

# Learning Sparsity in Neural Networks and Robust Statistical Analysis

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HKUST



Yuan Yao

**Wotao Yin**

DAMO Academy  
Alibaba



Wotao Yin

<https://sparse-learning.github.io>

# **Learning Sparsity in Neural Networks and Robust Statistical Analysis**

## **Lecture 1:**

**Yanwei Fu**

School of Data Science

Fudan University

<http://yanweifu.github.io>

# Motivation

# Overview of

ECCV2012  
tutorial:

## Sparse and Low-Rank Recovery of *Data*

**Sparse and Low-Rank Representation**  
**Lecture I: Motivation and Theory**

Tutorial on Sparse and Low-rank Modeling, European Conference on Computer Vision,  
Firenze, Italy, October 2012

**Yi Ma**

MSRA and UIUC

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Our CVPR2022 tutorial:

Prerecorded Sessions			
8:30 - 8:40	<b>Opening Remarks</b>	Virtual	Yanwei Fu
8:40 - 9:10	<b>Introduction</b>	Virtual	Yanwei Fu
9:10 - 9:30	<b>Sparsity Learning in Noisy Data Detection</b>	Virtual	Yanwei Fu and Yikai Wang
9:30 - 10:15	<b>Inverse Scale Space method and Statistical Properties</b>	On-site	Yuan Yao
10:15 - 10:30	<b>Break</b>		
10:30 - 11:00	<b>Sparsity Learning in Medical Imaging</b>	Virtual	Xinwei Sun
11:00 - 12:00	<b>Learning to Optimize</b>	On-site	Wotao Yin

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Our CVPR2022 tutorial:

## Learning Sparsity in Labels/Data

for Robust Statistical Analysis

Sparse data/label learning

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大数据学院  
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Our CVPR2022 tutorial:

## Learning Sparsity in Labels/Data

Sparse data/label learning

## Learning Sparsity in Deep Models

Learning the sparse model

for Robust Statistical Analysis

for Compressive Neural Networks

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# Sparse and Low-Rank Recovery of *Data*



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*Underdetermined  
Linear system*

$$y = Ax$$

# Sparse and Low-Rank Recovery of *Data*

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**Observation**  $y \in \mathbb{R}^m$

**Unknown**  $x \in \mathbb{R}^n$

$A \in \mathbb{R}^{m \times n}$

$m \ll n$

# Observations    # unknowns

Part of content adapted from Prof. Yi Ma's ECCV2012 tutorial

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[Wainwright, et al. IEEE TIT 2009]

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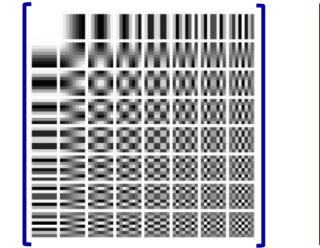
[Wainwright, et al. IEEE TIT 2009]

Compression:



(Patches of) ...  
input image

$\approx$



$A$  DCT basis



$x$  coefficients

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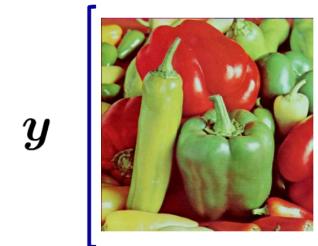
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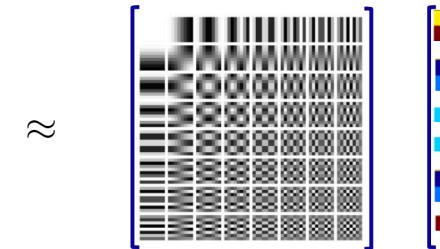
# Observations # unknowns

[Wainwright, et al. IEEE TIT 2009]

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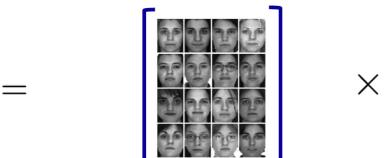


$A$  DCT basis       $x$  coefficients

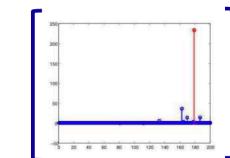
Recognition:



$y \in \mathbb{R}^m$   
Test image



$A = [A_1 \mid A_2 \mid \cdots \mid A_k]$   
Combined  
training  
dictionary



$x \in \mathbb{R}^n$   
coefficients



$e \in \mathbb{R}^m$   
corruption,  
occlusion

# Sparse and Low-Rank Recovery of *Data* (Cont.)

From recovering a *single sparse vector* to recovering *low-rank matrix* (many correlated vectors):

$$y = \begin{bmatrix} \text{Image} \\ A \end{bmatrix} x + e \quad \xrightarrow{\hspace{10em}} \quad Y = \begin{bmatrix} \text{Image} \\ \dots \\ \text{Image} \end{bmatrix} = \begin{bmatrix} \text{Image} \\ X \end{bmatrix} + \begin{bmatrix} \text{Image} \\ \dots \\ \text{Image} \end{bmatrix} E$$

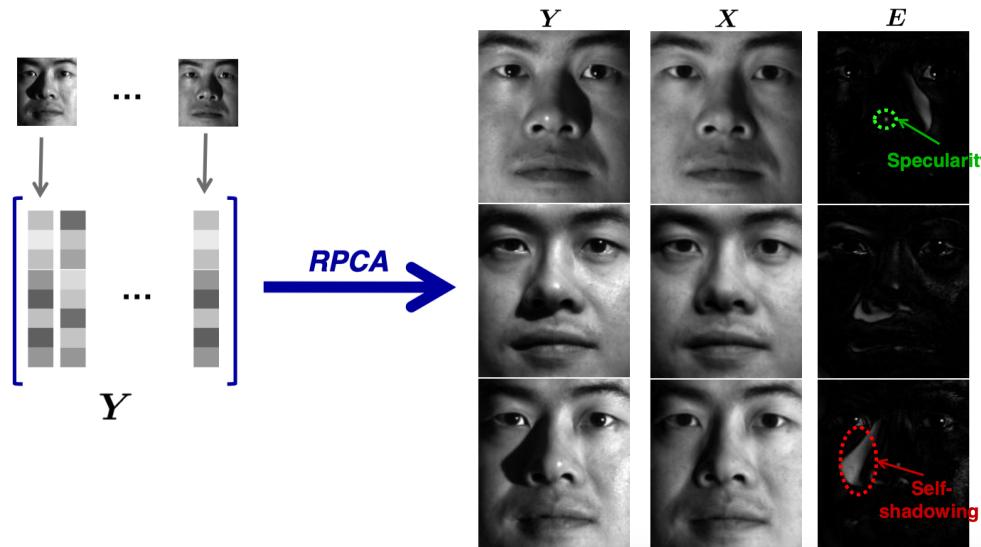
# Sparse and Low-Rank Recovery of *Data* (Cont.)

From recovering a *single sparse vector* to recovering *low-rank matrix* (many correlated vectors):

$$y = \begin{bmatrix} \text{Face Image} \\ A \end{bmatrix} x + e \quad \xrightarrow{\text{RPCA}} \quad Y = \begin{bmatrix} \text{Face Image} \\ E \end{bmatrix} = \begin{bmatrix} \text{Face Image} \\ X \end{bmatrix} + \begin{bmatrix} \text{Face Image} \\ E \end{bmatrix}$$

---

Faces under varying illumination:



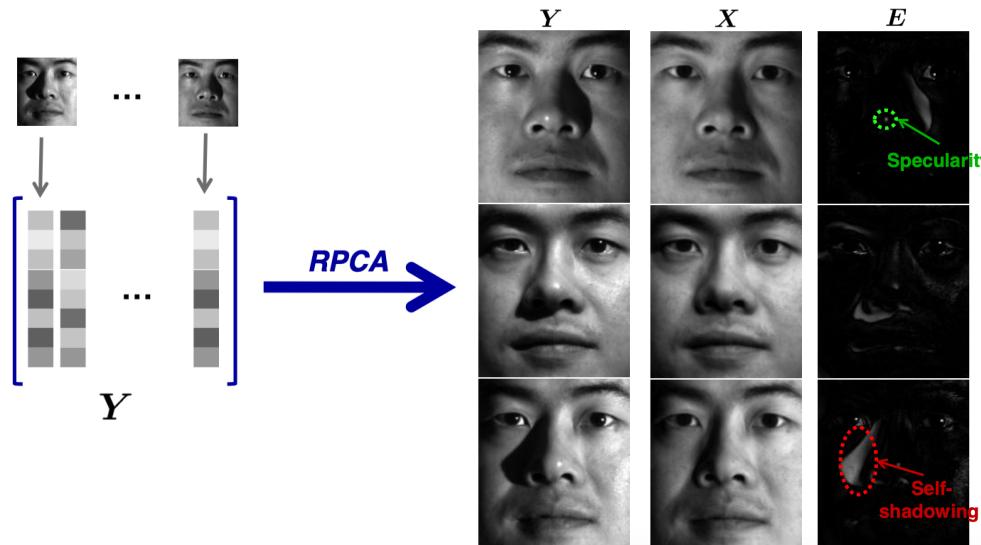
Part of content adapted from Prof. Yi Ma's ECCV2012 tutorial

# Sparse and Low-Rank Recovery of Data (Cont.)

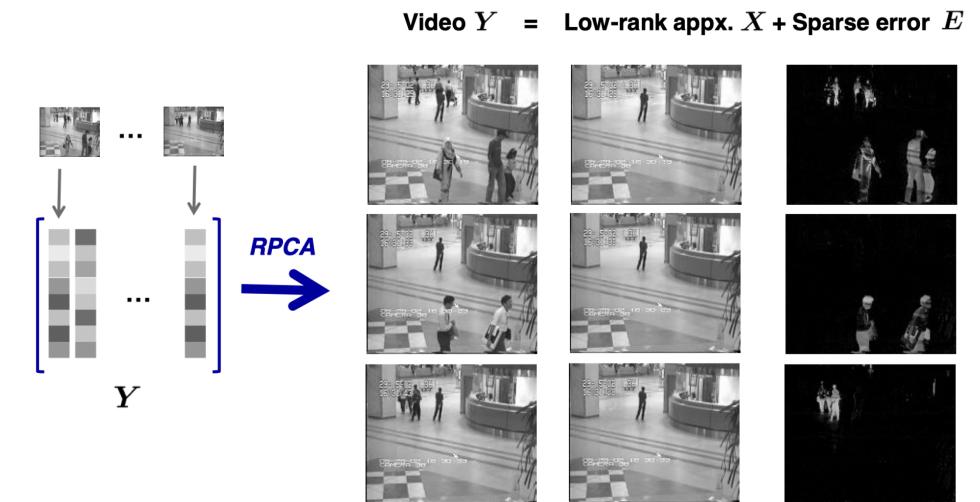
From recovering a *single sparse vector* to recovering *low-rank matrix* (many correlated vectors):

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Faces under varying illumination:



Background modeling from video :



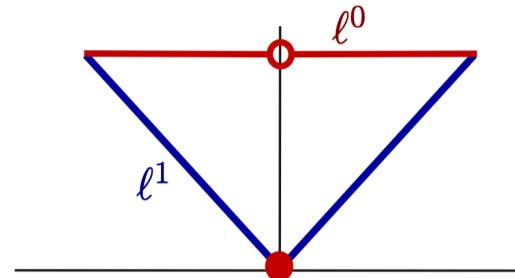
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# Sparse Optimization

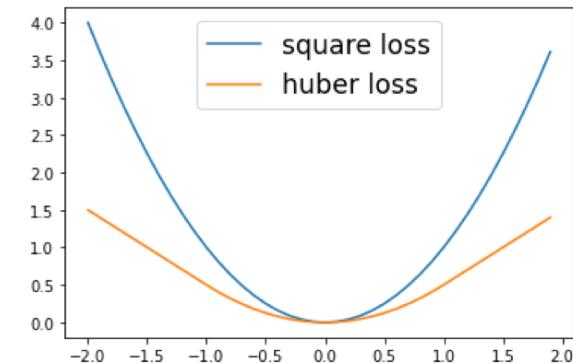
$$\text{minimize} \quad \|x\|_0 \quad \text{subject to} \quad Ax = y$$

- $L_1$  norm  $\|x\|_0 \rightarrow \|x\|_1$



- Huber-Loss:  $\|y - Ax\|_2^2 \rightarrow L_\delta(y - Ax)$

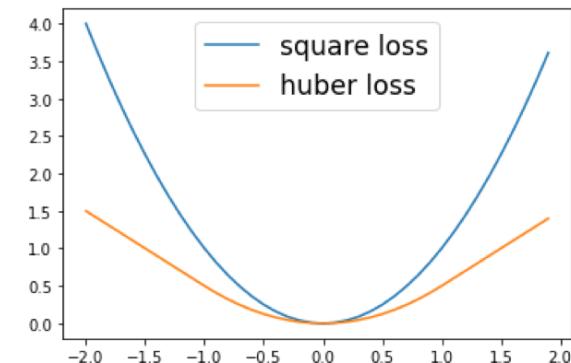
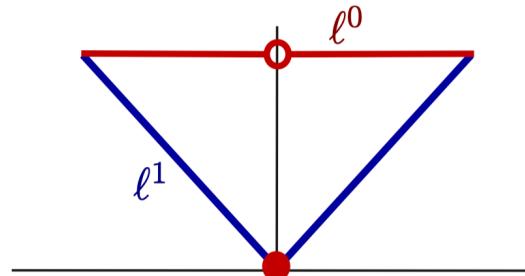
$$\text{where } L_\delta(x) = \begin{cases} \frac{1}{2}(x)^2 & \text{for } |x| \leq \delta \\ \delta \cdot (|x| - \frac{1}{2}\delta), & \text{otherwise} \end{cases}$$



# Sparse Optimization

minimize  $\|x\|_0$  subject to  $Ax = y$   
nonconvex  $\longrightarrow$  NP-hard!

- $L_1$  norm  $\|x\|_0 \rightarrow \|x\|_1$
- Huber-Loss:  $\|\mathbf{y} - A\mathbf{x}\|_2^2 \rightarrow L_\delta(\mathbf{y} - A\mathbf{x})$   
where  $L_\delta(x) = \begin{cases} \frac{1}{2}(x)^2 & \text{for } |x| \leq \delta \\ \delta \cdot (|x| - \frac{1}{2}\delta), & \text{otherwise} \end{cases}$



# Sparse Optimization

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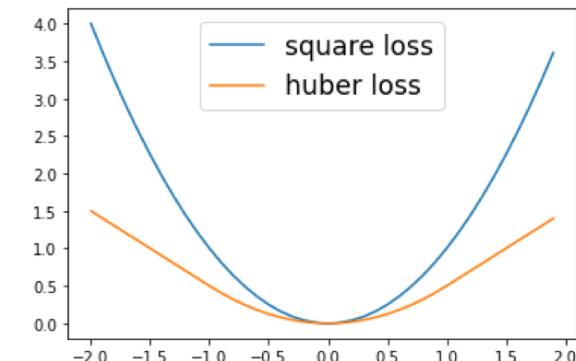
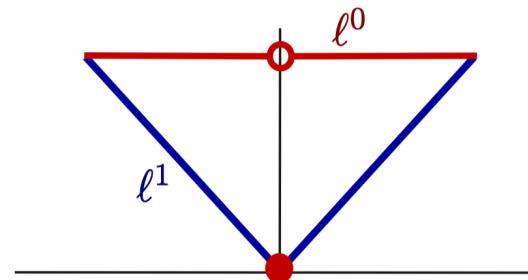


Relax the problem

- $L_1$  norm  $\|x\|_0 \rightarrow \|x\|_1$

- Huber-Loss:  $\|y - Ax\|_2^2 \rightarrow L_\delta(y - Ax)$

where  $L_\delta(x) = \begin{cases} \frac{1}{2}(x)^2 & \text{for } |x| \leq \delta \\ \delta \cdot (|x| - \frac{1}{2}\delta), & \text{otherwise} \end{cases}$



# Overview

- Sparse Learning in Data/Label
- Sparse Learning in Deep Models



# Sparse learning for Noisy Data/Labels

*Underdetermined Linear system*

$$y = Ax$$

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**Unknown**  $x \in \mathbb{R}^n$

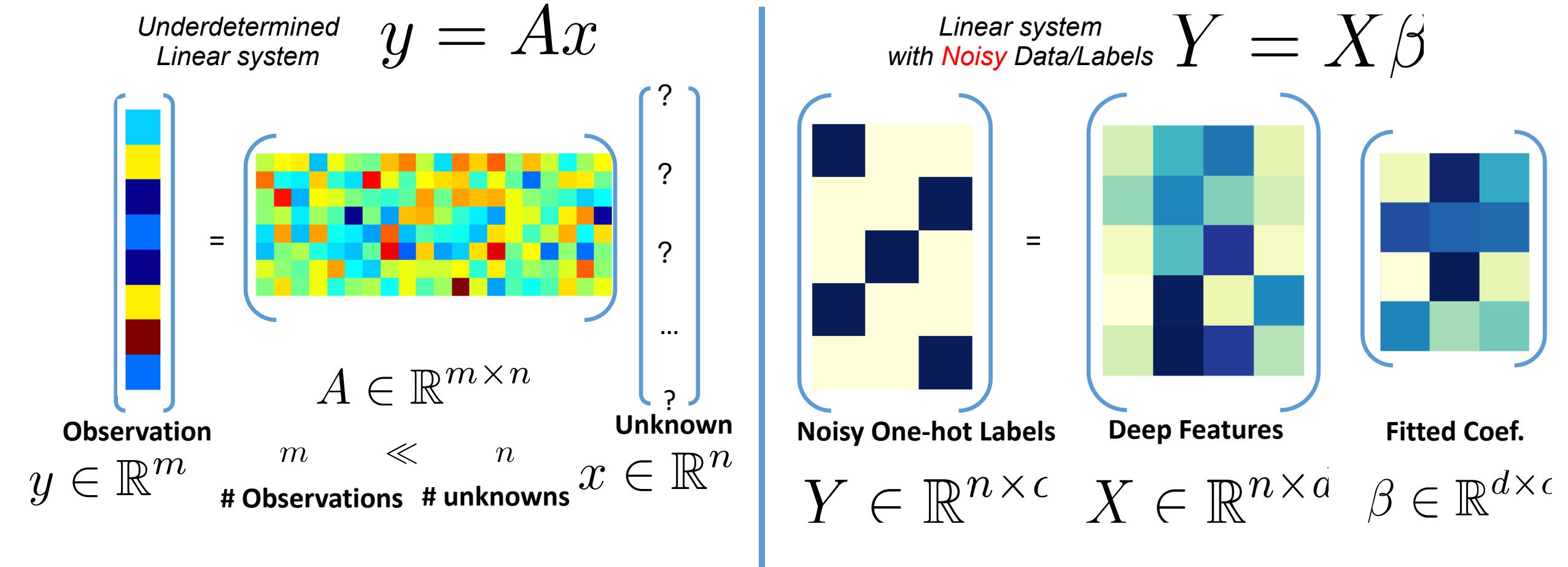
$A \in \mathbb{R}^{m \times n}$

$m \ll n$

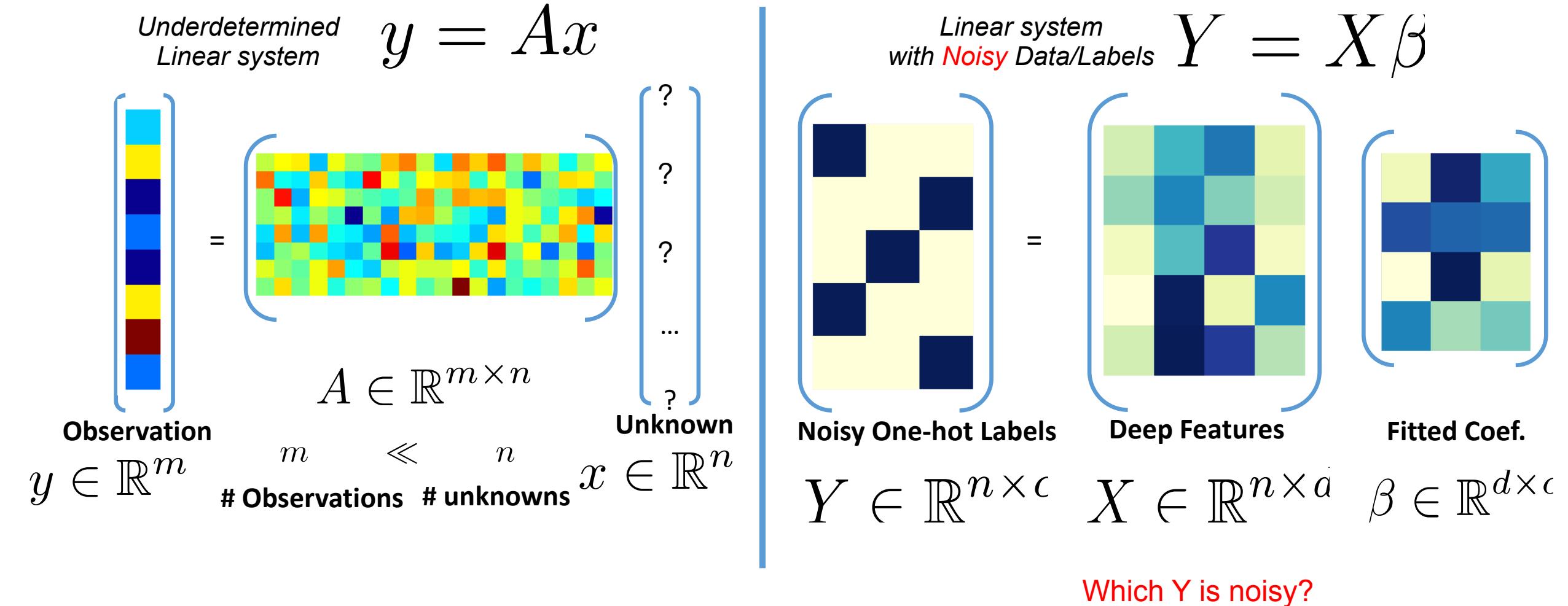
# Observations    # unknowns

The diagram illustrates an underdetermined linear system  $y = Ax$ . On the left, a vertical vector labeled "Observation" is shown with a blue bracket around its top half, which contains several colored squares (cyan, yellow, dark blue, light blue, red). This vector is followed by an equals sign. To the right of the equals sign is a large blue bracket enclosing a square matrix  $A$ , which is filled with a variety of colored squares (cyan, yellow, green, orange, red, dark blue) in a seemingly random pattern. Below the matrix  $A$  is the expression  $A \in \mathbb{R}^{m \times n}$ . To the right of the matrix  $A$  is another vertical vector labeled "Unknown" with a blue bracket around its bottom half, containing question marks above each entry. Below this vector is the expression  $x \in \mathbb{R}^n$ . At the bottom of the diagram, there is a horizontal line with two arrows pointing towards each other, and below this line is the text "# Observations" and "# unknowns".

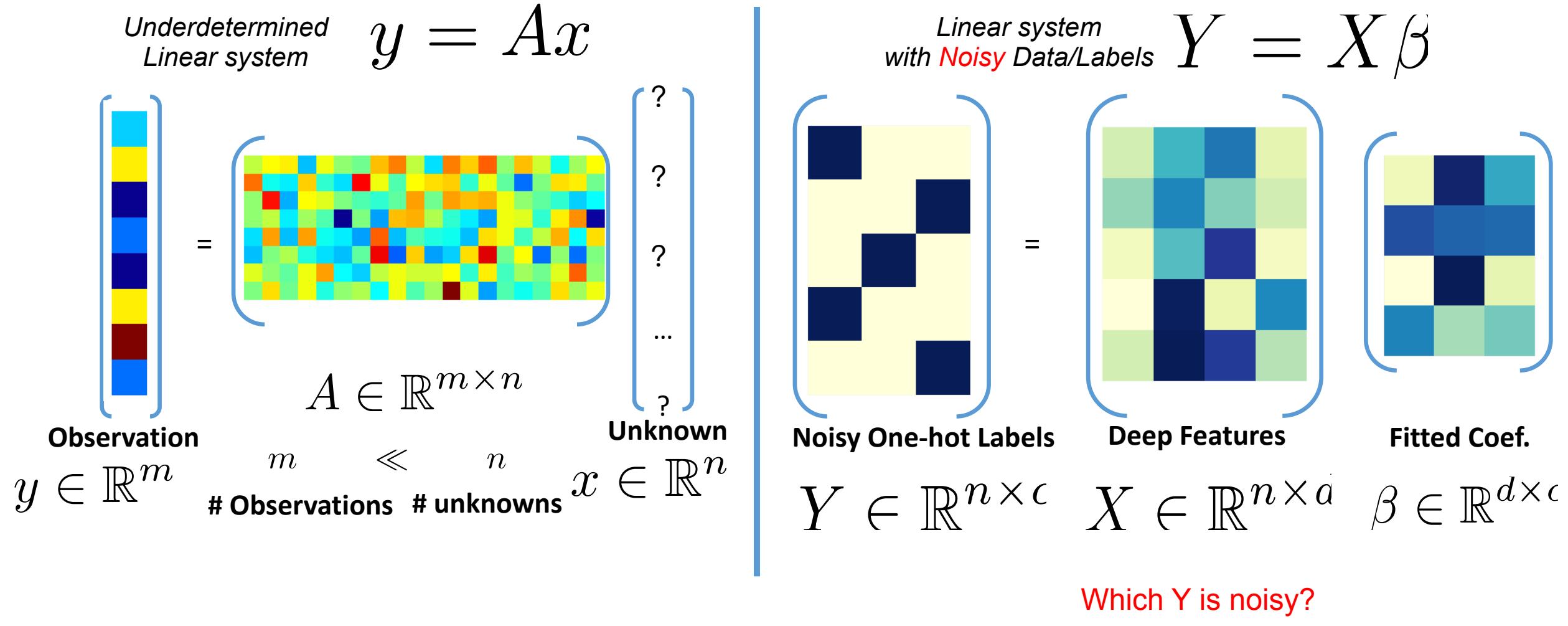
# Sparse learning for Noisy Data/Labels



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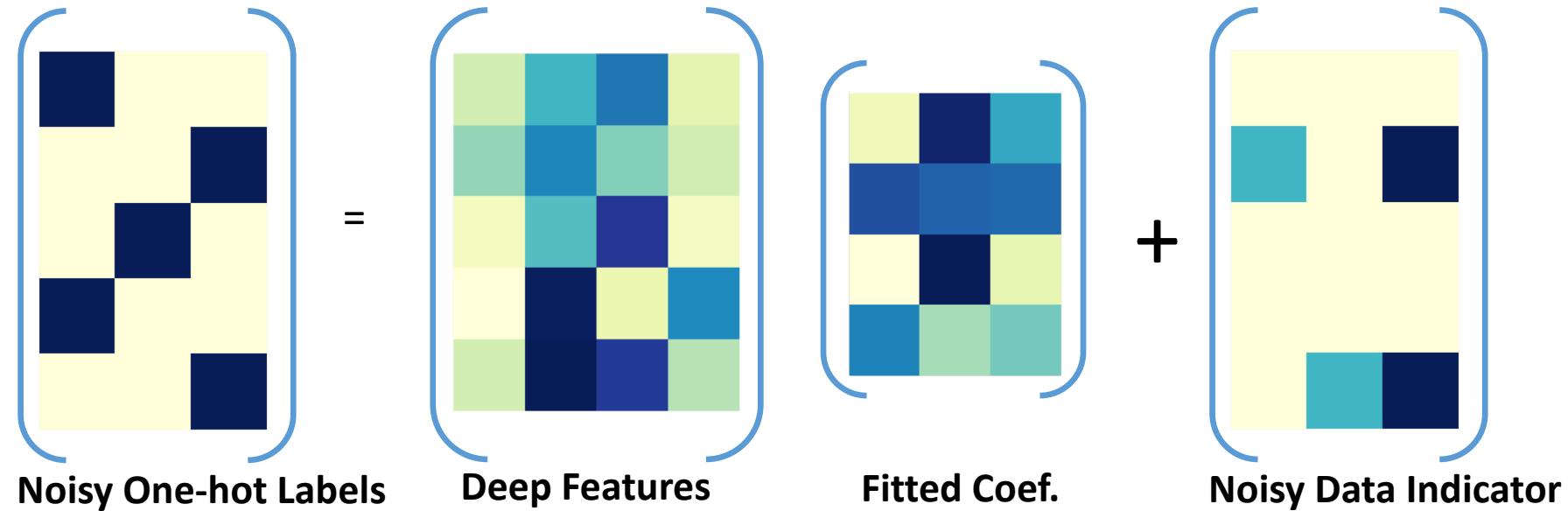


# Sparse learning for Noisy Data/Labels



# Sparse learning for Noisy Data/Labels: The Indicator

*Linear system  
with Noisy Data/Labels*  $Y = X\beta + \gamma$



$$Y \in \mathbb{R}^{n \times c} \quad X \in \mathbb{R}^{n \times a} \quad \beta \in \mathbb{R}^{d \times c} \quad \gamma \in \mathbb{R}^{n \times c}$$

# Sparsity in Data/Labels: Different Focus

Robust regression/  
Classification,  
[Wang et al. CVPR2020].

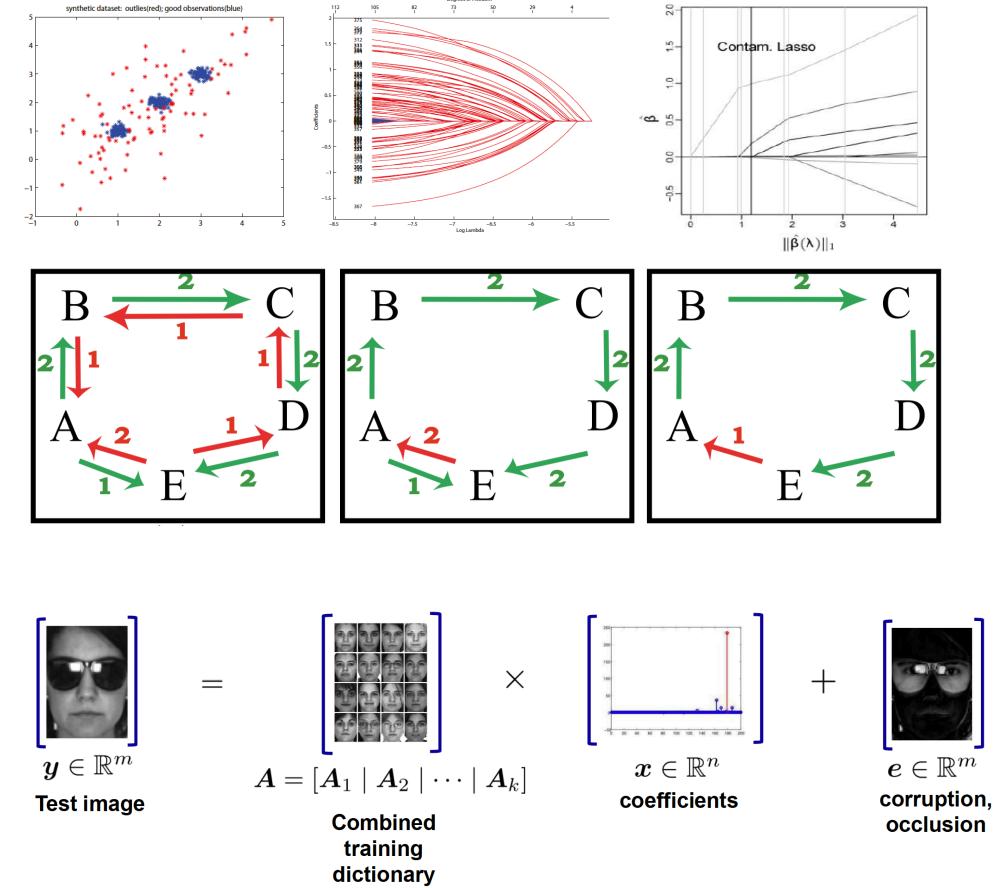
$$y = x^\top \beta + \epsilon + \gamma$$

Statistical robust ranking,  
[Fu et al. TPAMI 16]

$$y = x^\top \beta + \epsilon + \gamma$$

Face Recognition,  
[Wright et al. TPAMI 09]

$$y = (A, I) \begin{pmatrix} x \\ \gamma \end{pmatrix} + \epsilon$$



[Zhao et al. ICML 2018][Fu et al. ECCV 2014/TPAMI2016], [Wang et al. CVPR2020/TPAMI2021/CVPR2022] [Huang et al. ECCV2014]

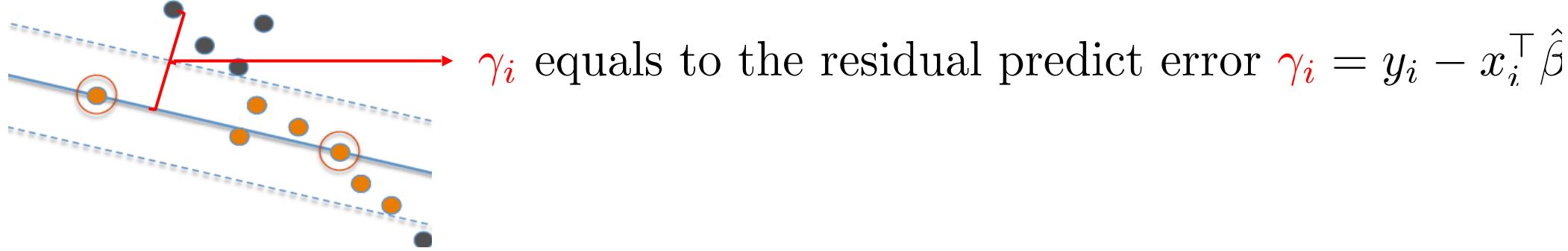
Figure of RANSAC is by Xavi.borras - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=37017886>

# Understanding $\gamma$ in Statistics

$$y = x^\top \beta + \epsilon + \gamma$$

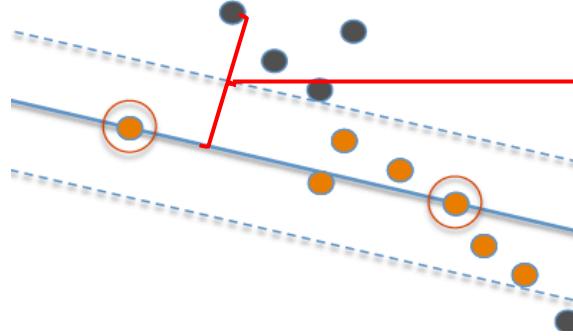
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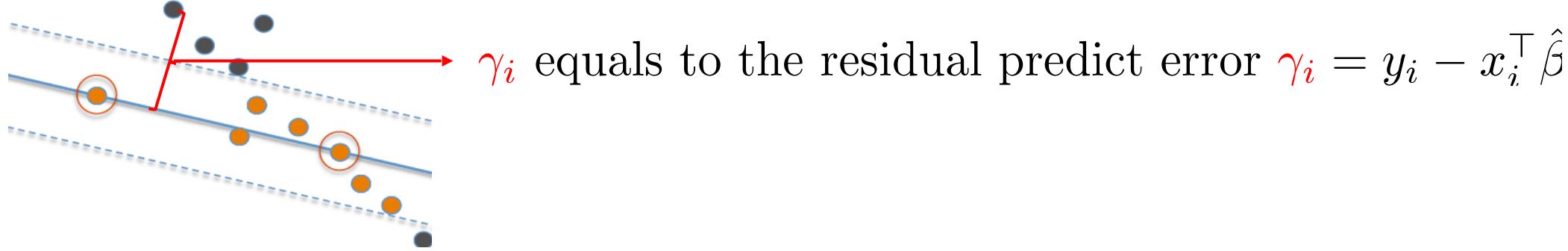


$\gamma_i$  equals to the residual predict error  $\gamma_i = y_i - x_i^\top \hat{\beta}$

Row residuals fail to detect outliers at *leverage points*.

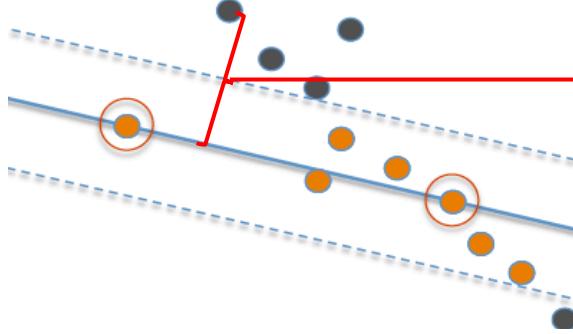
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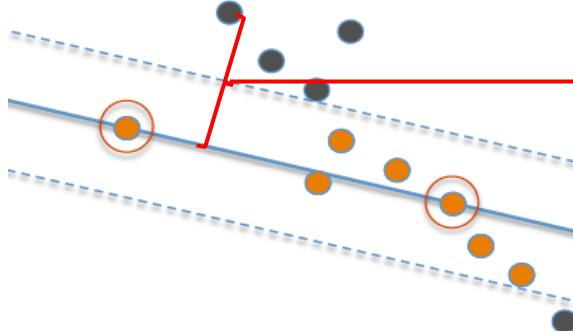


Leave-one-out externally studentized residual

$$t_i = \frac{y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}_{(i)}}{\hat{\sigma}_{(i)} (1 + \mathbf{x}_i (\mathbf{X}_{(i)}^\top \mathbf{X}_{(i)})^{-1} \mathbf{x}_i)^{1/2}}$$

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$$y = \mathbf{x}^\top \boldsymbol{\beta} + \epsilon + \gamma$$



$\gamma_i$  equals to the residual predict error  $\gamma_i = y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}$



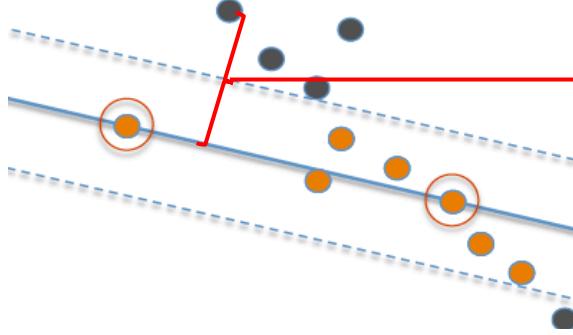
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$\Leftrightarrow$  test whether  $\gamma = 0$  in  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \gamma \mathbf{1}_i + \boldsymbol{\epsilon}$

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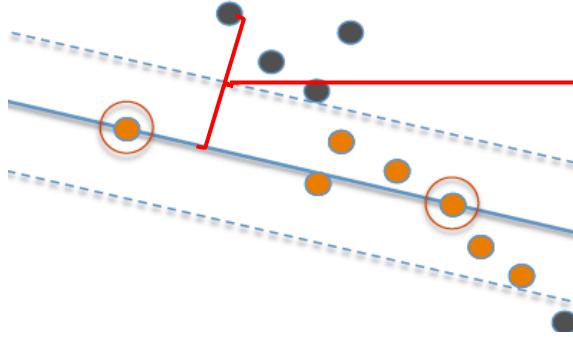
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When there are multiple outliers:

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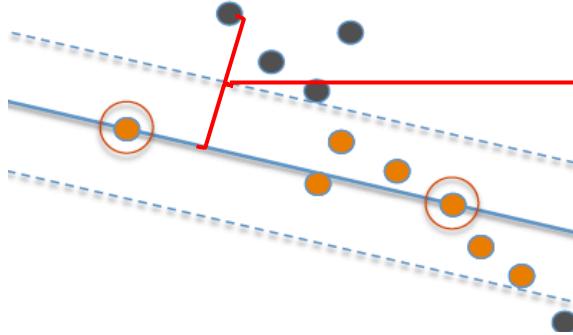
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When there are multiple outliers:

**1. masking:** multiple outliers may mask each other and being **undetected**;

# Understanding $\gamma$ in Statistics

$$y = \mathbf{x}^\top \boldsymbol{\beta} + \epsilon + \gamma$$



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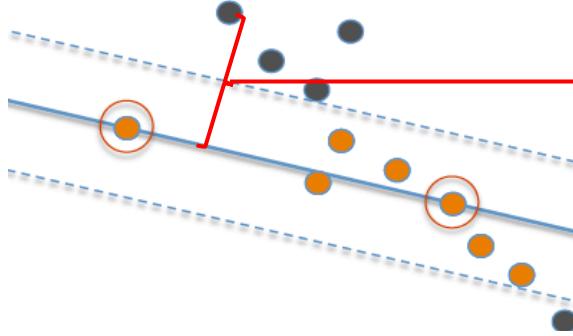
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When there are multiple outliers:

1. **masking**: multiple outliers may mask each other and being **undetected**;
2. **swamping**: multiple outliers may lead the **large  $t_i$  for clean data**.

