Unit-8: Algorithms for LTL

B. Srivathsan

Chennai Mathematical Institute

NPTEL-course

July - November 2015

Module 1:

Automata-based LTL model-checking

Does **Transition system** satisfy **LTL formula** ϕ ?

Does **Transition system** satisfy **LTL formula** ϕ ?

Negation $\neg\,\phi$

Does **Transition system** satisfy **LTL formula ϕ ?**

Negation $\neg\ \phi$

$\downarrow$

NBA $\mathscr{A}_{\neg\phi}$

Does **Transition system** satisfy **LTL formula ϕ ?**

Negation $\neg\,\phi$

$$\text{Transition system} \downarrow \qquad \text{Negation } \neg\phi \downarrow$$

NBA $\mathscr{A}_{T.S}$

NBA $\mathscr{A}_{\neg\phi}$

Does **Transition system** satisfy **LTL formula ϕ ?**

Negation $\neg\,\phi$

NBA $\mathscr{A}_{T.S}$

NBA $\mathscr{A}_{\neg\phi}$

Is $L(\mathscr{A}_{T.S.}) \cap L(\mathscr{A}_{\neg\phi})$ empty ?

Does **Transition system** satisfy **LTL formula ϕ ?**

Negation $\neg \phi$

NBA $\mathscr{A}_{T.S}$

NBA $\mathscr{A}_{\neg\phi}$

Is $L(\mathscr{A}_{T.S.}) \cap L(\mathscr{A}_{\neg\phi})$ empty ?

Is $L(\mathscr{A}_{T.S.} \times \mathscr{A}_{\neg\phi})$ empty ?

Here: Converting LTL formulas to NBA

Here: Converting LTL formulas to NBA

Coming next: Examples

Atomic propositions $\mathbf{AP} = \{ p_1, p_2 \}$

Alphabet:

$\{ \{ \}, \{ p_1 \}, \{ p_2 \}, \{ p_1, p_2 \} \}$

$\mathbf{F}\, p_1$ Words where p_1 occurs sometime

$$\{p_2\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\cdots$$

$$\{p_1,p_2\}\{\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\cdots$$

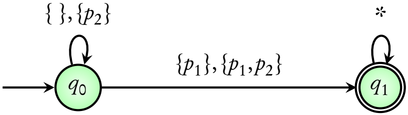
$$\vdots$$

$\mathbf{F}\ p_1$ Words where p_1 occurs sometime

$\{p_2\}\{\,\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\cdots$

$\{p_1,p_2\}\{\,\}\{\,\}\{p_1\}\{p_1\}\{p_1,p_2\}\cdots$

$\vdots$



$\mathbf{G}\, p_1$ Words where p_1 occurs always

$$\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1\}\{p_1\}\dots$$

$$\{p_1,p_2\}\{p_1,p_2\}\{p_1\}\{p_1\}\{p_1\}\{p_1,p_2\}\dots$$

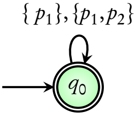
$$\vdots$$

$\mathbf{G}\ p_1$ Words where p_1 occurs always

$\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1\}\{p_1\}\cdots$

$\{p_1,p_2\}\{p_1,p_2\}\{p_1\}\{p_1\}\{p_1\}\{p_1,p_2\}\cdots$

$\vdots$



$p_1 \wedge \neg p_2$ Words starting with $\{p_1\}$

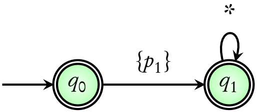
$$\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\dots$$
$$\{p_1\}\{\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\dots$$
$$\vdots$$

$p_1 \wedge \neg p_2$ Words starting with $\{p_1\}$

$\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{p_1\}\{\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$



$$p_1 \wedge \mathbf{X} \neg p_2$$

$$\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\cdots$$

$$\{p_1,p_2\}\{p_1\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\cdots$$

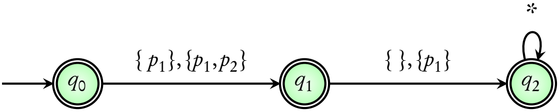
$$\vdots$$

$$p_1 \wedge \mathbf{X}\,\neg p_2$$

$$\{p_1\}\,\{\,\}\,\{p_2\}\,\{p_1,p_2\}\,\{p_2\}\,\{p_2\}\,\{p_2\}\cdots$$

$$\{p_1,p_2\}\,\{p_1\}\,\{\,\}\,\{p_1\}\,\{p_1\}\,\{p_1,p_2\}\cdots$$

$$\vdots$$



$$\mathbf{XX}\,(p_1 \;\wedge\; \mathbf{X}\,\neg p_2)$$

$$\{\ \}\{\ \}\{p_1\}\{\ \}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\dots$$

$$\{p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\dots$$

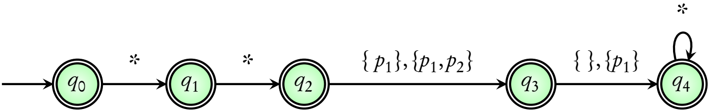
$$\vdots$$

$$\mathbf{XX}\,(p_1 \;\wedge\; \mathbf{X}\,\neg p_2)$$

$$\{\,\}\{\,\}\{p_1\}\{\,\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$$

$$\{p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{\,\}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$$

$$\vdots$$



$$p_1 \ \mathbf{U} \ p_2$$

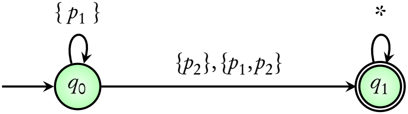
$$\{\, p_1 \,\} \, \{\, p_1 \,\} \, \{\, p_2 \,\} \, \{\, p_1, p_2 \,\} \, \{\, p_2 \,\} \, \{\, p_2 \,\} \, \{\, p_2 \,\} \cdots$$

$$\{\, p_1, p_2 \,\} \, \{\,\} \, \{\ \ \} \, \{\, p_1 \,\} \, \{\, p_1 \,\} \, \{\, p_1, p_2 \,\} \cdots$$

$$\vdots$$

$$p_1 \; \mathbf{U} \; p_2$$

$$\{p_1\}\{p_1\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\dots$$
$$\{p_1,p_2\}\{\,\}\{\,\}\{p_1\}\{p_1\}\{p_1,p_2\}\dots$$
$$\vdots$$



$(\mathbf{X}\ p_1)\ \mathbf{U}\ \ p_2$

$$\{p_2\}\ \{\ \}\ \{p_2\}\ \{p_1,p_2\}\ \{p_2\}\ \{p_2\}\ \{p_2\}\ \cdots$$

$$\{\ \}\ \{p_1\}\ \{p_1\}\ \{p_1\}\ \{p_1,p_2\}\ \{p_1,p_2\}\ \cdots$$

$$\{\ \}\ \{p_1,p_2\}\ \{\ \}\ \{\ \}\ \{p_2\}\ \{p_1,p_2\}\ \cdots$$

$$\vdots$$

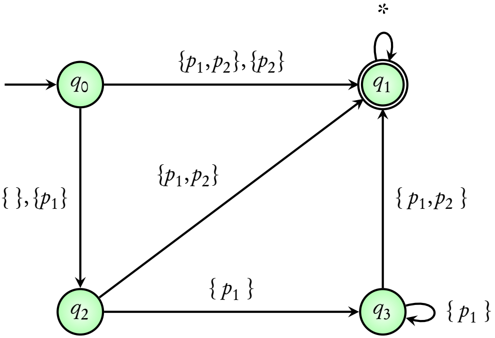
$(\mathbf{X}\ p_1)\ \mathbf{U}\ \ p_2$

$\{p_2\}\{\ \}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{\ \}\{p_1\}\{p_1\}\{p_1\}\{p_1,p_2\}\{p_1,p_2\}\ldots$

$\{\ \}\{p_1,p_2\}\{\ \}\{\ \}\{p_2\}\{p_1,p_2\}\ldots$

$\vdots$



$$\mathbf{F}\, p_1 \wedge \neg\, \mathbf{F}\, p_2$$

$$\{\ \}\{\ \}\{p_1\}\{p_1\}\{\ \}\{p_1\}\{p_1\}\ldots$$

$$\{p_1\}\{\ \}\{\ \}\{\ \}\{\ \}\{\ \}\ldots$$

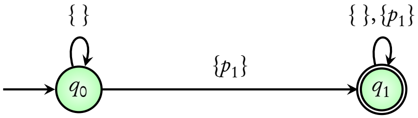
$$\vdots$$

$$\mathbf{F}\, p_1 \wedge \neg\, \mathbf{F}\, p_2$$

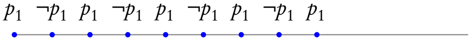
$\{\ \}\{\}\{p_1\}\{p_1\}\{\ \}\{p_1\}\{p_1\}\ldots$

$\{p_1\}\{\}\{\ \}\{\ \}\{\ \}\{\ \}\ldots$

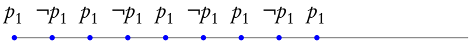
$\vdots$

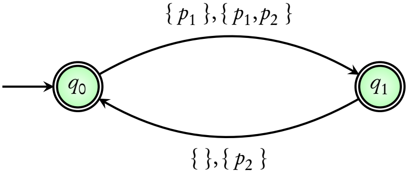


$$p_1 \wedge \mathbf{X}\neg p_1 \wedge \mathbf{G}(p_1 \leftrightarrow \mathbf{XX}\,p_1)$$



$$p_1 \wedge \mathbf{X}\neg p_1 \wedge \mathbf{G}(p_1 \leftrightarrow \mathbf{XX}\,p_1)$$





$\mathbf{G}\ \mathbf{F}\ p_1$ Words where p_1 occurs infinitely often

$$\{\ \}\,\{p_1\}\,\{p_2\}\,\{p_1, p_2\}\,\{p_2\}\,\{p_1\}\,\{p_2\}\cdots$$

$$\{\ \}\,\{\}\,\{\ \}\,\{p_1\}\,\{p_1\}\,\{p_1\}\cdots$$

$$\vdots$$

$\mathbf{G}\ \mathbf{F}\ p_1$ Words where p_1 occurs infinitely often

$\{\ \}\{p_1\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_1\}\{p_2\}\dots$

