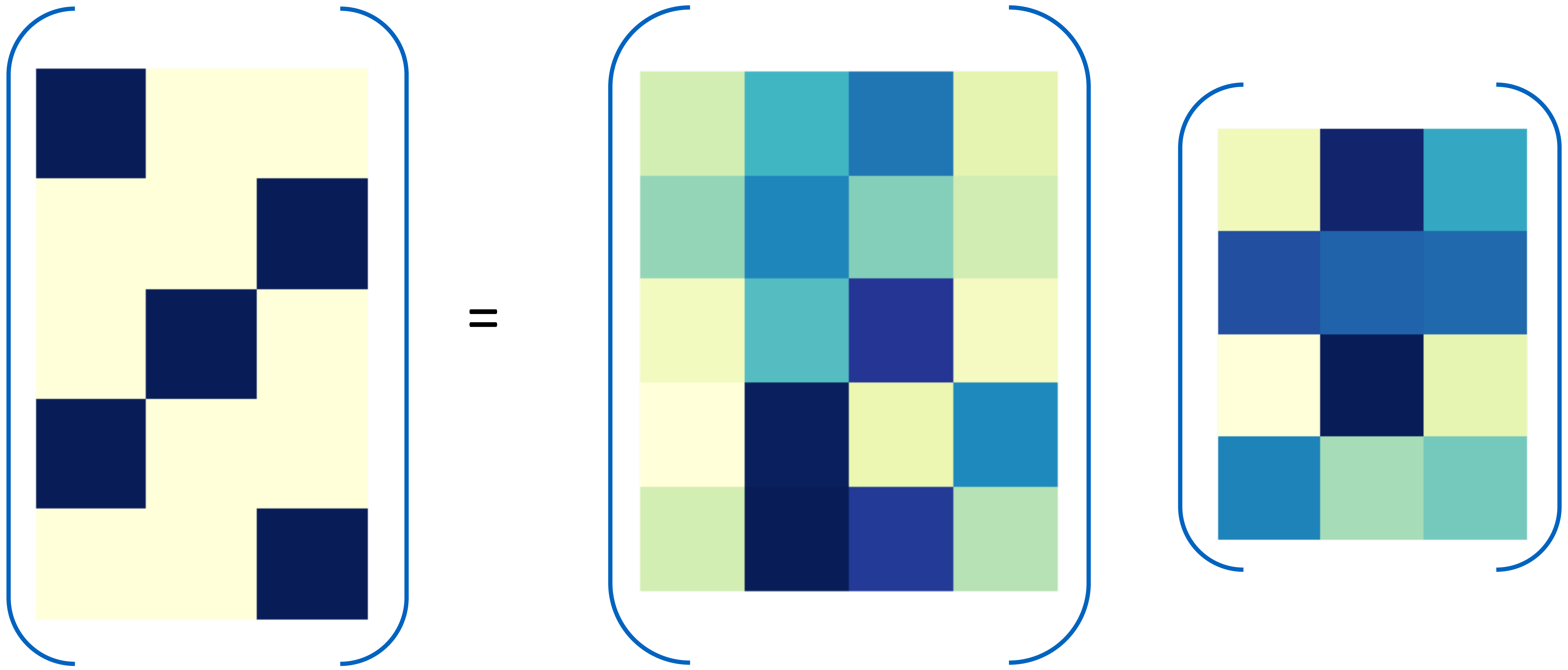


- Annotator mistakes

- Complex/Confusing items identified

Identify Noisy Data in Label Space

Linear system $$Y = X\beta$$

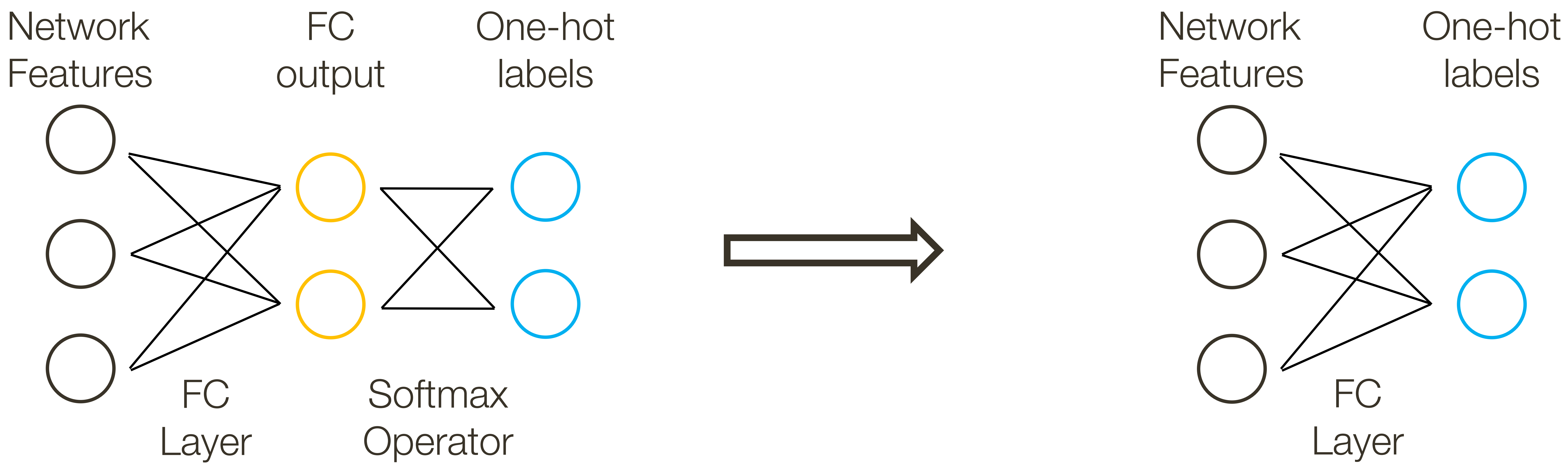


Noisy One-hot Labels **Deep Features** **Fitted Coef.**

$$Y \in \mathbb{R}^{n \times c} \quad X \in \mathbb{R}^{n \times d} \quad \beta \in \mathbb{R}^{d \times c}$$

β is sensitive to noisy data!

Approximated Linear Assumption in Networks



$$y_i = \mathrm{SoftMax}(\boldsymbol{x}_i^\top \beta)$$

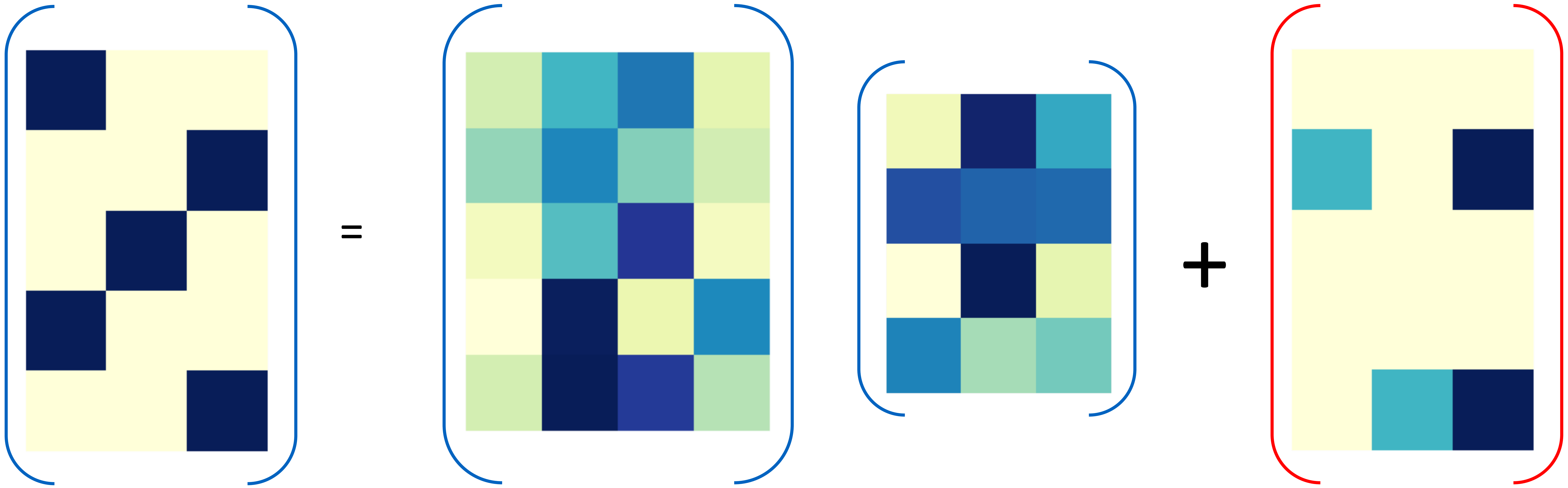
$$y_i = \boldsymbol{x}_i^\top \beta + \varepsilon$$

Wang et al. Scalable Penalized Regression for Noise Detection in Learning with Noisy Labels. CVPR 2022.

Identify Noisy Data in Label Space: The Indicator

Linear system
with Noisy Data/Labels

$$Y = X\beta + \gamma$$



Noisy One-hot Labels $Y \in \mathbb{R}^{n \times c}$

Deep Features $X \in \mathbb{R}^{n \times d}$

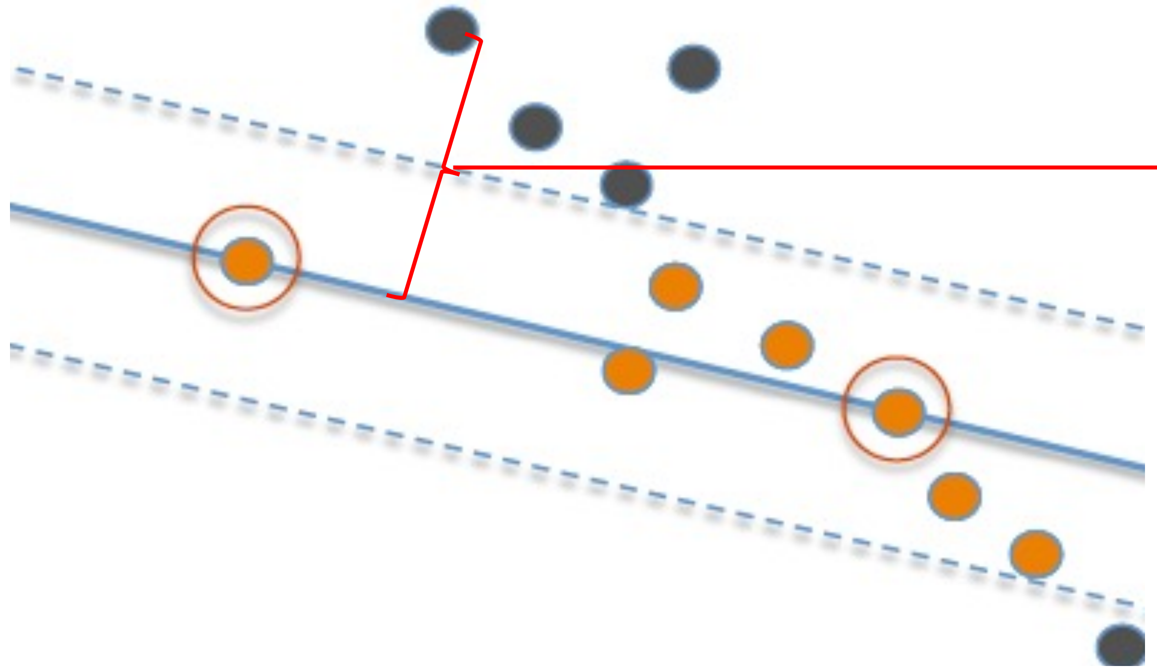
Fitted Coef. $\beta \in \mathbb{R}^{d \times c}$

Noisy Data Indicator $\gamma \in \mathbb{R}^{n \times c}$

[Wright et al. TPAMI 09] [She et al. JASA 11] [Fu et al. ECCV 14, TPAMI 16.] [Fan et al. Statistical Sinica 18] [Wang et al. CVPR 20, TPAMI 21, CVPR 22]

Understanding γ in Statistics

$$y = x^\top \beta + \varepsilon + \gamma$$



γ_i equals to the residual predict error $\gamma_i = y_i - x_i^\top \hat{\beta}$

Row residuals fail to detect outliers at *leverage points.*

[1] Yiyuan She and Art B Owen. Outlier detection using nonconvex penalized regression. Journal of the American Statistical Association, 2011.