# Understanding $\gamma$ in Statistics

$$y = \mathbf{x}^\top \boldsymbol{\beta} + \epsilon + \gamma$$



$\gamma_i$  equals to the residual predict error  $\gamma_i = y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}$



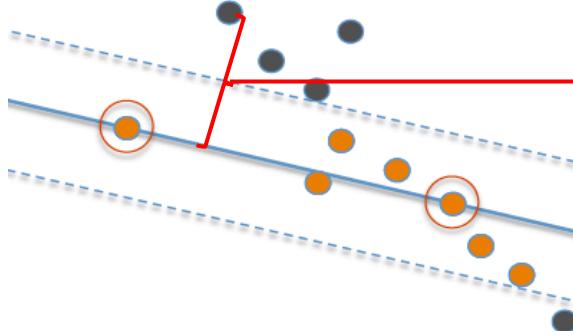
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Leave-one-out externally studentized residual

$$t_i = \frac{y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}_{(i)}}{\hat{\sigma}_{(i)}(1 + \mathbf{x}_i^\top (\mathbf{X}_{(i)}^\top \mathbf{X}_{(i)})^{-1} \mathbf{x}_i)^{1/2}}$$

$\Leftrightarrow$  test whether  $\gamma = 0$  in  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \gamma \mathbf{1}_i + \boldsymbol{\epsilon}$



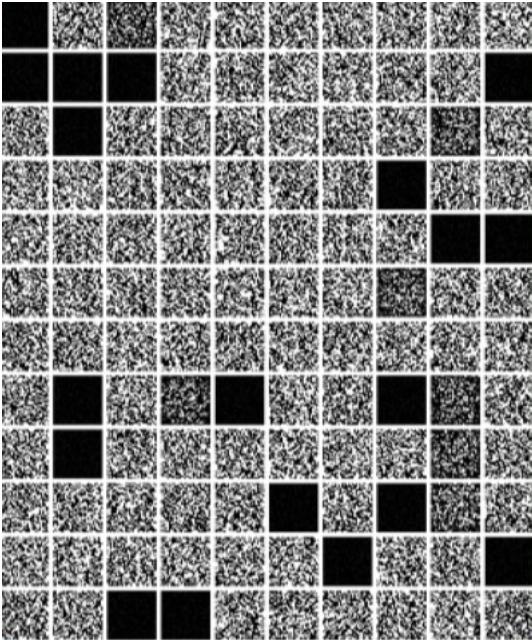
$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} + \gamma$$

# Overview

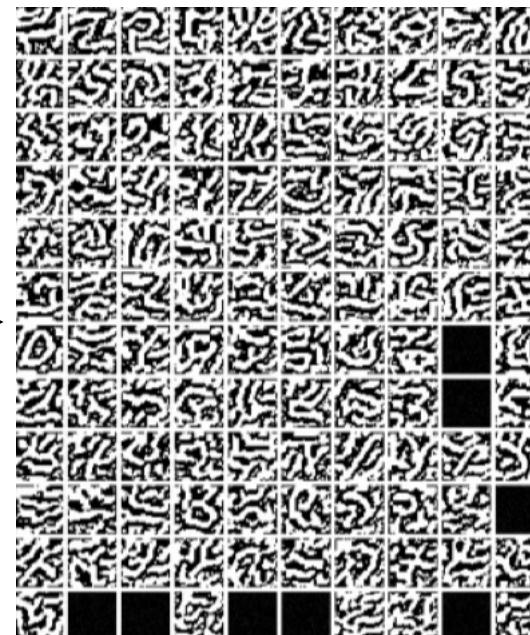
- Spare Learning in Data/Label
- Sparse Learning in Deep Models

# Learning Sparsity in Neural Networks

Random initialized weights  
in convolutional layers



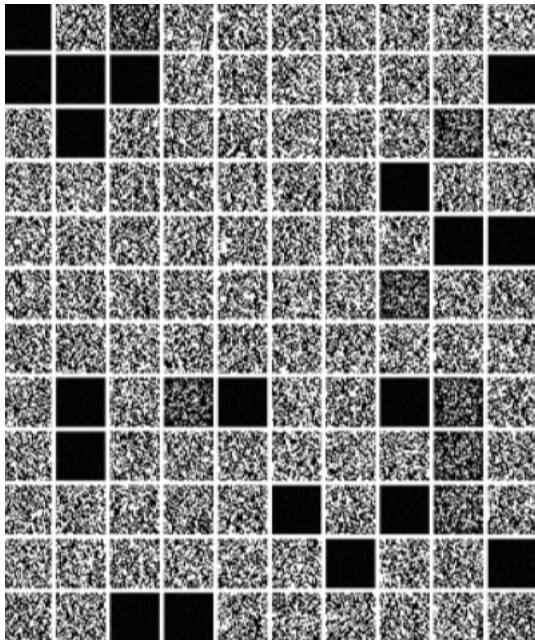
Trained CNN weights



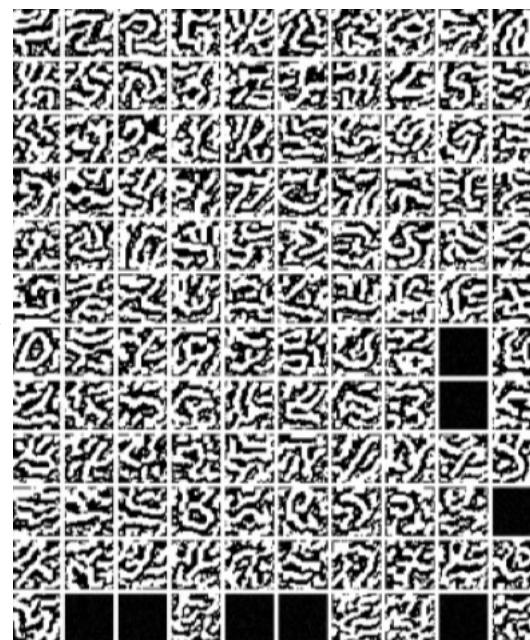
Densely trained model

# Learning Sparsity in Neural Networks

Random initialized weights  
in convolutional layers

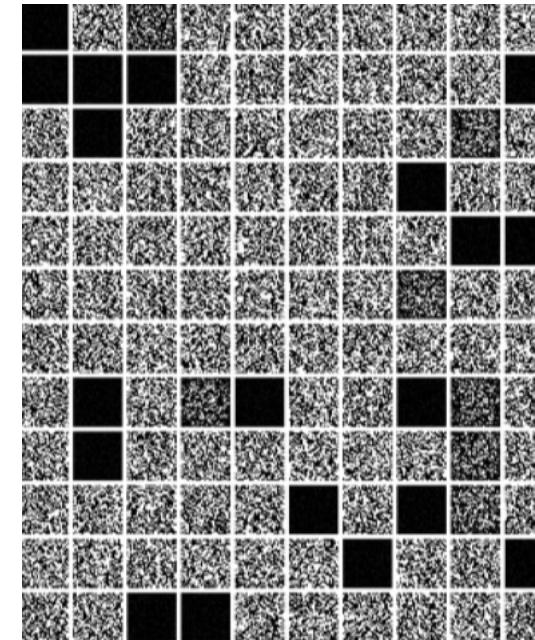


Trained CNN weights

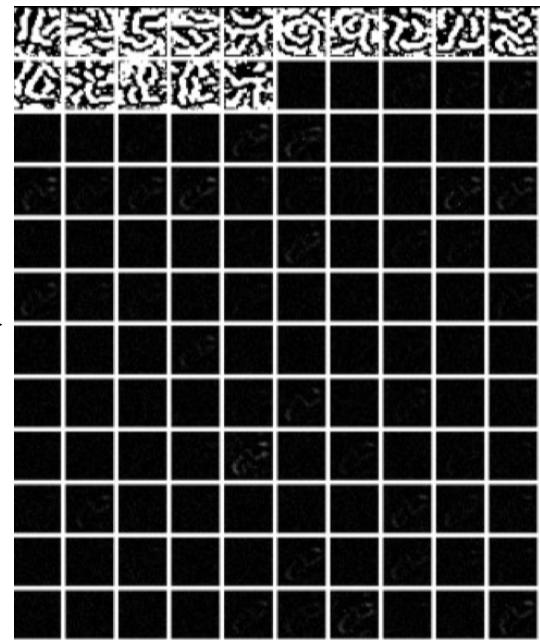


Densely trained model

Random initialized weights  
in convolutional layers

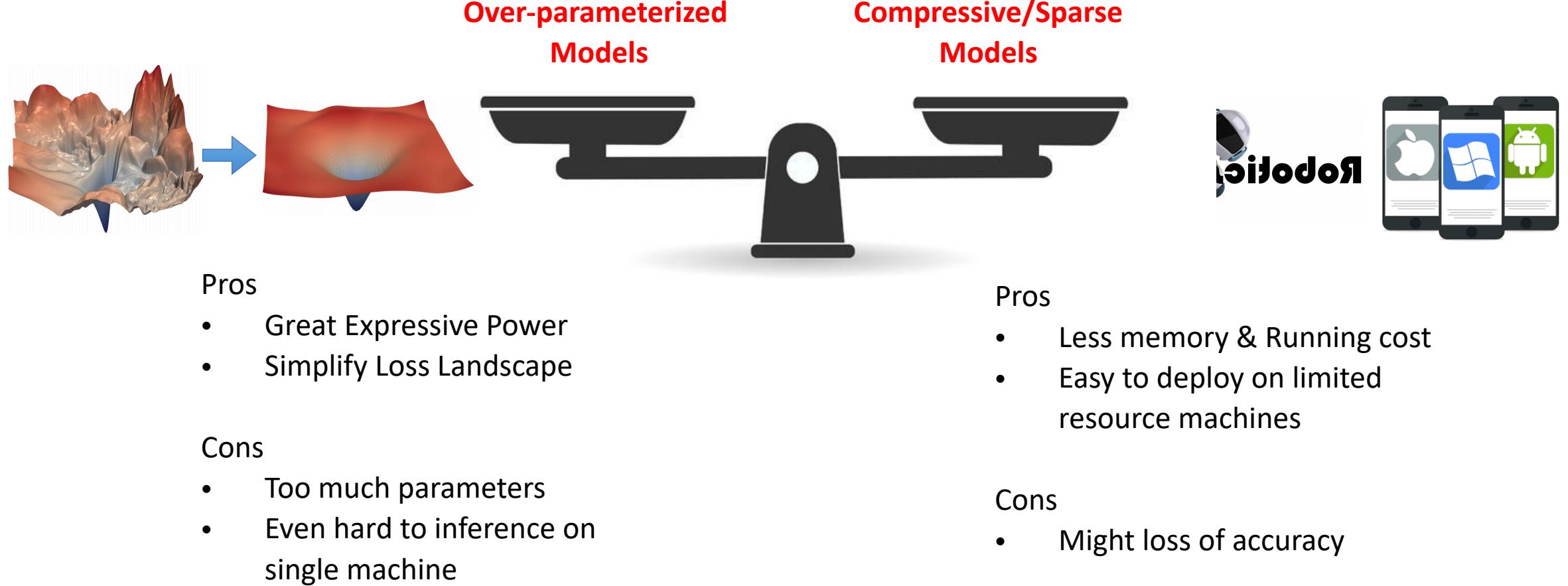


Trained CNN weights



Sparsified model

# Tradeoff between Overparameterized and Compressive models

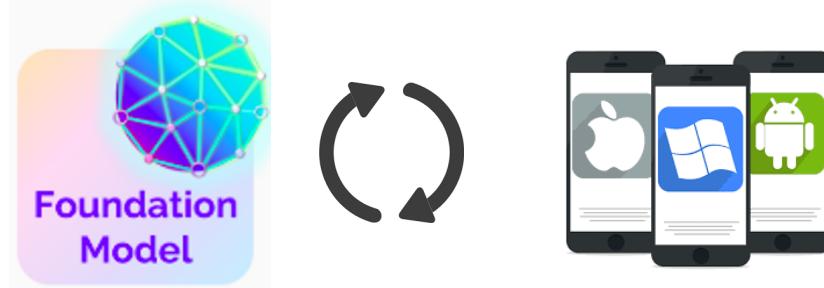


[Fu et al. ICML 2020/TPAMI2022]

Left figures from Li et al. Visualizing the loss landscape of neural nets. NeurIPS 2018

# Potential Connection to Foundation model

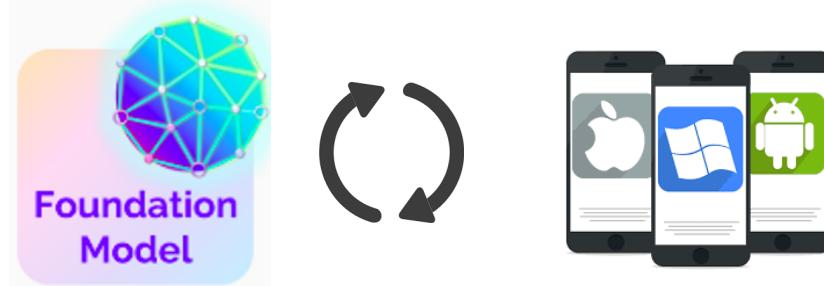
# Potential Connection to Foundation model



Training foundation model:

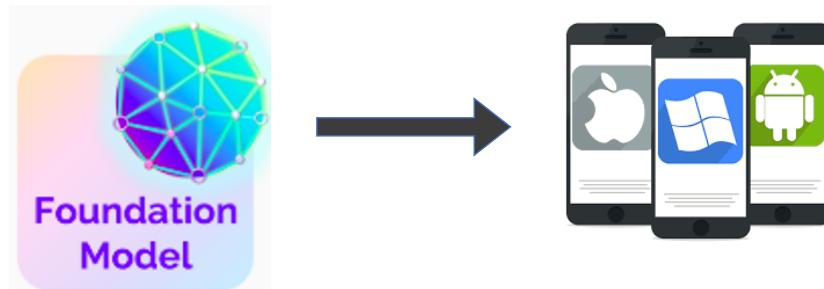
- A routine of compressing and growing network may be beneficial.

# Potential Connection to Foundation model



Training foundation model:

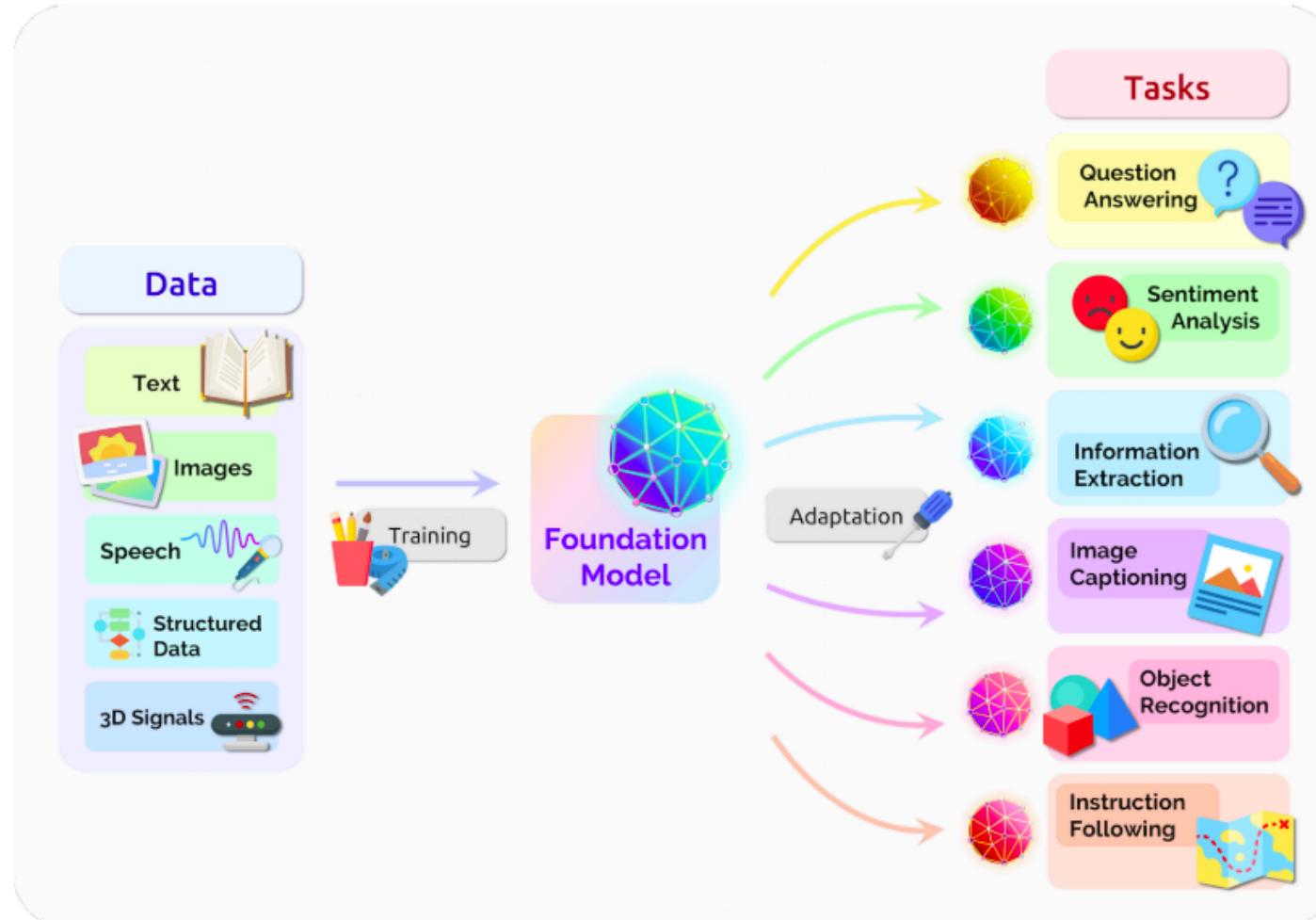
- A routine of compressing and growing network may be beneficial.



Deploying to downstream task:

- Desirable reduced model size for the task of limited resources

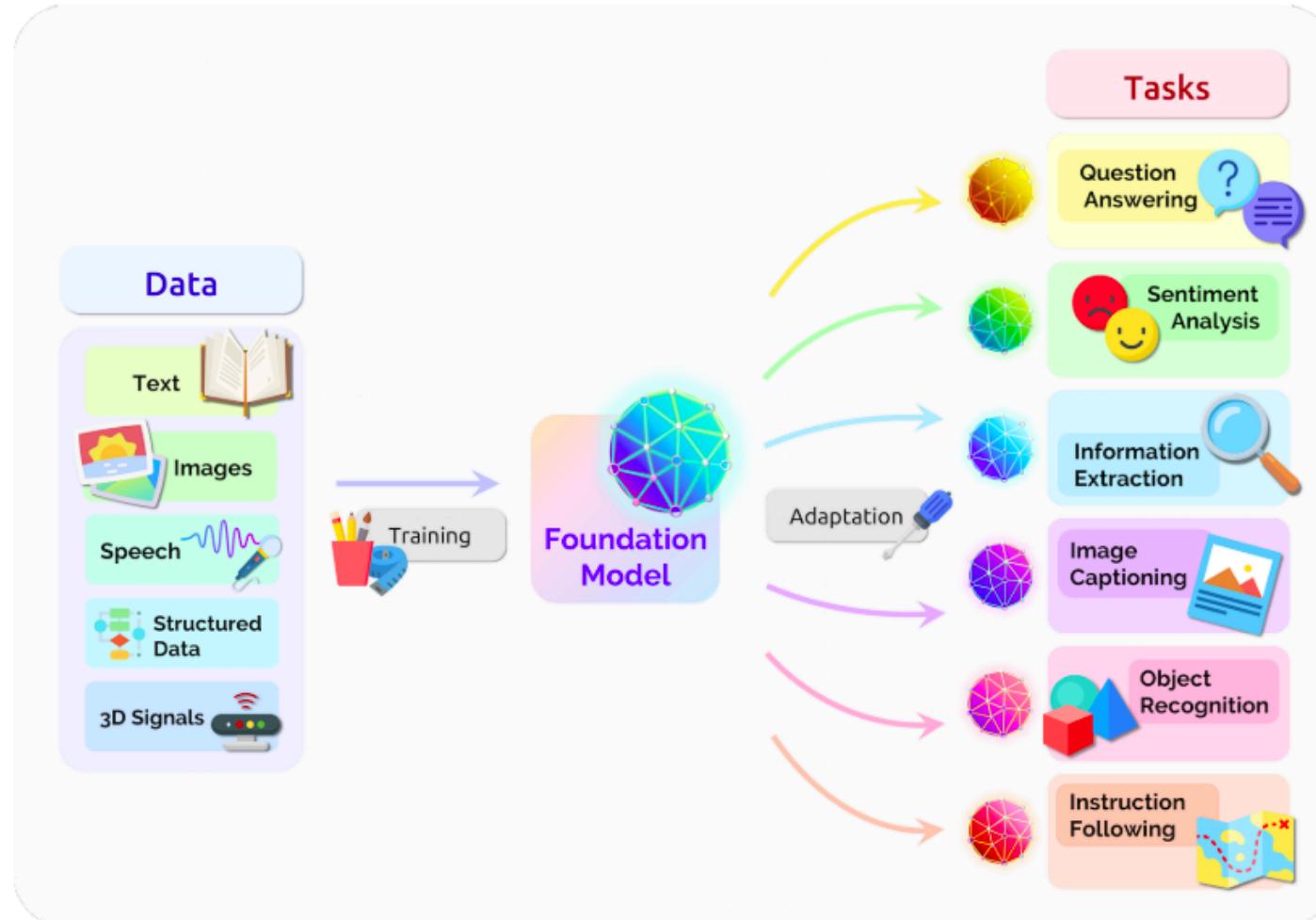
# The Dilemma of Foundation Models



The foundation model is *emergence* and *homogenization*, and should be adapted to different task deployed on various platforms. Figure modified from [Bommasani et al 2021].

# The Dilemma of Foundation Models

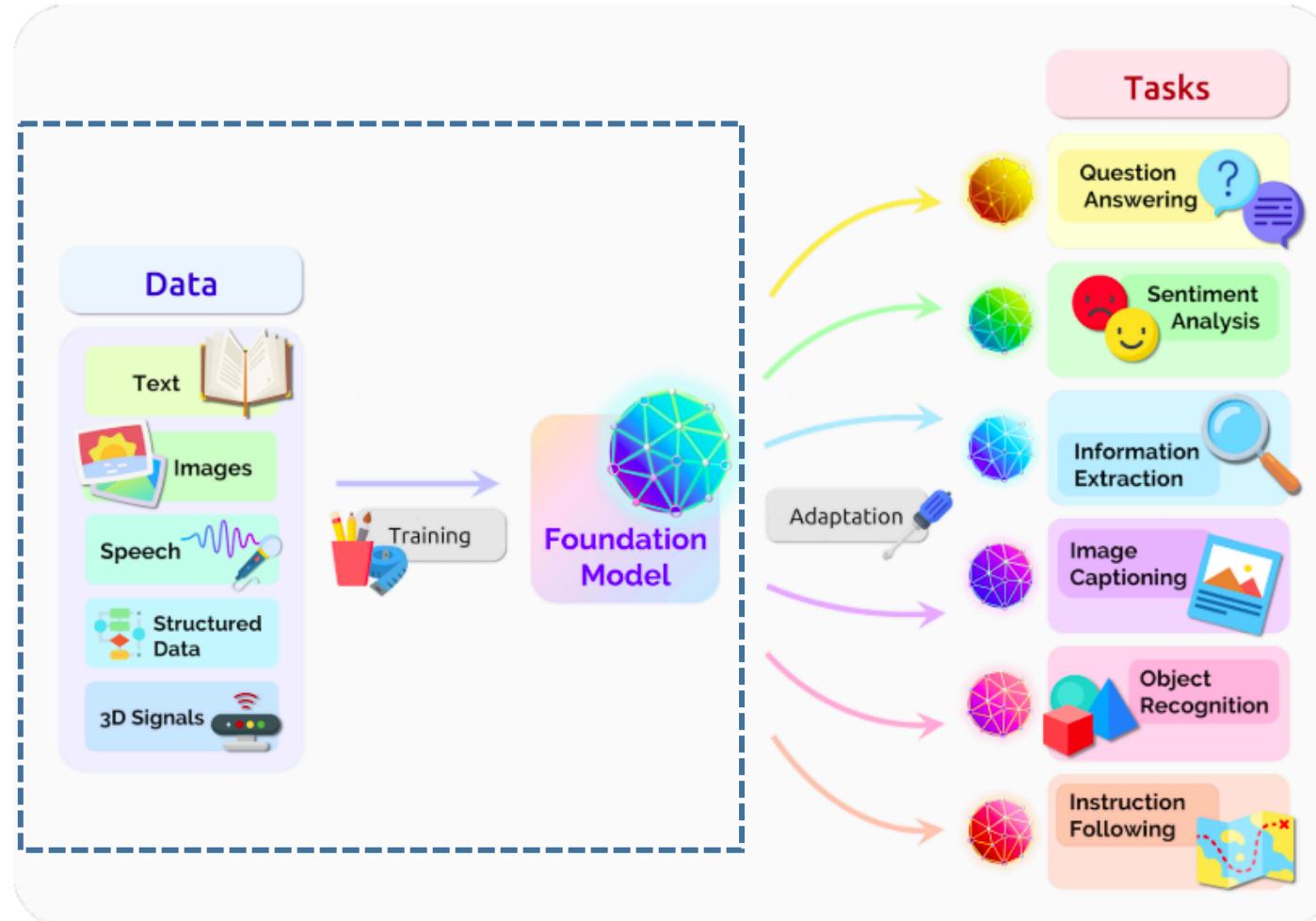
4B



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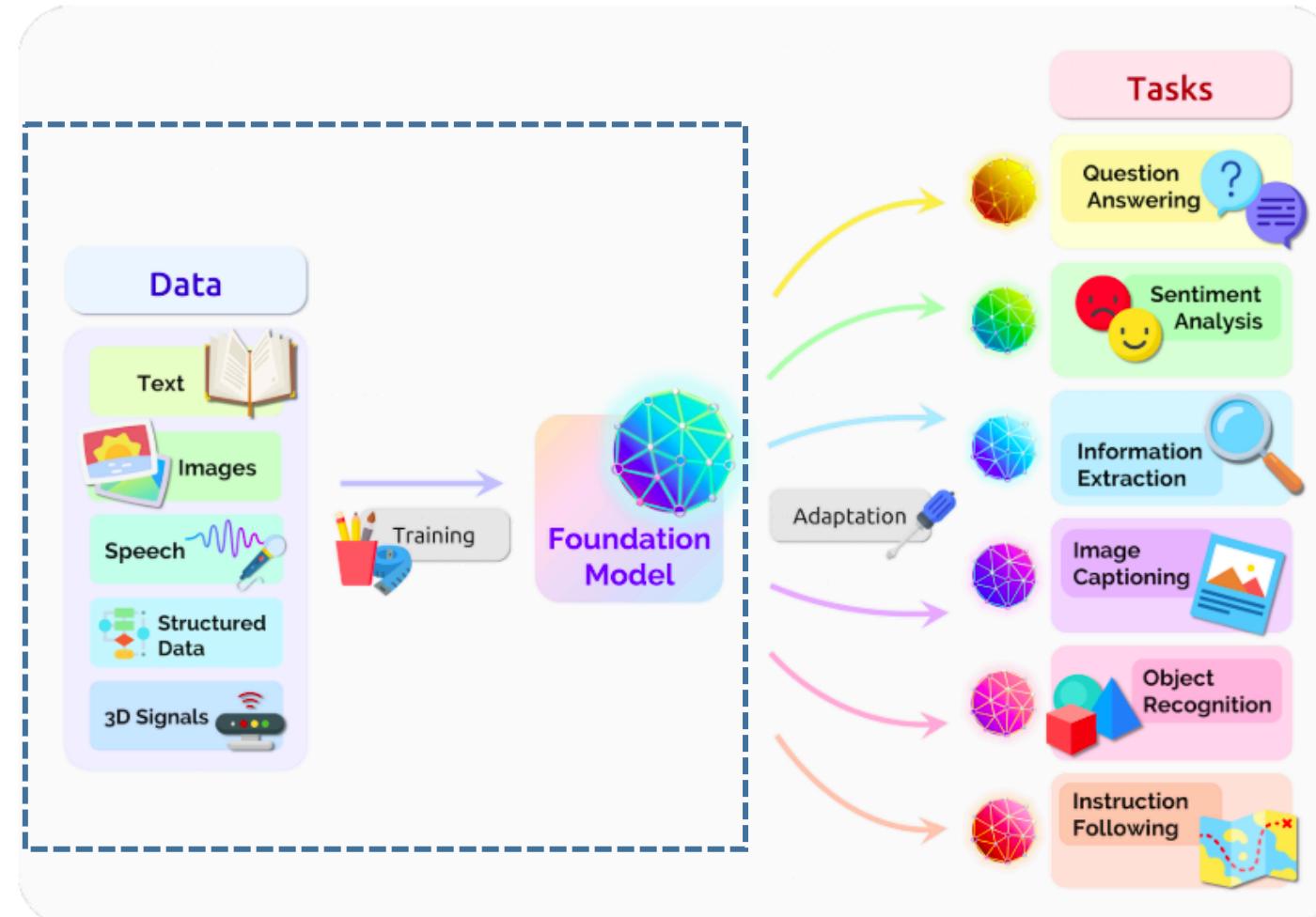
Big Data



Big Machines



Big Model



The foundation model is *emergence* and *homogenization*, and should be adapted to different task deployed on various platforms. Figure modified from [Bommasani et al 2021].

# The Dilemma of Foundation Models

4B

Big Data



Big Machines



Big Model



---

Money Is All You Need

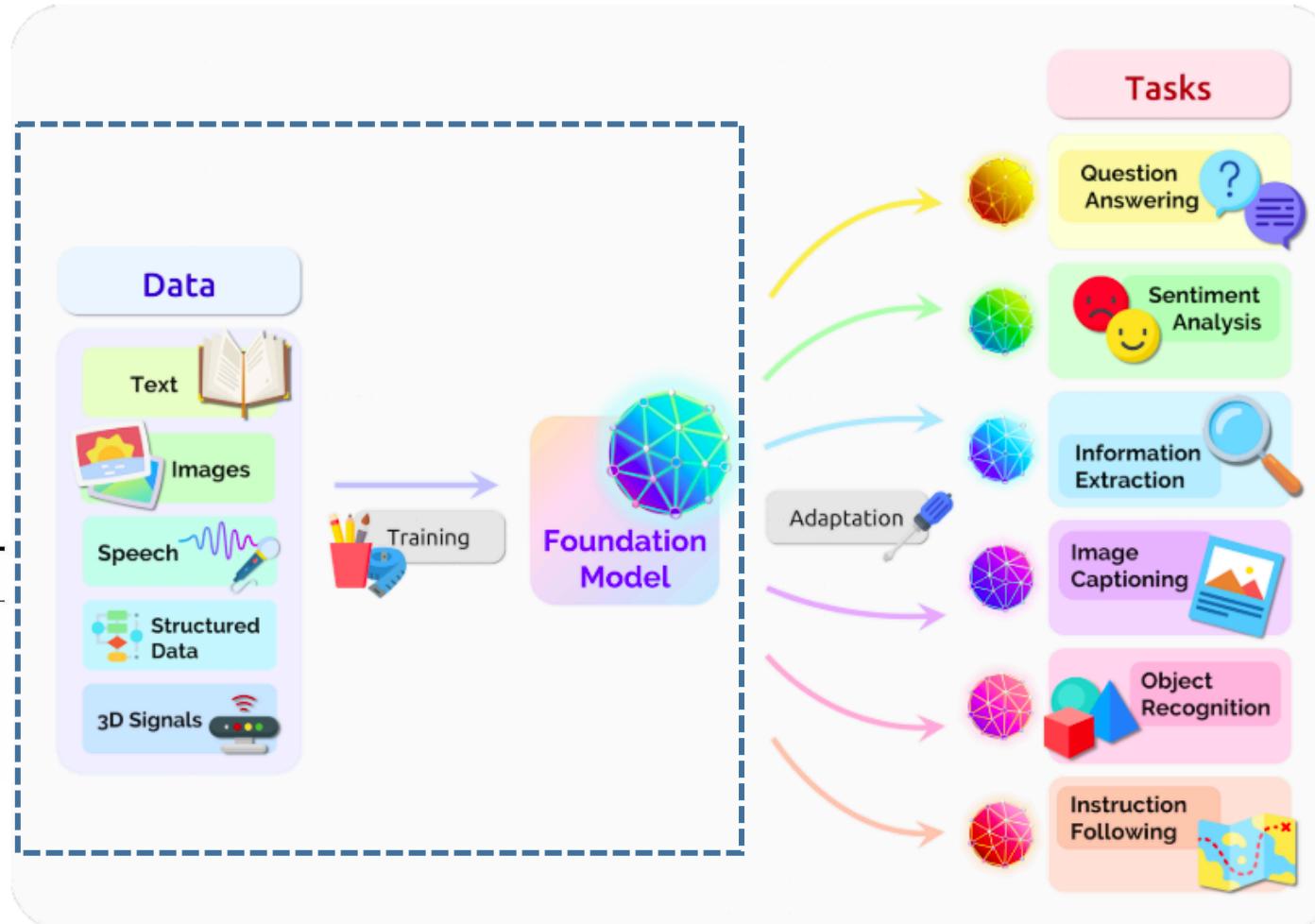
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Nick Debu  
Tokyo Institute of Bamboo Steamer

Abstract

Transformer-based models routinely achieve state-of-the-art results on a number of tasks but training these models can be prohibitively costly, especially on long sequences. We introduce one technique to improve the performance of Transformers. We replace NVIDIA P100s by TPUs, changing its memory from hoge GB to pivo GB. The resulting model performs on par with Transformer-based models while being much more "TSUYO TSUYO".

Big Money



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# The Dilemma of Foundation Models

4B

**Big Data**



**Big Machines**



**Big Model**



---

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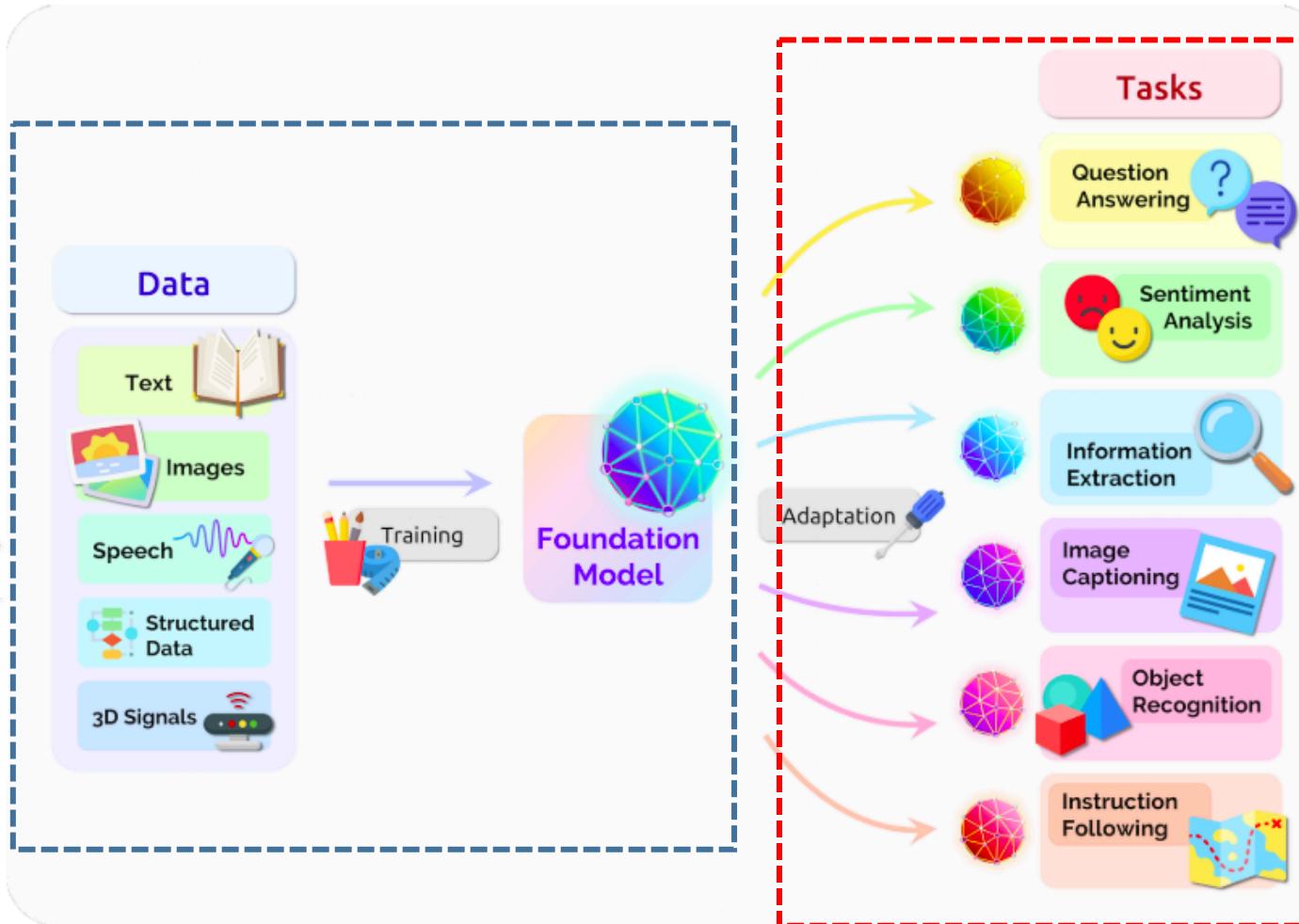
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4B

Big Data



Big Machines



Big Model



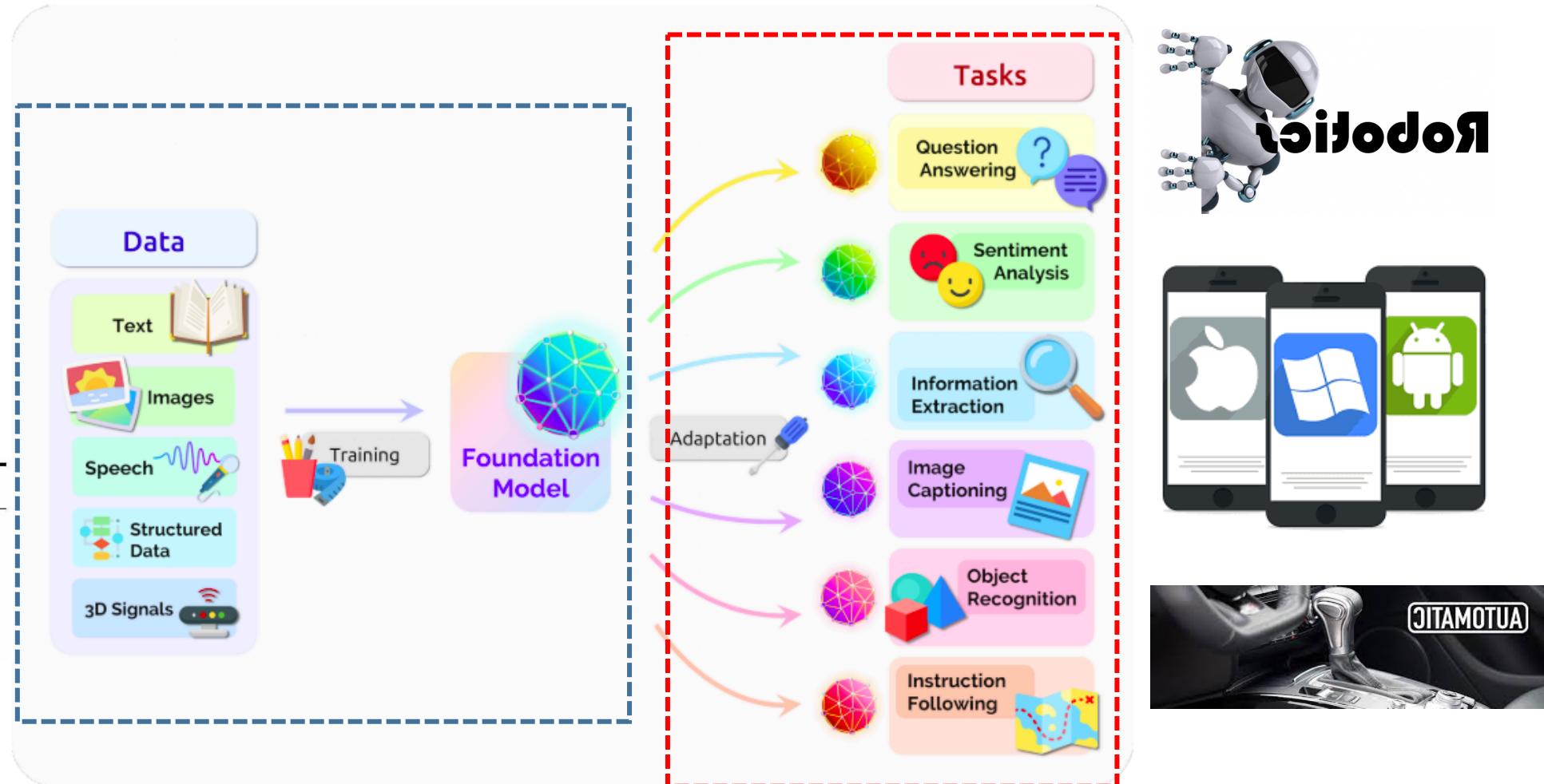
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# Foundation Models in Computer Vision

