Sharpe of a non-MVO portfolio

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The standard MVO portfolio u is $(\gamma\Sigma)^{-1}\mu$. So, the passive-asset ER's μ can be written as $\gamma\Sigma u$.

Now consider an arbitrary portfolio w. Its expected return is

$$w'\mu = \gamma w'\Sigma u.$$

Its variance is as usual

$$w'\Sigma w.$$

So, its Sharpe is

$$\begin{aligned}\gamma\frac{w'\Sigma u}{\sqrt{w'\Sigma w}} &= \gamma\frac{\mathrm{Cov}(w,\, u)}{\sigma_w} = \gamma\frac{\mathrm{Cov}(w,\, u)}{\sigma_w\sigma_u}\sigma_u = \gamma\frac{\mathrm{Cov}(w,\, u)}{\sigma_w\sigma_u}\sigma_u \\ &= \gamma\mathrm{Corr}(w,\, u)\sigma_u = \boxed{\mathrm{Corr}(w,\, u)\gamma\sigma_u}.\end{aligned}$$

For the MVO portfolio u, this is just $\gamma\sigma_u$. For any other portfolio, it is the same thing discounted by "transfer coefficient".