Applied Soft Computing Journal 93 (2020) 106367

Contents lists available at ScienceDirect

Applied Soft Computing Journal

journal homepage: www.elsevier.com/locate/asoc

A memory-based Grey Wolf Optimizer for global optimization tasks

Shubham Gupta *, Kusum Deep

Department of Mathematics, Indian Institute of Technology Roorkee, Uttarakhand 247667, India

ARTICLE INFO

Article history:
Received 14 November 2019
Received in revised form 29 March 2020
Accepted 28 April 2020
Available online 5 May 2020

Keywords:
Optimization
Swarm intelligence
Grey Wolf Optimizer
Exploration and exploitation

ABSTRACT

Grey Wolf Optimizer (GWO) is a new nature-inspired metaheuristic algorithm based on the leadership and social behaviour of grey wolves in nature. It has shown potential to solve several real-life applications, but still for some complex optimization tasks, it may face the problem of getting trapped at local optima and premature convergence. Therefore, in this study, to prevent from these drawbacks and to get a more stable sense of balance between exploitation and exploration, a new modified GWO called memory-based Grey Wolf Optimizer (mGWO) is proposed. In the mGWO, the search mechanism of the wolves is modified based on the personal best history of each individual wolves, crossover and greedy selection. These strategies help to enhance the global exploration, local exploitation and an appropriate balance between them during the search procedure. To investigate the effectiveness of the proposed mGWO, it has been tested on standard and complex benchmarks given in IEEE CEC 2014 and IEEE CEC 2017. Furthermore, some real engineering design problems and multilevel thresholding problem are also solved using the mGWO. The results analysis and its comparison with other algorithms demonstrate the better search-efficiency, solution accuracy and convergence rate of the proposed mGWO in performing the global optimization tasks.

© 2020 Elsevier B.V. All rights reserved.

1. Introduction

Mathematical optimization refers to the finding of an element within an available domain for a given function so that the function value is the maximum or minimum. Optimization problems exist in all fields of study and thus the development of optimization methods are essential. Most of the conventional optimization methods are deterministic and based on the derivative information of the functions involved in the problem. But, in real life, it is not possible always to calculate the derivative of the functions. Nature-inspired algorithms are becoming increasingly popular due to their characteristics of derivative-free mechanism and excellent optimization ability. Flexibility, easy implementation and the ability to escape from local optima are some other advantages with these algorithms.

In the category of the nature-inspired algorithm, swarm intelligence based algorithms are quite popular. The main advantage with these algorithms is their collaboration strength between search agents. This collaboration helps in exchanging the information available to search agents and participates in exploiting and exploring the promising search areas of the solution space. The exploration is the ability to discover new search regions and the exploitation is the process of evaluating the potential of available promising areas of the solution space. The exploration and exploitation are considered two foundation factors for any nature-inspired algorithm. These factors are conflicting [1] and therefore, maintaining an appropriate balance between them is a challenging task for the proper working of an algorithm. Thus, the main advantages with swarm intelligence based algorithms are

1. Swarm intelligence based algorithms preserve the potential of solution space during the iterations, which helps in discovering new promising regions.
2. Swarm intelligence based algorithms have fewer parameters to adjust.
3. Swarm intelligence based algorithms are easy to implement.
4. The collaborative strength among individual search agents helps to avoid the situation of getting trapped at local optima.

A large number of swarm intelligence based algorithms are developed in the literature and successfully applied to solve several real-life application problems. For example: PSO [2], ABC [3], ACO [4], CS [5], BA [6], FA [7]. Some of the other algorithms, which are recently added to this field are GWO [8], MFO [9], WOA [10], SSA [11], SS [12], HHO [13], and many others.

In the present article, the GWO developed by Mirjalili, Mirjalili, & Lewis [8] is focused on study. The reason for working with the GWO is its special and different search mechanism, which is based on the leadership hierarchy of grey wolves. In

* Corresponding author.
E-mail addresses: sgupta@ma.iitr.ac.in, g.shubh93@gmail.com (S. Gupta), kusumfma@iitr.ernet.in (K. Deep).

https://doi.org/10.1016/j.asoc.2020.106367
1568-4946/© 2020 Elsevier B.V. All rights reserved.

Nomenclature

N	Size of the population
D	Dimension of the problem
t	The current iteration number
T	The maximum iteration number
X_t	Position vector of the wolf
$X_{p,t}$	Position vector of the prey
$X_{\alpha,t}$	Position vector of alpha wolf
$X_{\beta,t}$	Position vector of beta wolf
$X_{\delta,t}$	Position vector of delta wolf
$X_{p\,best}$	Personal best position of wolf X
A_t	Coefficient vector at iteration t
D_t	Difference vector at iteration t
C_t	Coefficient vector at iteration t
a_t	Transition parameter at iteration t
r_1, r_2, r_3, k	Uniformly distributed random number from the interval (0,1)
CR	Crossover probability

Abbreviations

PSO	Particle Swarm Optimization
HS	Harmony Search
GSA	Gravitational Search Algorithm
ABC	Artificial Bee Colony
ACO	Ant Colony Optimization
CS	Cuckoo Search
BA	Bat Algorithm
FA	Firefly Algorithm
GWO	Grey Wolf Optimizer
MFO	Moth-flame Optimization
WOA	Whale Optimization Algorithm
SSA	Salp Swarm Algorithm
SS	Squirrel Search
HHO	Harris Hawks Optimization
SCA	Sine Cosine Algorithm
DE	Differential Evolution
CMA-ES	Covariance Matrix Adaptation Evolution Strategy (CMA-ES)
GABC	modified Artificial Bee Colony
modGWO	modified Grey Wolf Optimizer
EEGWO	Exploration-enhanced Grey Wolf Optimizer
i-GWO	Inspired Grey Wolf Optimizer
IGWO	Improved Grey Wolf Optimizer

the GWO, three different wolves (alpha, beta and delta) are used to guide the search activities. The search mechanism based on the leadership hierarchy maintains an appropriate exploration in the algorithm and prevents the search agents to stagnate at local optima. Since past few years, GWO has become quite popular. In the literature, GWO has been successfully used to solve several real-life application problems such as economic dispatch problem [14,15], training of q-Gaussian radial basis [16], power dispatch problem [17], scheduling problem [18], parameter estimation in surface waves [19], feature selection [20] and many others.

Despite the successful applicability of the GWO on several real-life problems arising in different fields, several works [21,22] criticized that it suffers from degraded performance when applying to solve highly multimodal problems. In order to deal such problems and to improve the solution accuracy, several attempts are made in the literature. Some of which are point out as follows:

In the GWO, control parameter a plays an important role in transition from exploration process to the exploitation process. Therefore, a non-linear formulation of this parameter has been adopted in [23–25] to establish an appropriate balance between exploitation and exploration, and to simulation the nature of non-linear search procedure of the GWO. Although, these variants have performed well on the test problems considered in the work, but in some cases, especially on multimodal problems they suffers from the problem of high diversity. This high diversity is the main cause of the skipping of true solutions during the search process. In order to speed up the convergence rate in GWO, opposition-based learning is employed in [26–29]. This opposition-based learning is effective on those cases where the optima is far away from the current regions which are explored. The search based on current solution and its opposite approximate although improve the search performance but it needs high computational effort as it requires function evaluations double of the population size. Levy-flight based search [21] and random walk search mechanisms [30,31] are also employed in the GWO to enhance its local exploitation and global exploration skills. These strategies generates long jump occasionally which is the cause of skipping of true solutions. The search equation of the GWO, which considers the mean of the positions obtained by the guidance of leading wolves is also modified to improve the search mechanism [32,33]. These variants focuses on exploiting the elite areas obtained by the wolves only and therefore, insufficient diversity is the main disadvantage with them. This degrades the performance of the algorithm on multimodal problems with massive number of optima. The parameter tuning of GWO parameters is also a key role to determine the best setting of these parameters. In this direction of study, fuzzy logic has been used in GWO to dynamically adopt these parameters [34,35]. In order to combine the advantages of different algorithms in a sense of maintaining comparatively better balance between exploitation and exploration, GWO has been hybridized with SCA [36] and DE [37]. The hybridization with SCA utilizes the exploration ability of the SCA, but since the SCA suffers from high diversity in its search mechanism [38] therefore, there may be a chance that this hybridized algorithm may experience the same problem. The hybridization with DE also adds the exploration ability of the DE in GWO. This variant is not able to increase the exploitation skills and therefore the results on unimodal problems are not good enough. In [39], various selection schemes are introduced in the GWO to analyse its performance. The results of this study provide that a tournament based selection is best among other selection scheme to select the leaders for the pack. But, the results indicated in the paper shows the low exploration and exploitation in the algorithm. The chaotic maps also used in GWO to increase the randomness during the search procedure [40,41]. This randomness perturb the wolves in a chaotic manner and therefore the algorithm may suffers from the skipping true solutions during the search procedure. In [42], different search strategies such as enhancement of global search ability, cooperative hunting strategy and disperse foraging strategies are employed in the GWO. The first strategy helps to increase the local exploitation around the best obtained solution, second strategy helps to increase the diversity by adopting one dimensional and total dimensional update alternatively. The third strategy avoids the local optima during the search procedure and speed up the convergence rate. In this variant extra parameters are used and therefore, their tuning is another tedious task for the users.

Although, the above modifications have tried to improve the search performance and solution accuracy of the GWO, but still in some situations such as for more complex optimization problems, the algorithm suffers from the situation of stagnation at local optima. Another reason for modifying the GWO can be answered with the fact provided by the "No Free Lunch" theorem [43], which states that there is no unique optimization algorithm is available or can be developed which is suitable for all optimization problems. Hence, this theorem open the area of research in improving the search mechanism of the existing algorithms. By motivating these facts, the present study proposed a modified GWO called, Memory-based Grey Wolf Optimizer (mGWO). To the best of our knowledge, this modified variant is the first version of the GWO, where the leading and personal best guidance of wolves are used simultaneously to guide the search procedure.

The mGWO algorithm tries to enhance the collaborative strength and exploration ability of wolves by employing four different strategies. In the first strategy, the information collected by each wolf in the past history of its search procedure is utilized to modify its hunting mechanism. This information is about the quality of the search regions they have visited. In the second strategy an additional search mechanism is proposed to contribute the information available at each wolves. This strategy has been employed to enhance the exploration strength of the algorithm in terms of personal best guidance based search. The third strategy applied the crossover between the positions obtained by modified hunting mechanism and a novel search equation based on personal best guidance. This crossover helps to preserve the features of leading and personal best guidance. In the fourth strategy, a greedy selection is applied between the positions of current and previous iteration to preserve the information of best obtained areas of the solutions space and to avoid the divergence of wolves from available promising areas of the search space. To investigate the effectiveness of the proposed mGWO, it has been tested on standard and complex benchmarks given in IEEE CEC 2014 [44] and IEEE CEC 2017 [45]. The efficacy of the proposed mGWO algorithm is evaluated on these benchmark problems using statistical, convergence and diversity analysis. In the paper, the mGWO is also used to find the solution of real engineering problems and to find the optimal thresholds for the segmentation of grey images.

The rest of the paper is structured as follows: Section 2 describes the classical version of GWO. Section 3 presents the main inspiration for this paper and proposes the Memory-based Grey Wolf Optimizer (mGWO). The experimental results of the mGWO algorithm on benchmark problems are presented and analysed in Section 4. Section 5 uses the mGWO for solving real engineering problems. In Section 6, the mGWO is used for multilevel thresholding of grey images. Finally, Section 7 concludes the work and discusses possible future research.

2. Overview of GWO

The GWO algorithm [8] mimics the behaviour of social interaction and dominant leadership characteristic of the grey wolf pack. Grey wolves give preference to live in a group (pack), where the leadership behaviour and discipline are maintained by dividing the wolves into four types, namely —

1. Alpha wolf — the most dominant wolf in the pack, which is responsible for all the crucial and major decision in the pack
2. Beta wolf — the secondary wolf, which acts as a leading wolf in the absence of alpha
3. Delta wolf — the caretakers, sentinels and elder wolves of the pack
4. Omega wolves — the remaining wolves of the pack which are allowed to eat in the end.

The wolves, alpha, beta and delta are called leading hunters and the prey hunting is fully depends on these hunters. Muro et al. [46] have explained that grey wolves complete their hunting in the following three steps:

I. Tracking and chasing prey
II. Encircling the prey
III. Attacking the prey.

The mathematical models of the leadership behaviour, encircling and attacking prey are provided as follows:

2.1. Leadership hierarchy

In an optimization problem, the top three fittest solutions are selected as leading wolves, alpha, beta and delta. This selection mimics the leadership hierarchy of grey wolves. The rest of the candidate solutions in optimization problem are called as omega wolves. All the omega wolves/solutions perform the search according to the guidance of leading hunters, alpha, beta and delta.

2.2. Encircling behaviour

When the prey location is captured by the grey wolves, encircling of prey is performed. This encircling stops the movement of the prey. To mimic the encircling behaviour, the following equations are used in the standard GWO

$$X_{t+1} = X_{p,t} - A_t \cdot D_t \tag{1}$$

$$D_t = |C_t \cdot X_{p,t} - X_t| \tag{2}$$

$$A_t = 2 \cdot a_t \cdot r_1 - a_t \tag{3}$$

$$C_t = 2 \cdot r_2 \tag{4}$$

where X_{t+1} is the wolf position at iteration $(t+1)$, $X_{p,t}$ is the prey position observed at iteration t. A_t is the coefficient vector which is responsible for exploration and exploitation, D_t is the difference vector which decides the movement of wolf either towards the neighbourhood regions of prey or opposite of it. C_t is the coefficient vector, which helps in exploring the solution space when the coefficient A_t fails. r_1 and r_2 are the random numbers which are uniformly distributed in the interval (0, 1). The transition parameter a_t decreases linearly with an increase of the iterations and can be formulated as follows —

$$a_t = 2 - 2 \cdot \left(\frac{t}{T}\right) \tag{5}$$

where t indicates the current iteration and T denotes the maximum number of iterations.

2.3. Hunting behaviour

In the wolf pack, it is assumed that the all leading wolves have enough ability to hunt the prey. Therefore, to approximate the position of prey, these leaders can be used simultaneously. The mathematical modelling of hunting strategy is given as follows:

$$D_{\alpha,t} = |C_{\alpha,t} \cdot X_{\alpha,t} - X_t| \tag{6}$$

$$D_{\beta,t} = |C_{\beta,t} \cdot X_{\beta,t} - X_t| \tag{7}$$

$$D_{\delta,t} = |C_{\delta,t} \cdot X_{\delta,t} - X_t| \tag{8}$$

where $X_{\alpha,t}$, $X_{\beta,t}$ and $X_{\delta,t}$ are the positions of leading hunters at tth iteration. $C_{\alpha,t}$, $C_{\beta,t}$ and $C_{\delta,t}$ are random numbers as defined in Eq. (4). After calculating the difference vectors $D_{\alpha,t}$, $D_{\beta,t}$ and

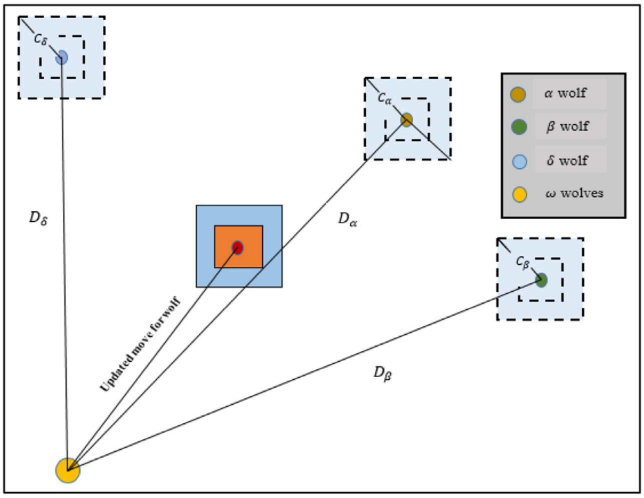

Fig. 1. Evolution of position in GWO.

$D_{\delta,t}$, the updated state of grey wolf for $(t+1)^{th}$ iteration can be calculated as follows:

$$Y_1 = X_{\alpha,t} - A_{\alpha,t} \cdot D_{\alpha,t} \quad (9)$$

$$Y_2 = X_{\beta,t} - A_{\beta,t} \cdot D_{\beta,t} \quad (10)$$

$$Y_3 = X_{\delta,t} - A_{\delta,t} \cdot D_{\delta,t} \quad (11)$$

$$X_{t+1} = (Y_1 + Y_2 + Y_3)/3 \quad (12)$$

where the vectors $A_{\alpha,t}$, $A_{\beta,t}$ and $A_{\delta,t}$ can be calculated from Eq. (3). In order to solve the optimization problems, the encircling and hunting mechanisms of grey wolves can be repeated. The graphical representation of hunting procedure is shown in Fig. 1.

2.4. Exploitation (attack on prey) and exploration (search for prey) in GWO

From the search equations, it is clear that the vectors A_t and C_t are introduced to keep the exploitation and exploration in the algorithm. When $|A_t| < 1$ and/or $C_t < 1$, search regions are exploited and this situation represents the behaviour of chasing of prey and when $|A_t| > 1$ and/or $C_t > 1$, the new search regions are explored, which prevents the pack from stagnation at local optima. This simulates the attacking prey behaviour by grey wolves. In GWO, when $T/2$ iterations passed, the coefficient A_t exploits the solution space. In this situation, the exploration is maintained in the algorithm by coefficient C_t. In GWO, the balance between the operator's exploitation and exploration is maintained with the decreasing nature of the transition parameter a_t. Step-wise description of the standard GWO is presented in Algorithm 1.

3. Proposed memory-based Grey Wolf Optimizer (mGWO)

3.1. Motivation of the work

The search equation of standard GWO indicate the dependency of the search directions towards the leading wolves only. The leading wolves sometimes get traps at local solutions particularly in multimodal problems where a large number of valleys are present. In the standard GWO, when the leading wolves get trapped at these local optima, then it is difficult for the pack to escape from them because of the dependency of the search on leading wolves. The stagnation at local optima is the cause of premature convergence. The collaborative and information-exchange mechanism between the individual wolves can help to alleviate these issues from the algorithm by exploring the solution space more efficiently. Therefore, in order to enhance the collaborative strength of the pack and to contribute the best knowledge of each individual wolf during the search, their personal best history is integrated into the search mechanism of the proposed algorithm. Thus, in the proposed algorithm, the leading and personal best guidance is used together to explore the elite and promising areas of the solution space. In the paper, the proposed algorithm is named as Memory-based Grey Wolf Optimizer (mGWO).

3.2. Framework of memory-based GWO

The proposed mGWO algorithm integrates the personal best information of wolves and best information of the pack simultaneously during the search to strengthen the collaboration among the wolves and to proceed the search in promising directions. The four different strategies, which are applied in the GWO can be summarized as follows:

1. In the first strategy, the hunting mechanism is modified with the help of personal best information collected by each wolf.
2. In the second strategy, a new search equation is proposed with the help of personal best guidance and random wolves to enhance the collaborative strength of the pack.
3. In the third strategy, a crossover is performed between the positions obtained through modified hunting mechanism and the position obtained through personal best guidance.
4. In the fourth strategy, a greedy selection is applied to restore the information of promising areas of the search space.

Since the hunting is performed by following the encircling process, therefore, first the encircling mechanism is modified. The modified encircling mechanism can be understood by the following equation:

$$X_{t+1} = X_\alpha - A_t \times |C_t \times X_\alpha - X_{pbest}| \quad (13)$$

where X_{pbest} is the personal best state saved in the memory of wolf X up to iteration t. The other symbols are same as defined for the standard GWO. After the encircling mechanism, the hunting is performed based on the leading search guidance provided by alpha, beta and delta wolves. The hunting mechanism utilizes the above proposed equation to approximate the prey location. In the newly proposed modified hunting mechanism, it has been assumed that the individual wolves can also share their best knowledge in terms of the quality of visited areas of the solution space. The proposed modified hunting mechanism can be explained by using the following mathematical equations:

$$Z_{i,t+1} = \frac{Y_1 + Y_2 + Y_3}{3} \quad (14)$$

where,

$$Y_1 = X_\alpha - A_{\alpha,t} \times |C_{\alpha,t} \times X_\alpha - X_{i_{pbest}}| \quad (15)$$

$$Y_2 = X_\beta - A_{\beta,t} \times |C_{\beta,t} \times X_\beta - X_{i_{pbest}}| \quad (16)$$

$$Y_3 = X_\delta - A_{\delta,t} \times |C_{\delta,t} \times X_\delta - X_{i_{pbest}}| \quad (17)$$

where $Z_{i,t+1}$ is the updated position of the wolf X_i through hunting mechanism, $X_{i_{pbest}}$ is the personal best state saved in the memory of wolf X_i up to iteration t. Rest of the symbols are same as defined for the standard GWO.

In order to explore and retrace the neighbourhood areas of the personal best states of wolves and to mimic the thought that the individual wolf may have the information about the prey, a novel search equation is proposed given by

$$\hat{X}_{i,t+1} = X_{i_{pbest}} + k \times (X_{r_1} - X_{r_2}) \quad (18)$$

Algorithm 1: pseudo code of standard GWO

Initialize the state of grey wolves randomly within the solution space
Initialize the parameters a and maximum number of iteration T
Evaluate the fitness of each grey wolf
Select the leaders as –
X_α – the fittest solution
X_β – second best solution
X_δ – third best solution
Initialize the iteration count $t = 0$
while $t < T$
    update the position of each wolf by equation (12)
    update the leaders and parameter a
    Evaluate the fitness of each grey wolf
    update the leading wolves X_α, X_β and X_δ
    $t = t + 1$
end while
Return the best solution X_α

where X_{r_1} and X_{r_2} are the wolves which are randomly selected from the pack. The parameter k is scaling factor which controls the effect of difference vector. The higher value of k leads to high exploration and the small values favour the exploitation. In the paper, the value of the parameter k is selected as a variable which decreases linearly from 1 to 0. This parameter value is chosen to control the difference vector.