Understanding γ in Statistics

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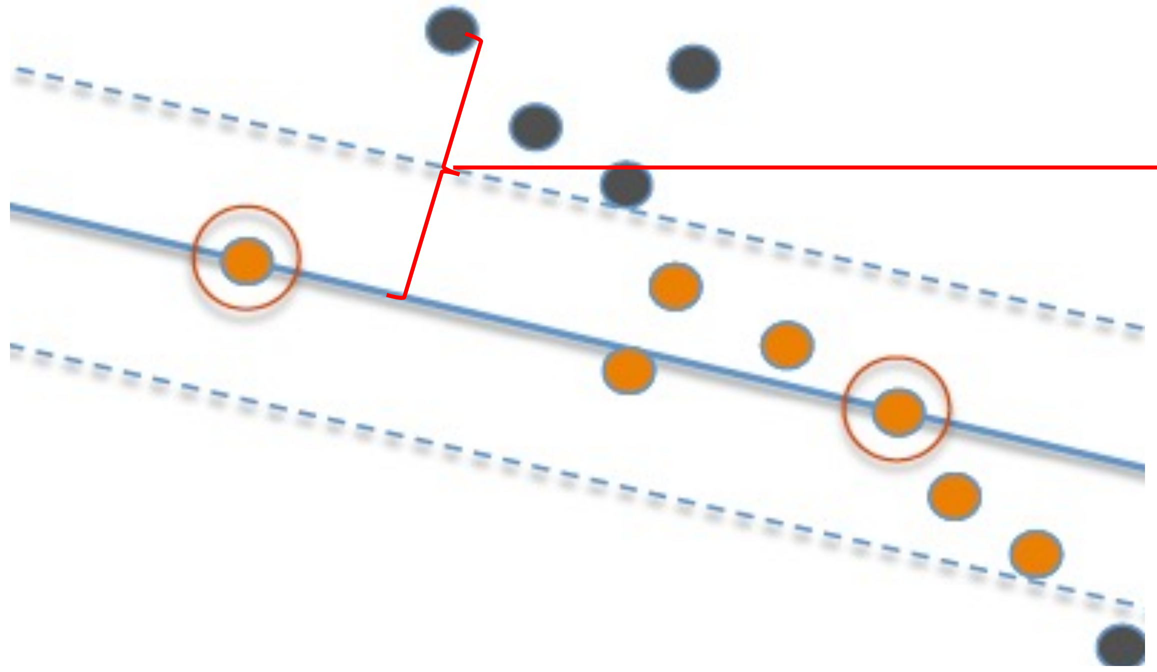

γ_i equals to the residual predict error $\gamma_i = y_i - x_i^\top \hat{\beta}$

$\downarrow$

Leave-one-out externally studentized residual:

$$t_i = \frac{y_i - \boldsymbol{x}_i^\top \hat{\beta}_{(i)}}{\hat{\sigma}_{(i)}(1+\boldsymbol{x}_i(\boldsymbol{X}_{(i)}^\top \boldsymbol{X}_{(i)})^{-1}\boldsymbol{x}_i)^{1/2}}$$

$\Leftrightarrow$ test whether $\gamma = 0$ in $\boldsymbol{y} = \boldsymbol{X\beta} + \gamma 1_i + \boldsymbol{\varepsilon}$.

[1] Yiyuan She and Art B Owen. Outlier detection using nonconvex penalized regression. Journal of the American Statistical Association, 2011.

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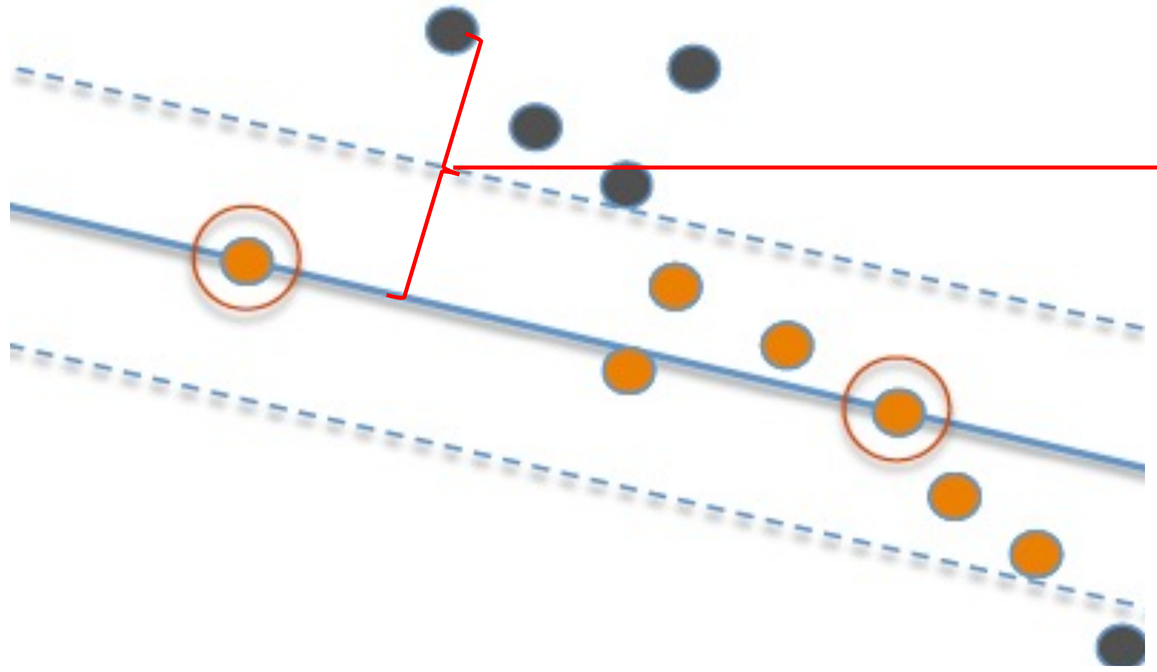

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$\Leftrightarrow$ test whether $\gamma = 0$ in $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \gamma 1_i + \boldsymbol{\varepsilon}$.

When there are multiple outliers:

1. masking: multiple outliers may mask each other and being undetected;

2. swamping: multiple outliers may lead the large t_i for clean data.

[1] Yiyuan She and Art B Owen. Outlier detection using nonconvex penalized regression. Journal of the American Statistical Association, 2011.

Understanding γ in Statistics

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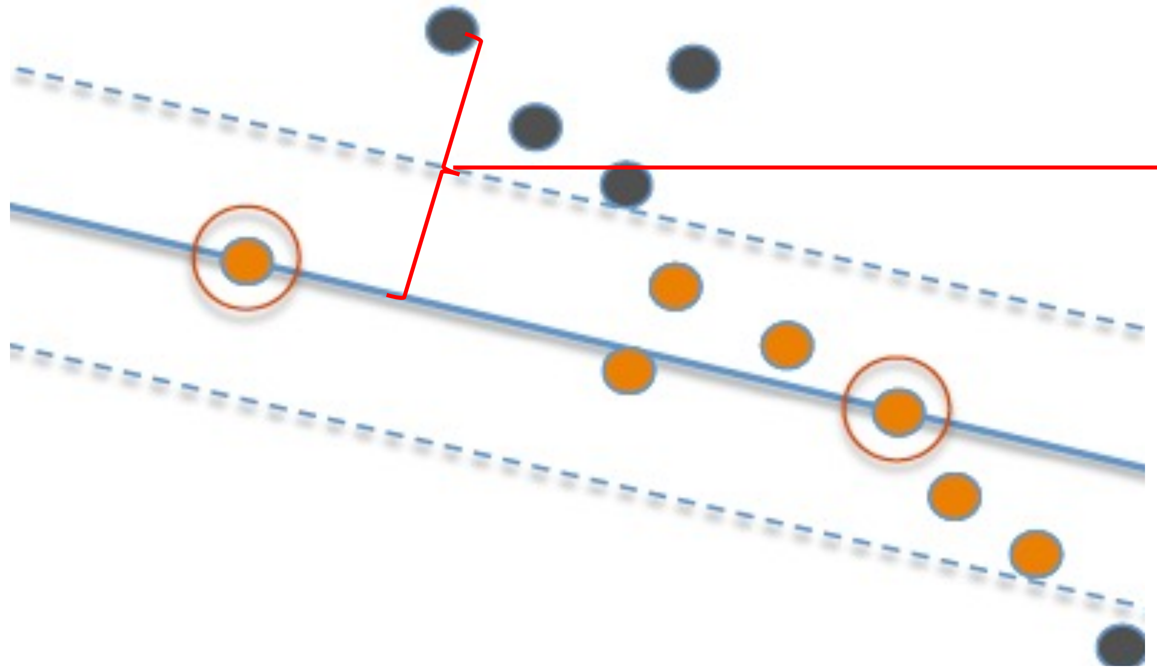

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$\Leftrightarrow$ test whether $\gamma = 0$ in $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \gamma 1_i + \boldsymbol{\varepsilon}$.

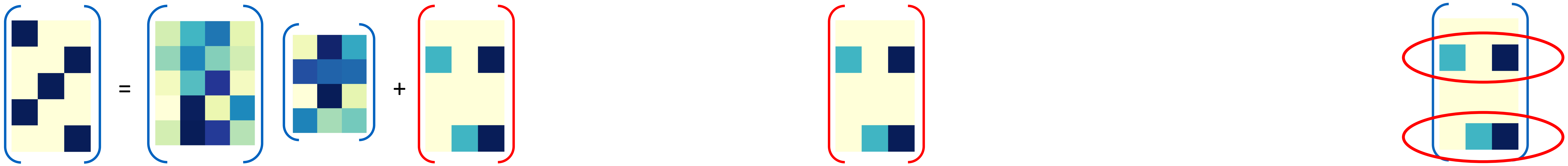
$\downarrow$

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} + \boldsymbol{\gamma}$$

[1] Yiyuan She and Art B Owen. Outlier detection using nonconvex penalized regression. Journal of the American Statistical Association, 2011.

