Unit-9: Computation Tree Logic

B. Srivathsan

Chennai Mathematical Institute

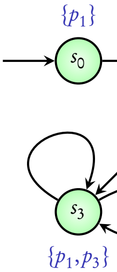
NPTEL-course

July - November 2015

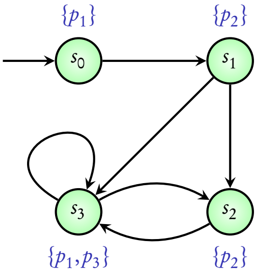
Module 1:

Tree behaviour of a transition system

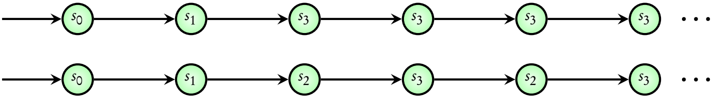
Transition System



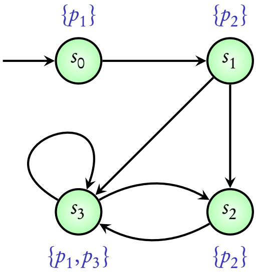
Transition System



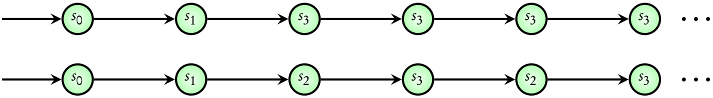
Paths



Transition System



Paths



Traces

$\{p_1\}\{p_2\}\{p_1, p_3\}\{p_1, p_3\}\{p_1, p_3\}\{p_1, p_3\} \cdots$

$\{p_1\}\{p_2\}\{p_2\}\{p_1, p_3\}\{p_2\}\{p_1, p_3\}\{p_2\}\{p_1, p_3\} \cdots$

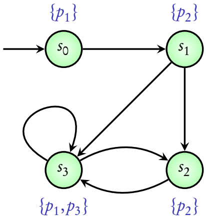
In this unit

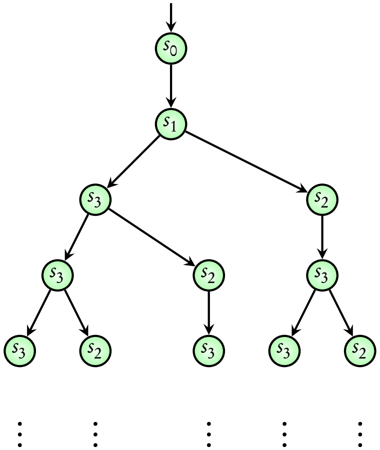
A **tree view** of the transition system ...

In this unit

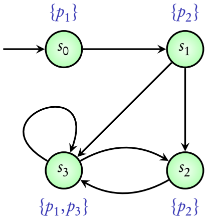
A **tree view** of the transition system ...

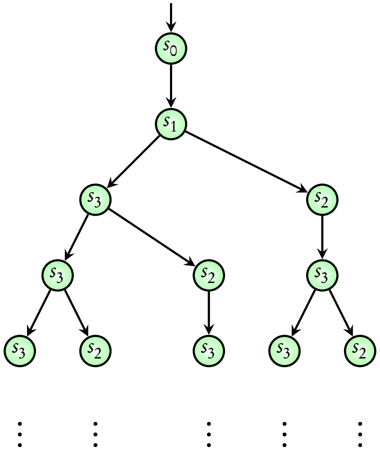
... obtained by repeatedly **unfolding** it


{p_1}
{p_2}
s_0
s_1
s_3
s_2
{p_1, p_3}
{p_2}


s_0
s_1
s_3
s_2
s_3
s_2
s_3
s_3
s_2
s_3
s_3
s_2

Computation tree





LTL talks about **properties of paths**

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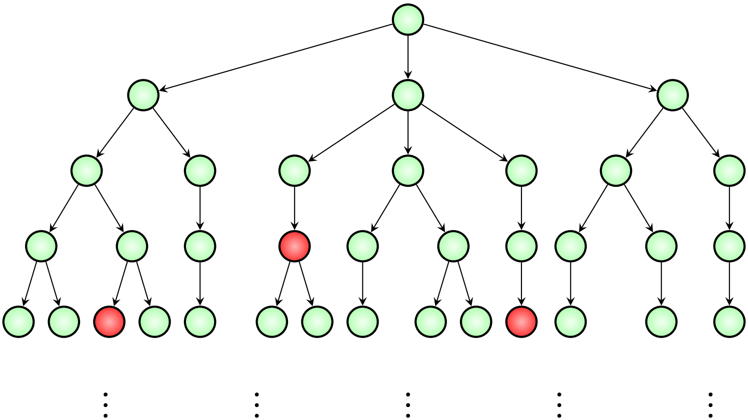
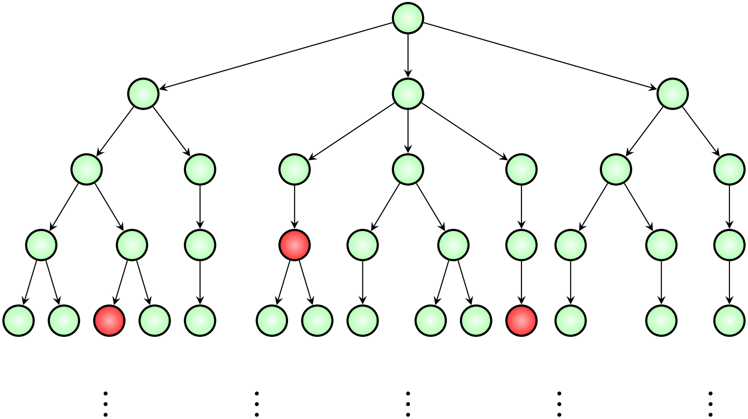
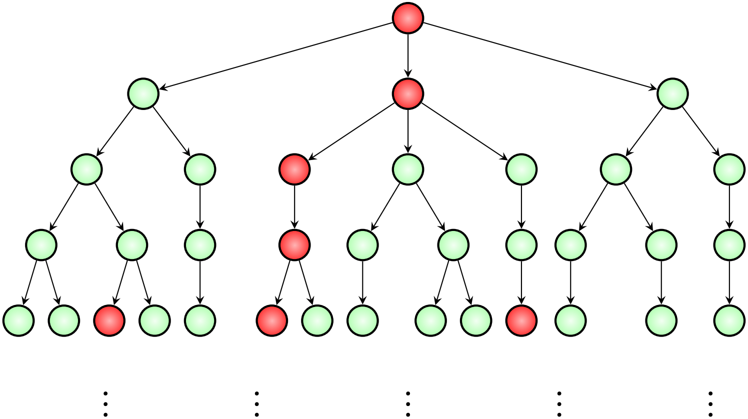
Coming next: Properties of **trees**

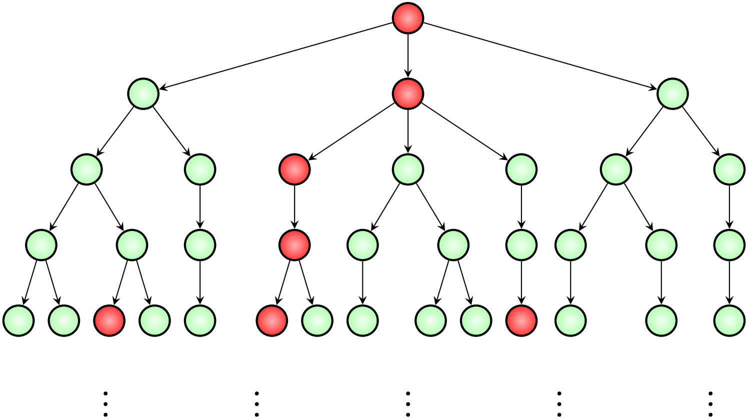


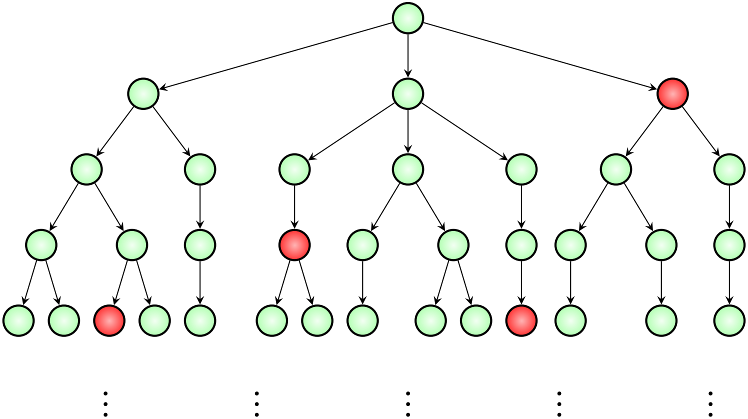
Exists a path satisfying **F**(*red*)



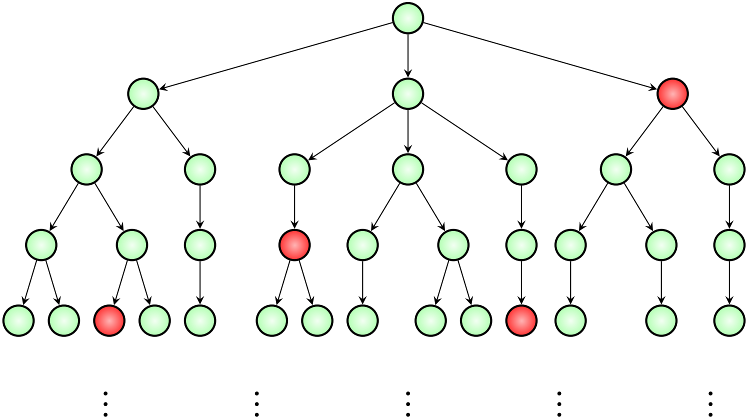


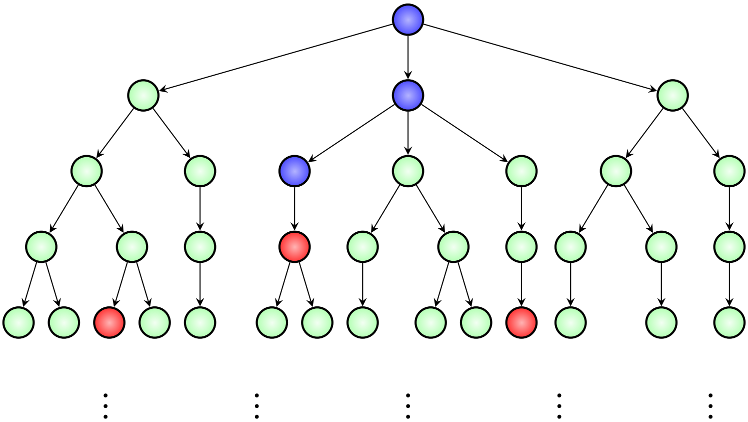
Exists a path satisfying **G**(*red*)



