Tarea 2 - Cómputo Numérico

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December 2, 2021

Ejercicio 1

1. Considere el metodo de la secante para encontrar la raiz $x = \alpha$ de la funcion $f(x)$, en donde el proceso iterativo esta dado por,

$$x_{i+1} = x_i - \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} f(x_i) \tag{1}$$

si $\epsilon_i = x_i - \alpha$ es el error asociado al proceso iterativo en el paso iésimo, demostrar que

$$|\epsilon_{i+1}| = C|\epsilon_i|^{\phi}$$

en donde C es una constante y $\phi = \frac{1+\sqrt{5}}{2}$ es el cociente dorado.

Demostración:

Si $\epsilon_i = x_i - \alpha$ entonces sabemos que:

$$\epsilon_{i+1} = x_{i+1} - \alpha$$

$$\epsilon_i = x_i - \alpha$$

$$\epsilon_{i-1} = x_{i-1} - \alpha$$

por lo que, al despejar las x_i obtenemos:

$$x_{i+1} = \epsilon_{i+1} + \alpha$$

$$x_i = \epsilon_i + \alpha$$

$$x_{i-1} = \epsilon_{i-1} + \alpha$$

al sustituir lo anterior en la ecuación 1 obtenemos:

$$\epsilon_{i+1} = \frac{\epsilon_{i-1} f(x_i) - \epsilon_i f(x_i)}{f(x_i) - f(x_{i-1})} \tag{2}$$

Por el teorema del valor medio sabemos que existe un valor a_i en el intervalo $[x_i, \alpha]$ tal que:

$$f'(a_i) = \frac{f(x_i) - f(\alpha)}{x_i - \alpha} \tag{3}$$

donde α al ser una raíz sabemos que $f(\alpha) = 0$ y donde $x_i - \alpha = \epsilon_i$, por lo que al simplificar la ecuación anterior y despejar $f(x_i)$ obtenemos:

$$f'(a_i) = \frac{f(x_i)}{\epsilon_i}, f(x_i) = \epsilon_i f'(a_i) \tag{4}$$

y dado lo anterior, similarmente para el punto x_{i-1} tenemos:

$$f(x_{i-1}) = \epsilon_{i-1} f'(a_{i-1}) \tag{5}$$

Usando 2, 4 y 5 entonces:

$$\epsilon_{i+1} = \epsilon_i \epsilon_{i-1} \frac{f'(a_i) - f'(a_{i-1})}{f(x_i) - f(x_{i-1})} \tag{6}$$

donde

$$\frac{f'(a_i) - f'(a_{i-1})}{f(x_i) - f(x_{i-1})}$$

no es mas que una constante C.

Al tener la ecuación 6 de la forma $\epsilon_{i+1} = C\epsilon_i\epsilon_{i-1}$, decimos que esta ecuación es de orden p si:

$$\epsilon_i = C\epsilon_{i-i}^p$$

$$\epsilon_{i+1} = C\epsilon_i^p$$

por lo que:

$$\epsilon_{i+1}^p = C\epsilon_i^{(p+1)/p} \tag{7}$$

de 7 entonces:

$$p = (p+1)/p$$

$$p^2 = p + 1$$

$$p^2 - p - 1 = 0$$

$$p = \frac{1 \pm \sqrt{1+4}}{2}$$

tomando la parte positiva, entonces

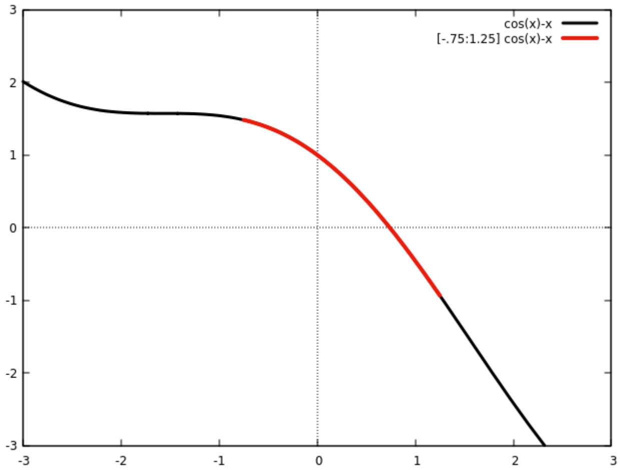
$$p = \frac{1+\sqrt{5}}{2} = \phi$$

por lo que se demuestra que $|\epsilon_{i+1}| = C|\epsilon_i|^\phi$

Ejercicio 2

2. Considere la función $f(x) = \cos(x) - x$, y los tres puntos $\alpha_1 = -0.75$, $\alpha_2 = 1.25$ y $\alpha_3 = 3.15$

a) Grafique la función y cerciórese de que existe un cambio de signo entre α_1 y α_2.



Se puede observar en el segmento rojo que cruza el 0 del eje y, por lo que si existe el cambio de signo.

b) Encuentre el cero de la función usando el método de bisección, correcto hasta 11 cifras significativas, tabulando los extremos del intervalo $[a, b]$ el punto medio c y $f(c)$ en cada paso del método para cuando los intervalos iniciales son: $[\alpha_1, \alpha_2]$ y $[\alpha_1, \alpha_3]$.

it	a	b	c	$f(c)$
0	-0.75	1.25	0.25	0.71891242171
1	0.25	1.25	0.75	-0.018311131126
2	0.25	0.75	0.5	0.37758256189
3	0.5	0.75	0.625	0.18596311951
4	0.625	0.75	0.6875	0.085334946152
5	0.6875	0.75	0.71875	0.033879372418
6	0.71875	0.75	0.734375	0.0078747254585
7	0.734375	0.75	0.7421875	-0.0051957117438
8	0.734375	0.7421875	0.73828125	0.0013451497518
9	0.73828125	0.7421875	0.740234375	-0.0019238727809
10	0.73828125	0.740234375	0.7392578125	-0.00028900914679
11	0.73828125	0.7392578125	0.73876953125	0.00052815843366
12	0.73876953125	0.7392578125	0.73901367188	0.00011959667132
13	0.73901367188	0.7392578125	0.73913574219	-8.4700731375e-05
14	0.73901367188	0.73913574219	0.73907470703	1.744934664e-05
⋮	⋮	⋮	⋮	⋮
34	0.73908513319	0.7390851333	0.73908513325	-5.1078363761e-11
35	0.73908513319	0.73908513325	0.73908513322	-2.3698820684e-12

Table 1: Tabla método Bisección, intervalos $[\alpha_1, \alpha_2]$

it	a	b	c	$f(c)$
0	-0.75	3.15	1.2	-0.83764224552
1	-0.75	1.2	0.225	0.74979410707
2	0.225	1.2	0.7125	0.044229923381
3	0.7125	1.2	0.95625	-0.3796620853
4	0.7125	0.95625	0.834375	-0.16273413814
5	0.7125	0.834375	0.7734375	-0.057924032117
6	0.7125	0.7734375	0.74296875	-0.0065052348029
7	0.7125	0.74296875	0.727734375	0.018948990059
8	0.727734375	0.74296875	0.7353515625	0.0062433917666
9	0.7353515625	0.74296875	0.73916015625	-0.00012556153348
10	0.7353515625	0.73916015625	0.73725585938	0.0030602574375
11	0.73725585938	0.73916015625	0.73820800781	0.0014676832421
⋮	⋮	⋮	⋮	⋮
35	0.73908513312	0.73908513323	0.73908513317	6.8257510755e-11
36	0.73908513317	0.73908513323	0.7390851332	2.0766610653e-11
37	0.7390851332	0.73908513323	0.73908513322	-2.9787283751e-12

Table 2: Tabla método Bisección, intervalos $[\alpha_1, \alpha_3]$

c) Encuentre el cero de la función usando el método de punto fijo, correcto hasta 11 cifras significativas, tabulando x_i y $f(x_i)$ en cada paso del método para cuando los puntos iniciales son: $x_1 = \alpha_1, x_1 = \alpha_2$ y $x_1 = \alpha_3$.

it	x_i	$f(x_i)$
0	-0.75	1.4816888689
1	0.73168886887	0.012358215915
2	0.74404708479	-0.0083134666015
3	0.73573361819	0.0056049807007
4	0.74133859889	-0.0037733025487
5	0.73756529634	0.002542763276
6	0.74010805962	-0.0017123684894
7	0.73839569113	0.0011536828824
8	0.73954937401	-0.00077703860825
9	0.7387723354	0.00052346602566
10	0.73929580143	-0.00035259325102
11	0.73894320817	0.00023752001104
12	0.73918072819	-0.00015999226962
13	0.73902073592	0.0001077745618
14	0.73912851048	-7.2597404055e-05
15	0.73905591307	4.8902864452e-05
16	0.73910481594	-3.2941385395e-05
17	0.73907187455	2.2189791661e-05
18	0.73909406434	-1.4947275101e-05
$\vdots$	$\vdots$	$\vdots$
52	0.73908513323	-2.1902590852e-11
53	0.73908513321	1.4753864797e-11
54	0.73908513322	-9.9383834495e-12

Table 3: Tabla método Punto Fijo, con $x_1 = \alpha_1$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376
1	0.3153223624	0.63537409394
2	0.95069645633	-0.36958001628
3	0.58111644006	0.25473385281
4	0.83585029287	-0.1653031719
5	0.67054712097	0.11293467316
6	0.78348179413	-0.075021234479
7	0.70846055965	0.050903876543
8	0.75936443619	-0.034090718339
9	0.72527371785	0.023044164066
10	0.74831788192	-0.015483451868
11	0.73283443005	0.010446786129
12	0.74328121618	-0.0070291132001
13	0.73625210298	0.0047384249578
14	0.74099052794	-0.0031902323872
15	0.73780029555	0.0021497094881
16	0.73995000504	-0.001447736235
17	0.7385022688	0.00097536332215
18	0.73947763213	-0.0006569478226
$\vdots$	$\vdots$	$\vdots$
63	0.73908513321	1.2473577726e-11
64	0.73908513322	-8.4022788727e-12

Table 4: Tabla método Punto Fijo, con $x_1 = \alpha_2$

it	x_i	$f(x_i)$
0	3.15	-4.1499646585
1	-0.99996465847	1.5402967029
2	0.5403320444	0.3172058737
3	0.85753791811	-0.20323655886
4	0.65430135924	0.13917195875
5	0.793473318	-0.09209952576
6	0.70137379224	0.062582652321
7	0.76395644456	-0.041851779297
8	0.72210466526	0.028311615784
9	0.75041628105	-0.019011228859
10	0.73140505219	0.012831628274
11	0.74423668046	-0.0086314831543
12	0.73560519731	0.0058195827248
13	0.74142478003	-0.0039176824758
14	0.73750709756	0.0026400987854
15	0.74014719634	-0.0017778983268
16	0.73836929802	0.0011978409988
17	0.73956713901	-0.00080677654677
62	0.73908513321	1.5318524227e-11
63	0.73908513322	-1.0318745858e-11
64	0.73908513321	6.9508843126e-12

Table 5: Tabla método Punto Fijo, con $x_1 = \alpha_3$

d) Encuentre el cero de la función con el método de Newton, correcto hasta 13 cifras significativas, tabulando x_i y $f(x_i)$ en cada paso del método para cuando los puntos iniciales son: $x_1 = \alpha_1, x_1 = \alpha_2$ y $x_1 = \alpha_3$. En cada caso, utilice la derivada:

i) Exacta, con $f'(x) = -\sin(x) - 1$.

ii) Numérica, calculando la derivada hacia adelante con

$$f'(x) \approx \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

con $\epsilon = 0.1$

iii) Numérica, calculando la derivada hacia atrás con

$$f'(x) \approx \frac{f(x) - f(x-\epsilon)}{\epsilon}$$

con $\epsilon = 0.1$

iv) Numérica, calculando la derivada con diferencias centrales con

$$f'(x) \approx \frac{f(x+\epsilon) - f(x-\epsilon)}{2\epsilon}$$

con $\epsilon = 0.1$

v) Repita los incisos **ii)**,**iii)** y **iv)** pero con $\epsilon = 10^{-2}$ y $\epsilon = 10^{-4}$.

