QEA2

Flatlands

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Gauntlet Map

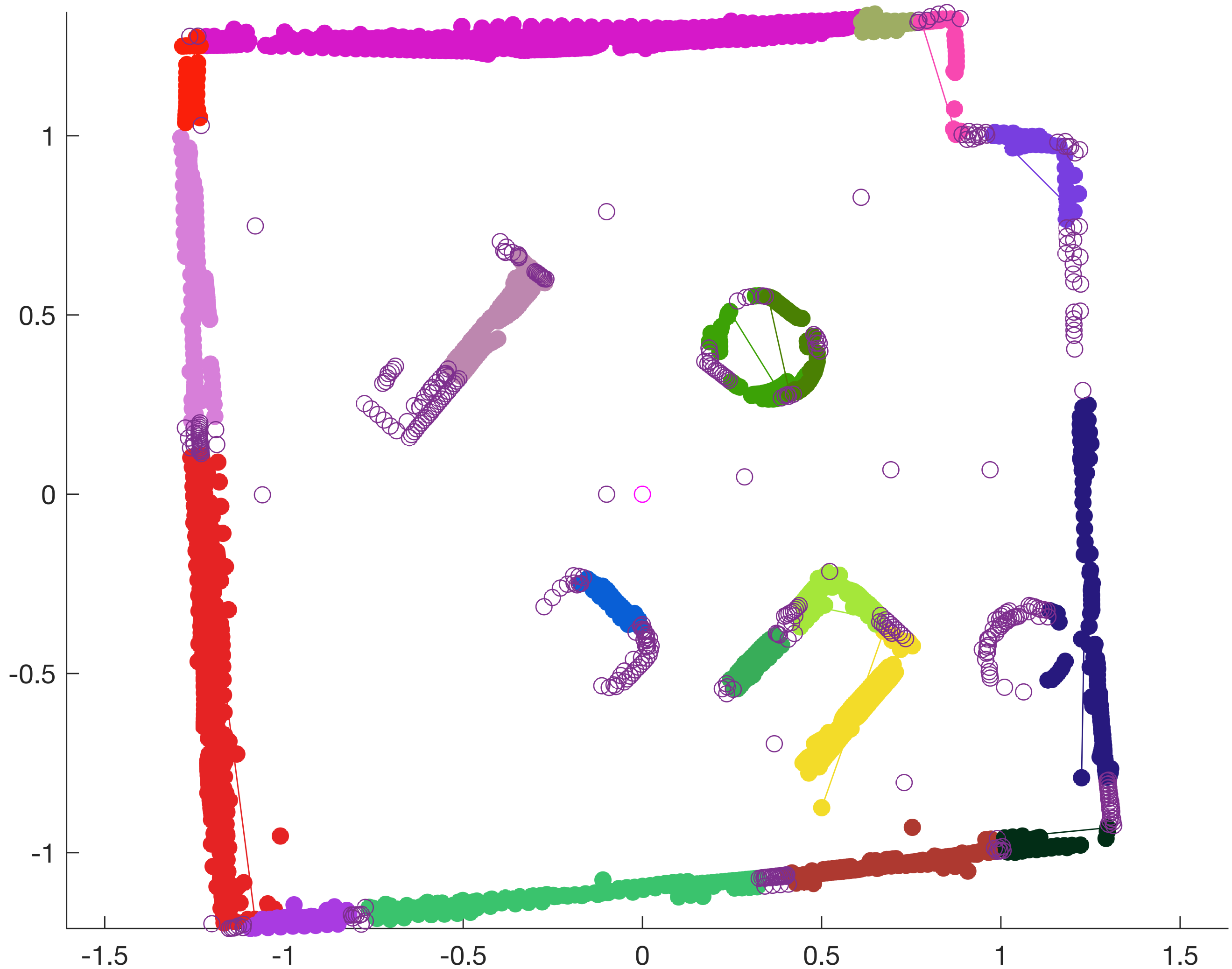

Figure 1: A map of the Gauntlet using LIDAR Data

Equation for Potential Field

Suppose we have a potential field V (x,y) over a line C. We perform a discrete summation to integrate it across this line which also results in a potential field. Therefore, the equation for the potential field due to any point (a,b) on a line segment is below where u represents the position along a line segment:

$$\int_0^1 V\,du = -0.1(\sqrt{(x-a)^2 + (y-b)^2}))^{-3.5},$$

$$\text{where } a(u) = (x_1 + (x_1 - y_1))u,\ b(u) = (x_2 + (x_2 - y_2))u,$$
$$\text{and, } x, y \text{ represent position vectors.}$$

