

CS161 – HW5

1.

(a) $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$

Smoke	Fire	$(\text{Smoke} \Rightarrow \text{Fire})$	$(\neg \text{Smoke} \Rightarrow \neg \text{Fire})$	$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

It is neither valid nor unsatisfiable.

(b) $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$

Smoke	Fire	Heat	$(\text{Smoke} \Rightarrow \text{Fire})$	$((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$	$(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow ((\text{Smoke} \vee \text{Heat}) \Rightarrow \text{Fire})$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

It is neither valid nor unsatisfiable.

(c) $((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$

Smoke	Fire	Heat	$((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

It is valid.

2. Using variables Immortal, Mythical, Mammal, and Horned.

(a)

- $\text{Mythical} \Rightarrow \text{Immortal}$
- $\neg \text{Mythical} \Rightarrow \neg \text{Immortal} \wedge \text{Mammal}$
- $\text{Immortal} \vee \text{Mammal} \Rightarrow \text{Horned}$
- $\text{Horned} \Rightarrow \text{Magical}$

(b) In CNF form, these are

$(\neg \text{Mythical} \vee \text{Immortal}) \wedge$
 $(\text{Mythical} \vee \neg \text{Immortal}) \wedge (\text{Mythical} \vee \text{Mammal}) \wedge$
 $(\neg \text{Immortal} \vee \text{Horned}) \wedge (\neg \text{Mammal} \vee \text{Horned}) \wedge$
 $(\neg \text{Horned} \vee \text{Magical})$

(c) First, to prove that it is Horned,

$(\text{Mythical} \vee \neg \text{Mythical})$ (1)

$(\text{Mythical} \wedge \text{Immortal})$
From point 1 of part (a) (2)

$(\neg \text{Mythical} \wedge (\neg \text{Immortal} \wedge \text{Mammal}))$
From point 2 of part (a) (3)

$(\text{Immortal} \vee (\neg \text{Immortal} \wedge \text{Mammal}))$
From (1) and (2) (4)

$(\text{Immortal} \vee \neg \text{Immortal}) \wedge (\text{Immortal} \vee \text{Mammal})$
Distributing (3) (5)

$(\text{Horned} \wedge (\text{Immortal} \vee \text{Mammal}))$
From point 3 of part (a) (6)

Since,
Left side of (5) is always true and given (6), we can conclude that it is Horned.

Further,
 $(\neg \text{Horned} \vee \text{Magical})$
From the CNF form (7)

Thus, we can conclude that it is also magical.

However, it is not possible to prove that it is mythical.

3.

(a) $P(A, B, B)$, $P(x, y, z)$

For unifying these,

$\{x=A, y=B, z=B\}$

(b) $Q(y, G(A, B)), Q(G(x, x), y)$.

For unifying these,

$\{y=G(x,x), G(A,B)=y\}$

$\Rightarrow \{y = G(x,x), y = G(A,B)\} \Rightarrow G(x,x) = G(A,B) \Rightarrow x = A \text{ and } x = B.$

However, this isn't possible. Thus, no unification exists.

(c) $\text{Older}(\text{Father}(y), y), \text{Older}(\text{Father}(x), \text{John})$.

For unifying these,

$\text{Older}(\text{Father}(y), y) = \text{Older}(\text{Father}(x), \text{John})$

$\Rightarrow \{y=\text{John}, \text{Father}(x) = \text{Father}(y)\}$

$\Rightarrow \{y=\text{John}, x=y\} \Rightarrow \{x = \text{John}, y = \text{John}\}$

(d) $\text{Knows}(\text{Father}(y), y), \text{Knows}(x, x)$.

For unifying these,

$\text{Knows}(\text{Father}(y), y) = \text{Knows}(x, x)$

$\Rightarrow \{x = \text{Father}(y), x=y\} \Rightarrow \{x=\text{Father}(x), y=x\}$

$\Rightarrow \{x = \text{Father}(\text{Father}(\dots))\}, y=x\}$

Since, there is recursion, we don't have a unifier.

4.