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.904112004911	-4.627210082022
2	-11.05834832415	11.12108114118
3	-5.492326215563	6.195561013016
4	-1.871219105005	1.575295029316
5	33.30061678569	-33.60938141361
6	16.0750934358	-17.00845471628
7	-10.45660443426	9.943352281013
8	-5.105646687587	5.488846162885
9	-2.252320137659	1.622342970371
10	5.010178240778	-4.716770754768
11	-102.158643887	102.101791912
12	63025.6761147	-63025.09907882
13	-280845.3779643	280846.2831857
14	-83752.01839899	83751.06301727
15	-19098.05385909	19097.10217375
16	-4487.489534782	4487.761631095
17	114456.7014808	-114457.2883032
18	51210.69540897	-51211.61394414
19	14508.78741367	-14508.17567477
20	6408.456989775	-6407.532851938
21	-3960.713005189	3960.042072903
22	11359.70237679	-11358.74605818
$\vdots$	$\vdots$	$\vdots$

Table 6: Tabla método de Newton, con derivada exacta y con $x_1 = \alpha_1$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7704284177913	-0.05281605077596
2	0.7392950057892	-0.000351261540583
3	0.7390851429387	-1.627347179234e-08
4	0.7390851332152	0

Table 7: Tabla método de Newton, con derivada exacta y con $x_1 = \alpha_2$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.035150251908	1.545546982848
2	9.999667303452	-10.83891978003
3	-13.75644872841	14.12803608088
4	183.5569691319	-183.3326921028
5	90.70797904375	-91.62973643093
6	24.68123952276	-23.7814466258
7	-17.50817752624	17.73558827065
8	-8.522668407894	7.90271229108
9	28.17209060215	-29.16686830971
10	1.706444704833	-1.841677466166
11	0.7813569779191	-0.07139842879085
12	0.7394624773729	-0.000631580334195
13	0.7390851646429	-5.259782454026e-08
14	0.7390851332152	-4.440892098501e-16

Table 8: Tabla método de Newton, con derivada exacta y con $x_1 = \alpha_3$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.411454689905	-4.37526237635
2	-2.969312004647	1.984115646138
3	-0.4243899932319	1.335680075135
4	1.680874899265	-1.790731297349
5	0.7795621521449	-0.0683407529021
6	0.7402254619365	-0.001908948233189
7	0.7391092588921	-4.037723825379e-05
8	0.7390856386037	-8.458244158405e-07
9	0.7390851437999	-1.771472413203e-08
10	0.7390851334368	-3.710108886779e-10
11	0.7390851332198	-7.770228904747e-12
12	0.7390851332153	-1.626476731076e-13
13	0.7390851332152	-3.441691376338e-15

Table 9: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_1$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	4.486708710703	-4.710478144037
2	-118.9514449484	119.8607955452
3	-31.45593421431	32.45513401389
4	4.203121316501	-4.690659290209
5	-26.51315346272	26.70238952451
6	2578.549628849	-2579.315796663
7	1043.1023136	-1042.106686454
8	44.45735657936	-43.56809103852
9	13.60620608599	-13.09984394566
10	6.469611153492	-5.486938181433
11	1.639331969828	-1.70781397194
12	0.7851737233301	-0.07790825686091
13	0.7385342654868	0.000921826698313
14	0.7390980251308	-2.157612644971e-05
15	0.7390848333736	5.018184585648e-07
16	0.7390851401899	-1.167304919392e-08
17	0.7390851330529	2.71531797047e-10
18	0.7390851332189	-6.316280831697e-12
19	0.7390851332151	1.468825061579e-13
20	0.7390851332152	-3.441691376338e-15

Table 10: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_1$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.887571213629	-4.621995328042
2	-10.44687422941	9.925295674093
3	-5.087012304405	5.452934307645
4	-2.260332601386	1.624153133966
5	4.809066747335	-4.712539511272
6	-739.9372393618	740.0292783589
7	-368.8272021941	368.5222058383
8	-179.915766145	179.252023561
9	-77.29340770838	76.97467923321
10	1355.249642433	-1355.590273094
11	-20733.91187052	20734.73740757
12	-7471.385547336	7472.164239894
13	12526.94165479	-12527.10003644
14	-865530.2418153	865529.3766906
15	868028.9328087	-868028.107155
$\vdots$	$\vdots$	$\vdots$
44	-2.560141836269	1.724475149017
45	1.257788327674	-0.9498664571031
46	0.770634043085	-0.05316489748262
47	0.7392762747323	-0.000319910242898
48	0.7390850130272	2.011479619535e-07
49	0.7390851332958	-1.349617084756e-10
50	0.7390851332151	9.059419880941e-14

Table 11: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_1$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.851075059578	-4.609774219226
2	-9.519261345977	8.523721581452
3	-1.694765226469	1.571113614516
4	220.4463119306	-219.5859546934
5	75.40789328121	-74.40794003138
6	2.075664288928	-2.559356168863
7	0.7090747454961	0.04988991792561
8	0.7392212455868	-0.0002278061486305
9	0.7390854362109	-5.070974210541e-07
10	0.7390851338807	-1.113868886016e-09
11	0.7390851332166	-2.446598479366e-12
12	0.7390851332152	-5.440092820663e-15

Table 12: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_1$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.958046003171	-4.642856032952
2	-12.94261091292	13.87266353194
3	9.150096208054	-10.11260777602
4	1.22515704063	-0.8863588160151
5	0.7680706413295	-0.04881821417724
6	0.7392036140125	-0.0001982960749334
7	0.7390848733763	4.348694726541e-07
8	0.7390851337919	-9.652877386301e-10
9	0.7390851332139	2.142730437527e-12
10	0.7390851332152	-4.884981308351e-15

Table 13: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_1$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.903945930775	-4.627158712855
2	-11.05198357989	11.10836296117
3	-5.493335107229	6.197286825938
4	-1.86969085075	1.575226926051
5	33.64572280106	-34.25804930566
6	14.51347332243	-14.88096113635
7	6.803182044416	-5.93536124354
8	2.837997954162	-3.7922659669
9	-0.08149347904222	1.078174722796
10	1.09222414733	-0.6317119116314
11	0.757566907151	-0.03105683371433
12	0.739158978098	-0.0001235896992847
13	0.7390851339238	-1.185909370705e-09
14	0.7390851332152	7.993605777301e-15

Table 14: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_1$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.903577223009	-4.627044594827
2	-11.04140368885	11.08721704898
3	-5.494889639004	6.199944609512
4	-1.867468344657	1.575129111409
5	34.20085775123	-35.13792544626
6	8.155418036917	-8.452310186383
7	3.831756090103	-4.602898061382
8	-8.837995100088	8.005268833262
9	9.100039320984	-10.04777346964
10	1.482397899159	-1.394114554923
11	0.7839786395834	-0.07586881624594
12	0.7395104730202	-0.0007119206609268
13	0.7390851825291	-8.253234795585e-08
14	0.7390851332163	-1.823208251039e-12
15	0.7390851332152	0

Table 15: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_1$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.90464687648	-4.627375390112
2	-11.0753272129	11.15499561999
3	-5.488939884478	6.189763116371
4	-1.876167263419	1.575520292109
5	32.16739504507	-31.43670795756
6	13.48482733025	-12.87778005621
7	6.309118940922	-5.309455198755
8	1.133609768334	-0.7102174515726
9	0.7609731994119	-0.03680798976261
10	0.7391880427727	-0.0001722345868066
11	0.7390851332813	-1.106427172104e-10
12	0.7390851332152	2.331468351713e-15

Table 16: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_1$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	-0.75	1.481688868874
1	3.904111988285	-4.62721007688
2	-11.05834768688	11.12107986789
3	-5.492326322269	6.195561195586
4	-1.871218943667	1.57529502209
5	33.30065302844	-33.60945212803
6	16.07499462156	-17.0083913657
7	-10.45278778456	9.936263784217
8	-5.099983895122	5.477946727653
9	-2.255510090066	1.623058771639
10	4.945286501259	-4.714488719974
11	-169.6766434648	170.6761740915
12	6.393506011222	-5.399585170724
13	1.529440190601	-1.488095842172
14	0.7850740363143	-0.07773809981705
15	0.7395299960881	-0.0007446009791902
16	0.7390851768925	-7.309894500818e-08
17	0.7390851332152	-5.551115123126e-16

Table 17: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_1$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7738904707096	-0.05869352107821
2	0.740031859657	-0.001584783883836
3	0.7391051286728	-3.346478611543e-05
4	0.7390855520677	-7.009967817329e-07
5	0.7390851419875	-1.468148114192e-08
6	0.7390851333989	-3.074837051642e-10
7	0.739085133219	-6.439848654338e-12
8	0.7390851332152	-1.347810751895e-13
9	0.7390851332152	-2.886579864025e-15

Table 18: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_2$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7661249409486	-0.04552210302764
2	0.7386389507405	0.0007466627787649
3	0.7390955631862	-1.745576513446e-05
4	0.7390848906272	4.059981804083e-07
5	0.7390851388581	-9.444119819513e-09
6	0.7390851330839	2.196836046409e-10
7	0.7390851332182	-5.110134537745e-12
8	0.7390851332151	1.189048859374e-13
9	0.7390851332152	-2.886579864025e-15

Table 19: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_2$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7700391143815	-0.05215567421902
2	0.739268870942	-0.0003075181447841
3	0.7390850173828	1.938584230921e-07
4	0.7390851332929	-1.300705099183e-10
5	0.7390851332151	8.726352973554e-14

Table 20: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_2$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7708121601317	-0.05346710051403
2	0.7393663471048	-0.0004706721698918
3	0.7390857680818	-1.062520624373e-06
4	0.7390851346097	-2.333964577161e-09
5	0.7390851332182	-5.126454816207e-12
6	0.7390851332152	-1.143529715364e-14

Table 21: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_2$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7700362641298	-0.05215083972952
2	0.739224457561	-0.0002331820741358
3	0.7390848283157	5.10283388655e-07
4	0.739085133892	-1.13269038593e-09
5	0.7390851332137	2.514100039264e-12
6	0.7390851332152	-5.662137425588e-15

Table 22: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_2$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7704245259574	-0.05280944850791
2	0.7392947415435	-0.000350819254853
3	0.739085141508	-1.387905956829e-08
4	0.7390851332151	9.303668946359e-14

Table 23: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_2$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7704322968175	-0.05282263132721
2	0.7392957154076	-0.0003524492765886
3	0.7390851476526	-2.416261435378e-08
4	0.7390851332155	-5.334621633324e-13
5	0.7390851332152	0

Table 24: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_2$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.770424537924	-0.05280946880857
2	0.7392942962502	-0.0003500739376863
3	0.7390851382559	-8.436308873705e-09
4	0.739085133215	1.862954235321e-13
5	0.7390851332152	0

Table 25: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_2$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	1.25	-0.9346776376047
1	0.7704284174021	-0.05281605011573
2	0.7392950057628	-0.000351261496335
3	0.7390851429386	-1.627323198417e-08
4	0.7390851332152	0

Table 26: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_2$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.257119293169	1.565677621999
2	22.53947444582	-23.39287281879
3	-30.99422652457	31.90662094839
4	-9.053394629514	8.121568409731
5	4.685261135334	-4.712385653187
6	-6945.1010271	6944.527777468
7	38341.6834777	-38341.7991551
8	19034.21379316	-19034.98114522
9	7150.884295541	-7150.070077882
10	2737.99514241	-2737.898150326
11	-241316.454188	241316.1844619
12	-117428.4738275	117428.0363197
13	1342644.279136	-1342645.265657
14	137484.1659894	-137484.3817655
15	67483.23386388	-67483.48404609
16	32948.7037712	-32947.75452938
17	-11976.01184202	11976.97806559
18	3165.650560765	-3165.174682563
19	-18554.08997212	18555.07779146
20	-3151.795692372	3151.083556782
21	-1259.588667885	1258.606661045
$\vdots$	$\vdots$	$\vdots$
65	-inf	-nan

Table 27: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_3$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-0.8343612978636	1.506012311257
2	5.805555948664	-4.917468806468
3	-4.094260238874	3.514749071697
4	-2.186722545912	1.609007760035
5	5.332613631039	-4.751395646837
6	-24.63122888277	25.50808538407
7	-6.86973223019	7.702589108963
8	12.11032182552	-11.21252219445
9	-9.641423551418	8.664799562396
10	-2.783036042682	1.846631725305
11	-0.1315164094572	1.122880584761
12	1.238595479976	-0.9124711437675
13	0.7651893478627	-0.04393813375748
14	0.7386484393829	0.0007307855689962
15	0.7390953403147	-1.708276309453e-05
16	0.7390848958103	3.97323639989e-07
17	0.7390851387375	-9.242336673765e-09
18	0.7390851330867	2.149898037374e-10
19	0.7390851332181	-5.001110636726e-12
20	0.7390851332151	1.164623952832e-13
21	0.7390851332152	-2.6645352591e-15

Table 28: Tabla método de Newton, con derivada numérica atrás con $x_1 = \alpha_3$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.035091142446	1.54553870303
2	9.885623878347	-10.78130051181
3	-9.503988293682	8.507123792172
4	-1.619690257031	1.570815805572
5	547.8254631633	-547.4522640462
6	263.6127032067	-262.6519467083
7	-99.63335263972	100.2568312034
8	-43.32630971243	44.11875574712
9	-15.90506313331	14.92442451003
10	-3.421227914911	2.460071742935
11	-1.492584761996	1.570716613852
12	331.4481177706	-331.4380251251
13	-192737.6642774	192738.2419028
14	848765.2556329	-848764.86402
15	406377.0242111	-406376.172682
16	-446340.113319	446340.2053321
$\vdots$	$\vdots$	$\vdots$
99	6.960374602119	-6.181037605396
$\vdots$	$\vdots$	$\vdots$

