**FOREIGN-EXCHANGE FORWARD POINTS**

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## THE FORWARD

Consider USD as domestic currency and GBP as foreign. We wish to go long a GBP-vs-USD forward contract, that is, we wish to enter an agreement today, to hand over at a chosen date in the future, a predetermined amount of USD to receive in exchange a predetermined amount of GBP. Suppose for simplicity WLOG the exchange rate between the currencies today, defined as the "price" of a GBP in USD, is $1$ USD/GBP. Suppose also that the prevailing 1-year T-Bill rate is $4\%$, and that the prevailing 1-year Gilt rate is $1\%$. Also suppose that we wish to receive a year from now exactly $1$ GBP.

Just as the forward price for a stock involves compounding at the risk-free rate minus the stock's dividend yield, the forward price for this GBP will involve compounding at the risk-free rate (that is, our, *domestic* risk-free rate) minus the GBP's "dividend yield" i.e. their risk-free rate. So, by no-arbitrage, we must at maturity hand over $F$ amount of USD[1]:

$$F(t,T) = X_t e^{(r_{\text{USD}} - r_{\text{GBP}})(T-t)}.$$

So, in our case, we must in one year hand over

$$1 \cdot e^{(0.04-0.01)(1-0)} = e^{0.03} = 1.03$$

USD, and we will receive exactly $1$ GBP.

By entering this agreement, we are effectively long GBP-vs-USD. The $3\%$ is called the "carry" or "forward points". It represents an in-some-sense "rational" expectation for terminal exchange rate due to arbitrage pressures arising from the risk-free-yield differential. But if the exchange rate rises more than that (GBP strengthens, i.e. price of GBP in USD increases) we profit. For example, suppose that $X_T$ crystallizes to $1.10$. Then, we pay out $1.03$ USD's and receive $1$ GBP, but can immediately convert that $1$ GBP to $1.10$ USD's, earning 7 cents of profit.

This dynamic—profiting when GBP strengthens—is why this contract can be used to hedge a British investor allocating to an American manager[2]. If the Brit handed the Yank $1$ GBP, then the Yank converted it to $1$ USD and invested it in her fund, then ended the year flat so that she still had a NAV of $1$ USD, but in the meantime the GBP strengthened so that $X_T = 2$, then she would have to covert that $1$ USD to $0.50$ GBP in order to hand it back. But with the contract, she will profit in her hedging sleeve due to the GBP's strengthening, offsetting her mark-to-GBP loss in the investment sleeve due to that same strengthening.

Of course, this is a symmetric hedge, not an option. If the GBP weakens, we'll see mark-to-GBP gains in the investment sleeve, but these will be offset by losses in the hedging sleeve. This symmetry is why it's free to enter this agreement, instead of costing an upfront premium like an option.

[1] Blyth, Steven (Harvard Statistics, Harvard Management Company, Oxford). 2014. "An Introduction to Quantitative Finance". OUP. §2.7.

[2] This leaves unresolved the choice of hedge-rebalance frequency. Condition on the forward points at $t = 0$, and fully hedge the initial principal (notional) for whatever length you like. The default choice is to hedge with a 3m IMM-dated forward. But suppose the forward points are against you (the hedge is "expensive") – Should you simply set a 1m forward, and wait to see if you can strike a new forward at more favorable forward points in a month from now? Perhaps, but you are then accepting the risk that the forward points blow out *even further* against you, and you end up regretting not setting a longer hedge at the less-unfavorable terms. Now leave aside the initial hedge, which is now fixed. (This is for practical purposes true even if you literally roll into fresh contracts every day, that is if tomorrow you close the $t + 3m$ contract and enter a $t + 1 + 3m$ contract. Suppose forward points haven't changed – Then, you're rolling into essentially an identical contract, just off by one day. Suppose on the other hand that forward points have become even more unfavorable for your side (the long side) – Then, when you sell to cover your current contract, you will make money (the forward struck at yesterday's forward price, which was free to enter yesterday, now has positive value given the new market reality—and hence new forward price—today), but in exchange you must strike the new forward at the new exactly-offsetting unfavorable terms. And finally if the forward points have moved in your favor, then you will have to pay someone cash to take your place in your now-unfavorable-looking contract from yesterday, but in exchange you can strike a new forward at the new exactly-offsetting favorable terms.) So, at this point, you have before you simply the following question: As you earn daily PNL, should you continue to enter fresh contracts to hedge those, too? For example, say you're up 1% tomorrow (without any move in $X$). A gap has opened up: You're hedged on $1$ GBP of notional, but the client now technically has $1.01$ GBP worth of assets with you, which means they have 1p worth of unhedged exposure. Of course if the forward points are in your favor, you'll happily roll out of yesterday's contract, into a fresh contract for $1.01$ GBP. But what if the forward points are against you? Should you sit on the unhedged exposure for a while, and hope that the forward points move into your favor soon, accepting the risk that $X$—the thing you're trying to hedge against—might move against you in the meantime? Well sure. But this starts to look more like an active macro strategy to me, than a hedging strategy.

