

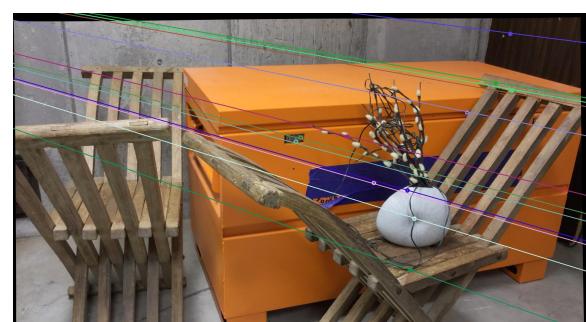
Calibration:

- 1) First, SIFT is used to detect matching points between left & right images.
- 2) Only a few (very similar) matches are selected so that the computational intensity of the pipeline remains tractable.
- 3) Then, the matching points are normalized to make the epipolar lines pass closely through the point correspondences.
- 4) Then the Fundamental matrix is calculated using the normalized 8-point algorithm.
 - a) It is the same as the 8-point algorithm except that normalized point correspondences are used.
 - b) In the 8-point algorithm, 8 point correspondences are taken to find the fundamental matrix which satisfies the equation $x_2^T F x_1 = 0$.
 - c) Representing the left point by $x_1 = (u, v)$ & $x_2 = (u', v')$, this leads to:

$$\begin{pmatrix} u'_1 u_1 & u'_1 v_1 & u'_1 & v'_1 u_1 & v'_1 v_1 & v'_1 & u_1 & v_1 & 1 \\ \vdots & \vdots \\ u'_n u_n & u'_n v_n & u'_n & v'_n u_n & v'_n v_n & v'_n & u_n & v_n & 1 \end{pmatrix} \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} = \mathbf{0}$$

$$\Leftrightarrow \mathbf{A}\mathbf{f} = \mathbf{0}$$

- d) Minimum required pairs of corresponding points to generate the Fundamental matrix is 8.
- e) A is solved by SVD to decompose it into U, S, V^T, i.e., $A = U^* S^* V^T$ & the elements of the fundamental matrix are given by the last column of V.
- f) Since, we have 8 equations for 9 unknowns, the smallest singular value of F should be 0. But, due to the noise in the correspondences, it'll be non-zero (rank of F != 2).
- g) So, to account for the noise, rank of F is forced to 2 by setting its smallest singular value to 0.
- h) So, F is decomposed by SVD into u, s, v^T & the smallest singular value is set to zero & F is recomputed as $F = u^* s^* v^T$.
- 5) A modified version of RANSAC (see Note I) is applied to increase the robustness of estimation of F.
- 6) The epipolar lines drawn using the obtained F are shown below:



- 7) Then, essential matrix is estimated using $E = K^T F K$ where K is the camera matrix.
- Again, the essential matrix is forced to have rank=2

b) By SVD, let $E = U^* D^* V^T$ & $W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Then the 4 possible camera configurations are given by:

- $C_1 = U(:, 3)$ and $R_1 = UWV^T$
- $C_2 = -U(:, 3)$ and $R_2 = UWV^T$
- $C_3 = U(:, 3)$ and $R_3 = UW^T V^T$
- $C_4 = -U(:, 3)$ and $R_4 = UW^T V^T$