In order to merge the information about the prey obtained from leading hunters and individual wolves, the crossover is performed between the positions obtained from Eqs. (14) and (18). This crossover is given by:

$$X_{i,t+1}^{j} = \begin{cases} Z_{i,t+1}^{j} & if\ r_3 < CR \\ \hat{X}_{i,t+1}^{j} & otherwise \end{cases} \tag{19}$$

where CR is the crossover probability fixed as 0.5 in our study, r_3 is a uniformly distributed random number within (0,1), $Z_{i,t+1}$ and $\hat{X}_{t+1}$ are the positions obtained from Eqs. (14) and (18) respectively.

When each wolf of the pack is updated by using Eq. (19), a greedy selection strategy is applied to select the best wolf between two consecutive iterations. The main steps of the mGWO are provided in detail in Algorithm 2. The flowchart of the proposed mGWO is presented in Fig. 2. In this figure the flow of the search procedure in the mGWO has been described.

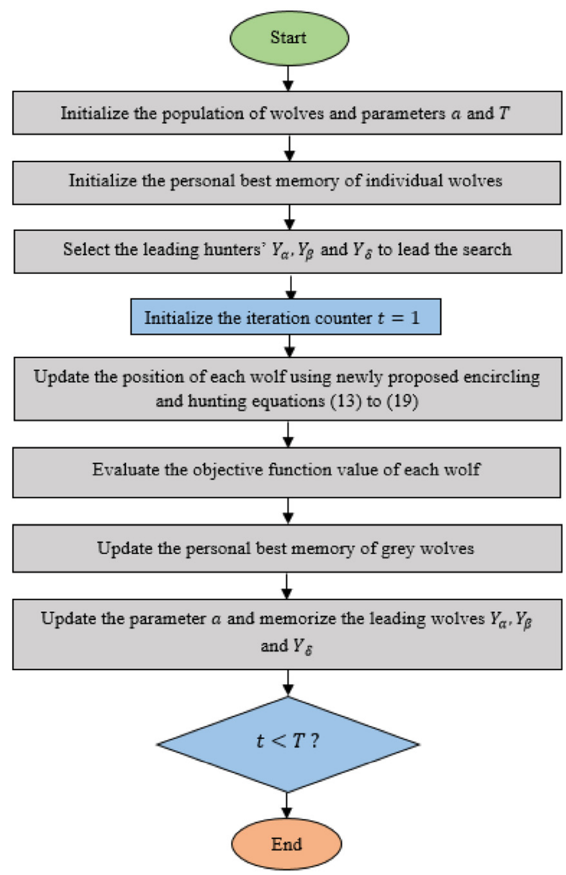

Fig. 2. Flow chart of the proposed mGWO.

3.3. Computational complexity

The time complexity of the standard GWO and the mGWO in terms of big–O notation is discussed as follows:

For standard GWO

1. The standard GWO initializes the wolf pack in $O(N \times D)$ time, where N the pack size and D represent the dimension of the problem.
2. Objective function evaluation at each solution (wolves) requires $O(N)$ time.
3. Selection of leading hunters in standard GWO requires $O(N)$ time.
4. Position update process in the standard GWO requires the $O(N \times D)$ time.

In summary, the total computational cost for the standard GWO is equal to $O(N \times D \times T)$ for maximum number of iterations T.

For mGWO

1. The mGWO initializes the wolf pack in $O(N \times D)$ time, where N the pack size and D represent the dimension of the problem.
2. Selection of leading hunters requires $O(N)$ time.
3. Position update in the mGWO requires the $O(N \times D)$ time.
4. Objective function evaluation at each solution (wolves) requires $O(N)$ time.
5. The greedy selection strategy requires an additional $O(N)$ time in the proposed mGWO.

Algorithm 2: Memory-based Grey Wolf Optimizer (mGWO)

1. $For \quad Min\, F(X) \quad s.t. \quad X_{\min} \leq X \leq X_{\max},\ X = (x_1, x_2, \ldots, x_D) \in R^D$
2. **Initialize** the grey wolf population $X_i\ (i = 1,2,\ldots,N)$
3. **Evaluate** the fitness of each grey wolf
4. **Initialize** the parameters CR and maximum number of iterations T
5. **Initialize** the memory matrix for grey wolf pack as $\left[X_{i_{pbest}}\right]_{i=1}^{N} = [X_i]_{i=1}^{N}$
6. ***Select*** $X_\alpha = fittest\ wolf\ of\ the\ pack$
   $X_\beta = second\ best\ wolf$
   $X_\delta = third\ best\ wolf$
7. Initialize the iteration count $t = 0$
8. ***while*** $t < T$
9.   ***for*** $i = 1,2,\ldots,N$
10.   Obtain the position $Z_{i,t+1}$ of wolf $X_{i,t}$ using modified hunting eq. (14)
11.   Obtain the position $\hat{X}_{i,t+1}$ of wolf $X_{i,t}$ using search eq. (18)
12.   Apply crossover as given by eq. (19) between the positions $Z_{i,t+1}$ and $\hat{X}_{i,t+1}$
    to obtain a new position $X_{i,t+1}$
13.   ***end for***
14.   **Evaluate** the fitness of each grey wolf
15.   ***for*** $i = 1,2,\ldots,N$
16.   ***if*** $F\left(X_{i,t+1}\right) > F(X_{i,t})$
17. $X_{i,t+1} = X_{i,t}$
18.   ***end if***
19.   ***end for***
20.   **Update** the leading wolves X_α, X_β and X_δ
21. $t = t + 1$
22. **end while**
23. $return\ the\ alpha\ wolf$

In summary, the total computational cost for the proposed mGWO algorithms is equal to $O(N \times D \times T)$ for maximum number of iterations T. Thus, the computational complexity for the standard GWO and the mGWO is same.

4. Experimental results and discussion

4.1. Benchmark suites and parameter setting

The benchmark suite is a collection of test problems which are used for the evaluation, characterization, and measurement of the performance of an optimization method. In the present paper, the search efficiency of the proposed Memory-based GWO (mGWO) algorithm is examined on standard and complex benchmark set CEC 2014 [44] and CEC 2017 [45]. These benchmarks sets contain 30 test problems with varying difficulty levels. These benchmark sets have four category of functions — unimodal, multimodal, composite and hybrid. The composite and hybrid function are designed by combining the unimodal and multimodal problems. The dimension of the problems is fixed to 10 and 30 in our study. The population size for the standard GWO and mGWO is fixed to 30 for both 10 and 30-dimensional problems. The stopping criteria for algorithms is fixed to $10^4 \times D$ function evaluations as suggested by CEC. Here, D represents the dimension of the problem. The discussion of results on each benchmark set is presented one by one as follows —

4.2. Benchmark suite I: CEC 2014

In this section, the numerical results are provided and discussed based on the CEC 2014 benchmark suite. The performance evaluation of the mGWO is performed through various metrics such as various statistics (median, average, standard deviation, maximum and minimum error in objective function values), statistical validation through Wilcoxon signed rank test, convergence analysis and comparison with variants of GWO and other optimization algorithms.

The obtained results are recorded in Tables 1 and 2 for 10 and 30-D problems. The various statistics of error in objective function values such as mean, standard deviation, median, maximum and minimum are presented in the same table.

Since, the unimodal test problems are used to evaluate the exploitation strength of wolves in the algorithm, therefore, the results on unimodal problems (F1 to F3) which are presented in Tables 1 and 2 verify the better exploitation strength of the mGWO algorithm as compared to standard GWO. On these test cases, the mGWO has outperformed standard GWO in terms of all the statistics. The performance on these problems also verifies the better convergence rate in the mGWO as compared to the standard GWO. Hence, the overall analysis of the results ensures that the proposed novel search equation based on the personal best guidance (given in (18)) and new hunting mechanism (when the parameter $a < 1$) given in (14) are impactful in exploiting the visited promising areas of the solution space.

The multimodal problems (F4–F16), which are used to analyse the exploration ability of an algorithm show that the mGWO has enhanced the wolves' exploration strength. On the 10 as well as 30-dimensional problems namely F6–F10, F14 and F15, the mGWO has outperformed standard GWO in terms of all the statistics such as minimum, mean, median, maximum and standard deviation of errors in objective function. In 10-dimensional F4, F5 and F16, the mGWO provides better results than the standard GWO in all the statistics except standard deviation value. In the remaining 10-dimensional problems, the mGWO is better than standard GWO in all the statistics except for maximum error in

Table 1
Results and comparison between standard GWO and the proposed mGWO algorithms on 10-dimensional benchmark suite I.

Problem	Algorithm	Median	Average	Minimum	Maximum	Std dev
F1	GWO	4.316E+06	5.157E+06	3.946E+04	1.610E+07	3.790E+06
	mGWO	**5.003E+04**	**1.529E+05**	**4.892E+03**	**2.837E+06**	**4.063E+05**
F2	GWO	1.836E+03	6.441E+07	2.190E+02	1.582E+09	2.432E+08
	mGWO	**1.078E+03**	**1.853E+03**	**1.177E+02**	**7.853E+03**	**1.860E+03**
F3	GWO	2.746E+03	4.299E+03	6.850E+02	1.319E+04	3.575E+03
	mGWO	**3.949E+01**	**1.017E+02**	**3.797E+00**	**9.305E+02**	**1.797E+02**
F4	GWO	3.516E+01	2.814E+01	3.224E−01	3.844E+01	**1.241E+01**
	mGWO	**3.499E+01**	**2.652E+01**	**1.318E−01**	**3.597E+01**	1.438E+01
F5	GWO	2.036E+01	2.036E+01	2.015E+01	2.053E+01	**8.222E−02**
	mGWO	**2.034E+01**	**1.757E+01**	**3.144E−02**	**2.051E+01**	7.055E+00
F6	GWO	2.261E+00	2.453E+00	1.584E−01	5.948E+00	1.450E+00
	mGWO	**1.310E−01**	**1.605E−01**	**8.369E−02**	**6.585E−01**	**9.572E−02**
F7	GWO	1.008E+00	1.947E+00	1.043E−01	3.125E+01	4.497E+00
	mGWO	**4.305E−01**	**3.787E−01**	**5.135E−02**	**6.651E−01**	**1.613E−01**
F8	GWO	8.955E+00	9.407E+00	1.991E+00	1.855E+01	4.408E+00
	mGWO	**9.954E−01**	**1.035E+00**	**1.592E−04**	**2.986E+00**	**8.665E−01**
F9	GWO	1.195E+01	1.296E+01	1.990E+00	2.434E+01	5.767E+00
	mGWO	**2.985E+00**	**3.696E+00**	**2.548E−04**	**1.167E+01**	**2.694E+00**
F10	GWO	2.991E+02	3.300E+02	2.724E+01	8.039E+02	1.977E+02
	mGWO	**7.866E+00**	**1.936E+01**	**3.521E−01**	**1.391E+02**	**3.411E+01**
F11	GWO	4.636E+02	4.715E+02	1.412E+01	**9.805E+02**	2.083E+02
	mGWO	**2.696E+01**	**6.751E+01**	**1.299E−01**	2.998E+02	**7.030E+01**
F12	GWO	**3.366E−01**	**6.072E−01**	**8.083E−03**	1.698E+00	5.576E−01
	mGWO	9.067E−01	8.922E−01	5.025E−01	**1.189E+00**	**1.748E−01**
F13	GWO	1.571E−01	1.690E−01	**6.027E−02**	2.699E−01	5.364E−02
	mGWO	**1.221E−01**	**1.182E−01**	6.804E−02	**1.740E−01**	**2.256E−02**
F14	GWO	1.881E−01	2.613E−01	7.593E−02	6.905E−01	1.904E−01
	mGWO	**9.746E−02**	**9.892E−02**	**4.578E−02**	**1.708E−01**	**3.480E−02**
F15	GWO	1.989E+00	1.858E+00	5.273E−01	3.737E+00	7.507E−01
	mGWO	**1.044E+00**	**1.117E+00**	**4.466E−01**	**2.055E+00**	**4.183E−01**
F16	GWO	2.509E+00	2.480E+00	1.097E+00	3.514E+00	**5.298E−01**
	mGWO	**1.091E+00**	**1.063E+00**	**1.863E−01**	**2.200E+00**	5.462E−01
F17	GWO	2.997E+03	1.071E+04	4.358E+02	3.410E+05	4.726E+04
	mGWO	**1.262E+03**	**1.584E+03**	**1.207E+02**	**6.528E+03**	**1.318E+03**
F18	GWO	7.106E+03	8.854E+03	9.942E+01	3.284E+04	6.983E+03
	mGWO	**6.083E+02**	**1.638E+03**	**5.570E+01**	**9.184E+03**	**2.241E+03**
F19	GWO	2.434E+00	3.431E+00	1.117E+00	5.249E+01	7.063E+00
	mGWO	**1.508E+00**	**1.385E+00**	**6.400E−02**	**1.661E+00**	**2.987E−01**
F20	GWO	1.082E+02	2.607E+03	3.039E+01	1.201E+04	3.708E+03
	mGWO	**4.313E+01**	**6.556E+01**	**9.757E+00**	**6.145E+02**	**9.482E+01**
F21	GWO	5.319E+03	6.272E+03	5.065E+02	1.265E+04	3.951E+03
	mGWO	**2.771E+02**	**5.523E+02**	**2.523E+01**	**4.657E+03**	**1.006E+03**
F22	GWO	4.518E+01	9.253E+01	2.177E+01	1.708E+02	6.117E+01
	mGWO	**2.158E+01**	**2.009E+01**	**1.033E+00**	**4.030E+01**	**7.497E+00**
F23	GWO	3.307E+02	3.321E+02	3.295E+02	3.379E+02	2.503E+00
	mGWO	**3.294E+02**	**3.294E+02**	**3.294E+02**	**3.294E+02**	**9.005E−03**
F24	GWO	1.252E+02	1.356E+02	1.108E+02	2.038E+02	2.907E+01
	mGWO	**1.086E+02**	**1.073E+02**	**1.000E+02**	**1.146E+02**	**4.813E+00**
F25	GWO	1.997E+02	1.954E+02	1.287E+02	2.034E+02	**1.434E+01**
	mGWO	**1.778E+02**	**1.609E+02**	**1.000E+02**	**2.016E+02**	3.952E+01
F26	GWO	1.001E+02	1.001E+02	1.001E+02	1.003E+02	4.130E−02
	mGWO	**1.001E+02**	**1.001E+02**	**1.001E+02**	**1.002E+02**	**1.986E−02**
F27	GWO	3.349E+02	2.929E+02	1.716E+00	4.117E+02	**1.395E+02**
	mGWO	**3.039E+02**	**2.004E+02**	**9.483E−01**	**4.003E+02**	1.850E+02
F28	GWO	4.732E+02	4.664E+02	3.569E+02	6.661E+02	6.708E+01
	mGWO	**3.694E+02**	**4.015E+02**	**3.569E+02**	**4.914E+02**	**5.551E+01**
F29	GWO	7.226E+02	3.952E+05	2.782E+02	4.327E+06	9.177E+05
	mGWO	**3.898E+02**	**4.310E+02**	**2.720E+02**	**7.832E+02**	**1.135E+02**
F30	GWO	8.714E+02	1.071E+03	**4.893E+02**	2.639E+03	5.426E+02
	mGWO	**7.001E+02**	**7.476E+02**	5.121E+02	**1.468E+03**	**1.844E+02**

F11, minimum, median and mean error in F12 and minimum error in F13. For the remaining 30-dimensional problems, the proposed mGWO is better than the standard GWO in terms of all the statistics except for minimum and median error in F5, standard deviation in F11, F16 and minimum, median, mean error in F12. The overall analysis of results on multimodal problems

Table 2
Results and comparison between standard GWO and the proposed mGWO algorithms on 30-dimensional benchmark suite I.

Problem	Algorithm	Median	Average	Minimum	Maximum	Std dev
F1	GWO	5.737E+07	6.735E+07	6.649E+06	2.192E+08	4.749E+07
	mGWO	**5.414E+06**	**5.579E+06**	**1.677E+06**	**1.340E+07**	**2.723E+06**
F2	GWO	1.891E+09	2.174E+09	9.447E+07	9.031E+09	1.944E+09
	mGWO	**3.725E+06**	**1.801E+07**	**2.344E+04**	**1.257E+08**	**2.633E+07**
F3	GWO	2.920E+04	3.018E+04	1.403E+04	4.767E+04	8.010E+03
	mGWO	**3.582E+02**	**4.500E+02**	**7.253E+01**	**1.556E+03**	**3.194E+02**
F4	GWO	2.129E+02	2.166E+02	1.137E+02	3.597E+02	5.544E+01
	mGWO	**1.367E+02**	**1.289E+02**	**8.575E+01**	**1.621E+02**	**2.294E+01**
F5	GWO	**2.094E+01**	**2.094E+01**	**2.062E+01**	2.105E+01	6.976E−02
	mGWO	2.095E+01	**2.094E+01**	2.075E+01	**2.103E+01**	**5.855E−02**
F6	GWO	1.293E+01	1.303E+01	8.822E+00	1.954E+01	2.302E+00
	mGWO	**2.040E+00**	**2.249E+00**	**9.513E−01**	**5.302E+00**	**9.791E−01**
F7	GWO	1.735E+01	2.155E+01	2.833E+00	7.743E+01	1.782E+01
	mGWO	**1.037E+00**	**1.023E+00**	**3.275E−01**	**2.397E+00**	**3.246E−01**
F8	GWO	8.170E+01	7.950E+01	3.924E+01	1.339E+02	1.903E+01
	mGWO	**1.793E+01**	**1.761E+01**	**8.971E+00**	**3.353E+01**	**4.953E+00**
F9	GWO	9.652E+01	9.858E+01	4.558E+01	2.254E+02	2.498E+01
	mGWO	**2.203E+01**	**2.476E+01**	**1.279E+01**	**4.337E+01**	**7.406E+00**
F10	GWO	2.277E+03	2.222E+03	1.445E+03	3.174E+03	4.042E+02
	mGWO	**3.514E+02**	**3.588E+02**	**1.302E+01**	**1.199E+03**	**2.091E+02**
F11	GWO	2.667E+03	2.828E+03	1.533E+03	7.034E+03	**8.259E+02**
	mGWO	**1.912E+03**	**2.279E+03**	**3.820E+02**	**6.330E+03**	1.411E+03
F12	GWO	**2.198E+00**	**1.482E+00**	**4.164E−02**	3.138E+00	1.223E+00
	mGWO	2.351E+00	2.407E+00	1.825E+00	**2.882E+00**	**2.616E−01**
F13	GWO	4.047E−01	5.568E−01	2.572E−01	2.651E+00	4.715E−01
	mGWO	**2.391E−01**	**2.394E−01**	**1.627E−01**	**3.076E−01**	**4.200E−02**
F14	GWO	8.381E−01	4.427E+00	1.865E−01	2.779E+01	6.959E+00
	mGWO	**2.191E−01**	**2.154E−01**	**1.354E−01**	**2.942E−01**	**3.346E−02**
F15	GWO	3.566E+01	2.211E+02	6.636E+00	2.532E+03	5.567E+02
	mGWO	**1.137E+01**	**9.220E+00**	**2.527E+00**	**1.499E+01**	**4.517E+00**
F16	GWO	1.103E+01	1.091E+01	9.508E+00	1.214E+01	**6.895E−01**
	mGWO	**9.511E+00**	**9.327E+00**	**6.009E+00**	**1.158E+01**	1.190E+00
F17	GWO	1.072E+06	1.898E+06	9.341E+04	8.342E+06	2.194E+06
	mGWO	**2.338E+05**	**2.535E+05**	**6.706E+04**	**5.689E+05**	**1.015E+05**
F18	GWO	1.277E+04	7.188E+06	3.732E+02	7.095E+07	1.839E+07
	mGWO	**1.142E+03**	**1.917E+03**	**3.634E+02**	**5.644E+03**	**1.563E+03**
F19	GWO	2.682E+01	3.887E+01	1.143E+01	9.032E+01	2.294E+01
	mGWO	**7.460E+00**	**7.487E+00**	**4.767E+00**	**9.724E+00**	**1.282E+00**
F20	GWO	1.228E+04	1.498E+04	3.947E+03	5.930E+04	9.682E+03
	mGWO	**2.308E+02**	**2.371E+02**	**1.258E+02**	**3.801E+02**	**5.677E+01**
F21	GWO	3.299E+05	1.296E+06	2.876E+04	1.052E+07	2.485E+06
	mGWO	**5.791E+04**	**7.592E+04**	**1.240E+04**	**2.274E+05**	**6.006E+04**
F22	GWO	3.918E+02	3.753E+02	5.994E+01	7.858E+02	1.592E+02
	mGWO	**1.550E+02**	**1.486E+02**	**3.448E+01**	**3.980E+02**	**6.572E+01**
F23	GWO	3.320E+02	3.347E+02	3.220E+02	3.923E+02	1.161E+01
	mGWO	**3.157E+02**	**3.157E+02**	**3.153E+02**	**3.172E+02**	**3.668E−01**
F24	GWO	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.000E+02**	**8.196E−04**
	mGWO	**2.000E+02**	**2.000E+02**	**2.000E+02**	**2.001E+02**	6.450E−03
F25	GWO	2.116E+02	2.102E+02	**2.000E+02**	2.186E+02	5.114E+00
	mGWO	**2.057E+02**	**2.056E+02**	2.040E+02	**2.096E+02**	**1.211E+00**
F26	GWO	1.005E+02	1.434E+02	1.003E+02	2.002E+02	4.981E+01
	mGWO	**1.002E+02**	**1.198E+02**	**1.001E+02**	**2.001E+02**	**4.004E+01**
F27	GWO	6.885E+02	6.534E+02	4.077E+02	8.611E+02	1.296E+02
	mGWO	**3.541E+02**	**3.635E+02**	**3.172E+02**	**4.060E+02**	**3.002E+01**
F28	GWO	1.018E+03	1.109E+03	8.099E+02	1.715E+03	2.233E+02
	mGWO	**8.623E+02**	**8.464E+02**	**6.499E+02**	**9.275E+02**	**6.040E+01**
F29	GWO	4.698E+04	9.120E+05	4.136E+03	7.431E+06	1.844E+06
	mGWO	**1.070E+04**	**1.146E+04**	**3.247E+03**	**2.295E+04**	**4.380E+03**
F30	GWO	4.407E+04	5.658E+04	9.884E+03	3.863E+05	5.953E+04
	mGWO	**5.963E+03**	**6.393E+03**	**4.042E+03**	**1.045E+04**	**1.537E+03**

verifies that the proposed strategies integrated in the mGWO effectively improve the exploration and local optima avoidance abilities of wolves to solve the multimodal problems.