(a)

- i. $A f, \text{Food}(f) \Rightarrow \text{Likes}(\text{John}, f)$
- ii. $\text{Food}(\text{Apples})$
- iii. $\text{Food}(\text{Chicken})$
- iv. $A f, g, \text{Eats}(g, f) \wedge \neg \text{Killed}(f, g) \Rightarrow \text{Food}(f)$
- v. $A f, g, \text{Killed}(f, g) \Rightarrow \neg \text{Alive}(f)$
- vi. $\text{Alive}(\text{Bill}) \wedge \text{Eats}(\text{Bill}, \text{Peanuts})$
- vii. $A f, \text{Eats}(\text{Bill}, f) \Rightarrow \text{Eats}(\text{Sue}, f)$

(b)

$(\neg \text{Food}(f) \vee \text{Likes}(\text{John}, f)) \wedge$
 $\text{Food}(\text{Apples}) \wedge$
 $\text{Food}(\text{Chicken}) \wedge$
 $(\neg \text{Eats}(f, g) \vee \text{Killed}(f, g) \wedge \text{Food}(f)) \wedge$
 $(\neg \text{Killed}(f, g) \vee \neg \text{Alive}(f)) \wedge$
 $\text{Alive}(\text{Bill}) \wedge$

$\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge$
 $(\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f))$

(c)

For a negated conclusion,

$\neg \text{Likes}(\text{John}, \text{Peanuts})$ (1)

$\neg \text{Food}(\text{Peanuts})$
 From (1) and (i) (2)

$\neg \text{Eats}(\text{g}, \text{Peanuts}) \wedge \neg \text{Killed}(\text{Peanuts}, \text{g})$
 From (iv) (3)

$\neg \text{Eats}(\text{Bill}, \text{Peanuts}) \wedge \neg \text{Killed}(\text{Peanuts}, \text{Bill})$
 From (vii) (4)

$\neg \text{Eats}(\text{Bill}, \text{Peanuts}) \vee \neg \text{Alive}(\text{Bill})$
 From (v) (5)

However, (5) is a direct contradiction to (vi).
 Hence, (1) is false.

$\Rightarrow \text{Likes}(\text{John}, \text{Peanuts}).$

(d) For a negated conclusion,

$\neg \text{Eats}(\text{Sue}, f)$
 $\text{Eats}(\text{Bill}, \text{Peanuts})$
 From (vi) (6)

$\text{Eats}(\text{Bill}, f) \Rightarrow \text{Eats}(\text{Sue}, f)$
 From (vii) (7)

$\text{Eats}(\text{Sue}, \text{Peanuts})$
 From (6) and (7)

Thus, the resolution provides us with a contradiction.

Hence,

A f, $\text{Eats}(\text{Sue}, f)$, where f could be Peanuts.

(e) The CNF for the new set of Axioms would be as follows:
 $(\neg \text{Food}(f) \vee \text{Likes}(\text{John}, f)) \wedge$

$\text{Food}(\text{Apples}) \wedge$
 $\text{Food}(\text{Chicken}) \wedge$
 $(\neg \text{Eats}(f, g) \vee \text{Killed}(f, g) \wedge \text{Food}(f)) \wedge$
 $(\neg \text{Kills}(x, y) \vee \neg \text{Alive}(y)) \wedge$
 $(\text{Eats}(a, b) \vee \text{Dead}(b)) \wedge$
 $(\neg \text{Dead}(c) \vee \neg \text{Alive}(c)) \wedge$
 $\text{Alive}(\text{Bill}) \wedge$
 $\text{Eats}(\text{Bill}, \text{Peanuts}) \wedge$
 $(\neg \text{Eats}(\text{Bill}, f) \vee \text{Eats}(\text{Sue}, f))$

Following the same process as above,
For a negated conclusion,

$\neg \text{Eats}(\text{Sue}, f)$
 $\neg \text{Eats}(\text{Bill}, f)$
 $\neg \text{Alive}(\text{Bill})$

Clearly this is a contradiction with $\text{Alive}(\text{Bill})$

Thus,

$\forall f, \text{Eats}(\text{Sue}, f)$ holds true and she eats whatever Bill does.