ASSET-BACKED SECURITIES
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TRANCHES

Suppose there are two i.i.d. loans for $10k each. Each independently has a 50% chance of defaulting. You buy them both at par for $20k and securitize the bundle. Jointly, there's a:

- 25% chance that both default (payout is zero),
- 50% chance that just one defaults (payout is $10k + interest),
- 25% chance that neither defaults (so payout is the full $20k + interest).

You decide to offer:

- $5k principal of A-notes, which get seniority: There's only a 25% chance of full default (since there's a 75% chance you collect at least $5k + interest from your bundle of loans). So, since the A-notes are much safer than the underlying individual loans, under you can sell the A-notes at 2 points above par for $5.1k.
- $10k principal of B-notes, which will have a
  - 25% chance of full default (happens if you collect zero payout – nobody gets paid),
  - 50% chance of half-default (happens if you collect $10k + interest – after the $5k of A-notes get paid, you're left with $5k to pay the B's),
  - 25% chance of no default.

  So, even though the expected payout of the $10k of B-notes is by Adam's law still $5k—exactly the same as if you had bought either of the underlying individual loans—the variance of outcomes is lower (because instead of a 50+50 all-or-nothing bet, you're taking a 25+25 all-or-nothing bet with 50% of "cushion" in the middle). A mean-variance investor will consider these notes safer than either underlying individual loan. So, you can sell the B's at 1 point above par for $10.1k.

Finally, you reserve $5k of principal for the residual ("equity certificate"), which gets paid last. Unlike the notes, which are debt (the securitization is legally like a regular company, and the notes are like loans from the investors to you, which you promise to repay and collateralize with the underlying individual loans), the residual is equity, and gets paid the analog of "retained earnings".

You could sell the residual shares (under no-arbitrage, this would necessarily have to be at $4.8k, since the loans themselves are selling for $20k, so the asset side of the balance sheet is $20k, and you have $15.2k worth of debt), but let's say you choose to keep them. There's a 75% chance that you will make nothing from this – The only way you make any money here is if neither loan defaults and you collect the full $20k + interest, leaving $5k + interest for yourself once the A's and B's get paid. Why would you take this deal, when you could have instead simply bought both loans and kept them for yourself which, thanks to diversification, would have given you a similar deal as the B's?

Well, now instead of taking your $20k and tying it all up in loans, you continue to have some exposure to the loans, but you now also have $15.2k of cash proceeds from selling the notes, and you can go invest it elsewhere or pile it up in your living room and look at it, or whatever else you want.

ABSBS: ASSET-BACKED-SECURITY-BACKED SECURITIES

In the Big Short, there's a patronizing pair of scenes where Anthony Bourdain and Ryan Gosling sneer that packaging B-notes from a random collection of existing securitizations into a single new securitization and selling A-notes off that because it's "suddenly diversified" is like taking a random collection of contaminated fish that they can't make into sushi and making it into a seafood stew. Yes! It is exactly like that – and that's why it makes sense.

Let me belabor their fish analogy. Suppose you have 7 fish, one from each of the seven seas, each weighing 100g. You know that each fish has a 20% chance of being tainted with mercury. The fish are independent. The extremely painful and lethal dose of tainted fish is exactly 100g. Assume you are averse to that fate. Would you rather have:

- Seven pieces of sashimi, all cut from the same fish (you can have your pick of which one); or
- Seven pieces of sashimi, each cut from a different fish?

The rational answer under any reasonable utility function is the second. In the first case, there's a one-in-five chance that your dinner will be painfully lethal; In the second, there's only a one-in-hundred-thousand chance, because it requires that *every single fish* ended up being tainted. This is what we call diversification:

- Compared to the one-in-five chance that the B-note from any single one of the underlying securitizations defaults;
- The chance that the B-notes from *every* one of the various underlying securitizations default is four orders of magnitude smaller.

Now notice that even this wasn't a free lunch (no pun intended). If you take the first option, there's an 80% chance that you get off scot-free with no mercury poisoning at all. But under the second option, there's only a 21% chance, because symmetrically as above it requires that *every single fish* ended up being safe. So under the second option, you effectively eliminate any chance of the painful death, but you must accept a heightened chance of mild-to-moderate mercury poisoning – Just like the B-noteholders in my sketch above.

2.1 The First Global Principal Component

Now let's make the argument a little more sophisticated, and account for "systemic" (AKA "undiversifiable") risk. Let's say that there's a 10% chance (I'm trying to match the once-every-ten-years pattern that recessions seem to follow) that there's been an insidious global calamity C and the entire global ocean has been contaminated with mercury. Other than that, each fish independently has an 11% chance of being tainted. (This is constructed so that, even though now the fish are no longer independent, marginally, each fish still has exactly a $0.10{+}0.11(0.90) = 20\%$ chance of being tainted.) You are *still* better off taking the mixed sashimi. Let L_n represent the event that fish n is tainted. In this case, by the law of total probability, your chance of having a lethal dinner L (no subscript) is

$$\begin{aligned}\Pr[L] &= \Pr[L \mid C]\Pr[C] + \Pr[L \mid \bar{C}]\Pr[\bar{C}] \\ &= 1.00(0.10) + \Pr[\cap_{n=1}^{n=7} L_n \mid \bar{C}](0.90) \\ &= 1.00(0.10) + 0.11^7(0.90) \\ &= \boxed{0.1000002}.\end{aligned}$$

This is barely more than $\Pr[C] = \boxed{0.10}$ by itself!

You understand now why, out of the 20% chance of any single fish being tainted, we call 10% "undiversifiable" or "systemic", and 10% "diversifiable" or "idiosyncratic". (It just ended up being half-and-half here because I decided that $\Pr[C] = 0.10$ – if I had set $\Pr[C] = 0.05$, then 5% of the chance would have been systemic and 15% idiosyncratic.) You can never have less than a 10% chance of fatality, but the other 10% pretty much vanishes if you diversify it away by sampling from each of the seas.

Once more for completeness let's analyze the opposite outcome. Under the first option, you still have an 80% chance of getting off scot-free F (meaning that you ingest not a single atom of mercury). But now, under the second option, keeping in mind that in the global calamity *every fish in the world* has become tainted, your chance is

$$\begin{aligned}\Pr[F] &= \Pr[F \mid C]\Pr[C] + \Pr[F \mid \bar{C}]\Pr[\bar{C}] \\ &= 0(0.10) + \Pr[\cap_{n=1}^{n=7} \bar{L_n} \mid \bar{C}](0.90) \\ &= 0(0.10) + 0.89^7(0.90) \\ &= 0.40.\end{aligned}$$

So in this case, you have a 40% chance of getting off scot-free, much higher than before. The reason is that by increasing the influence of the global factor while holding fixed the unconditional risk per fish, we were forced to make the fishes more correlated, which makes our scenario look more like the single-fish scenario. Take this to the extreme: If we max out the influence of the global factor, i.e. set $\Pr[C] = 0.20$ which forces $\Pr[L_n \mid \bar{C}] = 0$, then the fish are now completely dependent. It's all-or-nothing, and we have essentially recovered the first option, as if the 7 fish are simply perfectly-entangled clones of each other.