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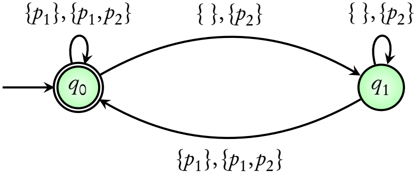
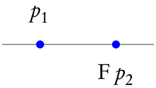
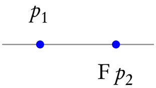
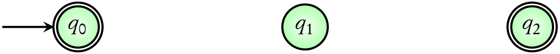


$$\mathbf{G}\,(p_1 \;\rightarrow\; \mathbf{XF}\; p_2)$$

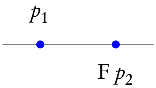


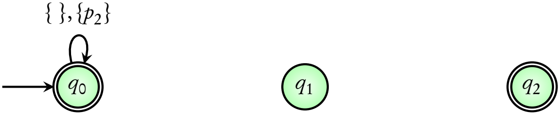
$\mathbf{G}\,(p_1 \rightarrow \mathbf{XF}\,p_2)$



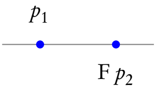


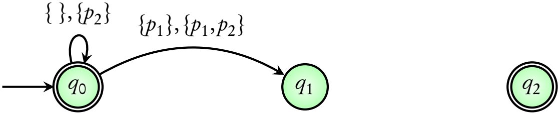
$\mathbf{G}\ (p_1 \rightarrow \mathbf{XF}\ p_2)$



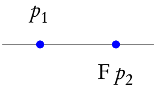


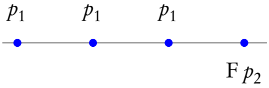
$\mathbf{G}\ (p_1 \rightarrow \mathbf{XF}\ p_2)$

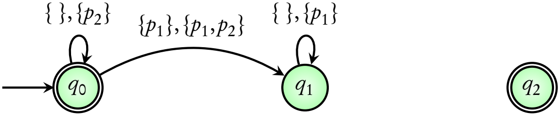




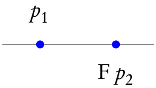
$$\mathbf{G}\,(p_1 \rightarrow \mathbf{XF}\,p_2)$$

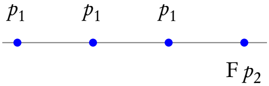


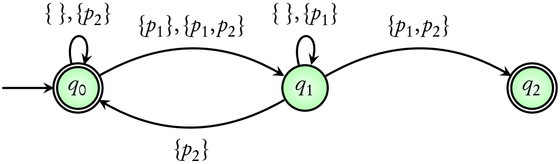




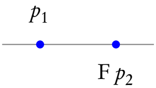
$$\mathbf{G}\,(p_1 \rightarrow \mathbf{XF}\,p_2)$$

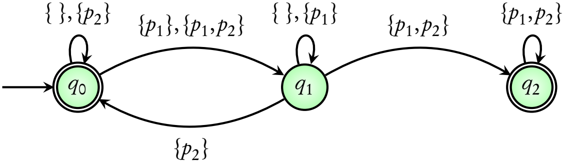




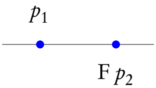


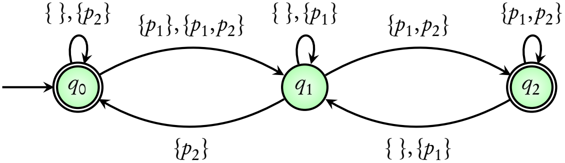
$\mathbf{G}\ (p_1 \rightarrow \mathbf{XF}\ p_2)$



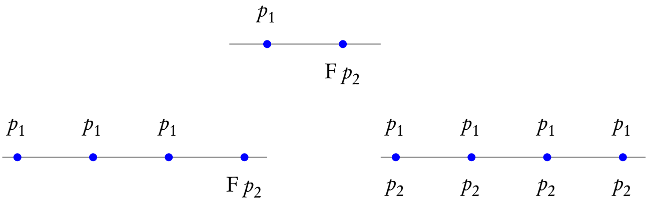


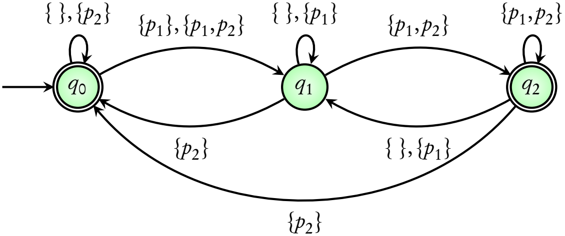
$\mathbf{G}\ (p_1 \rightarrow \mathbf{XF}\ p_2)$





$\mathbf{G}\,(p_1 \rightarrow \mathbf{XF}\,p_2)$





Summary

LTL model-checking

Method

LTL to NBA examples

Unit-8: Algorithms for LTL

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NPTEL-course

July - November 2015

Module 2:

LTL to NBA

Goal: Understand the **evaluation** of an LTL formula on an infinite word

$p_1 \ \mathbf{U} \ p_2$

$$p_1 \ \mathbf{U} \ p_2$$

$\{p_1\} \quad \{p_1\} \quad \{p_1\} \quad \{p_1\} \quad \{p_2\} \quad \{p_1\} \quad \{p_1\} \quad \{p_1\} \quad \{p_1, p_2\} \ \cdots$

$$p_1 \;\mathbf{U}\; p_2$$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\cdots$
p_1										
p_2										
$p_1 \; U \; p_2$										

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$ $\cdots$
p_1									
p_2									
$p_1 \ U \ p_2$									

$$p_1 \ \mathbf{U} \ p_2$$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$									

$$p_1 \ \mathbf{U} \ p_2$$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1

p_1									
p_2									
$p_1 \ U \ p_2$									

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1										
p_2										
$p_1 \ U \ p_2$										

p_1 **U** p_2

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1\ U\ p_2$	1	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{\ \}$	$\{p_1\}$	$\{\ \}$	$\{p_1\}$	$\{\ \}$	$\{p_1\}$	$\{\ \}$	$\{p_1\}$ $\cdots$
p_1	1	0	1	0	1	0	1	0	1
p_2	0	0	0	0	0	0	0	0	0
$p_1\ U\ p_2$									

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$ $\cdots$
p_1	1	1	1	1	0	1	1	1	1
p_2	0	0	0	0	1	0	0	0	1
$p_1 \ U \ p_2$	1	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$ $\cdots$
p_1	1	0	1	0	1	0	1	0	1
p_2	0	0	0	0	0	0	0	0	0
$p_1 \ U \ p_2$	0	0	0	0	0	0	0	0	0

G F p_1

$\mathbf{G\ F}\ p_1$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\;\mathit{true}\;\mathrm{U}\;\neg\phi$

$$\neg\;\; \mathit{true}\;\mathbf{U}\;\neg(\mathit{true}\;\mathbf{U}\;p_1)$$

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

$\{\,\}\quad \{\,\}\quad \{p_1\}\quad \{\,\}\quad \{\,\}\quad \{p_1\}\quad \{\,\}\quad \{\,\}\quad \{p_1\}$

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$
p_1									
true									
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$\mathbf{G}\,\mathbf{F}\,p_1$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$\neg\ \mathit{true}\,\mathbf{U}\,\neg(\mathit{true}\,\mathbf{U}\,p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true									
$\mathit{true}\,\mathrm{U}\,p_1$									
$\neg\,\mathit{true}\,\mathrm{U}\,p_1$									
$\mathit{true}\,\mathrm{U}\,\neg\,(\mathit{true}\,\mathrm{U}\,p_1)$									
$\neg\ \mathit{true}\,\mathrm{U}\,\neg\,(\mathit{true}\,\mathrm{U}\,p_1)$									

$\mathbf{G}\ \mathbf{F}\ p_1$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\;\mathrm{U}\,\neg\phi$

$$\neg\;\mathit{true}\;\mathbf{U}\,\neg(\mathit{true}\;\mathbf{U}\;p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\;\mathrm{U}\;p_1$	1	1	1	1	1	1	1	1	1
$\neg\,\mathit{true}\;\mathrm{U}\;p_1$									
$\mathit{true}\;\mathrm{U}\,\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$									
$\neg\;\mathit{true}\;\mathrm{U}\,\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\, \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\,(\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$
p_1	0	0	1	0	0	1	0	0	1
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	0	0	0	0	0	0	0
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1

$\mathbf{G}\,\mathbf{F}\,p_1$

$\mathbf{G}\ \mathbf{F}\ p_1$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$ and $\mathrm{G}\,\phi = \neg\,\mathit{true}\,\mathrm{U}\,\neg\phi$

$$\neg\ \mathit{true}\,\mathbf{U}\,\neg(\mathit{true}\,\mathbf{U}\,p_1)$$

$\{p_1\}$ $\{p_1\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$ $\{\,\}$

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1									
true									
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\ \}$	$\{\ \}$	$\{\ \}$	$\{\ \}$	$\{\ \}$	$\{\ \}$	$\{\ \}$
p_1	1	1	0	0	0	0	0	0	0
true									
$\mathit{true}\ \mathrm{U}\ p_1$									
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\,\mathbf{F}\,p_1$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi \quad$ and $\quad \mathrm{G}\,\phi = \neg\,\mathit{true}\;\mathrm{U}\;\neg\phi$

$$\neg\;\mathit{true}\;\mathbf{U}\;\neg(\mathit{true}\;\mathbf{U}\;p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\;\mathrm{U}\;p_1$									
$\neg\,\mathit{true}\;\mathrm{U}\;p_1$									
$\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$									
$\neg\;\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\quad \mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi \quad$ and $\quad \mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$									
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\ \phi = true\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ true\ \mathrm{U}\ \neg\phi$

$$\neg\ true\ \mathbf{U}\ \neg(true\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
$true$	1	1	1	1	1	1	1	1	1
$true\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ true\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$true\ \mathrm{U}\ \neg\ (true\ \mathrm{U}\ p_1)$									
$\neg\ true\ \mathrm{U}\ \neg\ (true\ \mathrm{U}\ p_1)$									