- ❖ **CLIP**: Learning Transferable Visual Models From Natural Language Supervision, arXiv Feb. 24, ICML2021, OpenAI
  - *Code/model:* <https://github.com/openai/CLIP>
- ❖ **ALIGN** : Scaling Up Visual and Vision-Language Representation Learning With Noisy Text Supervision, ICML2021, Google Research
  - *Code/model:* N/A
- ❖ **ALBEF** : Align before Fuse: Vision and Language Representation Learning with Momentum Distillation, NeurIPS 2021, *Salesforce Research*
  - *Code/model:* <https://github.com/salesforce/ALBEF>,
- ❖ **Florence**: A New Foundation Model for Computer Vision, arXiv, Nov. 22, 2021, Microsoft Cloud and AI, Microsoft Research Redmond
  - *Code/model:* N/A
- ❖ **NUWA**: Visual Synthesis Pre-training for Neural visUal World creAtion, arXiv Nov. 24, 2021, MSRA, Peking Univ.
  - *Code/model:* <https://github.com/microsoft/NUWA>
- ❖ **INTERN**: A New Learning Paradigm Towards General Vision, arXiv Nov. 16, 2021 *Shanghai AI Laboratory, SenseTime, CUKH, SJTU*
  - *Code/model:* N/A
- ❖ **Gopher**: Scaling Language Models: Methods, Analysis & Insights from Training Gopher, arXiv Dec. 8, 2021, *DeepMind*
  - *Code/model:* N/A
- ❖ **FLAVA**: A Foundational Language And Vision Alignment Model, CVPR 2022, FAIR
  - *Code/model:* <https://flava-model.github.io>
- ❖ **OPT**: Open Pre-trained Transformer Language Models. Meta AI 2022
  - *Code/model:* <https://github.com/facebookresearch/metaseq/tree/main/projects/OPT>



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  - *Code/model:* <https://github.com/facebookresearch/metaseq/tree/main/projects/OPT>

It urges us to study *learning sparsity in deep foundation models*



# Learning Sparsity in Data/Labels

# Few-shot Learning by Unlabeled Data

Wang et al. Instance Credibility Inference for Few-Shot Learning. CVPR 2020

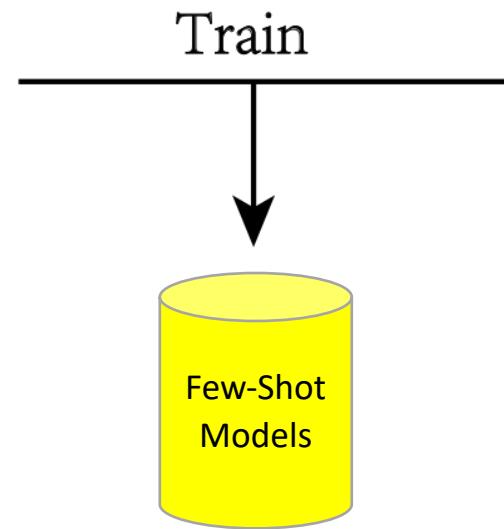
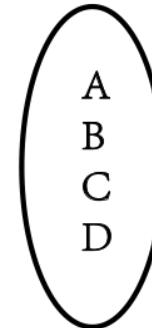
Wang et al. How to Trust Unlabeled Data? Instance Credibility Inference for Few-Shot Learning. IEEE TPAMI 2021

# Few-shot Learning by Unlabeled Data

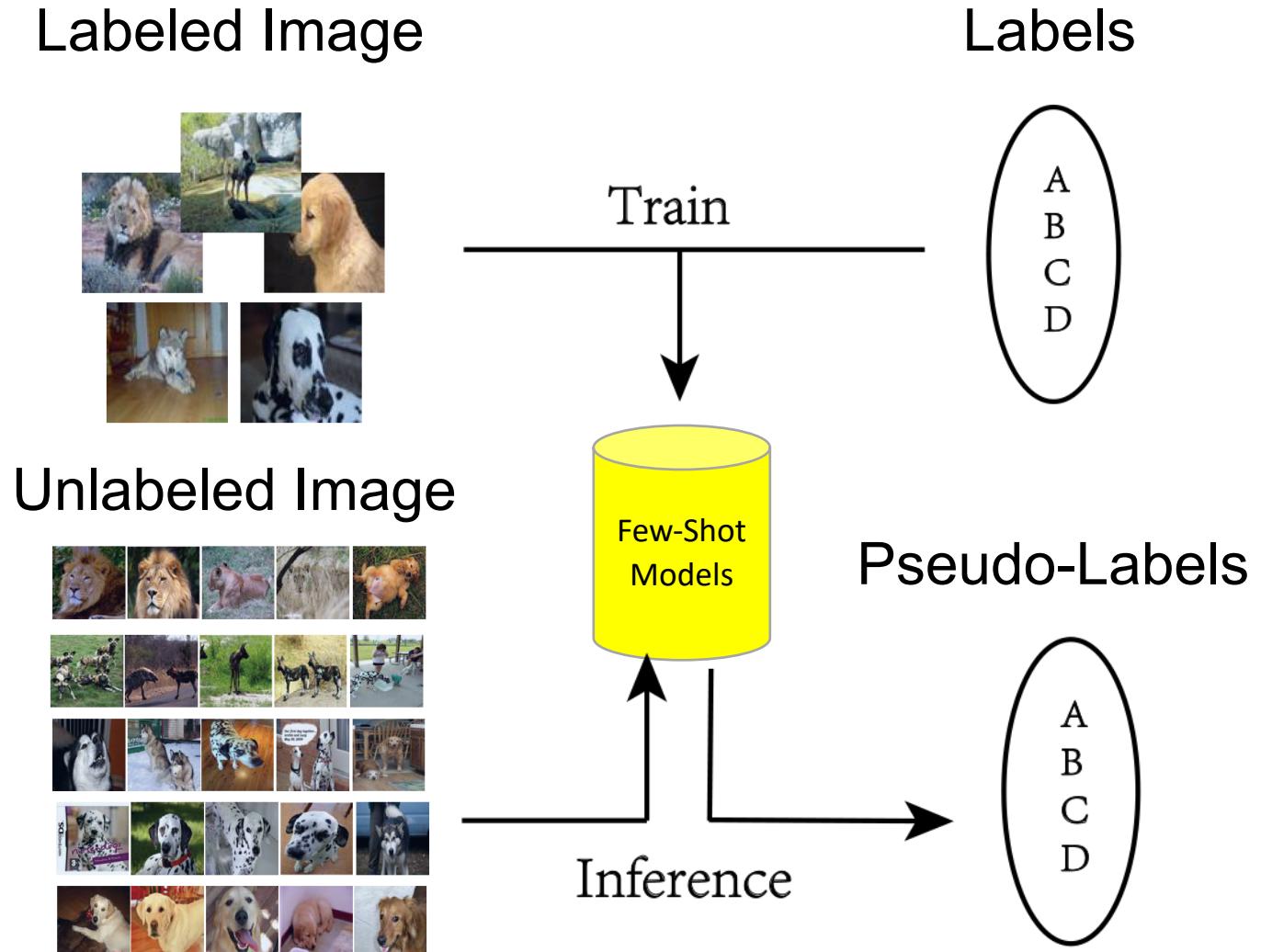
Labeled Image



Labels



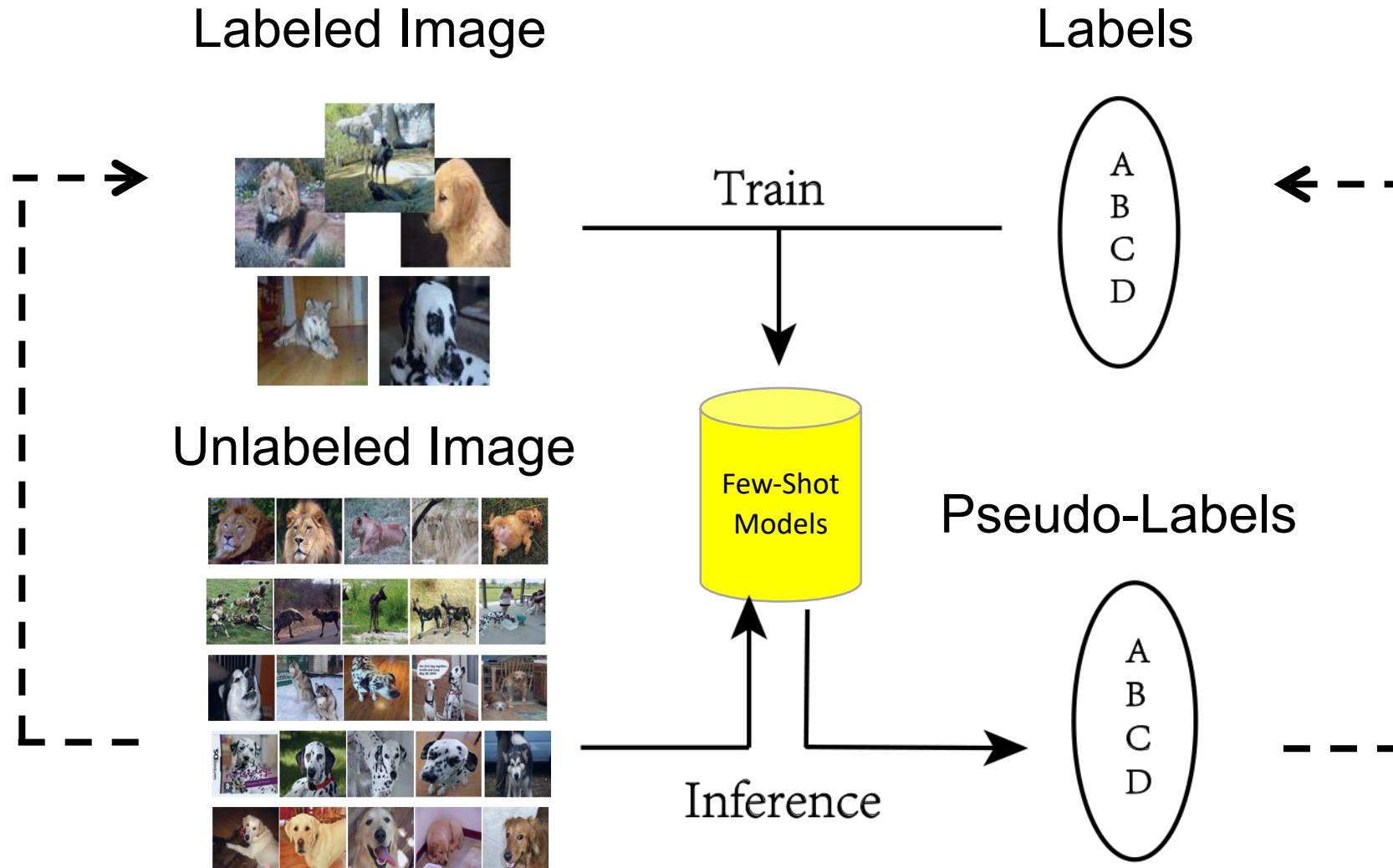
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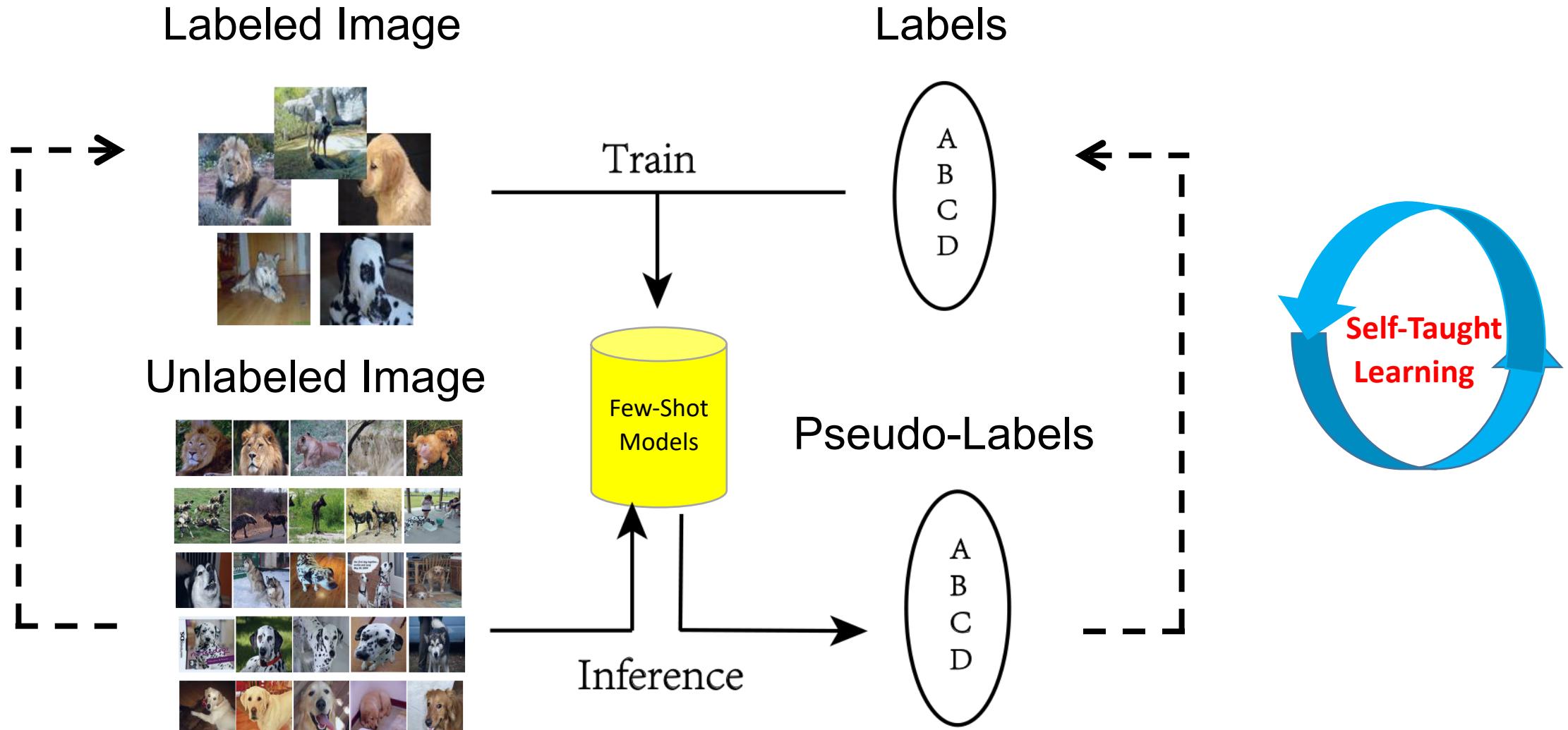
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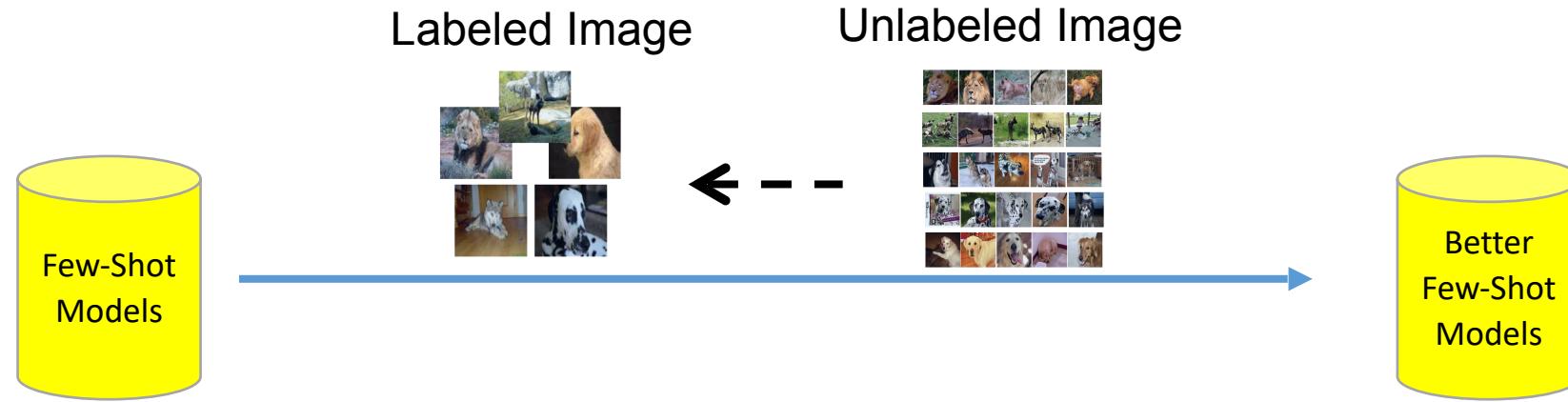
# Sparse Labels in Semi-supervised Few-Shot Learning



We will introduce the details in the next talk.

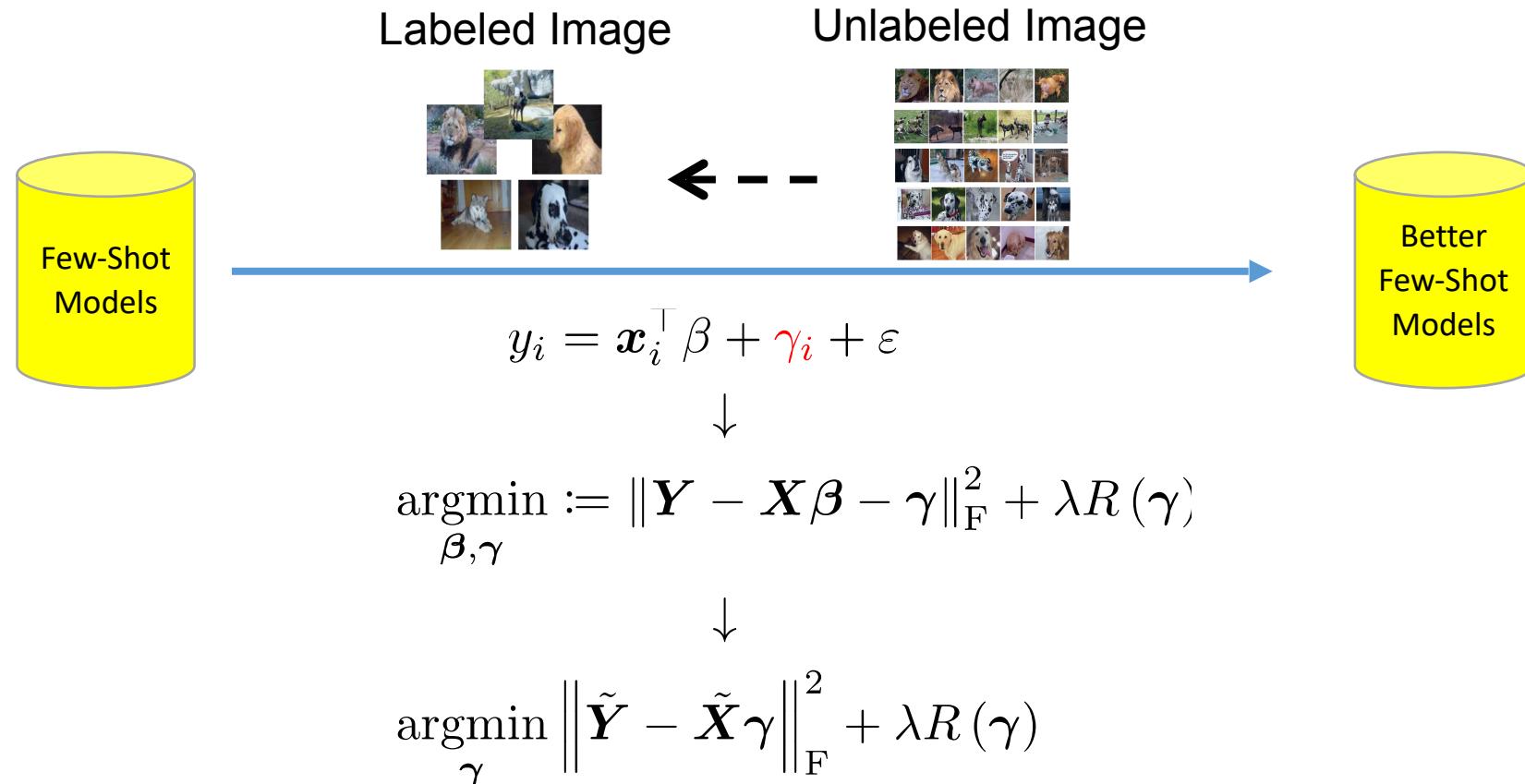
[Wang et al. CVPR2020/TPAMI2021]

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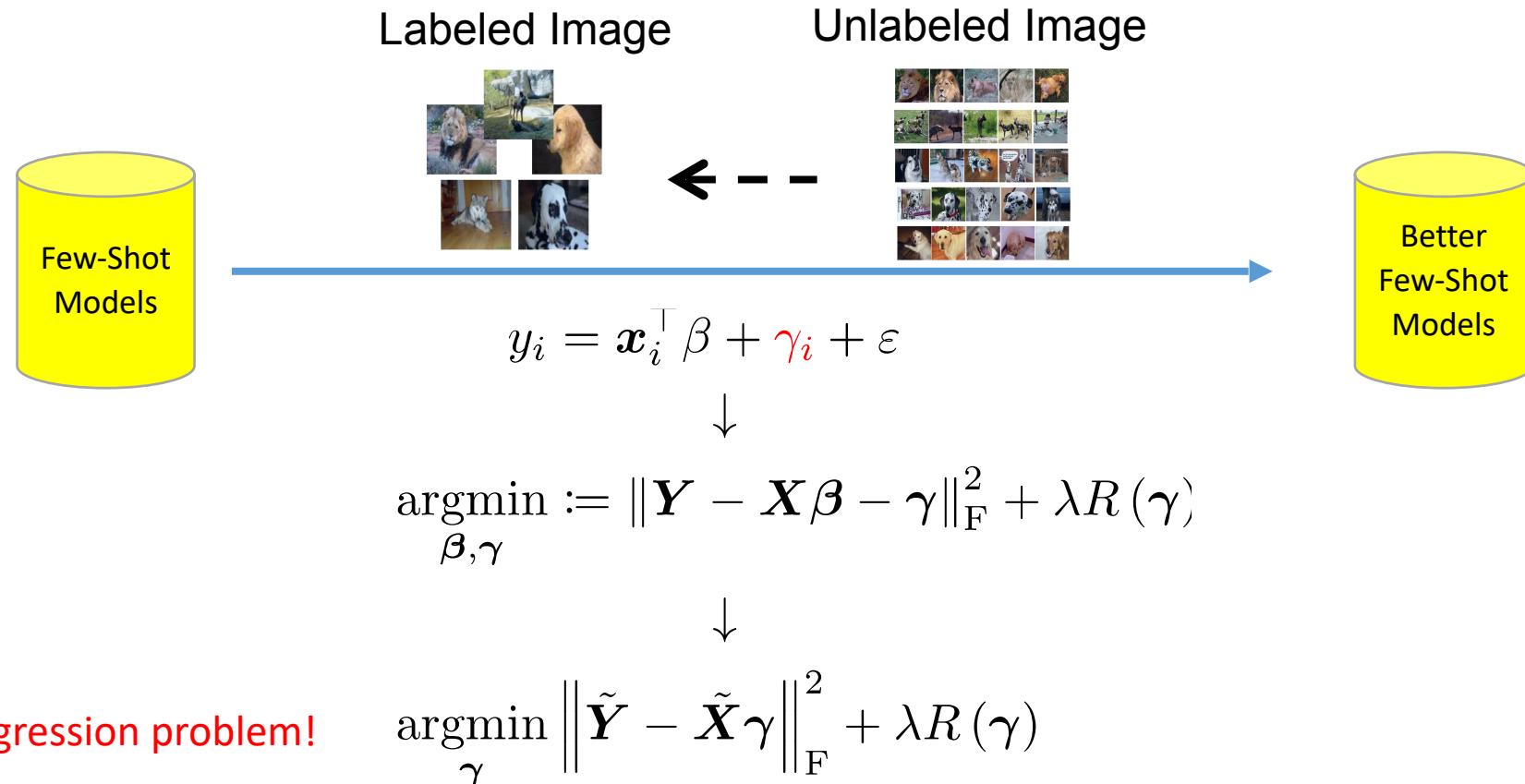
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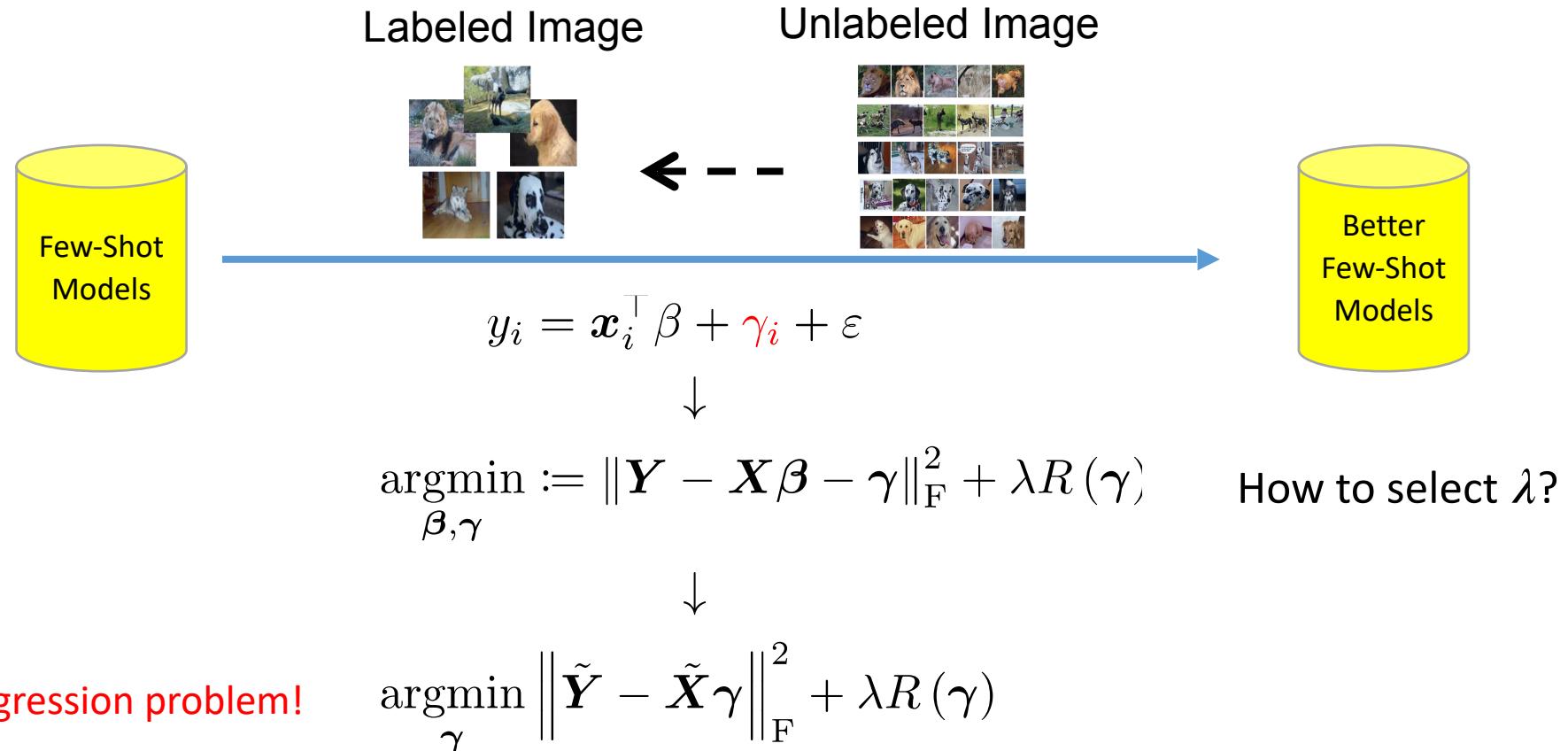
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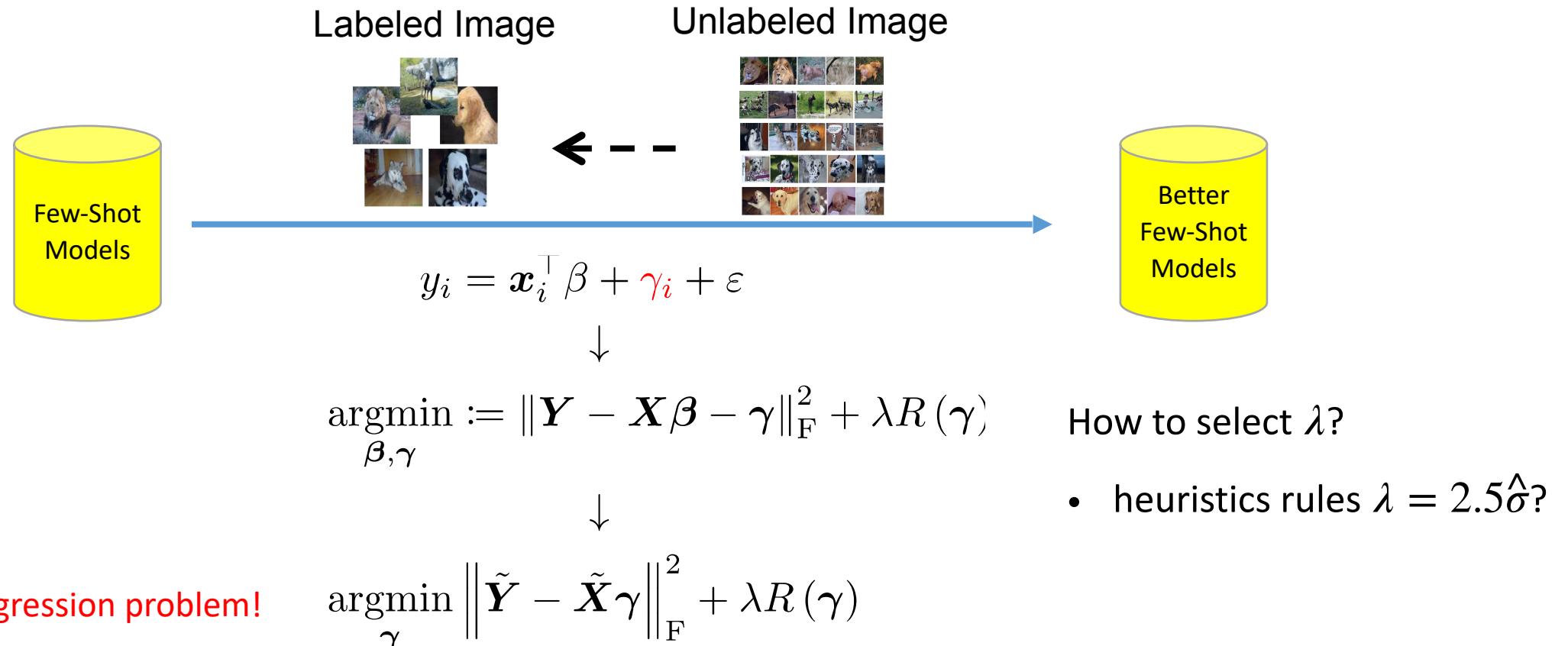
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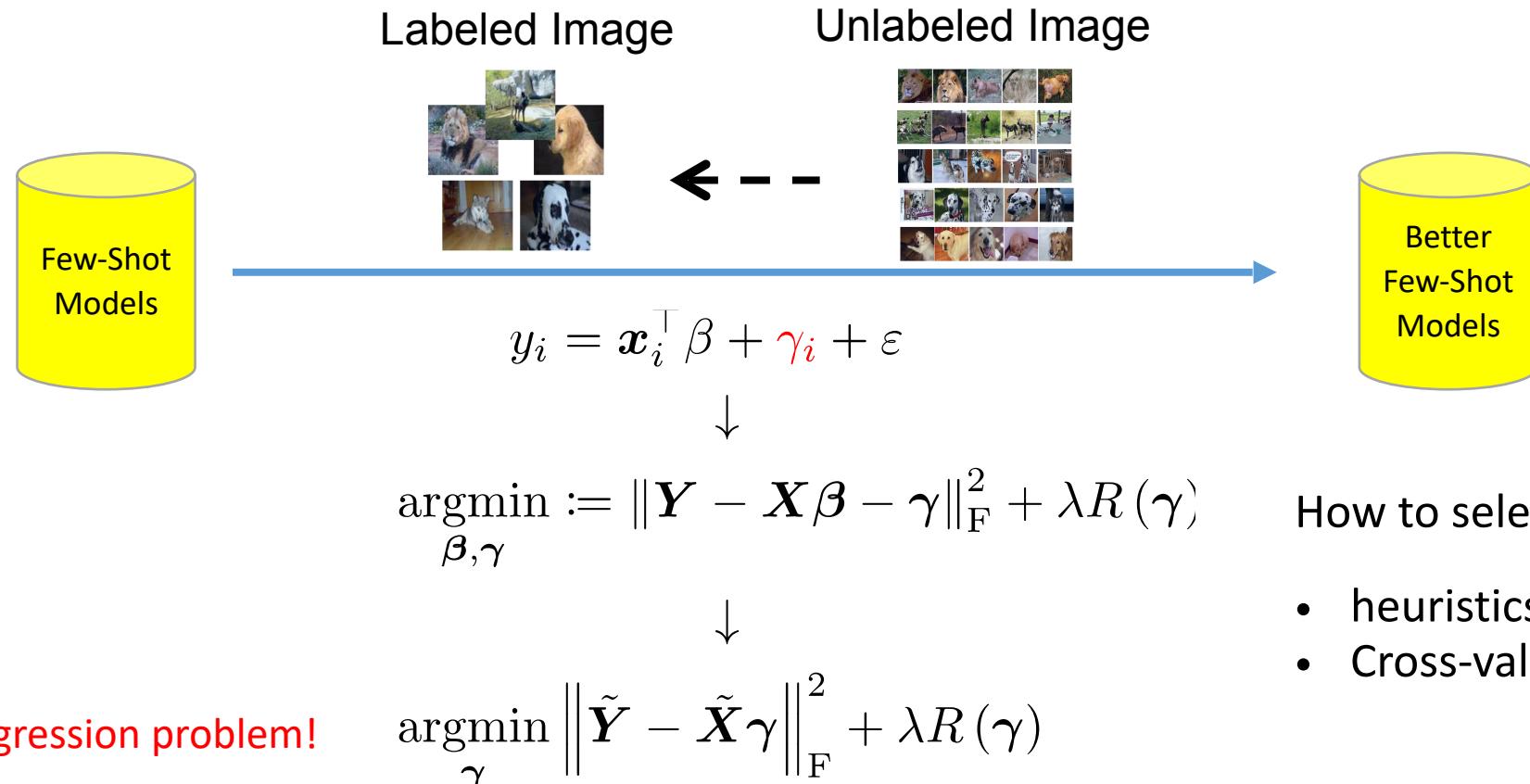
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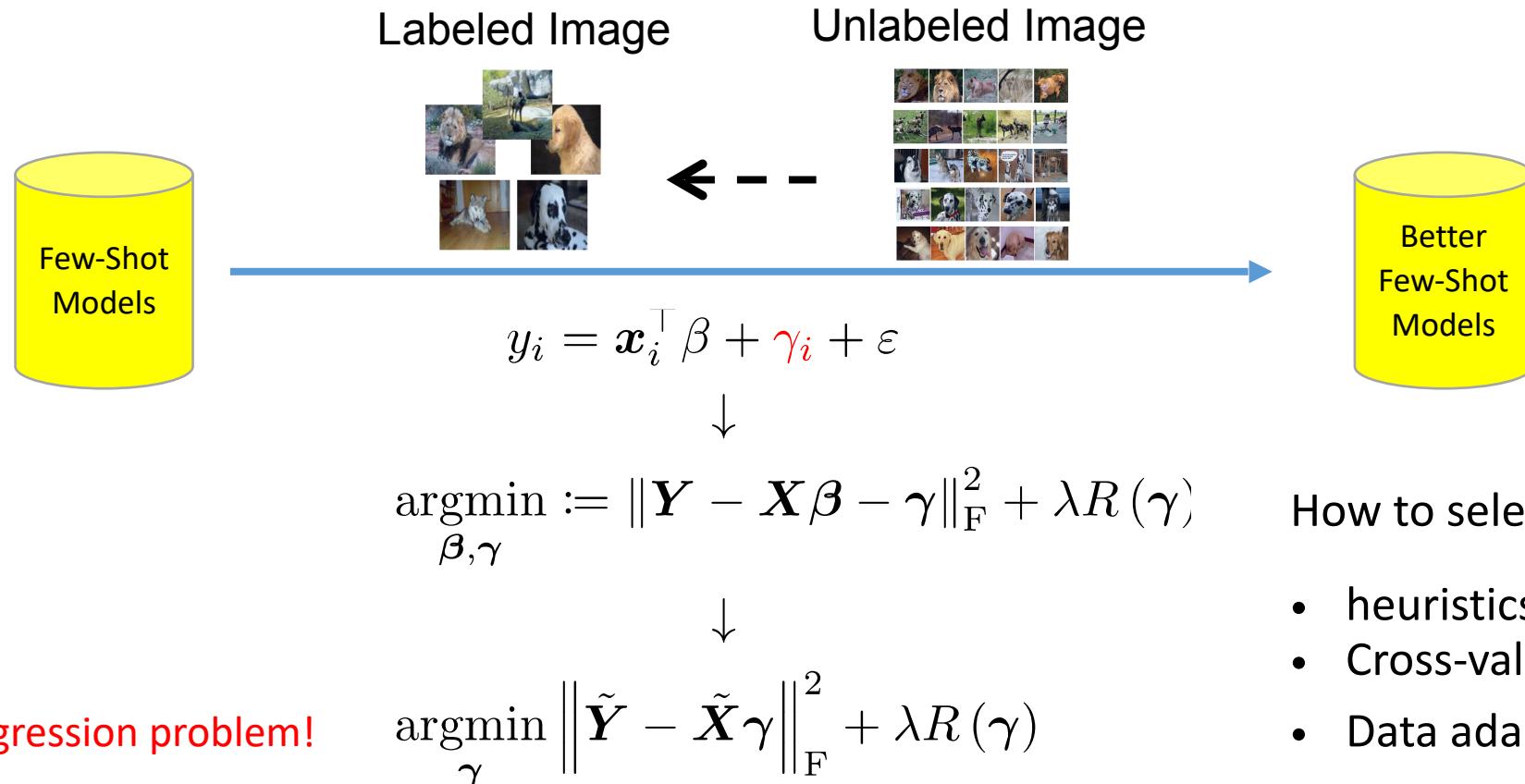


How to select  $\lambda$ ?

- heuristics rules  $\lambda = 2.5\hat{\sigma}$ ?
- Cross-validation?