The equation for the potential field due to any point (a,b) on a circle is below where u represents the position along a circle:

$$\int_0^1 V\,du = 1000(\sqrt{(x-a)^2 + (y-b)^2}))^{-4}$$

$$\text{where } a(u) = x + rad(cos(2\pi u),\ b(u) = y + rad(cos(2\pi u)$$
$$\text{and, } x, y \text{ represent position vectors.}$$

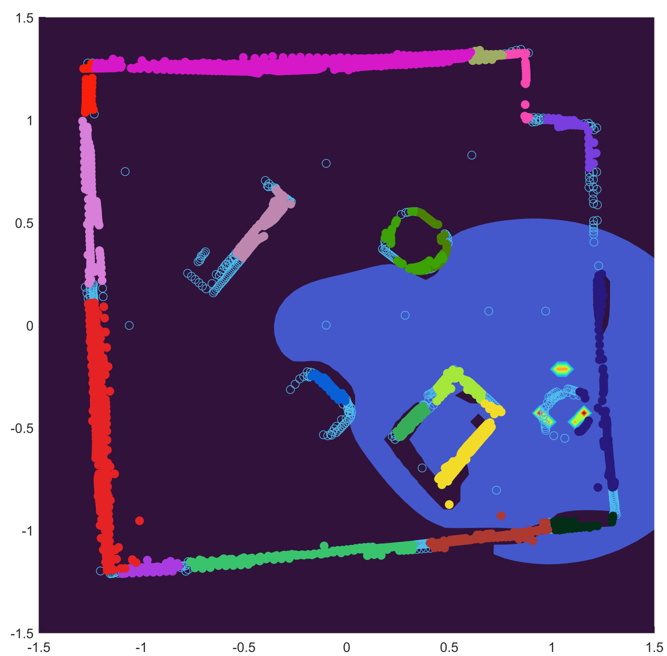

Figure 2: A contour plot of the potential field.

