Seq no:

Name: **SR No.:** **Dept.:**

Maximum Points: 15 **E2-243: Quiz 4** **Duration**: 40 minutes

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- Tick the correct answer **(1+1+1+1 points. -0.5 point for each negative answer)**

1. Variance of any random variable is
   a) Always positive
   b) Always zero
   c) Could be positive or negative
   d) Always non negative
   Answer: d. Since it is expectation of non-negative variable $(X - \mu_x)^2$
2. X and Y are random variables defined on the same probability space. It is given that $Var(X+Y) = Var(X) + Var(Y)$. Then
   a) X and Y are always independent
   b) $Cov(X,Y) = 0$
   c) Both (a) and (b) are true
   d) $E(X) = E(Y)$
   Answer: b. $Cov(X,Y) = 0$
3. Let W be a non empty subset of R^3.
   a) If null vector $\theta_3 \in W$ then W is always a subspace of R^3
   b) If $x \in W$, $\alpha \in R \Rightarrow \alpha * x \in W$ then W is always a subspace of R^3
   c) If W is a subspace of R^3 then null vector $\theta_3 \in W$ is always true
   d) Both (a) and (c) are always true
   Answer: c. null vector $\theta_3 \in W$ and $\alpha * x \in W$ are necessary conditions but not sufficient condition for W to be a subspace of R^3
4. Let W be a subspace in R^3. It is given that the vector $\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 \in W \forall \alpha_1, \alpha_2, \alpha_3 \in R$. Then
   a) Only one of $u_1, u_2, u_3 \in W$
   b) Exactly two amongst $u_1, u_2, u_3 \in W$
   c) All 3 $u_1, u_2, u_3 \in W$
   d) None of $u_1, u_2, u_3 \in W$
   Answer: c. Could be proven by making any two of the vectors α_1, α_2, $\alpha_3 = 0$

- Random variables X and Y are defined on the same probability space as follows:
  $X = 1$ and -1 with probability 0.05 each
  $X = 2$ and -2 with probability 0.1 each
  $X = 3$ and -3 with probability 0.15 each
  $X = 4$ and -4 with probability 0.2 each
  $Y = |X|$
  Answer the following questions (**6 points.**)

1. Plot the CDF of Y. (**1 point**)
   Ans:

$$F_Y(x) = \begin{cases} 0, & \text{if } x \in (-\infty, 1) \\ 0.1, & \text{if } x \in [1, 2) \\ 0.3, & \text{if } x \in [2, 3) \\ 0.6, & \text{if } x \in [3, 4) \\ 1, & \text{if } x \in [4, \infty) \end{cases}$$

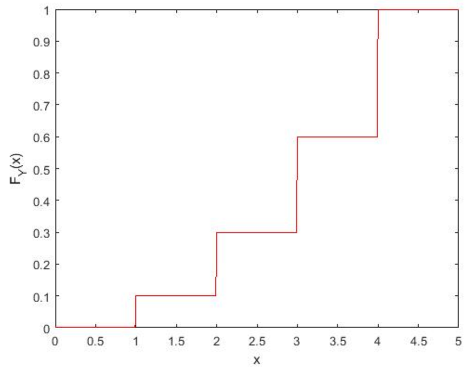

Figure 1: Optimal and approximate policy for $p = 0.7, \psi = 10, d = 21$.

2. Find the expectation of Y. (**1 point**)
   Ans: 3. (1*0.1)+(2*0.2)+(3*0.3)+(4*0.4)
3. Prove or disprove. X and Y are independent. (**1.5 points**)
   Ans: No. P(X=1,Y=2) =0. P(X=1).P(Y=2)=0.05*0.2. Since both are not equal, they are not independent.
4. Prove or disprove. X and Y are uncorrelated. (**1.5 points**)
   Ans: Yes. E(XY)=0=E(X)E(Y).
5. Can we apply Markov's inequality to find $P(X \geq 0)$. State your reason. (**1 point**)
   Ans: No. Since Markov's inequality could be applied only to non-negative random variables.

- State whether the following are true or false (**1+1+1 points. -0.5 point for each negative answer**)

1. A sequence, $\{X_n\}_{n \in \mathbb{N}}$, of random variables on a Probability space $(\Omega, \mathbb{B}, \mathbb{P})$ is said to converge "in probability" to a random variable X on $(\Omega, \mathbb{B}, \mathbb{P})$ if

$$\lim_{n \to \infty} P(w \in \Omega : |X_n(w) - X(w)| < \epsilon) = 1$$

   for every $\epsilon > 0$

   Ans: True. As the definition says,

$$\lim_{n \to \infty} P(w \in \Omega : |X_n(w) - X(w)| \geq \epsilon) = 0$$

   for every $\epsilon > 0$, it implies

$$\lim_{n \to \infty} P(w \in \Omega : |X_n(w) - X(w)| < \epsilon) = 1$$

   for every $\epsilon > 0$

2. If $x > y$, then $\forall\ a \in \mathbb{R}$, $ax > ay$.

   Ans: False. If $a > 0$, then $ax > ay$. Note that $3 > 2$ but $-1 \times 3 < -1 \times 2$.

3. $\lim_{n \to \infty} \mathrm{E}(|X_n - X|^k) = 0 \iff X_n \xrightarrow{\mathrm{p}} X$, where k is any positive integer.

   Ans: False. As $\lim_{n \to \infty} \mathrm{E}(|X_n - X|^k) = 0 \Rightarrow X_n \xrightarrow{\mathrm{p}} X$, where k is any positive integer. Find the following example

$$X_n = \begin{cases} n, & \text{w.p. } \frac{1}{n} \\ 0, & \text{w.p. } 1 - \frac{1}{n} \end{cases}$$

$$P(X_n > \epsilon) = \begin{cases} \frac{1}{n}, & \text{if } \epsilon \in [0, n) \\ 0, & \text{otherwise} \end{cases}$$

Therefore, $\lim_{n\to\infty} P(|X_n - 0| > \epsilon) = 0$. Hence it converges in probability. Let us look at k^{th} mean convergence

$$\lim_{n\to\infty} E[|X_n - 0|^k] = \lim_{n\to\infty} n^k * \frac{1}{n} + 0^2 * (1 - \frac{1}{2}) \neq 0$$

Therefore, convergence in probability does not imply convergence in k^{th} mean.

- Give an example where Markov inequality provides a tightest bound. . **(2 points)**
  Ans: Consider the following example where X, a non negative random variable that can take either 0 or $k > 0$.

  $$P(X = 0) = 1 - \frac{1}{k^2}, P(X = k) = \frac{1}{k^2}$$

  Let us evaluate Markov inequality as follows

  $$P(X \geq k) \leq \frac{E(X)}{k} = \frac{1}{k^2}$$

  Also, note that $P(X \geq k) = \frac{1}{k^2}$. Thus Markov inequality is tight for this example.