On the hybrid benchmark problems (F17–F22), the mGWO has provided better results as compared to the standard GWO in all the test problems. The results of the mGWO are better than the standard GWO in all statistics except for standard deviation value in 30-dimensional F16 of errors in objective function. In composite problems (F23–F30), the proposed mGWO has shown its better ability of search as compared to the standard GWO. On 10-dimensional composite problems, the mGWO provides better results in terms of all the statistics (except for standard deviation in F25, F27 and minimum error in F30) than the standard GWO. For the 30-dimensional composite problems, the mGWO has outperformed standard GWO in all the problems corresponding to all statistics (except for standard deviation in F24 and minimum error in F25). Hence, the results on hybrid and composite benchmarks show that the search strategies such as modified hunting mechanism, search mechanism based on personal best guidance, crossover and greedy selection strategy are successful to maintain a balance between exploration and exploitation.

Hence Tables 1 and 2 clearly demonstrate that the proposed mGWO outperforms standard GWO in most of the problems. The low standard deviation value in most of the problems indicates the better solution quality of the mGWO. Thus, the results clearly demonstrate the strength of the concept of hybridizing the personal best information of wolves, which is saved in their memory, with leading guidance in the mGWO. This hybridization explores the neighbourhood regions of the personal best history of individual wolves and provides an efficient direction of search from individual's best history to the leading best positions.

In order to analyse the difference in results statistically, a non-parametric Wilcoxon signed rank test [47] is applied at 5% significance level. The obtained p-values and corresponding outcomes are reported in Table A.1 of Appendix, which clearly demonstrate the outperformance of the proposed mGWO over the GWO. For the dimension 10, mGWO significantly superior to standard GWO on 26 test cases, while for dimension 30, it is superior on 27 test cases. Standard GWO is significantly better than the mGWO for only one case of dimension 10 and only two cases for dimension 30.

4.3. Benchmark suite II: CEC 2017

In the present section, the validation of the proposed algorithm mGWO is done on the latest collection of benchmarks CEC 2017 [45]. The experiments are conducted with the same parameter setting as followed on CEC 2014. In this benchmark suite, the functions from F1 to F3 are unimodal, F4 to F10 and multimodal, F11 to F20 are hybrid and F21 to F30 are composite. Thus, in this benchmark suite, a large number of hybrid and composite benchmarks are included as compared to the CEC 2014. The detailed comparative results obtained by the mGWO and standard GWO for 10 and 30-dimensional problems are shown in Tables 3 and 4. In these tables, the results on the problem F2 is not provided because this problem is currently deleted from the CEC 2017 due to its unstable behaviour.

From the comparison of results, it can be observed that in all the unimodal problems, the mGWO has outperformed standard GWO in all the statistics. Thus, the performance evaluation on unimodal benchmarks provided by CEC 2017 ensures that the mGWO is better than the standard GWO in terms of exploitation strength and convergence rate.

On the multimodal problems from F4 to F10, the proposed mGWO provides better results in all the statistics except for standard deviation in 30-dimensional F10. Hence, an overall comparison of results demonstrates the better ability of exploring the solution space in the mGWO than the standard GWO.

On the hybrid problems from F11 to F20, the mGWO provides better value of all the statistics as compared to standard GWO for the dimension 10. On the 30-dimensional hybrid problems, the mGWO provides better value of errors in all the statistics corresponding to each problem except for the minimum error in function F13.

On the 10-dimensional composite problems from F21 to F30, the mGWO is better than the standard GWO in terms of providing the better value of all the statistics (except for standard deviation in F21, F24 and F25). In all the 30-dimensional composite problems, the mGWO is better than the standard GWO in all statistics. Hence, the results obtained on hybrid and composite problems ensure that the proposed strategies which are integrated in the search mechanism of the mGWO are successful in terms of balancing the exploration and exploitation within the algorithm.

The comparison provided in Table 3 and Table 4 demonstrates the outperformance of the mGWO as compared to the standard GWO in all categories of benchmark problems. In order to statistically conclude the best performer algorithm for CEC 2017 benchmark suite, a non-parametric Wilcoxon signed rank test [47] is applied. The achieved statistical decisions along with the p-values are presented in Table A.2 of Appendix corresponding to the dimension 10 and 30. For the dimension 10, mGWO significantly superior to the standard GWO on 29 test cases, while for dimension 30, it is superior on all the 30 test cases. The standard GWO is not significantly better than the mGWO even on single problem. Hence, the statistical analysis ensures that the proposed method significantly beats the classical version of GWO on most of the test problems.

From all the experiments conducted on CEC 2014 and CEC 2017, it can be observed that the personal best history which has been introduced in the mGWO to enhance the search mechanism is very effective to produce better results on all category of benchmark problems such as unimodal, multimodal, hybrid and composite. The diversity analysis of the proposed mGWO can be performed by comparing the diversity between the wolves for standard GWO and mGWO algorithms. The diversity curves are plotted by considering the average distance between the wolves (solutions). The Euclidean distance between two wolves $X = (x_1, x_2, \ldots, x_D)$ and $Y = (y_1, y_2, \ldots, y_D)$ can be calculated as follows:

$$\|X - Y\|_2 = \sqrt{\sum_{j=1}^{D}(x_j - y_j)^2} \tag{20}$$

The diversity curves presented in Figs. 3 and 4 demonstrate that the diversity between the wolves is high at some initial iterations and decreases with the progress of iterations for standard GWO and mGWO. But, it is also evident from the figures that the average distance in initial iterations is higher in the mGWO than the standard GWO, which demonstrates the better exploration ability of search agents in the mGWO. This verifies the impact of enhancement in the search strategy based on the personal best history of wolves. The personal best history of wolves allows more exploration of the solution space. In the figures, it can be also observed that the transition from the phase of exploration to exploitation is faster in the mGWO as compared to the standard GWO.

4.4. Comparison with variants of GWO and other algorithms

The previous subsections empirically verify the better performance of the proposed mGWO algorithm as compared to the classical version of GWO in terms of exploitation, exploration and in maintaining an appropriate balance between exploitation and exploration. In the present section, the performance of the mGWO is compared with some modified versions of GWO and some other metaheuristic algorithms, which are population-based. 30-dimensional problems of CEC 2014 and CEC 2017 are

Table 3
Results and comparison between standard GWO and the proposed mGWO algorithms on 10-dimensional benchmark suite II.

Problem	Algorithm	Median	Average	Minimum	Maximum	Std dev
F1	GWO	6.032E+04	1.861E+07	7.974E+02	3.324E+08	5.064E+07
	mGWO	**2.079E+03**	**2.874E+03**	**3.980E+02**	**1.480E+04**	**2.568E+03**
F3	GWO	2.598E+02	1.563E+03	1.764E+01	9.686E+03	2.183E+03
	mGWO	**1.231E−01**	**1.823E+00**	**2.052E−03**	**3.213E+01**	**4.912E+00**
F4	GWO	7.392E+00	1.420E+01	1.567E−02	1.454E+02	2.278E+01
	mGWO	**5.967E+00**	**5.638E+00**	**3.125E+00**	**7.201E+00**	**1.213E+00**
F5	GWO	1.327E+01	1.488E+01	2.987E+00	3.852E+01	7.843E+00
	mGWO	**2.986E+00**	**3.169E+00**	**1.503E−03**	**7.960E+00**	**1.521E+00**
F6	GWO	3.004E−01	9.916E−01	1.745E−02	1.103E+01	1.955E+00
	mGWO	**2.003E−02**	**2.128E−02**	**6.774E−03**	**4.137E−02**	**8.047E−03**
F7	GWO	2.574E+01	2.815E+01	1.435E+01	5.370E+01	8.984E+00
	mGWO	**1.334E+01**	**1.479E+01**	**6.333E+00**	**2.907E+01**	**4.773E+00**
F8	GWO	1.227E+01	1.330E+01	3.980E+00	2.547E+01	5.206E+00
	mGWO	**2.985E+00**	**3.095E+00**	**2.925E−04**	**1.113E+01**	**1.957E+00**
F9	GWO	7.418E−01	1.234E+01	2.102E−02	7.290E+01	2.119E+01
	mGWO	**4.726E−02**	**8.534E−02**	**4.039E−03**	**1.001E+00**	**1.639E−01**
F10	GWO	5.684E+02	6.259E+02	6.998E+00	1.328E+03	3.300E+02
	mGWO	**1.189E+02**	**8.369E+01**	**1.918E−01**	**3.521E+02**	**7.762E+01**
F11	GWO	2.442E+01	3.581E+01	4.115E+00	1.454E+02	3.866E+01
	mGWO	**3.608E+00**	**3.601E+00**	**1.448E+00**	**6.933E+00**	**1.137E+00**
F12	GWO	1.453E+05	5.071E+05	9.833E+03	3.972E+06	8.660E+05
	mGWO	**1.842E+04**	**4.280E+04**	**9.522E+02**	**5.886E+05**	**8.843E+04**
F13	GWO	8.029E+03	9.805E+03	6.027E+02	2.610E+04	6.987E+03
	mGWO	**8.543E+02**	**1.544E+03**	**1.928E+02**	**8.117E+03**	**1.625E+03**
F14	GWO	1.102E+02	1.549E+03	4.312E+01	3.869E+03	1.777E+03
	mGWO	**4.236E+01**	**4.440E+01**	**1.813E+01**	**8.190E+01**	**1.306E+01**
F15	GWO	1.059E+03	1.872E+03	2.841E+01	6.282E+03	1.899E+03
	mGWO	**2.817E+01**	**2.963E+01**	**5.007E+00**	**6.709E+01**	**1.456E+01**
F16	GWO	1.349E+02	1.409E+02	7.099E+00	4.681E+02	1.345E+02
	mGWO	**3.393E+00**	**3.528E+00**	**1.501E+00**	**7.758E+00**	**1.196E+00**
F17	GWO	4.407E+01	5.124E+01	9.325E+00	1.642E+02	2.595E+01
	mGWO	**2.261E+01**	**2.061E+01**	**2.874E+00**	**4.388E+01**	**8.370E+00**
F18	GWO	2.985E+04	2.525E+04	9.577E+02	5.296E+04	1.574E+04
	mGWO	**8.041E+02**	**1.187E+03**	**1.029E+02**	**3.879E+03**	**9.751E+02**
F19	GWO	6.002E+01	4.297E+03	7.171E+00	1.207E+04	5.495E+03
	mGWO	**1.432E+01**	**1.623E+01**	**4.864E+00**	**3.580E+01**	**7.041E+00**
F20	GWO	4.945E+01	6.874E+01	1.765E+01	1.720E+02	4.772E+01
	mGWO	**2.106E+01**	**1.769E+01**	**2.027E−01**	**3.038E+01**	**8.145E+00**
F21	GWO	2.135E+02	2.065E+02	1.008E+02	2.371E+02	**3.093E+01**
	mGWO	**2.040E+02**	**1.725E+02**	**1.000E+02**	**2.139E+02**	4.930E+01
F22	GWO	1.067E+02	1.375E+02	1.011E+02	8.838E+02	1.372E+02
	mGWO	**1.019E+02**	**1.026E+02**	**1.001E+02**	**1.065E+02**	**1.856E+00**
F23	GWO	3.155E+02	3.181E+02	3.056E+02	3.491E+02	9.763E+00
	mGWO	**3.029E+02**	**3.029E+02**	**3.000E+02**	**3.112E+02**	**2.776E+00**
F24	GWO	3.411E+02	3.445E+02	3.271E+02	3.678E+02	**1.195E+01**
	mGWO	**3.304E+02**	**3.073E+02**	**1.003E+02**	**3.404E+02**	6.921E+01
F25	GWO	4.414E+02	4.282E+02	3.980E+02	4.572E+02	**2.011E+01**
	mGWO	**3.984E+02**	**4.182E+02**	**3.977E+02**	**4.463E+02**	2.327E+01
F26	GWO	3.718E+02	5.618E+02	5.621E+01	1.327E+03	4.120E+02
	mGWO	**3.001E+02**	**3.001E+02**	**3.000E+02**	**3.001E+02**	**1.341E−02**
F27	GWO	3.947E+02	4.044E+02	3.895E+02	4.975E+02	2.479E+01
	mGWO	**3.946E+02**	**3.951E+02**	**3.900E+02**	**3.979E+02**	**1.653E+00**
F28	GWO	5.838E+02	5.523E+02	3.641E+02	6.590E+02	8.989E+01
	mGWO	**3.001E+02**	**3.091E+02**	**3.000E+02**	**3.681E+02**	**2.294E+01**
F29	GWO	2.809E+02	2.893E+02	2.333E+02	4.120E+02	3.663E+01
	mGWO	**2.451E+02**	**2.463E+02**	**2.320E+02**	**2.772E+02**	**9.734E+00**
F30	GWO	1.223E+05	6.275E+05	2.210E+03	4.689E+06	8.633E+05
	mGWO	**4.832E+03**	**5.992E+03**	**9.786E+02**	**1.805E+04**	**3.727E+03**

considered in this section for the performance comparison. The obtained numerical results from state-of-the-art algorithms such as PSO [48], modified PSO (wPSO) [49], HS [50], GSA [51], ABC [3], GABC [52], FA [7], CMA-ES [53], JADE [54,55] and modified variants of GWO such as modGWO [23], i-GWO [33], OBGWO [22], IGWO [25], EEGWO [24] and standard GWO [8] are listed in Tables 5 and 6 corresponding to the CEC 2014 and CEC 2017 benchmark sets, respectively. The parameter setting for these algorithms is adapted same as reported in their original papers. In the tables, the results have been presented by conducting

Table 4
Results and comparison between standard GWO and the proposed mGWO algorithms on 30-dimensional benchmark suite II.

Problem	Algorithm	Median	Average	Minimum	Maximum	Std dev
F1	GWO	2.002E+09	2.373E+09	1.576E+08	7.153E+09	1.509E+09
	mGWO	**4.951E+06**	**1.064E+07**	**2.885E+04**	**5.780E+07**	**1.540E+07**
F3	GWO	3.108E+04	3.149E+04	1.415E+04	5.363E+04	1.006E+04
	mGWO	**4.468E+02**	**6.349E+02**	**7.082E+01**	**2.933E+03**	**6.100E+02**
F4	GWO	1.783E+02	2.363E+02	8.212E+01	2.621E+03	3.484E+02
	mGWO	**1.152E+02**	**1.133E+02**	**7.278E+01**	**1.365E+02**	**1.356E+01**
F5	GWO	9.517E+01	9.915E+01	5.995E+01	1.568E+02	2.134E+01
	mGWO	**2.500E+01**	**2.801E+01**	**1.390E+01**	**1.385E+02**	**1.727E+01**
F6	GWO	7.843E+00	8.076E+00	1.526E+00	2.115E+01	4.208E+00
	mGWO	**5.998E−02**	**7.131E−02**	**2.027E−02**	**2.125E−01**	**4.230E−02**
F7	GWO	1.522E+02	1.656E+02	1.003E+02	3.507E+02	5.098E+01
	mGWO	**5.838E+01**	**6.630E+01**	**4.852E+01**	**1.881E+02**	**3.044E+01**
F8	GWO	8.045E+01	8.470E+01	5.730E+01	1.184E+02	1.659E+01
	mGWO	**2.551E+01**	**2.554E+01**	**1.291E+01**	**4.391E+01**	**6.987E+00**
F9	GWO	7.231E+02	8.785E+02	1.634E+02	3.477E+03	5.987E+02
	mGWO	**1.481E+00**	**2.246E+00**	**1.529E−01**	**1.393E+01**	**2.538E+00**
F10	GWO	2.872E+03	3.072E+03	1.870E+03	7.635E+03	**8.250E+02**
	mGWO	**2.026E+03**	**2.495E+03**	**1.105E+03**	**6.685E+03**	1.393E+03
F11	GWO	3.886E+02	7.354E+02	1.746E+02	5.408E+03	9.510E+02
	mGWO	**8.618E+01**	**7.842E+01**	**1.403E+01**	**1.451E+02**	**3.130E+01**
F12	GWO	3.028E+07	8.145E+07	6.122E+05	1.846E+09	2.611E+08
	mGWO	**8.455E+05**	**1.261E+06**	**8.274E+04**	**4.764E+06**	**1.103E+06**
F13	GWO	6.796E+04	4.792E+06	**1.710E+04**	1.366E+08	2.010E+07
	mGWO	**5.267E+04**	**5.656E+04**	1.721E+04	**1.301E+05**	**2.686E+04**
F14	GWO	6.900E+04	3.068E+05	4.334E+03	1.740E+06	4.021E+05
	mGWO	**2.281E+03**	**3.274E+03**	**2.698E+02**	**1.192E+04**	**2.972E+03**
F15	GWO	4.327E+04	4.796E+05	8.351E+03	3.625E+06	9.986E+05
	mGWO	**1.612E+04**	**2.002E+04**	**3.802E+03**	**7.201E+04**	**1.302E+04**
F16	GWO	8.813E+02	8.319E+02	1.991E+02	1.526E+03	2.740E+02
	mGWO	**2.504E+02**	**2.794E+02**	**3.597E+01**	**9.649E+02**	**2.047E+02**
F17	GWO	2.496E+02	3.001E+02	8.572E+01	7.947E+02	1.567E+02
	mGWO	**6.348E+01**	**7.200E+01**	**3.314E+01**	**1.814E+02**	**3.155E+01**
F18	GWO	5.972E+05	9.310E+05	4.280E+04	9.788E+06	1.542E+06
	mGWO	**1.170E+05**	**1.359E+05**	**2.127E+04**	**6.035E+05**	**1.004E+05**
F19	GWO	3.784E+05	1.826E+06	3.385E+03	3.603E+07	6.646E+06
	mGWO	**1.123E+04**	**1.626E+04**	**6.754E+02**	**6.920E+04**	**1.528E+04**
F20	GWO	3.533E+02	3.725E+02	1.639E+02	6.948E+02	1.236E+02
	mGWO	**1.591E+02**	**1.445E+02**	**3.681E+01**	**2.228E+02**	**4.934E+01**
F21	GWO	2.875E+02	2.902E+02	2.436E+02	4.089E+02	3.001E+01
	mGWO	**2.291E+02**	**2.285E+02**	**2.158E+02**	**2.519E+02**	**6.546E+00**
F22	GWO	2.947E+03	2.486E+03	1.352E+02	4.257E+03	1.350E+03
	mGWO	**1.161E+02**	**1.167E+02**	**1.021E+02**	**1.535E+02**	**7.777E+00**
F23	GWO	4.472E+02	4.564E+02	4.070E+02	5.739E+02	3.618E+01
	mGWO	**3.730E+02**	**3.740E+02**	**3.526E+02**	**3.920E+02**	**9.238E+00**
F24	GWO	5.062E+02	5.177E+02	4.580E+02	6.853E+02	4.786E+01
	mGWO	**4.395E+02**	**4.412E+02**	**4.285E+02**	**4.727E+02**	**8.892E+00**
F25	GWO	4.723E+02	4.804E+02	4.216E+02	6.187E+02	4.222E+01
	mGWO	**3.894E+02**	**3.959E+02**	**3.848E+02**	**4.251E+02**	**1.175E+01**
F26	GWO	1.985E+03	2.003E+03	9.579E+02	3.164E+03	3.857E+02
	mGWO	**1.158E+03**	**1.087E+03**	**2.130E+02**	**1.385E+03**	**2.977E+02**
F27	GWO	5.463E+02	5.500E+02	5.124E+02	6.069E+02	2.180E+01
	mGWO	**5.179E+02**	**5.197E+02**	**4.937E+02**	**5.439E+02**	**1.039E+01**
F28	GWO	6.087E+02	6.077E+02	4.820E+02	7.725E+02	6.200E+01
	mGWO	**4.285E+02**	**4.306E+02**	**4.041E+02**	**4.898E+02**	**1.800E+01**
F29	GWO	8.304E+02	8.468E+02	6.102E+02	1.130E+03	1.252E+02
	mGWO	**4.858E+02**	**4.904E+02**	**4.525E+02**	**5.649E+02**	**2.451E+01**
F30	GWO	3.702E+06	4.694E+06	2.799E+05	3.003E+07	4.897E+06
	mGWO	**3.002E+05**	**4.115E+05**	**4.410E+04**	**1.708E+06**	**3.245E+05**

the experiments with the same parameter setting (30 population size and $10^4 \times D$ function evaluations). The outcomes obtained by applying the Wilcoxon signed rank test [47] test are also recorded in the same tables.