Identify Noisy Data in the Dataset

$$y_i = x_i^\top \beta + \varepsilon + \gamma_i \longrightarrow \hat{\gamma}_i \longrightarrow O = \{i : \hat{\gamma}_i \neq 0\}$$



$$\underset{\boldsymbol{\beta},\boldsymbol{\gamma}}{\operatorname{argmin}} L(\boldsymbol{\beta}, \boldsymbol{\gamma}) := \|\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{\gamma}\|_{\mathrm{F}}^2 + \lambda R(\boldsymbol{\gamma})$$

Wang et al. Instance Credibility Inference for Few-Shot Learning. CVPR 2020
Wang et al. How to Trust Unlabeled Data? Instance Credibility Inference for Few-Shot Learning. IEEE TPAMI 2021.
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Simplification

$$\underset{\boldsymbol{\beta},\boldsymbol{\gamma}}{\operatorname{argmin}} L(\boldsymbol{\beta},\boldsymbol{\gamma}) := \|\boldsymbol{Y}-\boldsymbol{X}\boldsymbol{\beta}-\boldsymbol{\gamma}\|_{\mathrm{F}}^{2}+\lambda R(\boldsymbol{\gamma})$$

$$\frac{\partial L}{\partial \beta}=0 \quad \Big\downarrow \quad \hat{\boldsymbol{\beta}}=\left(\boldsymbol{X}^{\top}\boldsymbol{X}\right)^{\dagger}\boldsymbol{X}^{\top}(\boldsymbol{Y}-\boldsymbol{\gamma})$$

$$\underset{\boldsymbol{\gamma}}{\operatorname{argmin}}\left\|\boldsymbol{Y}-\boldsymbol{X}\left(\boldsymbol{X}^{\top}\boldsymbol{X}\right)^{\dagger}\boldsymbol{X}^{\top}(\boldsymbol{Y}-\boldsymbol{\gamma})-\boldsymbol{\gamma}\right\|_{\mathrm{F}}^{2}+\lambda R(\boldsymbol{\gamma})$$

$$\boldsymbol{H}=\boldsymbol{X}\left(\boldsymbol{X}^{\top}\boldsymbol{X}\right)^{\dagger}\boldsymbol{X}^{\top} \quad \Big\downarrow \quad \tilde{\boldsymbol{X}}=\boldsymbol{I}-\boldsymbol{H},\tilde{\boldsymbol{Y}}=\tilde{\boldsymbol{X}}\boldsymbol{Y}$$

$$\underset{\boldsymbol{\gamma}}{\operatorname{argmin}}\left\|\tilde{\boldsymbol{Y}}-\tilde{\boldsymbol{X}}\boldsymbol{\gamma}\right\|_{\mathrm{F}}^{2}+\lambda R(\boldsymbol{\gamma})$$

A linear regression problem!

Wang et al. Instance Credibility Inference for Few-Shot Learning. CVPR 2020

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Solving Gamma in Linear Regression

$$\underset{\gamma}{\operatorname{argmin}} \left\| \tilde{\boldsymbol{Y}} - \tilde{\boldsymbol{X}}\gamma \right\|_{\mathrm{F}}^{2} + \lambda R(\gamma)$$

How to select λ?

- heuristics rules $\lambda = 2.5\hat{\sigma}$?
- Cross-validation?
- Data adaptive techniques?
- AIC, BIC?

It is hard to select a proper λ.

We regard $\hat{\gamma} = f(\lambda)$.

When $\lambda \to \infty$, $\hat{\gamma} \to 0$.

With $R(\gamma) = \sum_{i=1}^{n} \|\gamma_i\|_2$,
γ vanishes instance by instance.

$C_i = \sup\{\lambda : \|\hat{\gamma}_i(\lambda)\| \neq 0\}$

This can be sovled by GLMnet[1].

[1] Friedman, et al. 2010. "Regularization Paths for Generalized Linear Models via Coordinate Descent." Journal of Statistical Software.

Solving Gamma in Linear Regression

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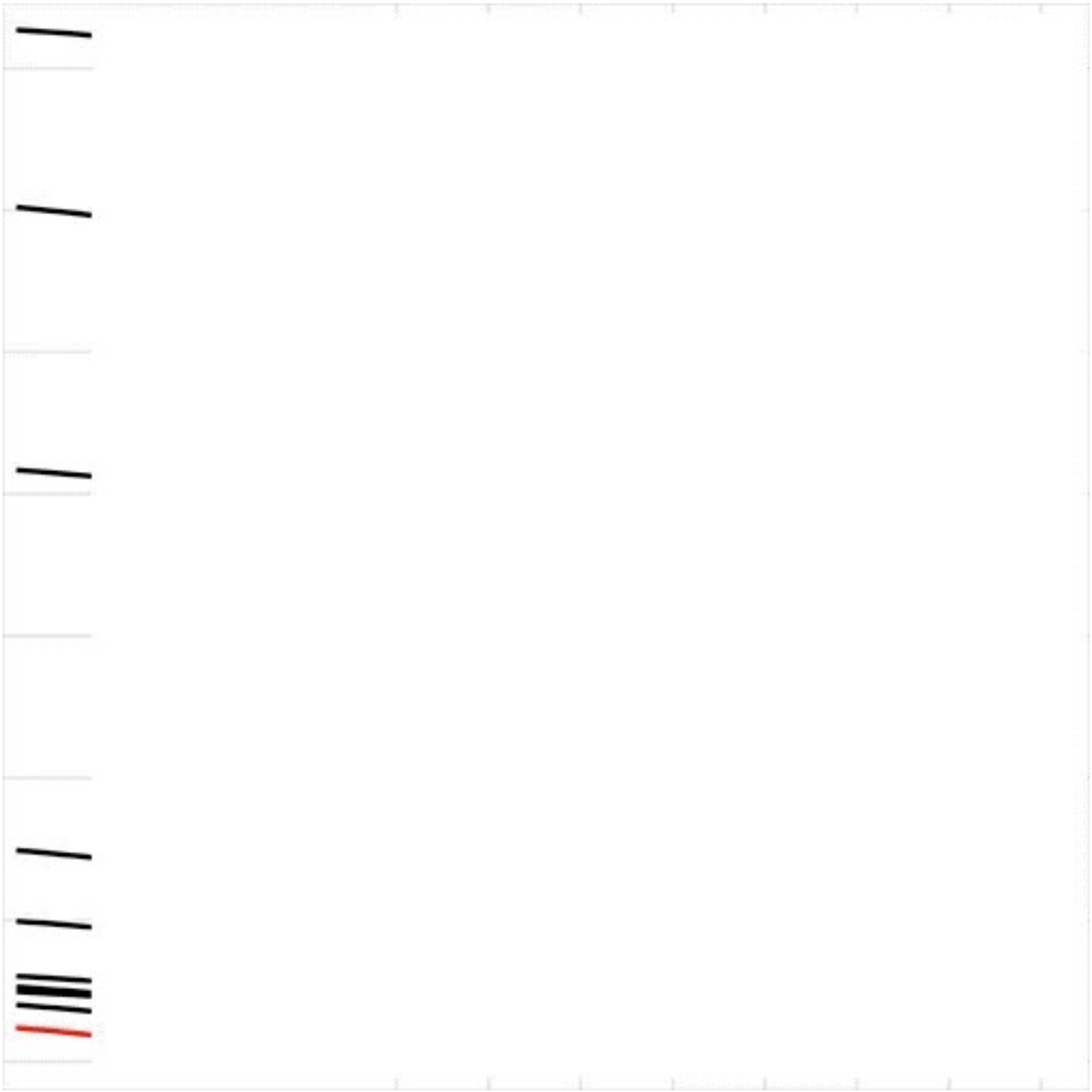

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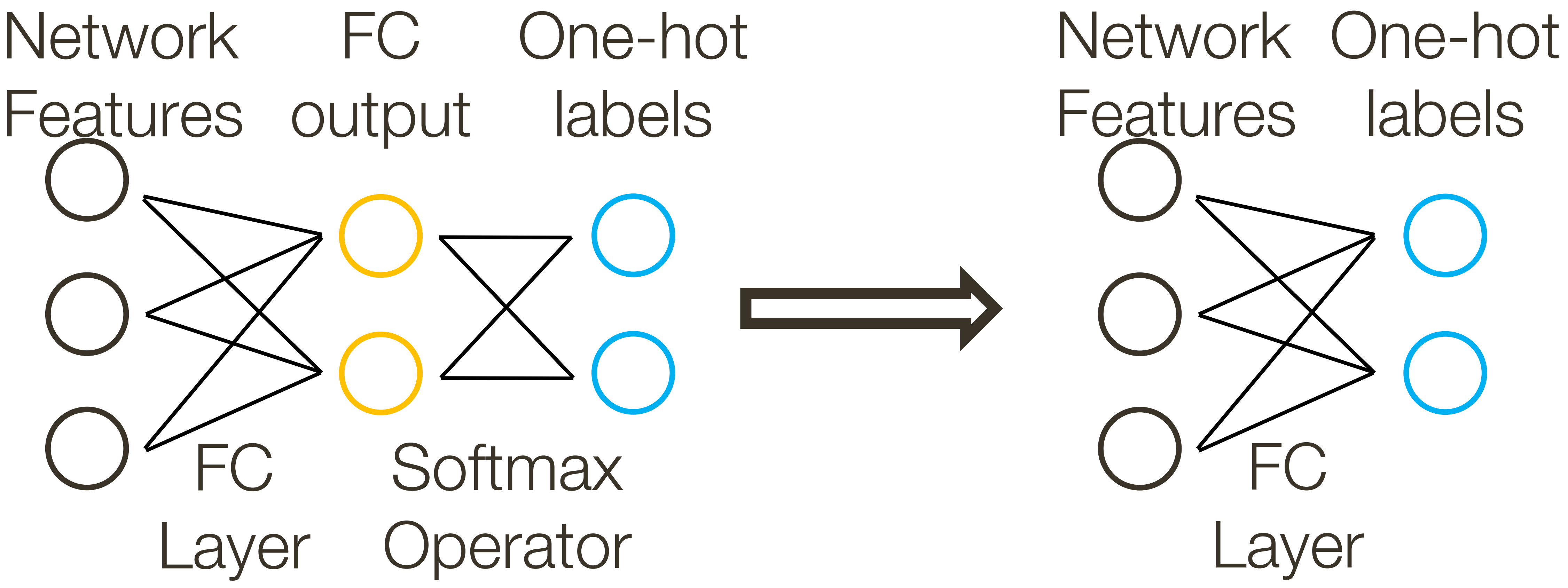
Instance Credibility Inference



$y_i = \mathrm{SoftMax}(\boldsymbol{x}_i^\top \beta)$ $\quad$ $y_i = \boldsymbol{x}_i^\top \beta + \varepsilon$