## The Swap

In fact, we can reconstruct the forward with a "three-legged trade". That is, we can at $t$:

1. Borrow $0.99$ USD from the bank.
2. Convert it at the prevailing exchange rate to $0.99$ GBP.
3. Enter a swap agreement with them:
   a. Today, hand them $0.99$ GBP and receive $0.99$ USD.
   b. In one year, hand them $1.03$ USD's and receive $1$ GBP[3].

In fact, this is the way large systematic macro hedge funds execute this for directional currency strategies in reality, as it is cheaper than entering the forward directly. The issue with the forward is that the bank, as our counterparty, is now exposed to the exactly symmetric directional risk as us: They are short GBP-vs-USD, and will immediately hedge that by entering a GBP-vs-USD forward agreement of their own. However, in the interim, they are exposed to slippage: The prevailing exchange rate can move against them in the minutes it takes them to find a counterparty (usually another speculator) for their own GBP-vs-USD forward. The bank will charge us tcost for taking on that risk.

By instead executing a swap, we hand them their hedge right there: As short-GBP-vs-USD, the bank will lose money if the exchange rate rises (GBP strengthens); But by handing them GBP right at the outset, we've hedged them, as they will make an equal amount of money by virtue of the fact that they hold newly-stronger hard currency.

Of course, as usual, there's no free lunch. We ourselves are exposed to slippage between legs 1 and 2, then again between legs 2 and 3a. Over the long term, the slippage should average out to zero (half the time in our favor, the other half against us), but for any single particular trade it could blow out one way or the other. So rather than paying a cost (a fixed tcost that we can infer based on the spread quoted to us), we choose to take on risk.

## Fairness

The forward points are *not* a cost, as I define it. I define a cost as something you pay a counterparty because they're charging you for providing liquidity or performing some other service. It can be high if they want to rip you off, or low if they're feeling magnanimous. But forward points are not a cost – They are simply a consequence of (a) monetary policy and (b) no-arbitrage.

In fact, *they are fair*. Why? Because the fairness is what engenders them in the first place! Picture the swap. You have 99 pence in your pocket. You can give that to His Majesty's Government, and receive in a year $0.99e^{0.01} =$ a quid. Or, you can enter a swap: Hand your 99 pence to the bank, receive 99 cents in return, give those 99 cents to Uncle Sam, receive in a year $0.99e^{0.04} =$ a-buck-three, hand that to the bank, and receive the same quid. The fact that the side that owes dollars must pay "more" is simply a reflection of the fact that they get the luxury of holding a higher-yielding currency for the next year – Yield that the side that had dollars but gave them away is now foregoing.

And it doesn't just have to come from govies. Even if you invest in a risky asset, USD-denominated risky assets should, in a uniform market, give higher expected returns than GBP-denominated risky assets. (If it helps, think about a DCF model: If risk-free rates are higher, that increases the discount rate, which lowers the NPV of future cashflows, and the NPV of future cashflows is the price PE analysts are willing to pay for a deal. I prefer to think about this like a quant: Quants look at excess-of-risk-free returns. If the risk-free rate rises, but the total ER stays flat, the higher risk-free rate eats into that and lowers excess ER. The quant will decrease the price she is willing to pay for the stock, thereby increasing the total return commensurately.)

So the bank, which was holding USD, could invest in a high-total-ER climate domestically. Once you force them to give up their USD and accept GBP, their asset universe shifts to lower-total-ER GBP-denominated assets. So it is only fair that you later swap back at a rate titled slightly against you, as you should have been taking advantage of the high ER's that the bank was missing out on.

[3] By no-arbitrage, this *must* be the swap rate, precisely because we can replicate the forward. I will not get into the detail here, but see Blyth 2014 for a treatment of replication and the Fundamental Theorem of Asset-Pricing.