The comparison presented in Tables 5 and 6 demonstrate that the proposed mGWO is significantly better than all the modified variants of the GWO and other metaheuristics such as PSO, wPSO, HS and FA on most of the test cases of unimodal, multimodal, hybrid and composite problems. This observation clearly

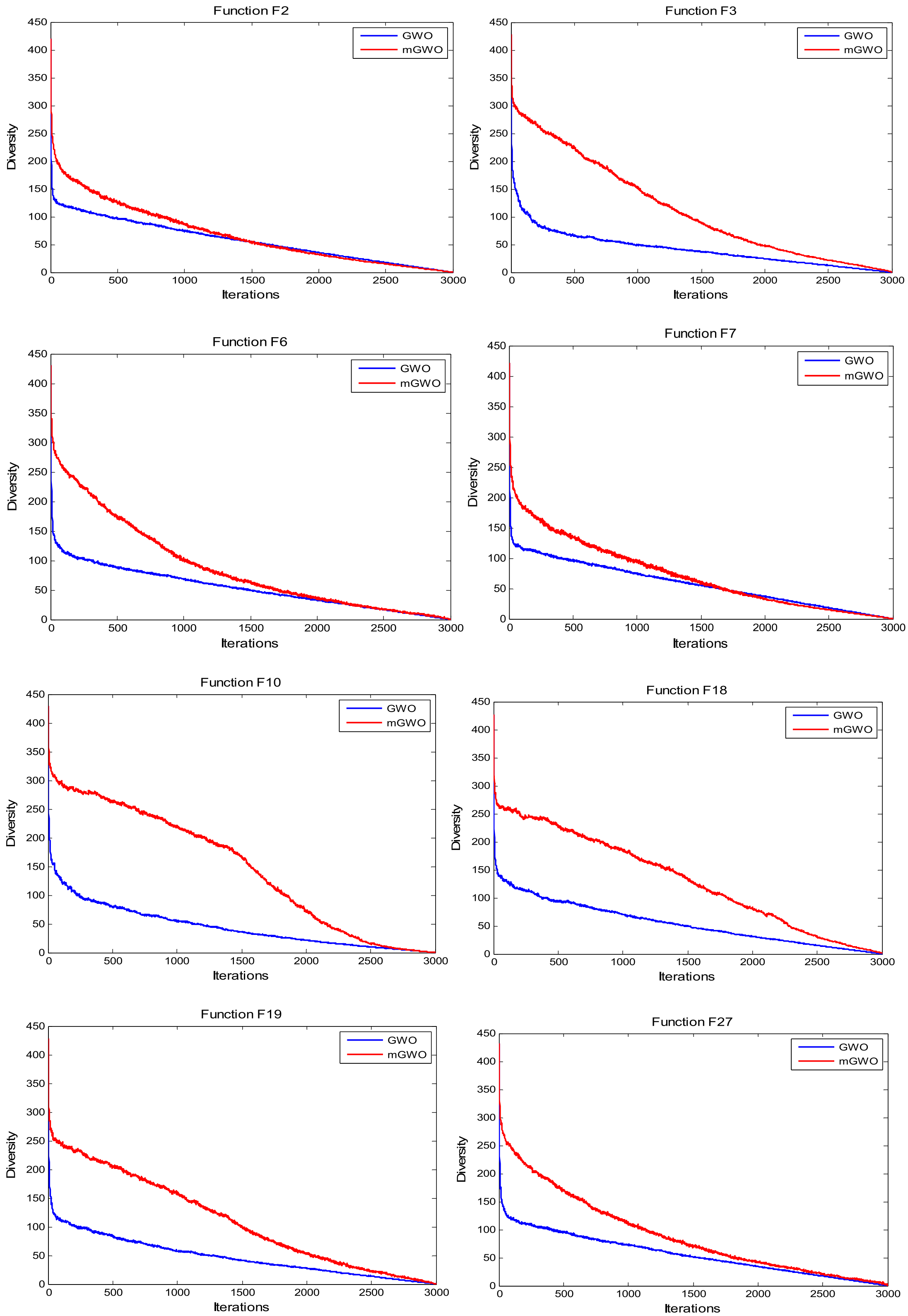

Fig. 3. Diversity curves for CEC 2014 benchmark problems.

demonstrates the better search ability of the mGWO in terms of exploration, exploitation and the ability of maintaining an appropriate balance between them. The comparison of the mGWO with GSA shown better solution accuracy of the mGWO on most of the test cases. But, the exploitation ability is very competitive in both of these algorithms. The comparison of the mGWO with ABC and GABC illustrate the competitive performance of both the algorithms. The mGWO performs poor as compared to the JADE and CMA-ES for unimodal problems. On multimodal problems CMA-ES and mGWO show competitive performance, while the JADE is superior in most of the problems. The comparison on hybrid and composite problems shows the competitive ability

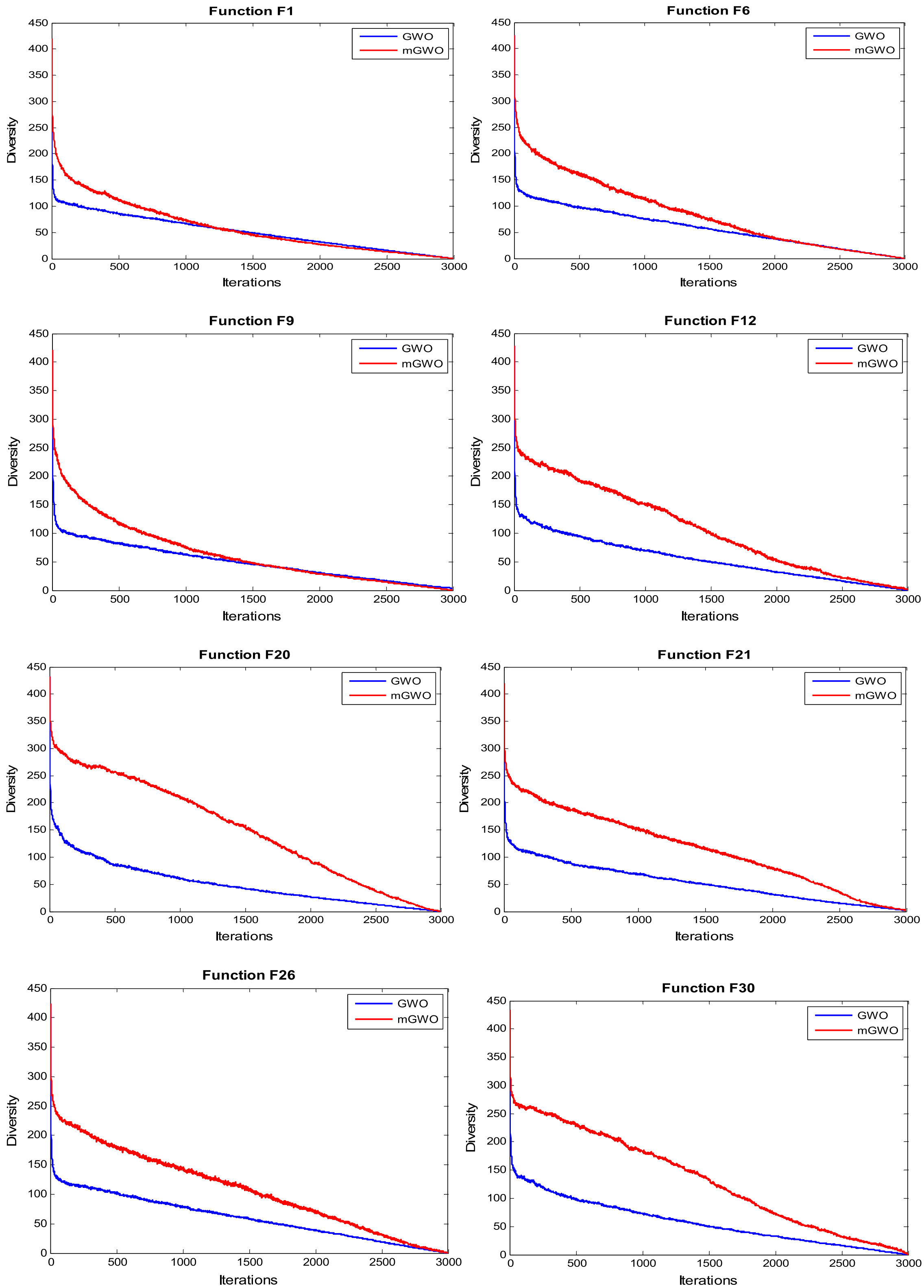

Fig. 4. Diversity curves for CEC 2017 benchmark problems.

of the CMA-ES and mGWO and superior ability of the JADE on most of the problems. Hence, the overall comparison shows that proposed mGWO has enhanced the exploration ability of the algorithm, while the comparison with CMA-ES and JADE shows that the exploitation ability of the mGWO is weak.

4.5. Convergence history analysis

The convergence behaviour of the proposed mGWO is compared with standard GWO, modGWO [23], EEGWO [24], i-GWO [33] and IGWO [25] for different category of problems such as unimodal, multimodal, hybrid and composite. The convergence

Table 5
Comparison of average error in objective function values on CEC 2014 benchmark problems.

Problem	PSO	wPSO	HS	GSA	ABC	GABC	FA	CMA-ES	JADE	modGWO	i-GWO	OBGWO	IGWO	EEGWO	mGWO
F1	8.85E+07	2.51E+07	2.39E+09	6.23E+06	7.53E+06	7.01E+06	3.65E+08	9.47E−15	4.18E+02	6.54E+06	2.53E+08	2.01E+07	1.76E+07	5.04E+08	5.58E+06
F2	9.10E+09	2.45E+09	1.31E+11	8.16E+03	9.83E+01	1.73E+01	2.07E+10	5.85E−14	2.01E−14	2.49E+07	1.29E+10	4.73E+08	5.20E+08	8.31E+09	1.80E+07
F3	3.53E+04	7.32E+03	2.48E+05	6.41E+03	4.76E+02	4.94E+02	7.95E+04	3.90E−14	4.14E−04	3.52E+03	5.30E+04	1.79E+04	8.41E+03	2.50E+04	4.50E+02
F4	4.96E+02	5.76E+02	2.67E+04	6.12E+01	1.24E+01	4.79E+00	2.48E+03	4.97E+00	8.36E−14	2.88E+01	7.67E+02	7.01E+01	4.78E+01	2.99E+03	1.29E+02
F5	2.09E+01	5.21E+02	2.11E+01	2.10E+01	2.11E+01	2.12E+01	2.29E+01	2.00E+01	2.03E+01	2.13E+01	2.10E+01	2.15E+01	2.14E+01	2.18E+01	2.09E+01
F6	2.72E+01	6.12E+02	4.48E+01	1.76E+01	1.38E+01	1.23E+01	3.64E+01	7.78E+00	8.92E+00	2.59E+00	2.82E+01	7.11E+00	4.42E+00	1.19E+01	2.25E+00
F7	8.17E+01	7.32E+02	1.07E+03	1.11E−13	2.84E−07	1.17E−07	2.22E+02	8.03E−14	1.11E−14	1.64E+00	1.22E+02	8.28E+00	1.28E+01	2.02E+02	1.02E+00
F8	1.90E+02	8.40E+02	4.99E+02	1.41E+02	1.09E−13	1.14E−13	2.26E+02	1.66E+02	0.00E+00	9.17E+00	2.03E+02	7.34E+01	2.49E+01	1.05E+02	1.76E+01
F9	2.21E+02	9.71E+02	5.77E+02	1.56E+02	7.94E+01	4.74E+01	2.55E+02	1.84E+02	2.52E+01	3.34E+01	2.95E+02	6.14E+01	2.55E+01	8.38E+01	2.48E+01
F10	6.11E+03	2.30E+03	8.95E+03	3.10E+03	1.77E−01	1.09E−01	5.44E+03	3.58E+03	6.53E−03	3.33E+02	5.08E+03	9.26E+02	6.42E+02	1.79E+03	3.59E+02
F11	6.70E+03	4.13E+03	8.38E+03	3.58E+03	3.80E+03	4.60E+03	6.04E+03	4.09E+03	1.58E+03	4.31E+03	6.02E+03	6.20E+03	7.78E+02	3.08E+03	2.28E+03
F12	2.49E+00	1.20E+03	3.66E+00	9.05E−04	1.71E−01	2.07E−01	2.14E+00	6.67E−02	2.63E−01	7.62E−01	2.63E+00	1.73E+00	9.93E−01	3.22E+00	2.41E+00
F13	1.74E+00	1.30E+03	1.00E+01	3.50E−01	2.93E−01	3.79E−01	4.01E+00	1.28E−01	2.25E−01	4.72E+00	2.74E+00	3.17E−01	3.06E−01	5.07E+00	2.39E−01
F14	2.42E+01	1.41E+03	3.84E+02	3.60E−01	3.80E−01	3.59E−01	9.01E+01	4.55E−01	2.35E−01	2.96E−01	4.12E+01	7.36E−01	7.91E−01	3.97E+01	2.15E−01
F15	1.47E+03	1.52E+03	1.89E+07	3.35E+01	6.76E+01	4.10E+01	2.38E+03	3.20E+00	3.13E+00	1.65E+01	2.99E+03	3.40E+01	2.32E+01	8.23E+03	9.22E+00
F16	1.24E+01	1.61E+03	1.37E+01	1.34E+01	9.49E+00	9.93E+00	1.32E+01	1.38E+01	9.41E+00	9.56E+00	1.25E+01	4.17E+01	3.07E+01	4.26E+01	9.33E+00
F17	2.52E+06	8.63E+05	1.57E+08	3.30E+05	1.99E+06	1.68E+06	1.26E+07	1.50E+03	1.15E+03	4.17E+05	9.14E+06	4.48E+05	8.83E+05	7.57E+05	2.54E+05
F18	6.44E+07	2.84E+05	7.52E+09	7.84E+02	3.86E+02	1.60E+03	2.13E+07	9.12E+01	2.44E+02	8.26E+03	3.94E+07	5.87E+04	8.70E+03	2.55E+06	1.92E+03
F19	4.50E+01	1.92E+03	8.51E+02	1.34E+01	6.89E+01	5.81E+01	1.49E+02	8.98E+00	4.42E+00	2.38E+01	1.00E+02	7.68E+01	3.90E+01	5.60E+01	7.49E+00
F20	8.36E+03	5.09E+03	2.71E+06	2.48E+04	6.22E+03	3.43E+03	1.05E+05	1.75E+02	3.45E+03	1.92E+03	3.90E+04	1.44E+04	4.60E+03	8.67E+05	2.37E+02
F21	7.67E+05	1.27E+05	5.13E+07	1.04E+05	1.71E+05	2.01E+05	5.57E+06	8.42E+02	1.68E+04	5.80E+05	2.44E+06	2.01E+05	1.91E+05	5.87E+06	7.59E+04
F22	6.39E+02	2.61E+03	1.25E+04	1.03E+03	2.37E+02	1.56E+02	1.14E+03	3.10E+02	1.69E+02	9.10E+02	6.68E+02	2.14E+02	1.94E+02	4.91E+02	1.49E+02
F23	3.71E+02	2.63E+03	1.75E+03	3.02E+02	3.15E+02	3.15E+02	4.21E+02	2.00E+02	3.15E+02	3.32E+02	3.69E+02	3.53E+02	3.37E+02	2.00E+02	3.16E+02
F24	2.76E+02	2.63E+03	5.28E+02	2.01E+02	2.26E+02	2.17E+02	2.05E+02	2.00E+02	2.26E+02	3.35E+02	2.00E+02	3.70E+02	3.46E+02	2.00E+02	2.00E+02
F25	2.14E+02	2.71E+03	3.36E+02	2.08E+02	2.07E+02	2.07E+02	2.14E+02	2.00E+02	2.04E+02	2.95E+02	2.07E+02	2.09E+02	1.98E+02	2.10E+02	2.06E+02
F26	1.51E+02	2.75E+03	2.32E+02	1.96E+02	1.42E+02	1.42E+02	1.91E+02	2.00E+02	1.02E+02	1.61E+02	1.83E+02	1.71E+02	1.71E+02	1.83E+02	1.20E+02
F27	9.12E+02	3.44E+03	1.56E+03	1.78E+03	4.07E+02	4.06E+02	1.05E+03	2.00E+02	3.48E+02	5.83E+02	9.56E+02	3.91E+02	3.94E+02	3.82E+02	3.64E+02
F28	1.86E+03	4.72E+03	7.35E+03	2.98E+03	9.56E+02	8.55E+02	5.96E+03	3.99E+03	7.95E+02	4.65E+02	4.23E+03	4.91E+02	3.85E+02	2.00E+02	8.46E+02
F29	1.21E+07	1.16E+07	5.80E+08	2.85E+03	9.49E+02	9.77E+02	3.49E+08	8.02E+02	7.34E+02	3.46E+05	3.81E+07	1.93E+05	1.11E+03	1.03E+06	1.15E+04
F30	5.89E+04	5.42E+04	6.06E+06	6.75E+03	6.99E+03	6.93E+03	9.99E+05	2.56E+03	1.51E+03	1.02E+04	4.54E+05	4.10E+04	1.83E+04	3.98E+05	6.39E+03
+/= /−	29/0/1	30/0/0	30/0/0	23/0/7	21/1/8	21/1/8	29/0/1	10/1/19	8/2/20	25/0/5	30/0/0	27/0/3	25/0/5	28/0/2	

Table 6
Comparison of average error in objective function values on CEC 2017 benchmark problems.

Problem	PSO	wPSO	HS	GSA	ABC	GABC	FA	CMA-ES	JADE	modGWO	i-GWO	OBGWO	IGWO	EEGWO	mGWO
F1	7.13E+09	1.36E+09	3.64E+10	2.31E+03	1.07E+02	7.75E+01	1.27E+10	3.56E−14	1.73E−20	2.01E+07	9.57E+09	8.43E+08	6.94E+08	1.52E+10	1.06E+07
F3	2.76E+04	4.86E+03	2.15E+05	6.51E+04	1.12E+05	9.44E+04	7.42E+04	4.45E−14	1.12E−27	1.55E+03	5.85E+04	8.68E+03	7.07E+03	1.61E+04	6.35E+02
F4	6.07E+02	5.95E+02	3.70E+04	7.32E+01	1.93E+01	5.96E+00	2.87E+03	5.61E+01	2.20E+01	1.15E+02	8.24E+02	4.95E+01	4.96E+01	1.49E+03	1.13E+02
F5	2.20E+02	5.75E+02	6.34E+02	1.89E+02	8.02E+01	4.51E+01	3.08E+02	2.80E+02	3.98E+01	3.60E+01	2.54E+02	5.94E+01	2.98E+01	1.42E+02	2.80E+01
F6	3.20E+01	6.02E+02	1.29E+02	2.81E+01	2.23E−15	0.00E+00	7.57E+01	6.37E+01	0.00E+00	8.41E−01	5.28E+01	3.35E+01	1.04E+01	7.56E+01	7.13E−02
F7	4.37E+02	8.08E+02	2.46E+03	6.81E+01	9.43E+01	7.01E+01	5.13E+02	6.37E+02	6.92E+01	7.08E+01	3.80E+02	1.08E+02	6.90E+01	1.50E+02	6.63E+01
F8	2.13E+02	8.68E+02	5.43E+02	1.40E+02	8.84E+01	5.12E+01	2.41E+02	1.95E+02	4.05E+01	3.38E+01	2.07E+02	4.09E+01	2.77E+01	8.44E+01	2.55E+01
F9	2.38E+03	1.05E+03	2.75E+04	5.49E+01	6.45E+02	7.84E+01	6.88E+03	4.50E+03	1.03E+00	1.22E+01	4.63E+03	1.43E+02	1.11E+02	1.36E+03	2.25E+00
F10	6.85E+03	4.14E+03	8.42E+03	3.67E+03	2.95E+03	2.85E+03	6.04E+03	4.31E+03	2.13E+03	5.56E+03	6.34E+03	1.34E+03	9.14E+03	2.55E+03	2.50E+03
F11	7.14E+02	1.32E+03	2.34E+04	1.00E+02	3.54E+02	2.12E+02	3.25E+03	1.06E+02	5.74E+01	3.48E+02	2.45E+03	3.39E+02	8.32E+01	1.01E+04	7.84E+01
F12	6.48E+08	8.54E+07	1.70E+10	5.09E+05	4.93E+05	3.23E+05	1.24E+09	1.49E+03	1.12E+03	4.79E+05	5.70E+08	2.69E+06	8.05E+05	1.09E+09	1.26E+06
F13	2.24E+08	1.08E+08	1.67E+10	3.62E+04	3.22E+03	9.23E+03	2.31E+08	9.88E+02	2.42E+02	8.83E+03	1.78E+08	7.55E+03	8.60E+03	6.09E+07	5.66E+04
F14	9.36E+04	3.41E+04	1.40E+07	7.72E+03	8.54E+04	6.46E+04	1.03E+06	1.39E+02	1.11E+02	9.07E+03	8.50E+05	3.76E+03	4.13E+03	4.37E+06	3.27E+03
F15	1.84E+07	2.32E+04	3.42E+09	5.50E+03	4.78E+02	1.15E+03	6.11E+05	2.00E+02	1.21E+02	1.22E+03	1.44E+06	2.08E+04	3.82E+03	8.69E+05	2.00E+04
F16	1.60E+03	2.40E+03	5.21E+03	1.47E+03	6.24E+02	4.82E+02	2.76E+03	4.22E+02	5.42E+02	1.59E+03	1.64E+03	5.58E+02	2.92E+02	8.89E+02	2.79E+02
F17	6.26E+02	1.98E+03	9.55E+03	1.03E+03	1.65E+02	1.06E+02	1.03E+03	2.28E+02	1.19E+02	9.46E+01	6.12E+02	1.58E+02	7.76E+01	2.97E+02	7.20E+01
F18	1.94E+06	4.19E+05	1.83E+08	9.85E+05	1.61E+05	1.27E+05	4.37E+06	1.48E+02	7.15E+01	2.32E+05	2.86E+06	7.36E+05	1.04E+05	9.98E+08	1.36E+05
F19	3.11E+07	5.02E+06	4.14E+09	3.58E+03	9.88E+02	3.71E+03	4.81E+06	1.09E+02	9.14E+01	2.76E+03	7.39E+06	1.45E+06	8.62E+03	5.48E+07	1.63E+04
F20	5.56E+02	2.27E+03	1.28E+03	1.00E+03	1.98E+02	1.47E+02	8.75E+02	1.22E+03	1.36E+02	6.41E+02	5.76E+02	1.94E+02	2.24E+02	4.09E+02	1.45E+02
F21	4.12E+02	2.39E+03	7.63E+02	3.89E+02	2.51E+02	2.34E+02	5.21E+02	2.29E+02	2.39E+02	2.98E+02	4.36E+02	2.62E+02	3.74E+02	3.32E+02	2.29E+02
F22	3.28E+03	4.09E+03	8.42E+03	1.64E+03	2.00E+02	1.23E+02	5.80E+03	2.74E+03	1.63E+02	1.31E+02	4.99E+03	1.40E+02	2.26E+02	1.36E+03	1.17E+02
F23	6.62E+02	2.88E+03	1.37E+03	8.63E+02	4.16E+02	3.95E+02	1.28E+03	1.40E+03	3.88E+02	3.98E+02	6.91E+02	3.76E+02	3.95E+02	5.73E+02	3.74E+02
F24	7.24E+02	3.08E+03	1.61E+03	6.63E+02	4.44E+02	4.73E+02	1.27E+03	4.43E+02	4.55E+02	4.44E+02	7.93E+02	4.93E+02	4.64E+02	6.04E+02	4.41E+02
F25	6.36E+02	2.91E+03	1.19E+04	4.87E+02	4.84E+02	4.84E+02	7.76E+02	3.88E+02	3.86E+02	4.31E+02	6.78E+02	4.48E+02	4.55E+02	1.10E+03	3.96E+02
F26	2.95E+03	4.40E+03	1.18E+04	1.29E+03	7.77E+02	6.42E+02	6.09E+03	2.86E+02	1.11E+03	6.09E+02	4.29E+03	4.57E+02	7.48E+02	1.99E+03	1.09E+03
F27	6.17E+02	3.30E+03	2.15E+03	8.68E+02	5.11E+02	5.05E+02	1.53E+03	3.01E+03	5.10E+02	4.04E+02	8.02E+02	4.02E+02	3.97E+02	7.41E+02	5.20E+02
F28	7.48E+02	3.34E+03	8.37E+03	3.28E+02	3.12E+02	3.10E+02	1.47E+03	3.53E+02	3.26E+02	5.42E+02	9.28E+02	5.40E+02	5.19E+02	1.13E+03	4.31E+02
F29	1.20E+03	3.63E+03	9.32E+03	1.40E+03	6.15E+02	5.21E+02	2.70E+03	6.63E+02	4.69E+02	5.84E+02	1.76E+03	4.06E+02	5.34E+02	1.01E+03	4.90E+02
F30	2.68E+07	2.68E+05	2.31E+09	3.09E+04	5.20E+03	5.24E+03	3.17E+07	2.45E+03	2.19E+03	4.91E+05	3.91E+07	3.40E+06	4.10E+05	9.79E+07	4.12E+05
+/=/−	29/0/0	28/0/1	29/0/0	21/0/8	17/2/10	14/4/11	29/0/0	15/1/13	10/0/19	21/2/6	29/0/0	22/1/6	20/1/8	28/1/0	

curves corresponding to the CEC 2014 are plotted in Figs. 5, 6 and for CEC 2017 convergence curves are plotted in Figs. 7, 8. In these figures, the growth of iterations is shown on the horizontal axis the vertical axis represents the average error in the objective function value. These curves identified the intervals that the mGWO outperforms other algorithms and converges to superior results as time continues. In addition, accelerated convergence trends can be realized in the convergence curves on the unimodal test problems for the mGWO algorithm as compared to other algorithms. This trend supports to the claim that the mGWO emphasis on the local search and further exploitation in the end of iterations. These curves also verify that the mGWO can effectively improve the fitness of all grey wolves and guarantee to exploit enhanced results. The convergence curves for multimodal problems show that the mGWO handle the exploration of the solution space more quickly as compared to the other metaheuristics. Convergence curves for hybrid and composite problems show that the mGWO maintains an appropriate balance between exploration and exploitation of the solution space by providing comparatively better transition from exploration to the exploitation phase as compared to other algorithms.