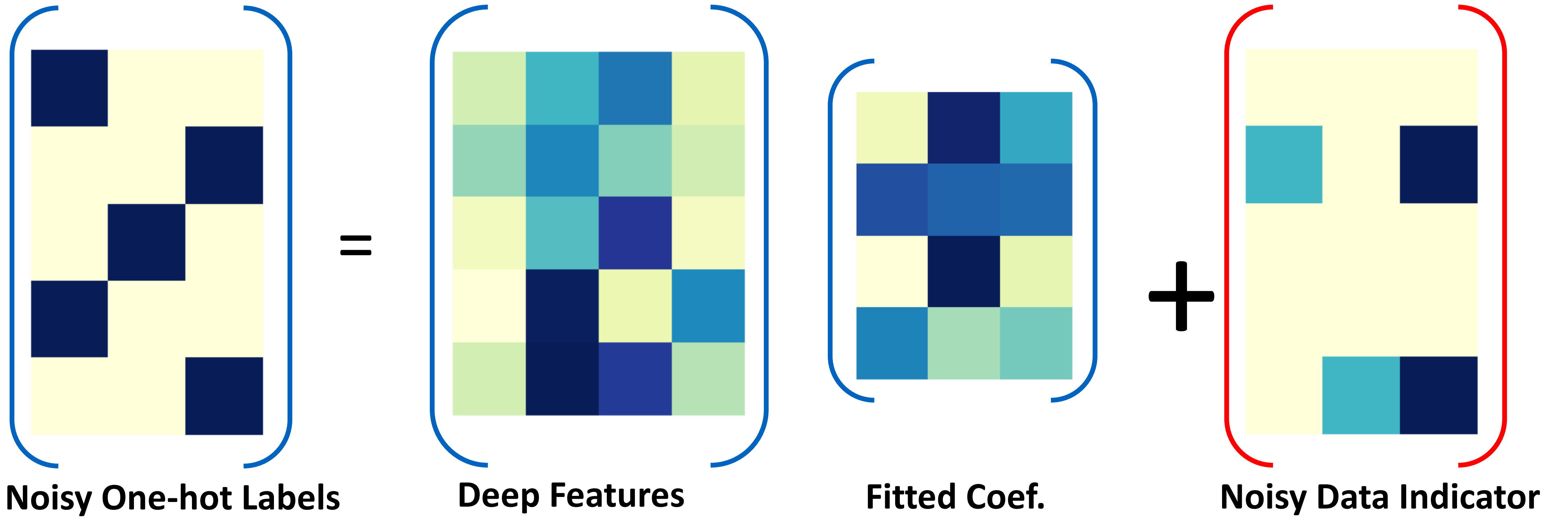
$$\underset{\boldsymbol{\beta},\boldsymbol{\gamma}}{\operatorname{argmin}} L(\boldsymbol{\beta},\boldsymbol{\gamma}) := \|\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{\gamma}\|_{\mathrm{F}}^2 + \lambda R(\boldsymbol{\gamma})$$

$$\downarrow$$

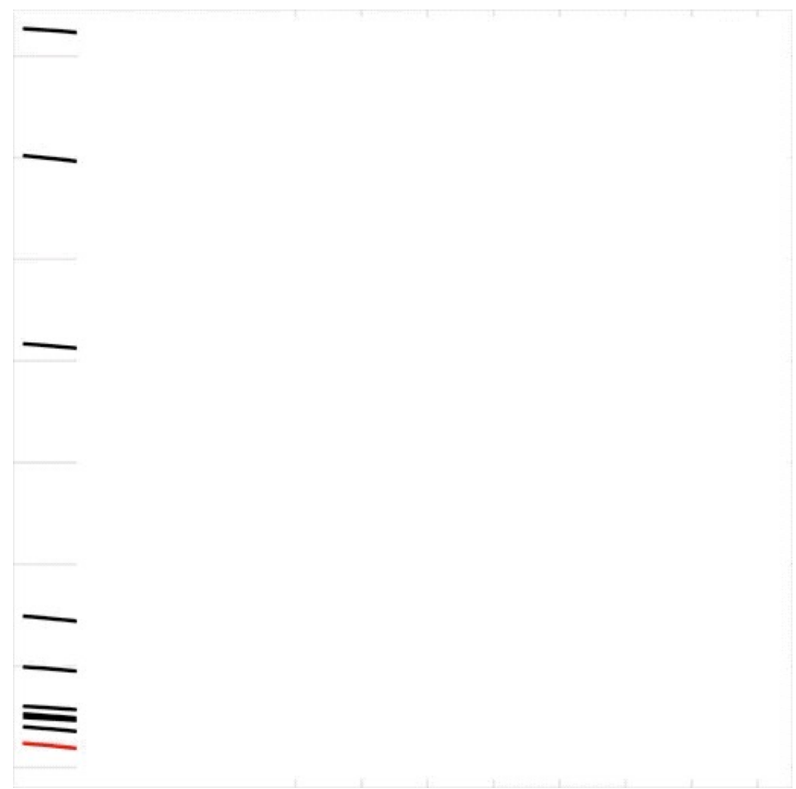
$$\underset{\boldsymbol{\gamma}}{\operatorname{argmin}} \left\|\tilde{\boldsymbol{Y}} - \tilde{\boldsymbol{X}}\boldsymbol{\gamma}\right\|_{\mathrm{F}}^2 + \lambda R(\boldsymbol{\gamma})$$

$$\downarrow$$

$$C_i = \sup\{\lambda : \|\hat{\gamma}_i(\lambda)\| \neq 0\}$$



$Y \in \mathbb{R}^{n\times c}$ $\quad$ $X \in \mathbb{R}^{n\times d}$ $\quad$ $\beta \in \mathbb{R}^{d\times c}$ $\quad$ $\gamma \in \mathbb{R}^{n\times c}$



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Noise Set Recovery

When will the model identify all the outliers?

Assume $\boldsymbol{\varepsilon}$ is i.i.d zero-mean sub-Gaussian noise. We give three conditions:

- (C1: Restricted eigenvalue)

$$\lambda_{\min}\left(\tilde{\boldsymbol{U}}_S^\top \tilde{\boldsymbol{U}}_S\right) = C_{\min} > 0.$$

- (C2: Irrepresentability) $\exists \eta \in (0,1]$,

$$\left\| \tilde{\boldsymbol{U}}_{S^c}^\top \tilde{\boldsymbol{U}}_S \left(\tilde{\boldsymbol{U}}_S^\top \tilde{\boldsymbol{U}}_S\right)^{-1} \right\|_\infty \le 1 - \eta.$$

- (C3: Large error)

$$\vec{\gamma}_{\min} := \min_{i \in S} |\vec{\gamma}^*| > h\left(\lambda, \eta, \tilde{\boldsymbol{U}}, \vec{\gamma}^*\right).$$

[1]: M. J. Wainwright, Sharp thresholds for high-dimensional and noisy sparsity recovery using l1-constrained quadratic programming. TIT 2009.

A non-asymptotic probabilistic result

Based on these conditions, we could provide the following theorem:

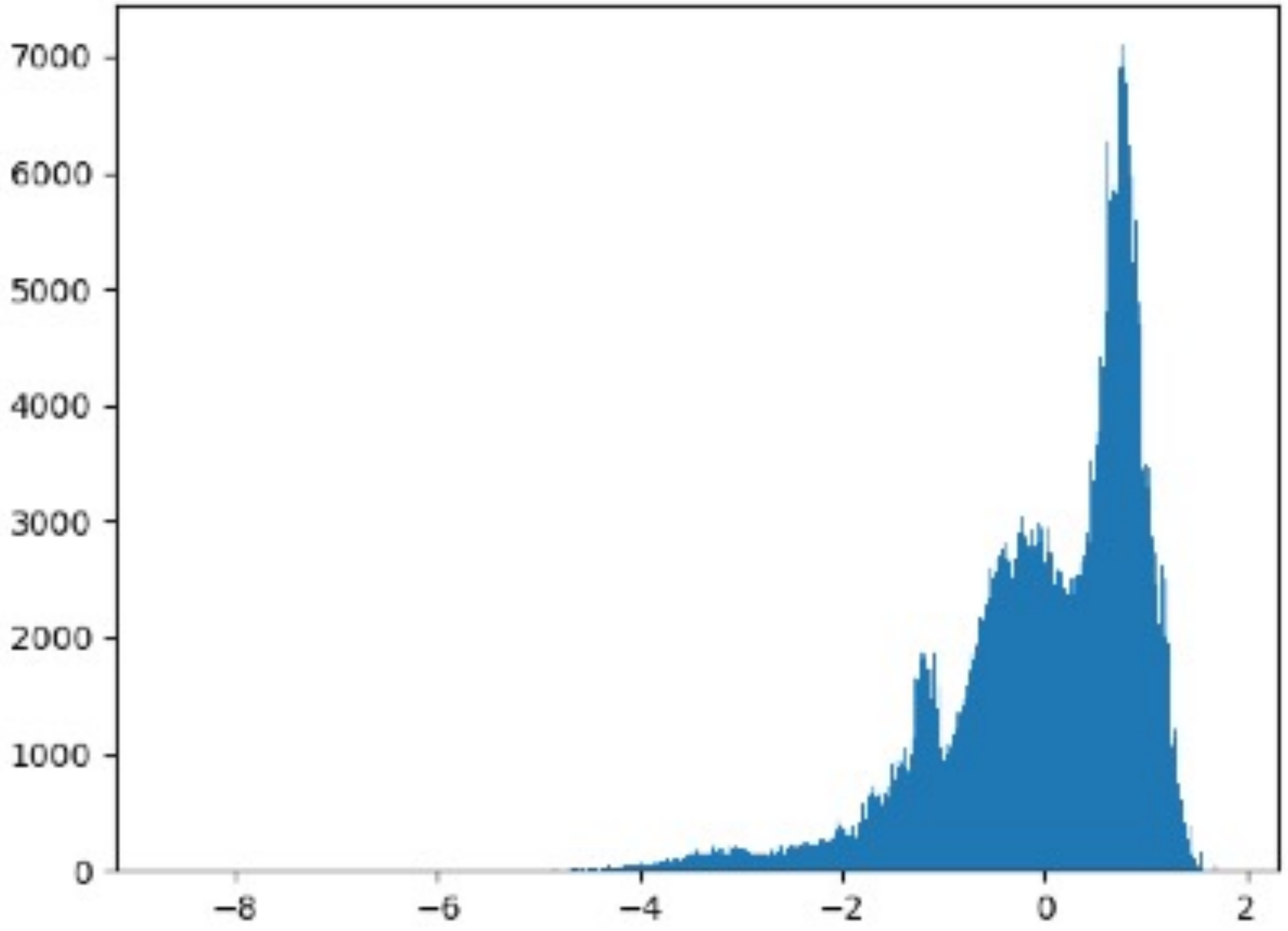
Theorem 1 (Identifiability of ICI). *Let* $\lambda \geq \frac{2\sigma\sqrt{\mu_{\tilde{U}}}}{\eta}\sqrt{\log cn}$. *Then with probability greater than* $1 - 2(cn)^{-1}$, *the problem has a unique solution* $\hat{\gamma}$ *satisfies the following properties:*