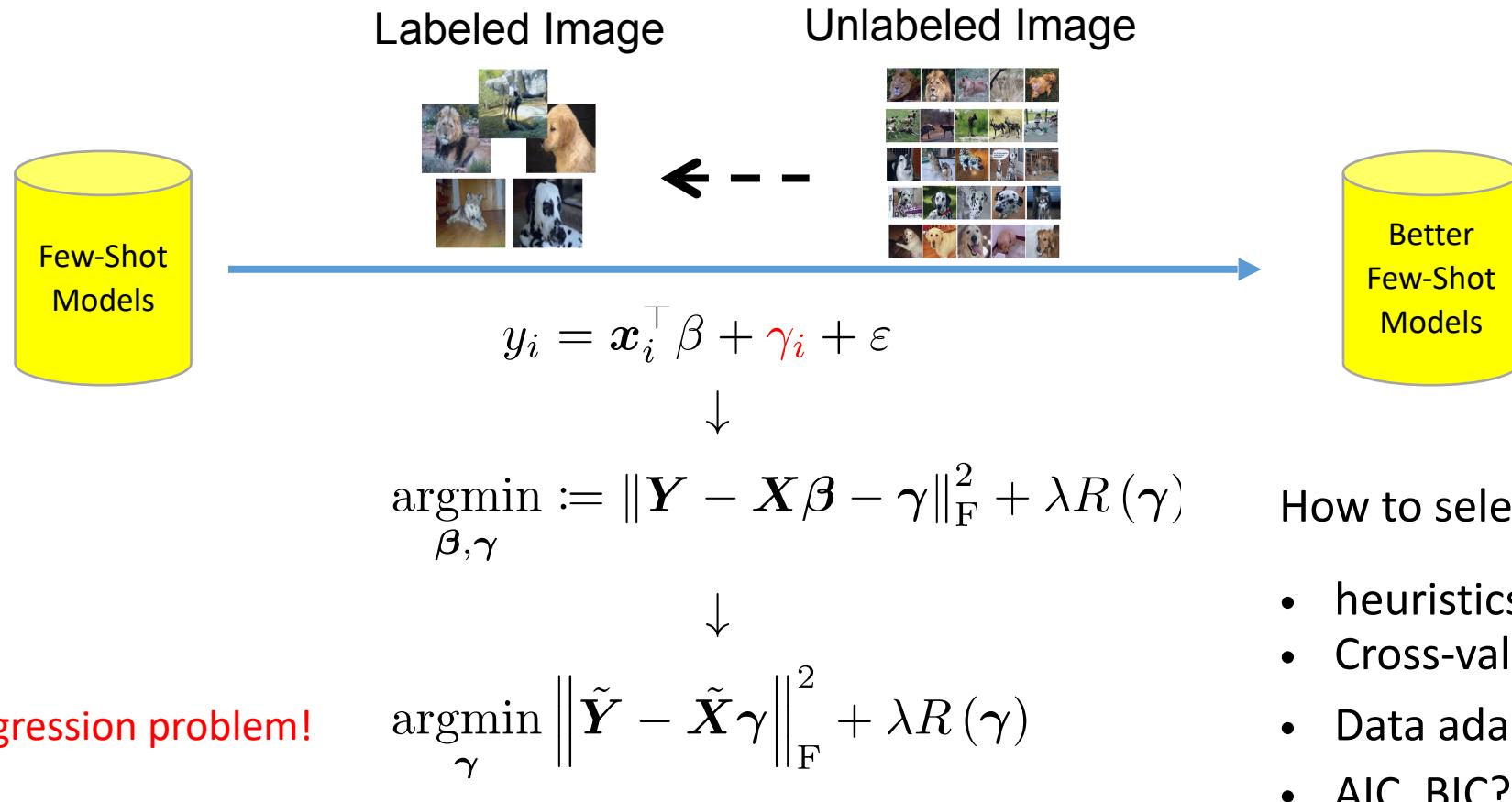
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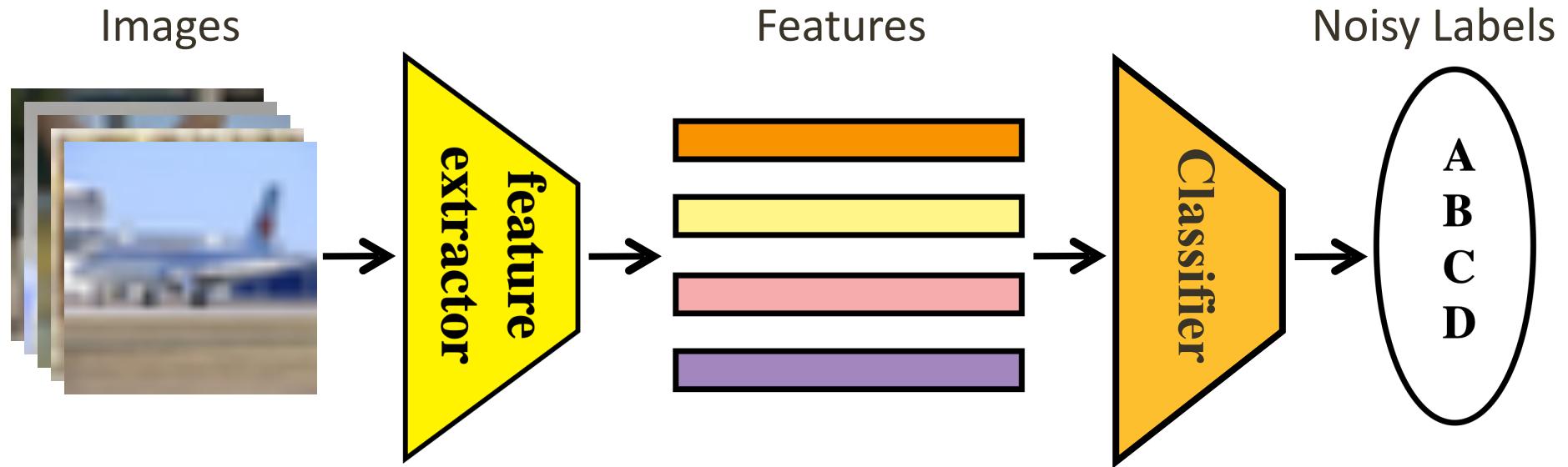
How to select  $\lambda$ ?

- heuristics rules  $\lambda = 2.5\hat{\sigma}$ ?
- Cross-validation?
- Data adaptive techniques?
- AIC, BIC?

We will introduce the details in the next talk.

# Learning Sparsity in Learning with Noisy Labels

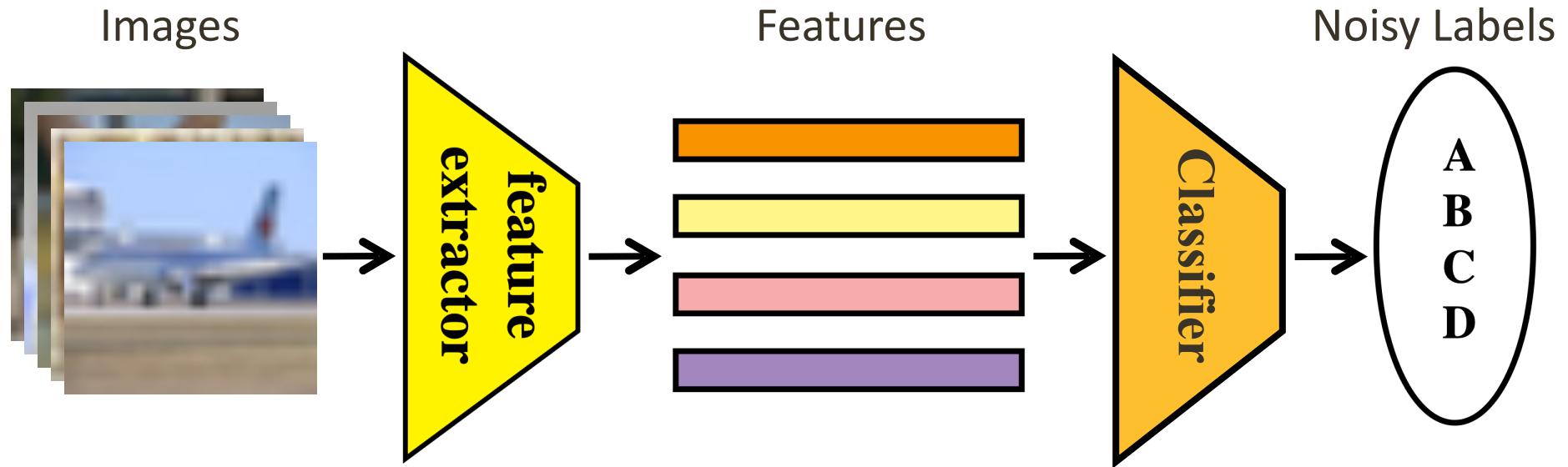
Stage 1:  
Feature Learning



Stage 2:  
Sample Selection

# Learning Sparsity in Learning with Noisy Labels

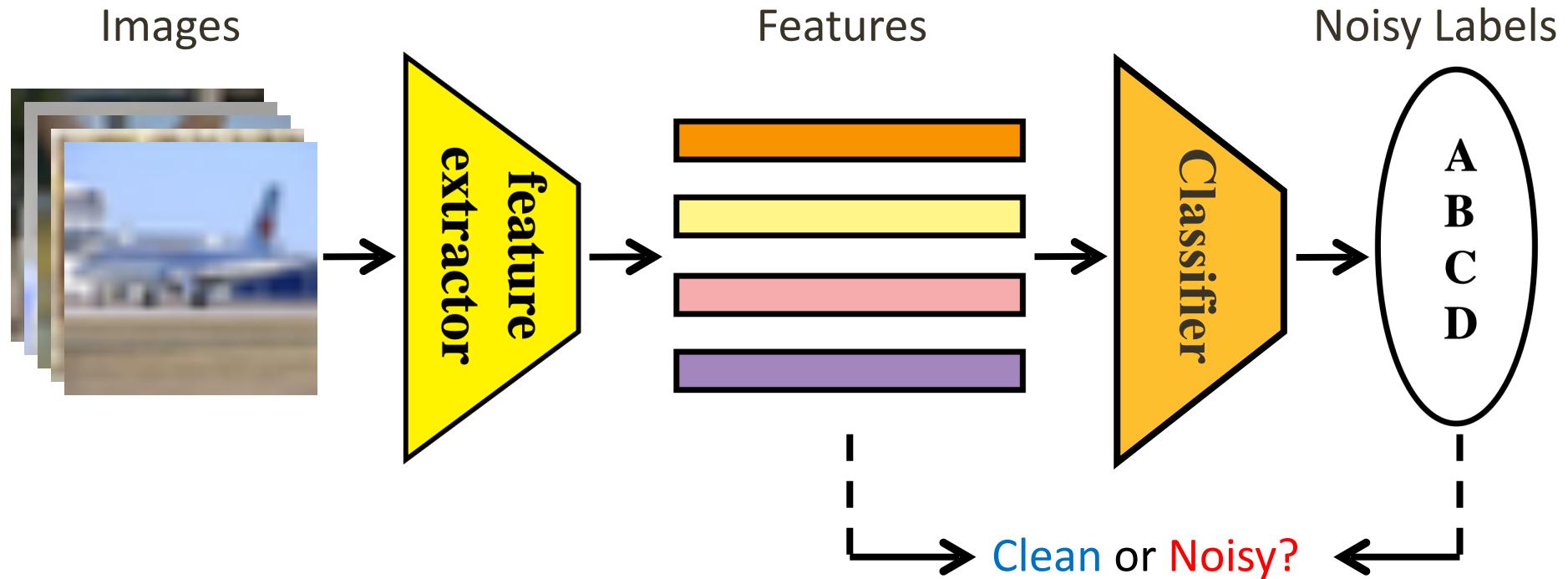
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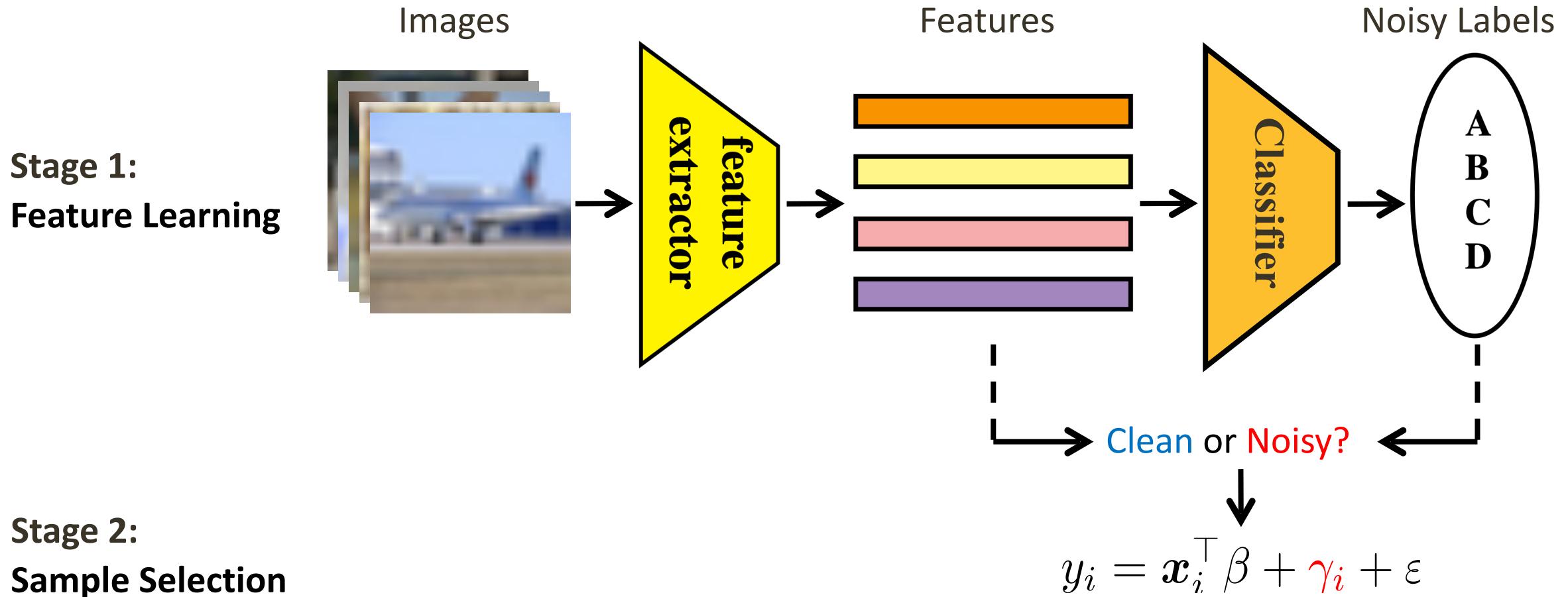
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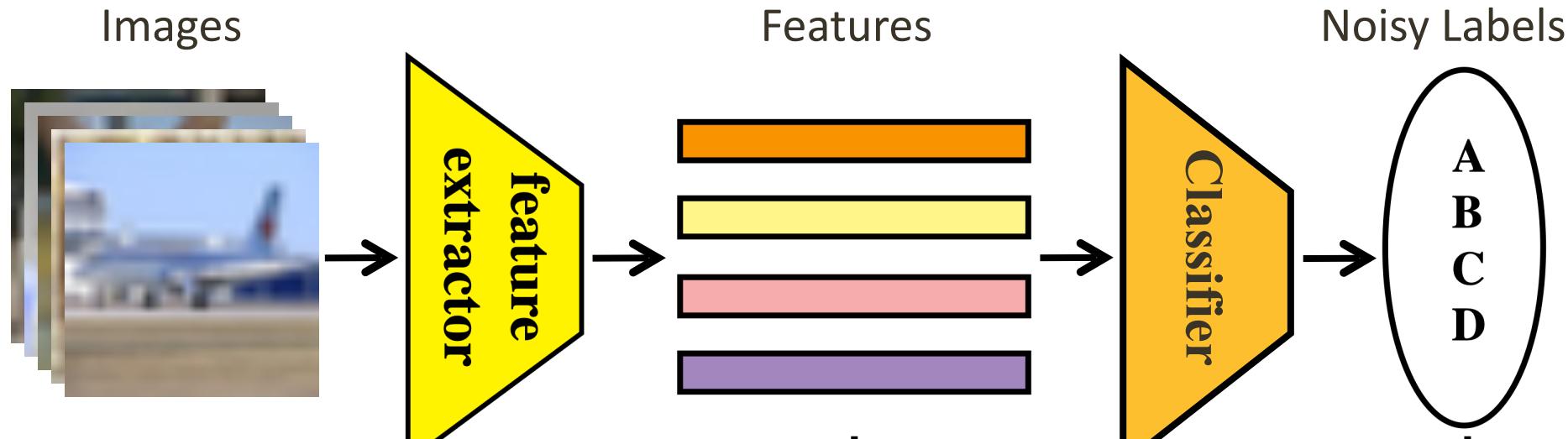
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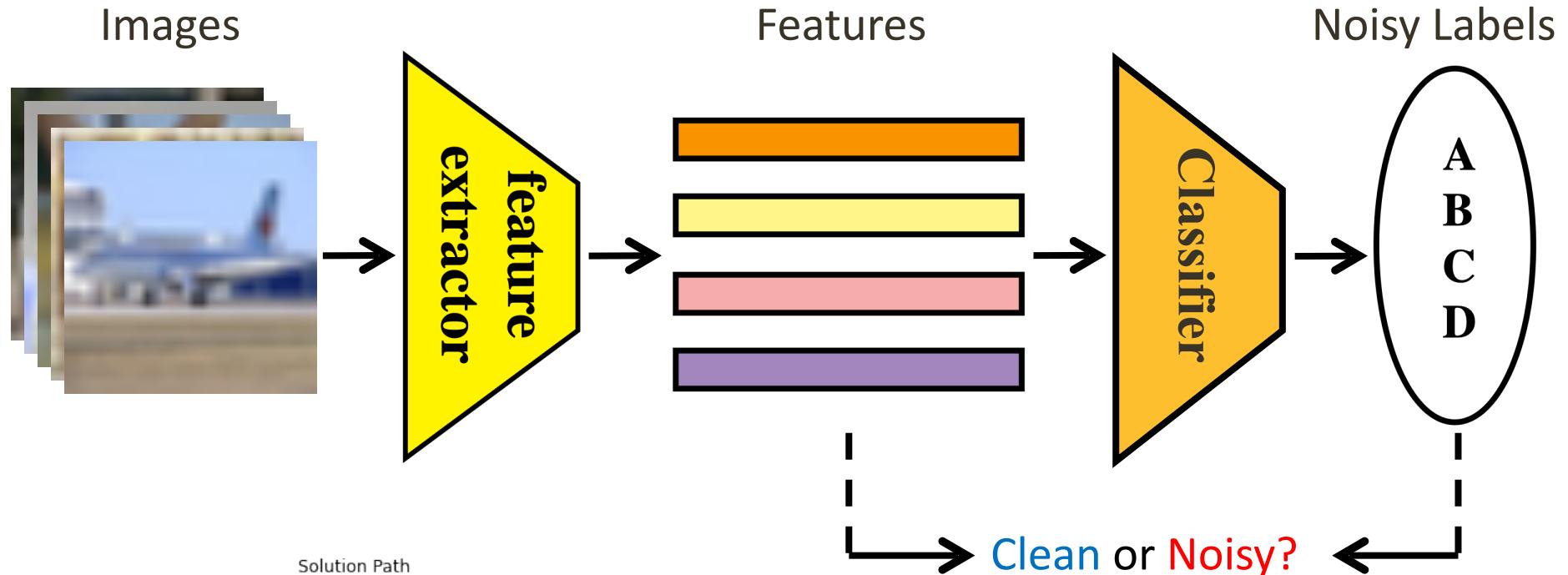
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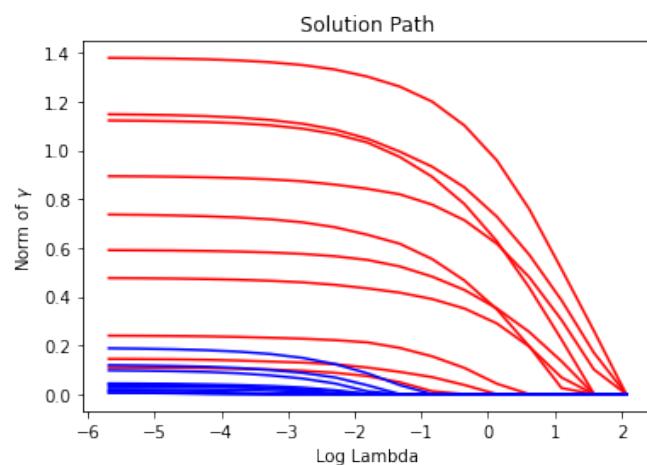
$$\begin{aligned}y_i &= \mathbf{x}_i^\top \boldsymbol{\beta} + \gamma_i + \varepsilon \\&\underset{\gamma}{\operatorname{argmin}} \left\| \tilde{\mathbf{Y}} - \tilde{\mathbf{X}} \gamma \right\|_{\text{F}}^2 + \sum_{i=1}^n P(\gamma_i; \lambda_i)\end{aligned}$$

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Stage 1:  
Feature Learning



Stage 2:  
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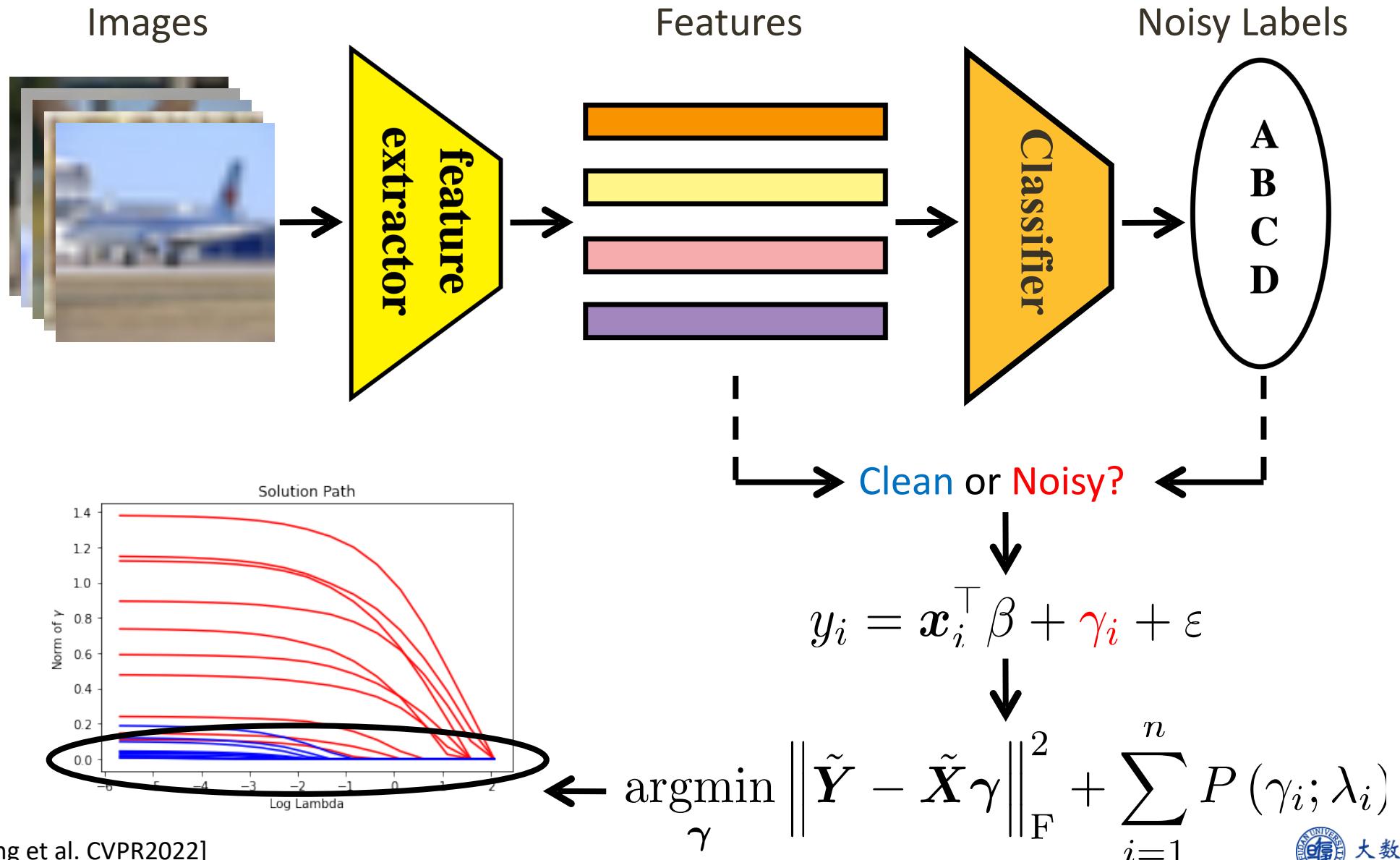


$$\begin{aligned} & y_i = \mathbf{x}_i^\top \boldsymbol{\beta} + \gamma_i + \varepsilon \\ & \underset{\gamma}{\operatorname{argmin}} \left\| \tilde{\mathbf{Y}} - \tilde{\mathbf{X}} \gamma \right\|_{\text{F}}^2 + \sum_{i=1}^n P(\gamma_i; \lambda_i) \end{aligned}$$

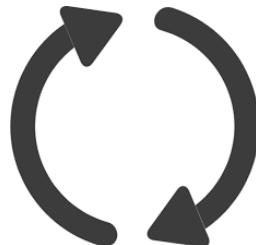
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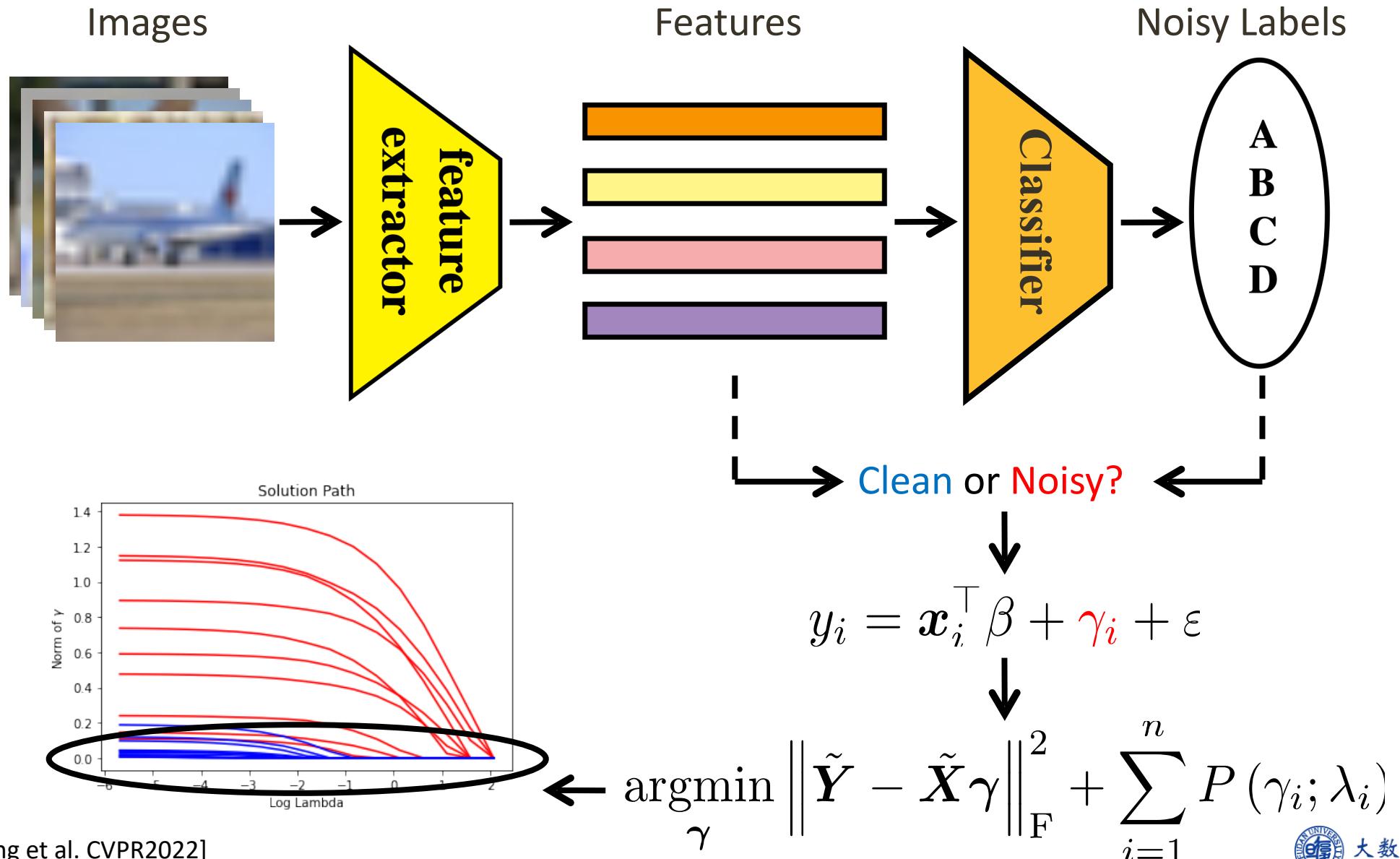
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Stage 1:  
Feature Learning  


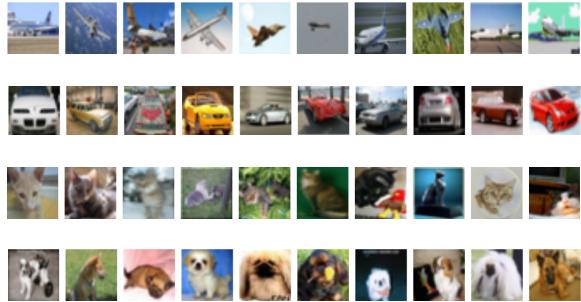
Stage 2:  
Sample Selection



# Make it scalable to large datasets

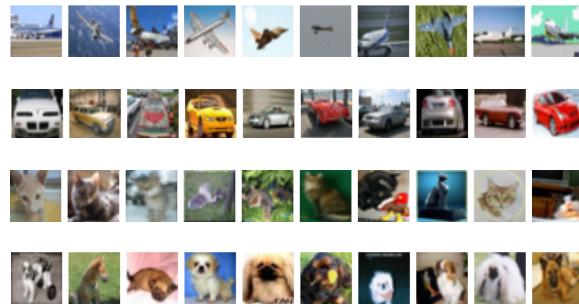
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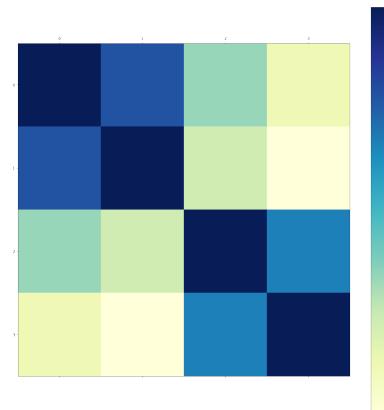


Compute  
Class Similarity →

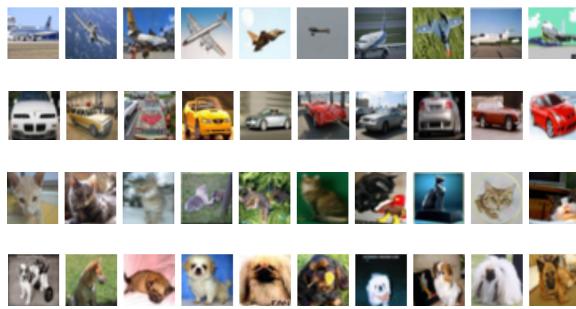
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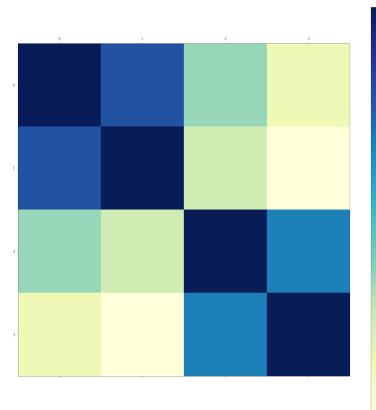
Compute  
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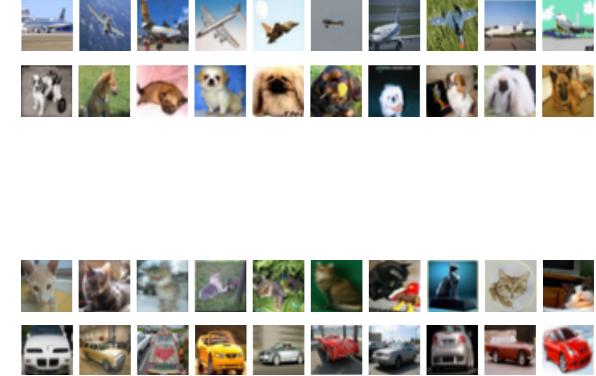
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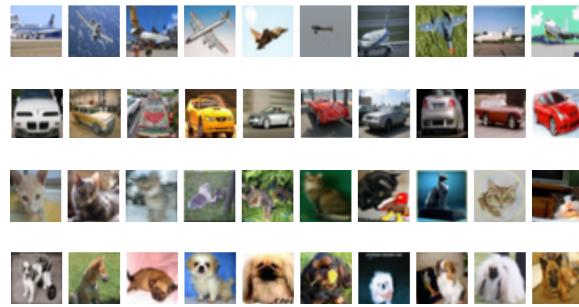
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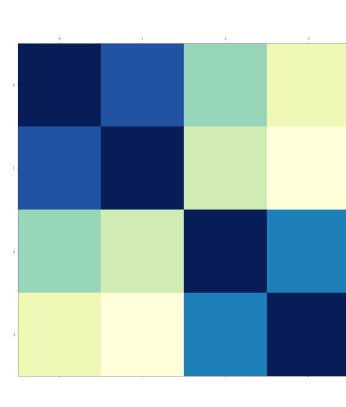
Group  
Dissimilar classes →



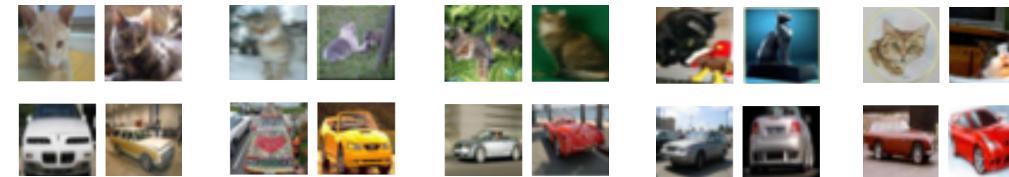
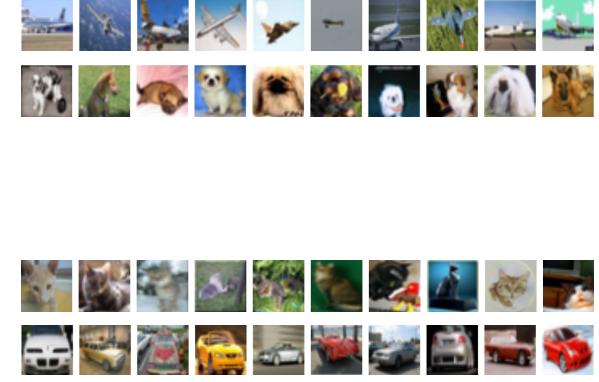
# Make it scalable to large datasets



Compute  
Class Similarity →

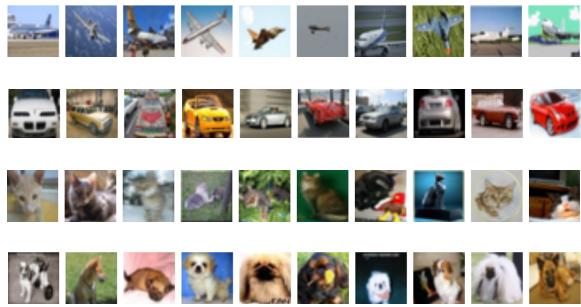


Group  
Dissimilar classes →

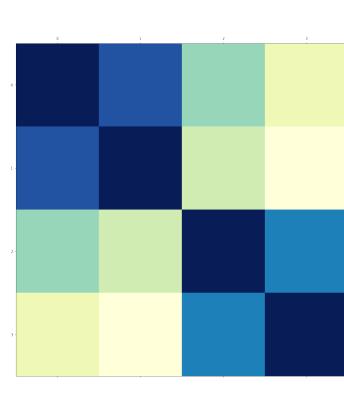


Split into  
pieces ↙

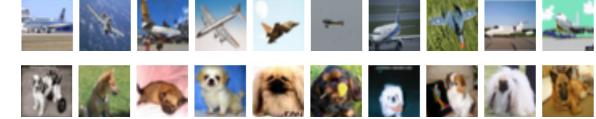
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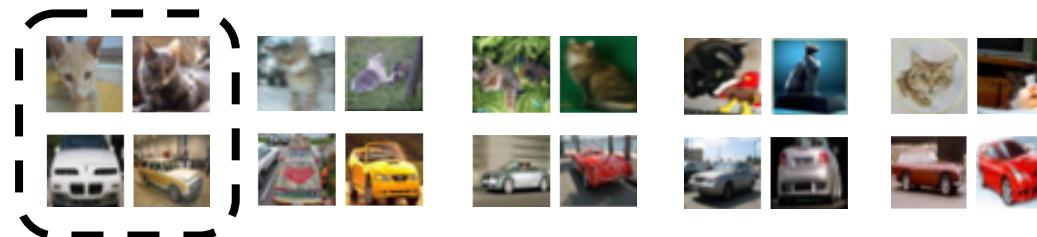
Compute  
Class Similarity →



→ Group  
Dissimilar classes



$$y_i = \mathbf{x}_i^\top \beta + \gamma_i + \varepsilon$$



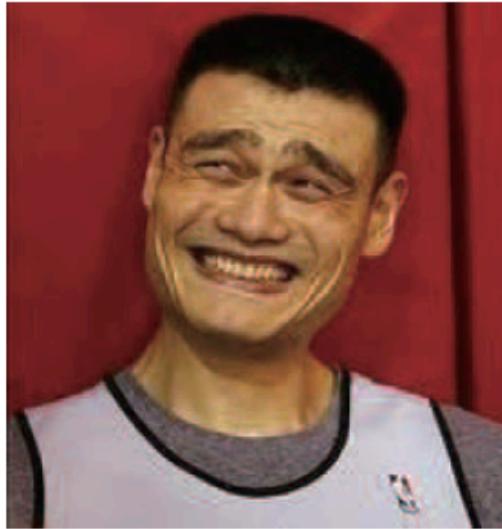
← Split into  
pieces

# Sparse Learning in Robust Ranking

Fu et al. Interestingness Prediction by Robust Learning to Rank. ECCV 2014

Fu et al. Robust Subjective Visual Property Prediction from Crowdsourced Pairwise Labels. IEEE TPAMI 2016

# Sparse Learning in Robust Ranking



?



Who is smiling more?

# Sparse Learning in Robust Ranking



?



Who is smiling more?



(a) Smiling



(b) ?