Table 29: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_3$ y $\epsilon = 0.1$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.056358844059	1.548404060515
2	10.68241699027	-10.99048090143
3	-222.6121629659	221.7078016033
4	167.2258143979	-167.9766937703
5	-332.9768565497	333.9763457212
6	-10.90311918378	10.81079574058
7	-5.48485527123	6.182758967422
8	-1.889536427883	1.576166057284
9	30.39703828165	-29.87272532655
10	-167.2914513142	166.5855069167
11	-69.57162399436	70.46918707543
12	55.46040791678	-54.99638038517
13	-416.5543809464	416.2652160548
14	9664.742886196	-9664.384150975
15	4670.79035934	-4671.516629111
16	1896.346752595	-1895.961403455
17	-22051.24996061	22050.33328883
18	-6245.136297137	6246.075704752
19	-1609.807163116	1610.063346601
20	44831.40865389	-44830.77264966
21	19572.53928723	-19571.62500155
22	5688.356276725	-5688.83813781
23	2652.404688095	-2651.783460829
24	1168.244738722	-1167.33482836
25	-811.1225760521	811.9525856547
26	1007.726031428	-1008.474391326
27	400.0429694271	-400.5312286274
28	-2807.424384338	2807.824229908
29	-1343.923227363	1344.70178735
30	-519.6872867417	519.4434719387
⋮	⋮	⋮
93	inf	-nan

Table 30: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_3$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.014153284643	1.542492295202
2	9.384275754316	-10.38345565207
3	-0.5474234793362	1.401291884629
4	2.401110238508	-3.139253424662
5	0.5306404618357	0.3318426569314
6	0.7516088909379	-0.02101765101399
7	0.7390920572636	-1.158818846125e-05
8	0.7390851178569	2.570379864508e-08
9	0.7390851332493	-5.705380612397e-11
10	0.7390851332151	1.266764471097e-13
11	0.7390851332152	-2.22044604925e-16

Table 31: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_3$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.035149660512	1.545546900017
2	9.998514465275	-10.83839323047
3	-13.70569258731	14.12390307332
4	140.3760646546	-140.9200601993
5	63.75051777062	-63.14363578661
6	28.56866757421	-29.52566322644
7	-13.02248069192	13.92025332949
8	11.85518732086	-11.09759728628
9	-20.10055705418	20.41492919924
10	382.4337490403	-381.7666993689
$\vdots$	$\vdots$	$\vdots$
54	116.3716728418	-117.3628751996
55	-18.894014304	19.89302619288
56	1.924237397102	-2.270365615609
57	0.7528418215372	-0.02309300030416
58	0.739126227143	-6.877591600563e-05
59	0.7390851333124	-1.626584422709e-10
60	0.7390851332152	1.110223024625e-15

Table 32: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_3$ y $\epsilon = 10^{-2}$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.035361286759	1.545576529225
2	10.0061457787	-10.8418580494
3	-14.04461423333	14.13703486326
4	3285.479306393	-3284.669285898
5	-4655.430020119	4656.347024309
6	-1326.908259818	1327.311116916
7	14333.23924786	-14332.96555141
8	7027.322259978	-7028.235142433
9	2036.304967864	-2035.4539776
10	701.7770801636	-702.1376492164
11	-9739.016634058	9740.013482904
12	839.614393437	-840.3045036981
13	-2202.08793577	2201.102100753
14	442.7276382103	-443.6997023053
15	83.3597137787	-83.46701525857
16	41.50527114773	-42.29245526088
17	-68.84900012679	69.81382017969
18	-13.5710383298	14.1074070169
19	76.83611856622	-76.70360800928
20	38.31459291787	-37.49809712567
21	14.54238400061	-14.93660225156
22	6.758838127016	-5.869844154478
23	2.732780703552	-3.650374446813
24	0.1206565848145	0.8720732358259
25	0.8990059130749	-0.2766175570338
26	0.7438416139727	-0.007968851886693
27	0.7390902080926	-8.493385376007e-06
28	0.7390851333329	-1.970442697896e-10
29	0.7390851332152	-4.329869796038e-15

Table 33: Tabla método de Newton, con derivada numérica hacia adelante con $x_1 = \alpha_3$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.034939238226	1.545517416708
2	9.993195550611	-10.83594939058
3	-13.4742949092	14.08967811232
4	53.067698814	-54.01066129304
5	12.54793720142	-11.54810709196
6	0.7823747424196	-0.07313331486973
7	0.7394795467498	-0.0006601527159749
8	0.739085158846	-4.289603039442e-08
9	0.7390851332146	9.470202400053e-13
10	0.7390851332152	0

Table 34: Tabla método de Newton, con derivada numérica hacia atrás con $x_1 = \alpha_3$ y $\epsilon = 10^{-4}$

it	x_i	$f(x_i)$
0	3.15	-4.149964658471
1	-1.035150251852	1.54554698284
2	9.999667188239	-10.83891972747
3	-13.75644364682	14.12803571702
4	183.5517615309	-183.3224126262
5	90.65240535123	-91.55120115217
6	27.00302641323	-27.29805826843
7	13.04330565468	-12.15489956577
8	4.712659319653	-4.712388980388
9	-123344835.7854	123344835.359
10	1169009852.928	-1169009853.671
11	-2358459719.225	2358459718.424
12	3529693275.314	-3529693275.973
13	1518197809.8	-1518197808.808
14	174783644.7626	-174783643.7875
15	31688222.28658	-31688222.06662
16	15647939.16596	-15647938.49311
17	-44487515.44906	44487516.44693
18	-2729256.956377	2729256.865868
19	-1361819.669341	1361819.176748
20	9134687.746267	-9134688.004564
21	-260102110.4885	260102109.654
$\vdots$	$\vdots$	$\vdots$
65	0.7390872908645	-3.611069561371e-06
66	0.7390851332162	-1.717848086003e-12
67	0.7390851332152	0

Table 35: Tabla método de Newton, con derivada numérica central con $x_1 = \alpha_3$ y $\epsilon = 10^{-4}$

e) Encuentre el cero de la función usando el método de la secante, correcto hasta 13 cifras significativas, tabulando x_i, x_{i-1} y $f(x_i)$ en cada paso del método para cuando cada par de puntos iniciales son: $x_0 = \alpha_1$, $x_1 = \alpha_2$; $x_0 = \alpha_1$ $x_1 = \alpha_3$; y $x_0 = \alpha_2$, $x_1 = \alpha_3$.

it	x_i	x_{i-1}	$f(x_i)$
1	1.25	-0.75	-0.9346776376047
2	0.4763775920592	1.25	0.4122842601182
3	0.7131714780688	0.4763775920592	0.04311931097567
4	0.74082954591	0.7131714780688	-0.002920593982821
5	0.7390750247705	0.74082954591	1.691757686395e-05
6	0.7390851293252	0.7390750247705	6.510327832387e-09
7	0.7390851332152	0.7390851293252	-1.454392162259e-14

Table 36: Tabla método Secante, con $x_0 = \alpha_1$, $x_1 = \alpha_2$

it	x_i	x_{i-1}	$f(x_i)$
1	3.15	-0.75	-4.149964658471
2	0.2760905718985	3.15	0.6860379120011
3	0.6837849009655	0.2760905718985	0.09140233460119
4	0.7464522133442	0.6837849009655	-0.01234964544888
5	0.7389928946041	0.7464522133442	0.0001543685049259
6	0.7390849837433	0.7389928946041	2.501579206005e-07
7	0.7390851332182	0.7390849837433	-5.095146526912e-12
8	0.7390851332152	0.7390851332182	1.110223024625e-16

Table 37: Tabla método Secante, con $x_0 = \alpha_1$, $x_1 = \alpha_3$

it	x_i	x_{i-1}	$f(x_i)$
1	3.15	1.25	-4.149964658471
2	0.6976737224628	3.15	0.06866502310845
3	0.7375893000855	0.6976737224628	0.002502617086188
4	0.7390991213355	0.7375893000855	-2.341075875145e-05
5	0.7390851285915	0.7390991213355	7.738282126191e-09
6	0.7390851332151	0.7390851285915	2.386979502944e-14

Table 38: Tabla método Secante, con $x_0 = \alpha_2$, $x_1 = \alpha_3$

f) Comente los resultados obtenidos.

Para el método de bisección para $[\alpha_1, \alpha_2]$ convergió a la iteración 36, y para $[\alpha_1, \alpha_3]$ también convergió a la iteración 38. Ambas ejecuciones tardaron iteraciones similares y ambas garantizaron la convergencia debido a que se cumplía la condición del cambio de signo.

Para el método de punto fijo, con las 3 condiciones iniciales las 3 convergieron, tardando 56, 65 y 65 iteraciones respectivamente, fue mas lento en general que bisección.

Para el método de Newton con la derivada analítica, para α_1 no logro converger, se encontró oscilando entre positivo y negativo "incrementando" en magnitud de la raíz. Para α_2 si converge, en tan solo 5 iteraciones, encontrando la raíz "exacta", siendo una solución muy apropiada para este problema. Para α_3 si converge, en 15 iteraciones.

Para el método de Newton con derivadas numéricas α_1 y ϵ_1 el mejor resultado lo dio la derivada hacia adelante con 14 iteraciones, seguido con la derivada hacia atrás con 21 y la derivada central con 51 iteraciones. Todas convergieron.

Para el método de Newton con α_2 y ϵ_1 el mejor resultado lo dio la derivada central con 6 iteraciones y en empate con 10 iteraciones las derivadas hacia adelante y hacia atrás. Todas convergieron.

Para Newton con derivadas numéricas con α_3 y ϵ_1 solo la derivada hacia atrás convergió con 22 iteraciones. La derivada hacia adelante divergió de modo oscilante en signo y de manera incremental en magnitud. Del mismo modo divergió la derivada numérica central.

Para las derivadas numéricas con α_1 y ϵ_2 todas las derivadas convergieron con iteraciones similares(11 en el mejor de los casos con la derivada hacia atrás y 15 en el peor de los casos con la derivada numérica central).

Para Newton con derivadas numéricas con α_2 y ϵ_2 obtuvo uno de los mejores resultados en general, con 7 iteraciones en el peor de los casos y 5 en el mejor. Todo convergió.

Para Newton con derivadas numéricas con α_3 y ϵ_2 divergió con la derivada hacia adelante(de manera oscilante en signo e incremental en magnitud). Convergió en los otros 2 casos, siendo el mejor la derivada hacia atrás.

Para las derivadas numéricas con α_1 y ϵ_3 todo convergió con iteraciones similares al rededor de las 15 iteraciones.

Para α_2 y ϵ_3 tuvo el mejor desempeño de las derivadas numéricas con 6 iteraciones con la derivada central y 7 iteraciones para los otros 2 casos.

Para α_3 y ϵ_3 logro converger en todos los casos, situación que no se dio con épsilon menos precisos, con un comportamiento inusual con la derivada central(parecía que iba a diverger por tener un comportamiento oscilante en signo y creciente , pero logró estabilizarse en la iteración 55).

En general de las derivadas numéricas con Newton, la derivada que se comporto mas estable fue la derivada hacia atrás, α_3 fue el que tuvo peor desempeño en tema de convergencia y α_3 fue el mejor desempeño en tema de convergencia y de menor cantidad de iteraciones.

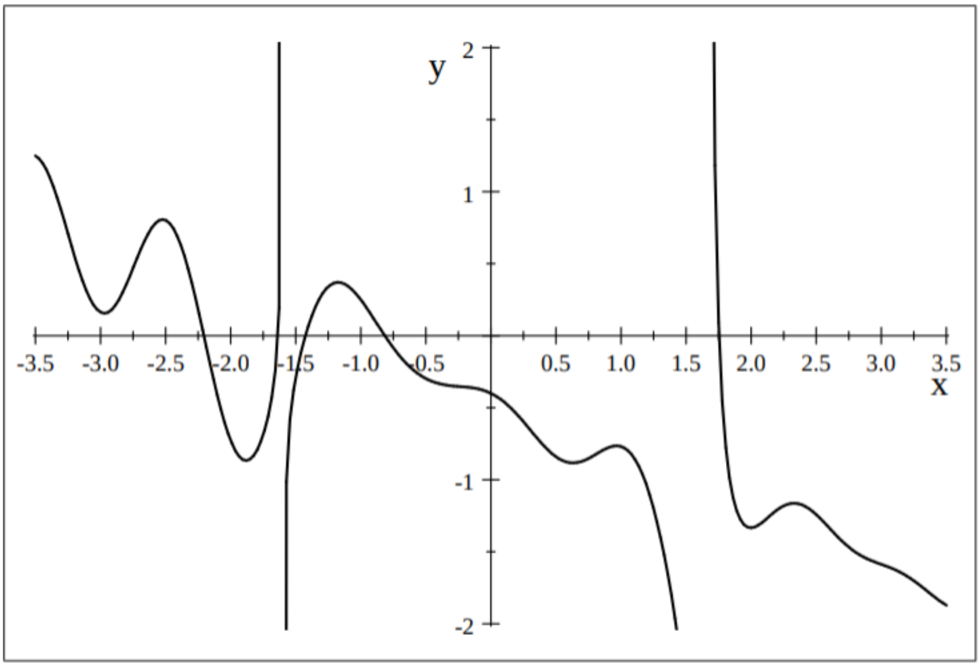
Para el método de la secante en general fue el mas mejor en tema de convergencia(todos los casos convergieron) y en tema de iteraciones (con 6 iteraciones en el mejor de los casos $[\alpha_2, \alpha_3]$ y 8 iteraciones en el peor de los casos $[\alpha_1, \alpha_3]$).