1) If C1 and C2 hold, the wrong-predicted instances indicated by ICI has no false positive error, i.e., $\hat{S} \subseteq S$ *and hence* $\hat{O} \subseteq O$, *and*

$$\left\|\hat{\vec{\gamma}}_S - \vec{\gamma}^*_S\right\|_\infty \leq h\left(\lambda, \eta, \tilde{\boldsymbol{U}}, \vec{\gamma}^*\right);$$

2) If C1, C2, and C3 hold, ICI will identify all the correctly-predicted instance, i.e., $\hat{S} = S$ *and hence* $\hat{O} = O$ *(in fact* $\operatorname{sign}\left(\hat{\vec{\gamma}}\right) = \operatorname{sign}\left(\vec{\gamma}^*\right)$*).*

Identifiability in reality: sub-Gaussian noise



$$\mathbb{E}\left[\hat{\varepsilon}\right] \approx 10^{-19}$$

$$\mathrm{Var}\left[\hat{\varepsilon}\right] \approx 0.99$$

Sparse Learning
in Few-Shot Learning

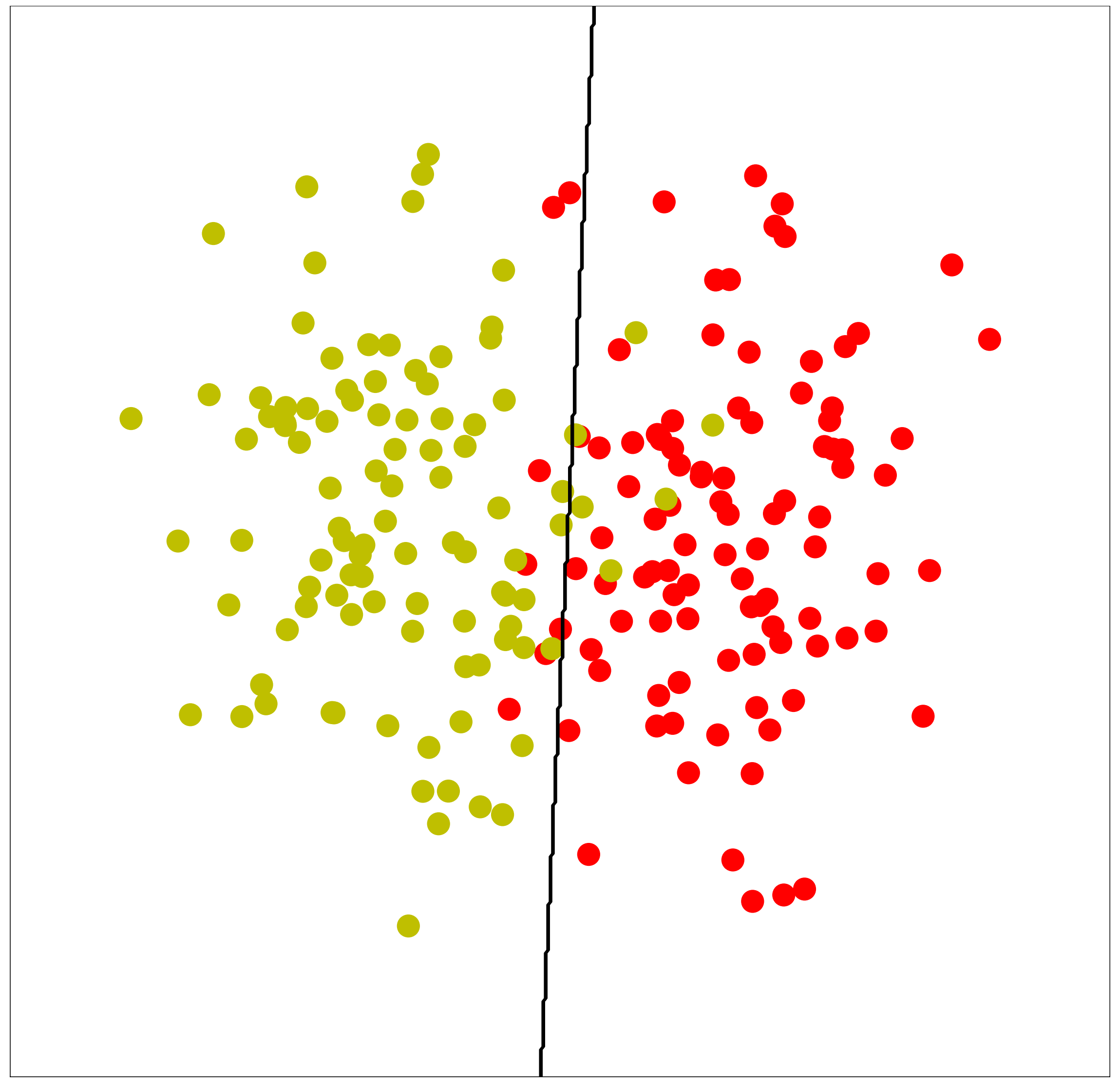
Definition of Few-Shot Learning

Tackle machine learning problem with only limited training data provided.