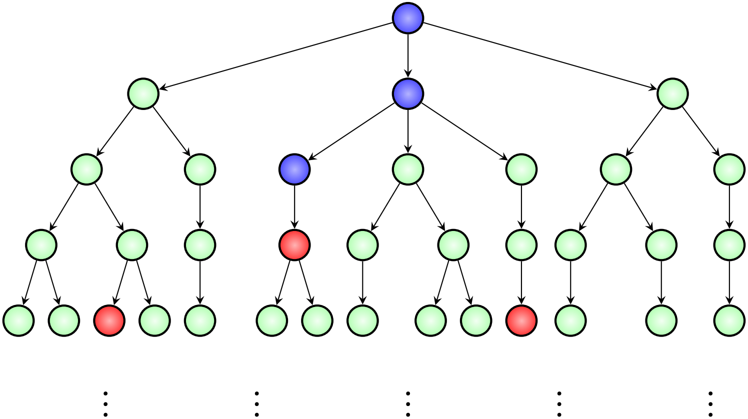


Exists a path satisfying **X**(*red*)





Exists a path satisfying *blue* **U** *red*



Properties of trees

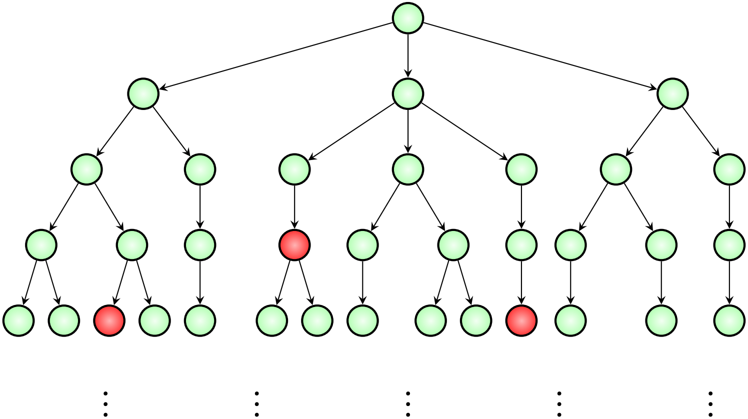
Type 1: **Exists a path** satisfying **LTL formula** ϕ

Properties of trees

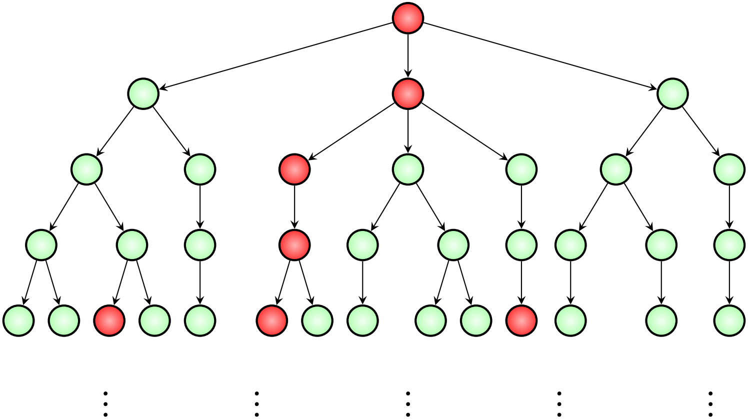
Type 1: **Exists a path** satisfying **LTL formula** ϕ

E operator: $\mathbf{E}\ \phi$

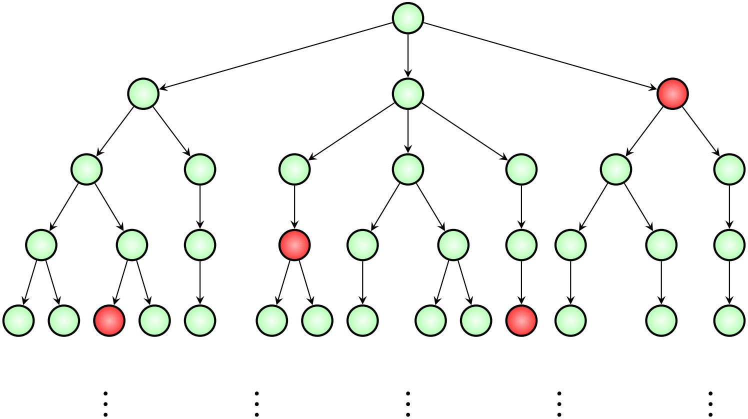
Exists a path satisfying $\mathbf{F}(\textit{red})$: $\quad \mathbf{E}\,\mathbf{F}\,(\textit{red})$



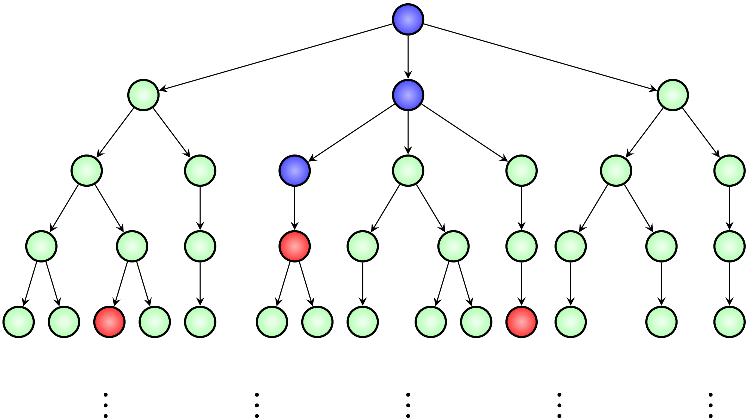
Exists a path satisfying $\mathbf{G}(\,\textit{red}\,)$: $\mathbf{E}\,\mathbf{G}\,(\,\textit{red}\,)$

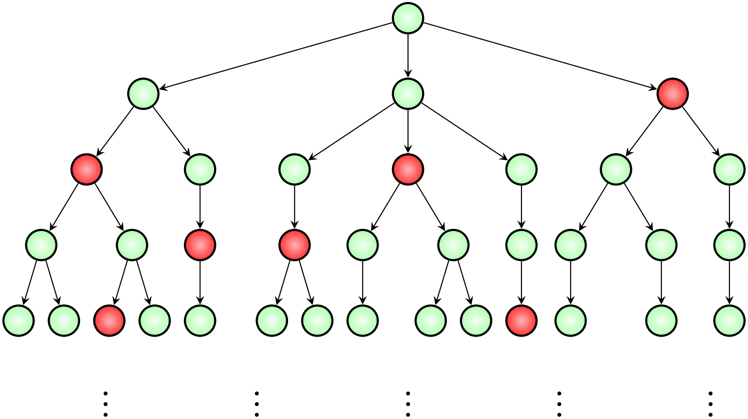


Exists a path satisfying $\mathbf{X}(\,red\,)$: $\mathbf{E}\ \mathbf{X}\ (\,red\,)$

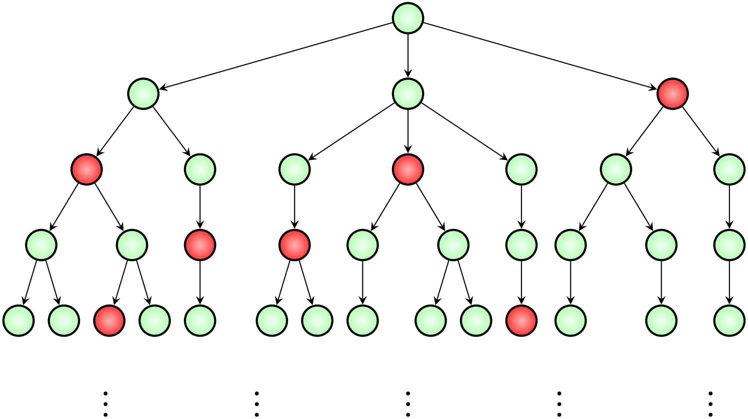


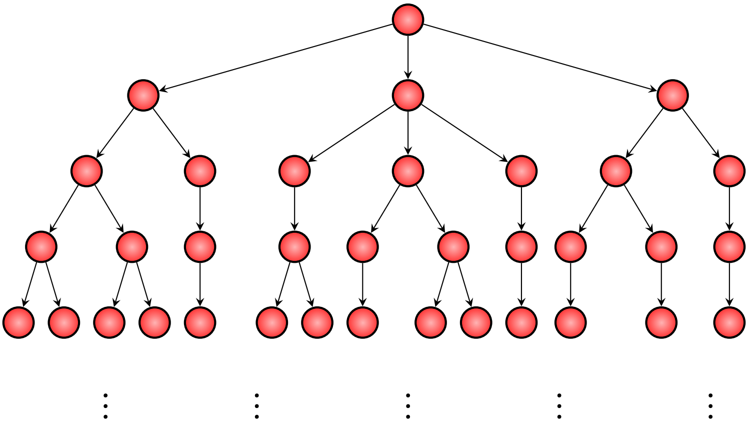
Exists a path satisfying *blue* **U** *red* : **E** (*blue* **U** *red*)