Ejercicio 3

3. Considere la función

$$f(x) = \frac{(\cos(2\pi x) - 6)x^3 - (2\cos(2\pi x) + 7)x^2 - 2(\cos(2\pi x) - 10)x - 3\cos(2\pi x) + 19}{15x^2 - x - 40}$$

cuya gráfica se muestra a continuación:



a) A partir de esta gráfica estime (la estimación debe ser tan solo de la información de la gráfica y con dos cifras significativas) el valor de las raíces y ordénelas en forma ascendente: $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5$.

$$\alpha_i = \begin{array}{ccccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \\ \text{-2.2} & \text{-1.6} & \text{-1.4} & \text{-0.76} & 1.7 \end{array}$$

Calcule $\beta_i = \lfloor \alpha \rfloor$ (la función Floor de la estimación de la raíz iésima) y $\gamma_i = \lceil \alpha \rceil$ (la función Ceiling de la estimación de la raíz iésima) para $i = 1, 2, 3, 4, 5$.

$$\beta_i = \begin{array}{ccccc} \lfloor \alpha_1 \rfloor & \lfloor \alpha_2 \rfloor & \lfloor \alpha_3 \rfloor & \lfloor \alpha_4 \rfloor & \lfloor \alpha_5 \rfloor \\ \text{-3} & \text{-2} & \text{-2} & \text{-1} & 1 \end{array}$$

$$\gamma_i = \begin{array}{ccccc} \lceil\alpha_1\rceil & \lceil\alpha_2\rceil & \lceil\alpha_3\rceil & \lceil\alpha_4\rceil & \lceil\alpha_5\rceil \\ -2 & -1 & -1 & 0 & 2 \end{array}$$

En los casos en que se pueda, utilizar el método de bisección directamente para encontrar una aproximación de las raíces con 5 cifras significativas utilizando como intervalo inicial $[\beta_i, \gamma_i]$.

it	a	b	c	$f(c)$
0	-3	-2	-2.5	0.80222222222
1	-2.5	-2	-2.25	0.18085106383
2	-2.25	-2	-2.125	-0.32895925119
3	-2.25	-2.125	-2.1875	-0.076045641319
4	-2.25	-2.1875	-2.21875	0.053498915288
5	-2.21875	-2.1875	-2.203125	-0.011198508772
6	-2.21875	-2.203125	-2.2109375	0.021193949618
7	-2.2109375	-2.203125	-2.20703125	0.0050055321585
8	-2.20703125	-2.203125	-2.205078125	-0.0030949264969
9	-2.20703125	-2.205078125	-2.2060546875	0.00095574219219
10	-2.2060546875	-2.205078125	-2.2055664062	-0.0010694884246
11	-2.2060546875	-2.2055664062	-2.2058105469	-5.6846420121e-05
12	-2.2060546875	-2.2058105469	-2.2059326172	0.00044945465556
13	-2.2059326172	-2.2058105469	-2.205871582	0.00019630579817
14	-2.205871582	-2.2058105469	-2.2058410645	6.9730107641e-05

Table 39: Tabla Método Bisección para $[\beta_1, \gamma_1]$

it	a	b	c	$f(c)$
0	-2	-1	-1.5	-0.28947368421
1	-1.5	-1	-1.25	0.34081632653
2	-1.5	-1.25	-1.375	0.14105260233
3	-1.5	-1.375	-1.4375	-0.037793666722
4	-1.4375	-1.375	-1.40625	0.058854977196
5	-1.4375	-1.40625	-1.421875	0.012445959988
6	-1.4375	-1.421875	-1.4296875	-0.012175377784
7	-1.4296875	-1.421875	-1.42578125	0.00025709215321
8	-1.4296875	-1.42578125	-1.427734375	-0.0059283661149
9	-1.427734375	-1.42578125	-1.4267578125	-0.002827985111
10	-1.4267578125	-1.42578125	-1.4262695312	-0.0012835386646
11	-1.4262695312	-1.42578125	-1.4260253906	-0.000512746938
12	-1.4260253906	-1.42578125	-1.4259033203	-0.00012770839196
13	-1.4259033203	-1.42578125	-1.4258422852	6.4721620897e-05

Table 40: Tabla Método Bisección para $[\beta_2, \gamma_2]$

Para el caso del intervalo $[\lfloor\alpha_2\rfloor, \lceil\alpha_2\rceil]$, los redondeos hacia arriba y hacia abajo son exactamente iguales que $[\lfloor\alpha_3\rfloor, \lceil\alpha_3\rceil]$, por lo que si bien el método “funciona”(converge a una raíz) no le es posible tomar la raíz α_2.

it	a	b	c	$f(c)$
0	-2	-1	-1.5	-0.28947368421
1	-1.5	-1	-1.25	0.34081632653
2	-1.5	-1.25	-1.375	0.14105260233
3	-1.5	-1.375	-1.4375	-0.037793666722
4	-1.4375	-1.375	-1.40625	0.058854977196
5	-1.4375	-1.40625	-1.421875	0.012445959988
6	-1.4375	-1.421875	-1.4296875	-0.012175377784
7	-1.4296875	-1.421875	-1.42578125	0.00025709215321
8	-1.4296875	-1.42578125	-1.427734375	-0.0059283661149
9	-1.427734375	-1.42578125	-1.4267578125	-0.002827985111
10	-1.4267578125	-1.42578125	-1.4262695312	-0.0012835386646
11	-1.4262695312	-1.42578125	-1.4260253906	-0.000512746938
12	-1.4260253906	-1.42578125	-1.4259033203	-0.00012770839196
13	-1.4259033203	-1.42578125	-1.4258422852	6.4721620897e-05

Table 41: Tabla Método Bisección para $[\beta_3, \gamma_3]$

it	a	b	c	$f(c)$
0	-1	0	-0.5	-0.2972027972
1	-1	-0.5	-0.75	-0.084178498986
2	-1	-0.75	-0.875	0.082494183321
3	-0.875	-0.75	-0.8125	-0.003771670383
4	-0.875	-0.8125	-0.84375	0.038917494974
5	-0.84375	-0.8125	-0.828125	0.017421756859
6	-0.828125	-0.8125	-0.8203125	0.006782591065
7	-0.8203125	-0.8125	-0.81640625	0.0014942871157
8	-0.81640625	-0.8125	-0.814453125	-0.0011415535443
9	-0.81640625	-0.814453125	-0.8154296875	0.00017565982032
10	-0.8154296875	-0.814453125	-0.81494140625	-0.00048312467133
11	-0.8154296875	-0.81494140625	-0.81518554688	-0.00015377674459
12	-0.8154296875	-0.81518554688	-0.81530761719	1.0930474775e-05
13	-0.81530761719	-0.81518554688	-0.81524658203	-7.1425902764e-05
14	-0.81530761719	-0.81524658203	-0.81527709961	-3.0248405699e-05
15	-0.81530761719	-0.81527709961	-0.8152923584	-9.6591383556e-06

Table 42: Tabla Método Bisección para $[\beta_4, \gamma_4]$

it	a	b	c	f(c)

Table 43: Tabla Método Bisección para $[\beta_5, \gamma_5]$

Para el caso del intervalo $[\lfloor \alpha_5 \rfloor, \lceil \alpha_5 \rceil]$ en la primera iteración de bisección, toma como punto central a 1.5, valor donde en la función se indetermina, por lo que no es posible aplicar el método de la bisección.

Notará que el método de bisección falla en algunos casos, explique el porqué de esta situación. En estos casos, alterar tan solo un extremo del intervalo inicial al intercambiar el valor de β_i o de γ_i por $\alpha_i \pm 0.05$ (i.e. según sea el caso se intercambia ya sea β_i por $\alpha_i - 0.05$ o γ_i por $\alpha_i + 0.05$). Después de este proceso se tendrán estimaciones a las raíces b_1, b_2, b_3, b_4 y b_5 correctas a 5 cifras.

it	a	b	c	$f(c)$
0	-2	-1.6	-1.8	-0.79876990751
1	-1.8	-1.6	-1.7	-0.50916595582
2	-1.7	-1.6	-1.65	-0.17696082432
3	-1.65	-1.6	-1.625	0.28642510539
4	-1.65	-1.625	-1.6375	-0.010308427494
5	-1.6375	-1.625	-1.63125	0.11211302339
6	-1.6375	-1.63125	-1.634375	0.046177460444
7	-1.6375	-1.634375	-1.6359375	0.016906434128
8	-1.6375	-1.6359375	-1.63671875	0.0030582700191
9	-1.6375	-1.63671875	-1.637109375	-0.0036833763396
10	-1.637109375	-1.63671875	-1.6369140625	-0.0003273570324
11	-1.6369140625	-1.63671875	-1.6368164063	0.0013617263081
12	-1.6369140625	-1.6368164063	-1.6368652344	0.0005162557679
13	-1.6369140625	-1.6368652344	-1.6368896484	9.4217607685e-05
14	-1.6369140625	-1.6368896484	-1.6369018555	-0.00011662759533
15	-1.6369018555	-1.6368896484	-1.636895752	-1.1219471693e-05
16	-1.636895752	-1.6368896484	-1.6368927002	4.1495447638e-05
17	-1.636895752	-1.6368927002	-1.6368942261	1.5137082998e-05
18	-1.636895752	-1.6368942261	-1.636894989	1.9585794183e-06

Table 44: Tabla Método Bisección para $[\beta_2, \alpha_2 + 0.05]$

Tabla Método Bisección para $[\lfloor\alpha_5\rfloor, \alpha_5 + 0.05]$, no converge ya que el intervalo $[\lfloor\alpha_5\rfloor, \alpha_5 + 0.05]$ no es continuo.

Tabla Método Bisección para $[\alpha_5 - 0.05, \lceil\alpha_5\rceil]$, no converge ya que el intervalo $[\alpha_5 - 0.05, \lceil\alpha_5\rceil]$ no es continuo.

b) En los casos en que se pueda, estime el valor de las raíces con 14 cifras significativas utilizando el método de Newton con la derivada analítica de $f(x)$. Mostrar en una sola tabla los resultados de cada iteración del método para las siguientes condiciones iniciales:

i Los valores iniciales de β_i

ii Los valores iniciales de γ_i

iii Los valores iniciales de α_i

iv Los valores iniciales de b_i

Al final para cada raíz se debe reportar una tabla de este tipo:

	Condición Inicial								
	β_i			γ_i		α_i		b_i	
Iteración	x_k	f_k	$\|x_{k+1} - x_k\|$	x_k	$\dots$	x_k	$\dots$	x_k	$\dots$
0	$x_0 = \beta_i$	f_0	$\|x_1 - x_0\|$	$x_0 = \gamma_i$	$\dots$	$x_0 = \alpha_i$	$\dots$	$x_0 = b_i$	$\dots$
1	$x_1 = \beta_i$	f_1	$\|x_2 - x_1\|$	$x_1 = \gamma_i$	$\dots$	$x_1 = \alpha_i$	$\dots$	$x_1 = b_i$	$\dots$
2	$x_2 = \beta_i$	f_2	$\|x_2 - x_1\|$	$x_2 = \gamma_i$	$\dots$	$x_2 = \alpha_i$	$\dots$	$x_2 = b_i$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$