Quiver Plot

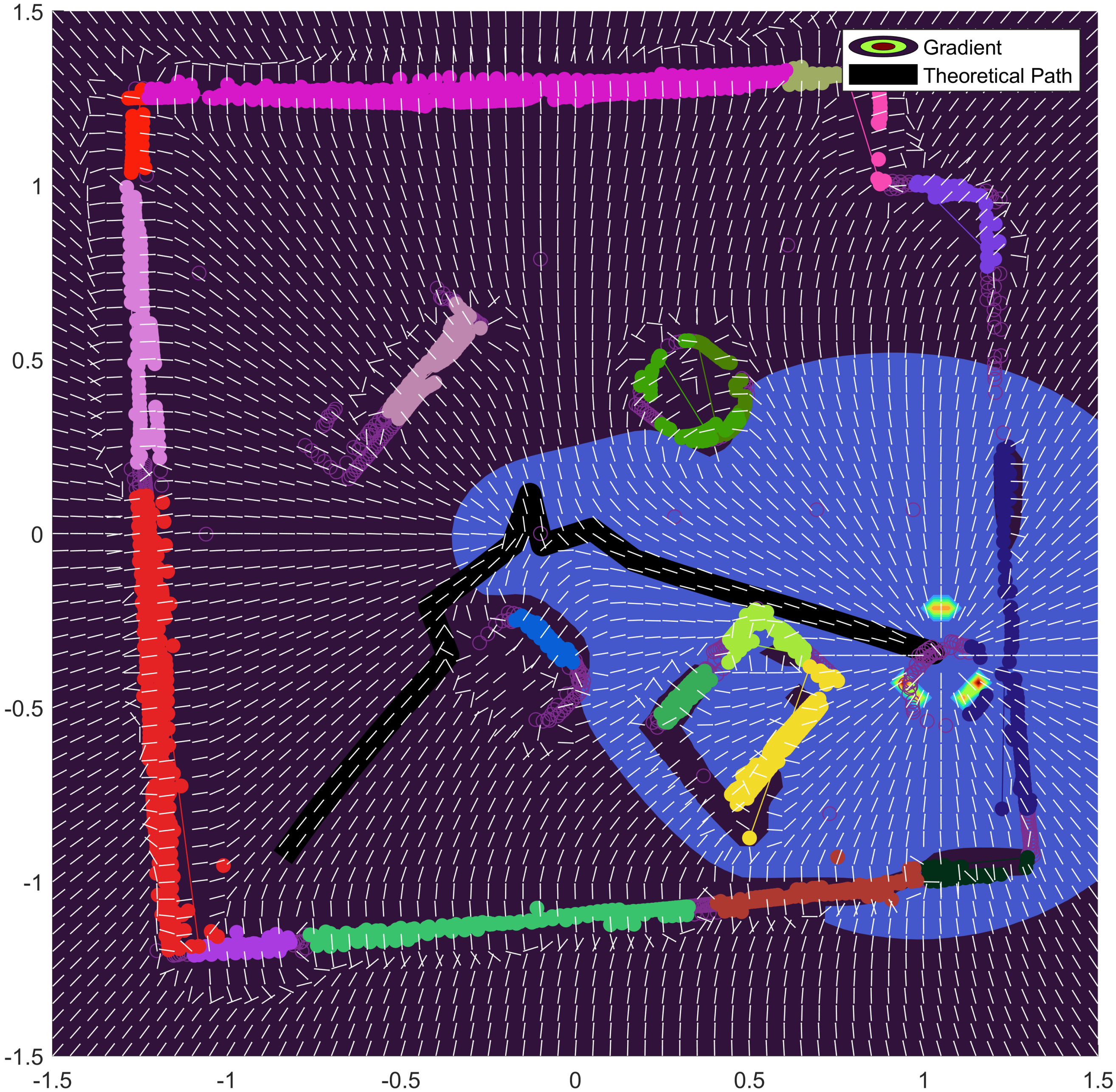

Figure 3: A quiver plot of the gradient of our potential field with theoretical path.