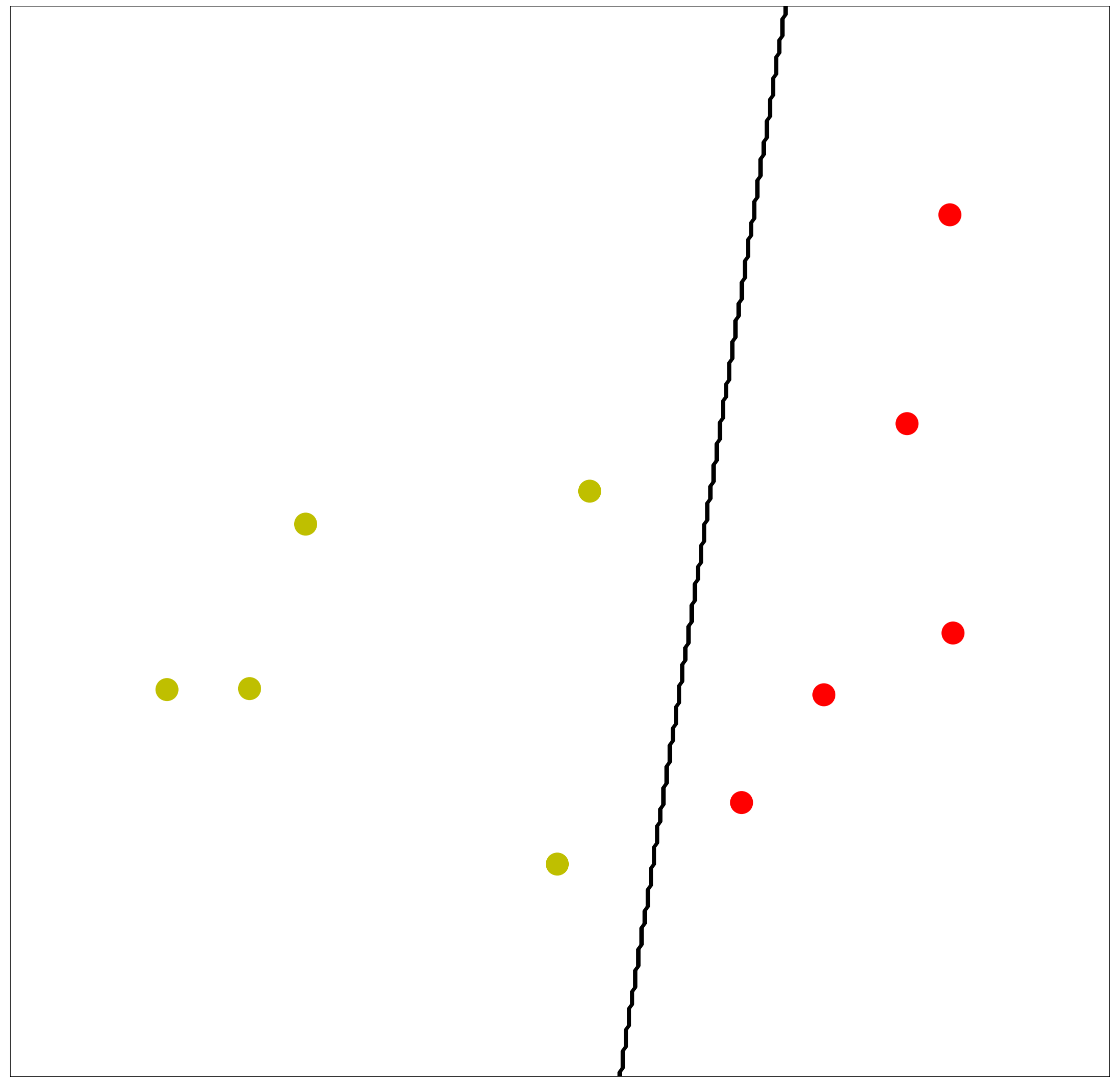
Few-Shot Learning

Low cost → **High cost**

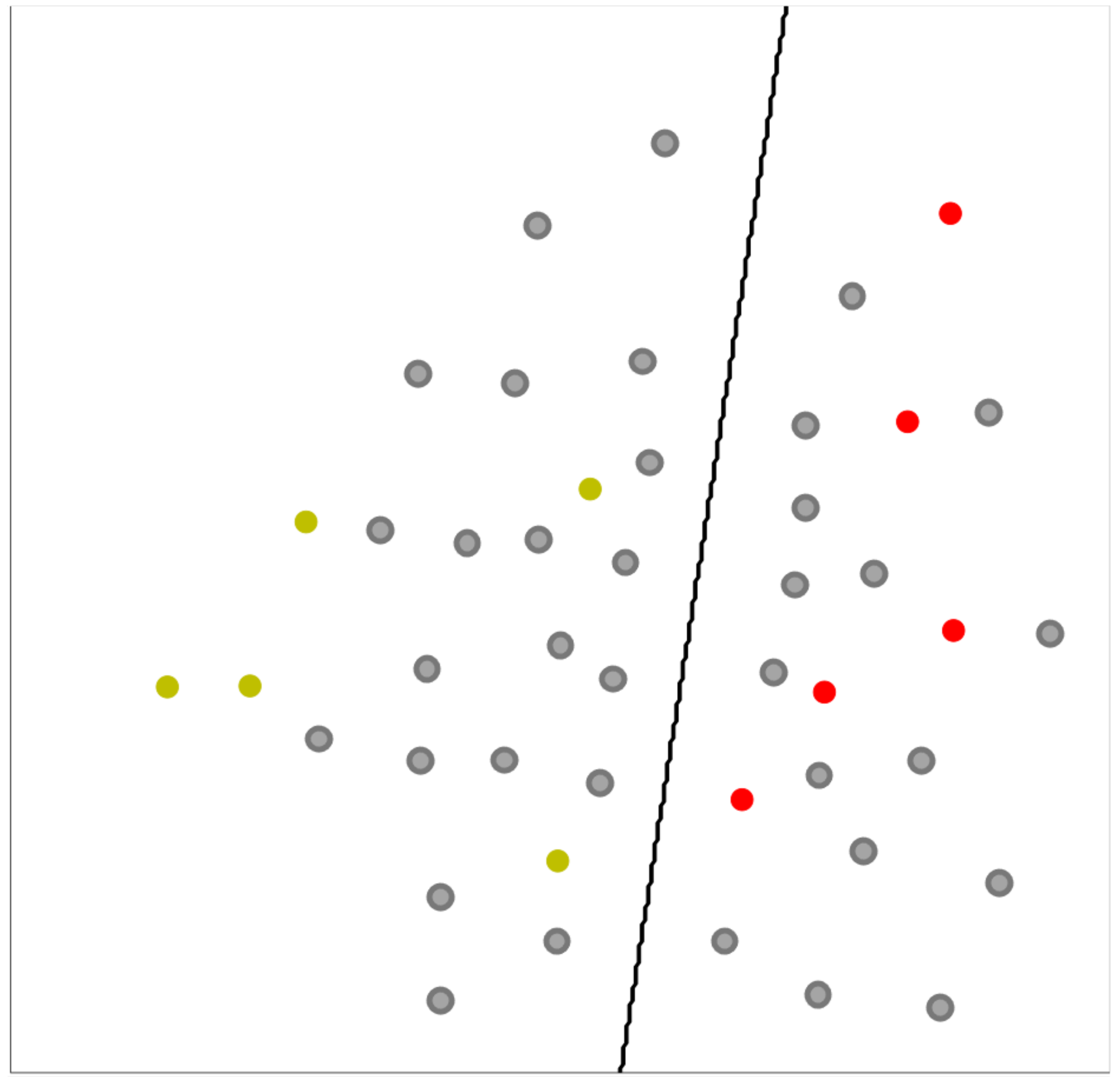
Unsupervised Learning | **Learning From Noisy Labels** | **Semi-Supervised Learning** | **Supervised Learning**



Binary classification with many labeled data

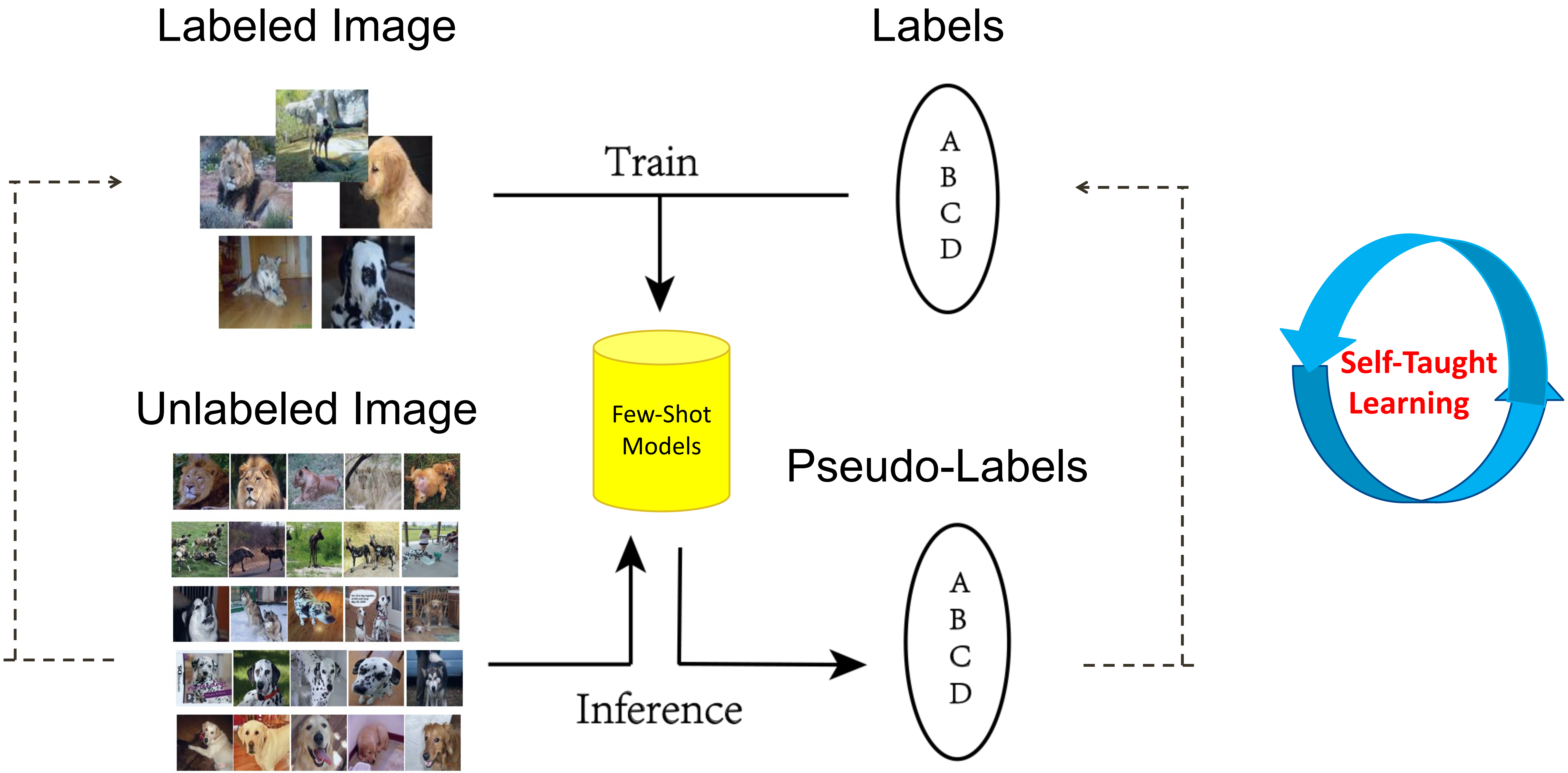


Few-shot binary classification



Few-shot binary classification with unlabeled data

Motivation



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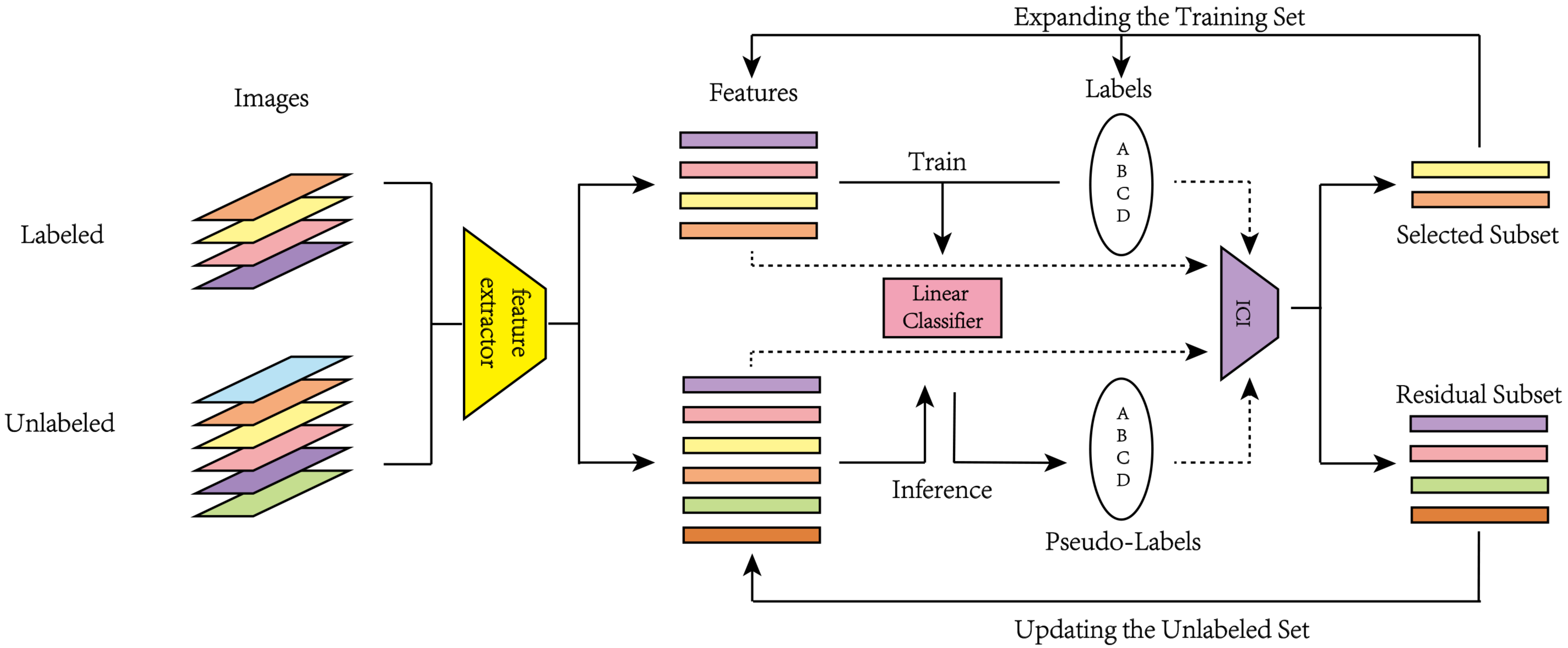
Wang et al. How to Trust Unlabeled Data? Instance Credibility Inference for Few-Shot Learning. IEEE TPAMI 2021

Framework



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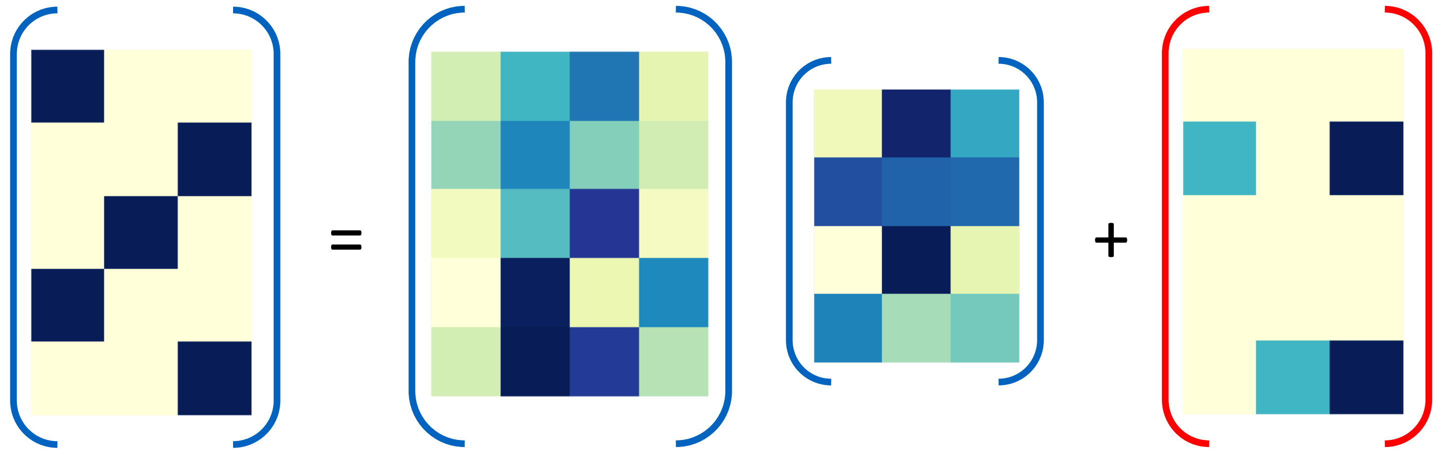
Sparse Learning in ICI

$$y_i = x_i^\top \beta + \varepsilon + \gamma_i$$



$$\underset{\boldsymbol{\beta},\boldsymbol{\gamma}}{\operatorname{argmin}} L(\boldsymbol{\beta},\boldsymbol{\gamma}) := \|\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{\gamma}\|_{\mathrm{F}}^2 + \lambda R(\boldsymbol{\gamma})$$