## PRACTICALS: THE "RETURN-ON-RETURN" PHENOMENON

Suppose a British investor wishes to invest in a US hedge fund. They hand us 1 GBP, and ask us to FX-hedge them. We enter a 1-year swap with the bank: We hand them 1 GBP today, and they hand us 1 USD; And we agree that we will hand them back 1.03 USD in a year, and they will hand us 1 GBP.

**Scenarios**

- We end the year up $+10\%$, so we have 1.10 USD. We hand the bank 1.03 USD, they hand us 1 GBP, and we have 7c left over.
  - GBP strengthens to 2 USD per GBP. Using dimensional analysis, We convert our 7c to 3.5p. We hand the investors 1.035 GBP, so instead of being up $+10\%$ they're up only $+3.5\%$. 300bps of that difference they're OK with because it reflects the different rate environments. But the other 350bps they're sad about, as it's a reflection of the ex-ante unknowable (and therefore unhedgeable) PNL.
  - GBP strengthens to 1.20 USD per GBP. We convert our 7c to 5.8p. We hand the investors 1.058 GBP, so instead of being up $+10\%$ they're up only $+5.8\%$. 300bps of that difference they're OK with because it reflects the different rate environments. But the other 120bps they're sad about, as it's a reflection of the ex-ante unknowable (and therefore unhedgeable) PNL.
  - GBP strengthens to 1.03 USD per GBP. We convert our 7c to 6.8p, and hand the investors 1.068 GBP. Instead of being up $+10\%$ they're up only $+6.8\%$, so the unhedged PNL hit them for just 20bps. They're happy.
  - GBP maintains at 1 USD per GBP. We convert our 7c to 7p, and the investors are up $+7\%$. Perfection.
  - GBP weakens to 0.97 USD per GBP. We convert our 7c to 7.2p, and the investors are up $+7.2\%$. They're pleasantly surprised.
  - GBP weakens to 0.70 USD per GBP. We convert our 7c to 10p, and the investors are up $+10\%$. This is a windfall. (Notice the magnitude of the weakening required to break even – The investors have unhedged exposure to what we call "return on return", that is "PNL from investments" times "change in exchange rate".)
  - GBP weakens to 0.50 USD to GBP. Investors are up $+14\%$. Drinks all around.
- We end the year down $-10\%$, so we have 0.90 USD. We borrow 13c and hand the bank 1.03 USD, they hand us 1 GBP, and we have 13c debt left over.
  - GBP strengthens to 2 USD per GBP. Thank god. Of the 1 GBP (100p) we hold, we're able to convert 6.5p to 13c, repay our debt, and hand the investors the remaining 0.935 GBP. They're down $-6.5\%$.
  - GBP strengthens to 1.20 USD per GBP. We convert 10.8p to 13c, repay our debt, and hand the investors the remaining 0.892 GBP. They're down $-10.8\%$.
  - GBP strengthens to 1.03 USD per GBP. We convert 12.6p to 13c, repay our debt, and hand the investors the remaining 0.874 GBP. They're down $-12.6\%$.
  - GBP maintains at 1 USD per GBP. We convert 13p to 13c, repay our debt, and hand the investors the remaining 0.87 GBP. They're down $-13\%$. You can see now why risk-free-relative hurdles are so important. It seems like when we were up $+10\%$ and $X$ didn't move, the investors got only $+7\%$; But when we were down $-10\%$, the investors lose fully $-13\%$. But that's not the right thinking. When we were up $+10\%$, with $r_{\text{USD}} = 4\%$, we outperformed cash by just $+6\%$; But when we were down $-10\%$, we *under*performed cash by fully $-14\%$. (The fact that they pick up $+100$bps in either case is a reflection of the fact that the terms of the swap "bake in" the fact that they could have just sat on the cash and earned $r_{\text{GBP}} = 1\%$.)
  - GBP weakens to 0.97 USD per GBP. We convert 13.4p to 13c, repay our debt, and hand the investors the remaining 0.866 GBP. They're down $-13.4\%$.
  - GBP weakens to 0.70 USD per GBP. We convert 18.6p to 13c, repay our debt, and hand the investors the remaining 0.814 GBP. They're down $-18.6\%$.
  - GBP weakens to 0.50 USD to GBP. Investors are down $-26\%$. They submit a full redemption and will probably never invest with a US manager again. Until 3-5 years from now when {`insert latest hot firm here`} gets lucky and strikes it up $+50\%$ by sheer dumb chance, and now the investors are clamoring to get in.