$\mathbf{G}\ \mathbf{F}\ p_1$

recall that $\mathrm{F}\ \phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\ \phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$									

$$\mathbf{G}\ \mathbf{F}\ p_1$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$ and $\mathrm{G}\,\phi = \neg\ \mathit{true}\ \mathrm{U}\ \neg\phi$

$$\neg\ \mathit{true}\ \mathbf{U}\ \neg(\mathit{true}\ \mathbf{U}\ p_1)$$

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ p_1$	1	1	0	0	0	0	0	0	0
$\neg\ \mathit{true}\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	1	1	1	1	1	1	1	1	1
$\neg\ \mathit{true}\ \mathrm{U}\ \neg\ (\mathit{true}\ \mathrm{U}\ p_1)$	0	0	0	0	0	0	0	0	0

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\,\mathbf{U}\,p_1))$$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\,\mathbf{U}\,p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\, \mathbf{U}\, p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\, \mathbf{U}\, p_1))$$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \,\mathbf{U}\, p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\,\mathrm{U}\,\phi$

$$\mathit{true}\,\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \,\mathbf{U}\, p_1))$$

$\{\}$ $\{p_2\}$ $\{\}$ $\{\}$ $\{p_1\}$ $\{p_1, p_2\}$ $\{p_1, p_2\}$ $\cdots$

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1								
p_2								
$\neg p_1$								
$\neg p_2$								
$\neg p_2\;\mathrm{U}\;p_1$								
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\ldots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$								
$\neg p_2$								
$\neg p_2\;\mathrm{U}\;p_1$								
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$								
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\mathit{true}\ \mathrm{U}\,(\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	…
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$					1	1	1	
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$								
$\mathit{true}\ \mathrm{U}\,(\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$			1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2\}$	$\dots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \;\mathrm{U}\; p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$								
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0							
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1						
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1					
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1				
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\dots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1			
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1		
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\ldots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$								
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0							
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1						
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \ \wedge\ \mathbf{X}\,(\neg\, p_2\ \mathbf{U}\ p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	1	1	0	
$\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1					
$\mathit{true}\ \mathrm{U}\,(\neg p_1\ \wedge\ \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\ldots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \;\mathrm{U}\; p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$	0	1	1	1				
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\ \mathrm{U}\ \phi$

$$\mathit{true}\ \mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\ \mathrm{U}\ p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1)$	0	1	1	1	0			
$\mathit{true}\ \mathrm{U}\,(\neg p_1 \wedge \mathrm{X}\,(\neg p_2\ \mathrm{U}\ p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0		
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\;\mathbf{U}\;p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$								

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

recall that $\mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$	1	1	1	1				

$$\mathbf{F}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

recall that $\quad \mathrm{F}\,\phi = \mathit{true}\;\mathrm{U}\;\phi$

$$\mathit{true}\;\mathbf{U}\,(\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2\; \mathbf{U}\; p_1))$$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\,(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2\;\mathrm{U}\;p_1))$	1	1	1	1	0	0	0	

$p_1 \; \mathbf{U} \; p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \,U\, p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \,U\, p_2$	0	0	0	0	0	0	0	0	0	

$\mathit{true}\; \mathbf{U}\; (\neg p_1 \;\wedge\; \mathbf{X}\,(\neg\, p_2 \;\mathbf{U}\; p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \;\mathrm{U}\; p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1)$	0	1	1	1	0	0	0	
$\mathit{true}\;\mathrm{U}\;(\neg p_1 \;\wedge\; \mathrm{X}\,(\neg p_2 \;\mathrm{U}\; p_1))$	1	1	1	1	1	1	0	

$\neg\; \mathit{true}\; \mathbf{U}\; \neg(\mathit{true}\; \mathbf{U}\; p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
$\mathit{true}\;\mathrm{U}\;p_1$	1	1	1	1	1	1	1	1
$\neg\;\mathit{true}\;\mathrm{U}\;p_1$	0	0	0	0	0	0	0	0
$\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$	0	0	0	0	0	0	0	0
$\neg\;\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true}\;\mathrm{U}\;p_1$	1	1	0	0	0	0	0	0	0
$\neg\;\mathit{true}\;\mathrm{U}\;p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$	1	1	1	1	1	1	1	1	1
$\neg\;\mathit{true}\;\mathrm{U}\;\neg\,(\mathit{true}\;\mathrm{U}\;p_1)$	0	0	0	0	0	0	0	0	0

Formula expansions

$p_1 \ \mathbf{U} \ p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \, U \, p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \, U \, p_2$	0	0	0	0	0	0	0	0	0	

$\mathit{true} \ \mathbf{U} \ (\neg p_1 \ \wedge \ \mathbf{X} \, (\neg \, p_2 \ \mathbf{U} \ p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \, \mathrm{U} \, p_1$	0	0	1	1	1	1	1	
$\mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \ \wedge \ \mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1)$	0	1	1	1	0	0	0	
$\mathit{true} \, \mathrm{U} \, (\neg p_1 \ \wedge \ \mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1))$	1	1	1	1	1	1	0	

$\neg \ \mathit{true} \ \mathbf{U} \ \neg(\mathit{true} \ \mathbf{U} \ p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
$\mathit{true} \, \mathrm{U} \, p_1$	1	1	1	1	1	1	1	1
$\neg \, \mathit{true} \, \mathrm{U} \, p_1$	0	0	0	0	0	0	0	0
$\mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	0	0	0	0	0	0	0	0
$\neg \ \mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	1	1	1	1	1	1	1	1

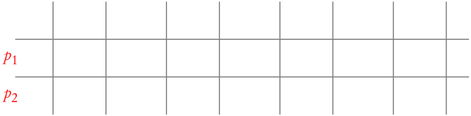
	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true} \, \mathrm{U} \, p_1$	1	1	0	0	0	0	0	0	0
$\neg \, \mathit{true} \, \mathrm{U} \, p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	1	1	1	1	1	1	1	1	1
$\neg \ \mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	0	0	0	0	0	0	0	0	0

Key idea: Construct automata whose **states are columns of the formula expansion**

Key idea: Construct automata whose **states are columns of the formula expansion**