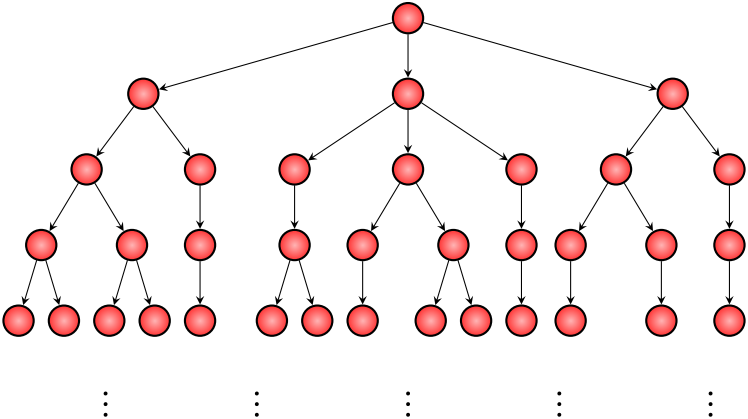


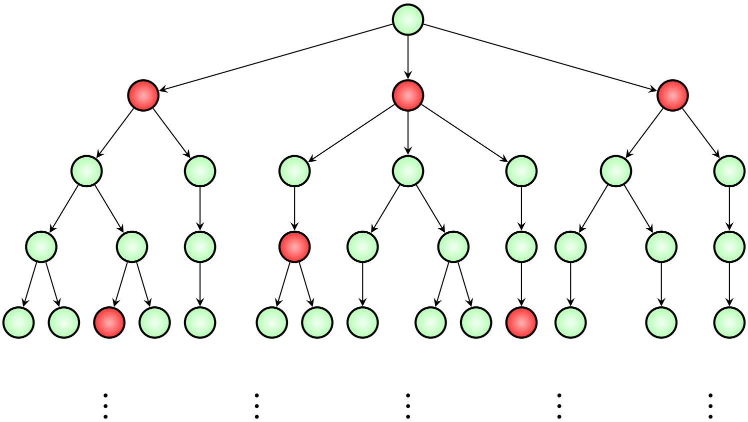
All paths satisfy **F**(*red*)



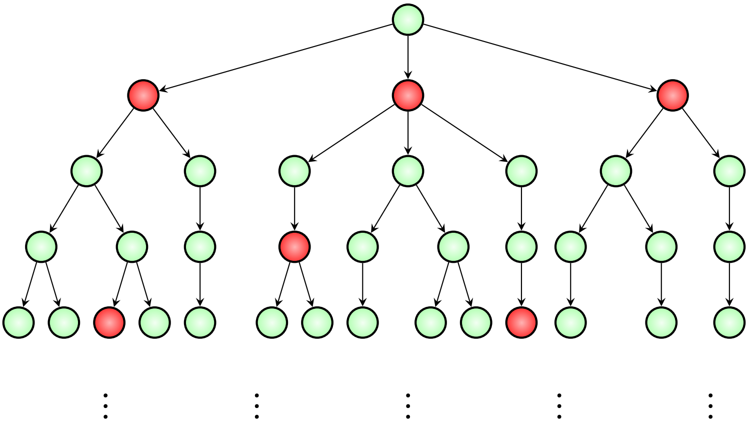


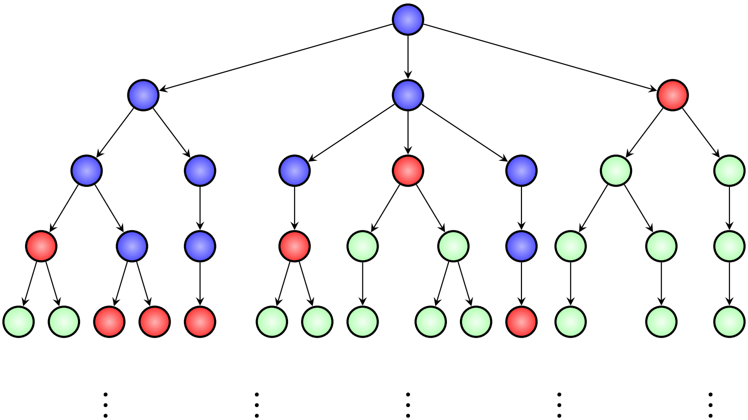
All paths satisfy **G**(*red*)



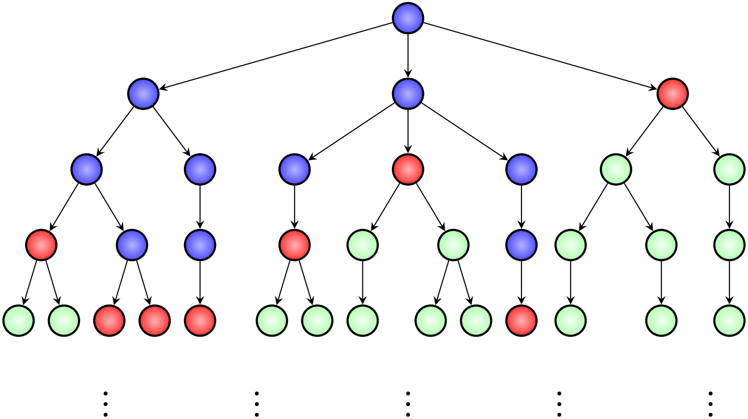


All paths satisfy **X**(*red*)





All paths satisfy *blue* **U** *red*



Properties of trees

Type 2: All paths satisfy **LTL formula** ϕ

Properties of trees

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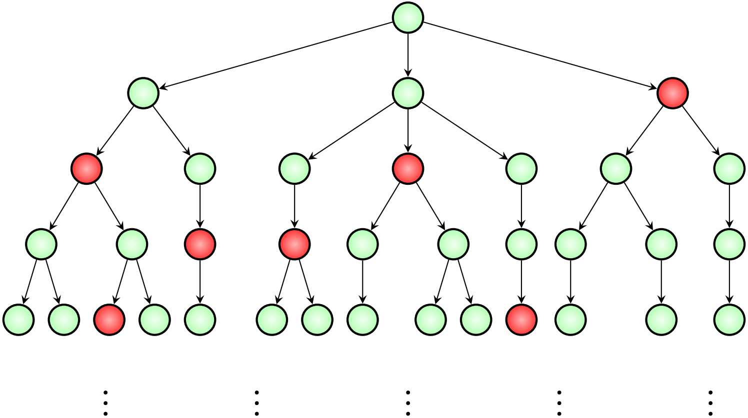
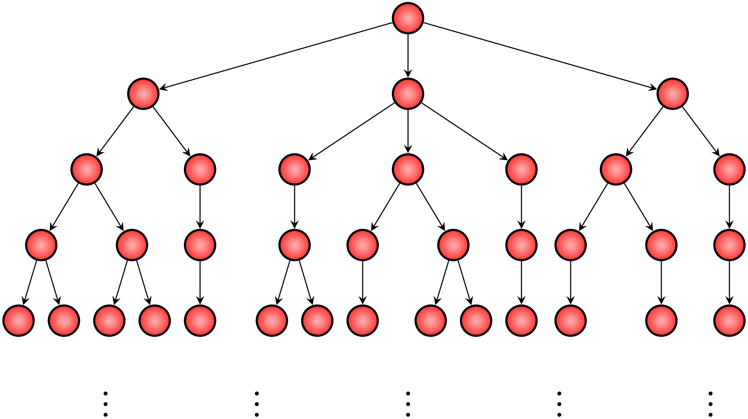
A operator: $\mathbf{A}\ \phi$

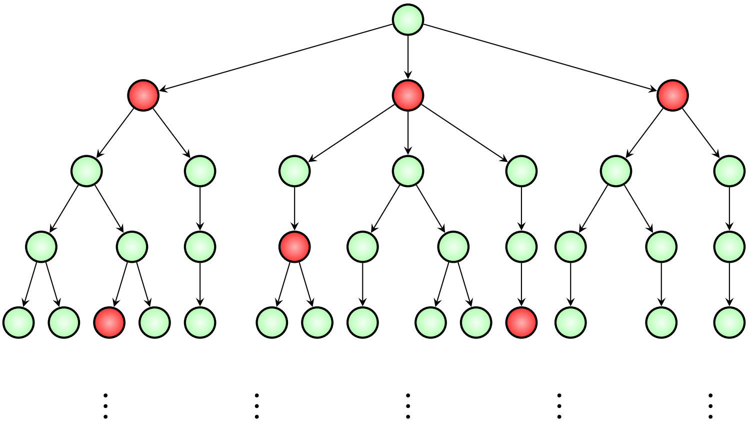
All paths satisfy **F**(*red*) :   **A F**(*red*)



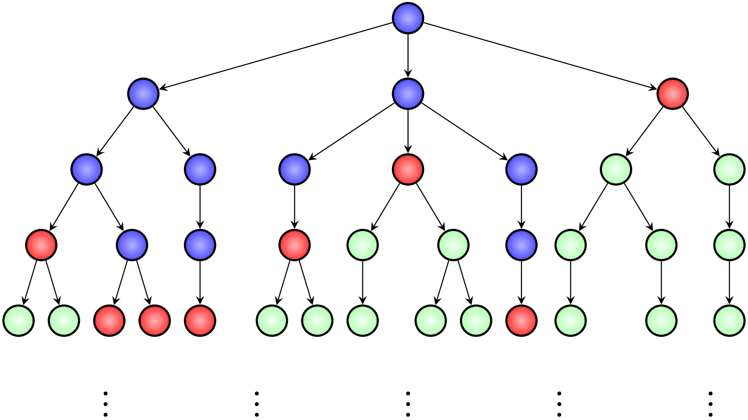
All paths satisfy **G**(*red*) :   **A G**(*red*)



All paths satisfy **X**(*red*) : **A X**(*red*)



All paths satisfy *blue* **U** *red* : **A** *blue* **U** *red*



Properties of trees

- **Exists a path** satisfying **path property** ϕ : **E** ϕ

- **All paths** satisfy **path property** ϕ : **A** ϕ

Properties of trees

- **Exists a path** satisfying **path property** ϕ : $\quad \mathbf{E}\ \phi$

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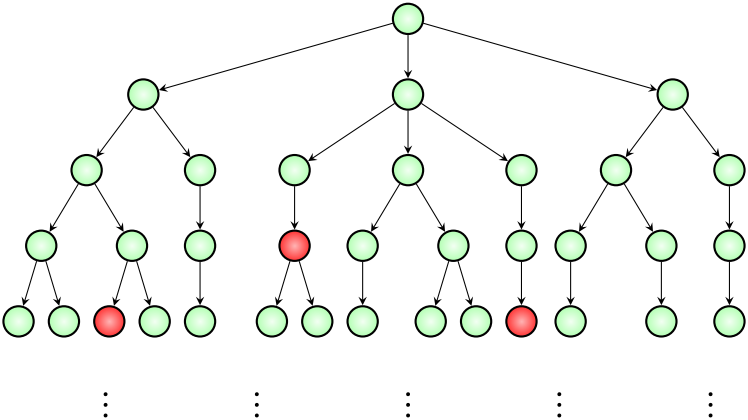
Coming next: Mixing **A** and **E**

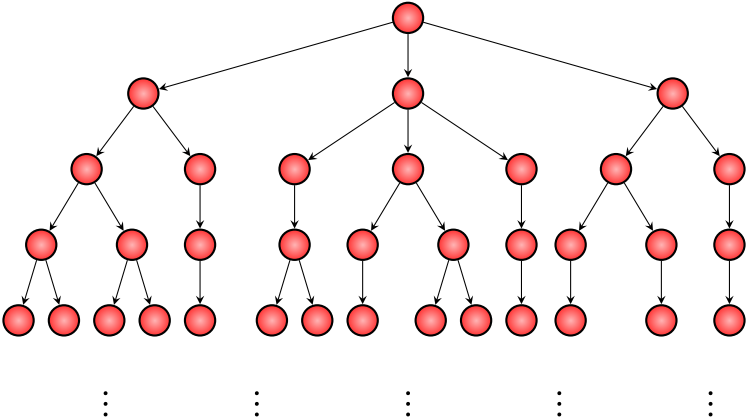
Recall...