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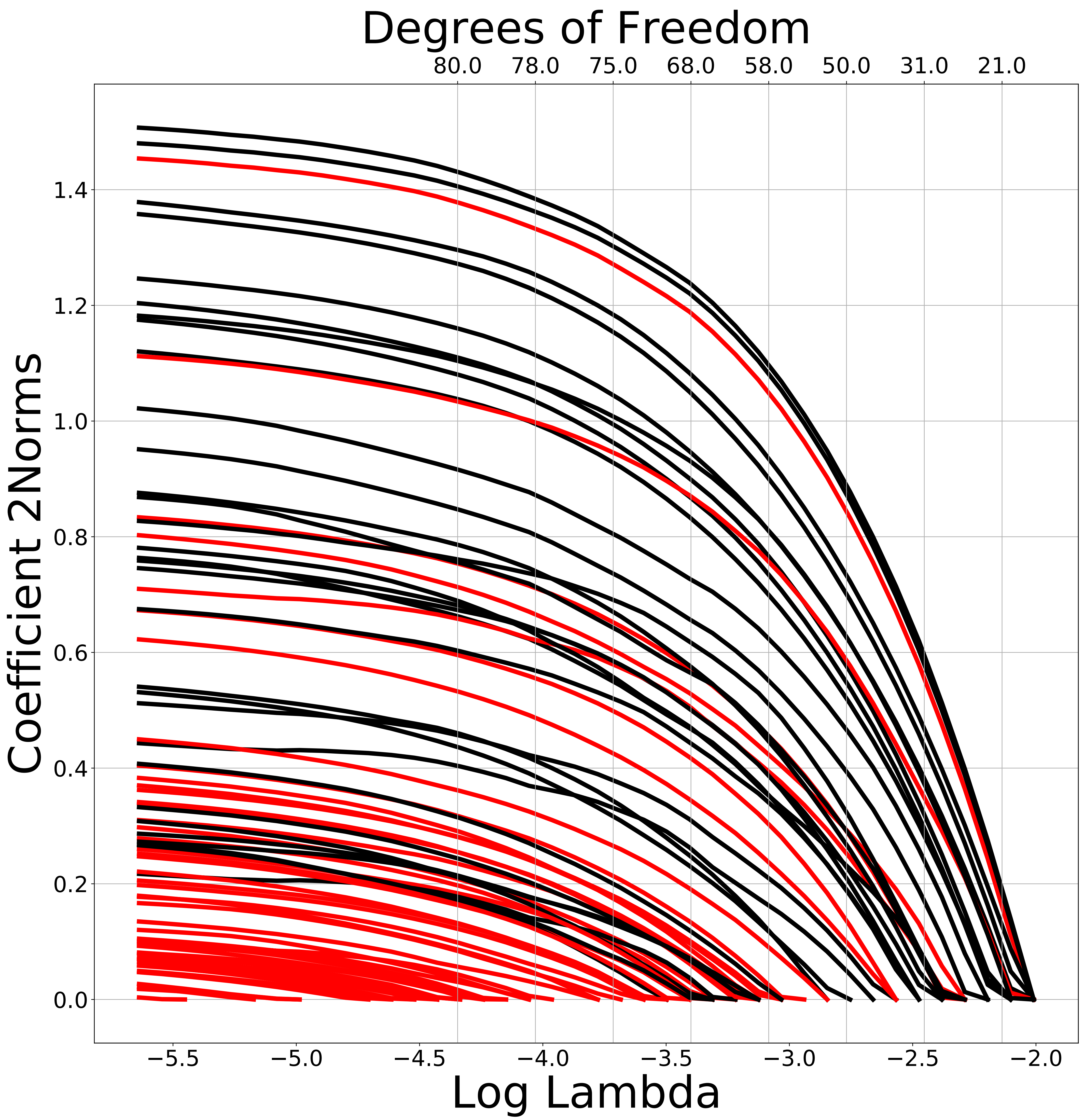

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Sparse Learning: Extend to Logistic Regression

$$\underset{\boldsymbol{\beta},\boldsymbol{\gamma}}{\operatorname{argmin}} L(\boldsymbol{\beta},\boldsymbol{\gamma}) := \|\boldsymbol{Y}-\boldsymbol{X}\boldsymbol{\beta}-\boldsymbol{\gamma}\|_{\mathrm{F}}^{2}+\lambda R(\boldsymbol{\gamma})$$

$$\downarrow$$

$$\underset{\boldsymbol{\gamma}}{\operatorname{argmin}} \left\|\boldsymbol{Y}-\boldsymbol{X}\left(\boldsymbol{X}^{\top}\boldsymbol{X}\right)^{\dagger}\boldsymbol{X}^{\top}(\boldsymbol{Y}-\boldsymbol{\gamma})-\boldsymbol{\gamma}\right\|_{\mathrm{F}}^{2}+\lambda R(\boldsymbol{\gamma})$$

$$\downarrow$$

$$\underset{\boldsymbol{\gamma}}{\operatorname{argmin}} \left\|\tilde{\boldsymbol{Y}}-\tilde{\boldsymbol{X}}\boldsymbol{\gamma}\right\|_{\mathrm{F}}^{2}+\lambda R(\boldsymbol{\gamma})$$

$$\boldsymbol{Y}_{i,c}=\frac{\exp\left(\boldsymbol{X}_{i,\cdot}\boldsymbol{\beta}_{\cdot,c}+\boldsymbol{\gamma}_{i,c}\right)}{\sum_{l=1}^{C}\exp\left(\boldsymbol{X}_{i,\cdot}\boldsymbol{\beta}_{\cdot,l}+\boldsymbol{\gamma}_{i,l}\right)}+\boldsymbol{\varepsilon}_{i,c}$$

$$\bar{\boldsymbol{X}}=(\boldsymbol{X},\boldsymbol{I}) \quad \downarrow \quad \bar{\boldsymbol{\beta}}=(\boldsymbol{\beta},\boldsymbol{\gamma})^{\top}$$

$$\boldsymbol{Y}_{i,c}=\frac{\exp\left(\bar{\boldsymbol{X}}_{i,\cdot}\bar{\boldsymbol{\beta}}_{\cdot,c}\right)}{\sum_{l=1}^{C}\exp\left(\bar{\boldsymbol{X}}_{i,\cdot}\bar{\boldsymbol{\beta}}_{\cdot,l}\right)}+\boldsymbol{\varepsilon}_{i,c}$$

Identifiability in Reality: Conditions and Accuracy

Satisfied Assumptions	None	C1	C1 and C2	All
Improved Episodes	0	424	1035	40
Total Episodes	0	793	1164	43
I/T	–	53.5%	88.9%	93.0%

1) In more than half of the experiments the assumptions C1-C2 are satisfied. Most of them (89.0%) will achieve better performance after self-taught with ICI.

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2) When all the assumptions are satisfied, we will get better performance in a high ratio (93.0%).

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Wang et al. How to Trust Unlabeled Data? Instance Credibility Inference for Few-Shot Learning. IEEE TPAMI 2021

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3) Even if C2-C3 are not satisfied, we still have the chance of improving the performance (53.5%).

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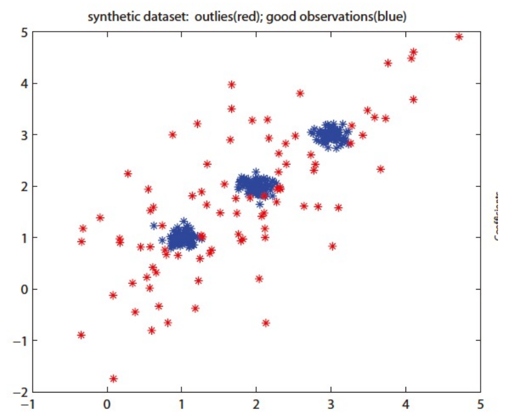
Sparse Learning

in Learning with Noisy Labels

Definition of learning with noisy labels

Deep Models



Shogun: Total War - IGN
ign.com

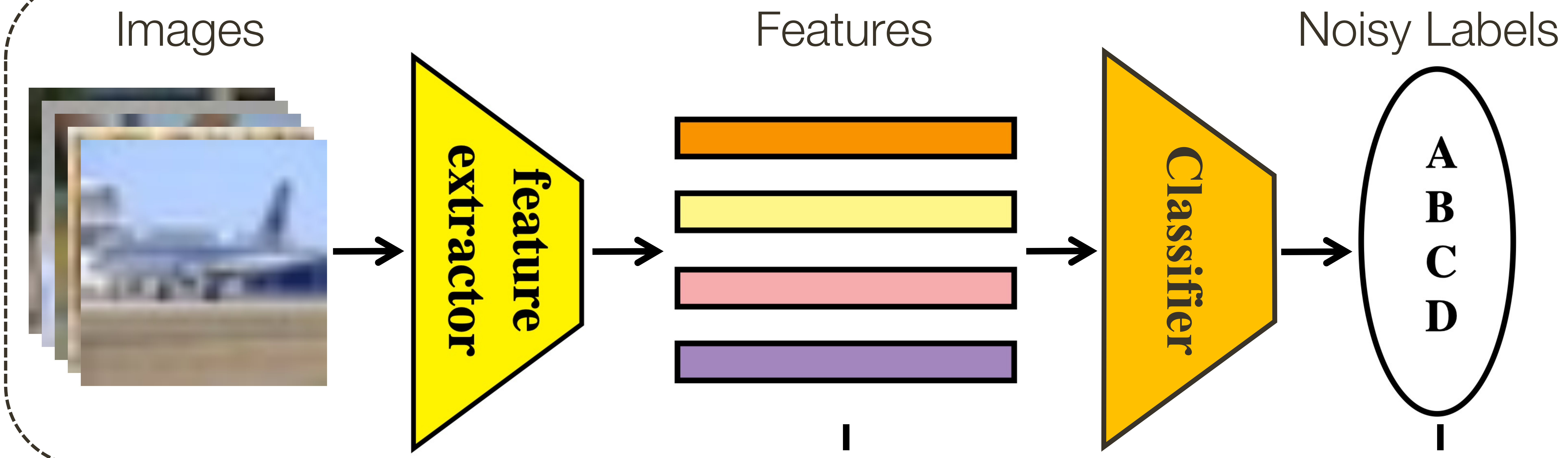
Aug 21, 1192 CE: First S...
nationalgeographic.org