Tablas Ejercicio 3 Método de Newton

	Condición Inicial											
	β_i			γ_i			α_i			b_i		
Iteración	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$
0	-3	0.16326530612245	0.97571483363588	-2	-0.72727272727273	0.71329162455228	-2.2	-0.024168783607154	0.0090423720435804	-2.25	0.18085106382979	0.065515195238621
1	-2.0242851663641	-0.66676253328831	0.52217349549069	-2.7132916245523	0.55274120045235	0.45469846801744	-2.2090423720436	0.013342439989964	0.0049391182900838	-2.1844848047614	-0.088542855135622	0.03390731758124
2	-2.5464586618548	0.80222617732085	2.5196589724163	-3.1679900925697	0.45940160249182	0.23637598607527	-2.2041032537535	-0.0071393590015309	0.0026575273339344	-2.2183921223426	0.052021701412609	0.019089877816773
3	-0.026799689438518	-0.38918886015487	1.0585152348081	-2.9316141064945	0.16822480321341	0.29521666372275	-2.2067607810874	0.0038839952626185	0.0014413487082718	-2.1993022445258	-0.027065303040078	0.010135269932103
4	-1.0853149242466	0.33506182478278	0.3796447477665	-3.2268307702172	0.63458082672942	0.27861178621412	-2.2053194323792	-0.0020939348783613	0.00077833023348139	-2.2094375144579	0.014979903590862	0.0055429734943662
5	-0.70567017648012	-0.13563207958057	0.13907714138005	-2.9482189840031	0.16005910623524	0.39602884670568	-2.2060977626126	0.0011343932774652	0.00042128679205922	-2.2038945409636	-0.0080053341668537	0.0029806137565438
6	-0.84474731786017	0.040297992664817	0.033680186954533	-3.3442478307088	0.98808696694456	0.4501002436761	-2.2056764758206	-0.00061293736764833	0.00022773965544687	-2.2068751547201	0.0043582704164756	0.0016171458432996
7	-0.81106713090564	-0.0056974820167015	0.0049099498575538	-2.8941475870327	0.20186054047316	0.22473767238185	-2.205904215476	0.00033165639570948	0.00012319640937752	-2.2052580088768	-0.0023487273341605	0.00087310028327803
8	-0.81597708076319	0.00091461784870433	0.00078406855342372	-3.1188852594145	0.33387621530144	0.22099153583015	-2.2057810190667	-0.00017931851384638	6.6618645388594e-05	-2.2061311091601	0.0012726946242791	0.00047263078130699
9	-0.81519301220977	-0.00014370454199824	0.00012329380440523	-2.8978937235844	0.19760019758211	0.22749805918447	-2.205847637712	9.699363900026e-05	3.6031381716928e-05	-2.2056584783788	-0.00068758716438348	0.00025548138828624
10	-0.81531630601418	2.2654992124739e-05	1.9434726725165e-05	-3.1253917827688	0.34901967870047	0.22164707356429	-2.2058116063303	-5.2452146020817e-05	1.9485823692822e-05	-2.2059139597671	0.00037207166689893	0.00013820745079629
11	-0.81529687128745	-3.5696584452537e-06	3.0623163000953e-06	-2.9037447092045	0.191330694309	0.23310365467374	-2.205831092154	2.8368496836858e-05	1.0538582014163e-05	-2.2057757523163	-0.00020116342539448	7.4734700808321e-05
12	-0.81529993360375	5.6250418210625e-07	4.8255602447256e-07	-3.1368483638783	0.37687400523349	0.22397599501973	-2.205820553572	-1.53419545496e-05	5.6994344954653e-06	-2.2058504870171	0.00010881152711189	4.0421382021094e-05
13	-0.81529945104773	-8.8637831059566e-08	7.603986318383e-08	-2.9128723688585	0.18251095011072	0.24573646553987	-2.2058262530065	8.297372308756e-06	3.0823988179662e-06	-2.2058100656351	-5.884244623671e-05	2.185984054659e-05
14	-0.81529952708759	1.3967329811352e-08	1.1982172254399e-08	-3.1586088343984	0.43358610295064	0.23181923200922	-2.2058231706077	-4.4873719670809e-06	1.6670239872951e-06	-2.2058319254756	3.1824826983966e-05	1.1822559104591e-05
15	-0.81529951510542	-2.2009365731632e-09	1.8881204377763e-09	-2.9267896023892	0.1713984856173	0.27830872249497	-2.2058248376317	2.4268788813247e-06	9.0156502441374e-07	-2.2058201029165	-1.7211122340655e-05	6.3938212360171e-06
16	-0.81529951699354	3.4681800436844e-10	2.9752522667792e-10	-3.2050983248842	0.56781721404913	0.26035655429362	-2.2058239360666	-1.3125071541131e-06	4.875858330422e-07	-2.2058264967378	9.3082864849292e-06	3.4579433361159e-06
17	-0.81529951669601	-5.4650667696885e-11	4.6883275039988e-11	-2.9447417705905	0.16141030592037	0.36706301396955	-2.2058244236525	7.0983365046149e-07	2.6369733552301e-07	-2.2058230387944	-5.0340888372948e-06	1.8701253581277e-06
18	-0.8152995167429	8.6118097814453e-12	7.3878680950656e-12	-3.3118047845601	0.89663857968154	0.38621204498407	-2.2058241599551	-3.8389351030681e-07	1.4261330960608e-07	-2.2058249089198	2.7225577543315e-06	1.0114071486989e-06
19	-0.81529951673551	-1.3571672267219e-12	1.1642908859244e-12	-2.925592739576	0.17224057948187	0.27465463932124	-2.2058243025684	2.0761816286327e-07	7.7128442743657e-08	-2.2058238975126	-1.4724161546732e-06	5.4699076246223e-07
20	-0.81529951673667	2.1391193116543e-13	1.8351986597054e-13	-3.2002473788973	0.55317606036803	0.25670591768086	-2.20582422544	-1.1228447494032e-07	4.1712763909629e-08	-2.2058244445034	7.9631617064669e-07	2.9582486993363e-07
21	-0.81529951673649	-3.3685960734972e-14	0.81529951673649	-2.9435414612164	0.16192124173181	0.35830361925392	-2.2058242671528	6.0725932230084e-08	2.2559186074744e-08	-2.2058241486785	-4.3066511177008e-07	1.5998858060584e-07
22				-3.3018450804703	0.867084106982	0.36979634609668	-2.2058242445936	-3.2841925929321e-08	1.2200506382953e-08	-2.2058243086671	2.3291329798473e-07	8.6525377085422e-08
23				-2.9320487343736	0.1679563678331	0.29693160277174	-2.2058242567941	1.7761640218958e-08	6.5983036812156e-09	-2.2058242221417	-1.2596463888263e-07	4.6794833341579e-08
24				-3.2289803371454	0.64126350945839	0.28059018172724	-2.2058242501958	-9.6058875674687e-09	3.5685085997272e-09	-2.2058242689366	6.8124470442178e-08	2.5307682172127e-08
25				-2.9483901554181	0.15999755833741	0.39761055922781	-2.2058242537643	5.1950766689704e-09	1.9299282172369e-09	-2.2058242436289	-3.6843218208143e-08	1.3686953792558e-08
26				-3.346000714646	0.992776495193	0.45409646278662	-2.2058242518344	-2.8096107374201e-09	1.0437473108027e-09	-2.2058242573158	1.9925627771767e-08	7.4022072915625e-09
27				-2.8919042518593	0.20450224458605	0.22335909571843	-2.2058242528781	1.5194985848901e-09	5.644813505512e-10	-2.2058242499136	-1.0776219034432e-08	4.003277265241e-09
28				-3.1152633475778	0.32566827144848	0.22086319382295	-2.2058242523136	-8.2177894610314e-10	3.0528424233012e-10	-2.2058242539169	5.8280173199324e-09	2.1650605752654e-09
29				-2.8944001537548	0.2015673439703	0.22490532040858	-2.2058242526189	4.4443713236676e-10	1.6510481870569e-10	-2.2058242517518	-3.1519191235324e-09	1.1709122560433e-09
30				-3.1193054741634	0.33483889459572	0.22101778679786	-2.2058242524538	-2.4036135861112e-10	8.9292129246132e-11	-2.2058242529227	1.7046280577896e-09	6.3325522603463e-10
31				-2.8982876873655	0.19716324105586	0.22782382813551	-2.2058242525431	1.2999198156202e-10	4.8291148857516e-11	-2.2058242522895	-9.218999826473e-10	3.424784900119e-10
32				-3.126111515501	0.35072557189639	0.22175102429225	-2.2058242524948	-7.0304203745086e-11	2.6117330520492e-11	-2.205824252632	4.9858454902347e-10	1.8522028355505e-10
33				-2.9043604912088	0.19069853545895	0.23379583447523	-2.2058242525209	3.8022228826815e-11	1.4125145497701e-11	-2.2058242524468	-2.6964675925976e-10	1.0017142670904e-10
34				-3.138156325684	0.3801465140816	0.22432800063473	-2.2058242525068	-2.0564018715395e-11	7.6392225878408e-12	-2.2058242525469	1.4583065644904e-10	5.4174886798819e-11
35				-2.9138283250493	0.18165637947944	0.24739032550854	-2.2058242525144	1.1121205626291e-11	4.1313619192351e-12	-2.2058242524927	-7.8868498226328e-11	2.9298785619858e-11
36				-3.1612186505578	0.4406914061182	0.23302100750039	-2.2058242525103	-6.0152578984607e-12	2.2346569039655e-12	-2.205824252522	4.2652573111291e-11	1.584510300745e-11
37				-2.9281976430574	0.17043559429968	0.28286649467676	-2.2058242525125	3.2539109699824e-12	1.2088108292119e-12	-2.2058242525062	-2.3067524167765e-11	8.5695894824767e-12
38				-3.2110641377342	0.58597112480418	0.26505652889644	-2.2058242525113	-1.7600658946605e-12	6.5369931689929e-13	-2.2058242525148	1.2476438193094e-11	4.6349590832051e-12
39				-2.9460076088378	0.16089618838808	0.37695126765822	-2.205824252512	9.5120688651318e-13	3.5349501104065e-13	-2.2058242525101	-6.7480970822054e-12	2.5068835896036e-12
40				-3.322958876496	0.92899104077699	0.40622538548714	-2.2058242525116	-5.1497626067788e-13	1.9140244944538e-13	-2.2058242525126	3.6492544392644e-12	1.3558043576722e-12
41				-2.9167334910088	0.17914103881414	0.25287017448865	-2.2058242525118	2.7894126803616e-13	1.0347278589506e-13	-2.2058242525113	-1.9734867562709e-12	7.3319128546245e-13
42				-3.1696036654975	0.46391637389894	0.23722457190512	-2.2058242525117	-1.5012148591019e-13	5.5955240441108e-14	-2.205824252512	1.0672052639917e-12	3.9657166439611e-13
43				-2.9323790935924	0.16775427263676	0.29825851836734	-2.2058242525118	8.2077180931386e-14	2.2058242525118	-2.2058242525116	-5.7797276855145e-13	2.1449508835758e-13
44				-3.2306376119597	0.64642269186264	0.28213735126225				-2.2058242525118	3.1215577298871e-13	1.1590728377087e-13
45				-2.9485002606975	0.15995821484566	0.39863649995392				-2.2058242525117	-1.6859642331541e-13	6.2616578588859e-14
46				-3.3471367606514	0.99580027904069	0.45672107671518				-2.2058242525118	9.1466083547153e-14	2.2058242525118
47				-2.8904156839362	0.20629211661056	0.2225499504909						
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
95				-2.9292403692158	0.16974194880682	0.28643364566179						
96				-3.2156740148776	0.60009805657412	0.26884823632391						
97				-2.9468257785537	0.16057741823464	0.3837279689008						
98				-3.3305537474545	0.95050333946367	0.42095512953249						
99				-2.909598617922	0.18553752668293	0.090401382078011						