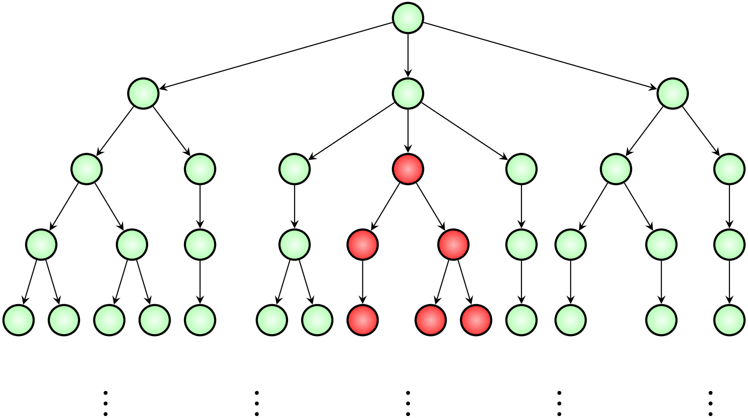
Exists a path satisfying **F**(*red*) :   **E F** (*red*)



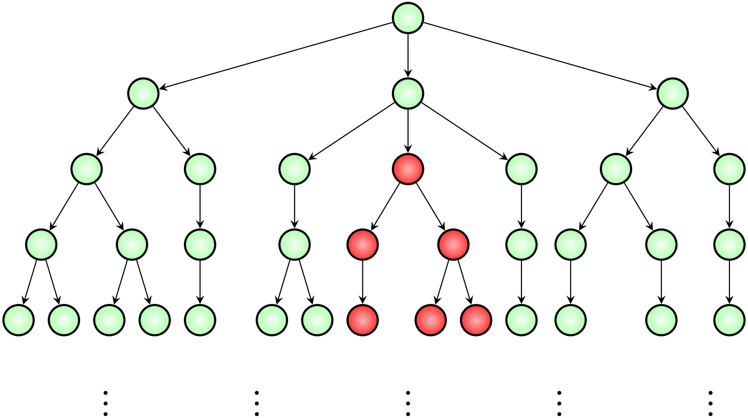
Recall...

All paths satisfy **G**(*red*) :   **A G**(*red*)

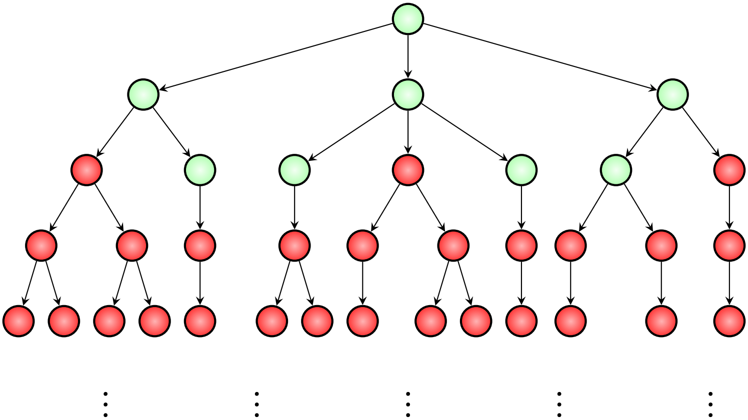




E F A G (*red*)

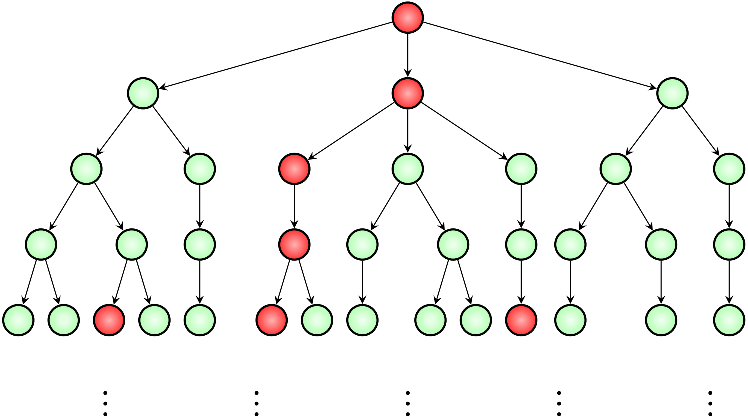


A F A G (*red*)



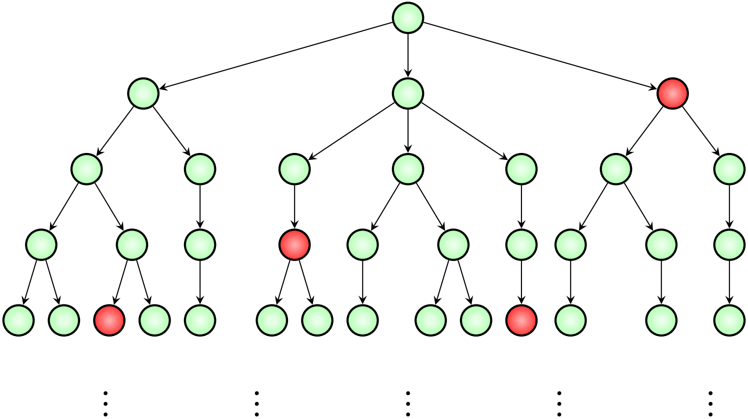
Recall...

Exists a path satisfying **G**(*red*) : **E G** (*red*)

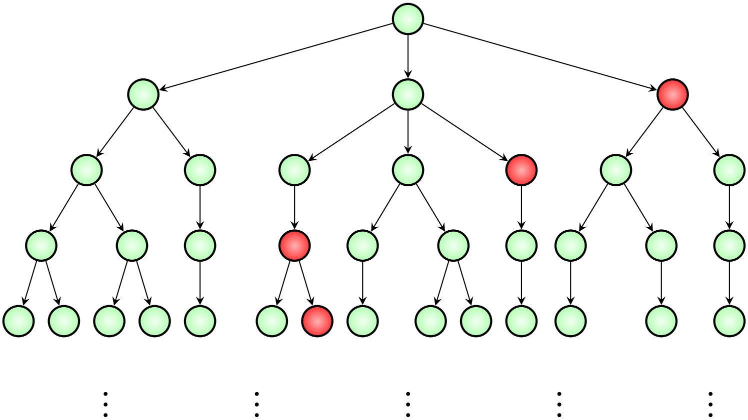


Recall...

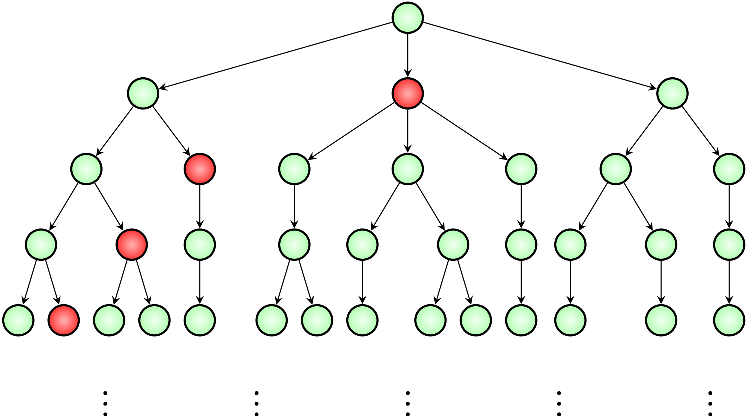
Exists a path satisfying $\mathbf{X}(\,\textit{red}\,)$: $\quad \mathbf{E}\,\mathbf{X}\,(\,\textit{red}\,)$



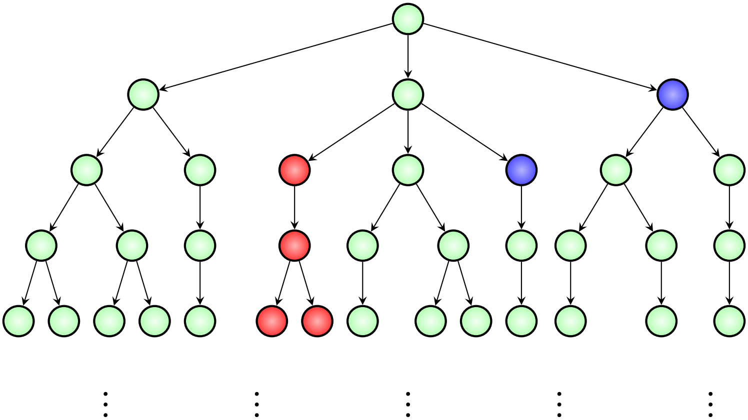
EGEX (*red*)



E G E X (*red*)



E (E X *blue*) U (A G *red*)



Summary

Transition system as a tree

Computation tree

E and A operators

Unit-9: Computation Tree Logic

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Module 2:

CTL*

Recap

- **Path formulae**
  - Express properties of paths
  - LTL
- **Properties on trees**
  - **A** and **E** operators
  - Mixing **A** and **E**