Next in this module: **understand properties** of formula expansions

Word compatibility



Word compatibility

	$\{\,\}$							
p_1	0							
p_2	0							

Word compatibility

	$\{\ \}$		$\{p_1\}$					
p_1	0		1					
p_2	0		0					

Word compatibility

	$\{\,\}$		$\{p_1\}$		$\{p_2\}$			
p_1	0		1		0			
p_2	0		0		1			

Word compatibility

	$\{\ \}$	$\{p_1\}$	$\{p_2\}$	$\{p_1, p_2\}$
p_1	0	1	0	1
p_2	0	0	1	1

AND-NOT-compatibility

ϕ		0		1	

$\neg\phi$		1		0	

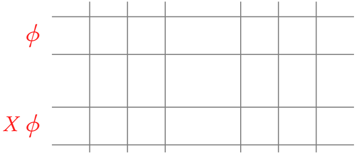
AND-NOT-compatibility

ϕ		0		1	

$\neg\phi$		1		0	

ϕ_1		1		0		1		0	
ϕ_2		1		1		0		0	
$\phi_1 \land \phi_2$		1		0		0		0	

X-compatibility



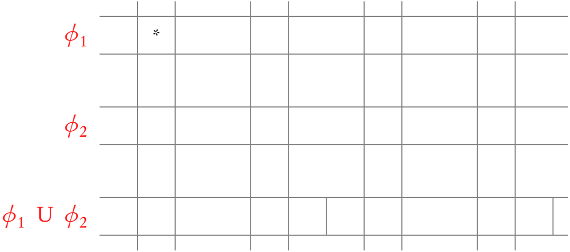
X-compatibility

ϕ			0				
$X\ \phi$		0					

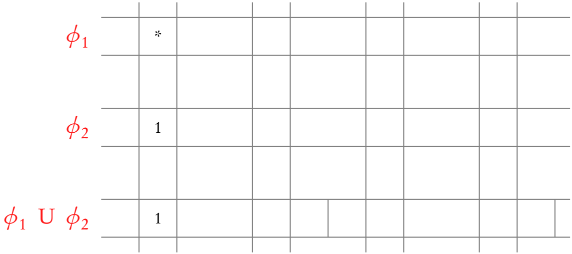
X-compatibility

ϕ			0			1	
$X\,\phi$		0			1		

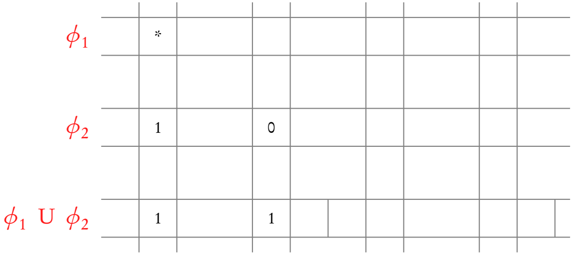
Until-compatibility



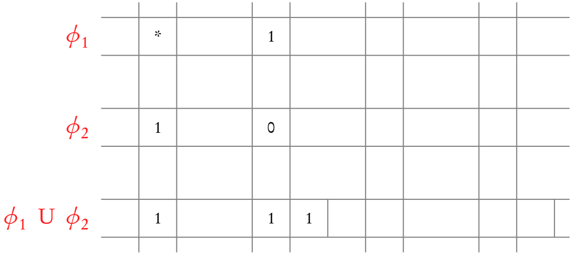
Until-compatibility



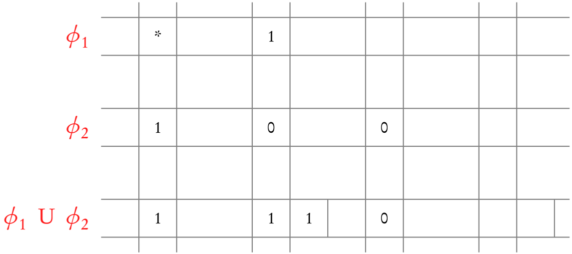
Until-compatibility



Until-compatibility



Until-compatibility



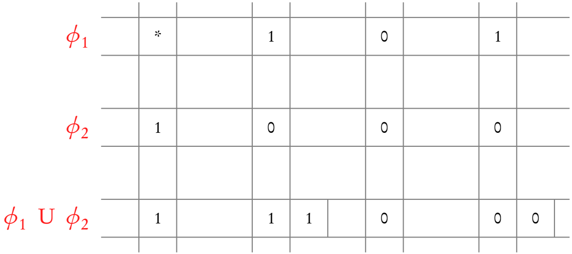
Until-compatibility

ϕ_1	*		1		0			
ϕ_2	1		0		0			
$\phi_1 \ \mathrm{U} \ \phi_2$	1		1	1	0			

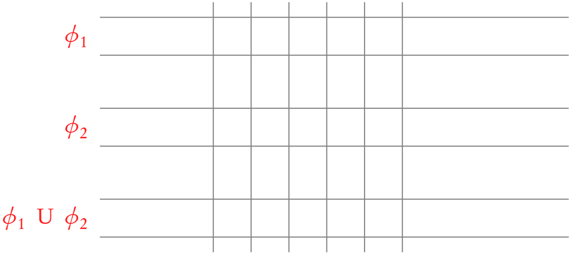
Until-compatibility

ϕ_1	*		1		0		1
ϕ_2	1		0		0		0
$\phi_1 \; \mathrm{U} \; \phi_2$	1		1	1	0		0

Until-compatibility



Until-compatibility: eventuality condition



Until-compatibility: eventuality condition

ϕ_1		1				
ϕ_2		0				
$\phi_1 \ U \ \phi_2$		1	1			

Until-compatibility: eventuality condition

ϕ_1	1	1			
ϕ_2	0	0			
$\phi_1 \ \mathrm{U} \ \phi_2$	1	1	1		

Until-compatibility: eventuality condition

ϕ_1	1	1	1		
ϕ_2	0	0	0		
$\phi_1 \mathbin{\mathrm{U}} \phi_2$	1	1	1	1	

Until-compatibility: eventuality condition

ϕ_1	1	1	1	1	
ϕ_2	0	0	0	0	
$\phi_1 \ \mathrm{U} \ \phi_2$	1	1	1	1	1

Until-compatibility: eventuality condition

ϕ_1		1	1	1	1	1	
ϕ_2		0	0	0	0	0	$\cdots$
$\phi_1 \ \mathrm{U} \ \phi_2$		1	1	1	1	1	

Until-compatibility: eventuality condition

ϕ_1		1	1	1	1	1	
ϕ_2		0	0	0	0	0	$\cdots$
$\phi_1 \ \mathrm{U} \ \phi_2$		1	1	1	1	1	

Cannot happen forever that $\phi_1 \ \mathrm{U} \ \phi_2 = 1$, $\phi_1 = 1$ but $\phi_2 = 0$

Accepting expansions

$p_1 \; \mathbf{U} \; p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \,\mathrm{U}\, p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \,\mathrm{U}\, p_2$	0	0	0	0	0	0	0	0	0	

$\mathit{true} \; \mathbf{U} \; (\neg p_1 \; \wedge \; \mathbf{X}\,(\neg \, p_2 \; \mathbf{U} \; p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \,\mathrm{U}\, p_1$	0	0	1	1	1	1	1	
$\mathrm{X}\,(\neg p_2 \,\mathrm{U}\, p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X}\,(\neg p_2 \,\mathrm{U}\, p_1)$	0	1	1	1	0	0	0	
$\mathit{true} \,\mathrm{U}\, (\neg p_1 \wedge \mathrm{X}\,(\neg p_2 \,\mathrm{U}\, p_1))$	1	1	1	1	1	1	0	

$\neg \; \mathit{true} \; \mathbf{U} \; \neg(\mathit{true} \; \mathbf{U} \; p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
$\mathit{true} \,\mathrm{U}\, p_1$	1	1	1	1	1	1	1	1
$\neg \, \mathit{true} \,\mathrm{U}\, p_1$	0	0	0	0	0	0	0	0
$\mathit{true} \,\mathrm{U}\, \neg\,(\mathit{true} \,\mathrm{U}\, p_1)$	0	0	0	0	0	0	0	0
$\neg \, \mathit{true} \,\mathrm{U}\, \neg\,(\mathit{true} \,\mathrm{U}\, p_1)$	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true} \,\mathrm{U}\, p_1$	1	1	0	0	0	0	0	0	0
$\neg \, \mathit{true} \,\mathrm{U}\, p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true} \,\mathrm{U}\, \neg\,(\mathit{true} \,\mathrm{U}\, p_1)$	1	1	1	1	1	1	1	1	1
$\neg \, \mathit{true} \,\mathrm{U}\, \neg\,(\mathit{true} \,\mathrm{U}\, p_1)$	0	0	0	0	0	0	0	0	0

Entry in **first column** of **last row** (corresponding to final formula) is **1**

Summary

LTL to NBA

Formula expansions

Properties

Columns as states of NBA

Unit-8: Algorithms for LTL

B. Srivathsan

Chennai Mathematical Institute

NPTEL-course

July - November 2015

Module 3:

Automaton construction

$p_1 \; \mathbf{U} \; p_2$

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \, U \, p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \, U \, p_2$	0	0	0	0	0	0	0	0	0	

$\mathit{true} \; \mathbf{U} \; (\neg p_1 \wedge \mathbf{X} \, (\neg p_2 \, \mathbf{U} \, p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1,p_2\}$	$\{p_1,p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \, \mathrm{U} \, p_1$	0	0	1	1	1	1	1	
$\mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1)$	0	1	1	1	0	0	0	
$\mathit{true} \, \mathrm{U} \, (\neg p_1 \wedge \mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1))$	1	1	1	1	1	1	0	

$\neg \; \mathit{true} \; \mathbf{U} \, \neg (\mathit{true} \; \mathbf{U} \; p_1)$

	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$
p_1	0	0	1	0	0	1	0	0
true	1	1	1	1	1	1	1	1
$\mathit{true} \, \mathrm{U} \, p_1$	1	1	1	1	1	1	1	1
$\neg \, \mathit{true} \, \mathrm{U} \, p_1$	0	0	0	0	0	0	0	0
$\mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	0	0	0	0	0	0	0	0
$\neg \, \mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{p_1\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$	$\{\,\}$
p_1	1	1	0	0	0	0	0	0	0
true	1	1	1	1	1	1	1	1	1
$\mathit{true} \, \mathrm{U} \, p_1$	1	1	0	0	0	0	0	0	0
$\neg \, \mathit{true} \, \mathrm{U} \, p_1$	0	0	1	1	1	1	1	1	1
$\mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	1	1	1	1	1	1	1	1	1
$\neg \, \mathit{true} \, \mathrm{U} \, \neg \, (\mathit{true} \, \mathrm{U} \, p_1)$	0	0	0	0	0	0	0	0	0

p_1 **U** p_2

	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_2\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\cdots$
p_1	1	1	1	1	0	1	1	1	1	
p_2	0	0	0	0	1	0	0	0	1	
$p_1 \, U \, p_2$	1	1	1	1	1	1	1	1	1	

	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{p_1\}$	$\cdots$
p_1	1	0	1	0	1	0	1	0	1	
p_2	0	0	0	0	0	0	0	0	0	
$p_1 \, U \, p_2$	0	0	0	0	0	0	0	0	0	

$true$ **U** $(\neg p_1 \wedge$ **X** $(\neg p_2$ **U** $p_1))$

	$\{\}$	$\{p_2\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2\}$	$\cdots$
p_1	0	0	0	0	1	1	1	
p_2	0	1	0	0	0	1	1	
$\neg p_1$	1	1	1	1	0	0	0	
$\neg p_2$	1	0	1	1	1	0	0	
$\neg p_2 \, \mathrm{U} \, p_1$	0	0	1	1	1	1	1	
$\mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1)$	0	1	1	1	1	1	0	
$\neg p_1 \wedge \mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1)$	0	1	1	1	0	0	0	
$true \, \mathrm{U} \, (\neg p_1 \wedge \mathrm{X} \, (\neg p_2 \, \mathrm{U} \, p_1))$	1	1	1	1	1	1	0	

$\neg$ $true$ **U** $\neg(true$ **U** $p_1)$

	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{p_1\}$	$\{\}$	$\{\}$
p_1	0	0	1	0	0	1	0	0
$true$	1	1	1	1	1	1	1	1
$true \, \mathrm{U} \, p_1$	1	1	1	1	1	1	1	1
$\neg \, true \, \mathrm{U} \, p_1$	0	0	0	0	0	0	0	0
$true \, \mathrm{U} \, \neg \, (true \, \mathrm{U} \, p_1)$	0	0	0	0	0	0	0	0
$\neg \, true \, \mathrm{U} \, \neg \, (true \, \mathrm{U} \, p_1)$	1	1	1	1	1	1	1	1

	$\{p_1\}$	$\{p_1\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$	$\{\}$
p_1	1	1	0	0	0	0	0	0	0
$true$	1	1	1	1	1	1	1	1	1
$true \, \mathrm{U} \, p_1$	1	1	0	0	0	0	0	0	0
$\neg \, true \, \mathrm{U} \, p_1$	0	0	1	1	1	1	1	1	1
$true \, \mathrm{U} \, \neg \, (true \, \mathrm{U} \, p_1)$	1	1	1	1	1	1	1	1	1
$\neg \, true \, \mathrm{U} \, \neg \, (true \, \mathrm{U} \, p_1)$	0	0	0	0	0	0	0	0	0

Construct an automaton with **states as column vectors** that can **guess accepting expansions**

Example 1: $p_1 \ \mathrm{U} \ p_2$

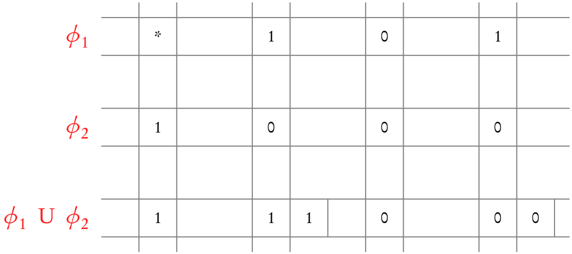
p_1	0	0	0	0	1	1	1	1
p_2	0	0	1	1	0	0	1	1
$p_1 \cup p_2$	0	1	0	1	0	1	0	1

p_1	0	0	0	0	1	1	1	1
p_2	0	0	1	1	0	0	1	1
$p_1 \, \mathrm{U} \, p_2$	0	1	0	1	0	1	0	1

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
$\phi_1 \; \mathrm{U} \; \phi_2$	1	1	1	0	0	0