(c) Not smiling



(d) Natural



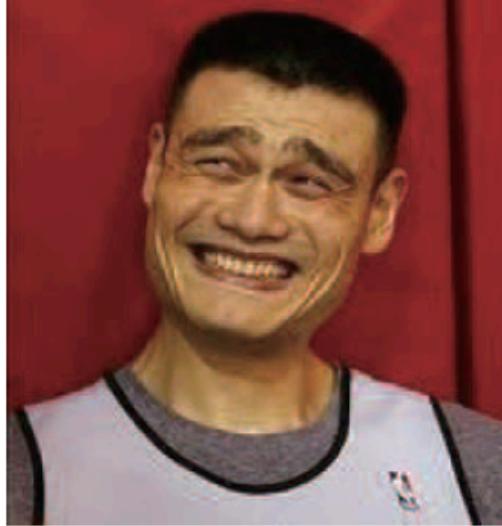
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# Sparse Learning in Robust Ranking



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1. Cultural factors
2. Psychological factors: Halo Effects
3. Ambiguous comparisons
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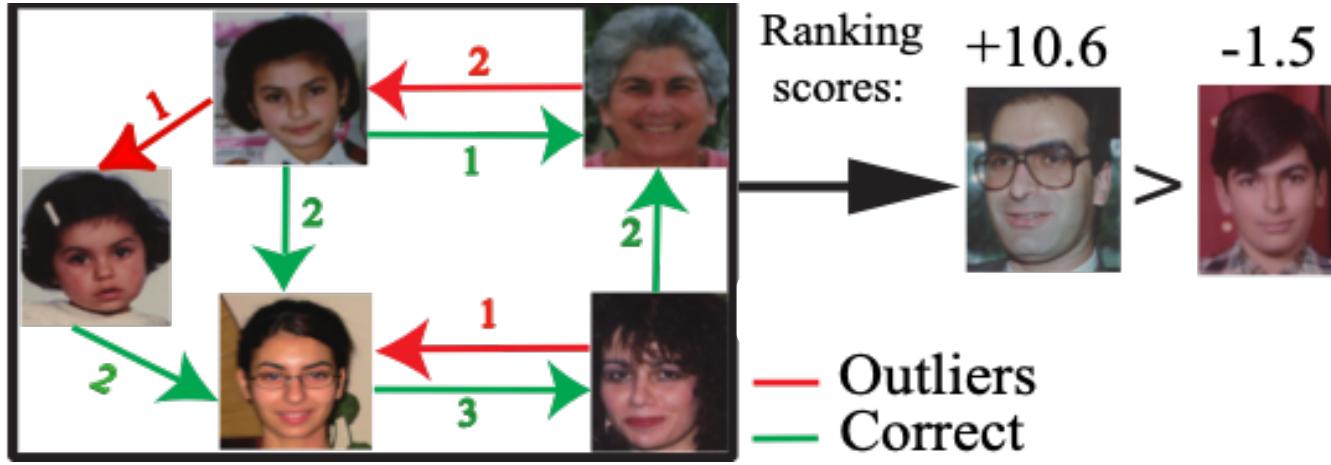


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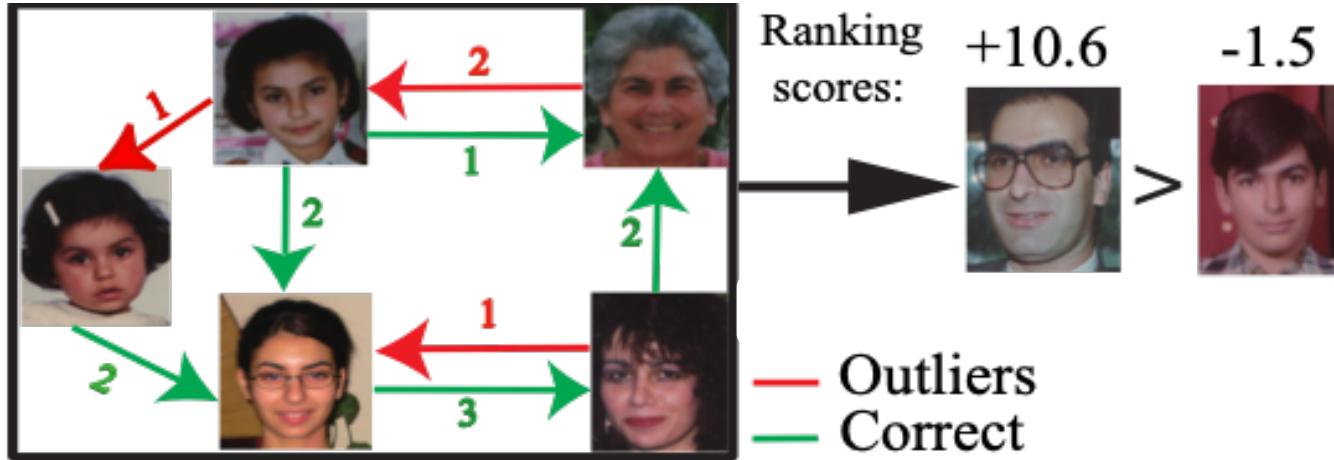
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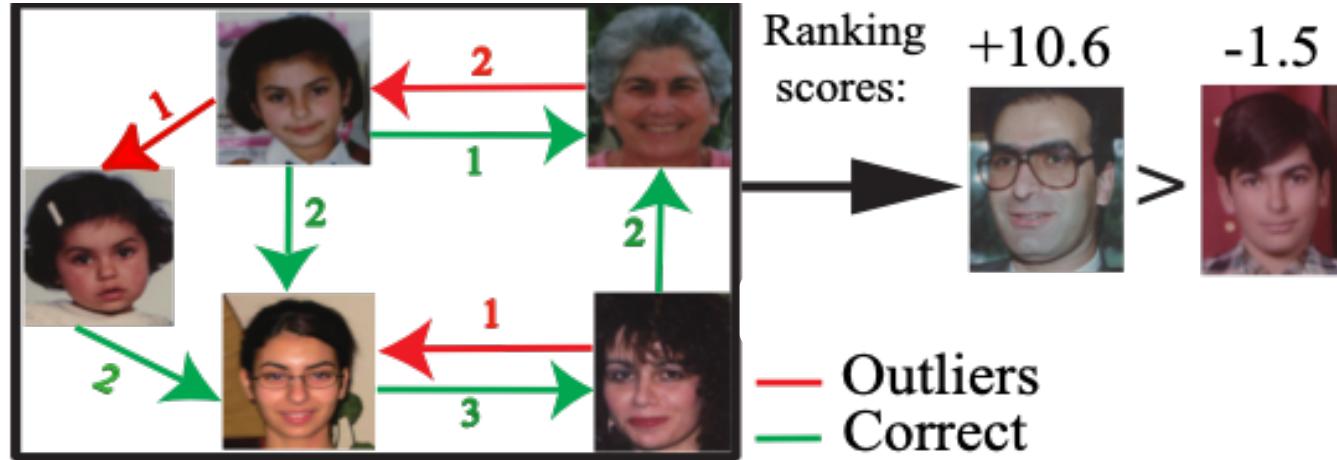


# Sparse Learning in Robust Ranking



Ranking age of images by learning from crowdsourced pairs as the directed graph  $G = (V, E)$

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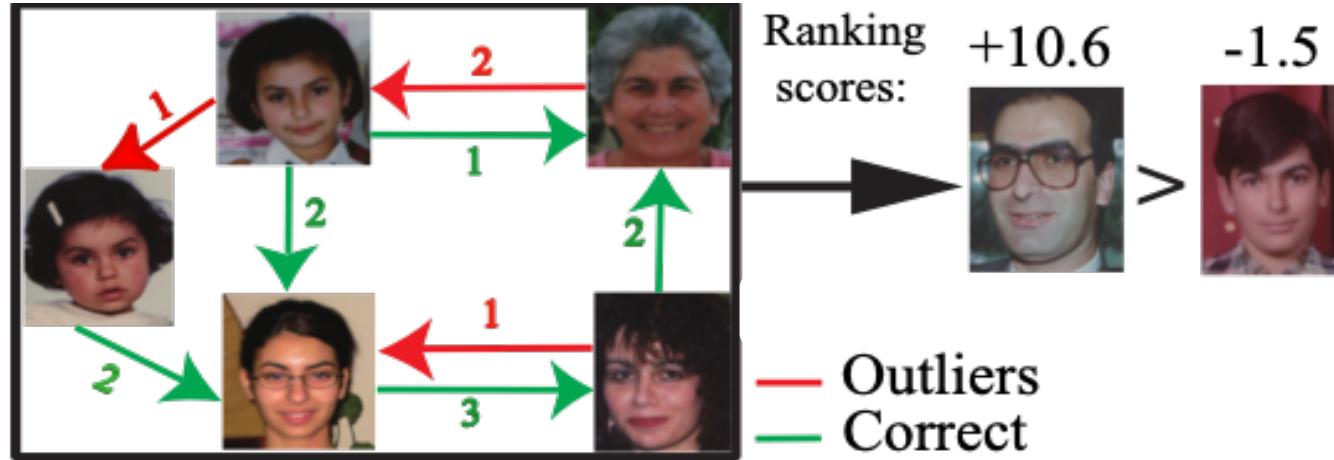


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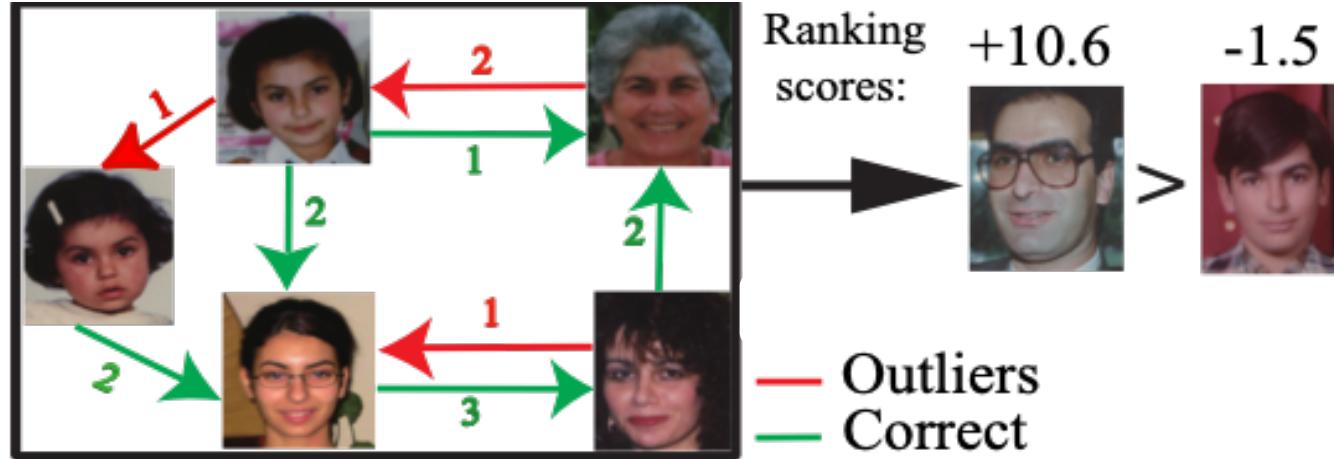
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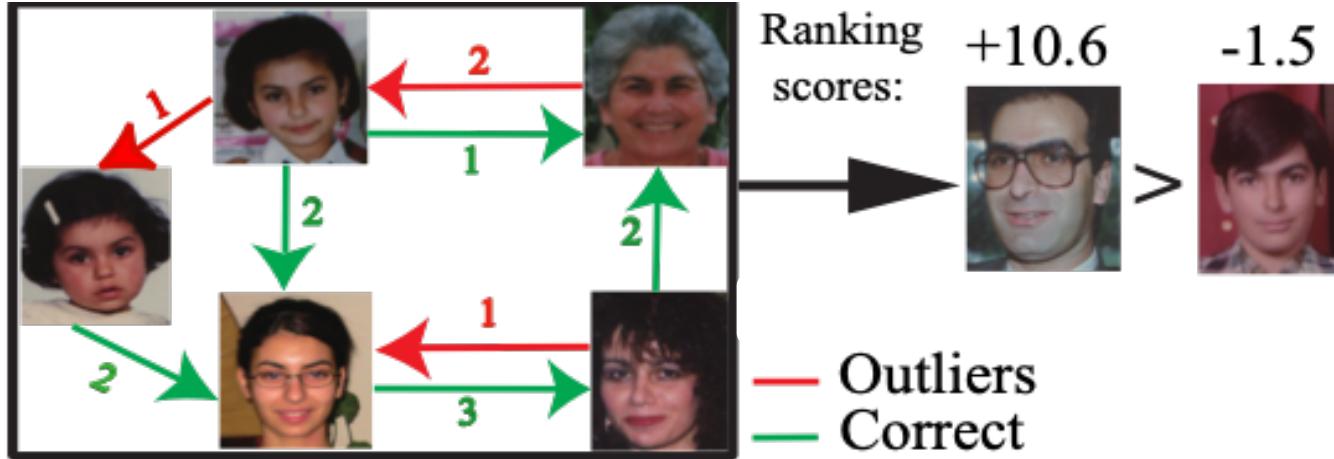
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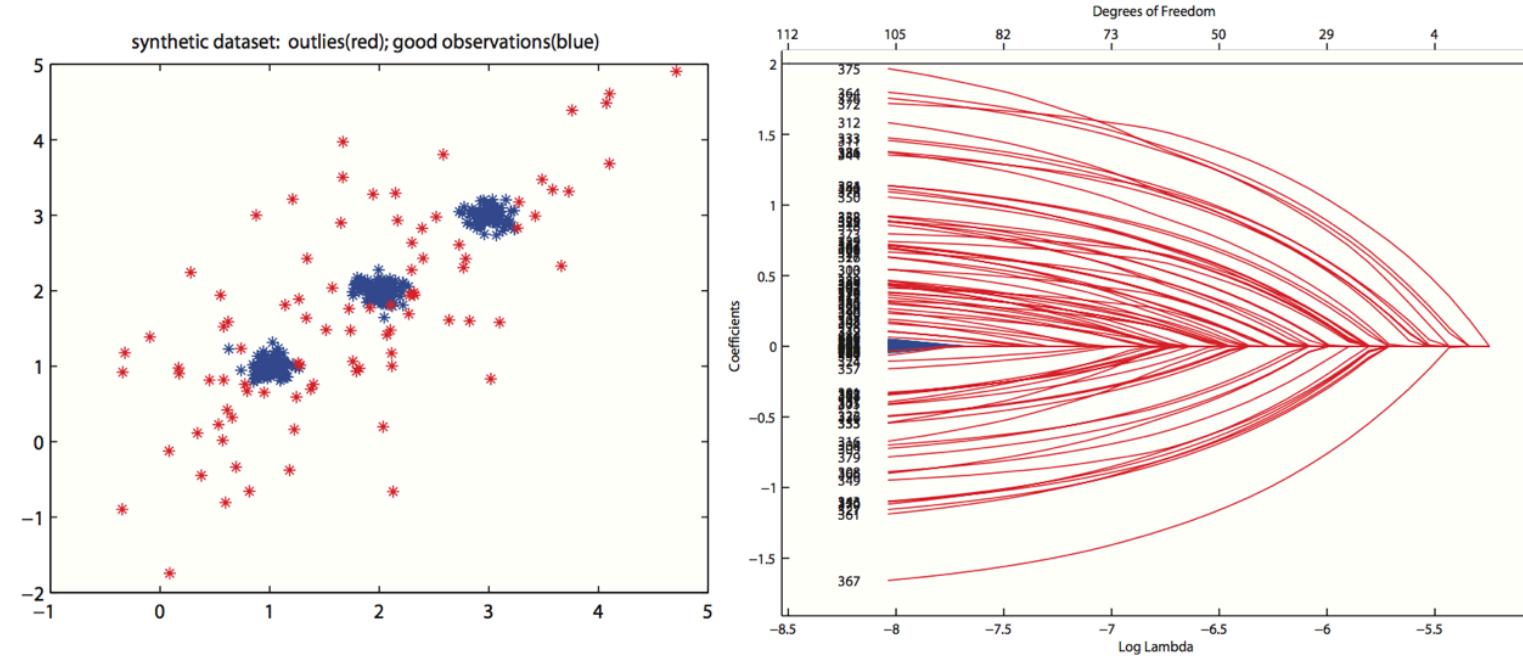
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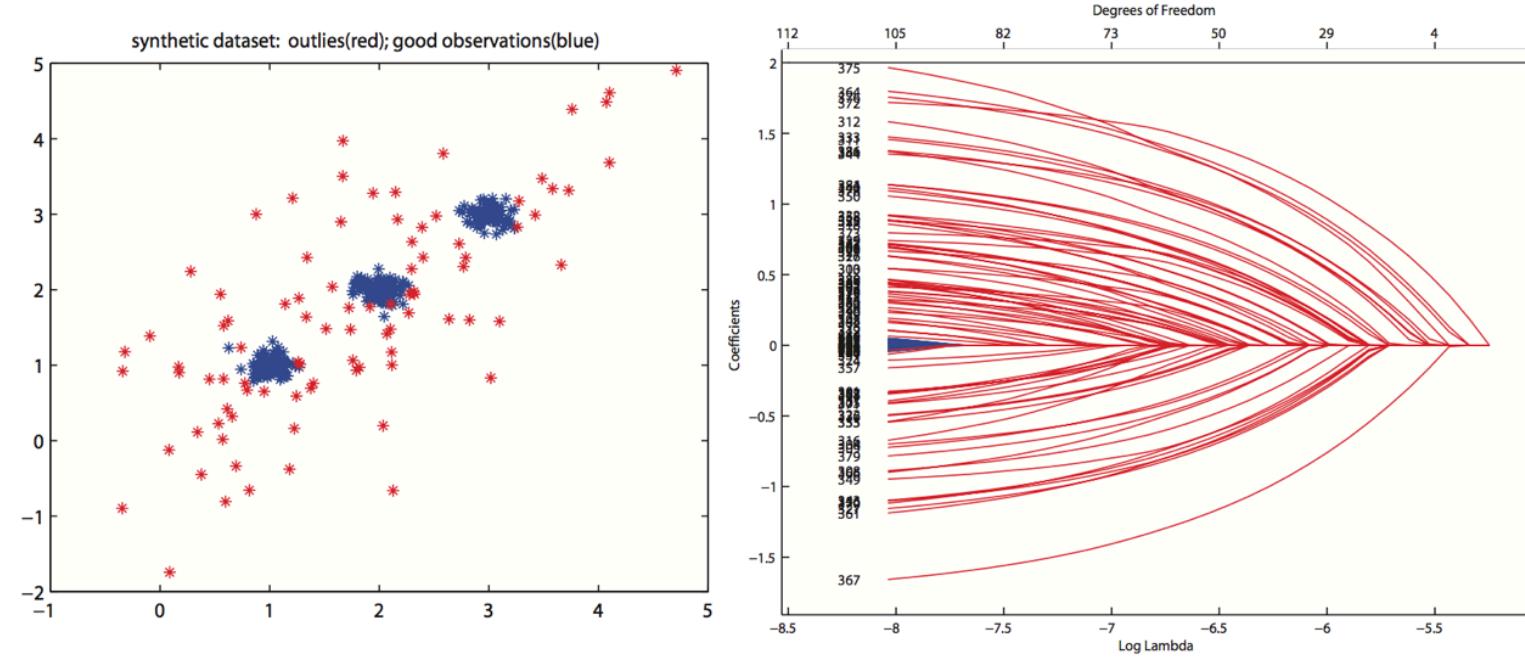


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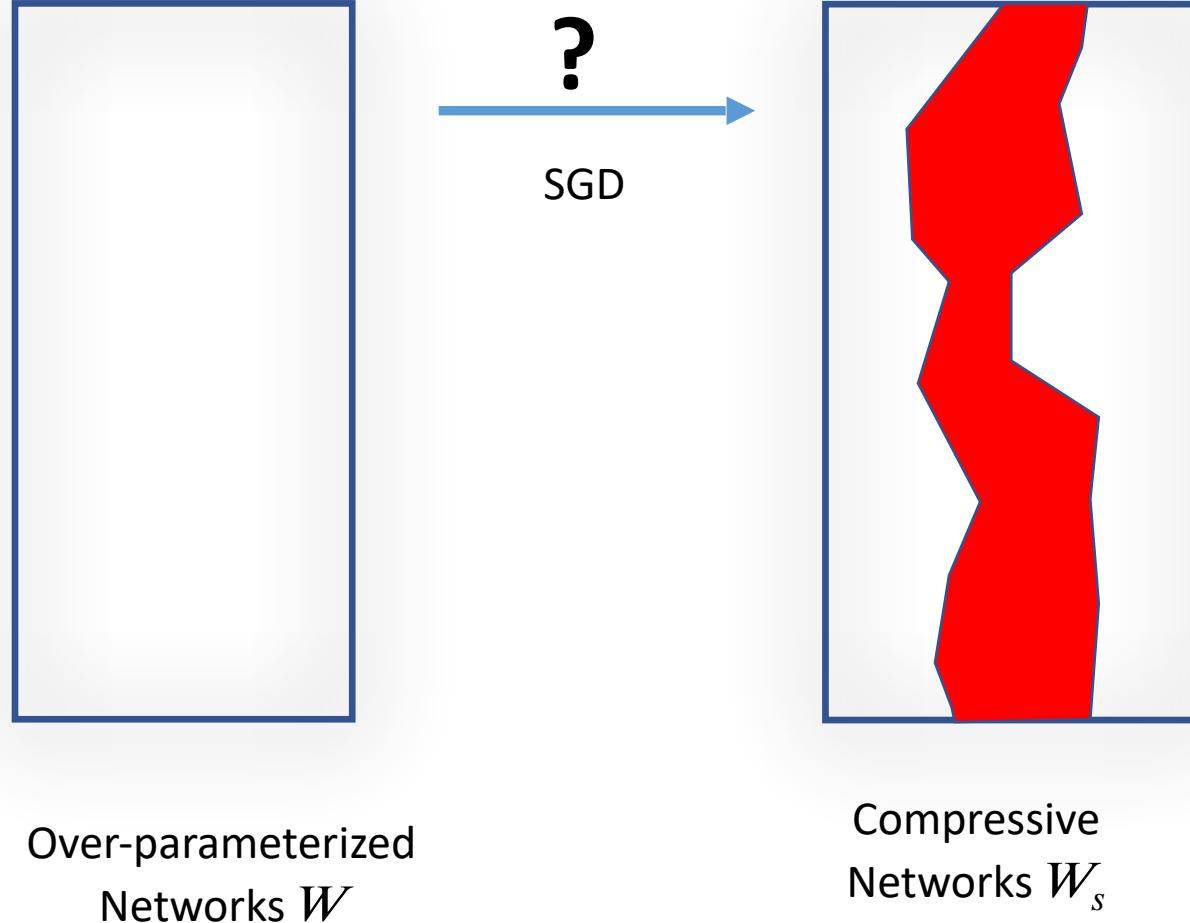


Red lines&points indicate outliers;  
Blue lines&points are inliers.

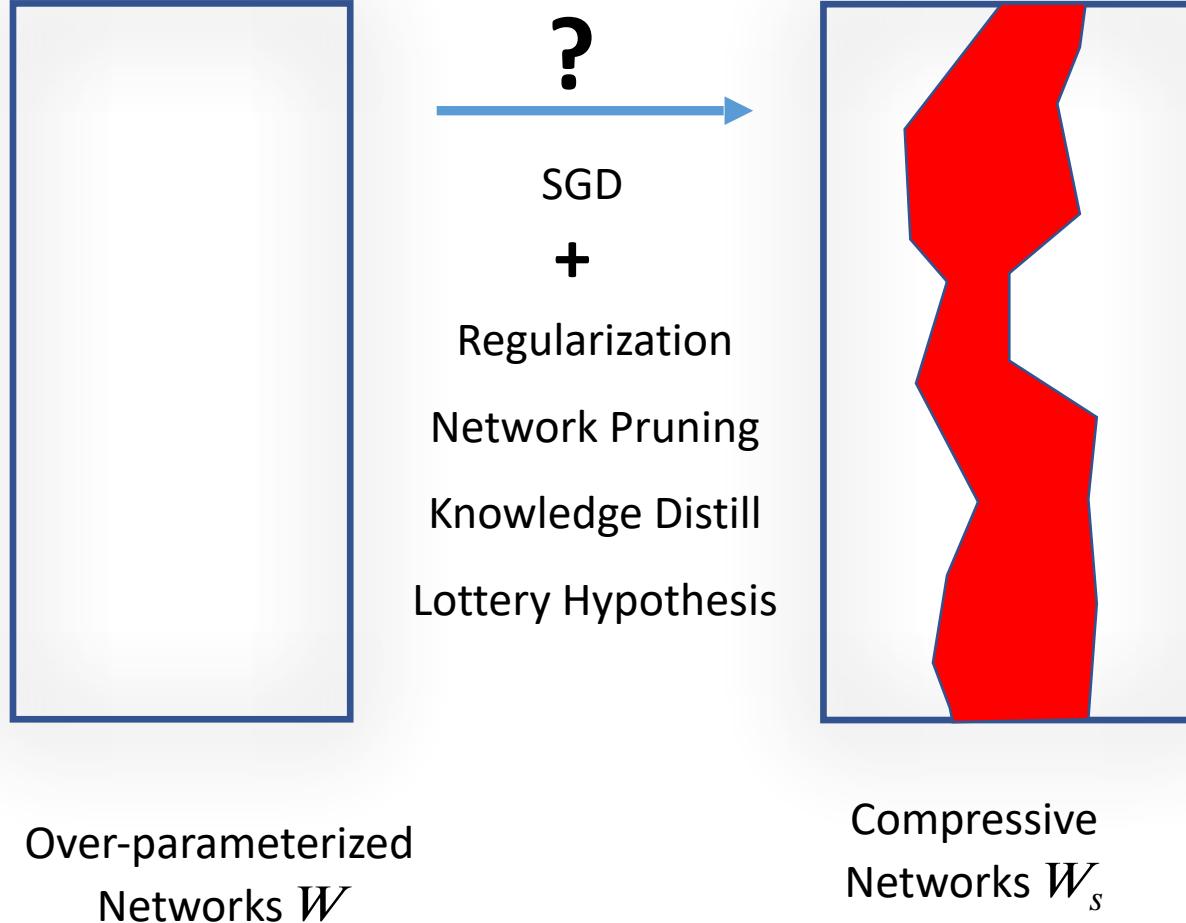
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# Learning Sparsity in Neural Network

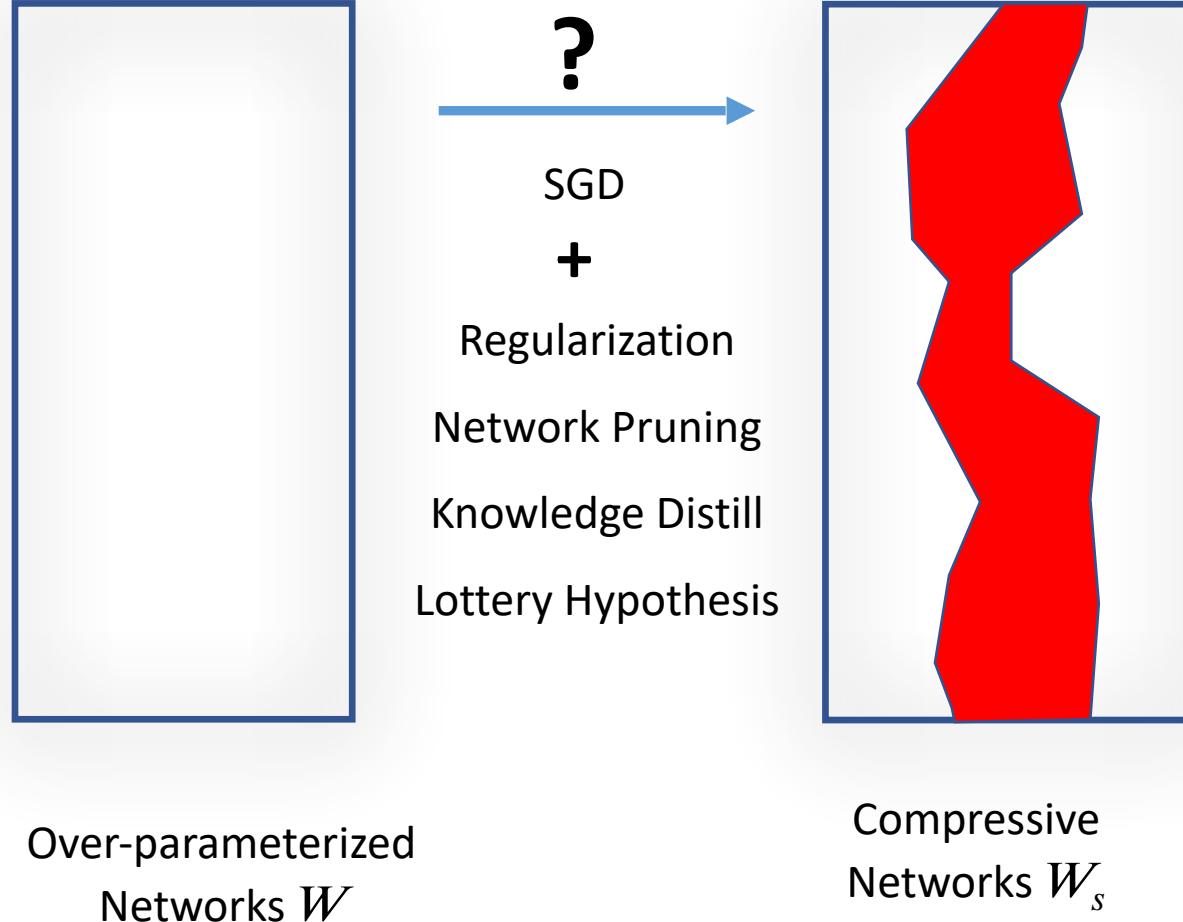
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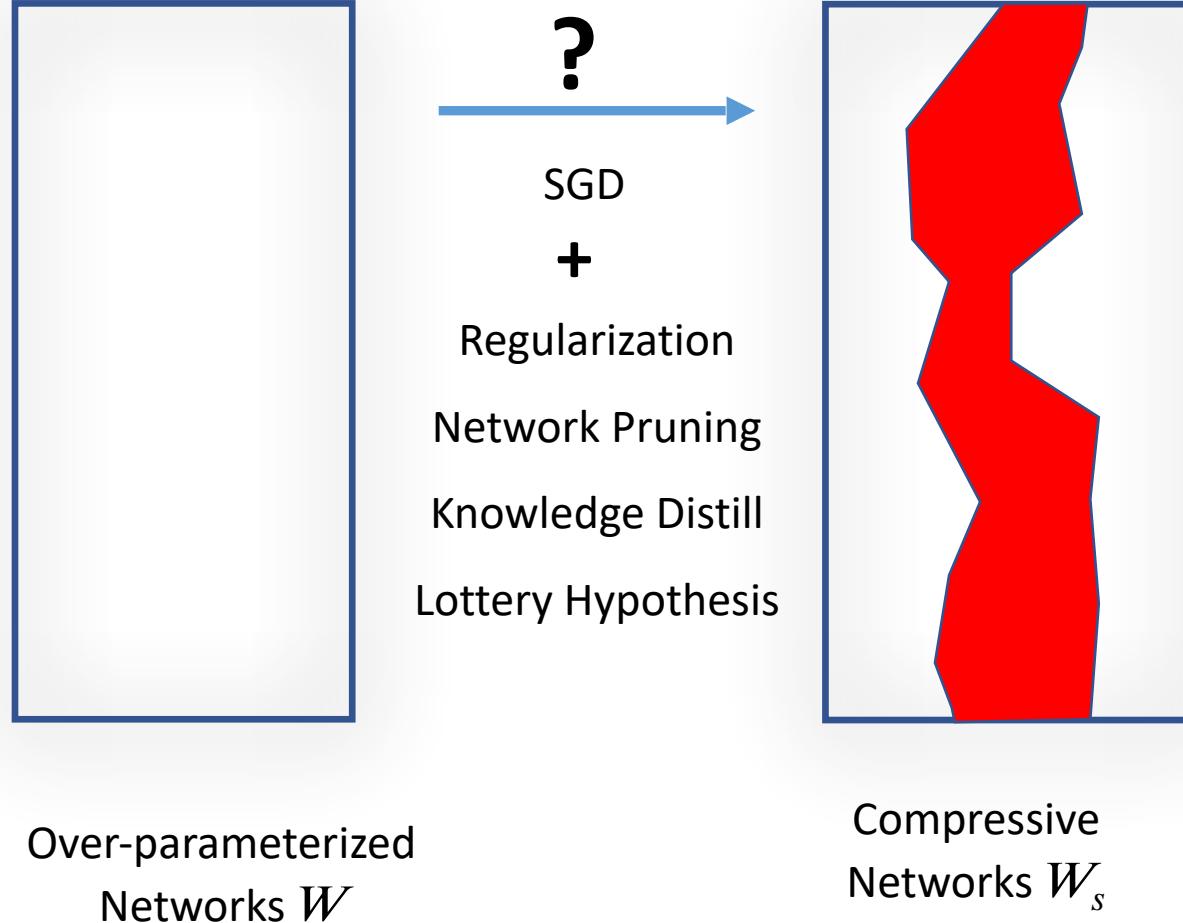
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Existing 2-Stage Approaches:

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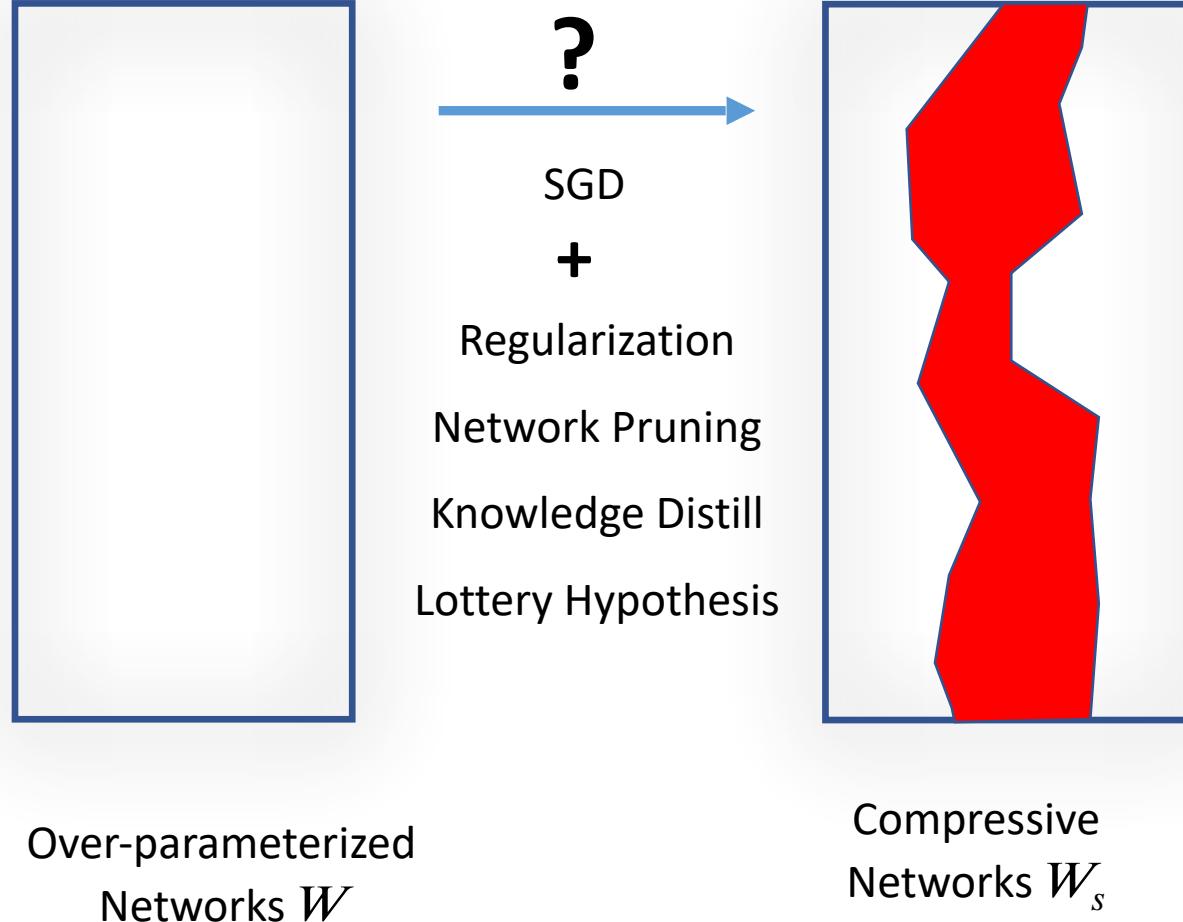
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Our method: 1-Stage Approach (end-to-end)  
Without fully training a dense model.

# Regularization and Overparameterization

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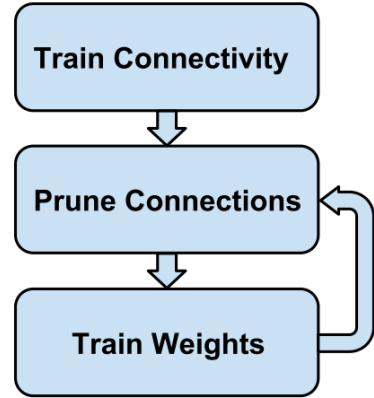
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However, *Larger Inference Time and Memory Cost!*

# Network Pruning (1)



Three-Step Training Pipeline

## Network Pruning:

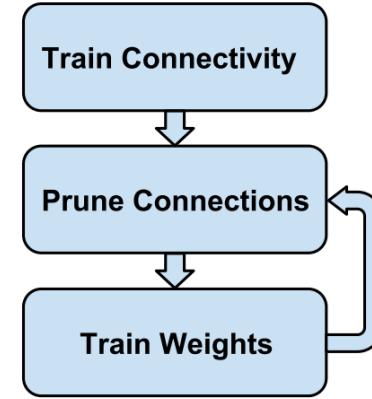
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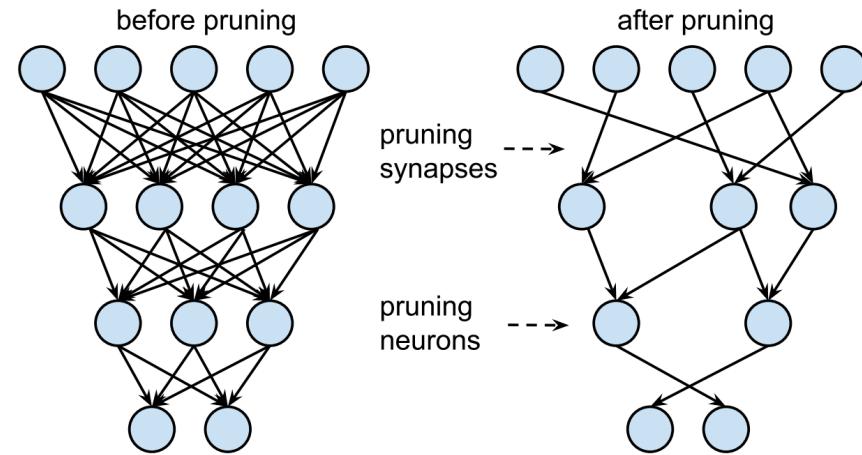
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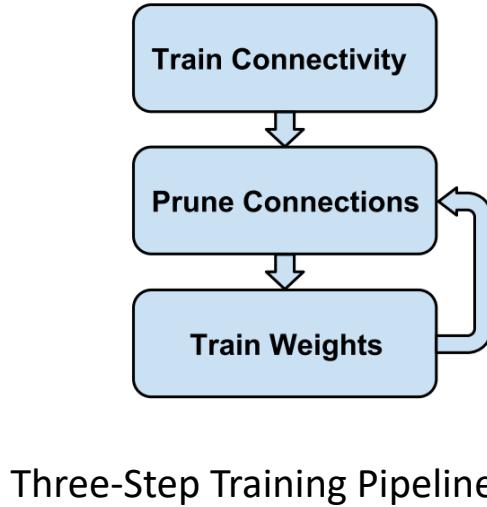
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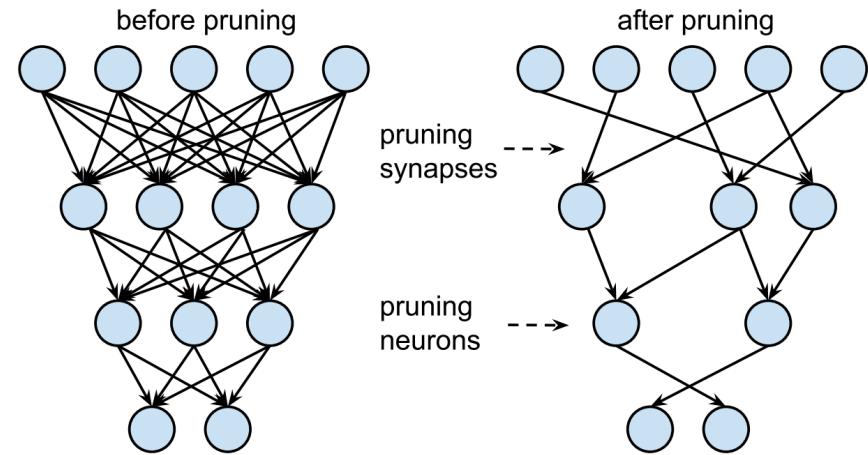
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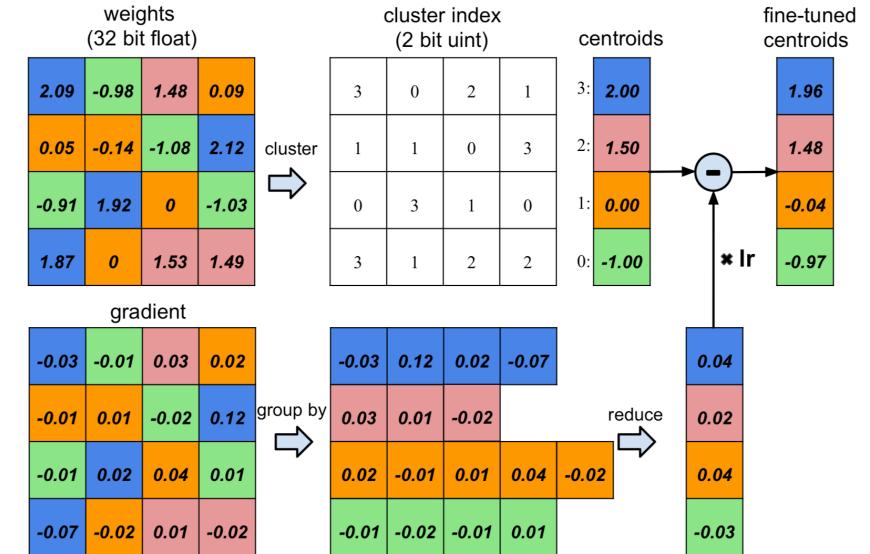
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## Hardware Acceleration



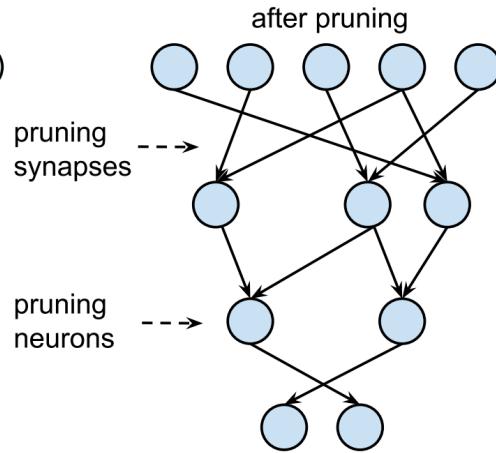
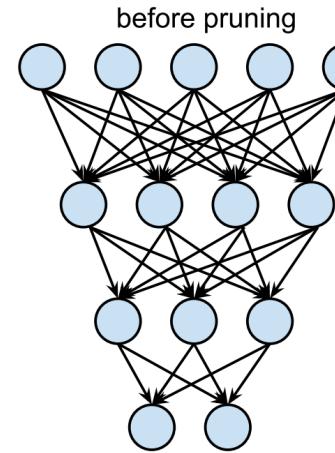
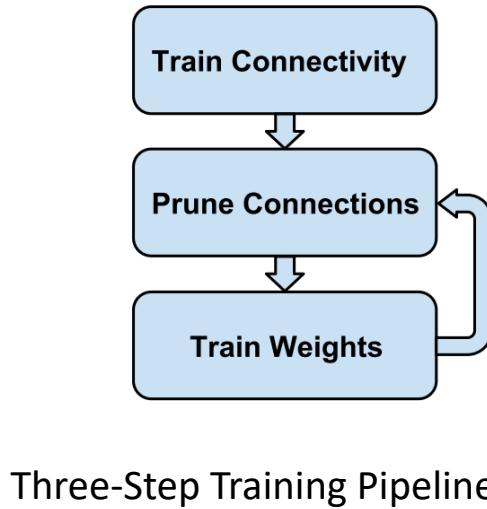
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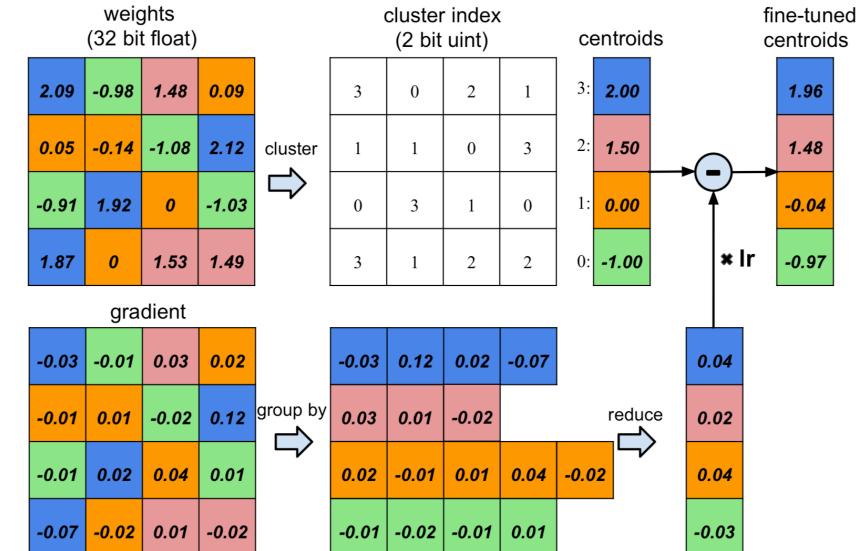


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Questions: Can we do sparsity in **weight level, filter level, and even layer level** with a **unified ‘algorithm’**?