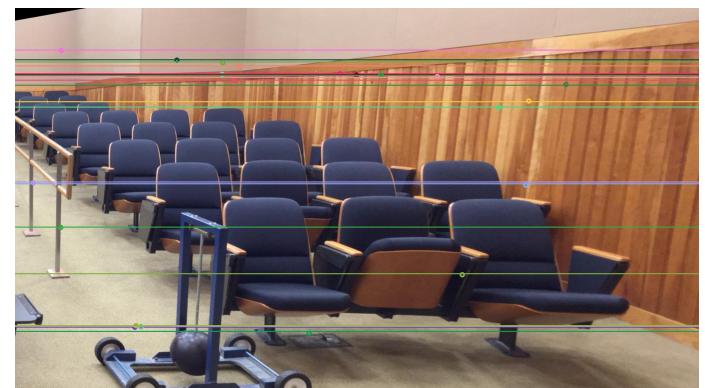
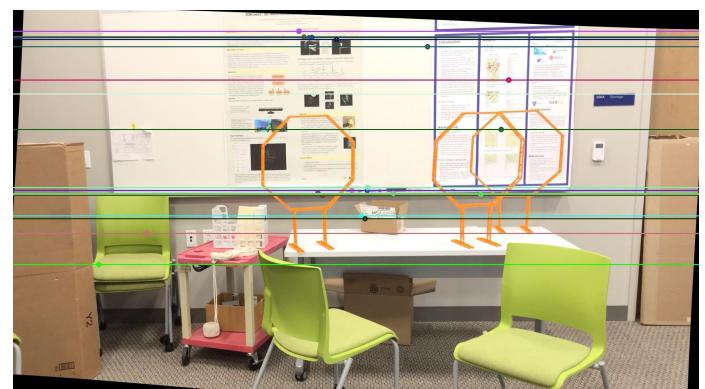
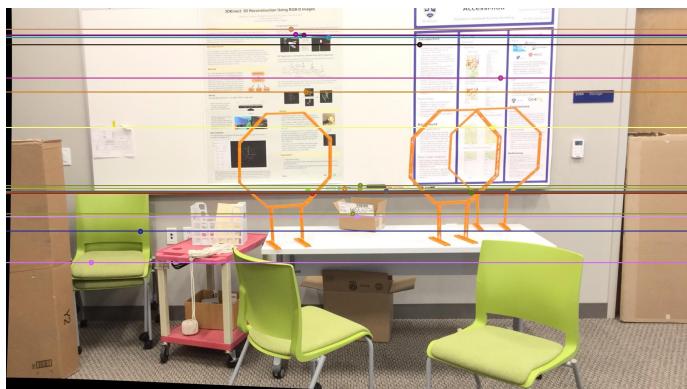
- Then, one of the above 4 possibilities is finally selected as the translation & Rotation matrices of the camera w.r.t scene.
 - This is done by checking the cheirality condition - reconstructed points must be in front of the camera.
 - But, due to the noise, not all triangulated points satisfy this condition.
 - So, that combination of C & R are selected which maximizes the number of points satisfying the cheirality condition.
 - This way C & R are obtained.

Note:

- Using the standard ransac, subsets containing 8 pairs of point-correspondences are considered to generate the Fundamental matrix. But multiple such subsets are having the same number of inliers - leading to different Fundamental Matrix on different runs. So, I improvised RANSAC such that only 1 solution (*the best*) is outputted in the following way:
 - Forming the subsets part is kept the same.
 - Instead of calculating the inliers & choosing the one with the most inliers, that subset is selected which minimizes the sum of $x_2^T F x_1$ for all the pairs of corresponding points.
- The fundamental matrix obtained by using the standard 8-point algorithm is generating epipolar lines which look almost parallel to each other (won't coincide to form epipoles or coincide very far from the image space). Used the normalized 8-point algorithm as described [here](#) to deal with this problem.