Figure 1: Método de Newton, raíz 1

	Condición Inicial											
	β_i			γ_i			α_i			b_i		
Iteración	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$
0	-2	-0.72727272727273	0.71329162455228	-1	0.25	0.22099687473515	-1.6	1.3371429661696e+14		-1.65	-0.17696082432197	0.0058975117531963
1	-2.7132916245523	0.55274120045235	0.45469846801744	-0.77900312526485	-0.047860495197067	0.042955240382728	-1.6	1.3371429661696e+14		-1.6558975117532	-0.23518714516854	0.0099353238784918
2	-3.1679900925697	0.45940160249182	0.23637598607527	-0.82195836564758	0.0090171965639797	0.0076835774562825	-1.6	1.3371429661696e+14		-1.6658328356317	-0.31616799086767	0.019099780498072
3	-2.9316141064945	0.16822480321341	0.29521666372275	-0.81427478819129	-0.0013819449337619	0.0011868137702213	-1.6	1.3371429661696e+14		-1.6849326161298	-0.4353069770249	0.047587580079906
4	-3.2268307702172	0.63458082672942	0.27861178621412	-0.81546160196151	0.0002187308432859	0.0001876110315101	-1.6	1.3371429661696e+14		-1.7325201962097	-0.63379259300086	0.2474002323978
5	-2.9482189840031	0.16005910623524	0.39602884670568	-0.81527399093	-3.4443023196028e-05	2.9548475506624e-05	-1.6	1.3371429661696e+14		-1.9799204286075	-0.76971885264923	0.94921719710611
6	-3.3442478307088	0.98808696694456	0.4501002436761	-0.81530353940551	5.4280414035977e-06	4.6565416748079e-06	-1.6	1.3371429661696e+14		-2.9291376257136	0.16980956044277	0.28607461317107
7	-2.8941475870327	0.20186054047316	0.22473767238185	-0.81529888286384	-8.553222569661e-07	7.3375696818534e-07	-1.6	1.3371429661696e+14		-3.2152122388847	0.5986793795423	0.26846209235314
8	-3.1188852594145	0.33387621530144	0.22099153583015	-0.8152996166208	1.3477989171812e-07	1.1562379875762e-07	-1.6	1.3371429661696e+14		-2.9467501465315	0.16060643934744	0.38308817493813
9	-2.8978937235844	0.19760019758211	0.22749805918447	-0.81529950099701	-2.1238265981698e-08	1.8219700748112e-08	-1.6	1.3371429661696e+14		-3.3298383214696	0.9484961619563	0.41952661534577
10	-3.1253917827688	0.34901967870047	0.22164707356429	-0.81529951921671	3.3466726007906e-09	2.8710146482425e-09	-1.6	1.3371429661696e+14		-2.9103117061239	0.18486514299987	0.24164089442934
11	-2.9037447092045	0.191330694309	0.23310365467374	-0.81529951634569	-5.2736028206458e-10	4.5240733381746e-10	-1.6	1.3371429661696e+14		-3.1519526005532	0.41574552889095	0.22899447031549
12	-3.1368483638783	0.37687400523349	0.22397599501973	-0.8152995167981	8.3100103278507e-11	7.1289196768021e-11	-1.6	1.3371429661696e+14		-2.9229581302377	0.17417032815168	0.2672652327206
13	-2.9128723688585	0.18251095011072	0.24573646553987	-0.81529951672681	-1.3094717978302e-11	1.1233569630065e-11	-1.6	1.3371429661696e+14		-3.1902233629583	0.52331455278035	0.24964742236008
14	-3.1586088343984	0.43358610295064	0.23181923200922	-0.81529951673804	2.0633563024947e-12	1.7701395904623e-12	-1.6	1.3371429661696e+14		-2.9405759405982	0.16328099596527	0.33892707784318
15	-2.9267896023892	0.1713984856173	0.27830872249497	-0.81529951673627	-3.2506344059409e-13	2.7888802378584e-13	-1.6	1.3371429661696e+14		-3.2795030184414	0.79901625065615	0.33723857303493
16	-3.2050983248842	0.56781721404913	0.26035655429362	-0.81529951673655	5.131940588505e-14	2.129442253854	-1.6	1.3371429661696e+14		-2.9422644454065	0.16248980697463	0.34958694087075
17	-2.9447417705905	0.16141030592037	0.36706301396955				-1.6	1.3371429661696e+14		-3.2918513862772	0.83689852440946	0.3545531923623
18	-3.3118047845601	0.89663857968154	0.38621204498407				-1.6	1.3371429661696e+14		-2.9372981939149	0.16494440690266	0.32068531263785
19	-2.925592739576	0.17224057948187	0.27465463932124				-1.6	1.3371429661696e+14		-3.2579835065528	0.73200000721301	0.31056711187121
20	-3.2002473788973	0.55317606036803	0.25670591768086				-1.6	1.3371429661696e+14		-2.9474163946816	0.16035391924882	0.3888211766715
21	-2.9435414612164	0.16192124173181	0.35830361925392				-1.6	1.3371429661696e+14		-3.3362375713531	0.9663000010463	0.43262465455214
22	-3.3018450804703	0.867084106982	0.36979634609668				-1.6	1.3371429661696e+14		-2.9036129168009	0.19146668170848	0.23295820241042
23	-2.9320487343736	0.1679563678331	0.29693160277174				-1.6	1.3371429661696e+14		-3.1365711192114	0.37618271192023	0.22390352735963
24	-3.2289803371454	0.64126350945839	0.28059018172724				-1.6	1.3371429661696e+14		-2.9126675918517	0.18269573247403	0.2453912910149
25	-2.9483901554181	0.15999755833741	0.39761055922781				-1.6	1.3371429661696e+14		-3.1580588828666	0.43209661292547	0.23157263838269
26	-3.346000714646	0.992776495193	0.45409646278662				-1.6	1.3371429661696e+14		-2.9264862444839	0.17160987358854	0.27736396260377
27	-2.8919042518593	0.20450224458605	0.22335909571843				-1.6	1.3371429661696e+14		-3.2038502070877	0.56403929410995	0.25940264052823
28	-3.1152633475778	0.32566827144848	0.22086319382295				-1.6	1.3371429661696e+14		-2.9444475665595	0.16153343178642	0.36486256844118
29	-2.8944001537548	0.2015673439703	0.22490532040858				-1.6	1.3371429661696e+14		-3.3093101350007	0.8892908309117	0.38197816596498
30	-3.1193054741634	0.33483889459572	0.22101778679786				-1.6	1.3371429661696e+14		-2.9273319690357	0.17102402465323	0.28003022233046
31	-2.8982876873655	0.19716324105586	0.22782382813551				-1.6	1.3371429661696e+14		-3.2073621913661	0.57468792940245	0.26211269070262
32	-3.126111515501	0.35072557189639	0.22175102429225				-1.6	1.3371429661696e+14		-2.9452495006635	0.16120104183224	0.37094559135469
33	-2.9043604912088	0.19069853545895	0.23379583447523				-1.6	1.3371429661696e+14		-3.3161950920182	0.90947365220277	0.39387250540128
34	-3.138156325684	0.3801465140816	0.22432800063473				-1.6	1.3371429661696e+14		-2.9223225866169	0.17465143356588	0.26560807926284
35	-2.9138283250493	0.18165637947944	0.24739032550854				-1.6	1.3371429661696e+14		-3.1879306658798	0.51656697056994	0.24812529698654
36	-3.1612186505578	0.4406914061182	0.23302100750039				-1.6	1.3371429661696e+14		-2.9398053688932	0.163656941017	0.33436321410205
37	-2.9281976430574	0.17043559429968	0.28286649467676				-1.6	1.3371429661696e+14		-3.2741685829953	0.78249578091744	0.33023371257021
38	-3.2110641377342	0.58597112480418	0.26505652889644				-1.6	1.3371429661696e+14		-2.9439348704251	0.1617512699046	0.36111169265847
39	-2.9460076088378	0.16089618838808	0.37695126765822				-1.6	1.3371429661696e+14		-3.3050465630835	0.87664645733014	0.374933356891
40	-3.322958876496	0.92899104077699	0.40622538548714				-1.6	1.3371429661696e+14		-2.9301132061925	0.16917406581943	0.28955255216569
41	-2.9167334910088	0.17914103881414	0.25287017448865				-1.6	1.3371429661696e+14		-3.2196657583582	0.61239209331239	0.2722453011333
42	-3.1696036654975	0.46391637389894	0.23722457190512				-1.6	1.3371429661696e+14		-2.9474204572249	0.16035240113429	0.38885681627267
43	-2.9323790935924	0.16775427263676	0.29825851836734				-1.6	1.3371429661696e+14		-3.3362772734976	0.96640938492409	0.43270822584222
44	-3.2306376119597	0.64642269186264	0.28213735126225				-1.6	1.3371429661696e+14		-2.9035690476554	0.19151200113091	0.23290999497879
45	-2.9485002606975	0.15995821484566	0.39863649995392				-1.6	1.3371429661696e+14		-3.1364790426342	0.37595330853764	0.2238796272257
46	-3.3471367606514	0.99580027904069	0.45672107671518				-1.6	1.3371429661696e+14		-2.9125994154085	0.18275738682245	0.24527707313081
47	-2.8904156839362	0.20629211661056	0.2225499504909				-1.6	1.3371429661696e+14		-3.1578764885393	0.43160321944295	0.23149137028573
48	-3.1129656344271	0.32054543820055	0.22087650852779				-1.6	1.3371429661696e+14		-2.9263851182536	0.17168065065397	0.27705186368886
49	-2.8920891258993	0.20428200186917	0.22346535722028				-1.6	1.3371429661696e+14		-3.2034369819424	0.5627901246286	0.25908905446481
50	-3.1155544831196	0.32632206461879	0.22086686917679				-1.6	1.3371429661696e+14		-2.9443479274776	0.16157544172676	0.36412535905561
51	-2.8946876139428	0.20123468459493	0.22509927581738				-1.6	1.3371429661696e+14		-3.3084732865332	0.88681745554841	0.38057662307302
84	-3.1539449754815	0.42104230896057	0.22980298383448				-1.6	1.3371429661696e+14		-2.8878723428374	0.20941782249579	0.22135154554748
85	-2.924141991647	0.17329028533354	0.27047905083228				-1.6	1.3371429661696e+14		-3.1092238883849	0.312345511414	0.22106595187741
86	-3.1946210424793	0.53634575305182	0.25266343912904				-1.6	1.3371429661696e+14		-2.8881579365075	0.20906261203178	0.22147492144734
87	-2.9419576033503	0.16263025165891	0.34757915457324				-1.6	1.3371429661696e+14		-3.1096328579548	0.31323307927687	0.22103473362486
88	-3.2895367579235	0.82984209080653	0.35118493659304				-1.6	1.3371429661696e+14		-2.88859812433	0.20851720927477	0.22167053962502
89	-2.9383518213305	0.16439139904444	0.32622531376525				-1.6	1.3371429661696e+14		-3.110268663955	0.31461718860697	0.22099141919699
90	-3.2645771350957	0.75262257597526	0.3183027825499				-1.6	1.3371429661696e+14		-2.889277244758	0.20768073823529	0.22198548531583
91	-2.9462743525458	0.16079109319238	0.37912609286749				-1.6	1.3371429661696e+14		-3.1112627300739	0.31679153378561	0.22093624862499
92	-3.3254004454133	0.93595507351234	0.41085777347881				-1.6	1.3371429661696e+14		-2.8903264814489	0.20640030567874	0.22250404482587
93	-2.9145426719345	0.18102646373438	0.24867302686915				-1.6	1.3371429661696e+14		-3.1128305262747	0.32024626800891	0.22087967237422
94	-3.1632156988037	0.44616887014525	0.23397532958786				-1.6	1.3371429661696e+14		-2.8919508539005	0.20444668415734	0.22338575967493
95	-2.9292403692158	0.16974194880682	0.28643364566179				-1.6	1.3371429661696e+14		-3.1153366135754	0.32583270338198	0.2208640069323
96	-3.2156740148776	0.60009805657412	0.26884823632391				-1.6	1.3371429661696e+14		-2.8944726066431	0.20148339405666	0.2249538892068
97	-2.9468257785537	0.16057741823464	0.3837279689008				-1.6	1.3371429661696e+14		-3.1194264958499	0.33511654341562	0.22102577390208
98	-3.3305537474545	0.95050333946367	0.42095512953249				-1.6	1.3371429661696e+14		-2.8984007219479	0.19703826303151	0.22791859724849
99	-2.909598617922	0.18553752668293	2.909598617922				-1.6	1.3371429661696e+14	0.6	-3.1263193191963	0.35121922590688	1.5263193191963