Hence, the overall analysis and comparison of results on two different benchmark set CEC 2014 and CEC 2017 indicate that the proposed mGWO is more efficient algorithm in providing the solution of optimization problems as compared to the standard GWO, other variants of GWO and some other state-of-the-art algorithms. The analysis of the results ensures that the integrated strategies such as personal best information of wolves, crossover and greedy selection strategy have improved the search efficiency of the wolf pack. The personal best information of each wolf is used in the modified hunting mechanism and in providing the search based on personal best guidance of each wolf. This information has improved the exploration skills of the wolves, which is helpful in avoiding the local optima and in locating the new promising areas of the solution space. The search based on personal best guidance of wolves is used to explore and exploit the search regions around the discovered promising and best available search regions, and helps to avoid the situation of skipping true solutions during the search process. The crossover combines the features of the personal best and leading guidance. A greedy selection strategy restores the information of best obtained areas of the solutions space. Thus, the proposed strategies, which are employed in the mGWO, have improved the search skills of the algorithm to find the solution of problems with better solution accuracy as compared to the standard GWO.

5. Engineering optimization problems

In this section, the efficiency of the proposed mGWO algorithm is investigated on real-engineering optimization problems. In these problems, constraints are handled by a simple constraint handling mechanism. This constraint handling mechanism, evaluates the constraint violation [56] value corresponding to each candidate solution (wolf). For a general optimization problem —

$$\textit{Minimize} \quad F(\boldsymbol{X}), \boldsymbol{X} = (x_1, x_2, \ldots, x_D) \in \boldsymbol{R^D} \tag{21}$$

$$s.t. \quad G_j(\boldsymbol{X}) \leq 0, \; j = 1, 2, \ldots, J \text{ (Inequality constraints)} \tag{22}$$

$$H_k(\boldsymbol{X}) = 0, \;\; k = 1, 2, \ldots, K \text{ (Equality constraints)} \tag{23}$$

$$lb_i \leq x_i \leq ub_i, \;\; i = 1, 2, \ldots, D \text{ (Bound constraints)} \tag{24}$$

the constraint violation [56] $viol_{\hat{X}}$, for a candidate solution (wolf) $\hat{X}$ can be calculated as follows:

$$viol_{\hat{X}} = \sum_{j=1}^{J} \eta_j(\hat{X}) + \sum_{k=1}^{K} \gamma_k(\hat{X}) \tag{25}$$

where,

$$\eta_j\left(\hat{X}\right) = \begin{cases} G_j\left(\hat{X}\right) & if \;\; G_j\left(\hat{X}\right) > 0 \\ 0 & otherwise \end{cases} \tag{26}$$

and

$$\gamma_k\left(\hat{X}\right) = \begin{cases} \left|H_k\left(\hat{X}\right)\right| & if \;\; \left|H_k\left(\hat{X}\right)\right| - \in > 0 \\ 0 & otherwise \end{cases} \tag{27}$$

Here, $\in$ is a tolerance parameter which should be predefined. In our experiments, the value of this parameter is fixed to 10^{-4}. In each iteration of the mGWO, first, the population of candidate solutions (wolves) is sorted according to constraint violation value. After that the feasible solution (i.e. the solutions corresponding to zero constraint violation value) are sorted according to their fitness value. Then from this sorted list the best solutions (alpha, beta and delta) can be selected to guide the search. Thus, this constraint handling technique is used only to elect the leading wolves of the pack. This constraint handling technique evaluates the guidance ability in the mGWO for constrained optimization problems. The engineering problems which are considered in the paper are presented as follows:

Problem 1. Design of gear train

The objective of this problem is to minimize the gear ratio by determining the optimal number of a tooth for four gears of a train. This problem is originally proposed by Sandgren [57]. The discrete variables y_1, y_2, y_3 and y_4 of this problem represent the teeth number for four gears. In this problem, the ratio $\frac{y_2 y_3}{y_1 y_4}$ represents the gear ratio. Mathematically, the problem is defined as follows —

$$\textit{Minimize} \quad F(\boldsymbol{Y}) = \left(\frac{1}{6.931} - \frac{y_2 y_3}{y_1 y_4}\right)^2, \quad \boldsymbol{Y} = (y_1, y_2, y_3, y_4) \tag{28}$$

$$s.t. \quad 12 \leq y_1, y_2, y_3, y_4 \leq 60 \tag{29}$$

In the paper, the proposed mGWO is applied to solve this problem using 30 independent trials and 130 function evaluations. The obtained numerical results are listed in Table 7. For the comparison, the results of standard GWO, modified variants of GWO such as modGWO [23], i-GWO [33], OBGWO [22], IGWO [25], EEGWO [24] and state-of-the-art algorithms such as PSO [48], modified PSO (wPSO) [49], HS [50], GSA [51], ABC [3], GABC [52], FA [7], CMA-ES [53] and JADE [54] are also listed in the same table by performing the same experiments with same parameter setting as used in the mGWO. The outcomes of the statistical results obtained by applying Wilcoxon test are also presented in Table 7 for the algorithms which are implemented in the paper. From the comparison of results between all these algorithm, it can be observed that the proposed mGWO performs better in terms of providing the better objective function value.

Problem 2. Parameters determination for Frequency-Modulated (FM) sound waves

In this problem [58], the aim is to determine the optimal values of six decision parameters namely y_1, y_2, y_3, y_4, y_5 and y_6 for FM synthesizer. This problem is multimodal in nature. Mathematical form of this problem is provided as follows:

$$\textit{Minimize} \quad F(\boldsymbol{Y}) = \sum_{t=1}^{100} \left(\boldsymbol{Y}(t) - \boldsymbol{Y}_0(t)\right)^2,$$

$$\boldsymbol{Y} = (y_1, y_2, y_3, y_4, y_5, y_6) \tag{30}$$

$$s.t. \quad -6.40 \leq y_i \leq 6.35 \text{ for all } i = 1, 2, 3, 4. \tag{31}$$

where

$$\boldsymbol{Y}(t) = y_1 \; \sin(y_2 t\phi + y_3 \sin(y_4 t\phi + y_5 \; \sin(y_6 t\phi))) \tag{32}$$

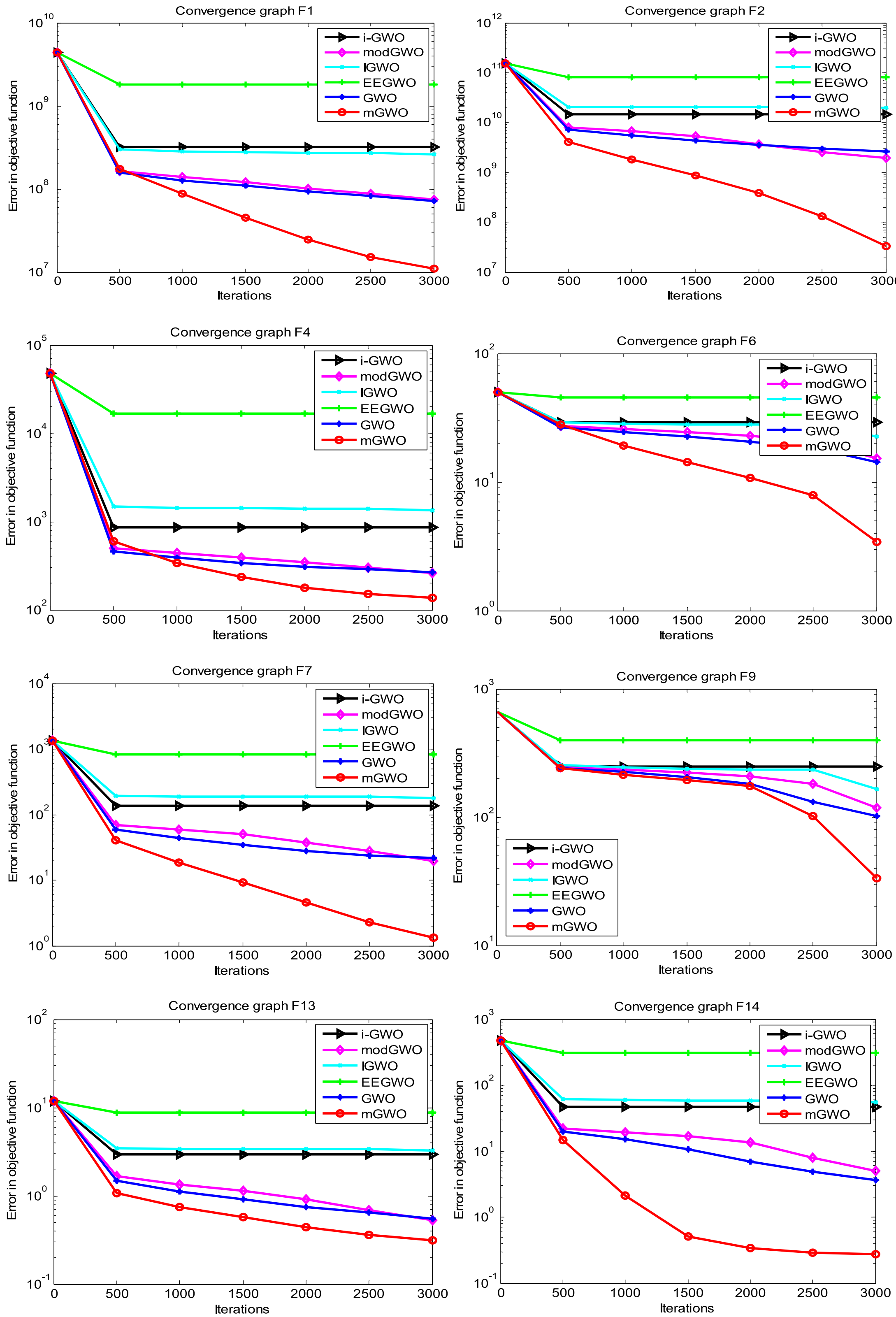

Fig. 5. Convergence history of alpha solution on CEC 2014 test problems.

$$\boldsymbol{Y}_0(t) = y_1 \sin(5t\phi + 1.5sin(4.8t\phi + 2sin(4.9t\phi))) \quad (33)$$

are the estimated sound and the target sound waves expressions. The value of ϕ is constant and equal to $2\pi/100$.

In order to determine the solution, 30 trials are performed with 2×10^5 function evaluations and the results are recorded in Table 8. To compare the results of the mGWO, the standard GWO, modGWO [23], i-GWO [33], IGWO [25], OBGWO [22], and EEGWO [24] are implemented with same parameter setting. In the same table, the results of some stat-of-the-art algorithms such as PSO [48], modified PSO (wPSO) [49], HS [50], GSA [51], ABC [3], GABC [52], FA [7], CMA-ES [53] and JADE [54] are

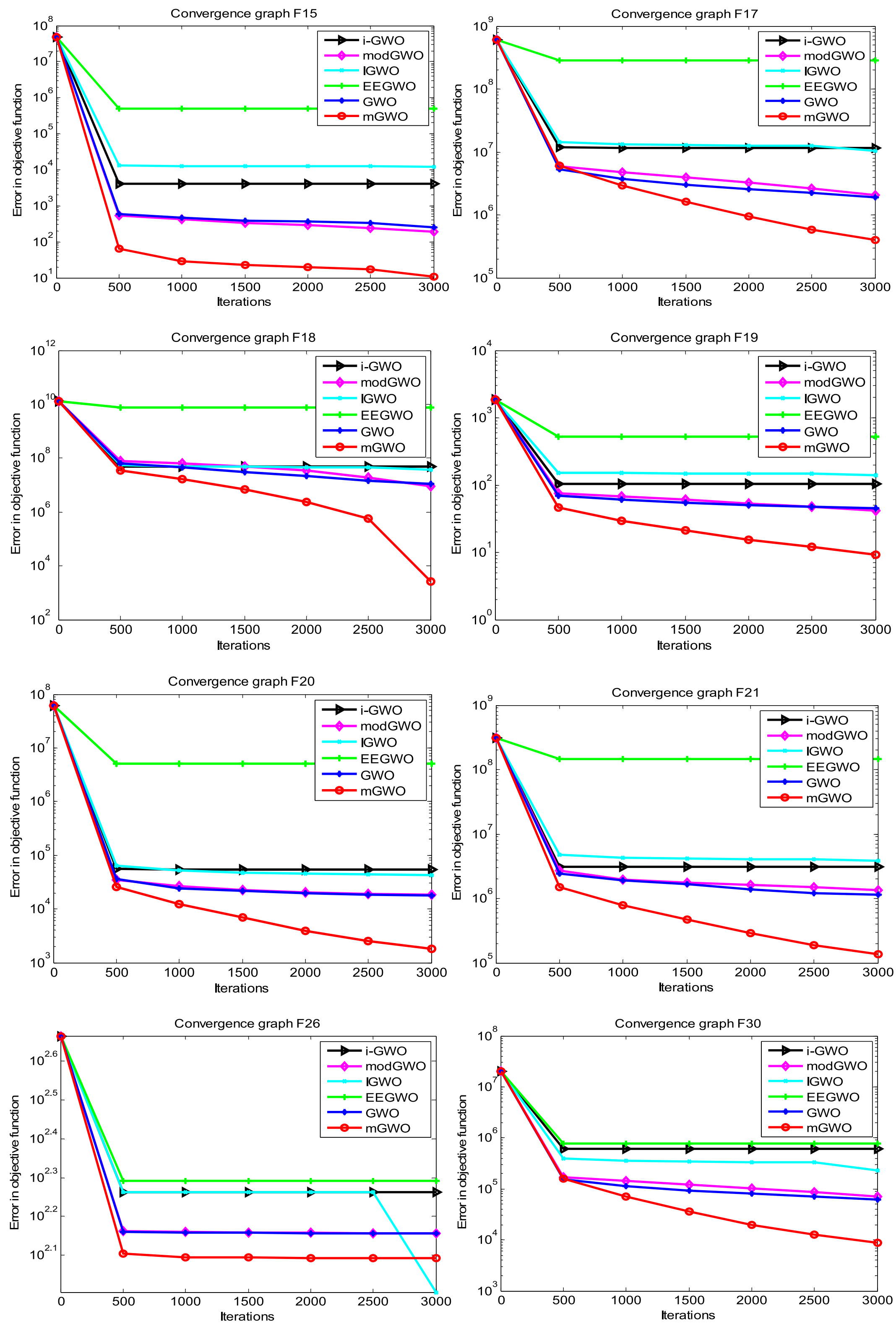

Fig. 6. Convergence history of alpha solution on CEC 2014 test problems.

also listed by using the same experiments with same parameter setting. The outcomes of the statistical results obtained by applying Wilcoxon test are also presented in Table 8 for the algorithms which are implemented in the paper. The comparison of results presented in table verifies the better search efficiency of the proposed mGWO algorithm in terms of various statistics as compared to the other algorithms.

Problem 3. Design of three-bar truss

Originally, this problem was introduced by Nowcki [59]. This case study considers a three-bar planar truss structure which is shown in Fig. 9. The aim of this study is to achieve the minimum volume of a three-bar truss, which is statically loaded and the constraints are applied on stress of each truss elements. In the

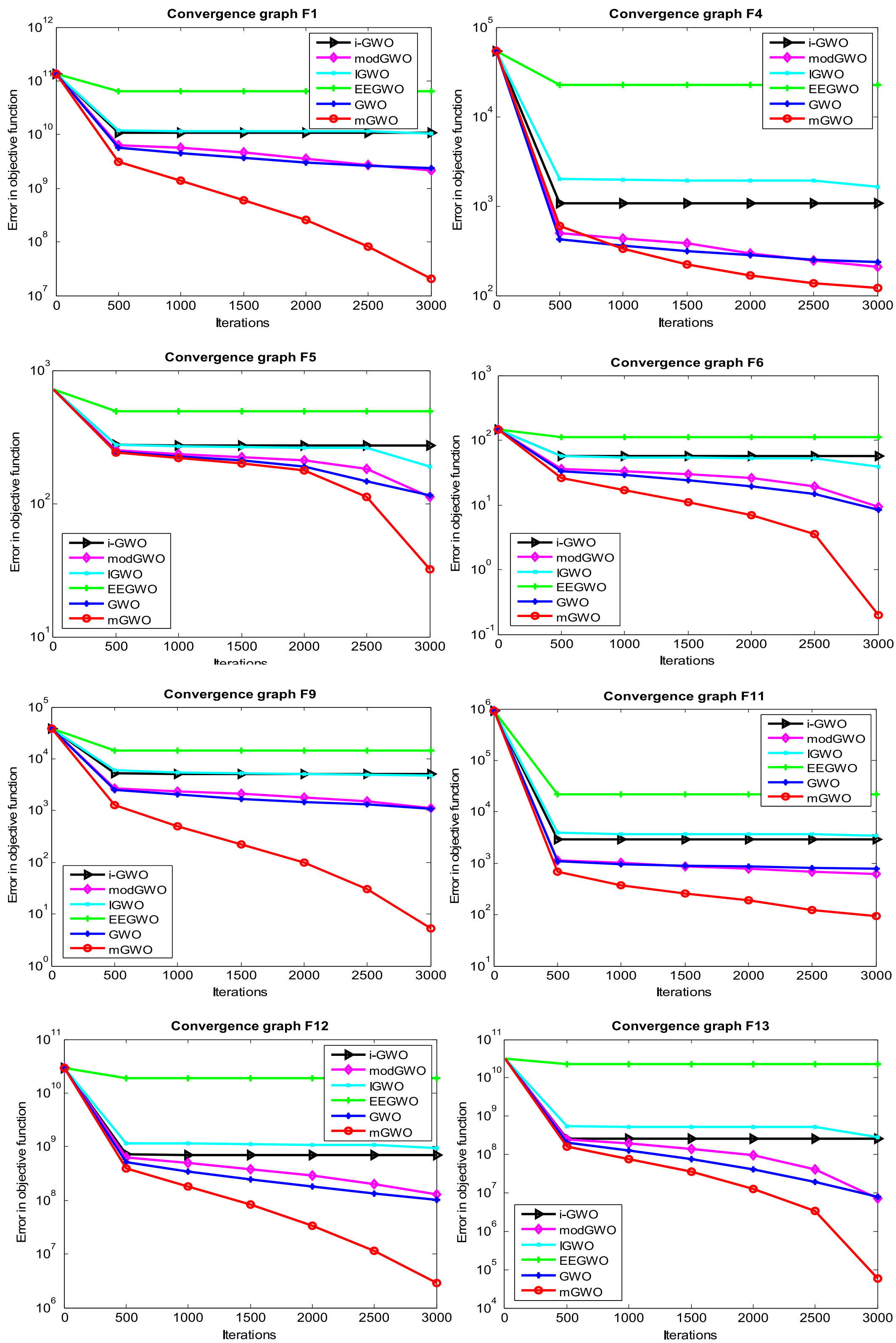

Fig. 7. Convergence history of alpha solution on CEC 2017 test problems.

