bond-ret-vs-yield

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BOND RETURN VS YIELD

1 Definitions

- For simplicity, all applicable measures are daily (not annualized).
- The current timestep is t.
- We consider a ZCB (zero-coupon bond) that pays out its face value $V := \$1$ on its maturity date $T := 261$.
- The bond's duration $\tau := T - t$.
- The bond's yield (rsp price, return) on day t is y_t (rsp Z_t, r_T).
- The bond's yield shift on day t is $\Delta_t[y] := y_t - y_{t-1}$.
- The bond's estimated yield volatility at day t is $\hat{\sigma}_t^y := |\Delta_t[y]|$.
- The bond's estimated return volatility at day t is $\hat{\sigma}_t^r := |r_t|$.

2 Logarithmic setting

We have

$$Z_t = \exp[-y_t(T - t)].$$

We define

$$r_t := \ln[Z_t/Z_{t-1}] = \ln[Z_t] - \ln[Z_{t-1}].$$

Notice that

$$\ln[Z_t] := \ln[\exp[-y_t(T - t)]] = -y_t(T - t).$$

Therefore,

$$r_t = -y_t(T - t) - -y_{t-1}(T - (t - 1)) = -y_t(T - t) + y_{t-1}(T - t + 1)$$

$$= -y_t(T - t) + y_{t-1}(T - t) + y_{t-1}$$

$$= -(y_t - y_{t-1})\tau + y_{t-1} = -\Delta_t[y]\tau + y_{t-1}.$$

Assume that the duration is very long (maturity date is very far away) $\tau \gg 0$, and that the yield did not literally stay flat $\Delta_t[y] \neq 0$. Then, we can approximate

$$\boxed{r_t \approx -\Delta_t[y]\tau}.$$

Notice that we can interpret the duration τ as the bond return's sensitivity to yield shifts.

So, assume that $\tau > 0$ (the bond has not matured yet) and suppose we are given the bond's estimated return volatility,

$$s := \hat{\sigma}_t^r = |r_t| \approx |-\Delta_t[y]\tau| = |\Delta_t[y]\tau| = |\Delta_t[y]|\tau.$$

We can recover the bond's estimated yield volatility as

$$s/\tau.$$