Recall Until-compatibility

p_1	0	0	0	1	1	1	1
p_2	0	1	1	0	0	1	1
$p_1 \mathrm{U} p_2$	0	0	1	0	1	0	1



Recall Until-compatibility

p_1	0	0	1	1	1	1
p_2	0	1	0	0	1	1
$p_1 \mathbin{\mathrm{U}} p_2$	0	1	0	1	0	1

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
$\phi_1 \mathbin{\mathrm{U}} \phi_2$	1	1	1	0	0	0

Recall Until-compatibility

p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \,\mathrm{U}\, p_2$	0	1	0	1	1

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
$\phi_1 \;\mathrm{U}\; \phi_2$	1	1	1	0	0	0

Recall Until-compatibility

p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \mathbin{\mathrm{U}} p_2$	0	1	0	1	1

Compatible states

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
$\phi_1 \mathbin{\mathrm{U}} \phi_2$	1	1	1	0	0	0

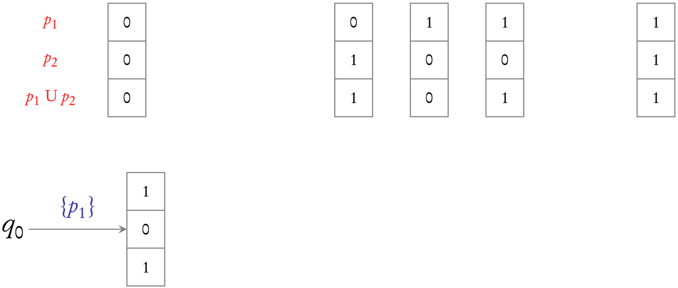
Recall Until-compatibility

p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \mathbin{U} p_2$	0	1	0	1	1

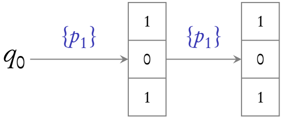
q_0

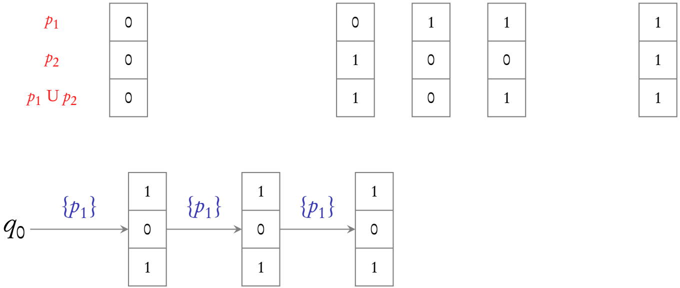
p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \mathrm{U}\, p_2$	0	1	0	1	1

$$q_0 \xrightarrow{\{p_1\}}$$

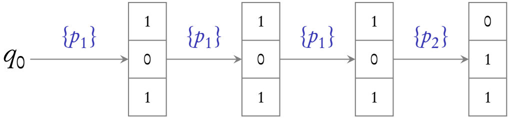

p_1
p_2
$p_1 \mathbf{U} p_2$
q_0
$\{p_1\}$

p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \mathsf{U}\, p_2$	0	1	0	1	1




p_1
p_2
$p_1 \mathbin{\mathsf{U}} p_2$
0 0 0
0 1 1
1 0 0
1 0 1
1 1 1
q_0
$\{p_1\}$
1 0 1
$\{p_1\}$
1 0 1
$\{p_1\}$
1 0 1

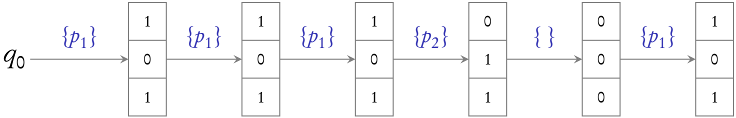
p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \,\mathrm{U}\, p_2$	0	1	0	1	1

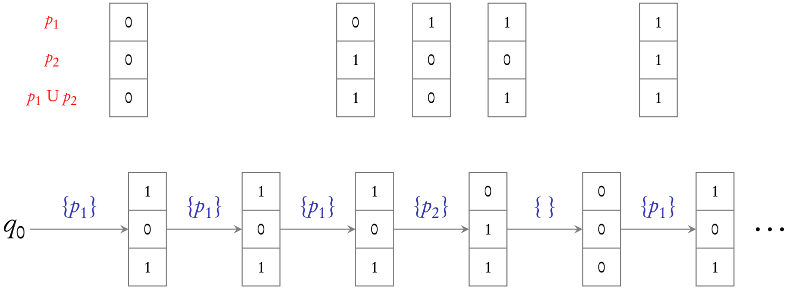


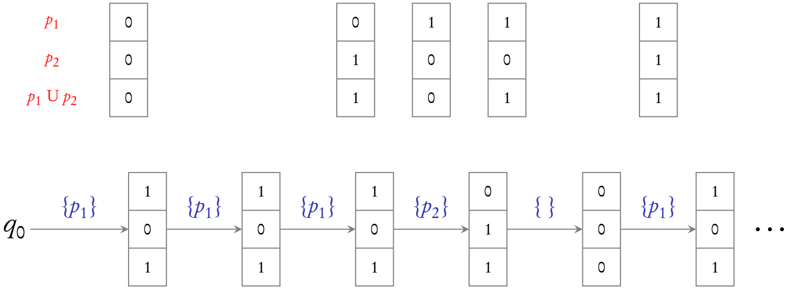
p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \, \mathrm{U} \, p_2$	0	1	0	1	1

$$q_0 \xrightarrow{\{p_1\}} \begin{array}{|c|}\hline 1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} \xrightarrow{\{p_1\}} \begin{array}{|c|}\hline 1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} \xrightarrow{\{p_1\}} \begin{array}{|c|}\hline 1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} \xrightarrow{\{p_2\}} \begin{array}{|c|}\hline 0 \\ \hline 1 \\ \hline 1 \\ \hline \end{array} \xrightarrow{\{\,\}} \begin{array}{|c|}\hline 0 \\ \hline 0 \\ \hline 0 \\ \hline \end{array}$$

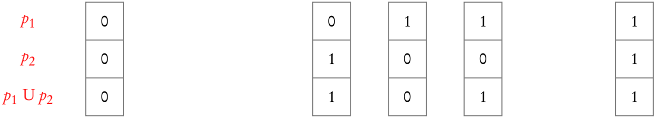
p_1	0	0	1	1	1
p_2	0	1	0	0	1
p_1 U p_2	0	1	0	1	1

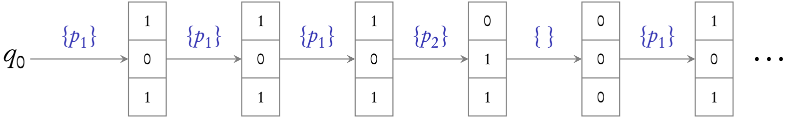


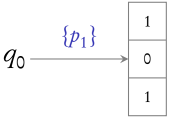

p_1
p_2
$p_1 \cup p_2$
q_0
$\{p_1\}$
$\{p_1\}$
$\{p_1\}$
$\{p_2\}$
$\{\}$
$\{p_1\}$



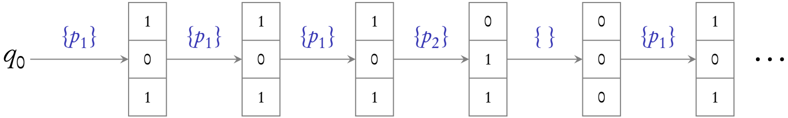
q_0

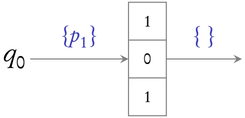

p_1
p_2
$p_1 \cup p_2$
0 0 0
0 1 1
1 0 0
1 0 1
1 1 1


q_0
$\{p_1\}$
1 0 1
$\{p_1\}$
1 0 1
$\{p_1\}$
1 0 1
$\{p_2\}$
0 1 1
$\{\}$
0 0 0
$\{p_1\}$
1 0 1
$\cdots$

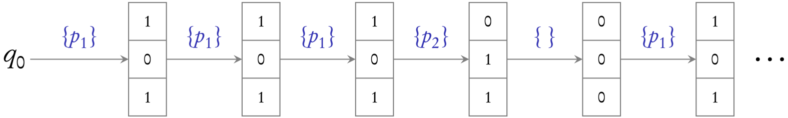

q_0
$\{p_1\}$
1 0 1

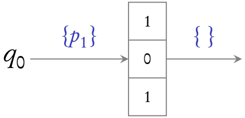
p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \cup p_2$	0	1	0	1	1





p_1	0	0	1	1	1
p_2	0	1	0	0	1
$p_1 \cup p_2$	0	1	0	1	1





No compatible transition

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

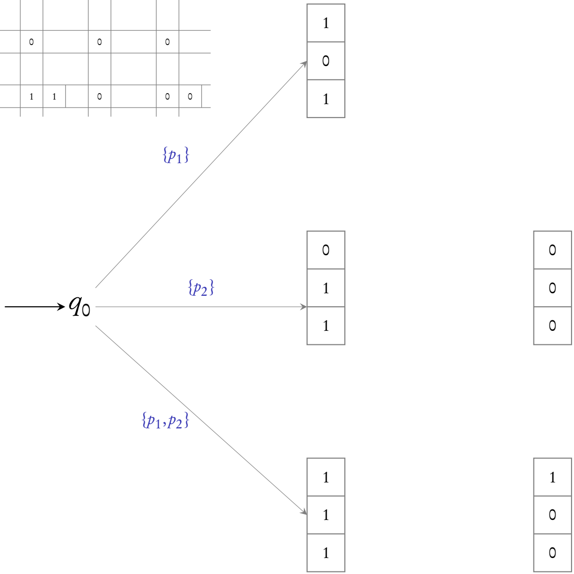
$$\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$\longrightarrow q_0$$