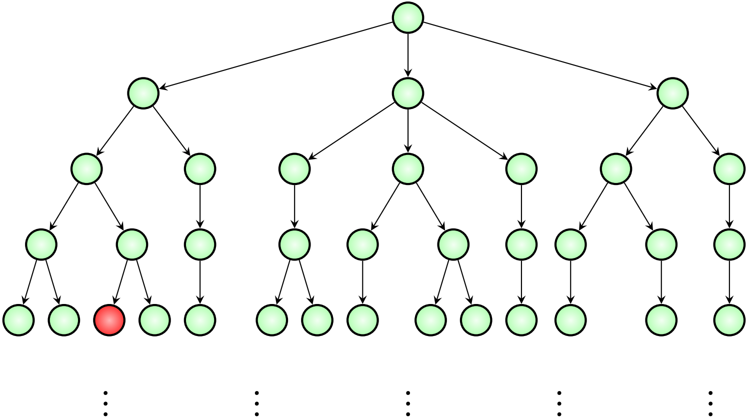
Recap

- **Path formulae**
  - Express properties of paths
  - LTL
- **Properties on trees**
  - **A** and **E** operators
  - Mixing **A** and **E**

Coming next: A **logic** for expressing **properties on trees**

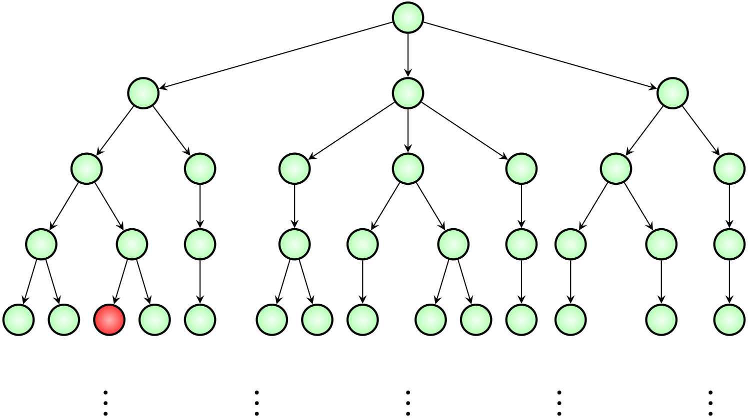
State formulae

$\phi :=$



State formulae

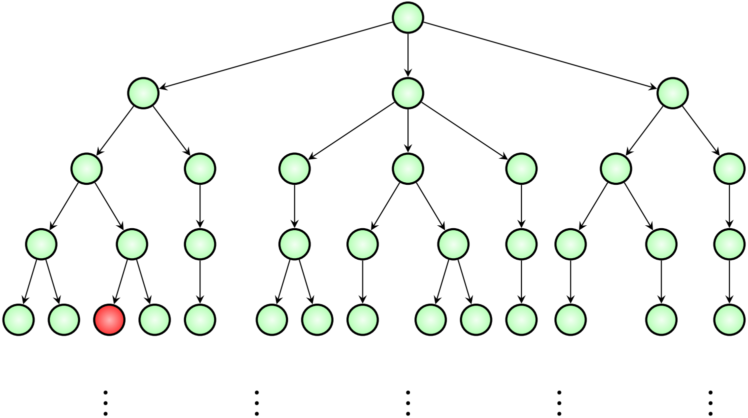
$\phi := \text{true} \mid$



State formulae

$$\phi := \ \text{true} \mid p_i \mid$$

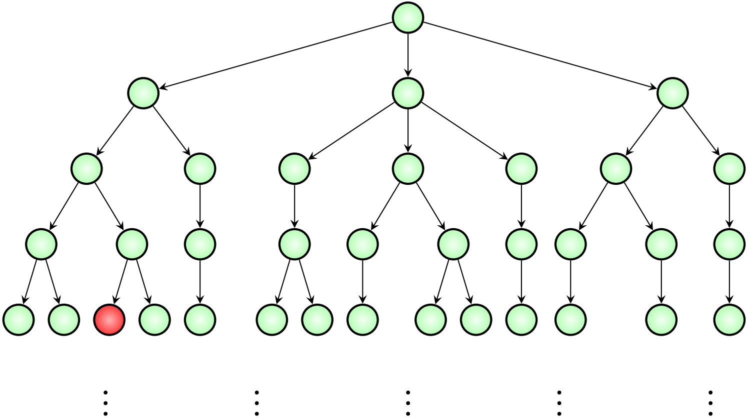
$p_i \in AP$



State formulae

$\phi :=$ true $\mid p_i \mid \phi_1 \wedge \phi_2 \mid$

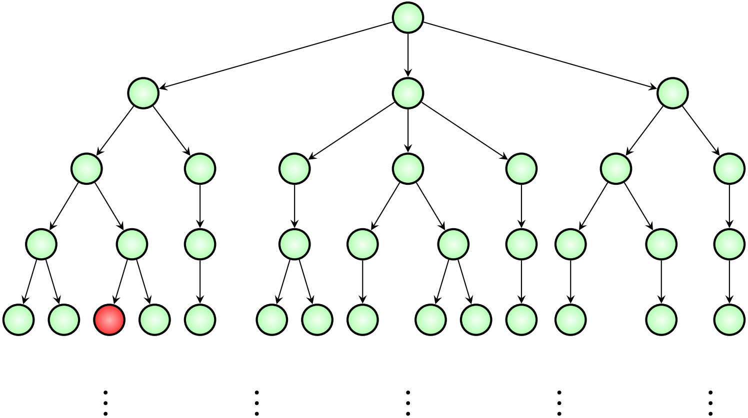
$p_i \in AP$ $\qquad$ ϕ_1, ϕ_2 : State formulae



State formulae

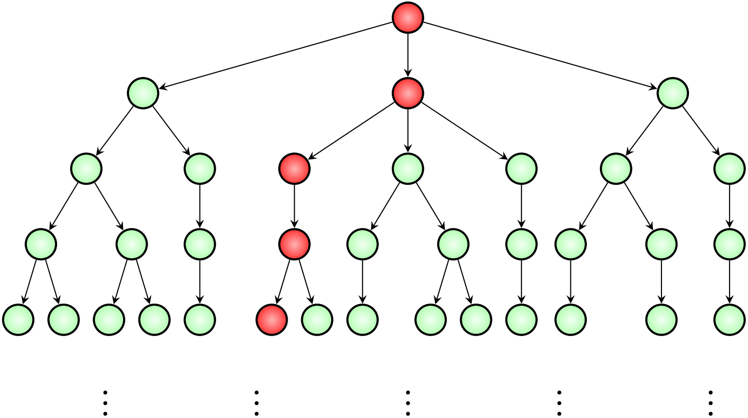
$$\phi := \text{true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1$$

$p_i \in AP$ ϕ_1, ϕ_2 : State formulae



Path formulae

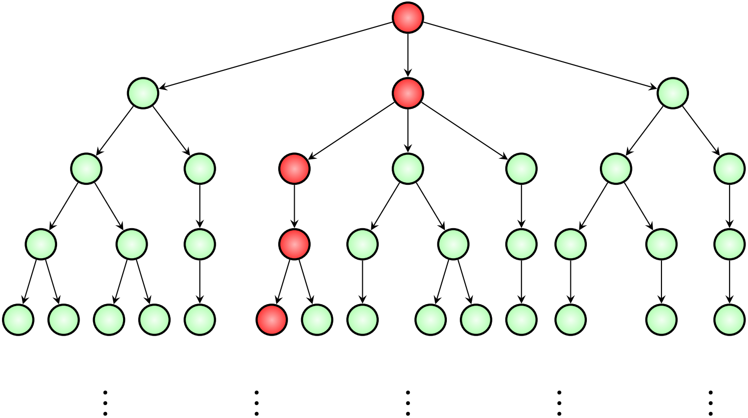
$\alpha :=$



Path formulae

$\alpha := \phi \mid$

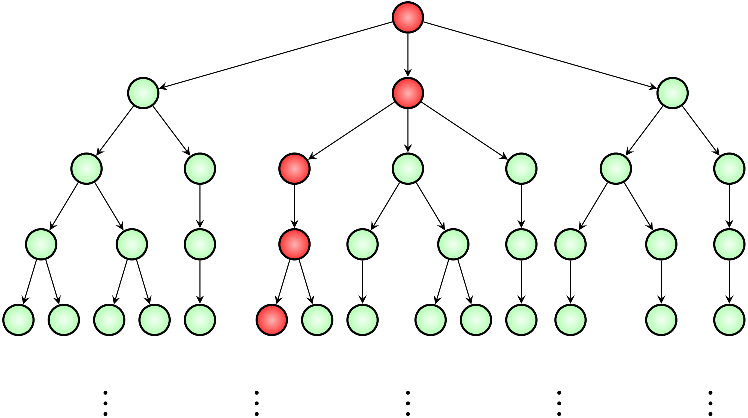
ϕ : State formula



Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid$$

ϕ : State formula α_1, α_2 : Path formulae



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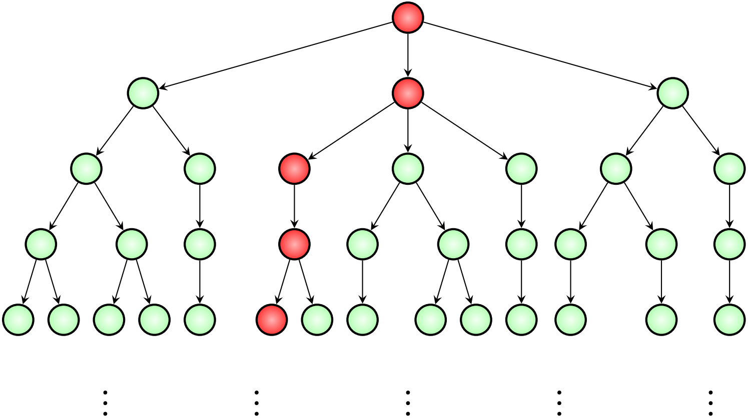
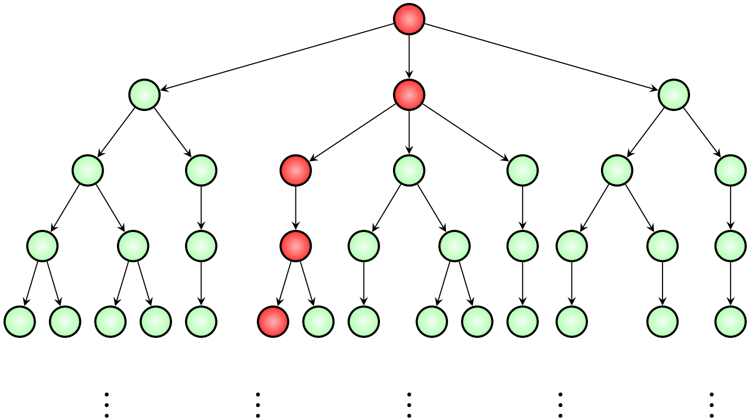


Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid \neg\alpha_1 \mid X\,\alpha_1 \mid$$

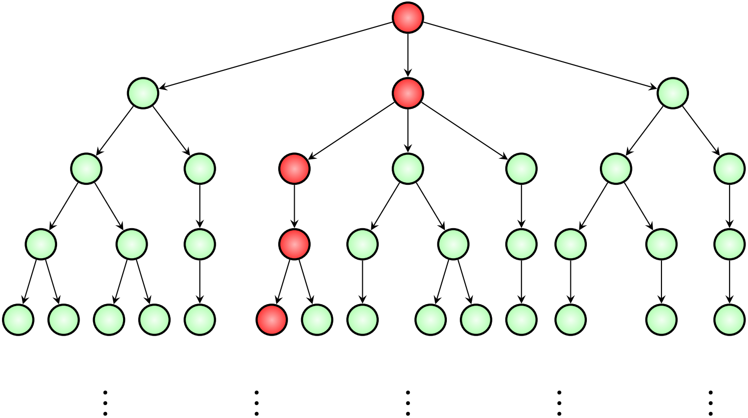
ϕ : State formula $\qquad$ α_1, α_2 : Path formulae



Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid \neg\alpha_1 \mid X\,\alpha_1 \mid \alpha_1\, U\, \alpha_2 \mid$$

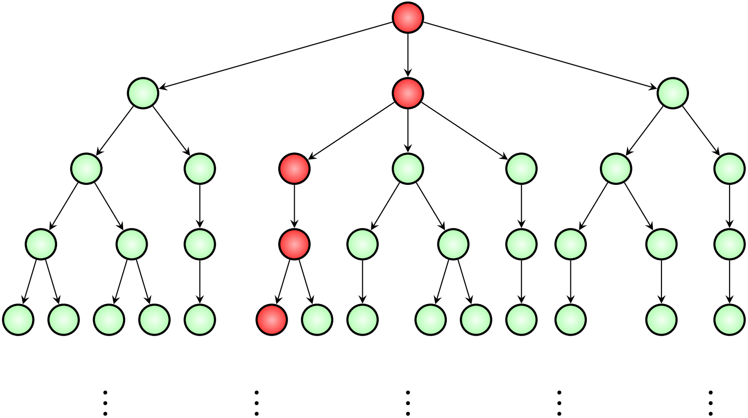
ϕ : State formula $\qquad$ α_1, α_2 : Path formulae



Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid \neg\alpha_1 \mid X\,\alpha_1 \mid \alpha_1\,U\,\alpha_2 \mid F\,\alpha_1 \mid$$

ϕ : State formula α_1, α_2 : Path formulae



Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid \neg\alpha_1 \mid X\,\alpha_1 \mid \alpha_1\, U\, \alpha_2 \mid F\,\alpha_1 \mid G\,\alpha_1$$

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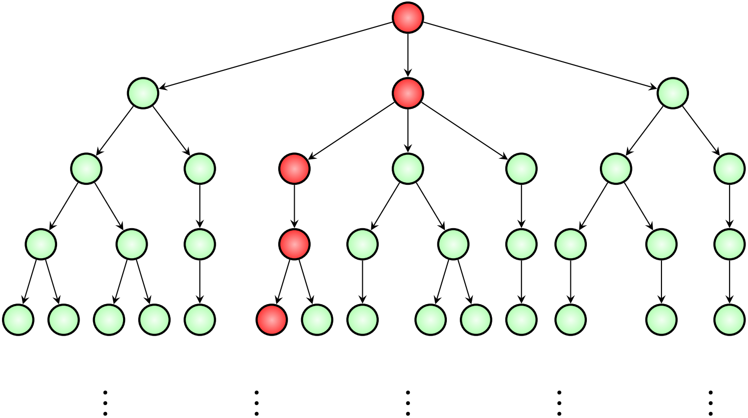
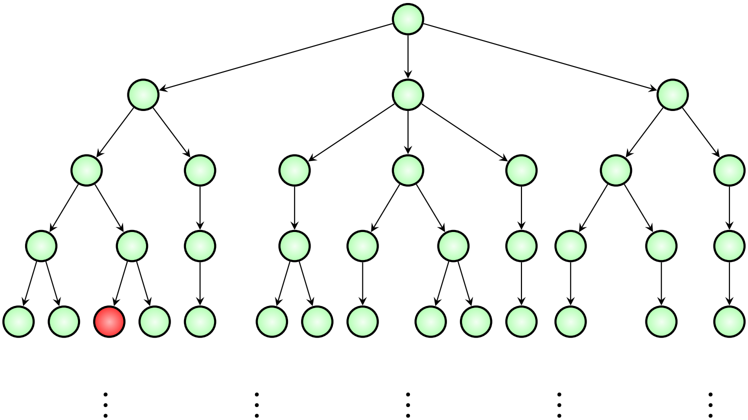


State formulae

$$\phi := \text{ true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1$$

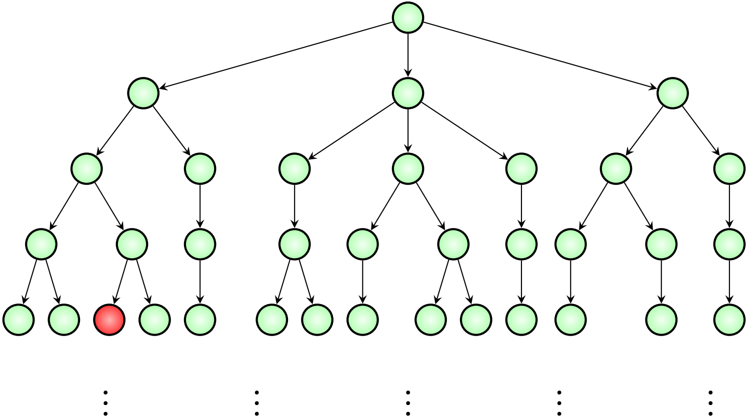
$p_i \in AP$ $\quad$ ϕ_1, ϕ_2 : State formulae



State formulae

$$\phi := \text{ true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid E\,\alpha \mid$$

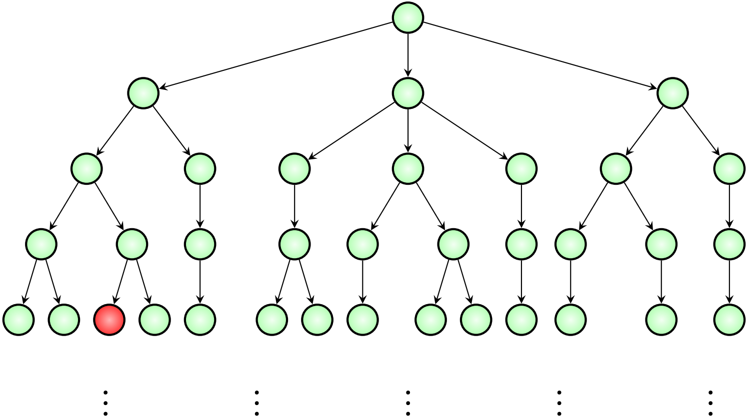
$p_i \in AP$ ϕ_1, ϕ_2 : State formulae α : Path formula



State formulae

$$\phi := \text{ true} \mid p_i \mid \phi_1 \wedge \phi_2 \mid \neg\phi_1 \mid E\,\alpha \mid A\,\alpha$$

$p_i \in AP$ ϕ_1, ϕ_2 : State formulae α : Path formula



CTL*

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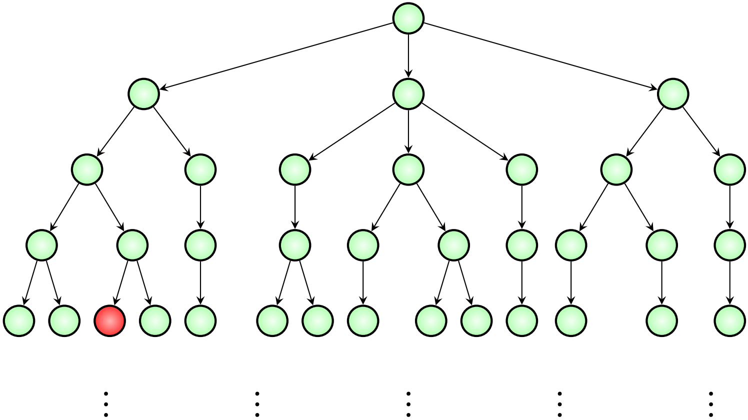
Examples: $\mathrm{E\,F}\,p_1$, $\mathrm{A\,F\,A\,G}\,p_1$, $\mathrm{A\,F\,G}\,p_2$, $\mathrm{A}\,p_1$, $\mathrm{A\,E}\,p_1$

When does a **state** in a tree **satisfy** a state formula?



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- **Every state** satisfies *true*

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$p_i \in AP$ $\quad$ ϕ_1, ϕ_2 : State formulae $\quad$ α : Path formula

- **Every state** satisfies *true*
- State satisfies p_i if its **label contains** p_i

State formulae

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$p_i \in AP$ ϕ_1, ϕ_2 : State formulae α : Path formula

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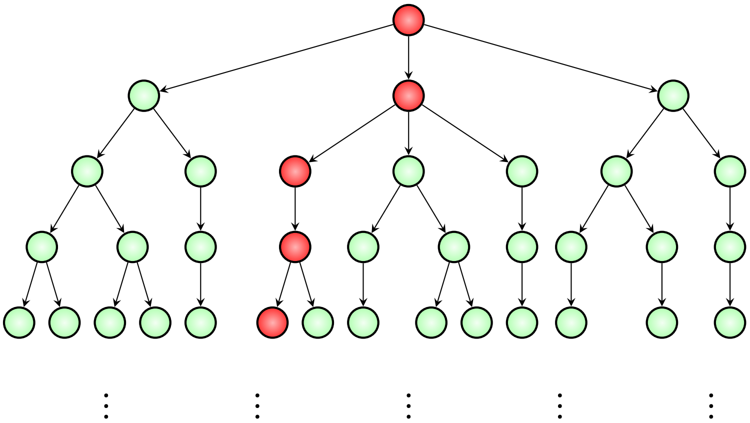
- State satisfies $\mathrm{A}\ \alpha$ if **all paths** starting from the state satisfy α

When does a **path** in a tree **satisfy** a path formula?