Gradient Descent

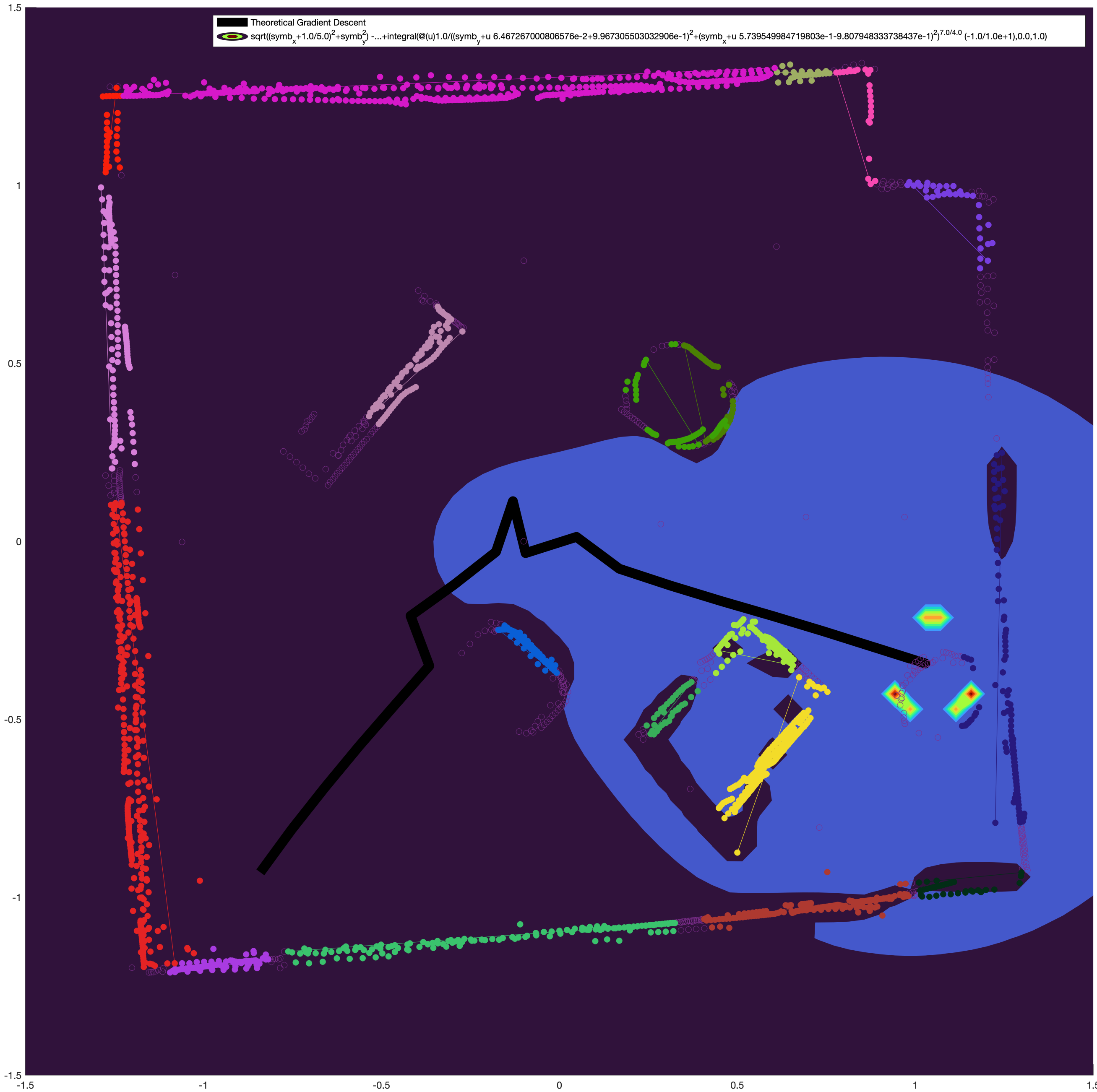

Figure 4: A theoretical path of gradient descent from the starting point to the BoB.

Introduction

Our mission was to move the neato through the gauntlet as accurately as possible through the gradient path without hitting obstacles to gently tap the Barrel of Benevolence.

Line Fitting

We randomly choose some data points to fit a line, which requires at least 2 points. Next, we define our initial candidate line as the line segment joining these 2 points.

We move the the start point of this line to the origin and calculate the distance of other points relative to it. To do this, we project the points in the direction of the end point. This gives us the horizontal distance, which we then use to find an orthogonal vector to get the vertical distance.

We then classify all the points into either of two categories, inliers and outliers, based on the vertical distance threshold and the longest gap threshold.

To determine the longest gap between inlier points and the candidate line, we use a direct comparison approach rather than looping through all pairs of points which is much more efficient.

We repeat this process until we find the best fit line.

Circle Fitting

Let the center of a circle be (x_c, y_c) with radius r. Any point (x, y) on the circle can be represented as $(x - x_c)^2 + (y - y_c)^2 = r^2$. Therefore, we need to minimize the following equation:

$$\min_{x_c, y_c, r} \left(\sum_{i=1}^{M} [(x_i - x_c)^2 + (y_i - y_c)^2 - r^2]^2\right),$$

where M is the number of points.

Now, we substitute $z = x_c^2 + y_c^2 - r^2$ and convert it to linear least squares problem.

$$\min_{x_c, y_c, r} \left(\sum_{i=1}^{M} [2x_i x_c) + 2y_i y_c) - z - x_i^2 - y_i^2]^2\right)$$

$$= \min_{w} ||Aw - b||^2,$$

$$\text{where } w = \begin{bmatrix} x_c \\ y_c \\ z \end{bmatrix}, b_i = \begin{bmatrix} x_{1,i}^2 + y_{1,i}^2 \\ \vdots \\ x_{M,i}^2 + y_{M,i}^2 \end{bmatrix}, \text{ and } A = \begin{bmatrix} 2x_{1,i} & 2y_{1,i} & -1 \\ \vdots & \vdots & \vdots \\ 2x_{M,i} & 2y_{M,i} & -1 \end{bmatrix}.$$

We use the RANSAC approach to fit a circle to three random points from the set. We perform linear regression using the above minimization equation to find x_c, y_c, z for the circle.

Then, we compute the radius $r = \sqrt{x_c^2 + y_c^2 - z}$ and the orthogonal distance of each point (x_i, y_i) from the the circle edge. We then set a threshold distance to find the number of inliers as points that have lower distance from the circle edge than this. If the circle has a radius close of the radius of BoB and consists of most inliers, then we classify it as the best fit. Otherwise, we repeat this process until we find the best fit.

Experimental Validation

It took around 1 minutes and 27 seconds for the neato to get to the BoB.

