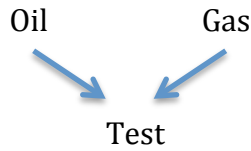


HW-6

1.

(a)

This can be formulated as a 3-way Bayesian network:



Given,

If T, $\Pr(\text{Oil}) = 0.5$ and thus, if F, $\Pr(\text{Oil}) = 0.5$

If T, $\Pr(\text{Gas}) = 0.2$ and thus, if F, $\Pr(\text{Gas}) = 0.8$

This can be represented with Test in the following table:

Test	Oil	Gas	$\Pr(\text{Test} \mid \text{Oil}, \text{Gas})$
T	T	T	0
T	T	F	0.9
T	F	T	0.3
T	F	F	0.1

(b)

In this question, we need to find

$\Pr(\text{Oil} \mid \text{Test})$

First, we find $\Pr(\text{Test})$

$\Pr(\text{Test}) = \Pr(\text{Test}, \text{Oil}) + \Pr(\text{Test}, \neg \text{Oil})$

Given the values above, we can compute this as

$$= 0.5 \cdot 0.9 + 0.4 \cdot 0.5$$

Now, $\Pr(\text{Oil} \mid \text{Test}) = \Pr(\text{Test} \mid \text{Oil}) \cdot \Pr(\text{Oil}) / \Pr(\text{Test}) = 0.9 \cdot 0.5 / (0.5 \cdot 0.9 + 0.4 \cdot 0.5) = \mathbf{0.69}$.

2.

(a)

$\Pr(A, B, C, D, E, F, G, H) =$

$$\Pr(A) \cdot \Pr(B) \cdot \Pr(C \mid A) \cdot \Pr(E \mid B) \cdot \Pr(G \mid F) \cdot \Pr(D \mid A, B) \cdot \Pr(F \mid C, D) \cdot \Pr(H \mid F, E)$$

(b)

$\Pr(E, F, G, H) =$

$$\Pr(E|B) * \Pr(G|F) * \Pr(F|C,D) * \Pr(H|F,E)$$

(c)

$\Pr(a, \neg b, c, d, \neg e, f, \neg g, h) =$

$$0.2 * 0.3 * 0.6 * 0.1 * \Pr(c|a) * \Pr(f|c,d) * \Pr(\neg g|f) * \Pr(h|f, \neg e)$$

The values in placeholders don't have assigned CPT values.

(d)

For $\Pr(\neg a|b)$, since a and b are independent,

$$\Pr(\neg a|b) = \Pr(\neg a) * \Pr(b) = 0.8 * 0.7 = 0.56$$

For $\Pr(\neg e|a)$, we know that e and a are independent. Thus,

$$\Pr(\neg e|a) = \Pr(\neg e) * \Pr(a) / \Pr(a) = \Pr(\neg e).$$

$$\begin{aligned} \text{Since e depends on b, } \Pr(\neg e) &= \Pr(\neg e, b) + \Pr(\neg e, \neg b) \\ &= 0.9 * 0.7 + 0.1 * 0.3 \text{ (Given values)} \\ &= 0.66 \end{aligned}$$

(e)

The assumptions are as follows:

$I(A, \text{NULL}, BE)$

$I(B, \text{NULL}, AC)$

$I(C, A, DBE)$

$I(D, AB, CE)$

$I(E, B, ACDFG)$

$I(F, CD, ABE)$

$I(H, EF, ABCG)$

(f)

The blanket is = {A, B, C, F}

(g)

A	B	D	P(D AB)
T	T	T	0.2
T	T	F	0.8
T	F	T	0.9
T	F	F	0.1

F	T	T	0.4
F	T	F	0.6
F	F	T	0.5
F	F	F	0.5

B	E	P(E B)
T	T	0.1
T	F	0.9
F	T	0.9
F	F	0.1

A	B	D	E	$P(D AB)^*P(E B)$
T	T	T	T	0.02
T	T	T	F	0.18
T	T	F	T	0.08
T	T	F	F	0.72
T	F	T	T	0.81
T	F	T	F	0.09
T	F	F	T	0.09
T	F	F	F	0.01
F	T	T	T	0.04
F	T	T	F	0.36
F	T	F	T	0.06
F	T	F	F	0.54
F	F	T	T	0.45
F	F	T	F	0.05
F	F	F	T	0.45
F	F	F	F	0.05