Path formulae

$$\alpha := \phi \mid \alpha_1 \wedge \alpha_2 \mid \neg\alpha_1 \mid X\,\alpha_1 \mid \alpha_1\, U\, \alpha_2 \mid F\,\alpha_1 \mid G\,\alpha_1$$

ϕ : State formula $\qquad$ α_1, α_2 : Path formulae

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ϕ : State formula α_1, α_2 : Path formulae

- **Path** satisfies ϕ if the **initial state** of the path satisfies ϕ

Path formulae

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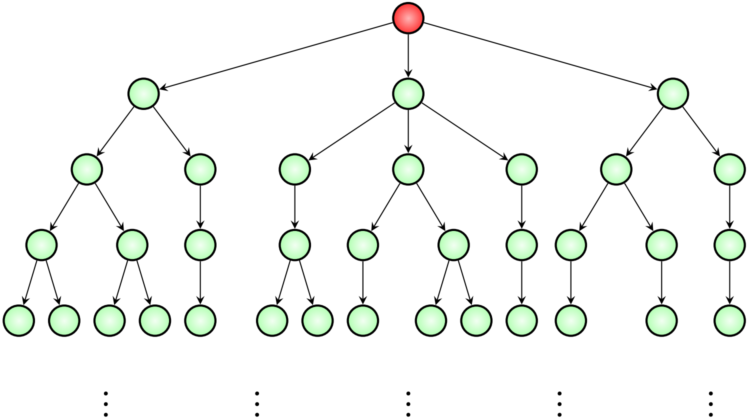
ϕ : State formula α_1, α_2 : Path formulae

- **Path** satisfies ϕ if the **initial state** of the path satisfies ϕ
- Rest **standard** semantics like LTL

A **tree** satisfies **state formula** ϕ if its **root** satisfies ϕ



- **E F** p_1: Exists a path where p_1 is true sometime

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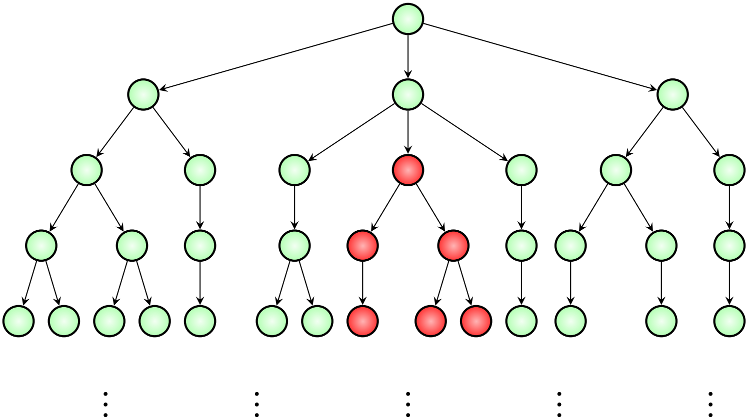
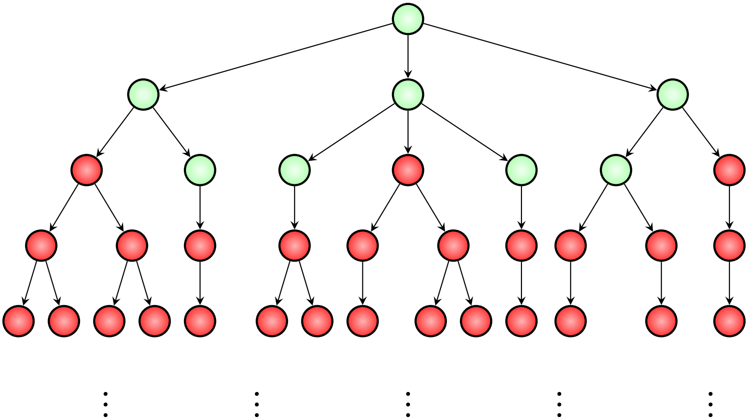
- **A F G** p_2: In all paths, there exists a state from which p_2 is true forever
- **A** p_1:
  - All paths satisfy p_1
  - All paths start with p_1
  - Same as p_1!

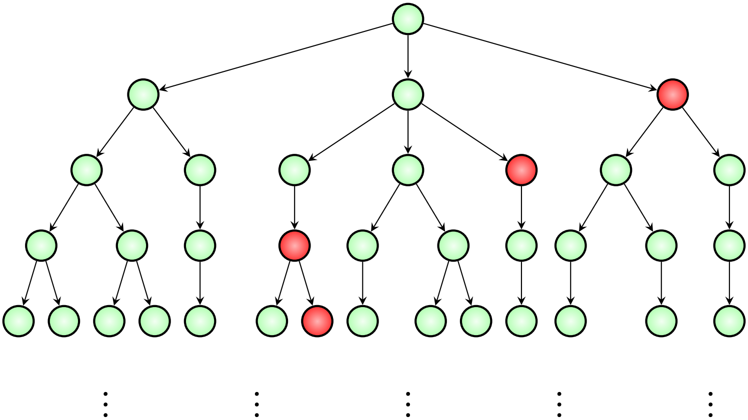
E F A G (*red*)



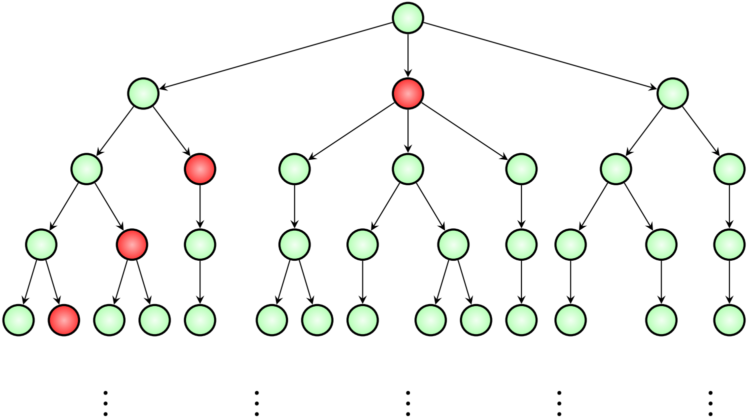
A F A G (*red*)



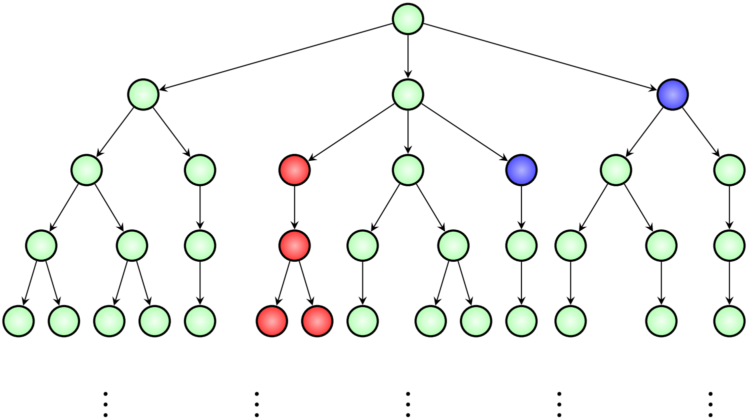
EGEX (*red*)



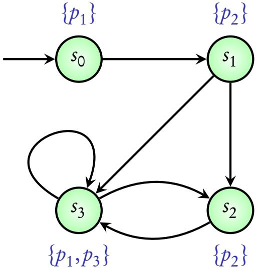
EGEX (*red*)



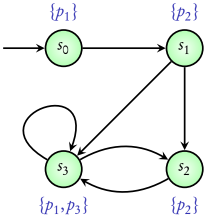
$\mathbf{E}\,(\mathbf{E}\,\mathbf{X}\,\textit{blue})\,\mathbf{U}\,(\mathbf{A}\,\mathbf{G}\,\textit{red})$

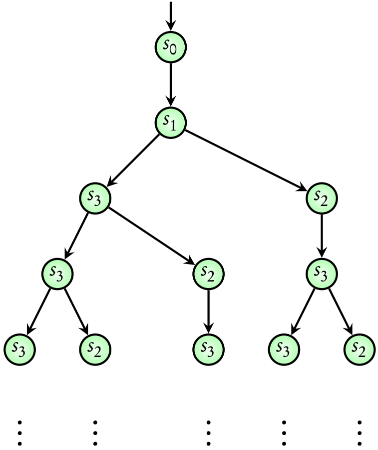


When does a **transition system** satisfy a CTL* formula?



Transition system satisfies CTL* formula ϕ if its **computation tree** satisfies ϕ





Can LTL properties be written using CTL*?

Transition System (TS) satisfies LTL formula ϕ if

$$\text{Traces(TS)} \subseteq \text{Words}(\phi)$$

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All paths in the **computation tree** of TS satisfy **path formula** ϕ

Equivalent CTL* formula: $\mathbf{A}\ \phi$

Can CTL* properties be written using LTL?

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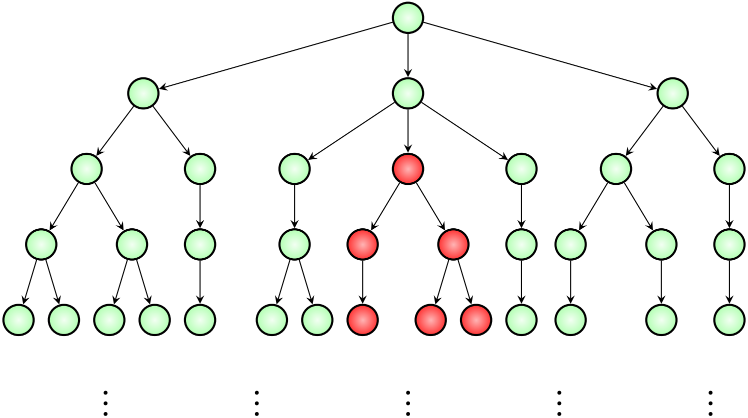
Answer: No

E F A G (*red*)



Cannot be expressed in LTL

Summary

CTL*

Syntax and semantics

State formulae, Path formulae

LTL properties $\subseteq$ CTL* properties

Unit-9: Computation Tree Logic

B. Srivathsan

Chennai Mathematical Institute

NPTEL-course

July - November 2015

Module 3:
CTL

In this module...