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# Learning Sparsity on Models

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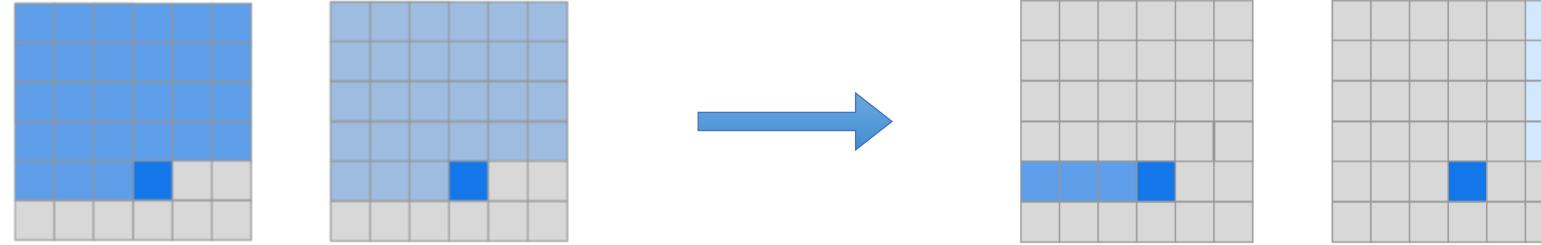
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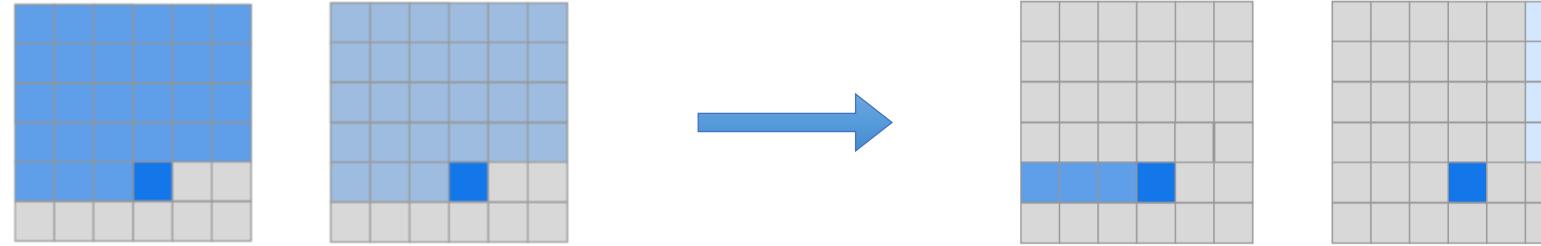
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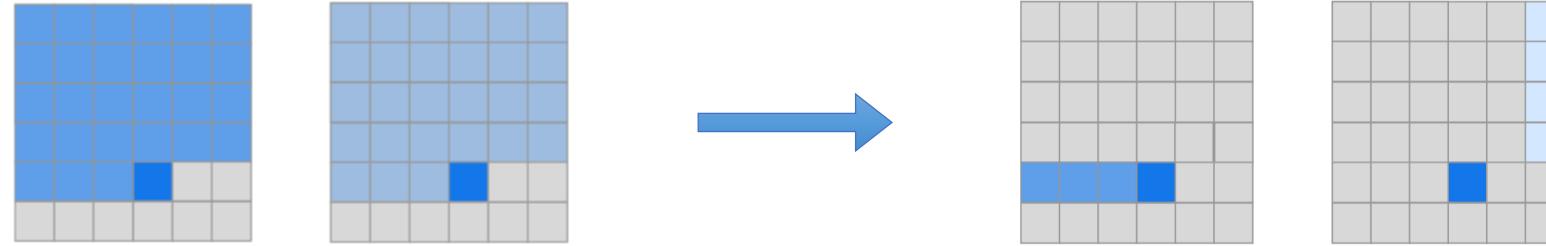
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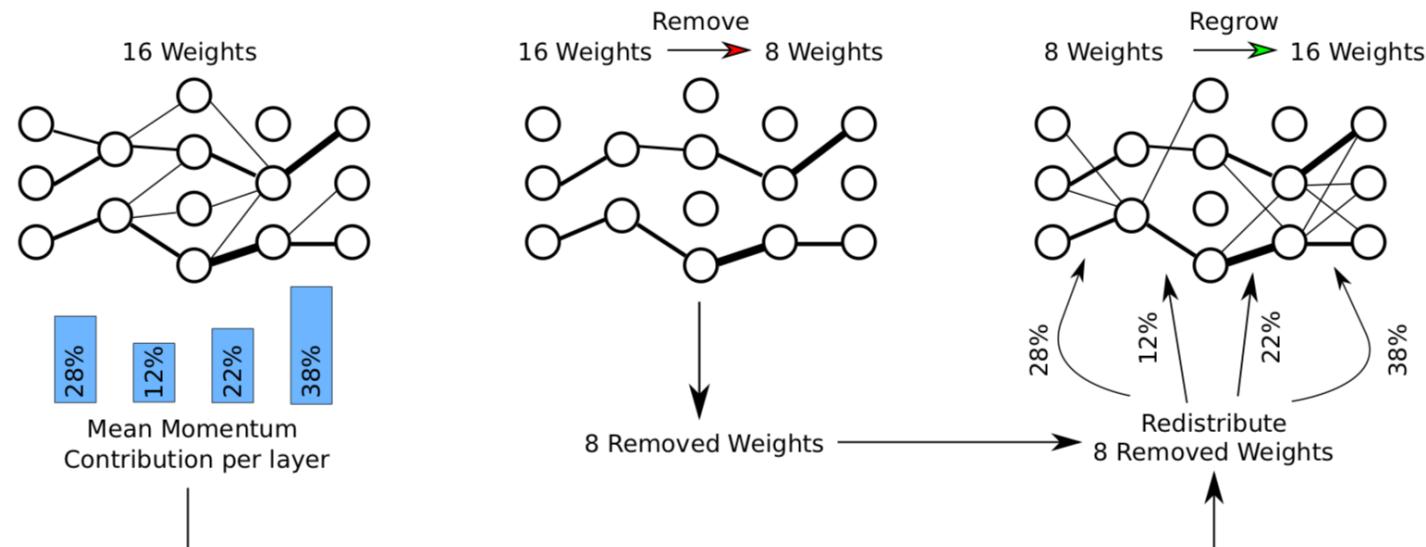
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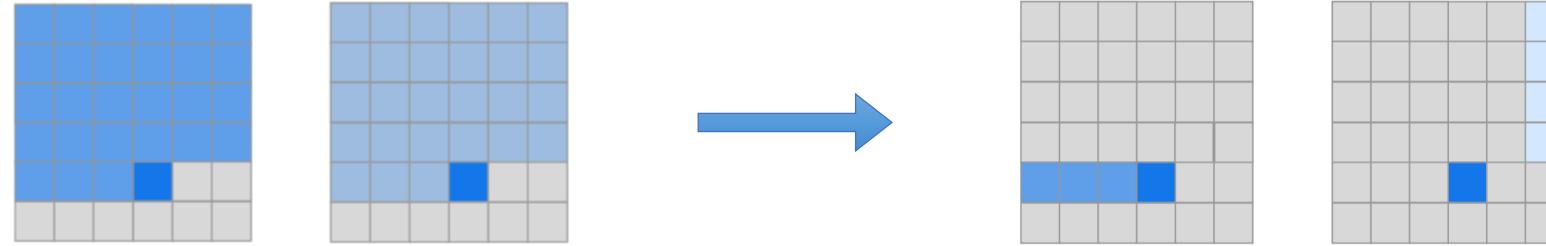


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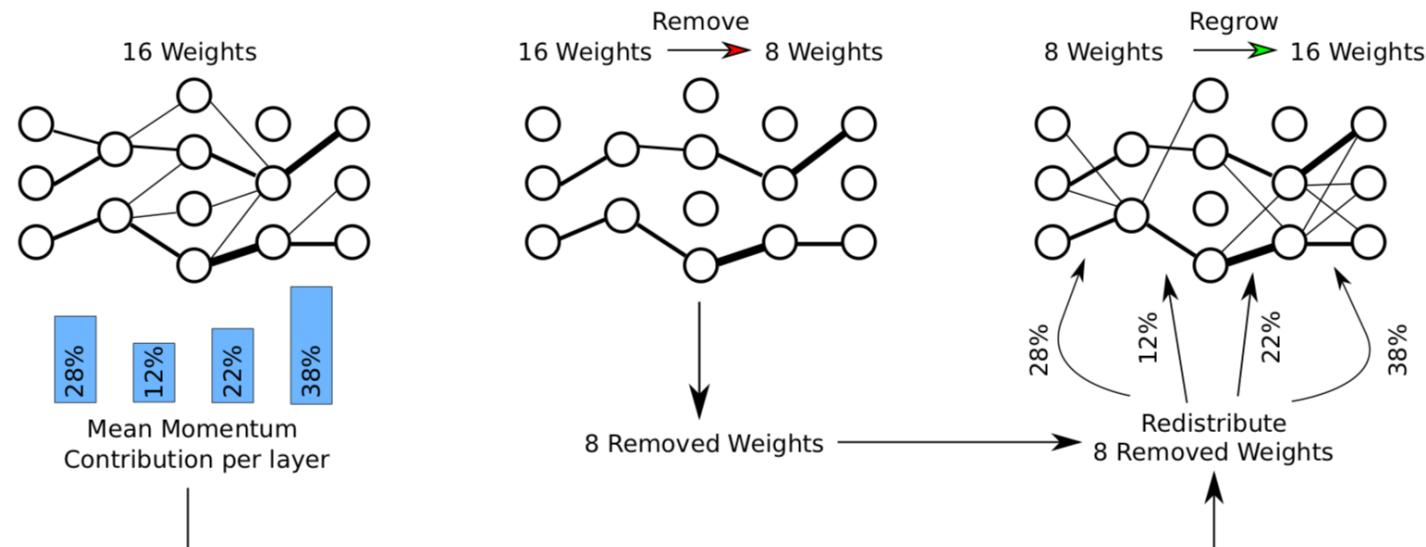
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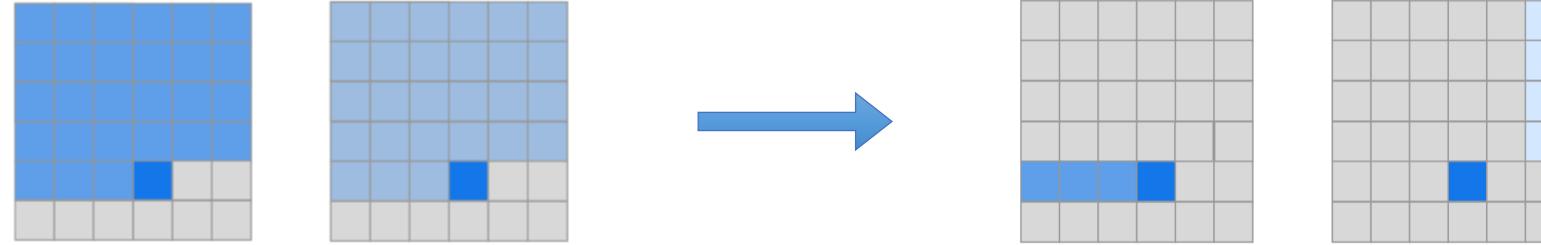


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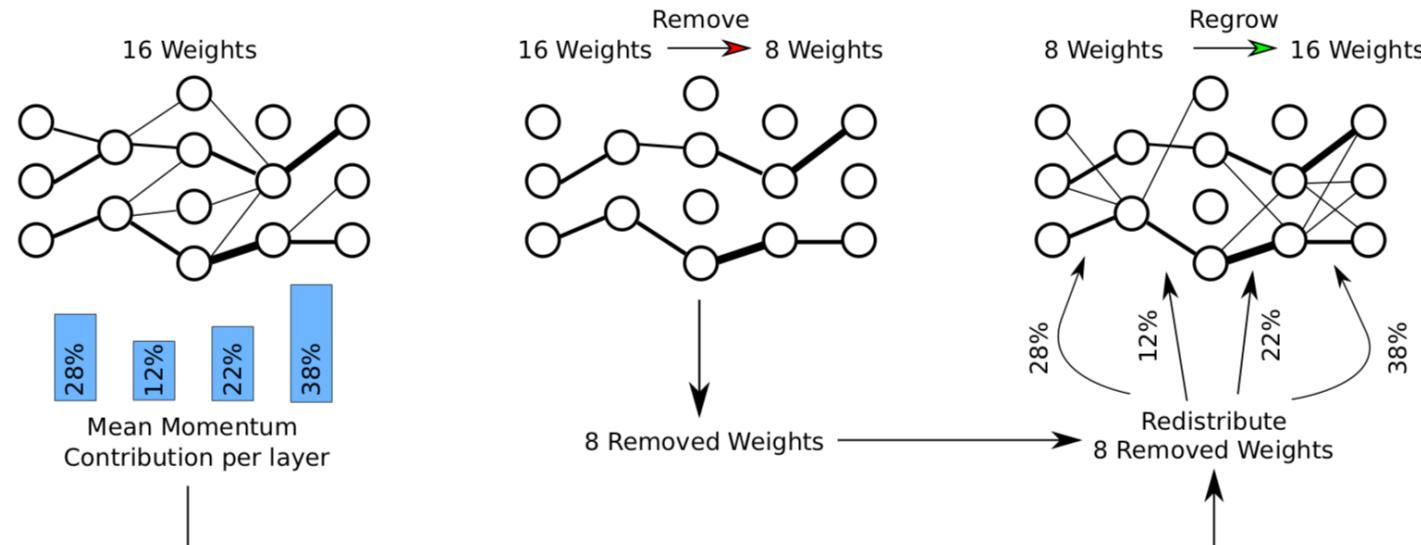
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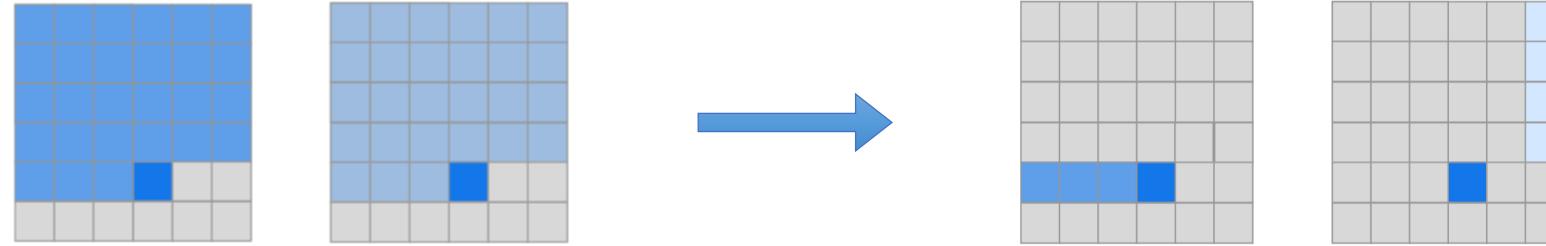


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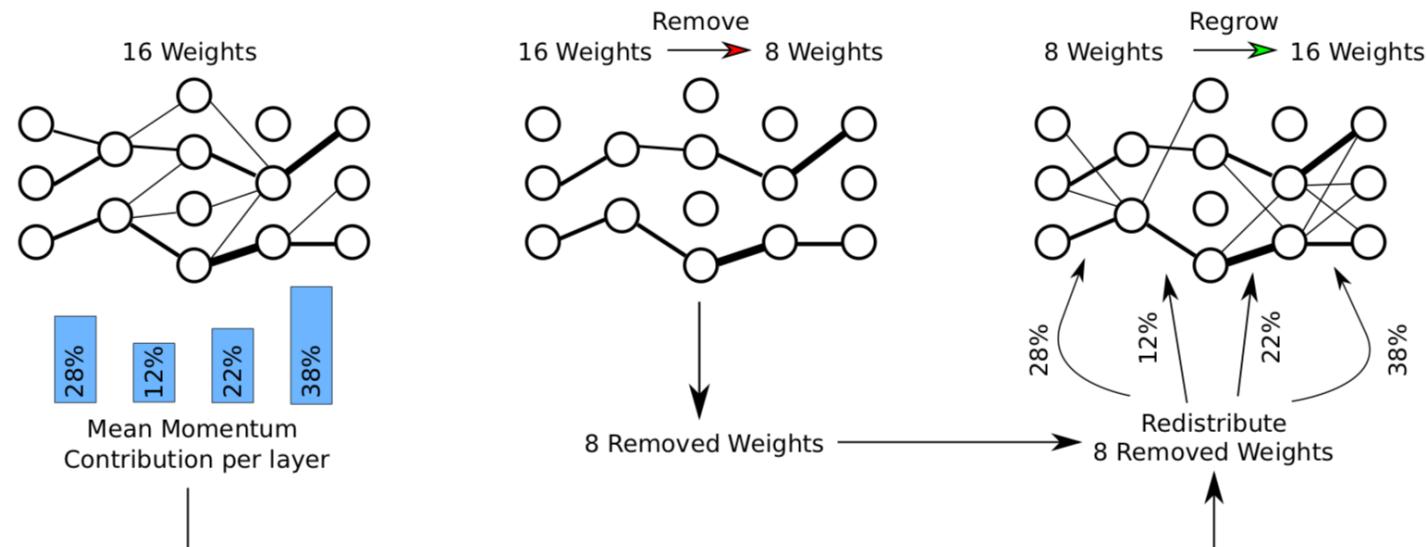
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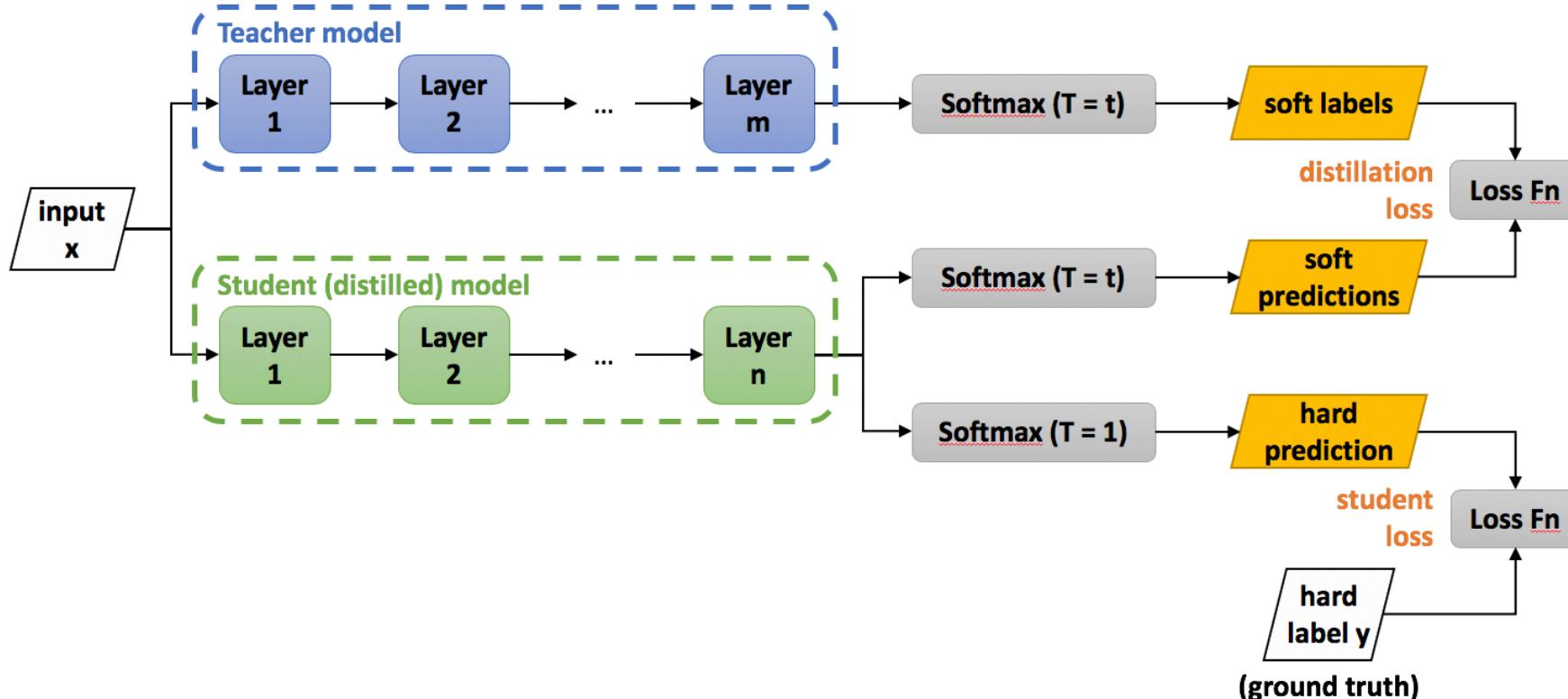
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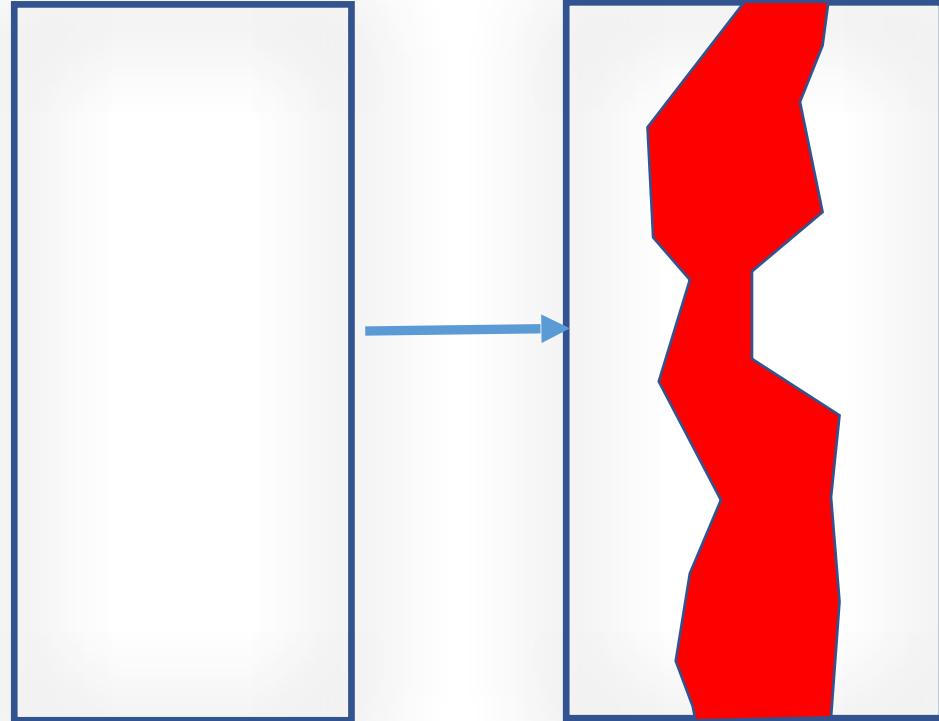
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# The Concept of Knowledge Distill



- Introducing the concept of "softmax temperature".
- Utilizing the “dark knowledge” of teacher model: which classes is more similar to the predicted class.

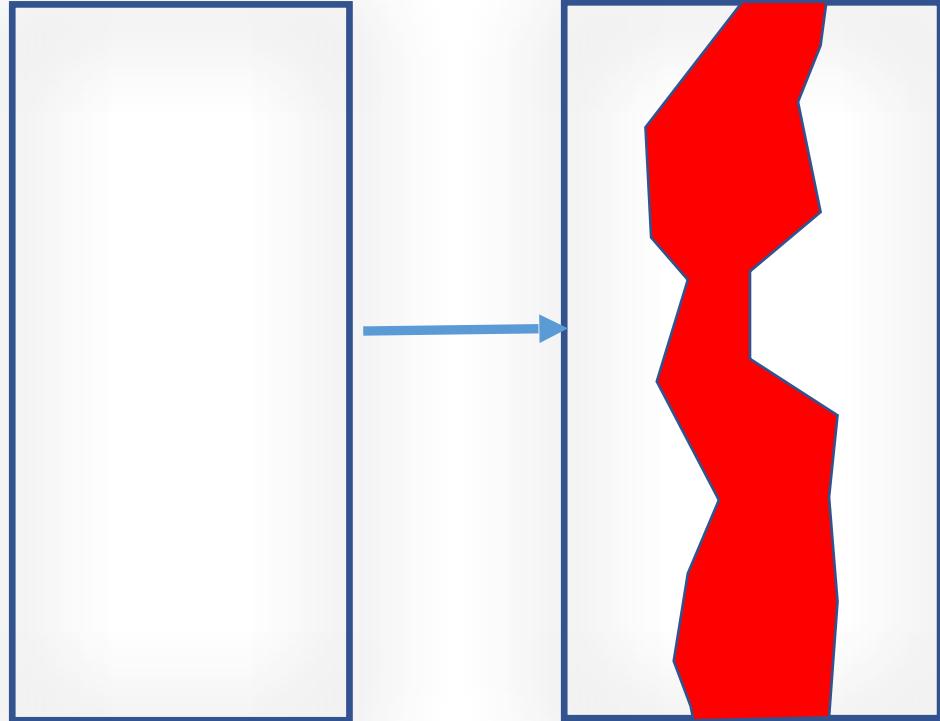
# Lottery Hypothesis



Over-parameterized  
Networks  $W$

Compressive  
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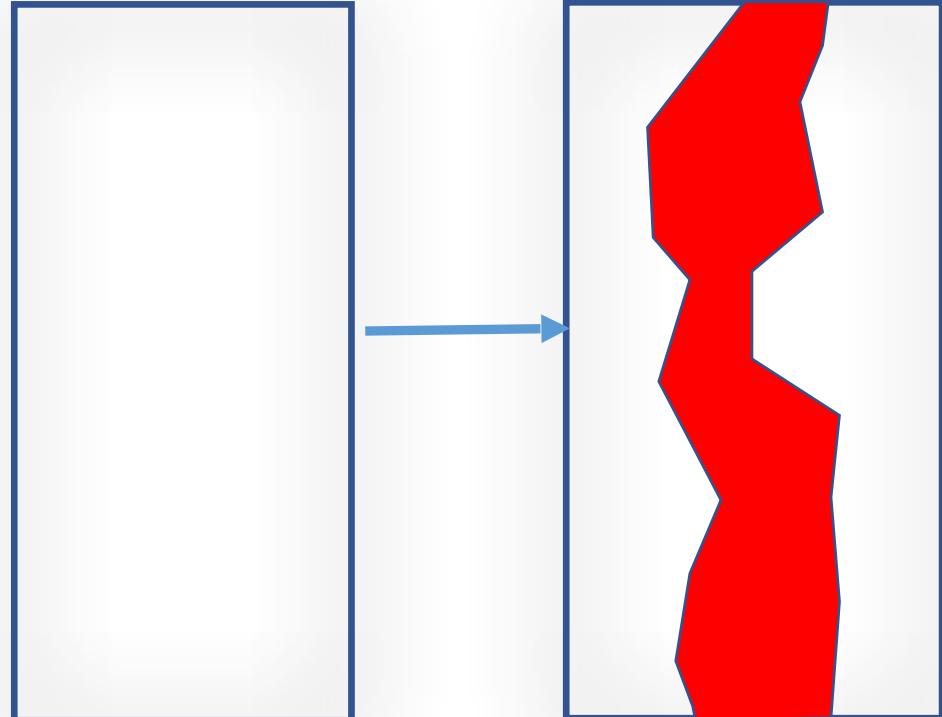
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- *Dense, randomly-initialized, feed-forward networks contain subnetworks (winning tickets) that – when trained in isolation – reach test accuracy comparable to the original network in a similar number of iterations. (Frankle & Carbin, 2019)*

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Rewinding the network from the initialization, and find “winning ticket” subnet



# The Key Idea of the Optimizer-DessiLBI

DessiLBI : Deep structurally splitting Linearized Bregman Iteration



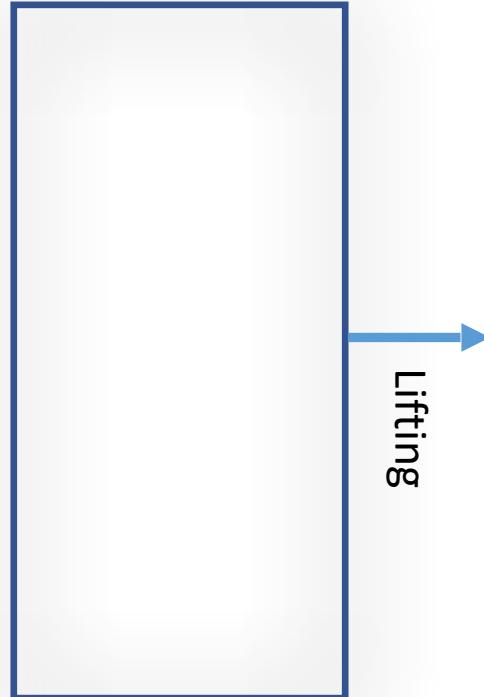
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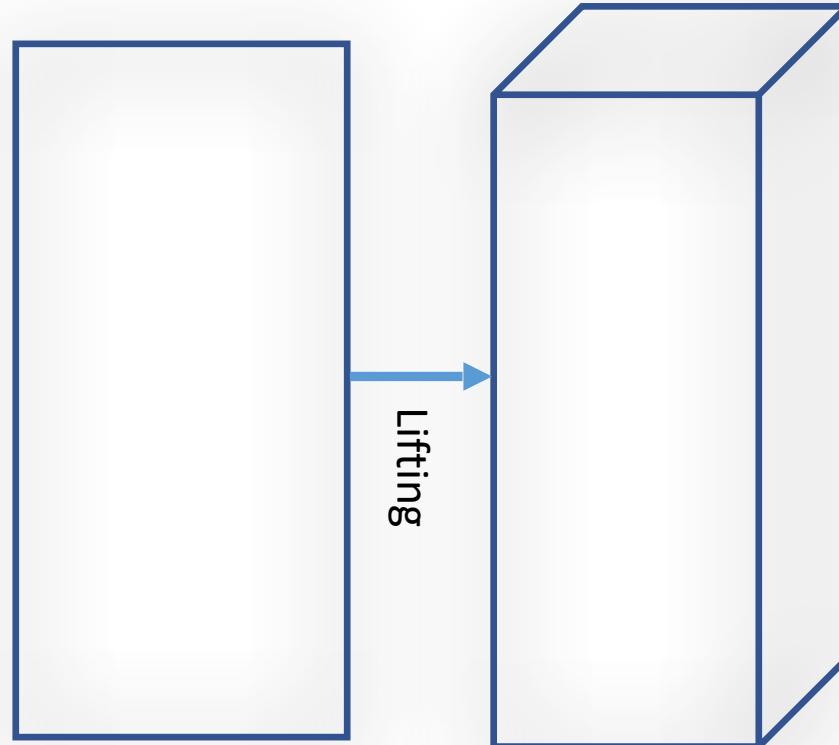
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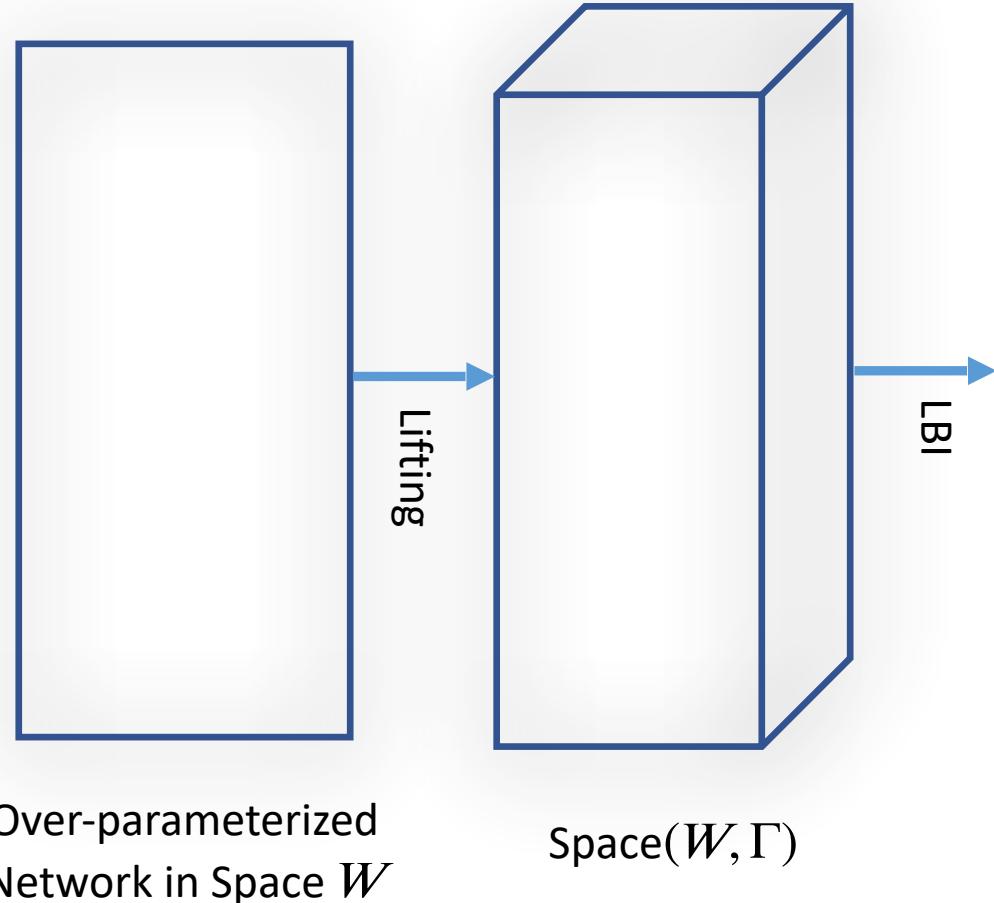
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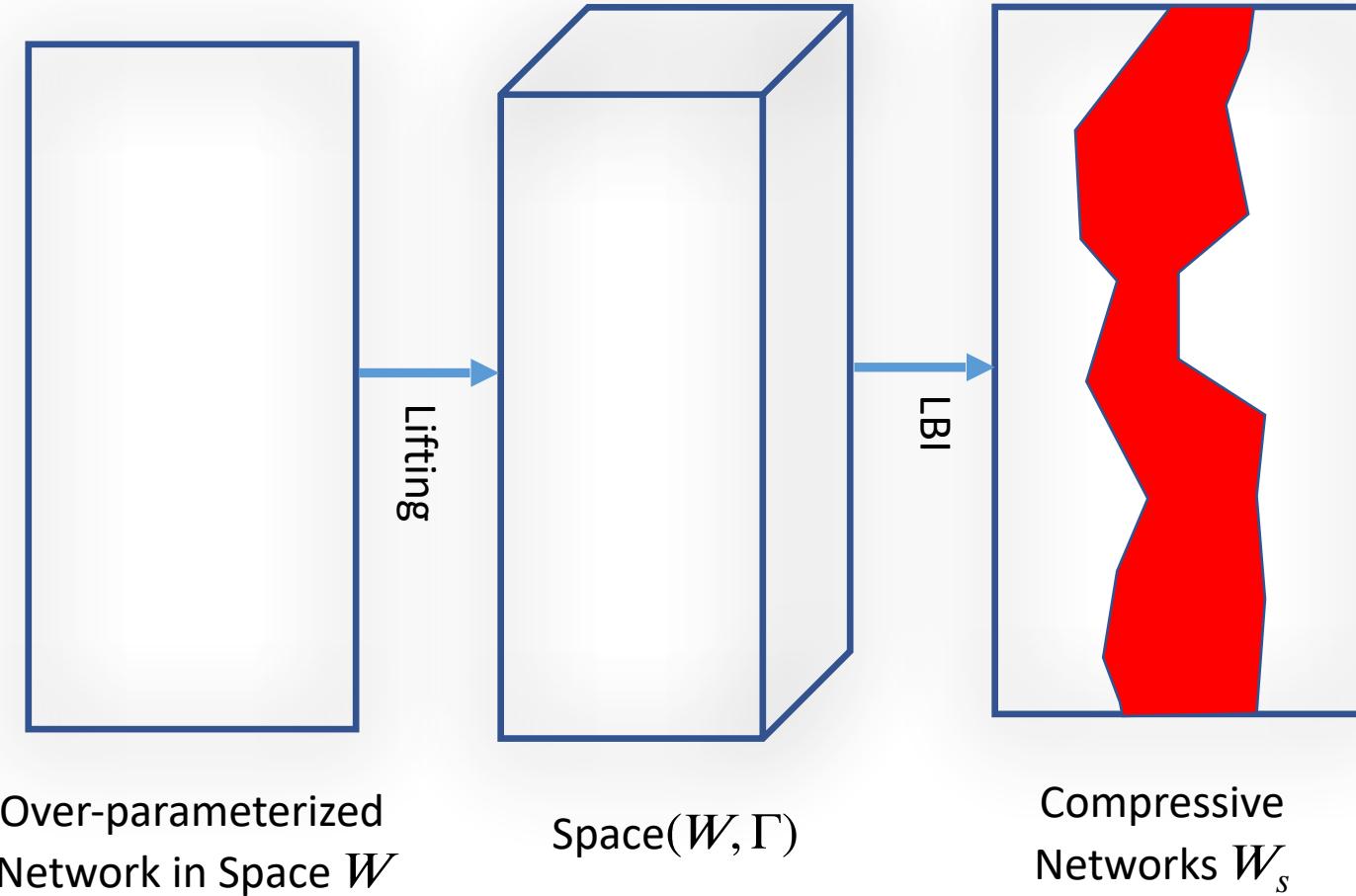


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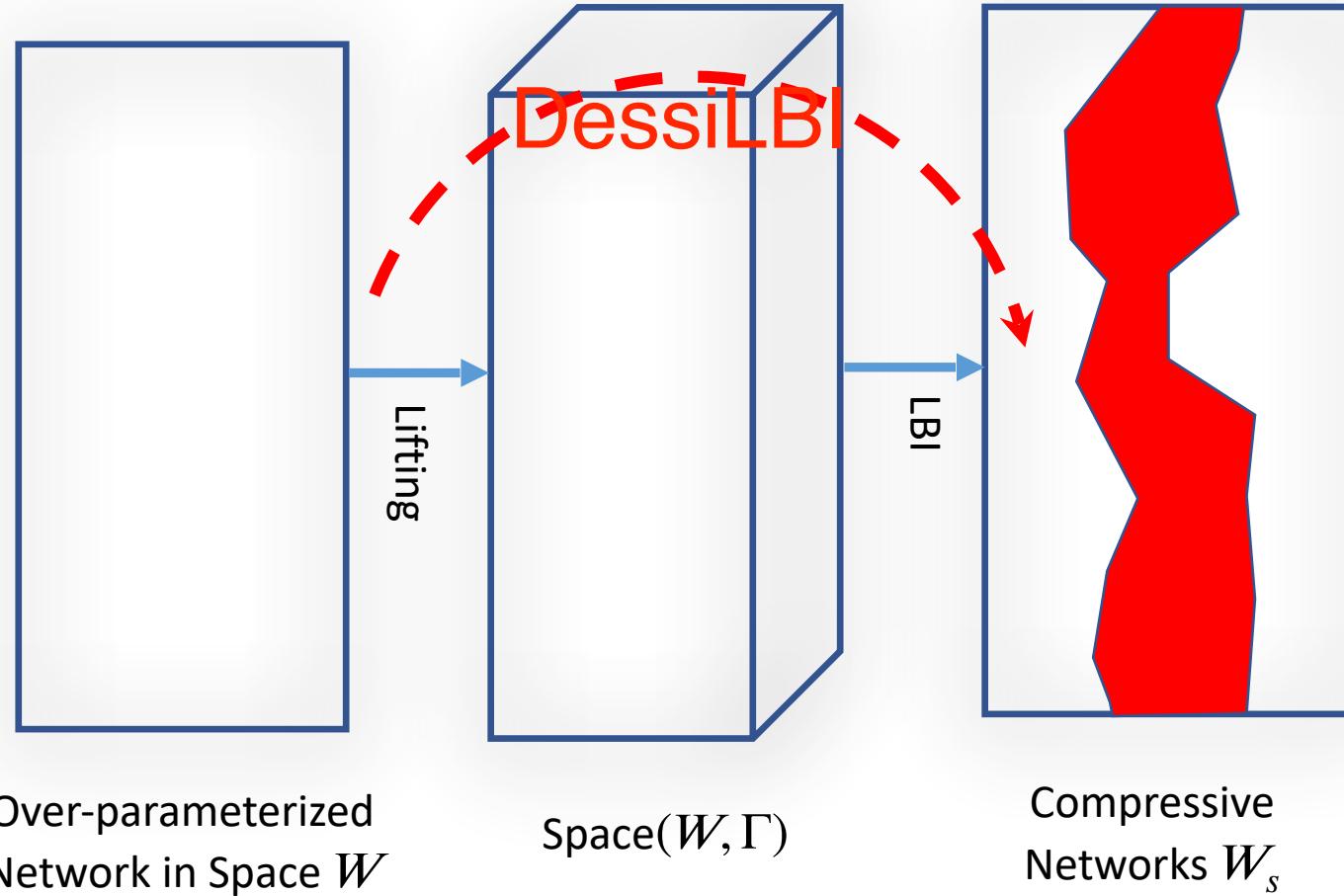


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Two Stage Method, Training Dense Network → Produce sparse subnet

One Stage Method, An end-to-end method, directly producing structurally sparse Subnet.