mathematical form, the bar truss problem is given by

$$Minimize \quad F(\boldsymbol{Y}) = \left(2\sqrt{2}y_1 + y_2\right) \times l \tag{34}$$

$$s.t. \quad G_1(\boldsymbol{Y}) = \frac{\sqrt{2}y_1 + y_2}{\sqrt{2}{y_1}^2 + 2y_1y_2} \times r - \rho \leq 0 \tag{35}$$

$$G_2(\boldsymbol{Y}) = \frac{y_2}{\sqrt{2}{y_1}^2 + 2y_1y_2} \times r - \rho \leq 0 \tag{36}$$

$$G_3(\boldsymbol{Y}) = \frac{1}{y_1 + \sqrt{2}y_2} \times r - \rho \leq 0 \tag{37}$$

$$0 \leq y_1, y_2 \leq 1 \tag{38}$$

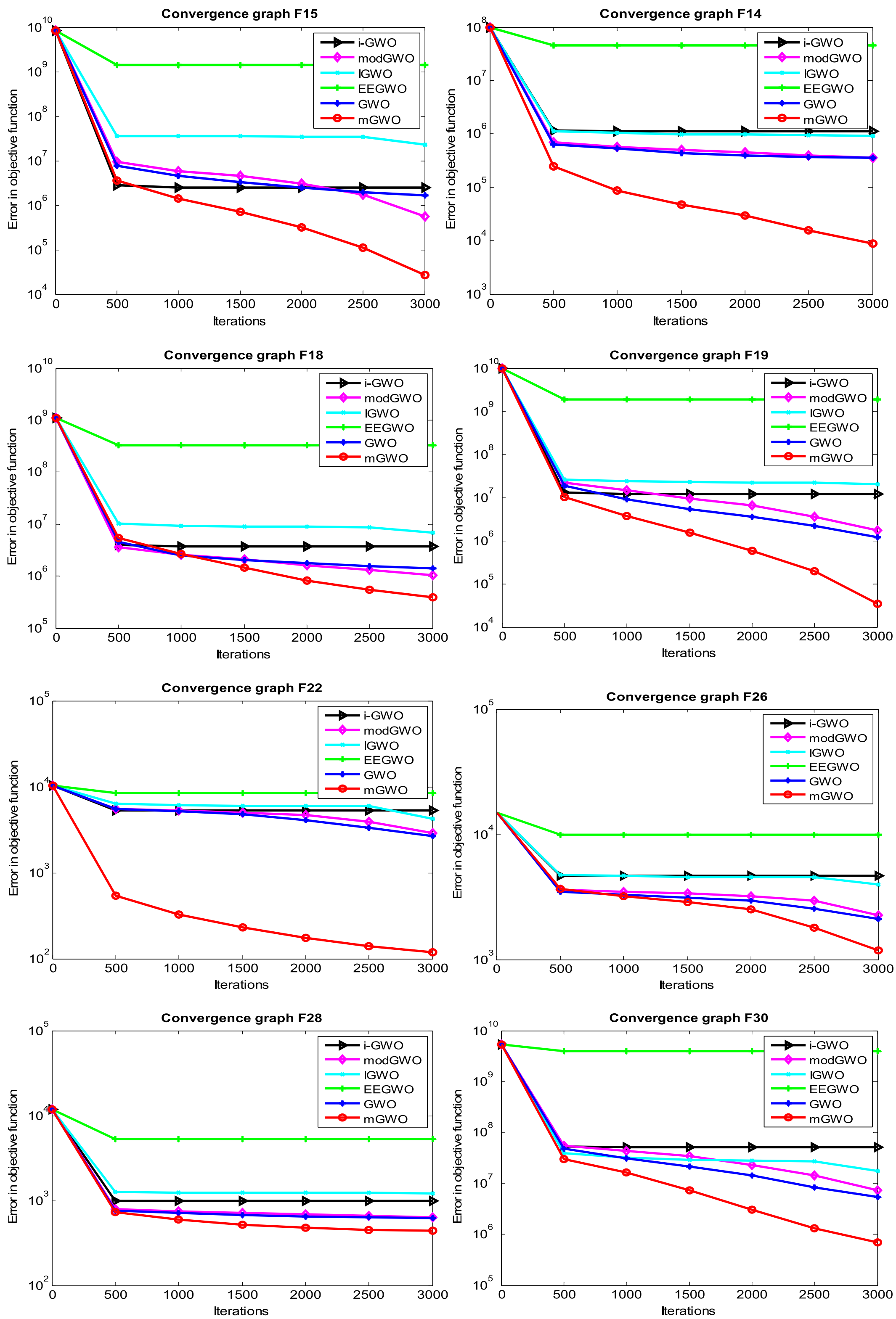

Fig. 8. Convergence history of alpha solution on CEC 2017 test problems.

$$l = 100 \text{ cm}, r = 2 \text{ KN/cm}^2 \text{ and } \rho = 2 \text{ KN/cm}^2 \quad (39)$$

In the present study, the proposed algorithm mGWO is applied to find the solution of this problem with the same parameter setting as used in [60].

The other modified variants of GWO such as modGWO [23], i-GWO [33], IGWO [25], OBGWO [22], and EEGWO [24] are also used for the comparison of results. Some other metaheuristics such as PSO [48], modified PSO (wPSO) [49], HS [50], GSA [51], ABC [3], GABC [52], FA [7], CMA-ES [53] and JADE [54] are also used to solve this problem and compared with the mGWO. The results of the mGWO and other methods are listed in Table 9. In the same table, the outcomes of the statistical results obtained by applying Wilcoxon test are also presented for the algorithms which are implemented in the paper. The comparison

Table 7
Comparison results for gear train problem.

Algorithm	Decision variables				Objective function value
	y_1	y_2	y_3	y_4	
mGWO (present study)	**43**	**16**	**19**	**49**	**2.7009E−12**
Standard GWO	55	28	17	60	1.3616E−09 (+)
modGWO	60	14	34	55	1.3616E−09 (+)
i-GWO	57	13	31	49	9.9399E−11 (+)
IGWO	57	19	16	37	1.8274E−08 (+)
OBGWO	55	17	14	30	1.3616E−09 (+)
EEGWO	37	13	18	47	9.4463E−05 (+)
PSO	57	13	31	49	9.9399E−11 (+)
wPSO	54	17	22	48	1.1661E−10 (+)
HS	52	13	31	56	3.4651E−05 (+)
GSA	38	12	26	57	5.5227E−08 (+)
ABC	44	16	19	49	2.7800E−11 (+)
GABC	46	12	26	47	9.9216E−10 (+)
FA	44	12	18	34	1.1173E−08 (+)
CMA-ES	43	16	19	49	2.7009E−12 (+)
JADE	43	16	19	49	2.7009E−12 (−)

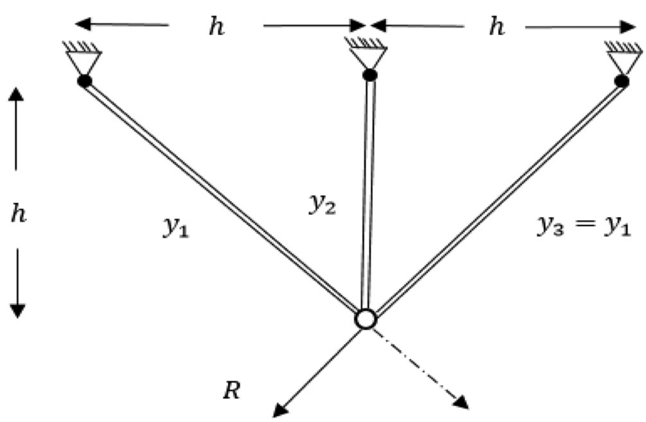

Fig. 9. Three bar truss design [60].

presented in table shows that the proposed mGWO is much better in providing the better objective function value than other algorithms.

Problem 4. Design of compression/tension spring

The goal of this problem is to achieve the minimum weight of spring with constraints on surge frequency, minimum deflection, and shear stress. In this problem, three decision variable namely wire diameter (y_1), coil diameter (y_2) and the number of active coils (y_3) are involved. Mathematically, the problem is stated as

Table 9
Comparison results for three-bar truss design problem.

Algorithm	Decision variables		Objective function value
	y_1	y_2	
mGWO (present study)	**0.7885845**	**0.4085071**	**263.8961**
Standard GWO	0.7898013	0.4050788	263.8974 (+)
modGWO	0.7878452	0.4106108	263.8974 (+)
i-GWO	0.7878452	0.4106108	263.8974 (+)
IGWO	0.8087516	0.3592847	264.6780 (+)
EEGWO	0.8087517	0.3592847	264.6780 (+)
OBGWO	0.7889477	0.4074816	263.8963 (=)
PSO	0.58959	0.20568	263.8994 (+)
wPSO	0.58959	0.20568	263.8994 (+)
HS	0.77806	0.44049	264.1631 (+)
GSA	0.78753	0.41150	263.8968 (=)
ABC	0.78497	0.41892	263.9138 (+)
GABC	0.78784	0.41062	263.8966 (+)
FA	0.81050	0.36013	265.2579 (+)
CMA-ES	0.78623	0.41608	263.9867(+)
JADE	0.78753	0.41150	263.8968 (+)

Table 10
Comparison results for spring design problem.

Algorithm	Decision variables			Objective function value
	y_1	y_2	y_3	
mGWO (present study)	**0.051640**	**0.355530**	**11.36064**	**0.012668**
Standard GWO	0.051129	0.343403	12.11483	0.012677 (+)
modGWO	0.051445	0.350456	11.69129	0.012699 (+)
i-GWO	0.051129	0.343403	12.11483	0.012677 (+)
IGWO	0.064660	0.560960	10.67685	0.029732 (+)
OBGWO	0.052334	0.372324	10.43529	0.012681 (+)
EEGWO	0.064660	0.560960	10.67685	0.029732 (+)
PSO	0.051129	0.343403	12.11483	0.012677 (+)
wPSO	0.051129	0.343403	12.11483	0.012677 (+)
HS	0.060180	0.578220	5.23780	0.015157 (+)
GSA	0.050000	0.315493	14.34524	0.012892 (+)
ABC	0.064316	0.672090	4.72910	0.018708 (+)
GABC	0.051111	0.342957	12.14630	0.012674 (=)
FA	0.075939	1.08710	2.2471	0.026625 (+)
CMA-ES	0.051129	0.343403	12.11483	0.012677 (+)
JADE	0.051129	0.343403	12.11483	0.012677 (+)

Table 8
Comparison results for FM-design problem.

Algorithm	Minimum	Maximum	Average	Std dev	Statistical outcome
mGWO (present study)	**1.94E−05**	**1.14E+01**	**1.80E+00**	4.12E+00	
Standard GWO	1.12E+01	2.51E+01	1.69E+01	5.05E+00	(+)
modGWO	1.04E−01	2.51E+01	1.38E+01	6.25E+00	(+)
i-GWO	1.02E+01	2.38E+01	1.57E+01	4.09E+00	(+)
IGWO	1.19E+01	2.51E+01	1.93E+01	4.67E+00	(+)
OBGWO	2.98E+01	2.98E+01	2.98E+01	**1.84E−03**	(+)
EEGWO	2.53E+01	3.01E+01	2.91E+01	1.40E+00	(+)
PSO	8.36E+00	2.27E+01	1.85E+01	3.31E+00	(+)
wPSO	2.53E−01	2.60E+01	1.33E+01	4.46E+00	(+)
HS	2.54E+01	3.05E+01	2.78E+01	1.69E+00	(+)
GSA	1.67E+01	2.75E+01	2.39E+01	2.77E+00	(+)
ABC	2.10E−01	1.46E+01	7.40E+00	4.65E+00	(+)
GABC	2.75E−03	1.16E+01	2.92E+00	4.62E+00	(=)
FA	1.59E+01	2.74E+01	2.41E+01	2.40E+00	(+)
CMA-ES	2.45E+01	2.95E+01	2.85E+01	1.02E+00	(+)
JADE	0.00E+00	1.7E+01	5.60E+00	6.45E+00	(−)

Table 11
Comparison results for pressure vessel design problem.

Algorithm	Decision variables				Objective function value
	y_1	y_2	y_3	y_4	
mGWO (present study)	0.8125	0.4375	42.09844	176.6366	6059.7140
Standard GWO	0.8125	0.4375	42.09840	176.6378	6059.7371 (+)
modGWO	0.8125	0.4375	42.09832	176.63837	6059.7362 (+)
i-GWO	0.8125	0.4375	42.09844	176.63660	6059.7144 (+)
IGWO	0.8125	0.4375	42.09757	176.64785	6059.8305 (+)
OBGWO	0.8125	0.4375	42.09809	176.64103	6059.7595 (+)
EEGWO	0.8125	0.4375	42.09758	176.64950	6059.8707 (+)
PSO	0.8125	0.4375	42.09130	176.74650	6061.0780 (+)
wPSO	0.8125	0.4375	42.09844	176.63650	6059.7143 (+)
HS	1.0000	0.5000	49.25302	105.6135	6705.8662 (+)
GSA	0.8125	0.4345	42.09844	176.6366	6059.7144 (+)
ABC	1.1250	0.5625	53.2127	78.2024	7395.4293 (+)
GABC	1.0625	0.5625	54.7796	68.0057	6934.8979 (+)
FA	0.8125	0.4375	42.09840	176.6378	6059.7360 (+)
CMA-ES	**0.8125**	**0.4375**	**42.09844**	**176.6366**	**6059.7140** (−)
JADE	0.8125	0.4345	42.09844	176.6366	6059.7144 (+)

follows:

$$\textit{Minimize} \quad \boldsymbol{F}(\boldsymbol{Y}) = (y_3+2)y_2y_1^2, \boldsymbol{Y} = (y_1, y_2, y_3) \tag{40}$$

$$s.t. \quad G_1(\boldsymbol{Y}) = 1 - \frac{y_2^3 y_3}{7} 1785 y_1^4 \le 0 \tag{41}$$

$$G_2(\boldsymbol{Y}) = \frac{1}{5108{y_1}^2} + \frac{4y_2^2 - y_1y_2}{12566(y_2y_1^3 - y_1^4)} - 1 \le 0 \tag{42}$$

$$G_3(\boldsymbol{Y}) = 1 - \frac{140.45y_1}{y_2^2y_3} \le 0 \tag{43}$$

$$G_4\,(Y) = \frac{y_1+y_2}{1.50} - 1 \le 0 \tag{44}$$

$$0.05 \le y_1 \le 2 \tag{45}$$

$$0.25 \le y_2 \le 1.30 \tag{46}$$

$$2 \le y_3 \le 15. \tag{47}$$

The obtained numerical results by the proposed mGWO and standard GWO are listed in Table 10. The results are achieved by conducting 30 runs and utilizing 2×10^5 functions evaluations. The modified variants of GWO such as modGWO [23], i-GWO [33], IGWO [25], OBGWO [22], and EEGWO [24] and some other metaheuristics such as PSO [48], modified PSO (wPSO) [49], HS [50], GSA [51], ABC [3], GABC [52], FA [7], CMA-ES [53] and JADE [54] are also employed to solve this problem with same parameter setting as used in the mGWO. The comparison of results between all these methods are presented in Table 10. In the same table, the outcomes of the statistical results obtained by applying Wilcoxon test are also presented for the algorithms which are implemented in the paper. The comparison shows that the proposed mGWO algorithm provides minimum weight for compression/tension spring.

Problem 5. Design of pressure vessel

The goal of this problem is to achieve the minimum cost of a cylindrical vessel which is shown in Fig. 10. The cost comprises of material, forming and welding of a pressure vessel. The vessel is capped at their ends by hemispherical heads. This problem was proposed by Kannan and Kramer [61]. The decision variables include in this problem are —

1. The thickness of the shell (y_1)
2. The thickness of the head (y_2)
3. Inner radius (y_3)
4. Length of the cylindrical section, without considering the head (y_4)

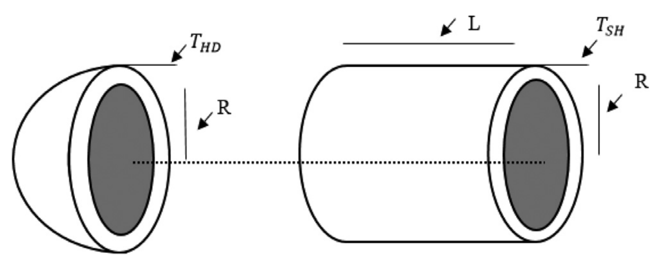

Fig. 10. Pressure vessel design [60].

In this problem, the thickness of the shell and head are discrete variables with multiple of 0.0625, while the others are continuous variables. Hence, this problem is a mixed-integer optimization problem. The mathematical formulation of this problem is as follows:

$$\textit{Minimize} \quad F(\boldsymbol{Y}) = 1.7781y_2y_3^2 + 3.1661y_1^2y_4 + 19.84y_1^2y_3 + 0.6224y_1y_3y_4 \tag{48}$$

$$s.t. \quad G_1(\boldsymbol{Y}) = 0.0193y_3 - y_1 \le 0 \tag{49}$$

$$G_2(\boldsymbol{Y}) = 0.00954y_3 - y_2 \le 0 \tag{50}$$

$$G_3(\boldsymbol{Y}) = 1.296 \times 10^6 - 4\pi y_3^3/3 - \pi y_3^2 y_4 \le 0 \tag{51}$$

$$G_4(\boldsymbol{Y}) = y_4 - 240 \le 0 \tag{52}$$

$$1 \times 0.0625 \le y_1, y_2 \le 99 \times 0.0625 \tag{53}$$

$$10 \le y_3, y_4 \le 200 \tag{54}$$

The proposed mGWO algorithm is employed to solve this problem using the same parameter setting as used in [59] by cuckoo search (CS) algorithm. The obtained results by the mGWO and standard GWO are listed in Table 11. In the same table, the comparison is also performed with other variants of GWO such as modGWO [23], i-GWO [33], IGWO [25], OBGWO [22], and EEGWO [24], and some state-of-the-art algorithms such as PSO [48], modified PSO (wPSO) [49], HS [50], GSA [51], ABC [3], GABC [52], FA) [7], CMA-ES [53] and JADE [54] by performing the same experiment with same parameter settings as used in the mGWO. In this table, the statistical results obtained by applying Wilcoxon test are also presented for the algorithms which are implemented in the paper. The comparison presented in Table 11 demonstrates that the mGWO provides a better minimum cost for a pressure vessel as compared to the other algorithms.

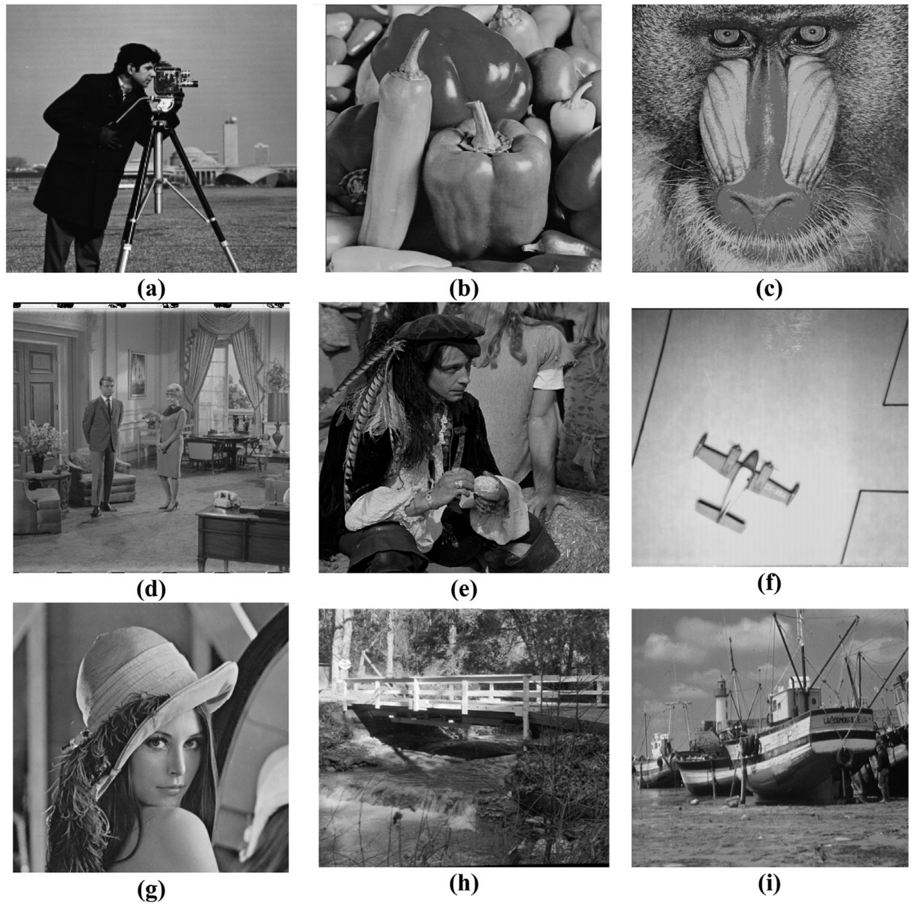

Fig. 11. Benchmark test images used for experimentation: **(a)** Cameraman **(b)** Peppers **(c)** Baboon **(d)** Couple **(e)** Male **(f)** Airplane **(g)** Lena **(h)** Bridge, and **(i)** Boat.

6. Image thresholding/segmentation

In this section, the proposed mGWO algorithm is employed to find the optimal thresholds for image segmentation. The multilevel thresholding is useful to achieve this objective. But, when the histogram of an image consists of many peaks with different heights and wide valleys then the multilevel thresholding is time consuming and tough task. In this situation, there is a higher chance of getting trapped in local optima for any search algorithm. In the literature, researchers have attempted to solve image thresholding problem using the nature-inspired algorithms. El Aziz, Ewees, and Hassanien [62] have employed the WOA and MFO algorithms for multilevel thresholding of grey images. In [63], a hybrid version of DE is implemented on grey images for their segmentation. The bacterial foraging algorithm [64] and the modified bees algorithm [65] are applied to determine the optimal number of thresholds. Dey et al. [66] have proposed quantum inspired meta-heuristics to solve the image segmentation problem.