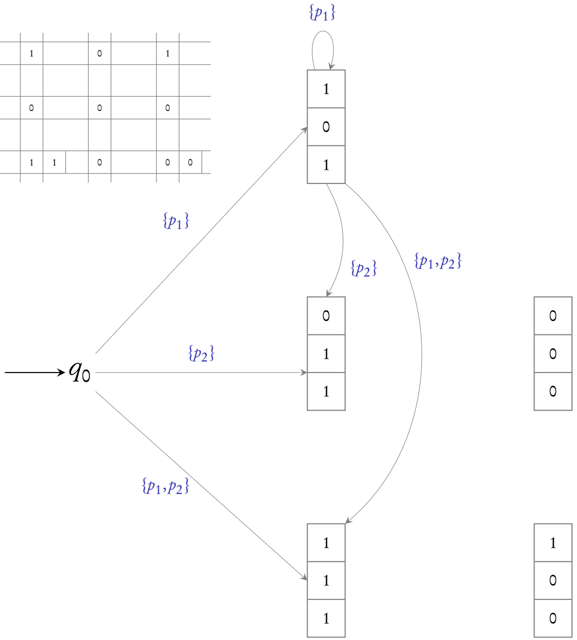
$$\begin{bmatrix}0\\1\\1\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad \begin{bmatrix}1\\0\\0\end{bmatrix}$$

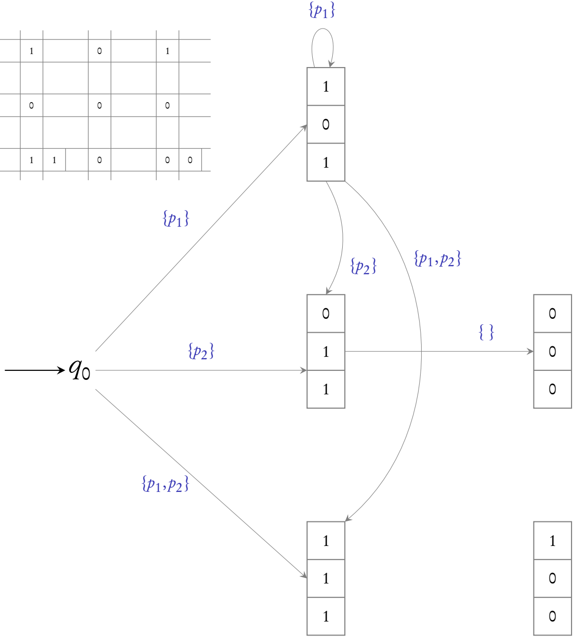
ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

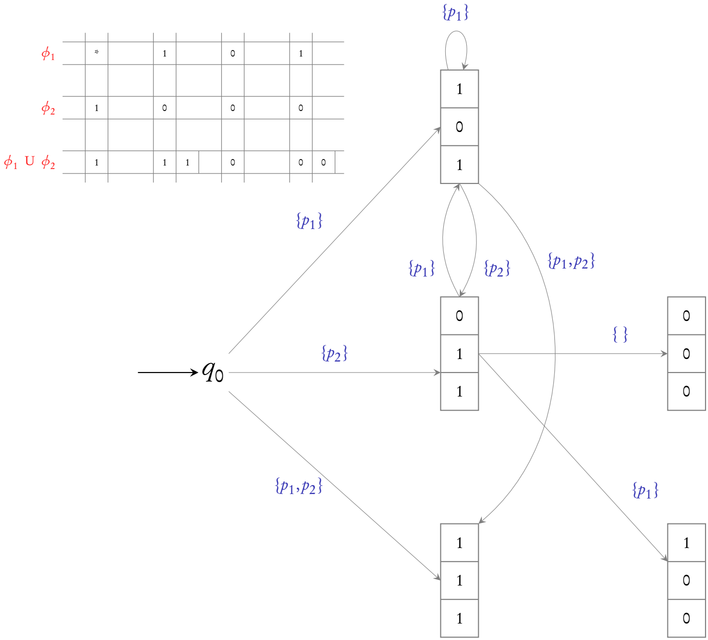


ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

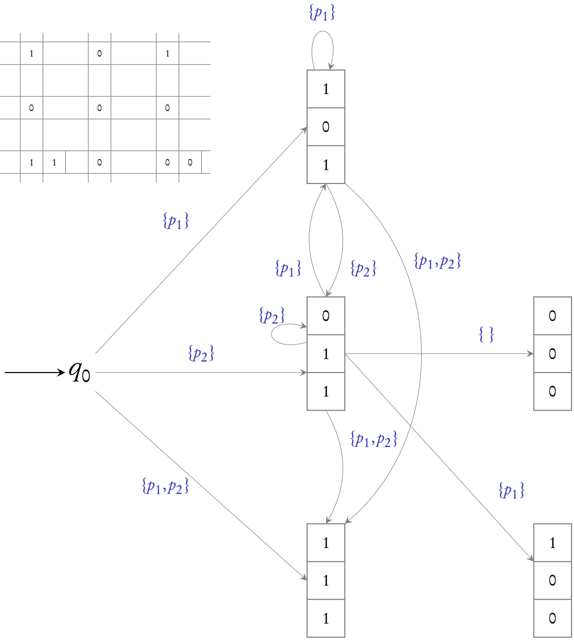


ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

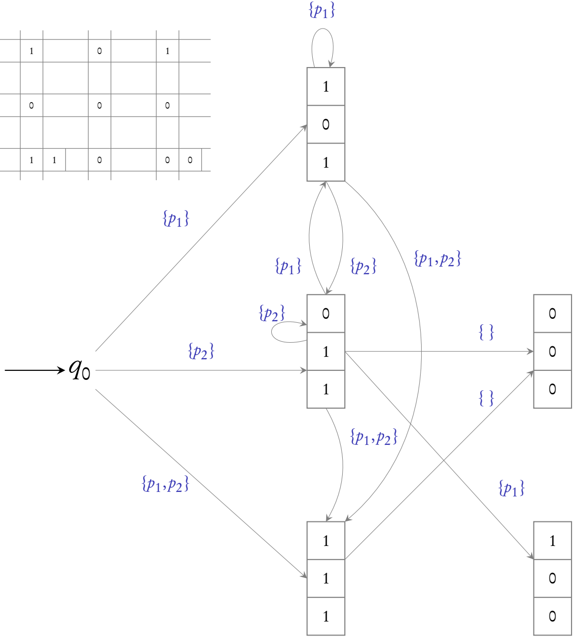



ϕ_1
ϕ_2
ϕ_1 U ϕ_2
q_0
$\{p_1\}$
$\{p_2\}$
$\{p_1,p_2\}$
$\{\}$

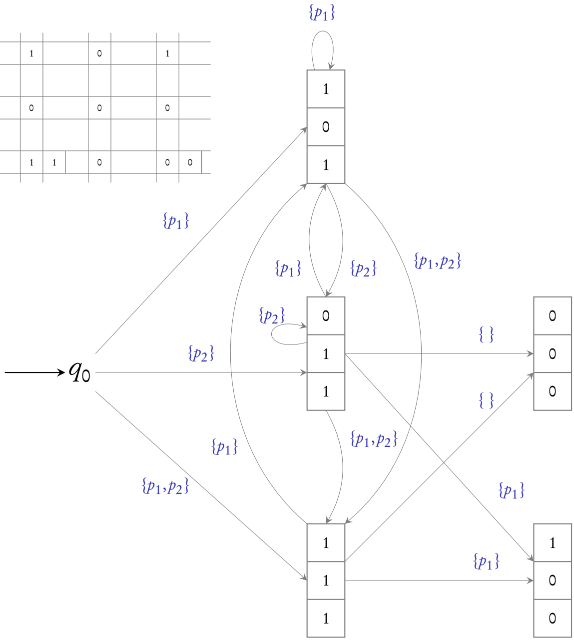
ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