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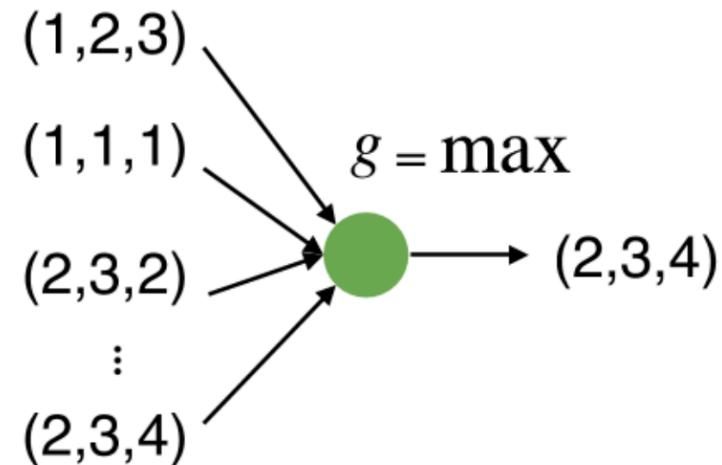
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Inspired by  
PointNet

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One Stage Method, An end-to-end method, directly producing structurally sparse Subnet.



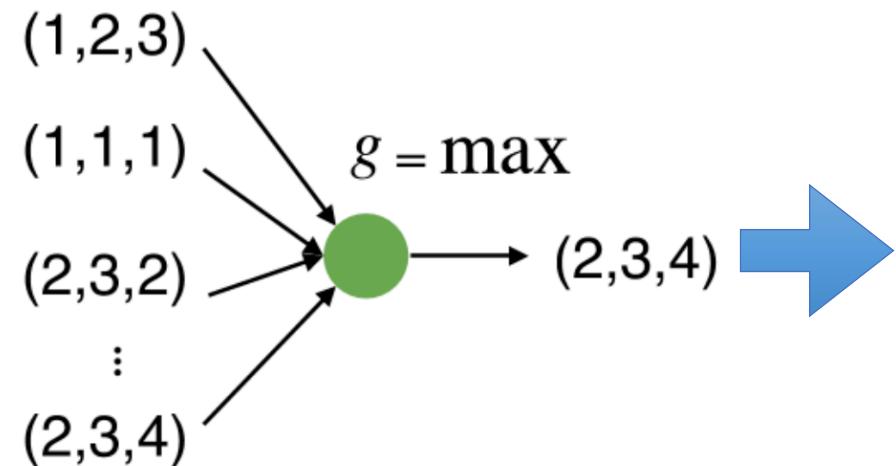
Inspired by  
PointNet

Discover naïve/extreme  
property of geometry

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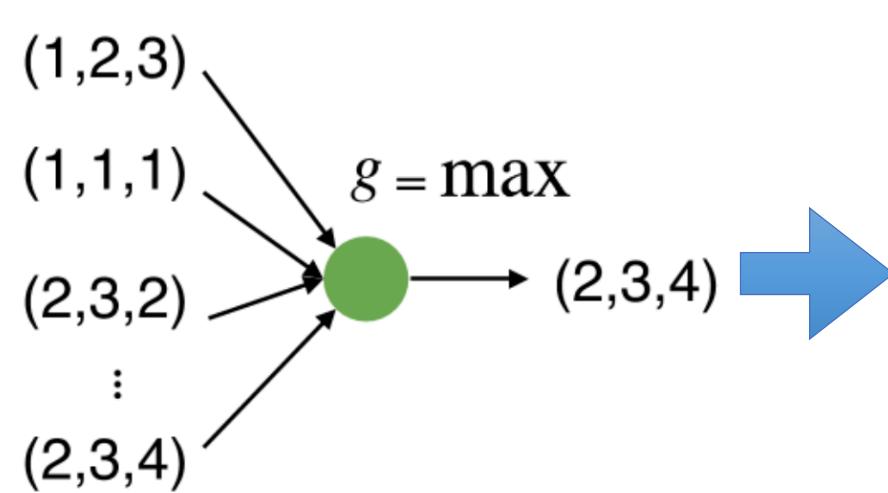
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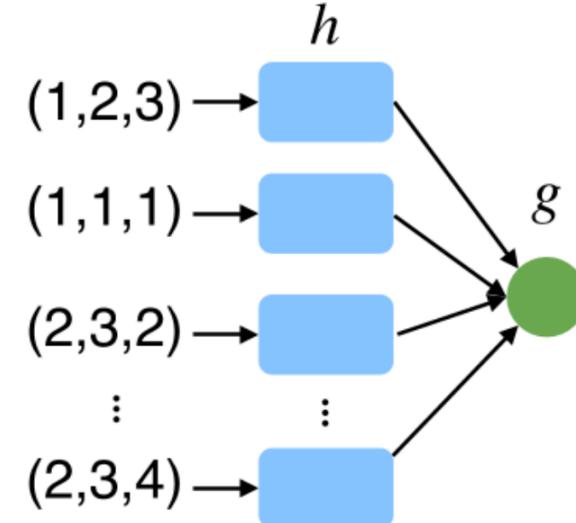
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Aggregation in **high-dim space** preserves  
interesting properties of the geometry

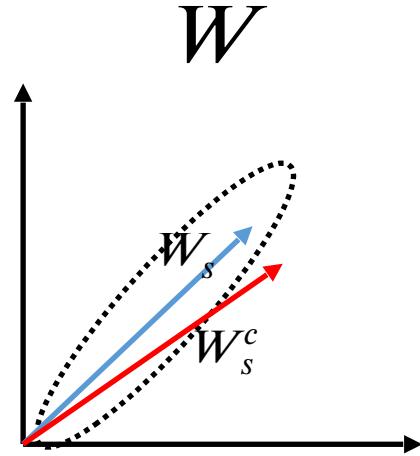
# Insights for the Inverse Scale Space

$W_s$ : wining ticket

$W$ : dense parameters

$W_s \cup W_{S^c} = W$

$W_s$  and  $W_{S^c}$  correlated



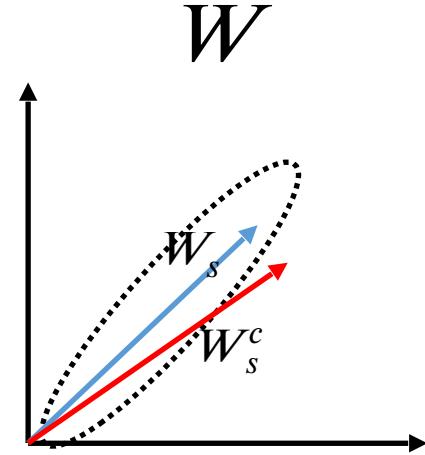
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GD of Weight Space

$$\dot{W}_t = -\nabla_W \hat{\mathcal{L}}_n (W)$$

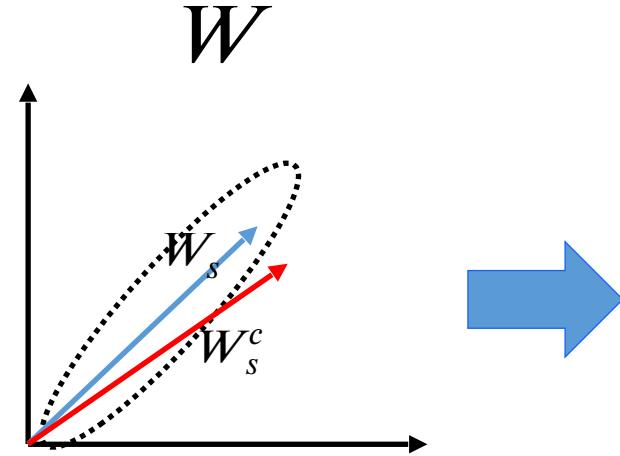
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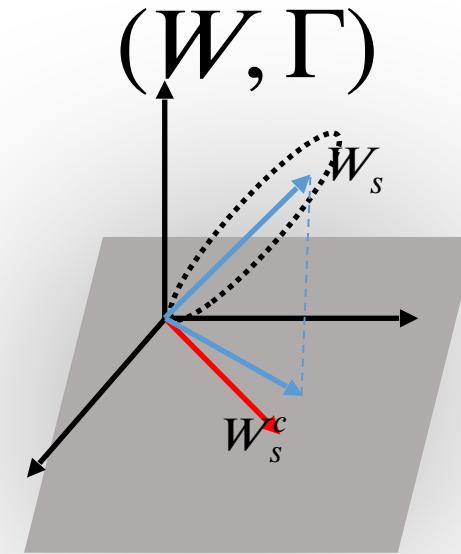
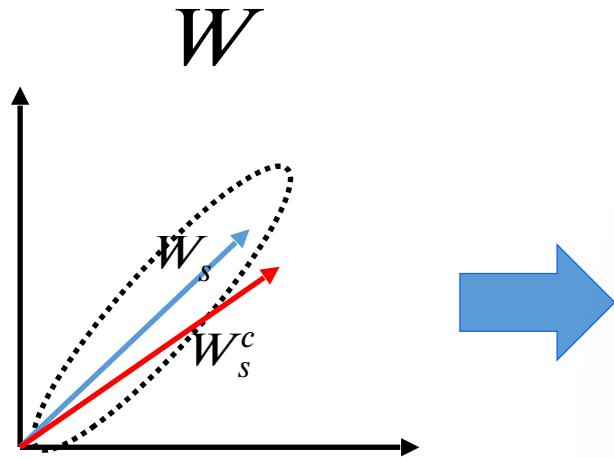
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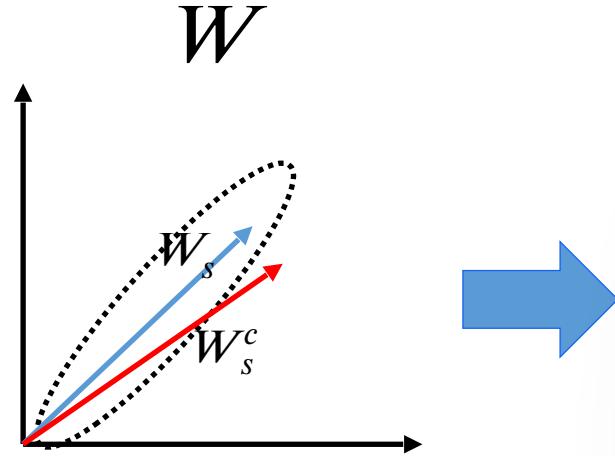
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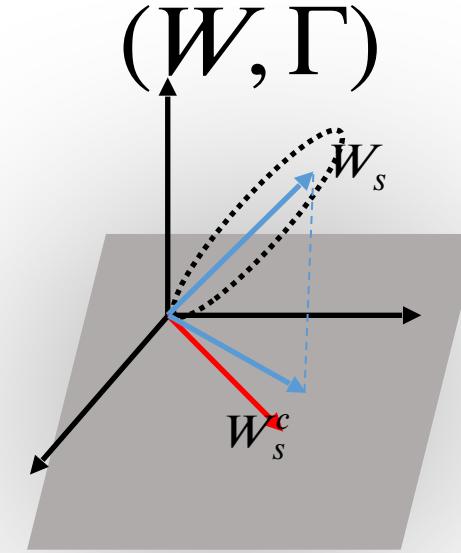
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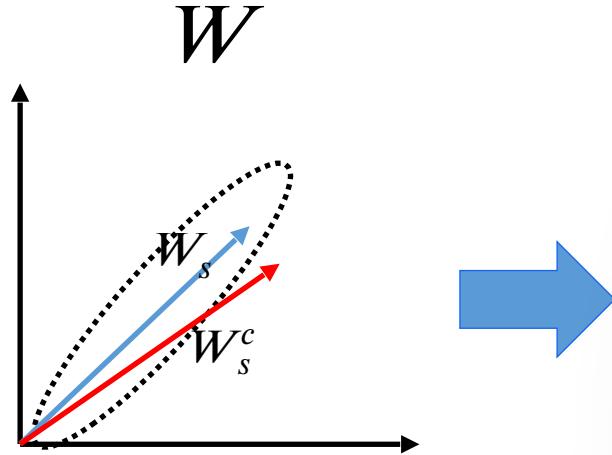
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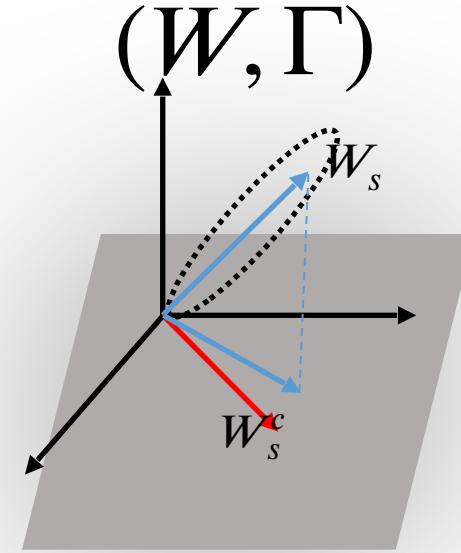
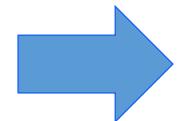
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$$\dot{V}_t = -\nabla_\Gamma \bar{\mathcal{L}}(W_t, \Gamma_t)$$

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Differential Inclusion of Inverse Scale Space

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Proximal Mapping

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$$\Omega(\Gamma) = \sum_g \|\Gamma^g\|_2 = \sum_g \sqrt{\sum_{i=1}^{|\Gamma^g|} (\Gamma_i^g)^2}$$

A close form solution:

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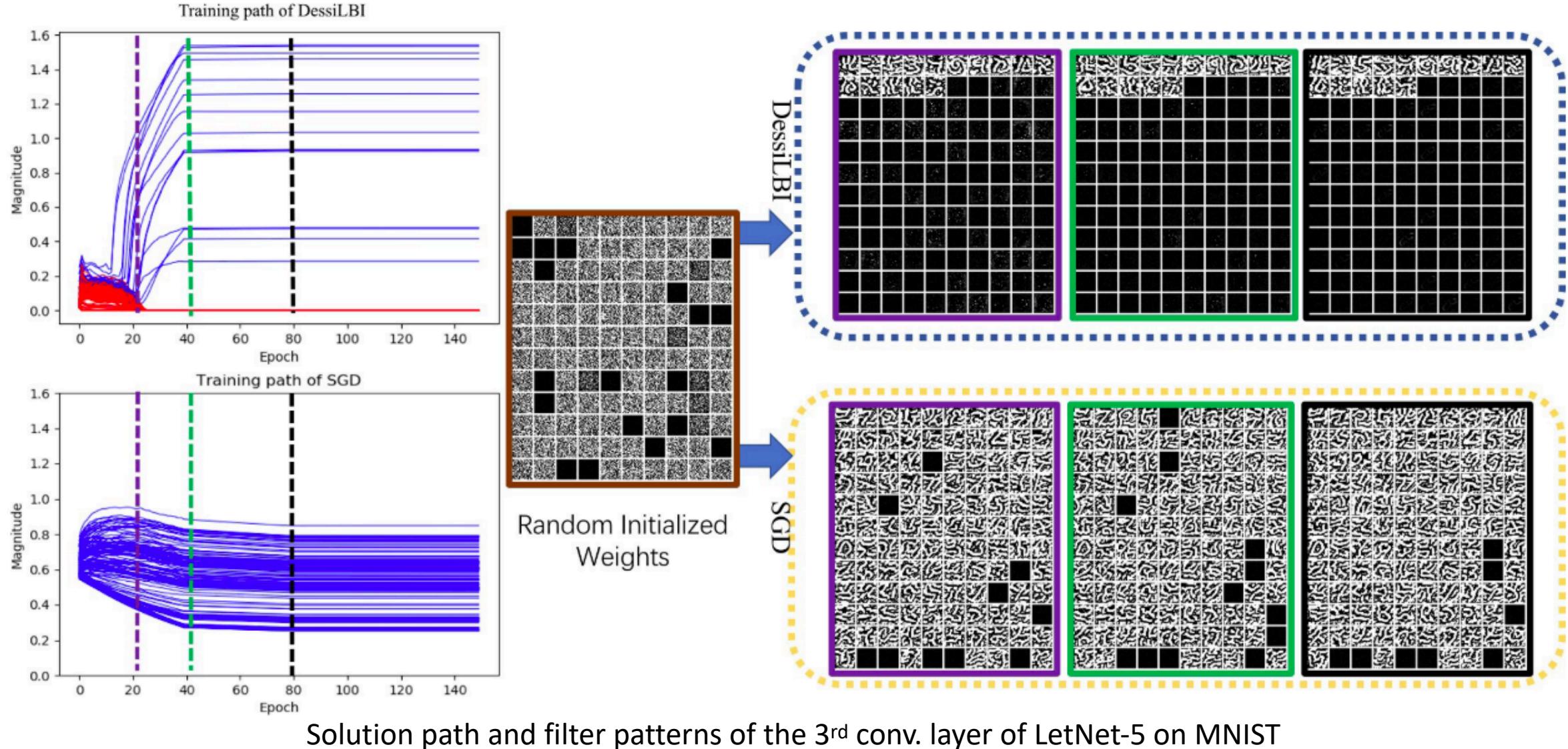
- (Batch) DessLBI w./w.o. Momentum and Weight-decay (Mom-Wd)
- We have a theorem that guarantees the **global convergence** of DessiLBI: **from any initialization**, DessiLBI sequence converges to a critical point.



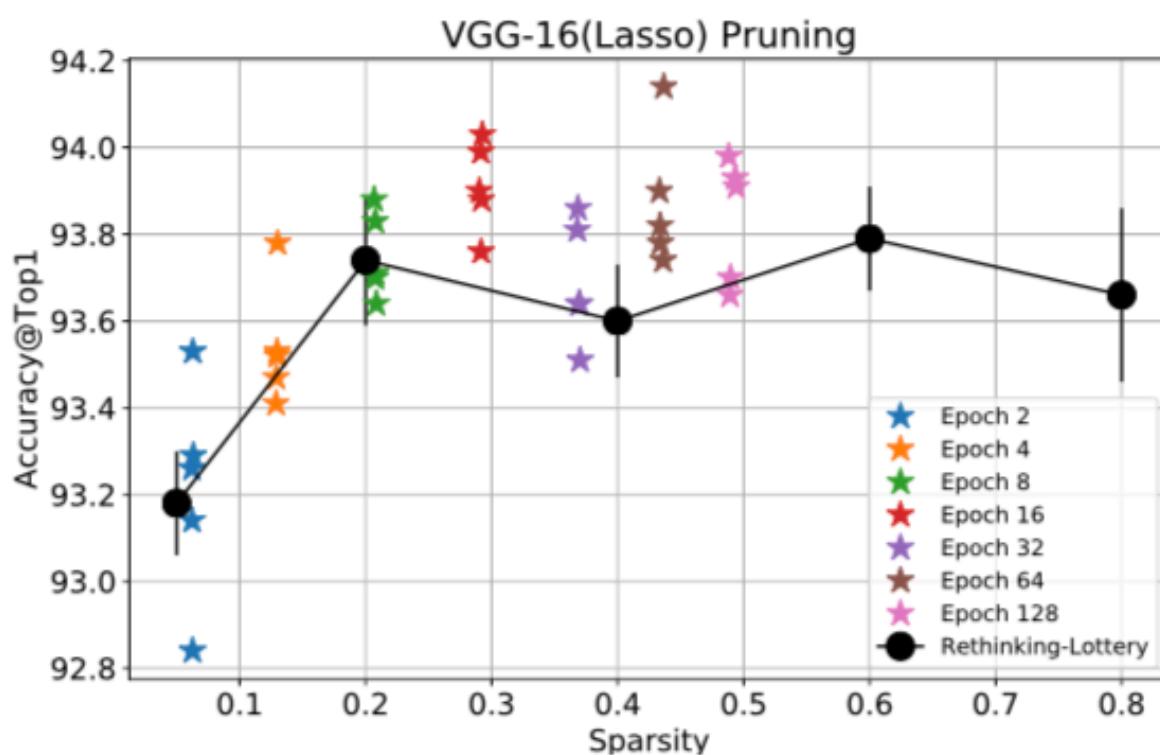
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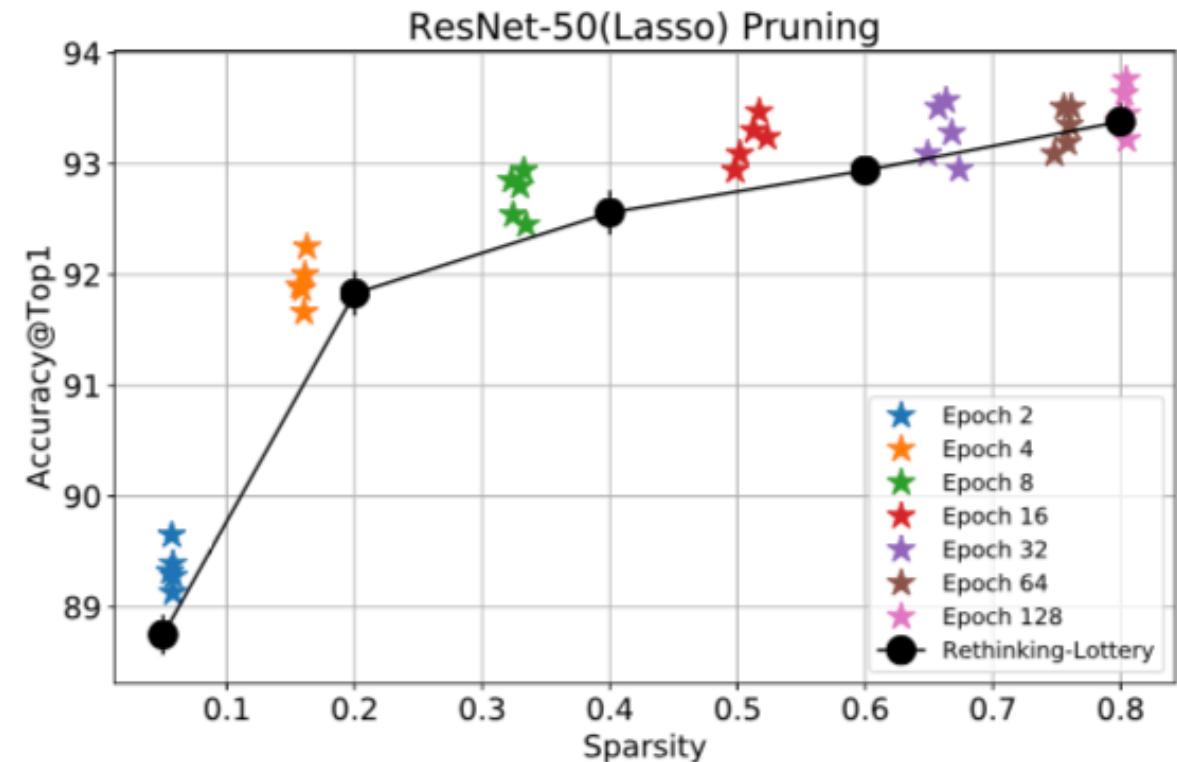
# Visualization of Sparse Filters



# Winning Tickets by DensiLBI

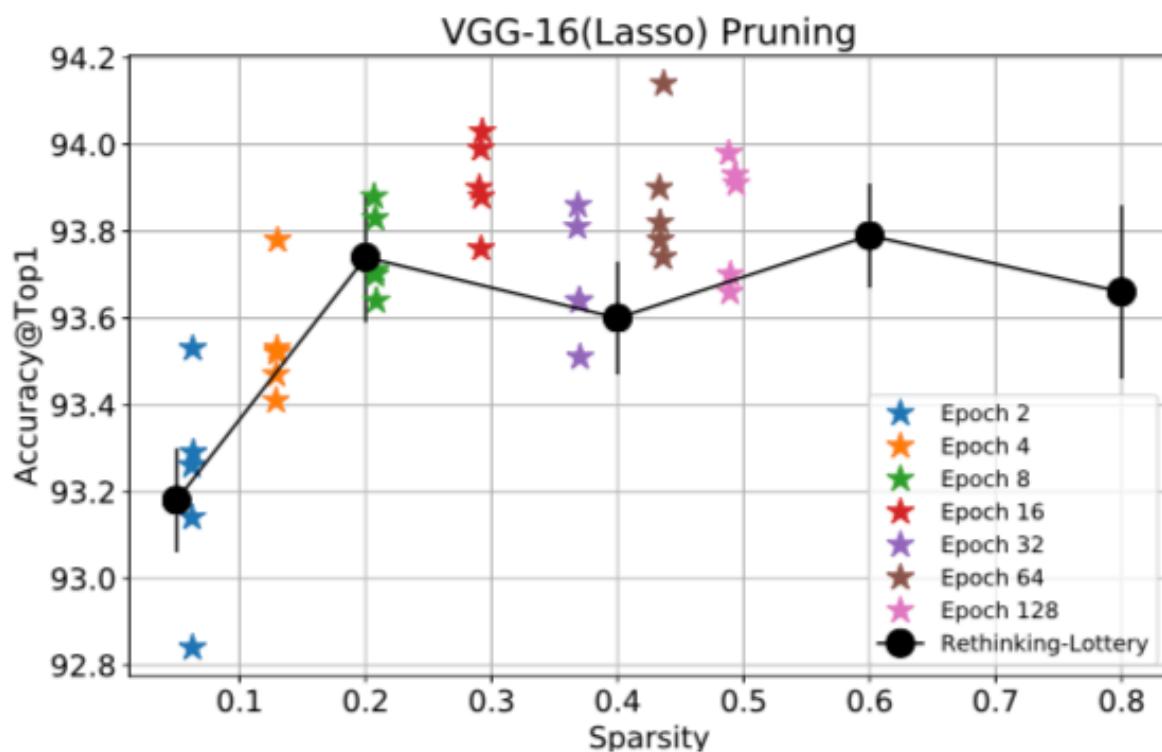


(c) VGG-16 (Lasso)



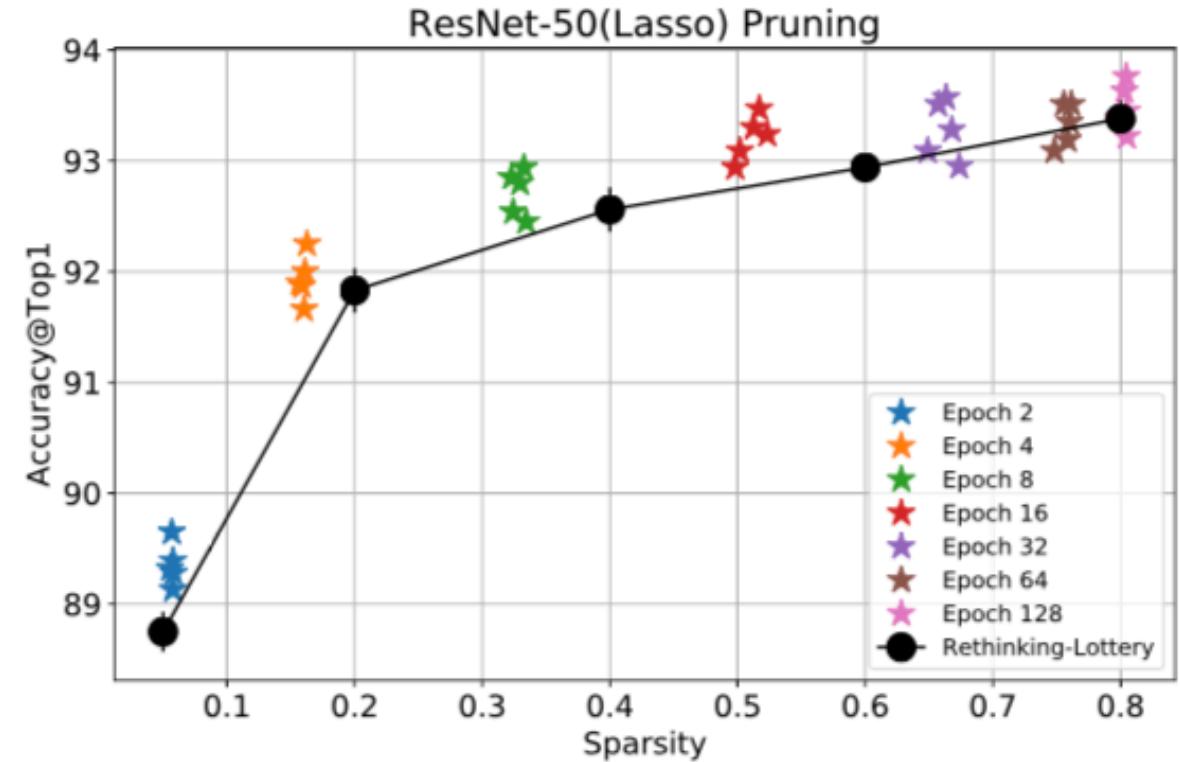
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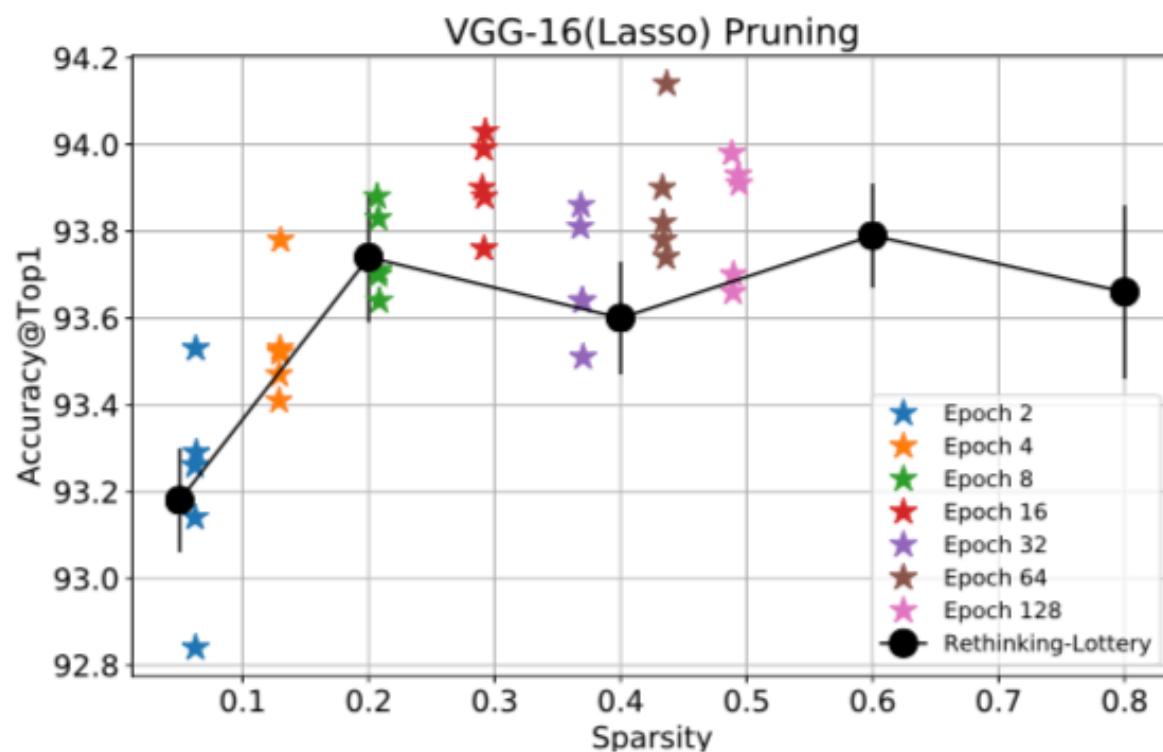
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Train DessimBI with Early Stopping

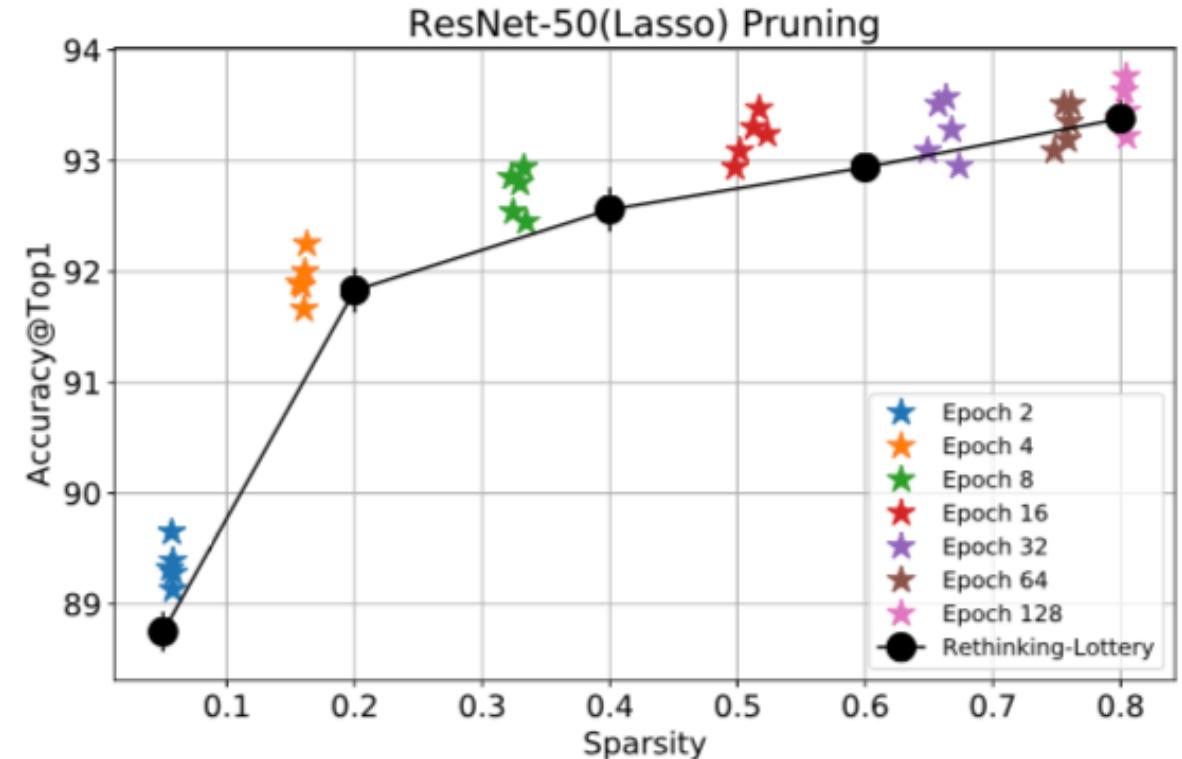


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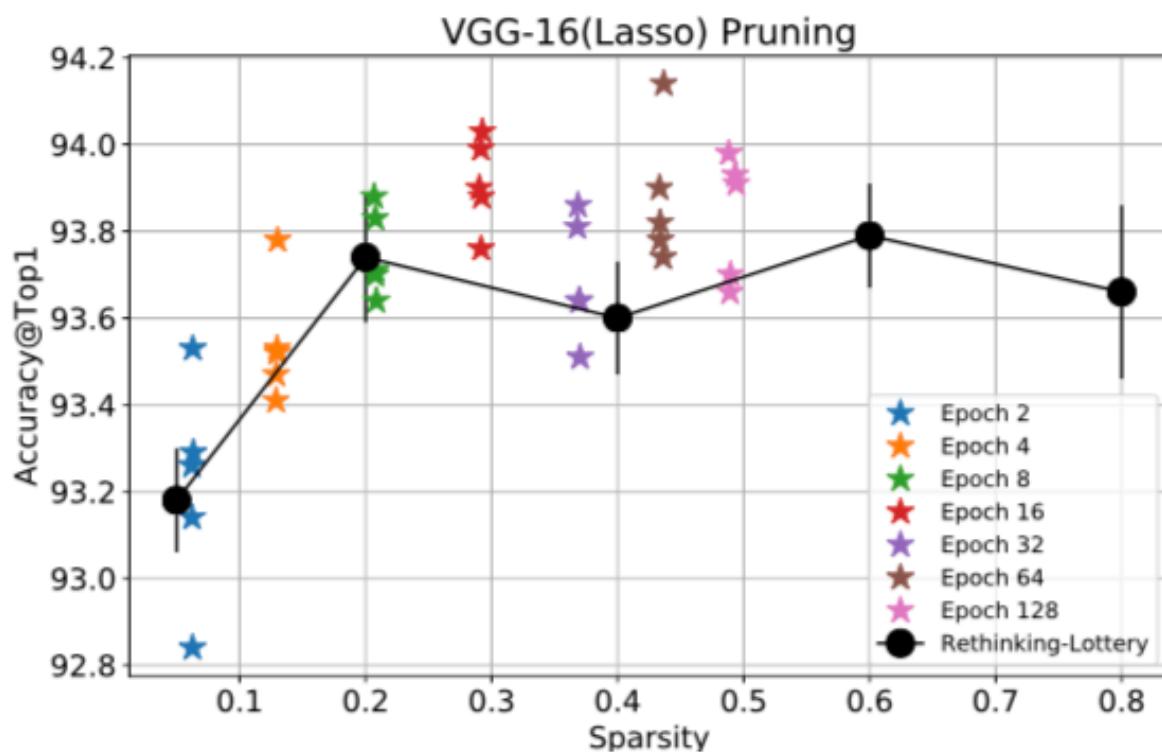


