E2-243
Programming Exercise - 1

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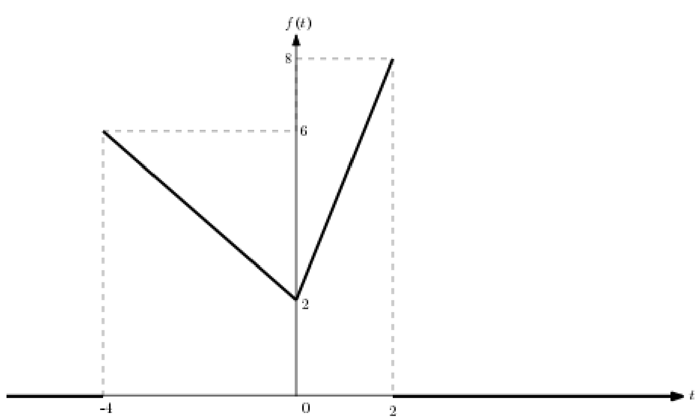
Instructions:

- **Attempt the following programming exercises using MATLAB.**
- **Do not submit your code, output files, etc.**
- **There will one lab exam towards the end of the semester that will test your understanding of the concepts taught in class, and your proficiency in using MATLAB to demonstrate your understanding of these concepts. The questions in the lab exam will be somewhat similar to these questions in both content and implementation complexity. If you do not program these exercises, handling the lab exam will not be easy! In a way, programming these assignments yourself will be your preparation for the lab exam.**
- **You are expected not to use built-in MATLAB functions, except for verifying the output.**
- **Plot for the sequence of numbers is a discrete plot. You may use any discrete plot command in MATLAB like 'stem' for plotting the sequence of numbers.**
- **Whenever $x \in$ interval, you may have to take appropriate discrete points in the interval to realize the functions in matlab. Please use appropriate commands for continuous plots when x is continuous.**

Functions

1. Let $f : \mathbb{R} \to \mathbb{R}$ be the function whose graph is sketched below:



Plot the following functions in MATLAB:

(a) $f(t)$

(b) $g(t) = f(t+2)$

(c) $g(t) = f(\frac{5}{4}t)$

(d) $g(t) = f(2t-4)$

(e) $g(t) = f(2-t)$

(You can use `subplot()` function in MATLAB to plot (a)-(e) in a single figure.)

2. Let T be a positve real number. The "ramp" function $R_T : \mathbb{R} \to \mathbb{R}$ is defined as follows:

$$R_T(t) = \begin{cases} \frac{t+T}{2T}, & \text{if } -T \leq t \leq T \\ 1, & \text{if } t \geq T \\ 0, & \text{if } t \leq T \end{cases}$$

Plot the following functions in MATLAB:

(a) $R_T(t)$

(b) $g(t) = R_T(t+T) - R_T(t-T)$

(Use T = 1, 5, 10).

3. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined as $f(t) = \exp(-|t|)$. Then answer the following by plotting the function.(Exercise-2 Q4)

(a) Range of f.

(b) From (a) comment if the function is onto or not.

(c) Is f one-one.

Convergence of sequences

4. **Sequences:** The following problems have been picked from previous homeworks, quizzes and midterm. Please match your theoretical results with simulations.

Let $\{f_n\}_{n\in\mathbb{N}}$ be the sequence of real numbers defined as follows

(a) $f_n = \frac{n^2+2}{2n^2+3}$ (Exercise-1, Q2)

(b) $f_n = \frac{3n^2-2n+\sin(n)}{5n^2}$ (Exercise-1, Q3(7))

(c) $f_n = \frac{(-1)^n}{n}$ (Exercise-1, Q4(a))

(d) $f_{2n-1} = \frac{1}{n+1}$
$f_{2n} = 1 - \frac{1}{n+1}, \ \forall\ n \in \{1, 2, ..\}$ (Exercise-1, Q6)

(e) $f_n = \sin(n\pi) + cos(n\pi)$ (Exercise-1, Q4(f))

(f) $f_n = \sin(\frac{n\pi}{2}) + cos(\frac{n\pi}{2})$

For the above sequences perform the following

(a) Plot the above sequences till

i. $n = 100$

ii. $n = 1000$

iii. $n = 10^5$

(b) Comment on convergence using above plots.

(c) Find the value of n for which $|f_n - f|$ is less than 10^{-4}.

(d) Find gub and lub, using (a)

(e) Is the sequence bounded.

Convergence of series

Note that we have used convergence of series results in proof of Borel Cantelli Lemma. Let us analyse the convergence for most commonly used series.

5. Let us define the following

$$S_N = \sum_{n=1}^{N} \frac{1}{n}$$

$$A_N = \sum_{n=1}^{N} \frac{1}{n^{1.1}}$$

$$Z_N = \sum_{n=1}^{N} \frac{1}{n^2}$$

Plot the sequence $\{S_n\}, \{A_n\}, \{Z_n\}$ till $n = 10^2, 10^4, 10^8$. Comment about the convergence of the sequences.

Remark The previous problem is an outcome of the following theorem "$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$."

Convergence of functions

6. Let us define the following sequence of functions $f : X \to R$ where $X = [0, 100]$

(a) $f_n(x) = \cos(nx)$

(b) $f_n(x) = \frac{\cos(nx)}{n}$ (Similar to Exercise-2 Q4(a))

For the above functions perform the following

(a) Plot $f_n(x)$ for $n = 1, 2, 3, 5, 7, 10, 50, 100, 10^3, 10^6$.

(b) Find the sequence of numbers generated at $x = 2, 10.35, 54.59, 95.19$ for the values of n given above .

(c) Using the above information comment about point-wise convergence of the sequence of functions.

Remark Note that to check for pointwise convergence of a sequence of functions, we fix the value of x and then observe corresponding values of the sequences of functions (this would be sequence of real numbers at a fixed x). This is iterated over all points in the domain.

7. Let us define the following (Exercise-3 Q13)

$$f_n(x) = \begin{cases} n(x + \frac{1}{n}) \text{ if } \frac{-1}{n} \leq x < 0 \\ -n(x - \frac{1}{n}) \text{ if } 0 \leq x < \frac{1}{n} \\ 0 \text{ otherwise} \end{cases}$$

For the above functions perform the following

(a) Plot $f_n(x)$ for $n = \{1, 2, .., 10\}$.

(b) Find the sequence of numbers generated at $x = 0.9, 0.1, 0.2, 0.001, 0.999$ for the values of n given above .

(c) Using the above information comment about point-wise convergence of the sequence of functions.