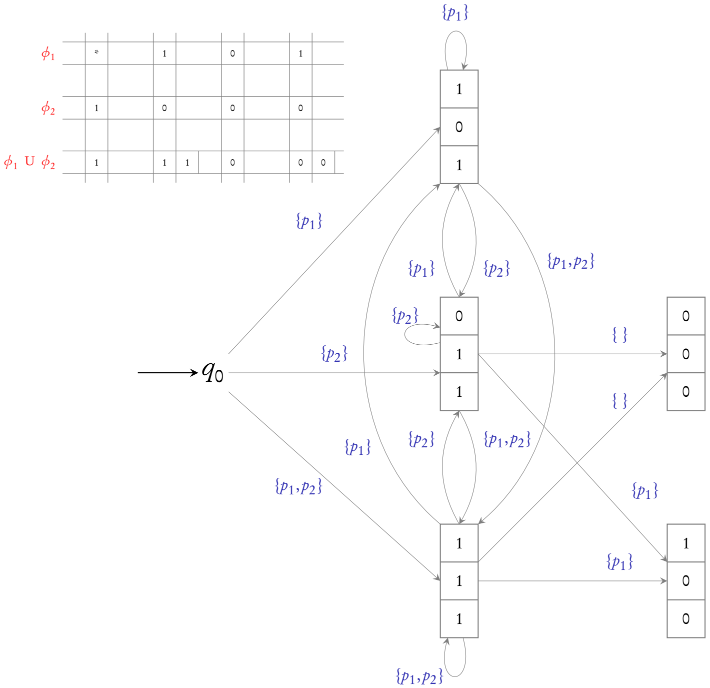


ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

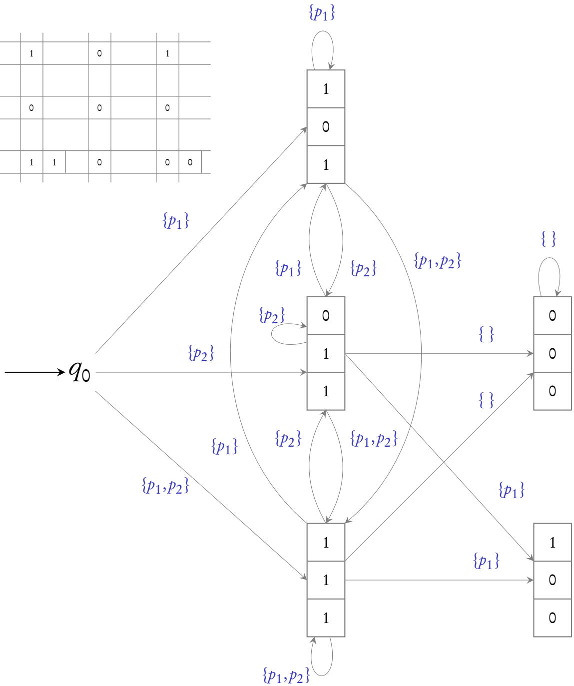


ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

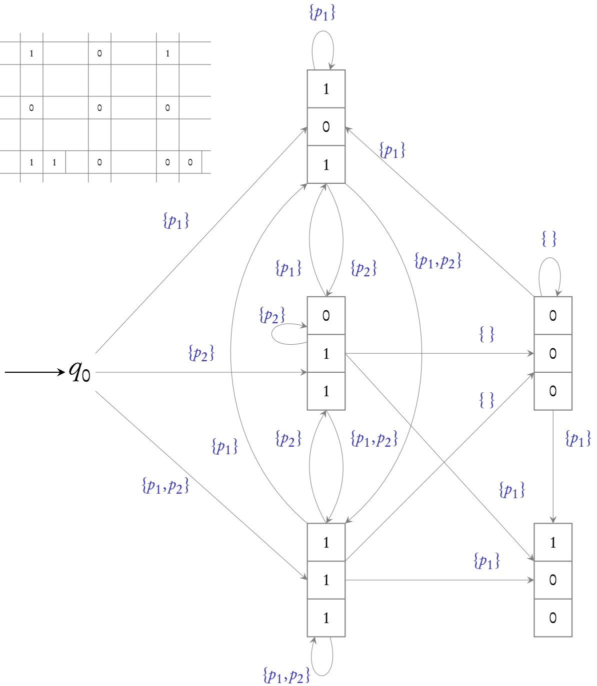


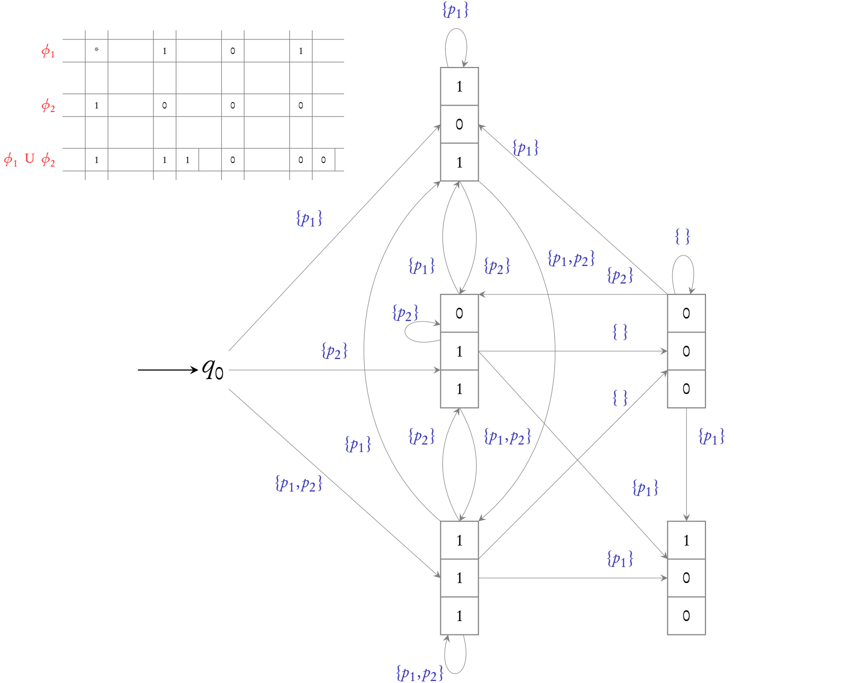

ϕ_1
ϕ_2
ϕ_1 U ϕ_2
*
1
0
1
q_0
$\{p_1\}$
$\{p_2\}$
$\{p_1, p_2\}$
$\{\ \}$

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0



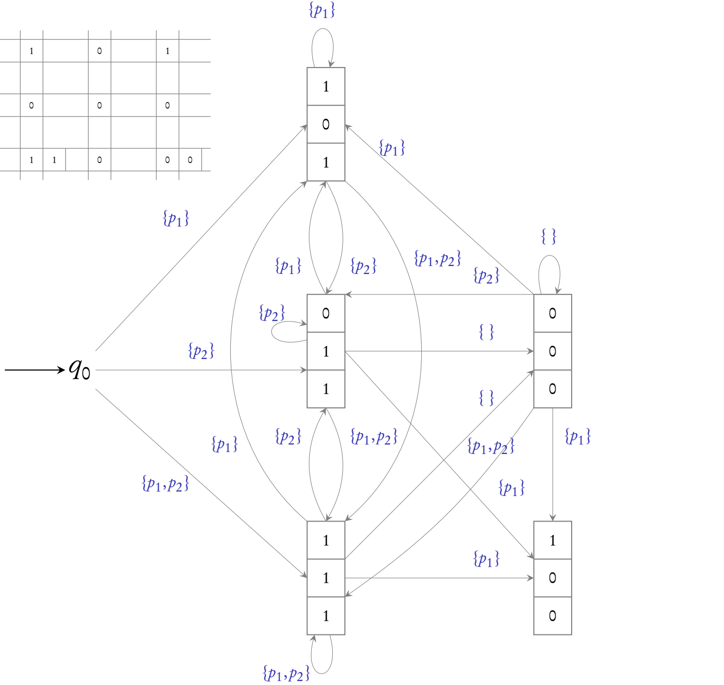
ϕ_1	*		1			0		1	
ϕ_2	1		0			0		0	
ϕ_1 U ϕ_2	1		1	1		0		0	0



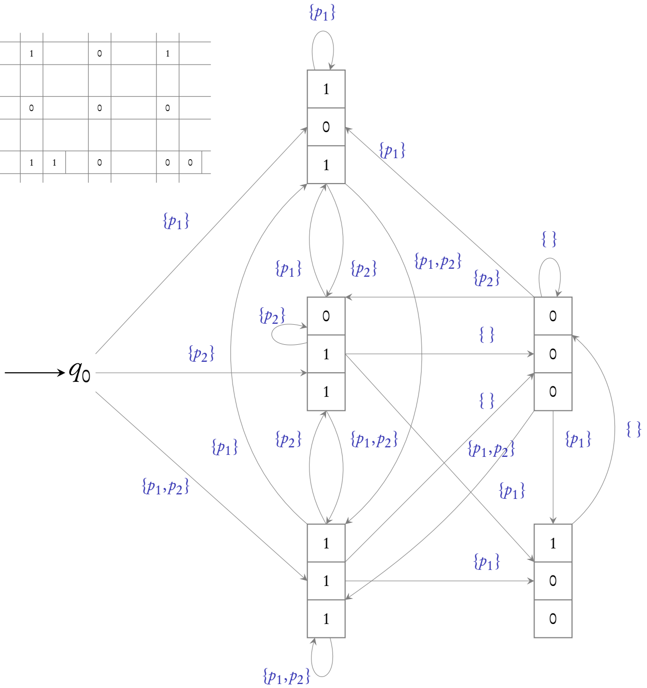


ϕ_1
ϕ_2
$\phi_1 \ \mathrm{U} \ \phi_2$
q_0
$\{p_1\}$
$\{p_2\}$
$\{p_1, p_2\}$
$\{\ \}$

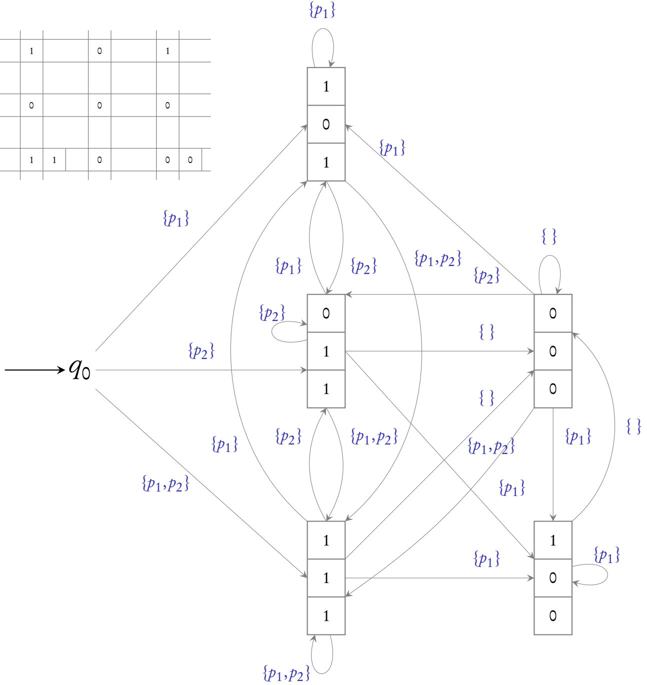
ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0



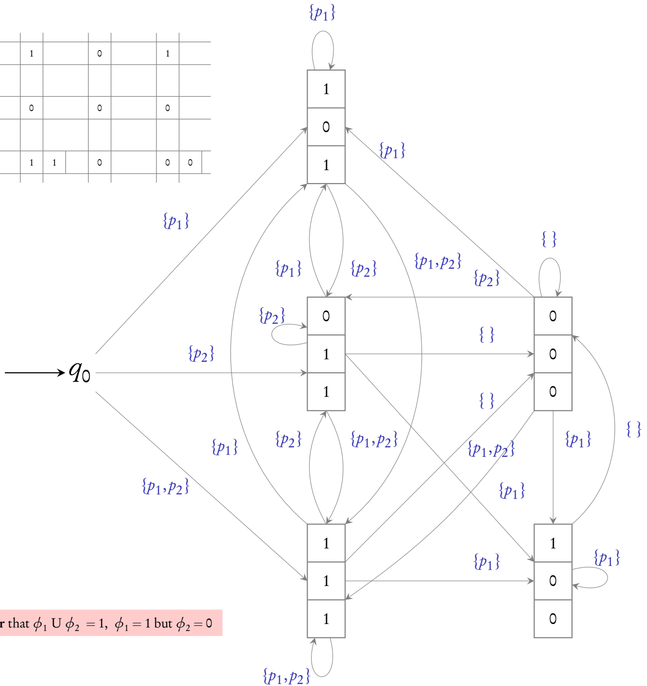
ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0



ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0

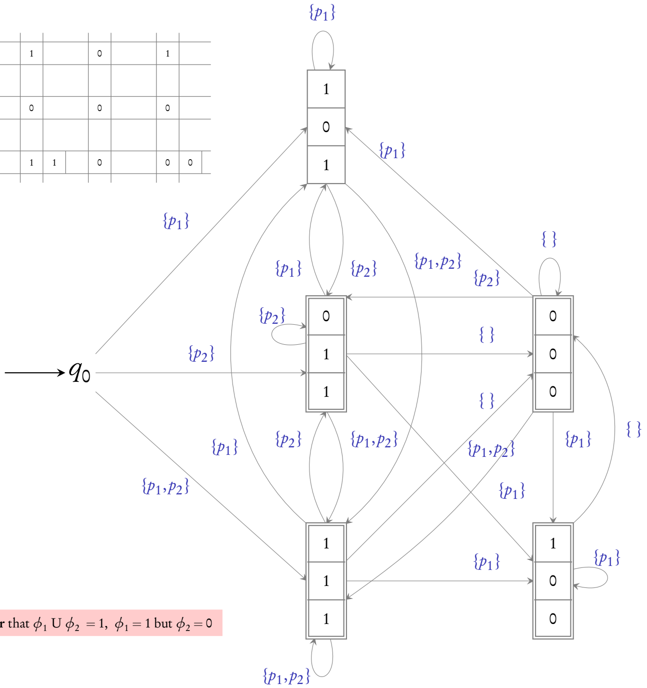


ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0



Cannot happen forever that ϕ_1 U $\phi_2 = 1$, $\phi_1 = 1$ but $\phi_2 = 0$

ϕ_1	*	1		0	1	
ϕ_2	1	0		0	0	
ϕ_1 U ϕ_2	1	1	1	0	0	0



Cannot happen forever that ϕ_1 U $\phi_2 = 1$, $\phi_1 = 1$ but $\phi_2 = 0$

Example 2: $(\mathrm{X}\ p_1)\ \mathrm{U}\ p_2$

p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\ \mathrm{U}\ p_2$	*

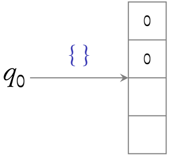
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\, \mathrm{U}\, p_2$	*

q_0

p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\ \mathrm{U}\ p_2$	*

$$q_0 \xrightarrow{\{\ \}}$$

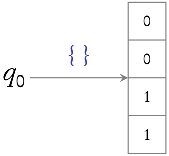
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\ \mathrm{U}\ p_2$	*



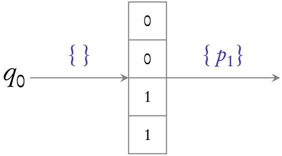
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\ \mathrm{U}\ p_2$	*

$$q_0 \xrightarrow{\{\,\}} \begin{array}{|c|}\hline 0 \\ \hline 0 \\ \hline 1 \\ \hline \ \\ \hline \end{array}$$

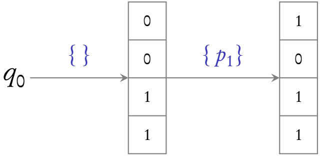
p_1	*
p_2	*
$X\,p_1$	*
$(X\,p_1)\;U\;p_2$	*



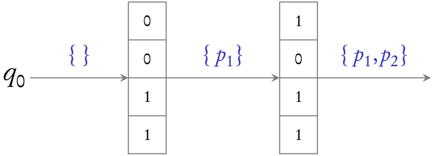
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\ \mathrm{U}\ p_2$	*



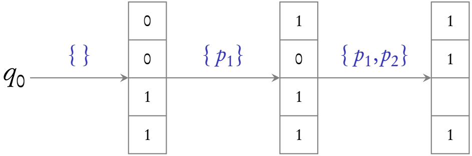
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\,\mathrm{U}\, p_2$	*



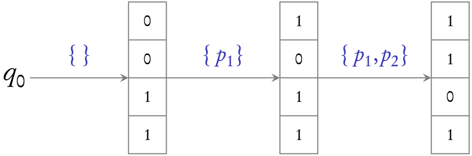
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\, \mathrm{U}\, p_2$	*



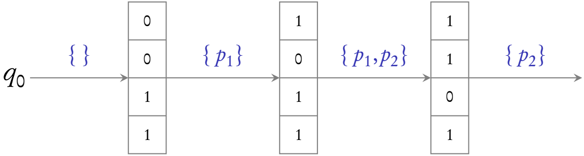
p_1	*
p_2	*
$X\, p_1$	*
$(X\, p_1)\, U\, p_2$	*



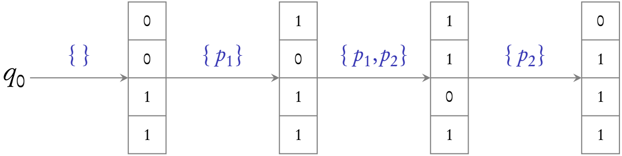
p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\, \mathrm{U}\, p_2$	*



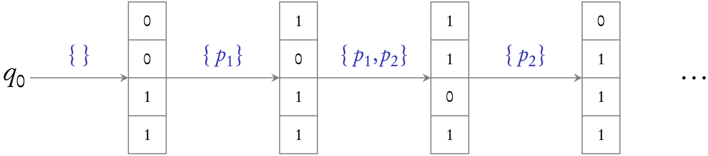
p_1	*
p_2	*
$X\, p_1$	*
$(X\, p_1)\, U\, p_2$	*



p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\, \mathrm{U}\, p_2$	*



p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1)\, \mathrm{U}\, p_2$	*



Coming next: Construction for an arbitrary LTL formula ϕ

Step 1: List down subformulae of ϕ

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p_1	*
p_2	*
$p_1 \,\mathrm{U}\, p_2$	*

p_1	*
p_2	*
$\mathrm{X}\, p_1$	*
$(\mathrm{X}\, p_1) \,\mathrm{U}\, p_2$	*

Step 2: Check **AND-NOT** and **Until** compatibility

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p_1	0
p_2	0
$p_1 \, \mathrm{U} \, p_2$	1

p_1	0
p_2	1
$\mathrm{X} \, p_1$	0
$(\mathrm{X} \, p_1) \, \mathrm{U} \, p_2$	0

Incompatible states!

Step 2: Check **AND-NOT** and **Until** compatibility

p_1	0
p_2	0
$p_1 \mathrm{U} p_2$	1

p_1	0
p_2	1
$\mathrm{X} p_1$	0
$(\mathrm{X} p_1) \mathrm{U} p_2$	0

Incompatible states!

Remove incompatible states and **add** a new state $\{q_0\}$

Step 3: Add transitions satisfying

Word, **X** and **Until** compatibility

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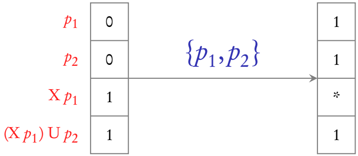
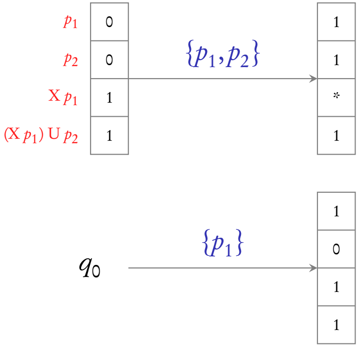
Word, **X** and **Until** compatibility



Step 3: Add transitions satisfying

Word, **X** and **Until** compatibility



From q_0 add compatible transitions to states where last entry is 1

Step 4: Accepting states should ensure Until-eventuality condition

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For **every** Until subformula $\phi_1 \mathrm{U} \phi_2$, define

$F_{\phi_1 \mathrm{U} \phi_2}$: set of states where $\phi_1 \mathrm{U} \phi_2 = 0$ or $\phi_2 = 1$

Step 4: Accepting states should ensure Until-eventuality condition

For **every** Until subformula $\phi_1 \; \mathrm{U} \; \phi_2$, define

$F_{\phi_1 \, \mathrm{U} \, \phi_2}$: set of states where $\phi_1 \; \mathrm{U} \; \phi_2 = 0$ or $\phi_2 = 1$

$F_{p_1 \, \mathrm{U} \, p_2}$:

0	1	0	1
0	0	1	1
0	0	1	1

Final automaton $\mathscr{A}_\phi$

Final automaton $\mathscr{A}_\phi$

- Compatible **states** + q_0

Final automaton $\mathscr{A}_\phi$

- Compatible **states** $+ q_0$

- Compatible **transitions**

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Final automaton $\mathscr{A}_\phi$

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Final automaton $\mathscr{A}_\phi$

- Compatible **states** + q_0
- Compatible **transitions**
- If no Until subformula, then every state is accepting
- Otherwise, accepting set for each Until subformula: $\{F_1, F_2, \dots, F_k\}$
- This will be a **Generalized NBA (GNBA)**

Every GNBA can be converted to an equivalent NBA (Unit 6 - Module 4)

In general, this algorithm gives NBA which is **exponential** in size of formula

Unit-8: Algorithms for LTL

B. Srivathsan

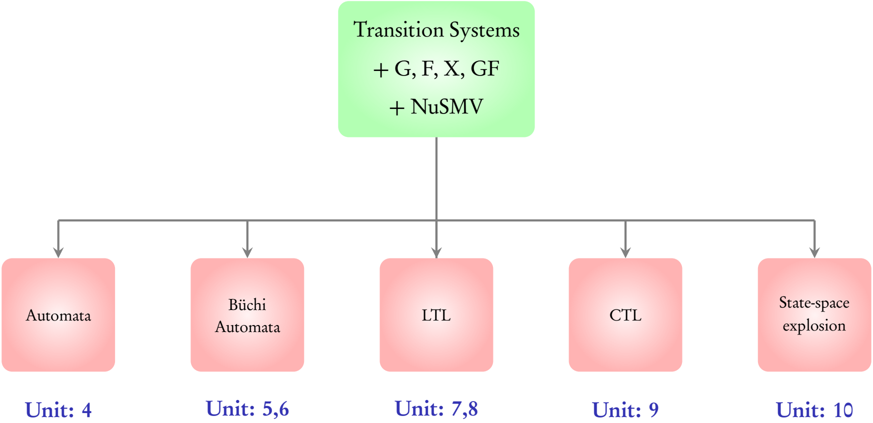
Chennai Mathematical Institute

NPTEL-course

July - November 2015

Summary

- Automata based LTL model-checking
- Algorithm for converting LTL formula to NBA


Transition Systems
+ G, F, X, GF
+ NuSMV
Automata
Büchi Automata
LTL
CTL
State-space explosion
Unit: 4
Unit: 5,6
Unit: 7,8
Unit: 9
Unit: 10