Restrict to a **subset** of CTL* which has **efficient model-checking algorithms**

CTL*

State formulae

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CTL

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Legal CTL formulae	Illegal CTL formulae

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Legal CTL formulae	Illegal CTL formulae
$\mathrm{E}\,\mathrm{F}\,p_1$	

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$\mathrm{A\,F}\,p_1 \wedge \mathrm{A\,G}\,p_2$	$\mathrm{A}\,(\mathrm{F}\,p_1 \wedge \mathrm{G}\,p_2)$
$\mathrm{A}\,(p_1\,\mathrm{U}\,(\mathrm{E\,G}\,p_2))$	$\mathrm{A}\,(p_1\,\mathrm{U}\,(\mathrm{G}\,p_2))$

Every temporal operator X, U, F, G has a **corresponding** A or E

CTL

Syntax: Restricted form of CTL*

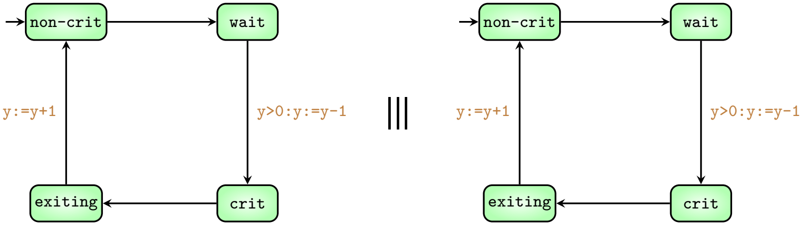
Semantics: Same as seen in CTL*

Example

Atomic propositions $\mathrm{AP} = \{ p_1, p_2, p_3, p_4 \}$

p_1: `pr1.location=crit` p_2: `pr1.location=wait`

p_3: `pr2.location=crit` p_4: `pr2.location=wait`



Mutual exclusion: $\mathbf{A}\,\mathbf{G}\,\neg(p_1 \wedge p_3)$

Can LTL properties be written using CTL?

Can LTL properties be written using CTL?

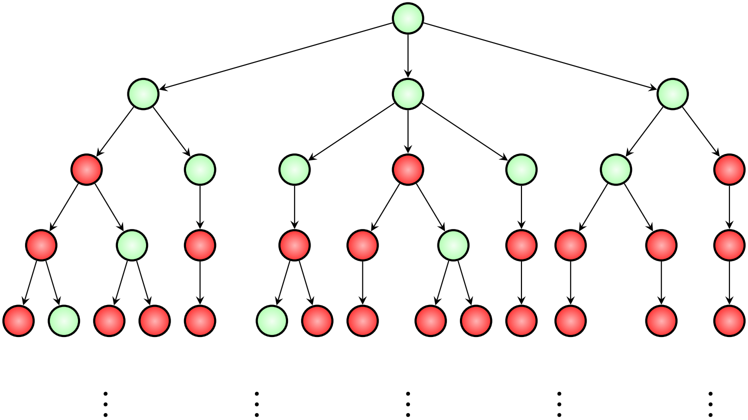
Answer: **No**

Can LTL properties be written using CTL?

Answer: **No**

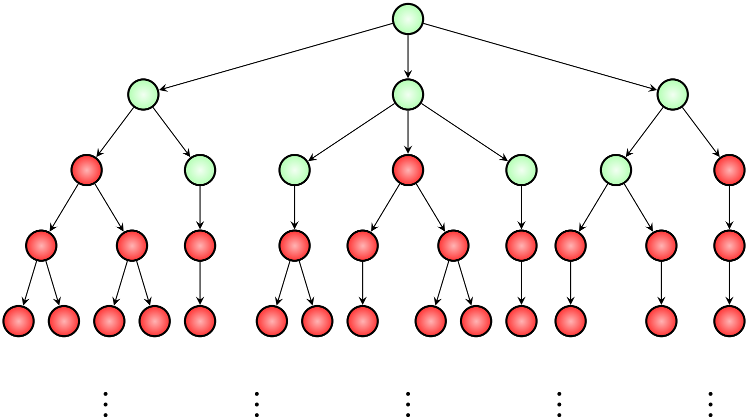
Property $\mathbf{A}\ \mathbf{F}\ \mathbf{G}\ p_1$ cannot be expressed in CTL

A F G (*red*)

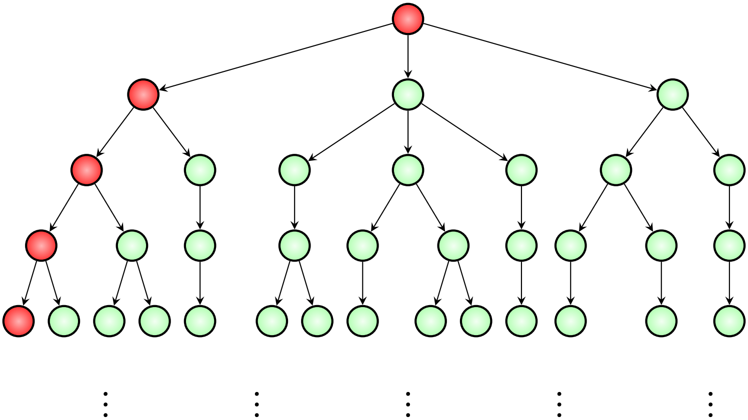


In **all paths**, **eventually** *red* is true **forever**

A F A G (*red*)



AFEG (*red*)



Can LTL properties be written using CTL?

Answer: **No**

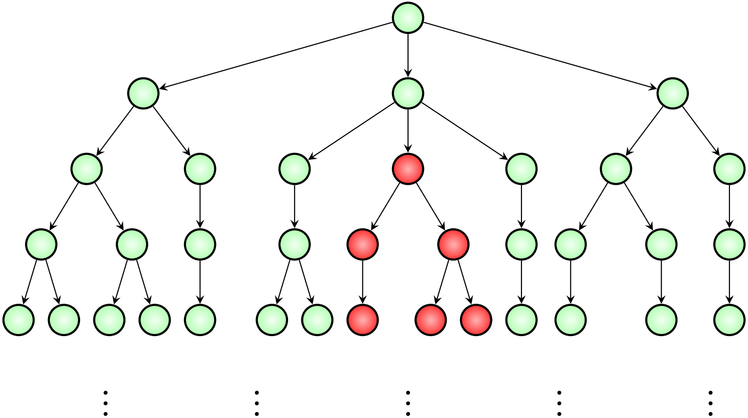
Property $\mathbf{A}\ \mathbf{F}\ \mathbf{G}\ p_1$ cannot be expressed in CTL

Can CTL properties be written using LTL?

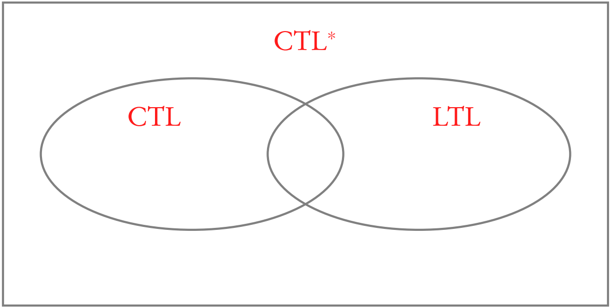
Can CTL properties be written using LTL?

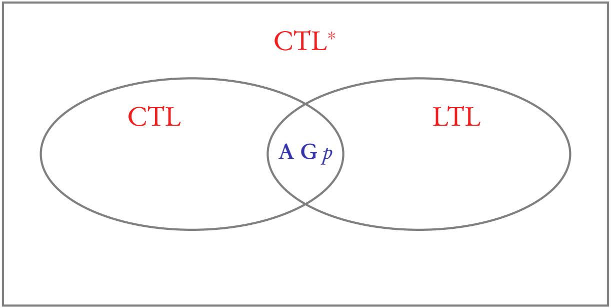
Answer: **No**

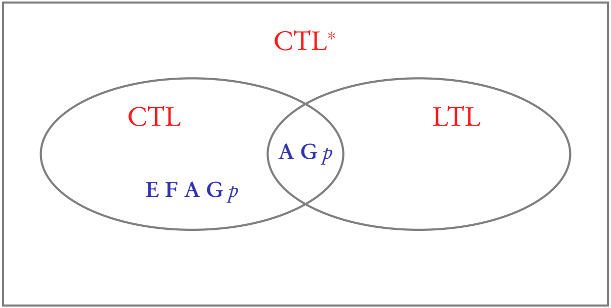
E F A G (*red*)

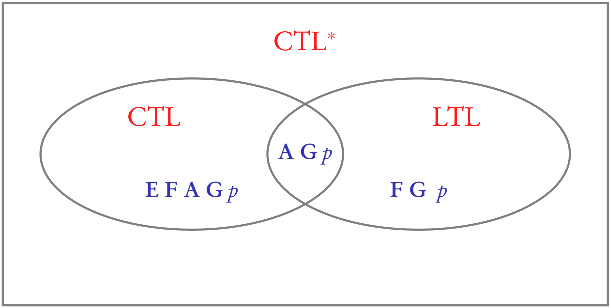


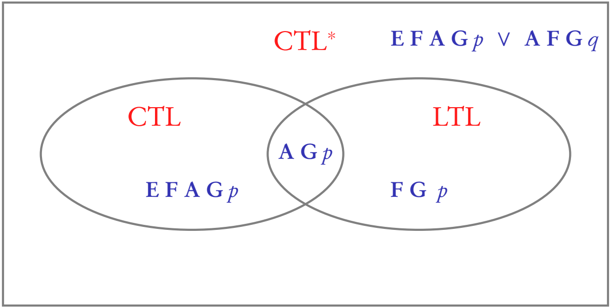
Cannot be expressed in LTL


CTL*
CTL
LTL


CTL*
CTL
LTL
A G p


CTL*
CTL
LTL
A G p
E F A G p


CTL*
CTL
LTL
A G p
E F A G p
F G p


CTL*
E F A G p ∨ A F G q
CTL
LTL
A G p
E F A G p
F G p

Summary

CTL

Subset of CTL*

Paired temporal and A-E operators

Expressive powers

Unit-9: Computation Tree Logic

B. Srivathsan

Chennai Mathematical Institute

NPTEL-course

July - November 2015

Summary

- Properties of **computation trees**
- CTL^* and CTL
- Relative **expressive powers**