

Name \rightarrow Aditya Singh
Section \rightarrow K19GA
Roll No \rightarrow RK19GAB63

CSE-322
Assignment (CA-3)

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1. Check whether the language is CFL or not, using pumping lemma. $L = \{a^{3i} \cdot b^{2i} \cdot c^{3i} \mid i \geq 0\}$

Ans:

$$L = \{a^{3i} \cdot b^{2i} \cdot c^{3i}\}$$

$$\text{Let } i = 2, \text{ Hence } S = a^6 b^4 c^6$$

dividing $S = uvxyz$

Case ① v and y each contain only one kind of symbol

$$\begin{array}{ccc} \underline{aaaaaa} & \underline{bbbb} & \underline{cccccc} \\ v & x & y \end{array}$$

$$v = a, u = aaaaa, x = bbbb, y = c, z = cccc$$

$$\text{Let } k = 3 \Rightarrow uv^k xy^k z = uv^3 xy^3 z$$

$$\Rightarrow aaaaaaa bbbb ccccccccc$$

$$\Rightarrow a^8 b^4 c^{10} \in L$$

Case ② v or y has more than 1 kind of symbol.

$$\begin{array}{ccc} \underline{aaaaaa} & \underline{bbbb} & \underline{cccccc} \\ u & v & x & y & z \end{array}$$

$$u = aaaaa, v = ab, x = bbb, y = c, z = ccccc$$

$$\text{Let } k = 3, uv^k xy^k z = uv^3 xy^3 z$$

$$= aaaaaababab bbb ccc ccccc$$

$\notin L$

As, we can observe that language does not satisfy the first property of pumping lemma. Hence not a CFL

2. Find a grammar in Chomsky Normal form equivalent to
 $S \rightarrow aAbB, A \rightarrow aA, A \rightarrow a, B \rightarrow bB, B \rightarrow b$

Ans: From the following grammar:

Step ① No Null productions in the grammar (No reduction)

Step ② No Unit production in the grammar (No reduction)

Step ③ Find out the production that has more than 2 variable in the RHS.

→ ie. $S \rightarrow aAbB$

Removing this we get Productions:-

$S \rightarrow XY, A \rightarrow aA, A \rightarrow a, B \rightarrow bB, B \rightarrow b, X \rightarrow aA, Y \rightarrow bB$

Step ④ Removing Productions, $A \rightarrow aA, B \rightarrow bB, X \rightarrow aA, Y \rightarrow bB$.
We get the following equation

$S \rightarrow XY, A \rightarrow ZA, B \rightarrow VB, X \rightarrow ZA, Y \rightarrow VB, Z \rightarrow a, V \rightarrow b.$

The above equation is in Chomsky Normal Form

3. Reduce the following grammars to Greibach normal form:

$S \rightarrow AB, A \rightarrow BSB, A \rightarrow BB, B \rightarrow aAb, B \rightarrow a, A \rightarrow b.$

Ans \rightarrow As we can see above grammar is already in CNF so we skip step 1.

Step ② $S = A_1, A = A_2, B = A_3$

$S \rightarrow AB, A \rightarrow BSB, A \rightarrow BS \mid b \mid, A_1 \rightarrow A_2 A_3, A_2 \rightarrow A_3 A_1 \mid b, A_3 \rightarrow A_1 A_2 \mid a$

Resulting Production are -

$A_3 \rightarrow A_2 A_3 A_2$

$A_3 = A_3 A_1 A_3 A_2 \mid b A_3 A_2$

Step ③: $A_3 \rightarrow a \mid b A_3 A_2, A_3 \rightarrow a Z_3 \mid b A_3, Z_3 \rightarrow A_1 A_3 A_2, Z_3 \rightarrow A_1 A_3 A_2 Z_3$

Step ④: (i) The $A_3 \rightarrow$ Productions are

$A_3 \rightarrow a \mid b A_3 A_2 \mid a Z_3 \mid b A_3 A_2 Z_3$

(ii) The resulting production

$A_2 \rightarrow a A_1 \mid b A_3 A_2 A_1 \mid a Z_3 A_1 \mid b A_3 A_2 Z_3 A_1$

The modified A_2 productions

$A_2 \rightarrow b \mid a A_1 \mid b A_3 A_2 A_1 \mid a Z_3 A_1 \mid b A_3 A_2 Z_3 A_1$

~~(iii) $A_1 \rightarrow b A_3 \mid a A_1 A_3 \mid b A_3 A_2 A_1 A_3 \mid a Z_3 A_1 A_3$~~

(iii) $A_1 \rightarrow b A_3 \mid a A_1 A_3 \mid b A_3 A_2 A_1 A_3 \mid a Z_3 A_1 A_3 \mid b A_3 A_2 Z_3 A_1 A_3$

Step ⑤: The Z_3 production to be modified are,

$Z_3 \rightarrow A_1 A_3 A_2 \mid A_1 A_3 A_2 Z_3$

$Z_3 \rightarrow b A_3 A_3 A_2 \mid b A_3 A_2 Z_3$

$Z_3 \rightarrow b A_3 A_2 a A_1 A_3 A_3 A_2 \mid a A_1 A_3 A_3 A_2 Z_3$

$Z_3 \rightarrow b A_3 A_2 A_1 A_3 A_3 A_2 \mid b A_3 A_2 A_1 A_3 A_3 A_2 Z_3$

$Z_3 \rightarrow a Z_3 A_1 A_3 A_3 A_2 \mid a Z_3 A_1 A_3 A_3 A_2 Z_3$

$Z_3 \rightarrow b A_3 A_2 Z_3 A_1 A_3 A_3 A_2 \mid b A_3 A_2 Z_3 A_1 A_3 A_3 A_2 Z_3$

Hence reduced to Greibach Normal Form