This section focuses on image segmentation using Otsu's method [67]. The Otsu method segments the image into various levels based on threshold selection. Usually, the segmentation is achieved with the help of the histogram of an image. The histogram is a distribution of grey level pixels in the image. Sometimes, the threshold selection is not an easy task because the histogram may contain many peaks with different heights and wide valleys. The Otsu method overcomes these difficulties because it works on the pixel probabilistic within the histogram. The brief description of the Otsu method is as follows:

6.1. Otsu's method

Otsu method is an unsupervised and non-parametric thresholding method. In this method, the optimal thresholds are determined by optimizing the between-class variance criterion. For an image which is represented in L grey levels $0, 1, 2, \ldots, L-1$, the image histogram $H = \{f_0, f_1, f_2, \ldots, f_{L-1}\}$ can be constructed. Here, f_i represent the frequency of occurrence of ith gray level in the image. Let N be the total number of pixels in an image. The occurrence probability of a ith gray level is given by:

$$p_i = \frac{f_i}{N}$$

Let us suppose that k thresholds namely $t_1, t_2, \ldots, t_k$ are to be computed. Obviously, these thresholds will divide the image into $k+1$ classes say $c_0, c_1, c_2, \ldots, c_k$. The objective function in the Otsu method which is to be maximized is as follows:

$$F(t_1, t_2, \ldots, t_k) = v_0^2 + v_1^2 + \cdots v_k^2$$

where,

$$v_0^2 = w_0 (u_0 - u_T)^2, \qquad w_0 = \sum_{i=0}^{t_1-1} p_i, \qquad u_0 = \sum_{i=0}^{j=t_1-1} \frac{ip_i}{w_0}$$

$$v_j^2 = w_j \left(u_j - u_T\right)^2, \qquad w_j = \sum_{i=t_j}^{t_{j+1}-1} p_i, \qquad u_j = \sum_{i=t_j}^{t_{j+1}-1} \frac{ip_i}{w_i},$$

$$j = 1, 2, \ldots, k-1$$

Table 12
The mean of Otsu's objective function value obtained from standard GWO, modified-GWO and other algorithms.

Benchmark image	Number of thresholds	Mean objective function value			
		GWO	PSO	ABC	mGWO
Cameraman	4	3782.0800	3737.8271	3771.8107	**3782.3175**
	5	3812.4041	3773.4042	3798.0389	**3813.4974**
	6	3830.0040	3791.6750	3816.6470	**3832.3969**
Peppers	4	2766.2282	2696.6530	2754.7740	**2766.2379**
	5	2808.8498	2748.5146	2787.9382	**2810.3364**
	6	2829.1899	2778.9001	2810.0898	**2831.2240**
Baboon	4	1692.9627	1619.0066	1677.8419	**1693.0133**
	5	1713.1540	1666.5360	1700.6944	**1718.2750**
	6	1730.8888	1683.6447	1715.7104	**1734.6678**
Couple	4	1448.8281	1391.1067	1438.7446	**1449.7373**
	5	1491.6446	1444.2419	1479.1913	**1497.2713**
	6	1521.4555	1469.8872	1506.5080	**1523.9345**
Male	4	3208.6258	3146.4332	3195.8096	**3208.7604**
	5	3253.9355	3199.1213	3237.6437	**3254.4513**
	6	3280.0402	3229.6044	3260.8185	**3283.3502**
Airplane	4	1010.9275	967.7415	1008.5726	**1020.9919**
	5	1037.2432	995.5473	1029.7934	**1038.3644**
	6	1052.9095	1011.5944	1039.9817	**1054.0449**
Lena	4	2191.7178	2097.1307	2176.3960	2191.6738
	5	**2216.7204**	2152.3601	2201.0520	**2216.4099**
	6	2234.8600	2166.3381	2216.7903	**2236.5722**
Bridge	4	2820.0613	2748.6460	2809.0664	**2822.4968**
	5	2869.7317	2810.8392	2853.9110	**2873.9506**
	6	2900.7254	2832.8291	2882.2413	**2905.0196**
Boat	4	2059.6621	2001.1972	2047.6841	**2059.8283**
	5	2090.9307	2039.8359	2079.0234	**2092.5989**
	6	2113.0638	2064.9620	2097.5841	**2116.3531**

Table 13
The best Otsu's objective function value obtained from standard GWO, modified-GWO and other algorithms.

Benchmark image	Number of thresholds	Best objective function value			
		GWO	PSO	ABC	mGWO
Cameraman	4	3782.3705	3776.6214	3780.8018	**3782.3976**
	5	3813.7391	3800.1559	3811.4167	**3813.7418**
	6	3833.6736	3818.4972	3828.4541	**3833.7146**
Peppers	4	2766.4586	2747.3818	2764.2174	2766.4586
	5	2810.8420	2793.1856	2803.7945	2810.8420
	6	2833.5842	2825.3933	2823.0225	**2833.5862**
Baboon	4	1693.1954	1688.4025	1691.6686	1693.1954
	5	1719.0301	1703.3024	1716.1659	**1719.0571**
	6	1736.5019	1722.6346	1728.1128	**1736.5923**
Couple	4	1449.8067	1437.6089	1448.8182	1449.8067
	5	1497.6073	1483.2765	1491.5670	**1497.6237**
	6	1525.7313	1505.8694	1522.9185	**1525.8209**
Male	4	3208.8095	3207.5599	3205.7587	3208.8095
	5	3254.7616	3244.5812	3252.7128	3254.7616
	6	3284.0619	3266.7235	3275.4299	**3284.0964**
Airplane	4	1021.2093	1011.7049	1018.8018	**1021.2194**
	5	1041.4812	1031.9770	1040.5528	**1041.5212**
	6	1055.1598	1041.5189	1049.6411	**1055.2440**
Lena	4	2191.8700	2180.8967	2191.0536	2191.8700
	5	2217.6837	2189.7892	2210.0882	2217.6837
	6	2238.2473	2222.5283	2231.5596	**2238.3458**
Bridge	4	2822.7086	2811.8618	2821.3861	2822.7086
	5	2874.2447	2851.2052	2864.2435	2874.2447
	6	2906.0065	2885.6842	2902.7476	**2906.0379**
Boat	4	2059.8663	2054.1751	2059.4383	2059.8663
	5	2092.7686	2076.2477	2091.3862	2092.7686
	6	2117.2700	2102.7990	2110.3698	**2117.2708**

Table 14
The location of thresholds obtained from standard GWO and proposed modified-GWO.

Benchmark image	Number of thresholds	Position of thresholds	
		GWO	mGWO
Cameraman	4	43, 96, 140, 170	42, 95, 140, 170
	5	36, 83, 122, 149, 173	36, 82, 122, 149, 173
	6	36, 80, 119, 146, 169, 202	36, 81, 120, 147, 170, 200
Peppers	4	47, 86, 126, 169	47, 86, 126, 169
	5	43, 79, 113, 146, 177	43, 79, 113, 146, 177
	6	41, 76, 104, 133, 160, 185	40, 75, 104, 130, 157, 183
Baboon	4	72, 106, 137, 168	72, 106, 137, 168
	5	68, 99, 125, 150, 175	68, 100, 126, 150, 175
	6	58, 87, 111, 133, 156, 179	60, 87, 110, 133, 156, 178
Couple	4	63, 103, 137, 179	63, 103, 137, 179
	5	59, 98, 129, 158, 205	59, 98, 129, 159, 205
	6	54, 89, 117, 139, 165, 209	55, 92, 119, 140, 166, 211
Male	4	35, 82, 124, 164	35, 82, 124, 164
	5	28, 65, 100, 134, 171	28, 65, 100, 134, 171
	6	24, 57, 89, 119, 145, 178	24, 56, 88, 118, 145, 178
Airplane	4	95, 154, 185, 210	94, 154, 185, 210
	5	93, 146, 172, 195, 214	84, 143, 171, 194, 213
	6	12, 103, 149, 174, 195, 214	37, 104, 148, 174, 196, 214
Lena	4	75, 114, 145, 180	75, 114, 145, 180
	5	72, 108, 136, 160, 188	72, 108, 136, 160, 188
	6	62, 90, 115, 140, 162, 189	63, 89, 115, 140, 162, 189
Bridge	4	65, 102, 146, 191	65, 102, 146, 191
	5	56, 86, 121, 155, 200	56, 86, 121, 155, 200
	6	49, 76, 98, 127, 160, 202	46, 77, 105, 131, 165, 203
Boat	4	65, 114, 147, 179	65, 114, 147, 179
	5	50, 90, 126, 152, 183	50, 90, 126, 152, 183
	6	48, 84, 118, 143, 162, 191	47, 82, 117, 142, 161, 190

Table 15
The mean PSNR value obtained from standard GWO, modified-GWO and other algorithms.

Benchmark image	No. of thresholds	PSNR value	
		GWO	mGWO
Cameraman	4	21.4137	**21.4871**
	5	23.1702	**23.2495**
	6	23.7079	**23.7767**
Peppers	4	20.6544	**20.6561**
	5	22.2486	**22.3225**
	6	23.3188	**23.3674**
Baboon	4	**20.3215**	20.2718
	5	21.7504	**21.8390**
	6	23.5768	**23.6722**
Couple	4	20.3594	**20.4075**
	5	20.7750	**21.4731**
	6	22.0446	**22.6521**
Male	4	20.9470	**20.9783**
	5	22.5353	**22.5801**
	6	23.7080	**23.9109**
Airplane	4	20.5148	**21.5133**
	5	22.8188	**23.1958**
	6	24.4135	**24.5911**
Lena	4	18.6423	**18.6414**
	5	19.6393	**19.6555**
	6	21.1039	**20.7395**
Bridge	4	18.9760	**18.9918**
	5	**20.5914**	20.5905
	6	22.0819	**22.1745**
Boat	4	20.2888	**20.2906**
	5	22.0240	**22.0545**
	6	23.2083	**23.2365**

$$v_k^2 = w_k\left(u_k - u_T\right)^2, \qquad w_k = \sum_{i=t_k}^{L-1} p_i, \qquad u_k = \sum_{i=t_k}^{L-1} \frac{ip_i}{w_i}$$

$v_0, v_1, \ldots, v_k$ are the variances, $w_0, w_1, \ldots, w_k$ are the class probabilities, $u_0, u_1, \ldots, u_k$ are the mean intensity values of the pixels in of the segmented classes $c_0, c_1, c_2, \ldots, c_k$. u_T is the average intensity for the image which can be calculated as: $u_T = \sum_{i=0}^{k} w_i u_i$ and $\sum_{i=0}^{k} w_i$.

Since the Otsu's method provides an objective function which is to be maximized. This objective function can be transformed into minimization type as follows:

$$F'\left(t_1, t_2, \ldots, t_k\right) = \frac{1}{1 + F\left(t_1, t_2, \ldots, t_k\right)}$$

6.2. Experiments and results

This section presents the experimental setup for the proposed mGWO algorithm to apply it on an image thresholding problem. The proposed mGWO is used to optimize the class variance defined by the Otsu method. The benchmark test images are introduced first followed by the adopted parameter settings. Results and discussion are presented later on in this section.

6.2.1. Benchmark test images

In this paper, the proposed mGWO algorithm is evaluated on nine well-known benchmark grey images, collected from the USC-SIPI image database. The selected test images are presented in Fig. 11.

6.2.2. Experimental setting

The proposed mGWO algorithm is implemented on the benchmark images with 12 wolf pack size and 100 iterations. The

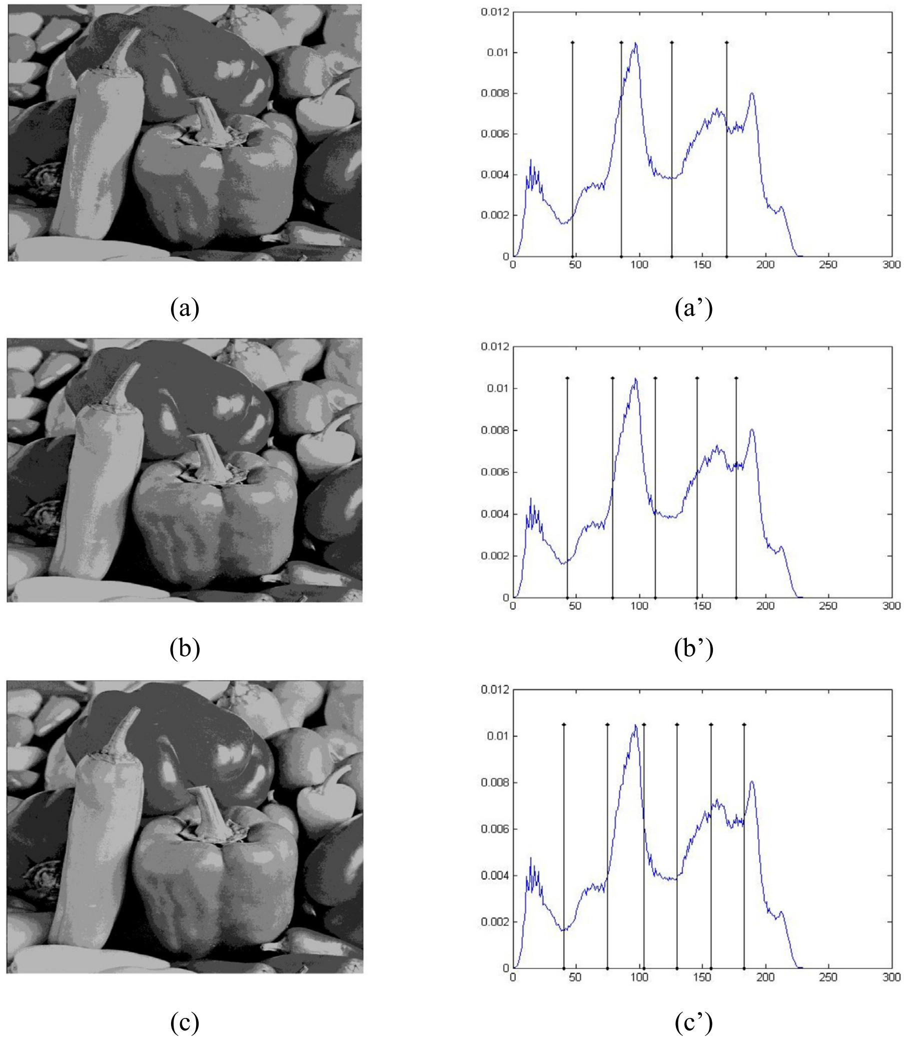

Fig. 12. Segmented results for Peppers test image using proposed mGWO. (a)–(c) represent the segmented image into 5, 6 and 7 levels, and (a')–(c') represent the histogram with location of thresholds.

Table A.1
Statistical outcomes along with *p*-values for benchmark suite I.

Problem	Statistical results		Problem	Statistical results	
	D = 10	D = 30		D = 10	D = 30
F1	5.80E−10 (+)	5.14E−10 (+)	F16	7.35E−10 (+)	7.73E−09 (+)
F2	1.39E−03 (+)	5.14E−10 (+)	F17	6.35E−08 (+)	5.85E−09 (+)
F3	5.14E−10 (+)	5.14E−10 (+)	F18	1.04E−09 (+)	4.41E−09 (+)
F4	0 (=)	5.14E−10 (+)	F19	1.57E−09 (+)	5.14E−10 (+)
F5	0 (=)	0 (=)	F20	1.06E−07 (+)	5.14E−10 (+)
F6	5.14E−10 (+)	5.14E−10 (+)	F21	1.11E−09 (+)	1.76E−09 (+)
F7	4.63E−08 (+)	5.14E−10 (+)	F22	5.79E−10 (+)	3.94E−09 (+)
F8	5.14E−10 (+)	5.14E−10 (+)	F23	9.30E−10 (+)	5.14E−10 (+)
F9	7.35E−10 (+)	5.14E−10 (+)	F24	5.14E−10 (+)	5.14E−10 (−)
F10	9.30E−10 (+)	5.14E−10 (+)	F25	2.10E−06 (+)	3.48E−06 (+)
F11	5.14E−10 (+)	6.45E−04 (+)	F26	0 (=)	6.24E−06 (+)
F12	9.68E−04 (−)	1.41E−04 (−)	F27	5.06E−03 (+)	5.14E−10 (+)
F13	6.14E−07 (+)	7.35E−10 (+)	F28	9.93E−07 (+)	1.04E−09 (+)
F14	7.31E−09 (+)	1.20E−08 (+)	F29	2.57E−08 (+)	8.24E−08 (+)
F15	2.77E−06 (+)	2.36E−09 (+)	F30	5.81E−04 (+)	5.14E−10 (+)

experiments are conducted on test images with a various number of threshold values 4, 5 and 6. The threshold values lower than these, all the presented algorithms perform equal. For a fair comparison with the standard GWO and other metaheuristic algorithms, the same parameter setting is used. The 30 trials of each algorithm are performed on each test images. The best and mean value of the objective function is used to analyse the search-accuracy of the proposed algorithm. In order to analyse the quality of the images, the Peak Signal-to-Noise Ratio (PSNR) metric is used. The mean of objective function value represents the robustness of the algorithm in finding the thresholds. The

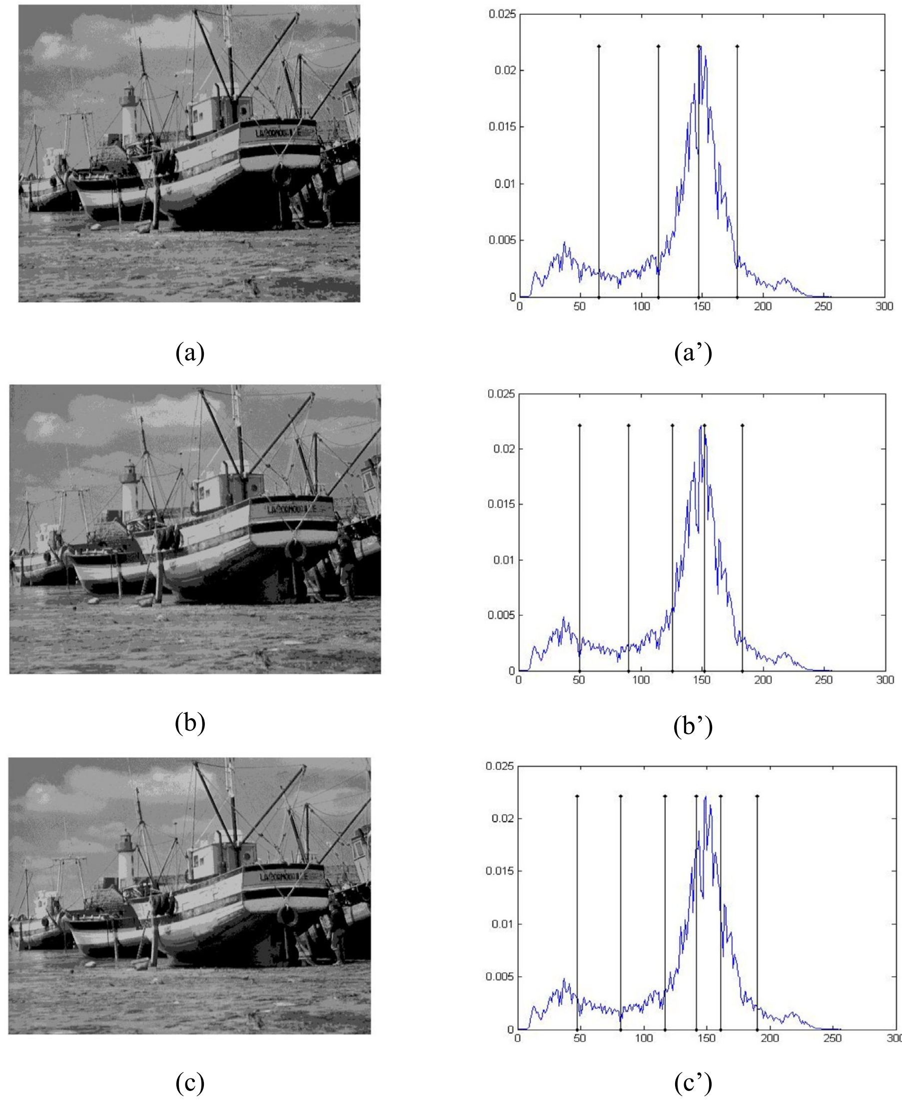

Fig. 13. Segmented results for Boat test image using proposed mGWO. (a)–(c) represent the segmented image into 5, 6 and 7 levels, and (a')–(c') represent the histogram with location of thresholds.

Table A.2
Statistical outcomes along with *p*-values for benchmark suite II.