Baal Ascension Materials: What To Farm For G...
forbes.com

Robustly Trained Models

Framework

Stage 1:
Feature Learning

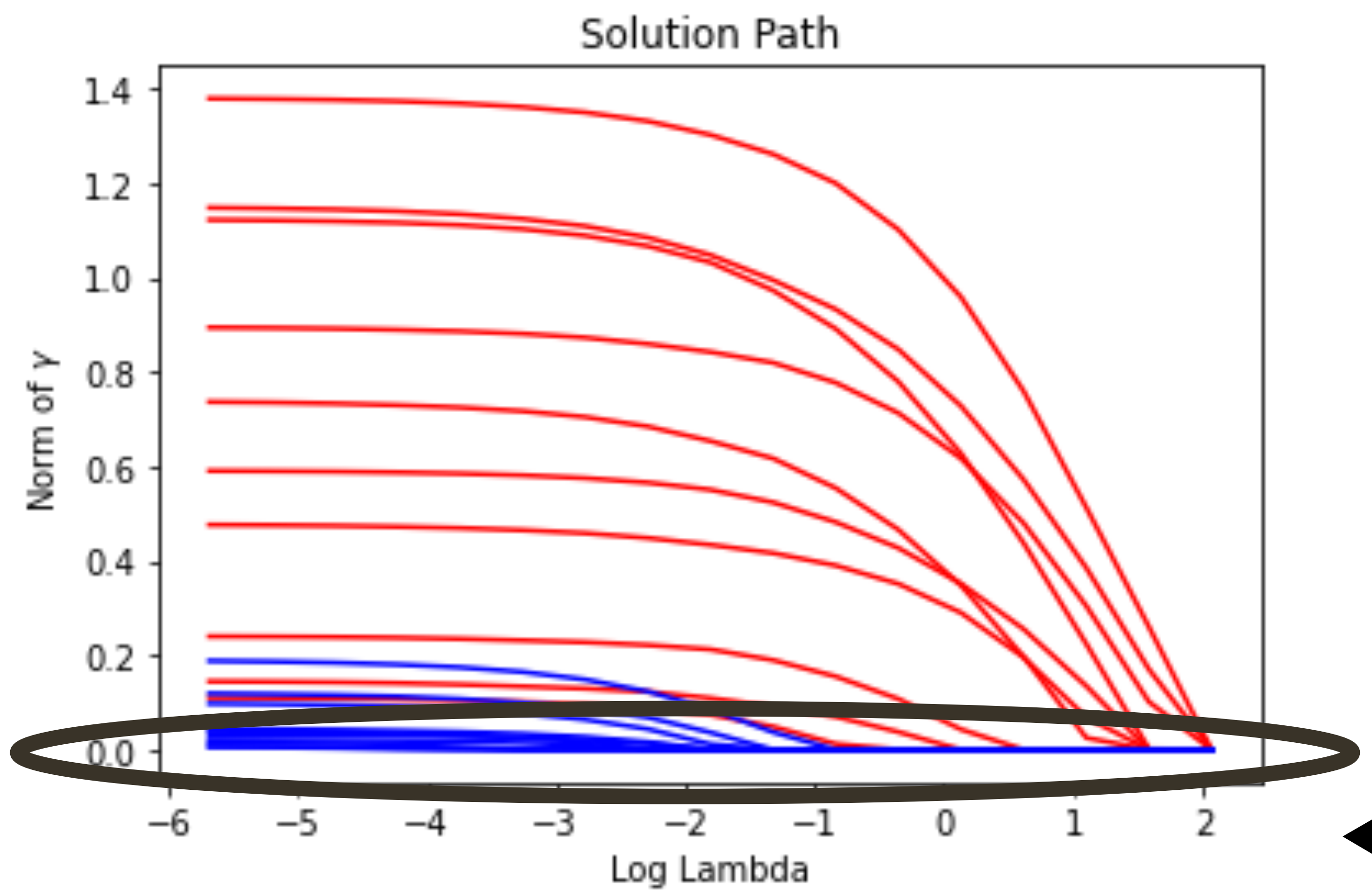


Stage 2:
Sample Selection

Clean or Noisy?

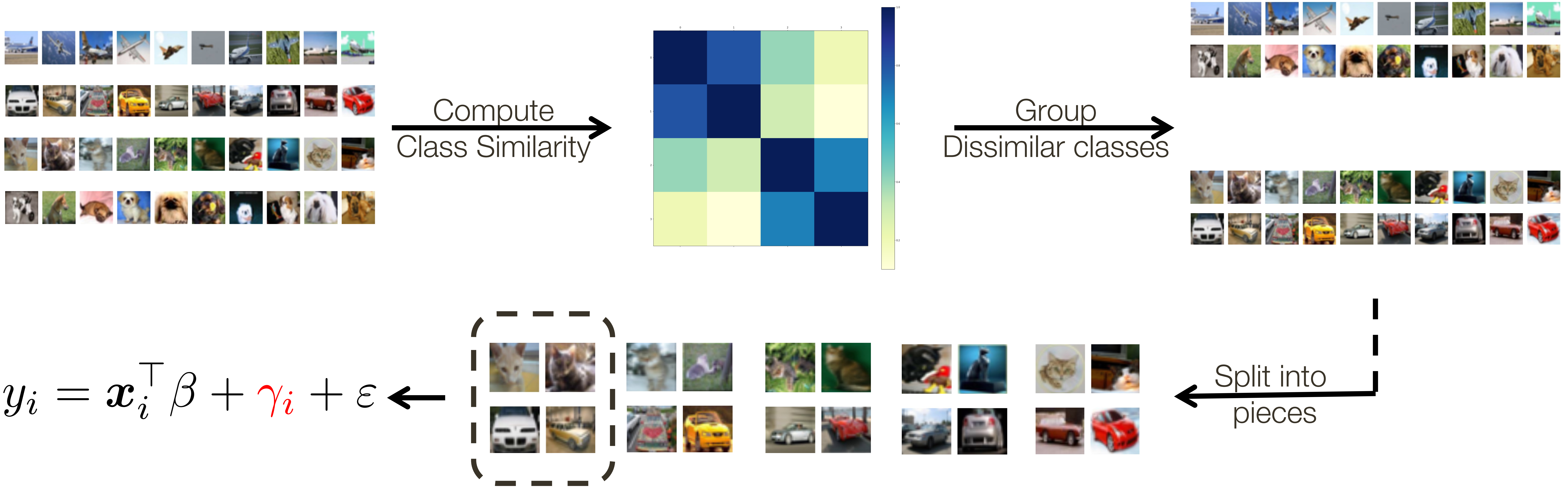
$$y_i = \boldsymbol{x}_i^\top \beta + \gamma_i + \varepsilon$$

$$\underset{\gamma}{\operatorname{argmin}} \left\| \tilde{\boldsymbol{Y}} - \tilde{\boldsymbol{X}} \boldsymbol{\gamma} \right\|_{\mathrm{F}}^2 + \sum_{i=1}^{n} P(\gamma_i; \lambda_i)$$



Wang et al. Scalable Penalized Regression for Noise Detection in Learning with Noisy Labels. CVPR 2022.

Make it scalable to large datasets



$$y_i = \boldsymbol{x}_i^\top \beta + \gamma_i + \varepsilon$$

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Strategies to help train the network

- Append a $\ell_q (q < 1)$ penalty to encourage the linear relation between feature and one-hot encoded vector:

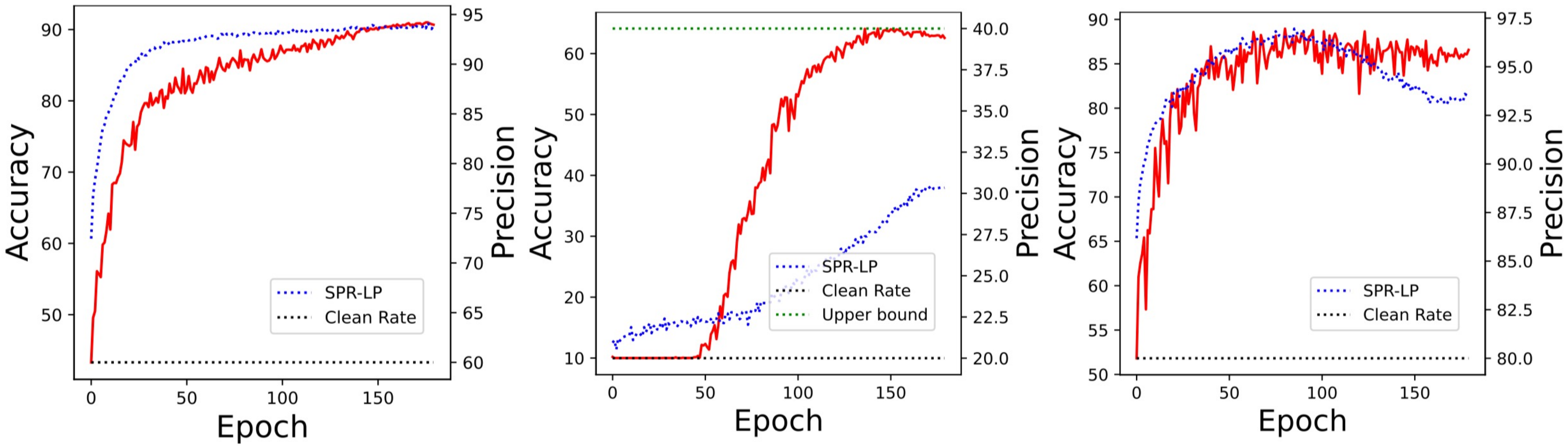
$$\mathcal{L}\left(\boldsymbol{x}_i, \boldsymbol{y}_i\right) = 1_{i \notin O}\left(\mathcal{L}_{\text{CE}}\left(\boldsymbol{x}_i, \boldsymbol{y}_i\right) + \lambda \left\|\boldsymbol{x}_i^{\top} W_{\text{fc}}\right\|_q\right)$$

- Use CutMix to further exploit the support of noisy data

$$\tilde{\boldsymbol{x}} = \boldsymbol{M} \odot \boldsymbol{x}_{\text{clean}} + (1 - \boldsymbol{M}) \odot \boldsymbol{x}_{\text{noisy}}$$
$$\tilde{\boldsymbol{y}} = \lambda \boldsymbol{y}_{\text{clean}} + (1 - \lambda) \boldsymbol{y}_{\text{noisy}}$$

Wang et al. Scalable Penalized Regression for Noise Detection in Learning with Noisy Labels. CVPR 2022.

Label precision performance



(a) Symmetric-40% (b) Symmetric-80% (c) Asymmetric-40%

Wang et al. Scalable Penalized Regression for Noise Detection in Learning with Noisy Labels. CVPR 2022.

THANKS