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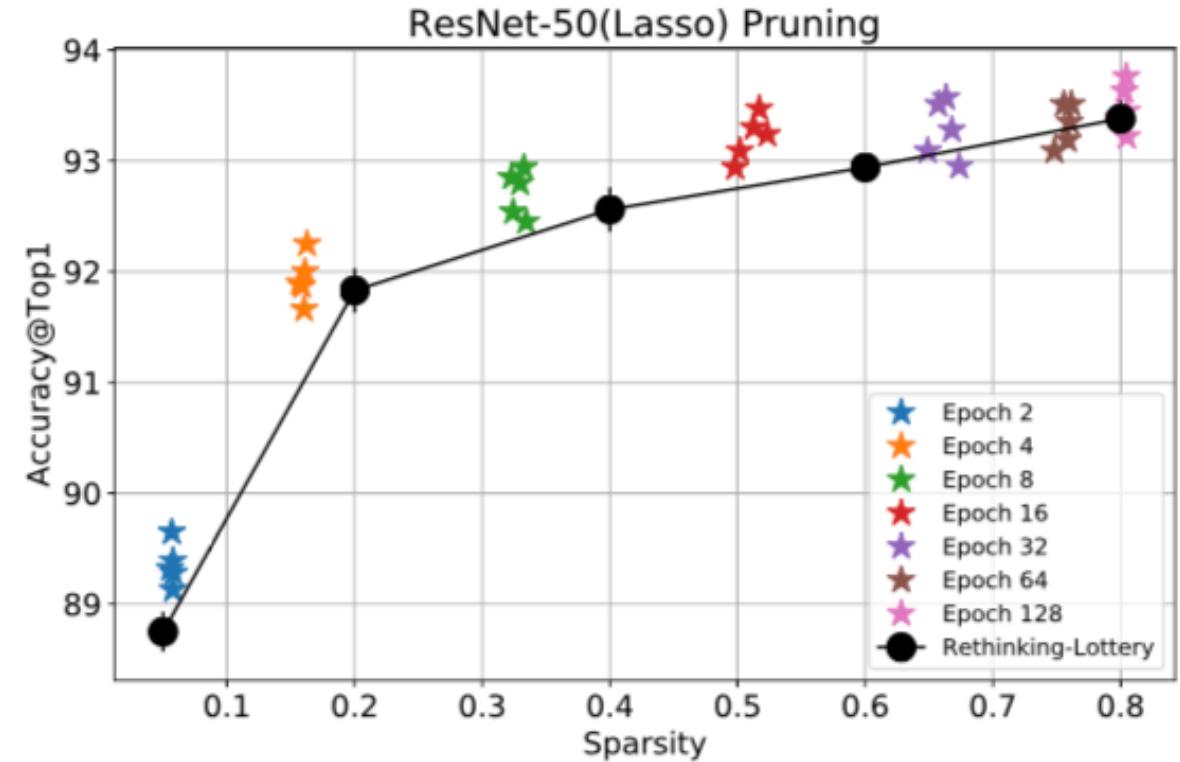


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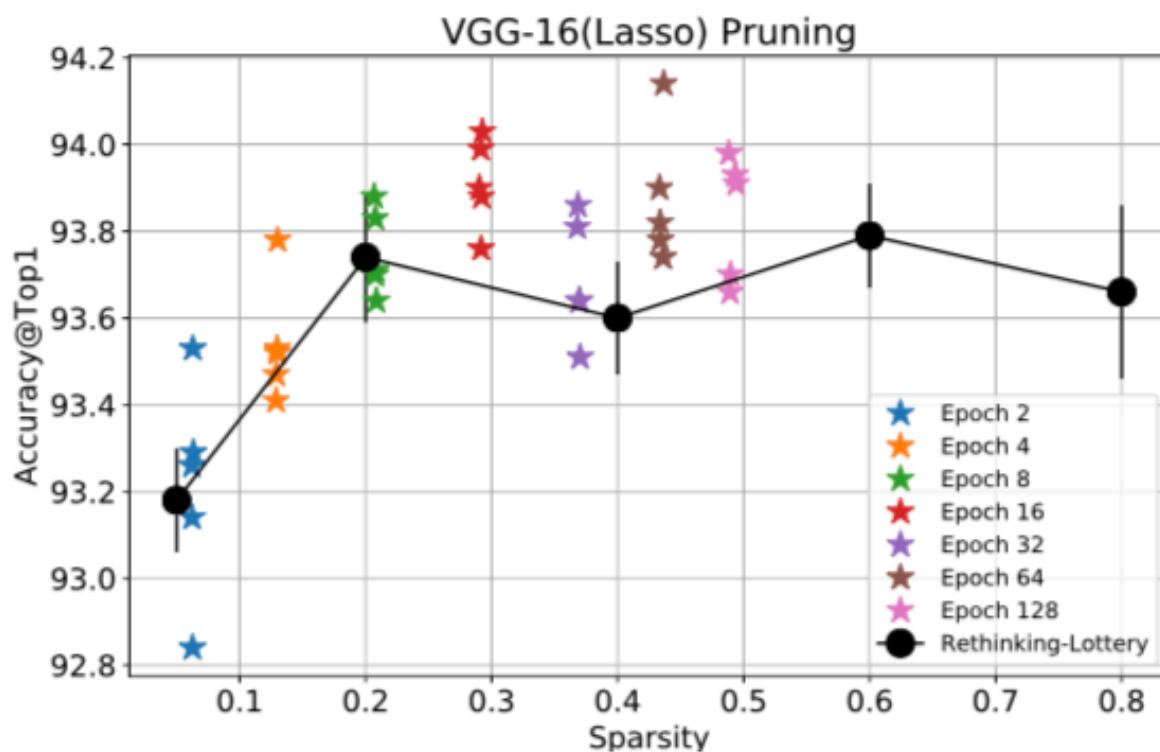
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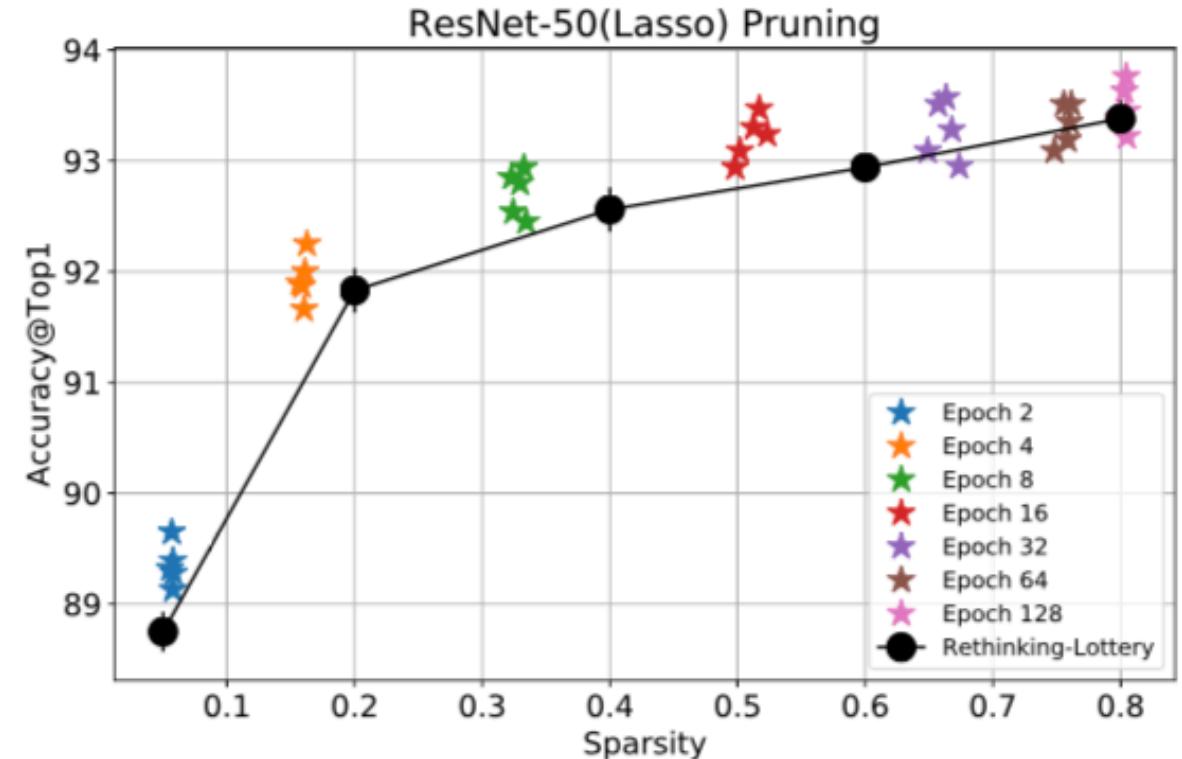
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Extract  $\Gamma$  as subnetwork structure

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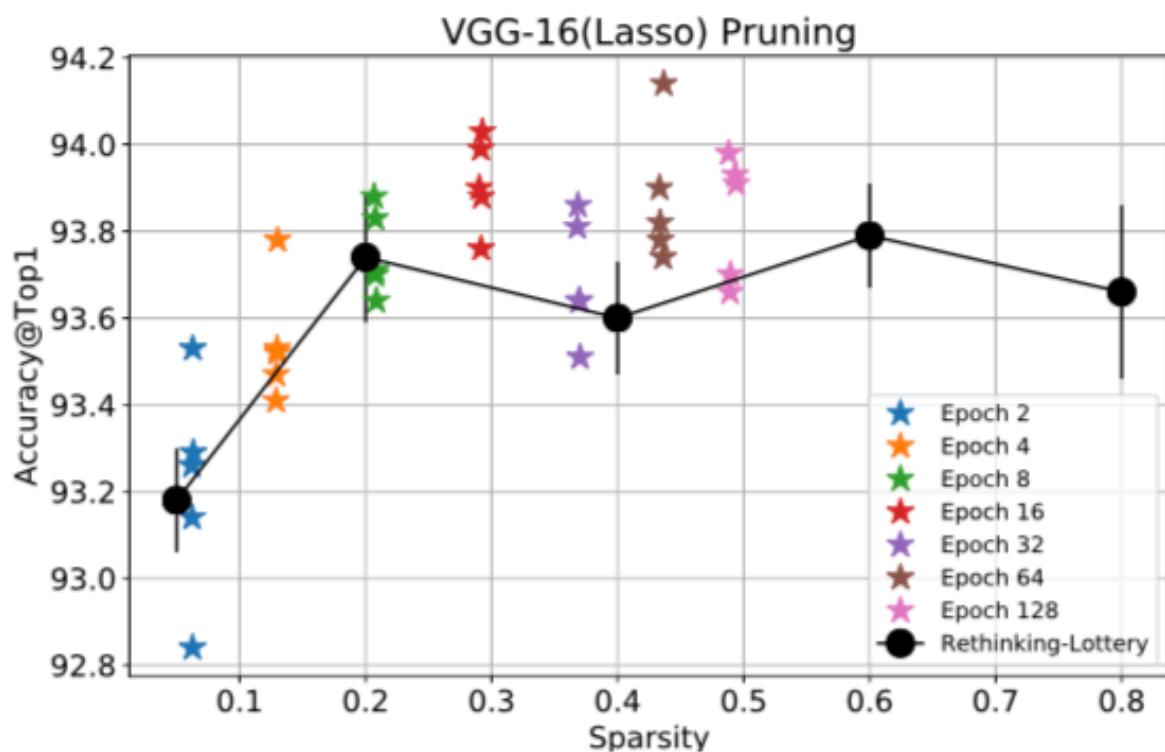


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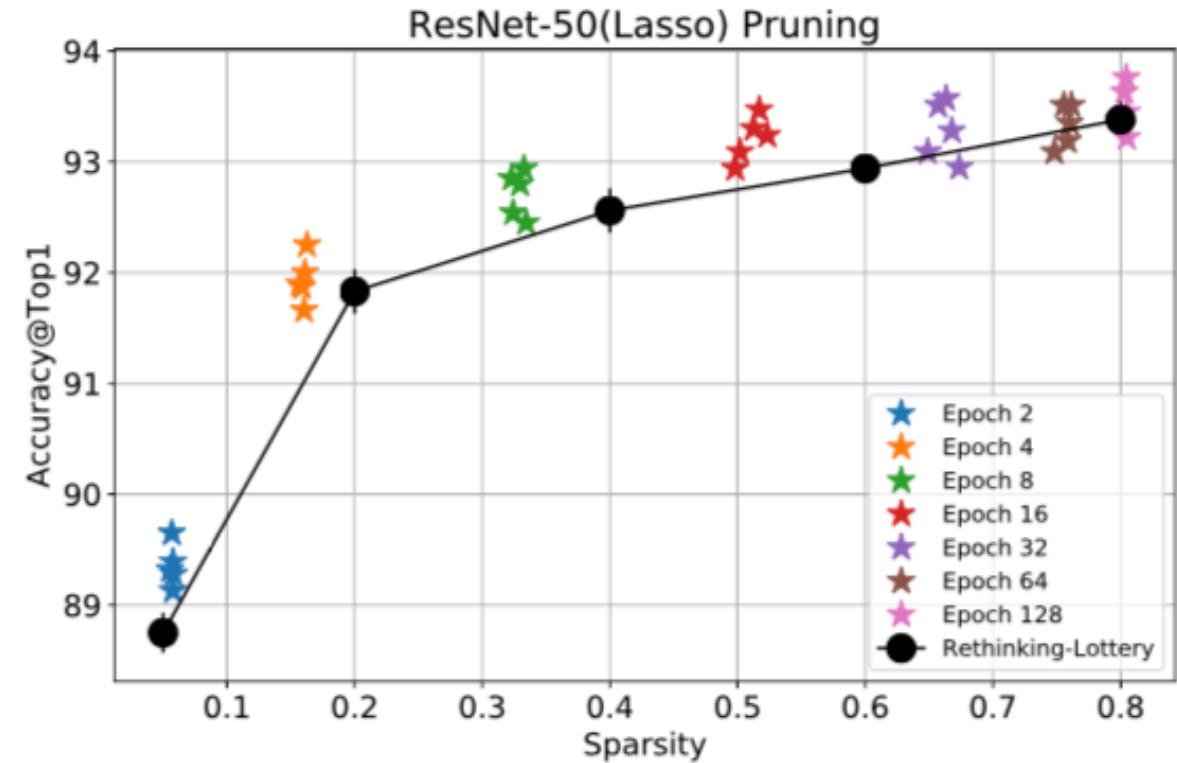


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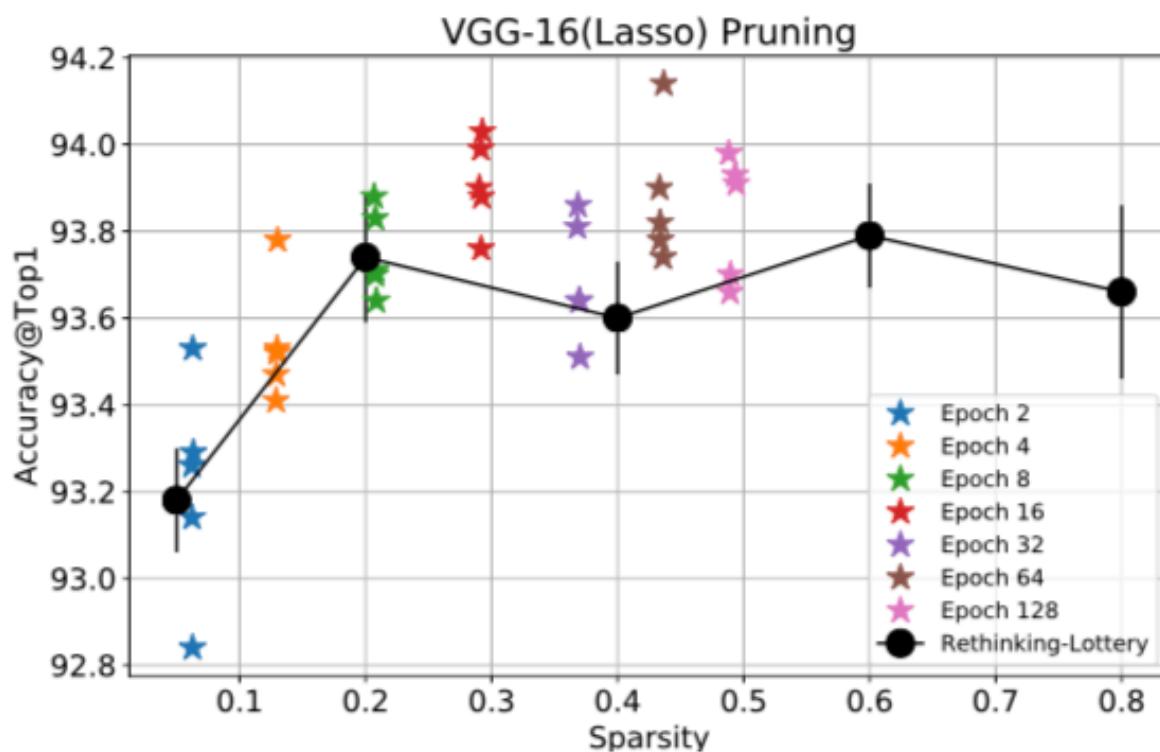
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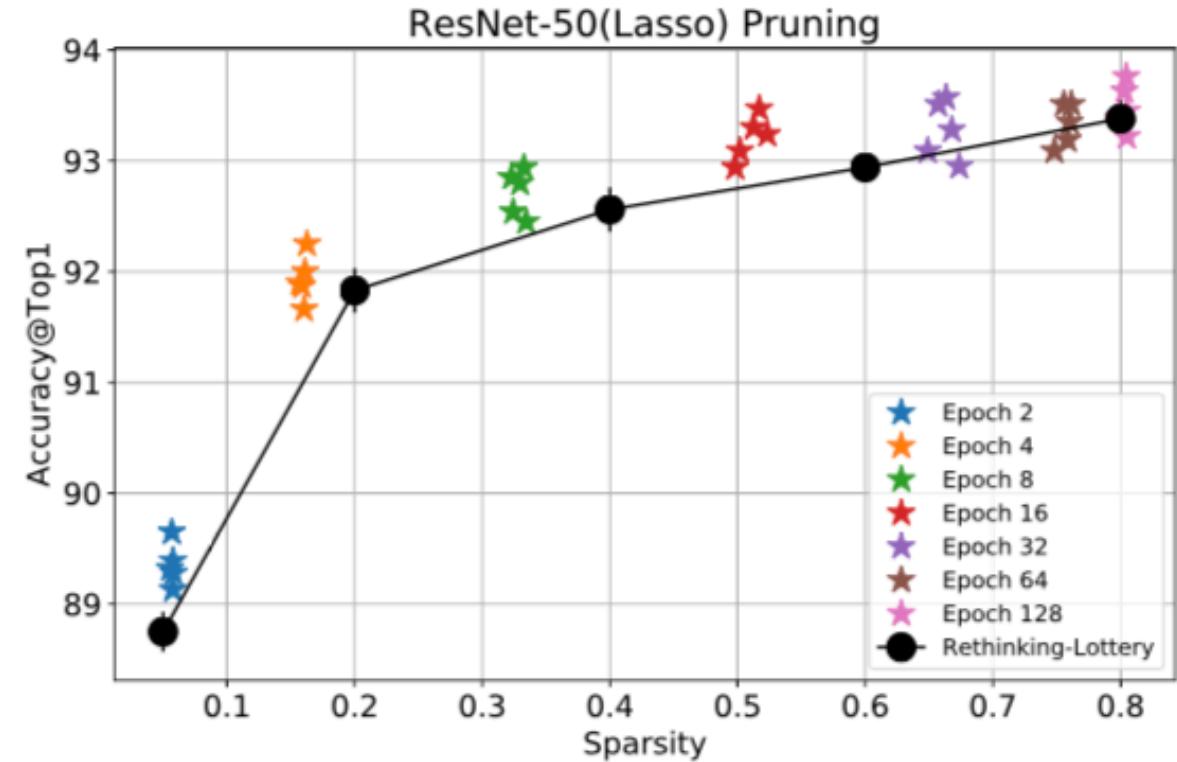
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Retrain subnetwork

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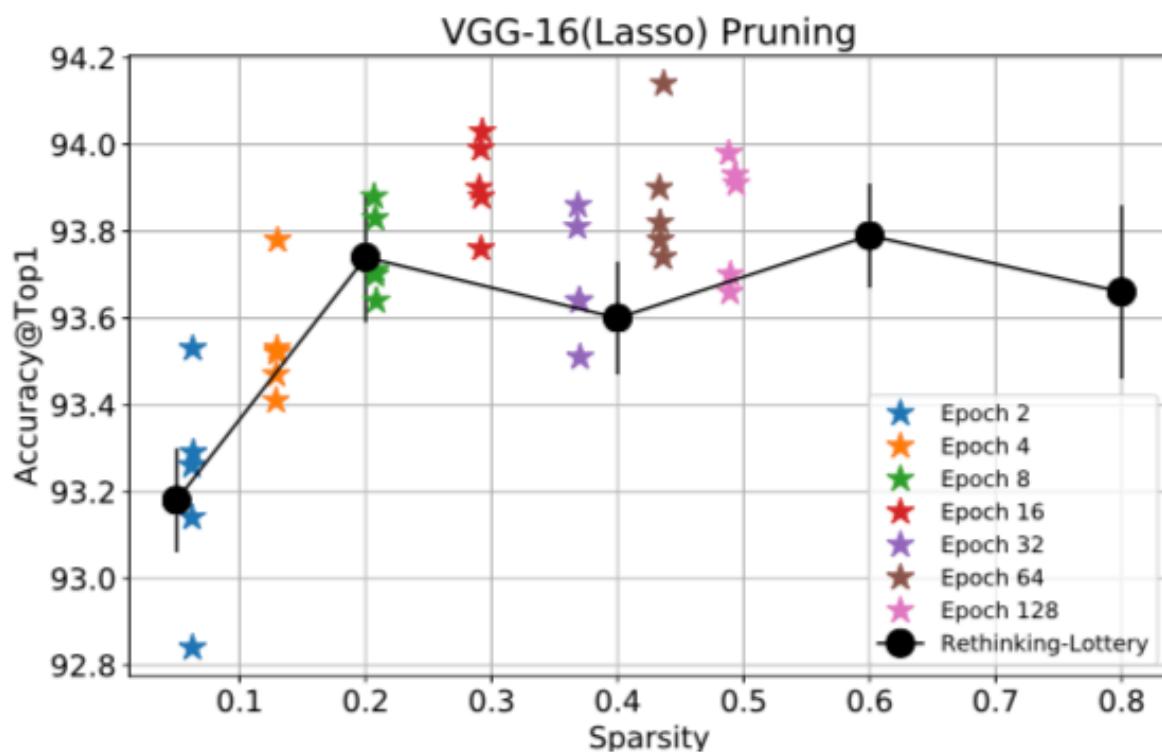


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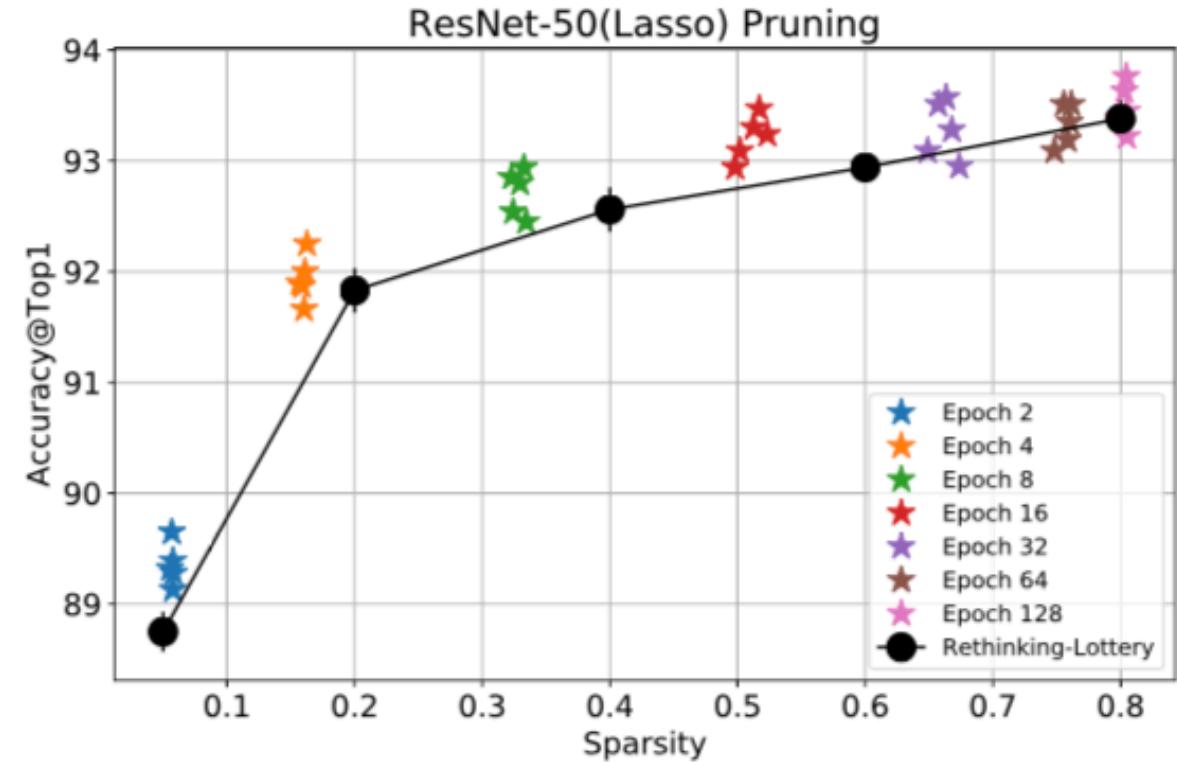


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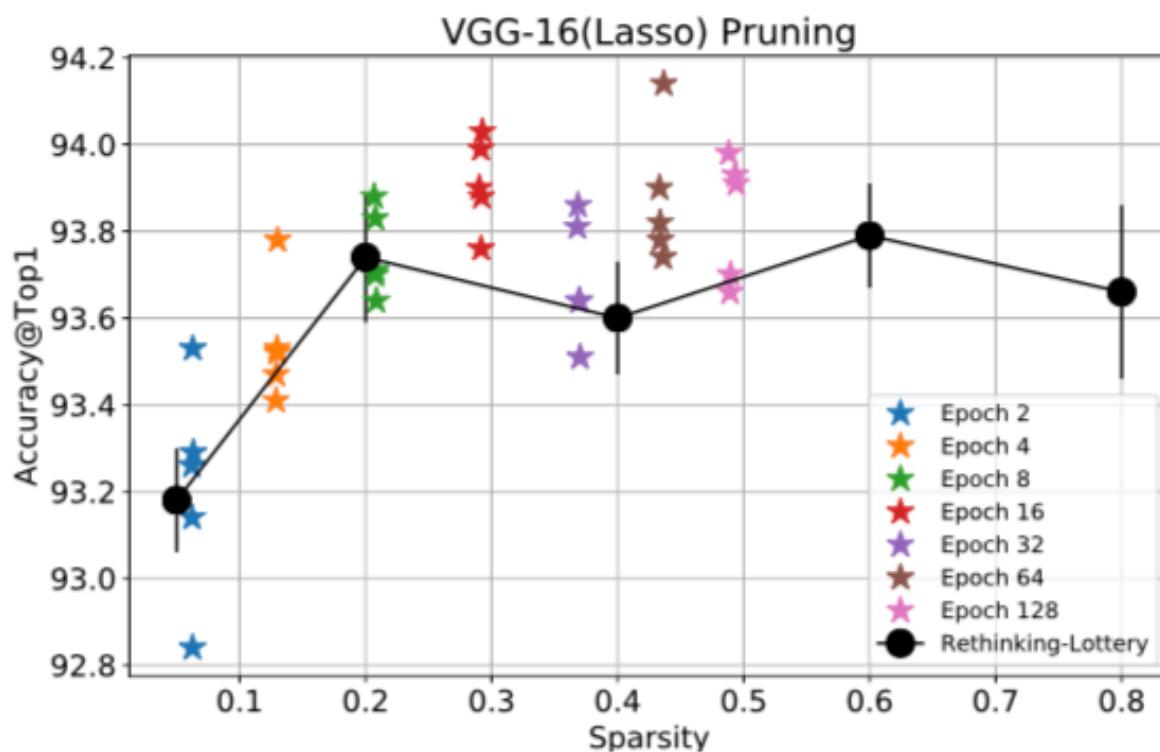
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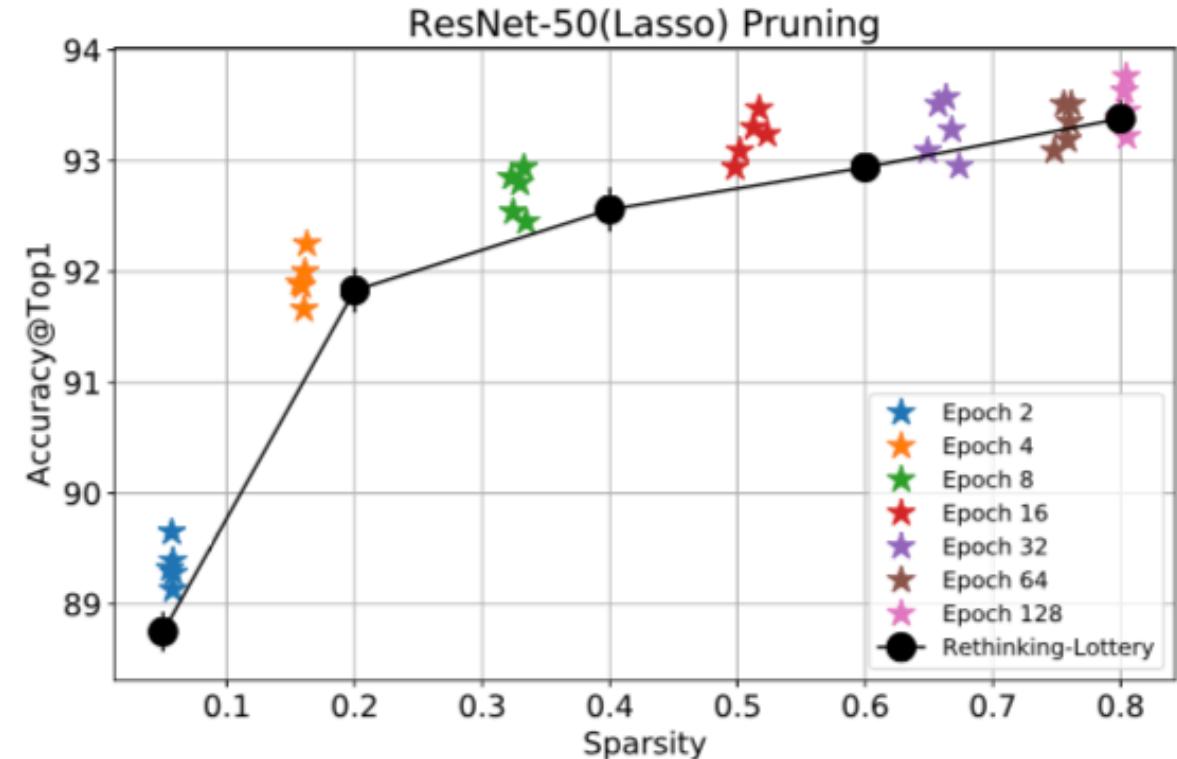
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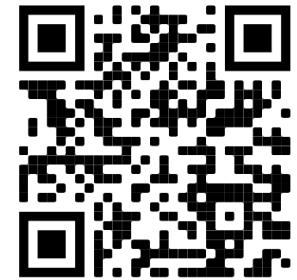
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# Toolbox: Very Easy to Use

<https://github.com/DessiLBI2020/DessiLBI>



It is install-free, put slbi\_opt.py and slbi\_toolbox.py into the project folder and import them.

Quick Example to Start with,

```
python ./example/train/train_lenet.py
```

To initialize the toolbox, the following codes are needed.

```
from slbi_toolbox import SLBI_ToolBox
import torch
optimizer = SLBI_ToolBox(model.parameters(), lr=args.lr, kappa=args.kappa, mu=args.mu, weight_decay=0)
optimizer.assign_name(name_list)
optimizer.initialize_slbi(layer_list)
```

For training a neural network, the process is similar to one that uses built-in optimizer

```
optimizer.zero_grad()
loss.backward()
optimizer.step()
```

# Training Neural Network

The training process is **the same as original Pytorch Optimizer**

## ImageNet Training Example

This part of code is included in example/imagenet. To do this demo, run

```
python train_imagenet_slbi.py
```

```
for ep in range(args.epoch):
    model.train()
    descent_lr(args.lr, ep, optimizer, args.interval)
    loss_val = 0
    correct = num = 0
    for iter, pack in enumerate(train_loader):
        data, target = pack[0].to(device), pack[1].to(device)
        logits = model(data)
        loss = F.nll_loss(logits, target)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
        _, pred = logits.max(1)
        loss_val += loss.item()
        correct += pred.eq(target).sum().item()
        num += data.shape[0]

    if 'z_buffer' in param_state:
        new_grad = d_p * lr_kappa + (p.data - param_state['gamma_buffer']) * lr_kappa / mu
        last_p = copy.deepcopy(p.data)
        p.data.add_(-new_grad)
        param_state['z_buffer'].add_(-lr_gamma, param_state['gamma_buffer'] - last_p)
        if len(p.data.size()) == 2:
            param_state['gamma_buffer'] = kappa * self.shrink(param_state['z_buffer'], 1)
        elif len(p.data.size()) == 4:
            param_state['gamma_buffer'] = kappa * self.shrink_group(param_state['z_buffer'])
        else:
            pass
    else:
        p.data.add_(-lr_kappa, d_p)#for bias update as vanilla sgd
```

We record the path via **two buffer** during training

<https://github.com/DessiLBI2020/DessiLBI>

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It is a network optimizer with

- finding important structural sparsity in model learning,
- Shorter training time,
- Exploring regularization path,
- Nice theoretical properties,
- Good interpretation of important parameters

```
for ep in range(args.epoch):
    model.train()
    descent_lr(args.lr, ep, optimizer, args.interval)
    loss_val = 0
    correct = num = 0
    for iter, pack in enumerate(train_loader):
        data, target = pack[0].to(device), pack[1].to(device)
        logits = model(data)
        loss = F.nll_loss(logits, target)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
        _, pred = logits.max(1)
        loss_val += loss.item()
        correct += pred.eq(target).sum().item()
        num += data.shape[0]

if 'z_buffer' in param_state:
    new_grad = d_p * lr_kappa + (p.data - param_state['gamma_buffer']) * lr_kappa / mu
    last_p = copy.deepcopy(p.data)
    p.data.add_(-new_grad)
    param_state['z_buffer'].add_(-lr_gamma, param_state['gamma_buffer'] - last_p)
    if len(p.data.size()) == 2:
        param_state['gamma_buffer'] = kappa * self.shrink(param_state['z_buffer'], 1)
    elif len(p.data.size()) == 4:
        param_state['gamma_buffer'] = kappa * self.shrink_group(param_state['z_buffer'])
    else:
        pass
else:
    p.data.add_(-lr_kappa, d_p)#for bias update as vanilla sgd
```

We record the path via **two buffer during training**

<https://github.com/DessiLBI2020/DessiLBI>

# Pruning Neural Network

We can **prune the network** according to the information of augmented variable  $\Gamma$

For pruning a neural network, the code is as follows.

```
optimizer.update_prune_order(epoch)
optimizer.prune_layer_by_order_by_list(percent, layer_name)
```

## Filter Pruning

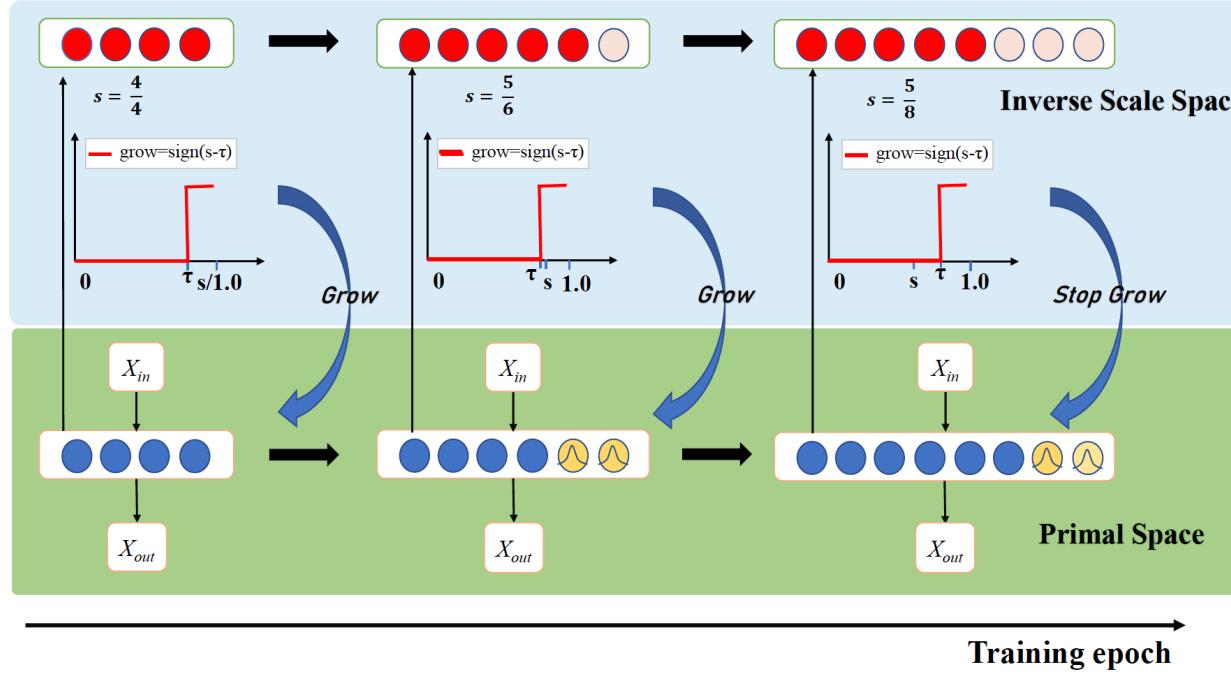
```
ts_reshape = torch.reshape(param_state['w_star'], (param_state['w_star'].shape[0], -1))
ts_norm = torch.norm(ts_reshape, 2, 1)
num_selected_filters = torch.sum(ts_norm != 0).item()
param_state['original_params'] = copy.deepcopy(p.data)
p.data = param_state['w_star']
```

## Weight Pruning

```
num_selected_units = (param_state['w_star'] > 0.0).sum().item()
param_state['original_params'] = copy.deepcopy(p.data)
p.data = param_state['w_star']
```

# Growing Neural Network

We add new filters according to the support set of augmented  $\Gamma$ , to **enlarge the model capacity**.



Dataset	Method	Params.	Acc(%)
CIFAR10	AutoGrow [124]	4.06 M	94.27
	Ours	2.69 M	94.82
CIFAR100	AutoGrow [124]	5.13 M	74.72
	Ours	3.37 M	76.86

```
def grow_filter(model, new_arc, NET, args, logger, topk_dict=None):
    # new_arc: [basic_block, [block_num list], [filter_num list]]
    # layer_name: the layer to be grown
    old_params = {}
    for n, p in model.named_parameters():
        if 'module' in n:
            n = '.'.join(n.split('.')[1:])
        old_params[n] = p.data

    new_net = NET(new_arc[0], new_arc[1], new_arc[2], num_classes=new_arc[3], resolution=new_arc[4])

    for n, p in new_net.named_parameters():
        if n in old_params.keys():
            if p.data.size() != old_params[n].size(): #this layer grown
                old_size = old_params[n].size()
                if len(old_size) == 4:
                    try:
                        filter_idx = topk_dict[n]
                        n_out, n_in, k1, k2 = old_size
                        for idx in filter_idx:
                            p.data[idx, :n_in, :k1, :k2] = old_params[idx, :, :, :]
                    except: #shortcut weight
                        n_out, n_in, k1, k2 = old_size
                        p.data[:n_out, :n_in, :k1, :k2] = old_params[n]
                elif len(old_size) == 2:
                    num_out, num_in = old_size
                    p.data[:num_out, :num_in] = old_params[n]
                elif len(old_size) == 1:
                    a, = old_size
                    p.data[:a] = old_params[n]
            else: #this layer did not grow
                p.data = old_params[n]
            #logger.info('{} has succeed parameters from last model!'.format(n))
        else:
            pass

    return new_net
```

# Different from ADMM

$$W_{k+1} = \operatorname{argmax}_W \mathcal{L}(W) + \frac{\rho}{2} \|W - \Gamma_k + U_k\|^2$$

$$\Gamma_{k+1} = \operatorname{argmax}_W \Omega(\Gamma) + \frac{\rho}{2} \|W_{k+1} - \Gamma + U_k\|^2$$

$$U_{k+1} = U_k + W_{k+1} - \Gamma_{k+1}$$

- Different from ours
  - ADMM targets on **convergence result**, with objective function:  $\mathcal{L}(W) + \lambda \cdot \Omega(W)$ 
    - DensiLBI is discretization of Differential Inclusion
  - DensiLBI cares the **regularized solution path**; it returns a sequence of models from simple to complex, corresponding to different regularization parameters;



THANKS

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# Learning for Sparse Optimization

# Prof. Yuan Yao: Inverse Scale Space Method and Statistical Properties

## Outlines

1. Inverse scale space method, differential inclusions, linearized bregman iterations and mirror descent
2. Structural sparsity and splitting method
3. Statistical regularization path and model selection consistency
4. Huber's robust statistics and outlier detection
5. False discovery rate control and (split) Knockoffs

# Prof. Wotao Yin: Learning to Optimize (L2O)

## Outlines

1. L2O idea and typical work flow.
2. Unrolling a classic algorithm and learning its parameters
3. Generalization and convergence safeguard
4. Learning regularization and plug-and-play
5. Fixed-point network and Jacobian-free back propagation