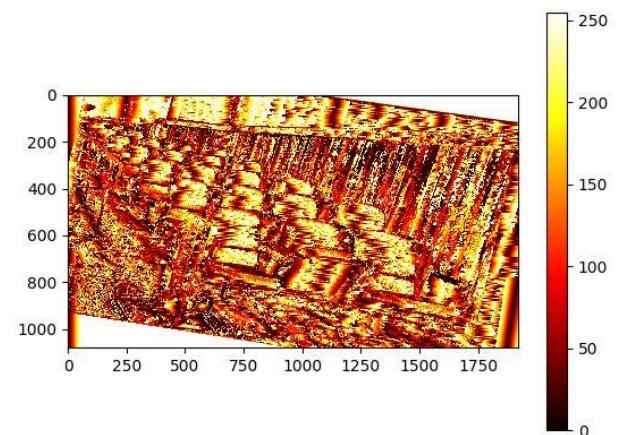
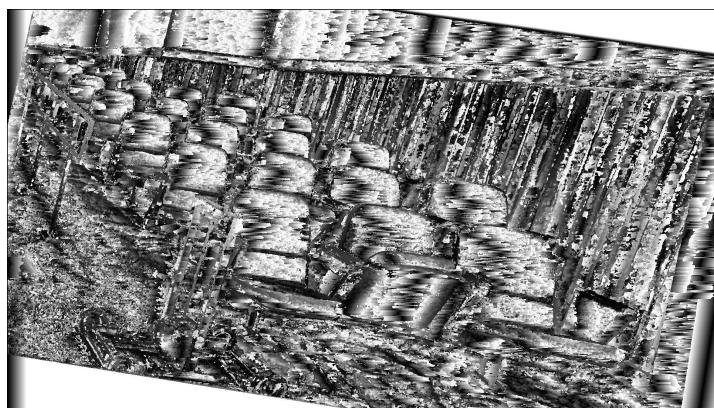
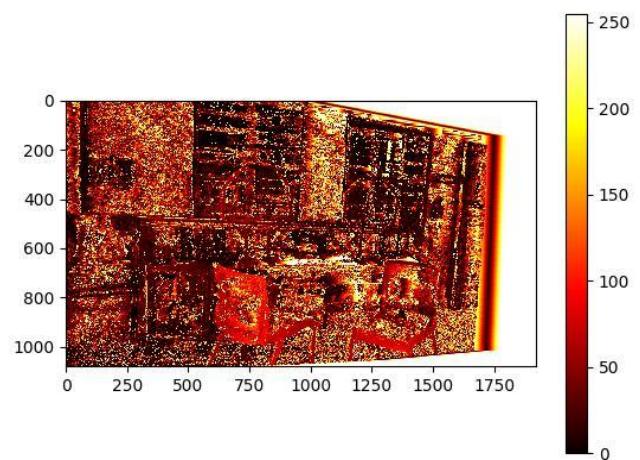
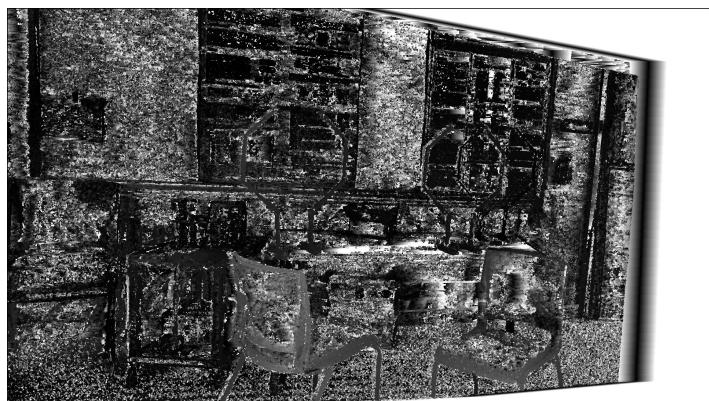
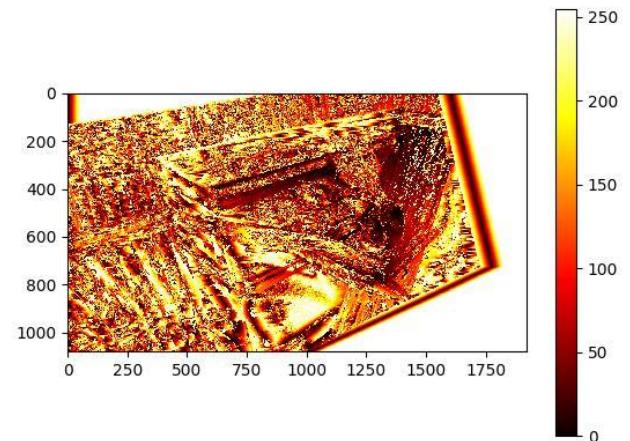
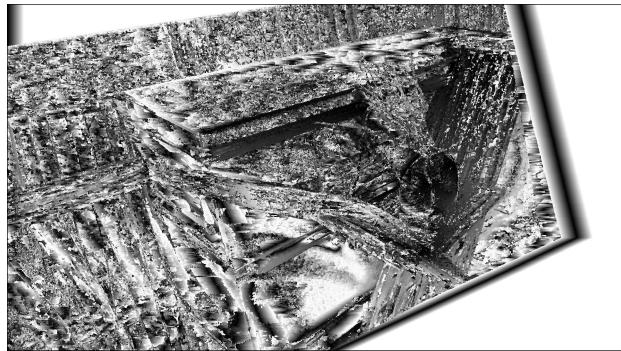
Rectification:

- 1) Then, rectification is applied to reproject image planes onto a common plane parallel to the line between camera centers.
- 2) OpenCV's stereoRectifyUncalibrated is used to find the rectification transform.
- 3) This returns the Homography matrices for the left & right images.
- 4) The rectified Fundamental matrix is computed & the below are the epipolar lines along with feature points:



Correspondence:

- 1) Then, disparity maps is calculated using the SSD (sum of squared differences) similarity.
- 2) For a given block on left image, similarity is checked only with a neighborhood of blocks, to reduce the computation intensity.
- 3) Below are the disparity maps generated.



Depth Image:

- 1) Then, depth map is computed using disparity using: $(\text{baseline} * \text{focal_len}) / (\text{disparity})$
- 2) Those depths which fall above the median depth are set to the median depth.