- What about buying a 1-year call option at today's spot price (i.e. at-the-money), assuming that options-implied volatility is at its long-term average of 9%[4]? By Black-Scholes, this will cost[5]:

$$\begin{aligned}
d_0 &:= \ln(X_t/K) + (r_{\text{USD}} - r_{\text{GBP}} + \sigma^2/2)(T-t) \\
&= \ln(1/1) + (0.04 - 0.01 + 0.09^2/2)(1-0) \\
&= 0.03405 \\
d_1 &:= \frac{d_0}{\sigma\sqrt{T-t}} \\
&= \frac{0.03405}{0.09\sqrt{1-0}} \\
&= 0.378 \\
d_2 &:= d_1 - \sigma\sqrt{T-t} \\
&= 0.378 - 0.09 \\
&= 0.288 \\
C &= X_t\Phi(d_1) - Ke^{-(r_{\text{USD}}-t_{\text{GBP}})(T-t)}\Phi(d_2) \\
&= \Phi(0.378) - 0.97\Phi(0.288) \\
&= \Phi(0.378) - 0.97\Phi(0.288) \\
&= 0.6473 - 0.97(0.6133) \\
&= 5.2\%
\end{aligned}$$

  Which is fully 2.2pp *more* than the 3% you "pay" under the forward agreement. Of course, the extra cost comes because, unlike with the forward, if GBP weakens, you lose nothing.

- OK, What about buying a 1-year call option at the forward price, still assuming that options-implied volatility is 9%? By Black-Scholes, this will cost:

$$\begin{aligned}
d_0 &:= \ln(X_t/K) + (r_{\text{USD}} - r_{\text{GBP}} + \sigma^2/2)(T-t) \\
&= \ln(1/1.03) + (0.04 - 0.01 + 0.09^2/2)(1-0) \\
&= 0.00449 \\
d_1 &:= \frac{d_0}{\sigma\sqrt{T-t}} \\
&= \frac{0.00449}{0.09\sqrt{1-0}} \\
&= 0.0499 \\
d_2 &:= d_1 - \sigma\sqrt{T-t} \\
&= 0.0499 - 0.09 \\
&= -0.04 \\
C &= X_t\Phi(d_1) - Ke^{-(r_{\text{USD}}-t_{\text{GBP}})(T-t)}\Phi(d_2) \\
&= \Phi(0.0499) - 1.03(0.97)\Phi(-0.04) \\
&= \Phi(0.0499) - \Phi(-0.04) \\
&= 0.5199 - (0.4840) \\
&= 3.6\%
\end{aligned}$$

  So not only are you paying 3.6% upfront (60bps more than the 3% you "pay" under the forward agreement), but even despite that you're still on the hook for any strengthening up to 1.03 USD-per-GBP!

[4]Getting an updated timeseries for BPVIX (the Cboe/CME FX British Pound Volatility Index) is expensive, but NYU Stern's V-Lab runs a great AGARCH estimator (link, archive). Remember to scale up by $\approx$ 1.1x to reflect the at-the-money volatility risk premium. Of course, during very quiet times, the option price will subside, while during very stormy times, it will blow out.

[5]You can play around with these using Drexel's calculator. Caution: I'm being loosey-goosey with rates here, which is fine for small rates, but to be precise, you must use logarithmic—not geometric—rates. So, input E.g. A $+9.5\%$ logarithmic rate if you mean a $+10\%$ geometric rate; Or, a $-10.5\%$ logarithmic rate if you mean a $-10\%$ geometric rate.

- Fine. What about striking a forward, but not at the forward price, rather at today's spot price? Well, by the forward-valuation formula, at the forward price, we have that the contract costs:

$$\begin{aligned}
V_t &= X_t - Fe^{-(r_{\text{USD}} - r_{\text{GBP}})(T-t)} \\
V_0 &= 1 - 1.03e^{-(0.04-0.01)(1-0)} \\
&= 1 - 1 \\
&= 0
\end{aligned}$$

  So by design, it's free to enter; But at today's spot price, we have

$$\begin{aligned}
V_t &= X_t - Ke^{-(r_{\text{USD}} - r_{\text{GBP}})(T-t)} \\
V_0 &= 1 - 1e^{-(0.04-0.01)(1-0)} \\
&= 1 - 0.97 \\
&= 3\%
\end{aligned}$$

  So that you pay a premium of exactly the $3\%$ you were effectively going to when striking at $F$. No-arb strikes (no pun intended) again!