Figure 1: Método de Newton, raíz 2

	Condición Inicial											
	β_i			γ_i			α_i			b_i		
Iteración	x_k	f_k	$\lvert x_{k+1}-x_k \rvert$	x_k	f_k	$\lvert x_{k+1}-x_k \rvert$	x_k	f_k	$\lvert x_{k+1}-x_k \rvert$	x_k	f_k	$\lvert x_{k+1}-x_k \rvert$
0	-2	-0.72727272727273	0.71329162455228	-1	0.25	0.22099687473515	-1.4	0.07640297289243	0.17091458596548	-1.45	-0.080992937560207	0.24399829749688
1	-2.7132916245523	0.55274120045235	0.45469846801744	-0.77900312526485	-0.047860495197067	0.042955240382728	-1.5709145859655	-0.98485757323773	0.012086752437376	-1.6939982974969	-0.48132745394779	0.067902428934524
2	-3.1679900925697	0.45940160249182	0.23637598607527	-0.82195836564758	0.0090171965639797	0.0076835774562825	-1.5830013384029	-1.4975124937963	0.005989934016706	-1.7619007264314	-0.71875755176616	0.63606129575366
3	-2.9316141064945	0.16822480321341	0.29521666372275	-0.81427478819129	-0.0013819449337619	0.0011868137702213	-1.5889912724196	-2.1372625014993	0.0035202314922029	-2.3979620221851	0.66494056729333	0.32831969875255
4	-3.2268307702172	0.63458082672942	0.27861178621412	-0.81546160196151	0.0002187308432859	0.0001876110315101	-1.5925115039118	-2.97643283165	0.0022472957222175	-2.0696423234325	-0.52975078384918	0.30347884097297
5	-2.9482189840031	0.16005910623524	0.39602884670568	-0.81527399093	-3.4443023196028e-05	2.9548475506624e-05	-1.594758799634	-4.0937905423883	0.0015059349306403	-2.3731211644055	0.60389640051153	0.26930504474237
6	-3.3442478307088	0.98808696694456	0.4501002436761	-0.81530353940551	5.4280414035977e-06	4.6565416748079e-06	-1.5962647345646	-5.5900814463433	0.0010408892906704	-2.1038161196631	-0.40934183400824	0.19816791188547
7	-2.8941475870327	0.20186054047316	0.22473767238185	-0.81529888286384	-8.553222569661e-07	7.3375696818534e-07	-1.5973056238553	-7.5987645337403	0.00073456638473712	-2.3019840315486	0.37945853634306	0.14235612502254
8	-3.1188852594145	0.33387621530144	0.22099153583015	-0.8152996166208	1.3477989171812e-07	1.1562379875762e-07	-1.59804019024	-10.298403877452	0.00052590472956626	-2.159627906526	-0.19083851183763	0.076912149119094
9	-2.8978937235844	0.19760019758211	0.22749805918447	-0.81529950099701	-2.1238265981698e-08	1.8219700748112e-08	-1.5985660949696	-13.928717937551	0.00038036366986405	-2.2365400556451	0.12647014972234	0.045923398455181
10	-3.1253917827688	0.34901967870047	0.22164707356429	-0.81529951921671	3.3466726007906e-09	2.8710146482425e-09	-1.5989464586395	-18.811937144329	0.00027711223842242	-2.19061665719	-0.06311809614174	0.023931059642609
11	-2.9037447092045	0.191330694309	0.23310365467374	-0.81529951634569	-5.2736028206458e-10	4.5240733381746e-10	-1.5992235708779	-25.381436970189	0.00020295655480473	-2.2145477168326	0.03613622442664	0.013304628846895
12	-3.1368483638783	0.37687400523349	0.22397599501973	-0.8152995167981	8.3100103278507e-11	7.1289196768021e-11	-1.5994265274327	-34.220213891181	0.00014921769727705	-2.2012430879857	-0.019008849315393	0.0071005798070107
13	-2.9128723688585	0.18251095011072	0.24573646553987	-0.81529951672681	-1.3094717978302e-11	1.1233569630065e-11	-1.59957574513	-46.112629424446	0.00011001734325755	-2.2083436677927	0.010446553300779	0.0038699957644939
14	-3.1586088343984	0.43358610295064	0.23181923200922	-0.81529951673804	2.0633563024947e-12	1.7701395904623e-12	-1.5996857624732	-62.114018773332	8.1283401732524e-05	-2.2044736720282	-0.0056025252432157	0.002084550360328
15	-2.9267896023892	0.1713984856173	0.27830872249497	-0.81529951673627	-3.2506344059409e-13	2.7888802378584e-13	-1.599767045875	-83.644343360548	6.0145930347533e-05	-2.2065582223885	0.0030440096872395	0.0011298864595441
16	-3.2050983248842	0.56781721404913	0.26035655429362	-0.81529951673655	5.131940588505e-14	2.129442253854	-1.5998271918053	-112.61419903145	4.4555471493402e-05	-2.205428335929	-0.0016421970258825	0.00061033952130085
17	-2.9447417705905	0.16141030592037	0.36706301396955				-1.5998717472768	-151.59436239905	3.3033818676032e-05	-2.2060386754503	0.00088933287411335	0.00033029929549411
18	-3.3118047845601	0.89663857968154	0.38621204498407				-1.5999047810955	-204.04392110179	2.4506732373863e-05	-2.2057083761548	-0.00048062184992405	0.00017857070904892
19	-2.925592739576	0.17224057948187	0.27465463932124				-1.5999292878278	-274.61723232389	1.818910832041e-05	-2.2058869468638	0.00026003315509882	9.6593285466628e-05
20	-3.2002473788973	0.55317606036803	0.25670591768086				-1.5999474769362	-369.57694905422	1.3504712039758e-05	-2.2057903535784	-0.00014060180086543	5.2234437533105e-05
21	-2.9435414612164	0.16192124173181	0.35830361925392				-1.5999609816482	-497.34976595434	1.002926340643e-05	-2.2058425880159	7.6049305553597e-05	2.8251103464161e-05
22	-3.3018450804703	0.867084106982	0.36979634609668				-1.5999710109116	-669.27420154796	7.4496224851117e-06	-2.2058143369124	-4.1126589334181e-05	1.5278363868276e-05
23	-2.9320487343736	0.1679563678331	0.29693160277174				-1.5999784605341	-900.60677471117	5.5342660505442e-06	-2.2058296152763	2.224291671889e-05	8.2630108386716e-06
24	-3.2289803371454	0.64126350945839	0.28059018172724				-1.5999839948001	-1211.8758632271	4.1117885345443e-06	-2.2058213522655	-1.2029242438946e-05	4.4687798177456e-06
25	-2.9483901554181	0.15999755833741	0.39761055922781				-1.5999881065887	-1630.7033854312	3.0551667384859e-06	-2.2058258210453	6.5057442632145e-06	2.4168263830227e-06
26	-3.346000714646	0.992776495193	0.45409646278662				-1.5999911617554	-2194.2559602495	2.2701987736351e-06	-2.2058234042189	-3.5184316313826e-06	1.3070697968409e-06
27	-2.8919042518593	0.20450224458605	0.22335909571843				-1.5999934319542	-2952.5430610244	1.6869852887957e-06	-2.2058247112887	1.9028510947454e-06	7.0689322573259e-07
28	-3.1152633475778	0.32566827144848	0.22086319382295				-1.5999951189395	-3972.8548399864	1.2536387583584e-06	-2.2058240043955	-1.0291023441003e-06	3.8230319976407e-07
29	-2.8944001537548	0.2015673439703	0.22490532040858				-1.5999963725782	-5345.7334347747	9.3163052494738e-07	-2.2058243866987	5.5656176985288e-07	2.0675810707971e-07
30	-3.1193054741634	0.33483889459572	0.22101778679786				-1.5999973042088	-7193.0076480586	6.9234502664273e-07	-2.2058241799406	-3.0100077377682e-07	1.118193311278e-07
31	-2.8982876873655	0.19716324105586	0.22782382813551				-1.5999979965538	-9678.6039976536	5.1452569493904e-07	-2.2058242917599	1.627879134133e-07	6.0474373952957e-08
32	-3.126111515501	0.35072557189639	0.22175102429225				-1.5999985110795	-13023.09349754	3.8238050170669e-07	-2.2058242312855	-8.8039292223149e-08	3.2705876673589e-08
33	-2.9043604912088	0.19069853545895	0.23379583447523				-1.59999889346	-17523.265068703	2.8417607755848e-07	-2.2058242639914	4.7613599258026e-08	1.7688061682009e-08
34	-3.138156325684	0.3801465140816	0.22432800063473				-1.5999991776361	-23578.462477698	2.1119401205638e-07	-2.2058242463033	-2.5750485371373e-08	9.5660945831355e-09
35	-2.9138283250493	0.18165637947944	0.24739032550854				-1.5999993888301	-31726.021992538	1.5695581168096e-07	-2.2058242558694	1.3926432561958e-08	5.1735558059818e-09
36	-3.1612186505578	0.4406914061182	0.23302100750039				-1.5999995457859	-42688.955403083	1.166472425318e-07	-2.2058242506959	-7.5317234911354e-09	2.7979734085193e-09
37	-2.9281976430574	0.17043559429968	0.28286649467676				-1.5999996624331	-57440.109886439	8.6690697953529e-08	-2.2058242534938	4.0733227941689e-09	1.5132060049439e-09
38	-3.2110641377342	0.58597112480418	0.26505652889644				-1.5999997491238	-77288.498102354	6.4427492230124e-08	-2.2058242519806	-2.2029438657804e-09	8.1837558951747e-10
39	-2.9460076088378	0.16089618838808	0.37695126765822				-1.5999998135513	-103995.45956347	4.7881800968597e-08	-2.205824252799	1.1914012633166e-09	4.4259573783734e-10
40	-3.322958876496	0.92899104077699	0.40622538548714				-1.5999998614331	-139930.96154075	3.5585257851523e-08	-2.2058242523564	-6.4433625824105e-10	2.3936586046602e-10
41	-2.9167334910088	0.17914103881414	0.25287017448865				-1.5999998970184	-188283.90878735	2.6446612233499e-08	-2.2058242525958	3.4847285589819e-10	1.2945466920655e-10
42	-3.1696036654975	0.46391637389894	0.23722457190512				-1.599999923465	-253345.12589175	1.9654870619945e-08	-2.2058242524663	-1.8846142310381e-10	7.0011996200492e-11
43	-2.9323790935924	0.16775427263676	0.29825851836734				-1.5999999431199	-340888.12497325	1.4607318243876e-08	-2.2058242525363	1.0192421054098e-10	3.7863934210236e-11
44	-3.2306376119597	0.64642269186264	0.28213735126225				-1.5999999577272	-458681.44275368	1.0856026966266e-08	-2.2058242524985	-5.5123054906108e-11	2.0477841644606e-11
45	-2.9485002606975	0.15995821484566	0.39863649995392				-1.5999999685832	-617178.02347418	8.0681028524765e-09	-2.205824252519	2.9812491615091e-11	1.1075140804451e-11
46	-3.3471367606514	0.99580027904069	0.45672107671518				-1.5999999766513	-830442.79395724	5.9961433596101e-09	-2.2058242525079	-1.612347160183e-11	5.9898752624576e-12
47	-2.8904156839362	0.20629211661056	0.2225499504909				-1.5999999826475	-1117400.8147895	4.4562815659788e-09	-2.2058242525139	8.7207761908817e-12	3.2396307858562e-12
48	-3.1129656344271	0.32054543820055	0.22087650852779				-1.5999999871037	-1503516.6276875	3.3118701114887e-09	-2.2058242525106	-4.7162577913888e-12	1.7519319328585e-12
49	-2.8920891258993	0.20428200186917	0.22346535722028				-1.5999999904156	-2023054.1035784	2.4613533433637e-09	-2.2058242525124	2.5507528332227e-12	9.4768637382003e-13
50	-3.1155544831196	0.32632206461879	0.22086686917679				-1.599999992877	-2722116.6971435	1.8292567460776e-09	-2.2058242525114	-1.3800677285766e-12	5.1247894816697e-13
51	-2.8946876139428	0.20123468459493	0.22509927581738				-1.5999999947062	-3662739.0729721	1.3594880776679e-09	-2.2058242525119	7.4444911708224e-13	2.7666757773659e-13
52	-3.1197868897602	0.33594441288223	0.2210506832999				-1.5999999960657	-4928391.8120707	1.0103600178724e-09	-2.2058242525117	-4.0210751740441e-13	1.4921397450962e-13
53	-2.8987362064603	0.19666836077393	0.22820331528913				-1.5999999970761	-6631388.9117765	7.5089090501024e-10	-2.2058242525118	2.1574284827843e-13	7.9936057773011e-14
54	-3.1269395217494	0.35269554716761	0.22187800974992				-1.599999997827	-8922852.7844076	5.5805582377388e-10	-2.2058242525117	-1.1508977399974e-13	4.2632564145606e-14
55	-2.9050615119995	0.1899853477198	0.23460946188595				-1.599999998385	-12006128.048804	4.1474224055094e-10	-2.2058242525118	6.2390772220907e-14	0.71289760831877
56	-3.1396709738854	0.38395903382044	0.22475621495017				-1.5999999987998	-16154821.785474	3.0823299468352e-10			
57	-2.9149147589353	0.18070129931	0.24935746435825				-1.599999999108	-21737090.468849	2.2907609142919e-10			
58	-3.1642722232935	0.44908059599627	0.23449220254256				-1.5999999993371	-29248298.162599	1.7024737175575e-10			
93	-2.9145426719345	0.18102646373438	0.24867302686915				-1.6	-955102118692.99	5.1070259132757e-15			
94	-3.1632156988037	0.44616887014525	0.23397532958786				-1.6	-1285714390548.1	3.7747582837255e-15			
95	-2.9292403692158	0.16974194880682	0.28643364566179				-1.6	-1692586033126.5	2.8865798640254e-15			
96	-3.2156740148776	0.60009805657412	0.26884823632391				-1.6	-2266344010457.4	2.2204460492503e-15			
97	-2.9468257785537	0.16057741823464	0.3837279689008				-1.6	-3038961286749.5	1.5543122344752e-15			
98	-3.3305537474545	0.95050333946367	0.42095512953249				-1.6	-4051948382332.5	1.3322676295502e-15			
99	-2.909598617922	0.18553752668293	2.909598617922				-1.6	-5571429025707	0.6			