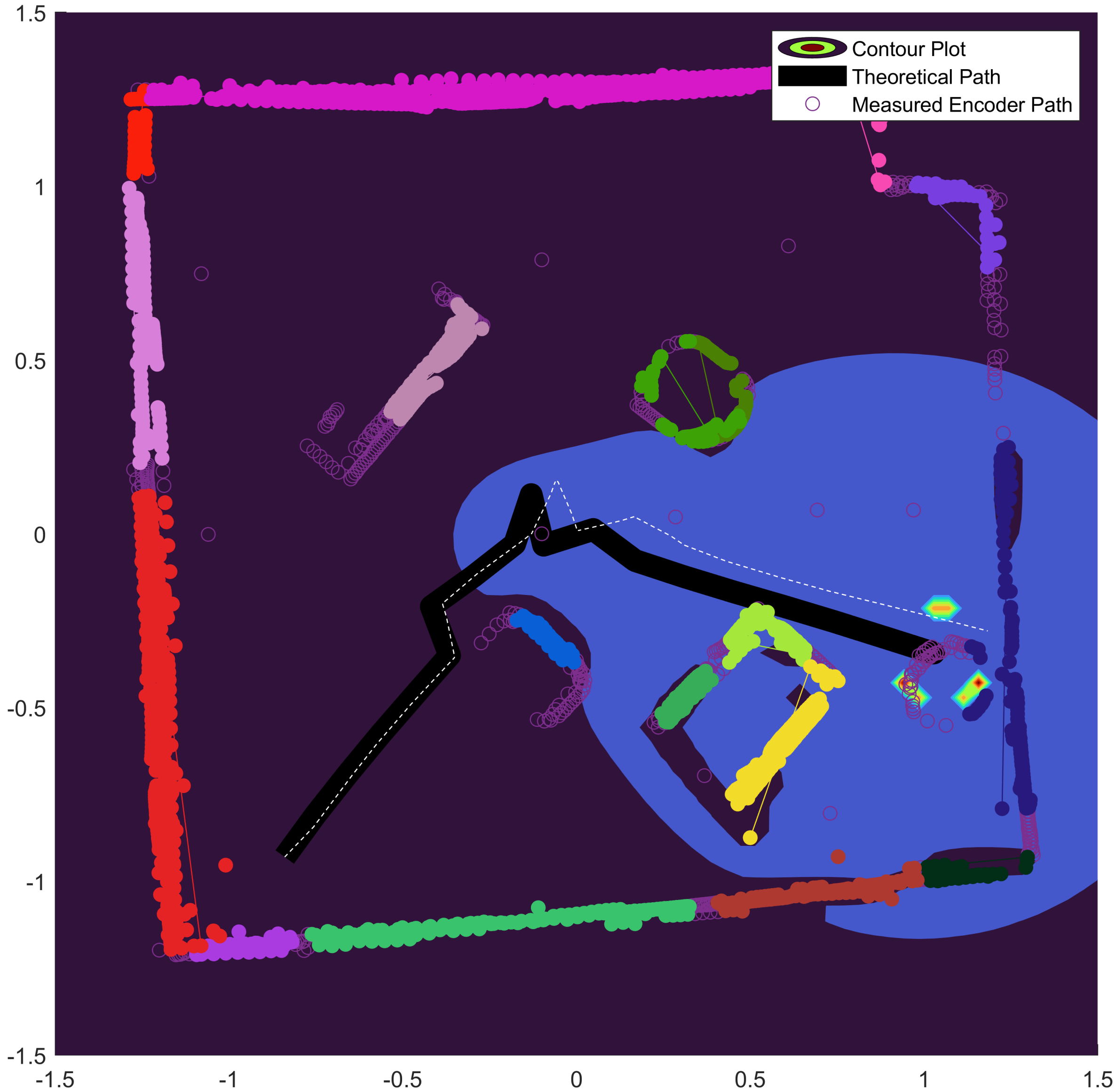

Figure 5: The measured path of gradient descent from the starting point to the BoB.

We used 14 scans of LIDAR data points for creating the Gauntlet map. The measured path was pretty similar to the theoretical path. The robot took a longer time to get to the BoB because we made it move slowly so that the path would be more accurate.

The RANSAC approach for line fitting and circle fitting described in the above section was used to achieve this level of accuracy and successfully move the neato in the Gaunlet.

Video

Click here to watch the neato traverse the Gauntlet. (YouTube)
Click here to watch the neato traverse the Gauntlet. (Google Drive)

Code

Click here to view the repository of code to conquer the Gauntlet.