Problem	Statistical results		Problem	Statistical results	
	D = 10	D = 30		D = 10	D = 30
F1	1.57E−09 (+)	5.14E−10 (+)	F17	3.72E−09 (+)	5.14E−10 (+)
F3	5.14E−10 (+)	5.14E−10 (+)	F18	9.30E−10 (+)	6.19E−09 (+)
F4	3.75E−08 (+)	2.96E−09 (+)	F19	6.92E−10 (+)	3.94E−09 (+)
F5	6.15E−10 (+)	6.15E−10 (+)	F20	6.92E−10 (+)	5.79E−10 (+)
F6	1.48E−09 (+)	5.14E−10 (+)	F21	1.27E−08 (+)	5.14E−10 (+)
F7	4.94E−09 (+)	1.48E−09 (+)	F22	2.79E−07 (+)	5.14E−10 (+)
F8	5.14E−10 (+)	5.14E−10 (+)	F23	5.79E−10 (+)	5.14E−10 (+)
F9	6.52E−10 (+)	5.14E−10 (+)	F24	1.01E−07 (+)	5.14E−10 (+)
F10	7.35E−10 (+)	5.24E−04 (+)	F25	3.77E−03 (+)	5.14E−10 (+)
F11	5.14E−10 (+)	5.14E−10 (+)	F26	2.17E−05 (+)	5.14E−10 (+)
F12	1.12E−07 (+)	5.79E−10 (+)	F27	0 (=)	2.80E−09 (+)
F13	1.57E−09 (+)	3.45E−03 (+)	F28	5.14E−10 (+)	5.14E−10 (+)
F14	9.87E−10 (+)	5.14E−10 (+)	F29	2.96E−09 (+)	5.14E−10 (+)
F15	6.92E−10 (+)	1.52E−06 (+)	F30	3.94E−09 (+)	7.35E−10 (+)
F16	5.14E−10 (+)	1.40E−09 (+)			

PSNR value of an image is calculated as follows:

$$PSNR = 10 \log_{10} \frac{255^2}{ME}$$

where

$$ME = \frac{1}{mn} \sum_{j=1}^{m} \sum_{k=1}^{n} (I(j, k) - S(j, k))^2$$

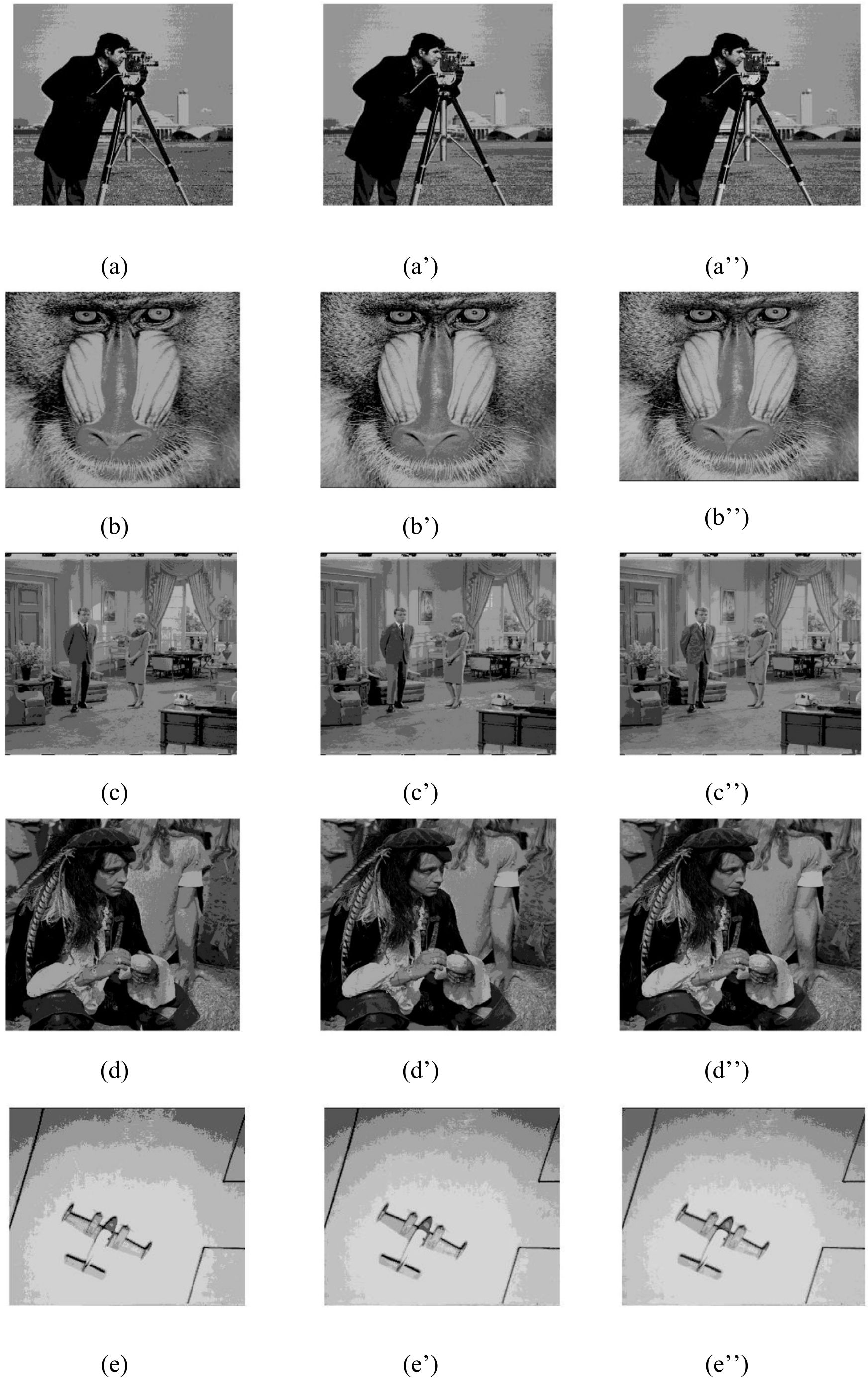

Fig. 14. Segmented test images using the proposed mGWO algorithm. (a)–(g) represent the segmentation into 5-levels, (a')–(g') represent the segmentation into 6-levels, and (a'')–(g'') represent the segmentation into 7-levels respectively.

where S and I are the segmented and original images respectively.

6.2.3. Results and discussion

This section provides the numerical results achieved by minimizing between-class variance criteria used in Otsu method. The mean and best objective function values corresponding to the test images are presented in Tables 12 and 13. The threshold positions obtained from the proposed mGWO algorithm and standard GWO are presented in Table 14. From the table, it can be seen that the mGWO obtains a better mean objective fitness as compared to the standard GWO in all the test images. In most of the test images, the obtained best objective function value is preferable in the mGWO than the standard GWO. This shows the superior performance of the proposed algorithm. In Tables 12 and 13 the results of the mGWO are compared with ABC and PSO. The obtained PSNR value is presented in Table 15. The PSNR value shows that the mGWO is a more reliable algorithm than standard GWO to segment the images in multiple thresholds. The histogram of an image and the threshold positions are shown for the test image Peppers and Boat in Figs. 12 and 13. In Fig. 14, the segmented test images are shown corresponding to the five, six and seven levels. Thus from analysing the experimental results through various metric, it can be concluded that the mGWO algorithm determines

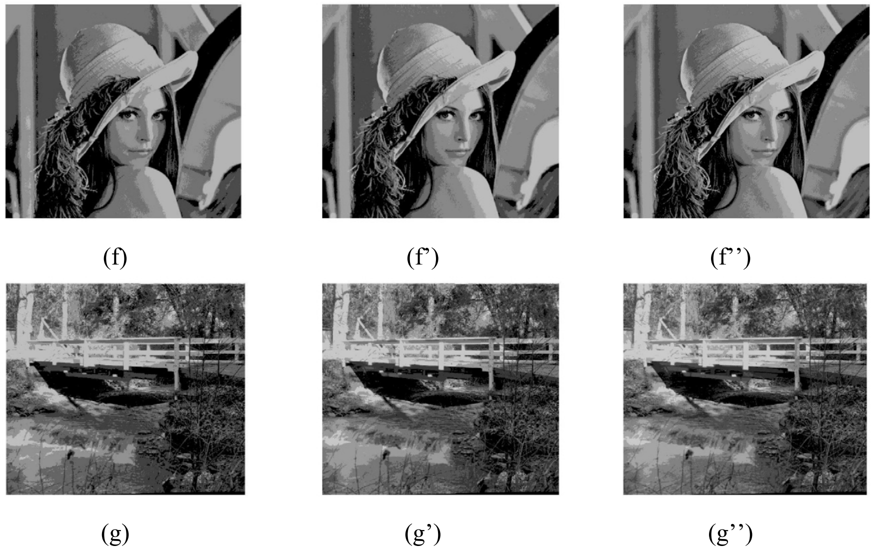

Fig. 14. (*continued*).

the threshold locations more accurately than the standard GWO and other metaheuristics.

7. Conclusions

The present study proposes a modified version of GWO called Memory-based GWO (mGWO). In the mGWO, the personal best history of each wolf and the elite wolf of the pack are simultaneously used to update their positions. The crossover and greedy selection mechanisms are also employed during the search to include the contribution of each wolf and to avoid the ignorance of obtained promising areas of the search space. These strategies enhance the collaborative strength of the pack and maintain and appropriate balance between exploitation and exploration. The investigation of the proposed mGWO is done on standard and complex benchmarks IEEE CEC 2014 and IEEE CEC 2017. The analysis of the results in terms of statistical analysis, diversity analysis and convergence analysis demonstrate the superior performance and better solution accuracy of the mGWO than the standard GWO in all type of problems such as unimodal, multimodal, hybrid and composite. Also, the comparison with variants of the GWO and other metaheuristics illustrate the competitive and better search ability of the mGWO. Further, the proposed mGWO is used to solve some real engineering design problems. The results and their comparison with other algorithms also shows the better solution accuracy of the mGWO than others. In the end, the mGWO is employed on the problem of determining the optimal thresholds for multilevel thresholding. The results ensure that the mGWO can be used for multilevel thresholding as it has outperformed standard GWO and other algorithms.

Overall, from the numerical experiments and analysis on benchmark problems and real-world application problems, it can be concluded that the mGWO has enhanced the search efficiency of the wolf pack in terms of providing better search directions and collaborative strength. The performance comparison of the mGWO with other algorithms shows its the enhanced exploration ability. When the comparison is analysed with CMA-ES and JADE, it has been observed that the exploitation ability can be enhanced further to tackle unimodal problems with high accuracy.

In the future, the proposed mGWO algorithm can be applied to solve various real-life problems like image enhancement, watermarking, etc. The capability of the leading wolves can be improved through various local and global search operators. Since the present work proposes the mGWO for solving single objective optimization problems only. Therefore, in future, this method can be extended to solve multiobjective optimization and binary optimization problems.

CRediT authorship contribution statement

Shubham Gupta: Conceptualization, Methodology, Writing - original draft, Validation, Writing - review & editing. **Kusum Deep:** Writing - review & editing, Visualization, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

The first author would like to thank Ministry of Human Resources, Government of India for funding this research. Grant No. MHR-02-41-113-429.

Appendix

See Tables A.1 and A.2.

References

[1] M. Črepinšek, S.H. Liu, M. Mernik, Exploration and exploitation in evolutionary algorithms: A survey, ACM Comput. Surv. 45 (3) (2013) 35.
[2] R. Eberhart, J. Kennedy, A new optimizer using particle swarm theory, in: Micro Machine and Human Science, 1995. MHS'95. Proceedings of the Sixth International Symposium on, IEEE, 1995, pp. 39–43.
[3] D. Karaboga, B. Basturk, A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm, J. Global Optim. 39 (3) (2007) 459–471.
[4] M. Dorigo, M. Birattari, Ant Colony Optimization, Springer, US, 2010, pp. 36–39.
[5] X.S. Yang, S. Deb, Cuckoo search via Lévy flights, in: 2009 World Congress on Nature & Biologically Inspired Computing, NaBIC, IEEE, 2009, pp. 210–214.
[6] X.S. Yang, A new metaheuristic bat-inspired algorithm, in: Nature Inspired Cooperative Strategies for Optimization, NICSO 2010, Springer, Berlin, Heidelberg, pp. 65-74.
[7] X.S. Yang, Firefly algorithm, stochastic test functions and design optimisation, 2010, arXiv preprint arXiv:1003.1409.

[8] S. Mirjalili, S.M. Mirjalili, A. Lewis, Grey wolf optimizer, Adv. Eng. Softw. 69 (2014) 46–61.

[9] S. Mirjalili, Moth-flame optimization algorithm: A novel nature-inspired heuristic paradigm, Knowl.-Based Syst. 89 (2015) 228–249.

[10] S. Mirjalili, A. Lewis, The whale optimization algorithm, Adv. Eng. Softw. 95 (2016) 51–67.

[11] S. Mirjalili, A.H. Gandomi, S.Z. Mirjalili, S. Saremi, H. Faris, S.M. Mirjalili, Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems, Adv. Eng. Softw. 114 (2017) 163–191.

[12] M. Jain, V. Singh, A. Rani, A novel nature-inspired algorithm for optimization: Squirrel search algorithm, Swarm Evol. Comput. 44 (2019) 148–175.

[13] A.A. Heidari, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, H. Chen, Harris hawks optimization: Algorithm and applications, Future Gener. Comput. Syst. 97 (2019) 849–872.

[14] N. Jayakumar, S. Subramanian, S. Ganesan, E.B. Elanchezhian, Grey wolf optimization for combined heat and power dispatch with cogeneration systems, Int. J. Electr. Power Energy Syst. 74 (2016) 252–264.

[15] V.K. Kamboj, S.K. Bath, J.S. Dhillon, Solution of non-convex economic load dispatch problem using grey wolf optimizer, Neural Comput. Appl. 27 (5) (2016) 1301–1316.

[16] N. Muangkote, K. Sunat, S. Chiewchanwattana, An improved grey wolf optimizer for training q-Gaussian Radial Basis Functional-link nets, in: Computer Science and Engineering Conference (ICSEC) 2014 International, IEEE, pp. 209–214, IEEE.

[17] M.H. Sulaiman, Z. Mustaffa, M.R. Mohamed, O. Aliman, Using the gray wolf optimizer for solving optimal reactive power dispatch problem, Appl. Soft Comput. 32 (2015) 286–292.

[18] G.M. Komaki, V. Kayvanfar, Grey wolf optimizer algorithm for the two-stage assembly flow shop scheduling problem with release time, J. Comput. Sci. 8 (2015) 109–120.

[19] X. Song, L. Tang, S. Zhao, X. Zhang, L. Li, J. Huang, W. Cai, Grey wolf optimizer for parameter estimation in surface waves, Soil Dyn. Earthq. Eng. 75 (2015) 147–157.

[20] E. Emary, H.M. Zawbaa, A.E. Hassanien, Binary grey wolf optimization approaches for feature selection, Neurocomputing 172 (2016) 371–381.

[21] A.A. Heidari, P. Pahlavani, An efficient modified grey wolf optimizer with Lévy flight for optimization tasks, Appl. Soft Comput. 60 (2017) 115–134.

[22] M. Pradhan, P.K. Roy, T. Pal, Oppositional based grey wolf optimization algorithm for economic dispatch problem of power system, Ain Shams Eng. J. 9 (4) (2017) 2015–2025.

[23] N. Mittal, U. Singh, B.S. Sohi, Modified grey wolf optimizer for global engineering optimization, Appl. Comput. Intell. Soft Comput. (2016) 8, http://dx.doi.org/10.1155/2016/7950348.

[24] W. Long, J. Jiao, X. Liang, M. Tang, An exploration-enhanced grey wolf optimizer to solve high-dimensional numerical optimization, Eng. Appl. Artif. Intell. 68 (2018) 63–80.

[25] W. Long, X. Liang, S. Cai, J. Jiao, W. Zhang, A modified augmented Lagrangian with improved grey wolf optimization to constrained optimization problems, Neural Comput. Appl. 28 (1) (2017) 421–438.

[26] R.A. Ibrahim, M.A. Elaziz, S. Lu, Chaotic opposition-based grey-wolf optimization algorithm based on differential evolution and disruption operator for global optimization, Expert Syst. Appl. 108 (2018) 1–27.

[27] W. Long, J. Jiao, X. Liang, S. Cai, M. Xu, A random opposition-based learning grey wolf optimizer, IEEE Access 7 (2019) 113810–113825.

[28] R. Salgotra, U. Singh, S. Sharma, On the improvement in grey wolf optimization, Neural Comput. Appl. (2019) 1–40, http://dx.doi.org/10.1007/s00521-019-04456-7.

[29] A. Saxena, B.P. Soni, R. Kumar, V. Gupta, Intelligent grey wolf optimizer–development and application for strategic bidding in uniform price spot energy market, Appl. Soft Comput. 69 (2018) 1–13.

[30] S. Gupta, K. Deep, A novel random walk grey wolf optimizer, Swarm Evol. Comput. 44 (2019) 101–112.

[31] S. Gupta, K. Deep, Enhanced leadership-inspired grey wolf optimizer for global optimization problems, Eng. Comput. (2019) 1–24, http://dx.doi.org/10.1007/s00366-019-00795-0.

[32] L. Rodríguez, O. Castillo, J. Soria, P. Melin, F. Valdez, C.I. Gonzalez, J. Soto, A fuzzy hierarchical operator in the grey wolf optimizer algorithm, Appl. Soft Comput. 57 (2017) 315–328.

[33] W. Long, J. Jiao, X. Liang, M. Tang, Inspired grey wolf optimizer for solving large-scale function optimization problems, Appl. Math. Model. 60 (2018) 112–126.

[34] L. Rodríguez, O. Castillo, J. Soria, Grey wolf optimizer with dynamic adaptation of parameters using fuzzy logic, in: 2016 IEEE Congress on Evolutionary Computation, CEC, IEEE, pp. 3116-3123.

[35] L. Rodríguez, O. Castillo, M. García, J. Soria, A comparative study of dynamic adaptation of parameters in the GWO algorithm using type-1 and interval type-2 fuzzy logic, in: Fuzzy Logic Augmentation of Neural and Optimization Algorithms: Theoretical Aspects and Real Applications, Springer, Cham, 2018, pp. 3–16.

[36] N. Singh, S.B. Singh, A novel hybrid GWO-SCA approach for optimization problems, Eng. Sci. Technol. Int. J. 20 (6) (2017) 1586–1601.

[37] M.A. Tawhid, A.M. Ibrahim, A hybridization of grey wolf optimizer and differential evolution for solving nonlinear systems, Evol. Syst. 1–23 (2019) http://dx.doi.org/10.1007/s12530-019-09291-8.

[38] S. Gupta, K. Deep, Improved sine cosine algorithm with crossover scheme for global optimization, Knowl.-Based Syst. 165 (2019) 374–406.

[39] A. Al-Betar, M.A. Awadallah, H. Faris, I. Aljarah, A.I. Hammouri, Natural selection methods for grey wolf optimizer, Expert Syst. Appl. 113 (2018) 481–498.

[40] M. Kohli, S. Arora, Chaotic grey wolf optimization algorithm for constrained optimization problems, J. Comput. Des. Eng. 5 (4) (2018) 458–472.

[41] M.A. Saxena, R. Kumar, S. Das, β-chaotic map enabled grey wolf optimizer, Appl. Soft Comput. 75 (2019) 84–105.

[42] Q. Tu, X. Chen, X. Liu, Multi-strategy ensemble grey wolf optimizer and its application to feature selection, Appl. Soft Comput. 76 (2019) 16–30.

[43] D.H. Wolpert, W.G. Macready, No Free Lunch Theorems for Search, vol. 10, Technical Report SFI-TR-95-02-010, Santa Fe Institute, 1995.

[44] J.J. Liang, B.Y. Qu, P.N. Suganthan, Problem Definitions and Evaluation Criteria for the CEC 2014 Special Session and Competition on Single Objective Real-Parameter Numerical Optimization, Technical Report, Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou China, Nanyang Technological University, Singapore, 2013.

[45] N.H. Awad, M.Z. Ali, J.J. Liang, B.Y. Qu, P.N. Suganthan, Problem Definitions and Evaluation Criteria for the CEC 2017 Special Session and Competition on Single Objective Bound Constrained Real-Parameter Numerical Optimization, Technical Report, Nanyang Technological University Singapore, 2016.

[46] C. Muro, R. Escobedo, L. Spector, R.P. Coppinger, Wolf-pack (Canis lupus) hunting strategies emerge from simple rules in computational simulations, Behav. Process. 88 (3) (2011) 192–197.

[47] J. Derrac, S. García, D. Molina, F. Herrera, A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms, Swarm Evol. Comput. 1 (1) (2011) 3–18.

[48] J. Kennedy, Particle swarm optimization, in: Encyclopedia of Machine Learning, Springer, Boston, MA, 2011, pp. 760–766.

[49] Y. Shi, R. Eberhart, A modified particle swarm optimizer, in: Evolutionary Computation Proceedings, 1998. IEEE World Congress on Computational Intelligence, the 1998 IEEE International Conference on, IEEE, pp. 69–73.

[50] Z.W. Geem, J.H. Kim, G.V. Loganathan, A new heuristic optimization algorithm: harmony search, Simulation 76 (2) (2001) 60–68.

[51] E. Rashedi, H. Nezamabadi-Pour, S. Saryazdi, GSA: a gravitational search algorithm, Inform. Sci. 179 (13) (2009) 2232–2248.

[52] W. Gao, S. Liu, L. Huang, A global best artificial bee colony algorithm for global optimization, J. Comput. Appl. Math. 236 (11) (2012) 2741–2753.

[53] N. Hansen, A. Ostermeier, Completely derandomized self-adaptation in evolution strategies, Evol. Comput. 9 (2) (2001) 159–195.

[54] J. Zhang, A.C. Sanderson, JADE: adaptive differential evolution with optional external archive, IEEE Trans. Evol. Comput. 13 (2009) 945–958.

[55] S.Y. Du, Z.G. Liu, An improved particle swarm optimization by hybriding with jade, in: 2017 International Symposium on Intelligent Signal Processing and Communication Systems, ISPACS, IEEE, 2017, pp. 439–443.

[56] K. Deb, An efficient constraint handling method for genetic algorithms, Comput. Methods Appl. Mech. Engrg. 186 (2–4) (2000) 311–338.

[57] E. Sandgren, Nonlinear integer and discrete programming in mechanical design, in: Proceeding of the ASME Design Technology Conference, 1988, pp. 95–105.

[58] S. Das, P.N. Suganthan, Problem Definitions and Evaluation Criteria for CEC 2011 Competition on Testing Evolutionary Algorithms on Real World Optimization Problems, Technical Report, Jadavpur University, Nanyang Technological University, 2010, pp. 341–359.

[59] H. Nowcki, Optimization in pre-contract ship design, in: Y. Fujita, K. Lind, T.J. Williams (Eds.), Computer Applications in the Automation of Shipyard Operation and Ship Design: Vol. 2, Elsevier, North Holland New York, 1974, pp. 327–338.

[60] A.H. Gandomi, X.S. Yang, A.H. Alavi, Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems, Eng. Comput. 29 (1) (2013) 17–35.

[61] B.K. Kannan, S.N. Kramer, An augmented Lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to mechanical design, J. Mech. Des. 116 (2) (1994) 405–411.

[62] M.A. El Aziz, A.A. Ewees, A.E. Hassanien, Whale optimization algorithm and moth-flame optimization for multilevel thresholding image segmentation, Expert Syst. Appl. 83 (2017) 242–256.

[63] U. Mlakar, B. Potočnik, J. Brest, A hybrid differential evolution for optimal multilevel image thresholding, Expert Syst. Appl. 65 (2016) 221–232.

[64] P.D. Sathya, R. Kayalvizhi, Optimal multilevel thresholding using bacterial foraging algorithm, Expert Syst. Appl. 38 (12) (2011) 15549–15564.

[65] W.A. Hussein, S. Sahran, S.N.H.S. Abdullah, A fast scheme for multilevel thresholding based on a modified bees algorithm, Knowl.-Based Syst. 101 (2016) 114–134.
[66] S. Dey, I. Saha, S. Bhattacharyya, U. Maulik, Multi-level thresholding using quantum inspired meta-heuristics, Knowl.-Based Syst. 67 (2014) 373–400.
[67] N. Otsu, A threshold selection method from gray-level histograms, IEEE Trans. Syst. Man Cybern. 9 (1) (1979) 62–66.