Figure 1: Método de Newton, raíz 3

	Condición Inicial											
	β_i			γ_i			α_i			b_i		
Iteración	x_k	f_k	$\|x_{k+1} - x_k\|$	x_k	f_k	$\|x_{k+1} - x_k\|$	x_k	f_k	$\|x_{k+1} - x_k\|$	x_k	f_k	$\|x_{k+1} - x_k\|$
0	-1	0.25	0.22099687473515		-0.4	0.90909090909091	-0.76	-0.07187424423847	0.066417364622902	-0.81	-0.0071296522431815	0.0061513858379981
1	-0.77900312526485	-0.047860495197067	0.042955240382728	-0.90909090909091	0.13020836229111	0.106590061313	-0.8264173646229	0.015089486810406	0.012803331517959	-0.816151385838	0.0011500151795475	0.00098568716893666
2	-0.82195836564758	0.0090171965639797	0.0076835774562825	-0.80250084777791	-0.017142100934338	0.014917871884157	-0.81361403310494	-0.0022722005519605	0.0019527380057091	-0.81516569866906	-0.00018055546890473	0.00015491514360544
3	-0.81427478819129	-0.0013819449337619	0.0011868137702213	-0.81741871966207	0.0028628734691351	0.0024505797799501	-0.81556677111065	0.00036067538721634	0.00030932641702031	-0.81532061381267	2.8467885067977e-05	2.4421242888373e-05
4	-0.81546160196151	0.0002187308432859	0.0001876110315101	-0.81496813988212	-0.00044706508788011	0.00038365800821183	-0.81525744469363	-5.6769025135287e-05	4.8702672985068e-05	-0.81529619256978	-4.4854900423285e-06	3.8479869344243e-06
5	-0.81527399093	-3.4443023196028e-05	2.9548475506624e-05	-0.81535179789033	7.054813654601e-05	6.0517906307567e-05	-0.81530614736662	8.9471409173326e-06	7.6754426505676e-06	-0.81530004055671	7.0682232491622e-07	6.0636230581057e-07
6	-0.81530353940551	5.4280414035977e-06	4.6565416748079e-06	-0.81529127998402	-1.1114295503289e-05	9.5347184018157e-06	-0.81529847192397	-1.4098276441149e-06	1.2094521049111e-06	-0.81529943419441	-1.1137902794873e-07	9.5548888023878e-08
7	-0.81529888286384	-8.553222569661e-07	7.3375696818534e-07	-0.81530081470242	1.7514247745957e-06	1.5024951309783e-06	-0.81529968137607	2.2215808362584e-07	1.9058302758168e-07	-0.8152995297433	1.7550832536876e-08	1.5056356605925e-08
8	-0.8152996166208	1.3477989171812e-07	1.1562379875762e-07	-0.81529931220729	-2.7598355575629e-07	2.3675844307292e-07	-0.81529949079304	-3.5007085624439e-08	3.0031577447076e-08	-0.81529951468694	-2.7656157904719e-09	2.3725426068211e-09
9	-0.81529950099701	-2.1238265981698e-08	1.8219700748112e-08	-0.81529954896574	4.3488830938355e-08	3.7307821942889e-08	-0.81529952082462	5.516328795515e-09	4.7323005514244e-09	-0.81529951705948	4.3579880084003e-10	3.7385927686984e-10
10	-0.81529951921671	3.3466726007906e-09	2.8710146482425e-09	-0.81529951165792	-6.8528593689817e-09	5.8788719448089e-09	-0.81529951609232	-8.6924919088266e-10	7.4570394303919e-10	-0.81529951668562	-6.8671932009343e-11	5.8911653333382e-11
11	-0.81529951634569	-5.2736028206458e-10	4.5240733381746e-10	-0.81529951753679	1.0798561292282e-09	9.263776412638e-10	-0.81529951683802	1.3697404812861e-10	1.1750611594863e-10	-0.81529951674453	1.0820976433861e-11	9.2830187981008e-12
12	-0.8152995167981	8.3100103278507e-11	7.1289196768021e-11	-0.81529951661041	-1.701606783101e-10	1.459760090583e-10	-0.81529951672052	-2.1584066523191e-11	1.8516299604698e-11	-0.81529951673525	-1.7050933410272e-12	1.4627188349436e-12
13	-0.81529951672681	-1.3094717978302e-11	1.1233569630065e-11	-0.81529951675639	2.681341669567e-11	2.3002488802604e-11	-0.81529951673903	3.4010659345742e-12	2.9176661087149e-12	-0.81529951673671	2.6851480614741e-13	2.3037127760972e-13
14	-0.81529951673804	2.0633563024947e-12	1.7701395904623e-12	-0.81529951673338	-4.2252166778798e-12	3.6246561307962e-12	-0.81529951673612	-5.3593512259557e-13	4.5974335449728e-13	-0.81529951673648	-4.2198658393629e-14	1.391258705652
15	-0.81529951673627	-3.2506344059409e-13	2.7888802378584e-13	-0.81529951673701	6.6557134694074e-13	5.7098770156472e-13	-0.81529951673658	8.4397316787265e-14	0.78452767506872			
16	-0.81529951673655	5.131940588505e-14	2.129442253854	-0.81529951673644	-1.0470618120148e-13	8.9817042692175e-14						
17				-0.81529951673653	1.6417345484554e-14	2.4965052678236						
18												

Figure 1: Método de Newton, raíz 4

	Condición Inicial											
	β_i			γ_i			α_i			b_i		
Iteración	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$	x_k	f_k	$\|x_{k+1}-x_k\|$
0	1	-0.76923076923077	3.4302392435587	2	-1.3333333333333	0.68953056180636	1.7	3.3561372715896	0.020262433903739	1.65	-12.618932875964	0.014610722948711
1	-2.4302392435587	0.72869371506764	0.42853729390076	2.6895305618064	-1.4178760324302	13.112069531374	1.7202624339037	1.3095825604503	0.023821692764215	1.6353892770513	-7.5904869771828	0.029227100334617
2	-2.0017019496579	-0.72335177313934	0.69717864891965	-10.422538969568	4.3880555611007	1.5801405345592	1.744084126668	0.26463690868971	0.012721264456927	1.6061621767167	-4.7710690639886	0.062412787301583
3	-2.6988805985776	0.58414628119399	0.51202151958149	-8.8423984350088	2.6155962171639	0.85780439437427	1.7568053911249	-0.068704118134231	0.0052715241114405	1.5437493894151	-3.1096075414035	0.14392327596375
4	-3.2109021181591	0.58547611356206	0.26492581003893	-9.7002028293831	3.6032918179184	0.99164783422833	1.7515338670134	0.057349547242315	0.0036309128537186	1.3998261134513	-1.870642167408	0.3197765356037
5	-2.9459763081201	0.16090859470561	0.3766982096378	-10.691850663611	4.0643964611788	1.0343664138304	1.7551647798672	-0.031080933873881	0.0022465495468793	1.0800495778476	-0.82327540369287	0.9927033930202
6	-3.3226745177579	0.92817708777755	0.40569200534537	-11.726217077442	4.3132107993936	0.92490854965603	1.7529182303203	0.022757373669098	0.0015155813611947	0.087346184827437	-0.44951944365998	0.64628258981125
7	-2.9169825124126	0.17893118068031	0.25337348520263	-12.651125627098	5.1177091676675	1.2906182354711	1.7544338116815	-0.013865381062984	0.00097586853460063	-0.55893640498381	-0.26419005035035	0.42410384616493
8	-3.1703559976152	0.46602855882168	0.23762656138413	-13.941743862569	4.0729047537003	2.0379891492803	1.7534579431469	0.0095693213774073	0.0006499665512345	-0.98304025114874	0.22918426267752	0.19741285000778
9	-2.9327294362311	0.16754178672486	0.29968823703626	-15.979733011849	4.6947454436637	6.5563308113668	1.7541079096981	-0.006097179829567	0.00042406344055435	-0.78562740114096	-0.039308339582087	0.034953549917229
10	-3.2324176732673	0.6519702788371	0.28382052783874	-22.536063823216	10.102789886908	6.5636204026347	1.7536838462576	0.0040977694179739	0.00028063083729957	-0.82058095105819	0.0071468142464752	0.0060980811237883
11	-2.9485971454286	0.15992375533515	0.39954482154845	-29.099684225851	9.4607636422297	1.2642917142648	1.7539644770949	-0.0026599683129189	0.00018403811432677	-0.8144828699344	-0.0011014538319567	0.00094571954473477
12	-3.348141966977	0.9984655204594	0.45906655530779	-27.835392511586	9.6168909429183	0.96809448294614	1.7537804389805	0.0017668611783124	0.00012142829478345	-0.81542858947914	0.00017417798349559	0.00014940207518965
13	-2.8890754116693	0.20792870419932	0.22189020464747	-28.803486994532	10.343496014258	0.91713398545375	1.7539018672753	-0.0011560521695249	7.9802688878639e-05	-0.81527918740395	-2.7431285097254e-05	2.3533016500821e-05
14	-3.1109656163167	0.316140332882	0.22095114913188	-29.720620979986	11.774655506616	0.98024206680584	1.7538220645864	0.00076397105345605	5.2583982886434e-05	-0.81530272042045	4.3229311739197e-06	3.7085064349407e-06
15	-2.8900144671848	0.20677955712966	0.22234573287166	-30.700863046792	12.440942083744	1.0375761220567	1.7538746485693	-0.00050157430717355	3.4589511825001e-05	-0.81529901191401	-6.8118719320928e-07	5.8437130323608e-07
16	-3.1123602000565	0.31920661642312	0.22089278886861	-31.738439168848	12.357660423078	0.94604665477319	1.7538400590575	0.00033072431983373	2.2778631213694e-05	-0.81529959628532	1.0733999321954e-07	9.2083899838613e-08
17	-2.8914674111879	0.20502446266466	0.22311312277125	-32.684485823622	13.49729968694	1.1014455394813	1.7538628376887	-0.00021745302592404	1.4989515852548e-05	-0.81529950420142	-1.6914359517413e-08	1.4510345036634e-08
18	-3.1145805339591	0.32413903141004	0.2208592552658	-33.785931363103	12.480678580976	0.91362008327798	1.7538478481729	0.00014324381088352	9.8687167886524e-06	-0.81529951871176	2.6653222364345e-09	2.2865036530817e-09
19	-2.8937212786933	0.20235737113021	0.22446050919297	-34.699551446381	14.141392376013	1.0439826318851	1.7538577168896	-9.4243753184905e-05	6.4952167109222e-06	-0.81529951642526	-4.1999485579018e-10	3.6030156636002e-10
20	-3.1181817878863	0.33226942612922	0.22095278049945	-35.743534078266	13.899575611402	0.94109938351134	1.7538512216729	6.2055556085252e-05	4.2758149800814e-06	-0.81529951678556	6.6181603120161e-11	5.6775251167096e-11
21	-2.8972290073869	0.19834225723829	0.22696418657812	-36.684633461777	15.202570081648	1.1020577433971	1.7538554974879	-4.0839226099351e-05	2.8143845378636e-06	-0.81529951672879	-1.0428662681618e-11	8.9463991770344e-12
22	-3.124193193965	0.34619226537227	0.22148743742048	-37.786691205174	14.009746695101	0.91706751911056	1.7538526831034	2.6886023971423e-05	1.8526268474872e-06	-0.81529951673773	1.6433154780223e-12	1.409761196669e-12
23	-2.9027057565445	0.19240930266195	0.23198228658695	-38.703758724285	15.758559043717	1.0317058840047	1.7538545357302	-1.7696013898245e-05	1.2194560581325e-06	-0.81529951673632	-2.5878600882314e-13	2.2204460492503e-13
24	-3.1346880431314	0.37150949850288	0.22343150412232	-39.73546460829	15.648402702549	0.95586567220391	1.7538533162742	1.1649043981872e-05	8.0271556091027e-07	-0.81529951673654	4.0739338795004e-14	1.3905263043087
25	-2.9112565390091	0.18398548916529	0.2430972295076	-40.691330280494	16.801819115934	1.0758125363833	1.7538541189897	-7.6676397369791e-06	5.2837931296956e-07			
26	-3.1543537685167	0.42213373422755	0.22997283544206	-41.767142816877	15.894576721923	0.9221565847747	1.7538535906104	5.0473299398486e-06	3.478062930995e-07			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
54	-3.1463847352569	0.40114687401558	0.22690959073305	-69.756871660005	27.199739770914	0.93735800853123	1.7538538003532	4.1491046487204e-11	2.8590463330147e-12			
55	-2.9194751445238	0.17688089450144	0.25872778673537	-70.694229668536	29.443398937814	1.0693782536222	1.753853800356	-2.7312407759911e-11	1.8820500713446e-12			
56	-3.1782029312592	0.48831995812939	0.24205496120868	-71.763607922159	27.789226104925	0.93300147330031	1.7538538003542	1.7976618559813e-11	1.2387868508767e-12			
57	-2.9361479700505	0.16556784850464	0.31495646781919	-72.696609395459	30.220193439322	1.0611624364361	1.7538538003554	-1.1828086907503e-11	8.1512574467979e-13			
58	-3.2511044378697	0.71044684270416	0.30287803219156	-73.757771831895	28.760258354183	0.93728059241528	1.7538538003546	7.7871141224285e-12	5.366818101038e-13			
59	-2.9482264056782	0.16005642794445	0.39609709529933	-74.69505242431	31.110625086498	1.06676006539	1.7538538003551	-5.1287712318396e-12	3.5349501104065e-13			
60	-3.3443235009775	0.9882900036207	0.45027143347771	-75.7618124897	29.424173592417	0.93471596085705	1.7538538003548	3.3759820742888e-12	2.3270274596143e-13			
61	-2.8940520674998	0.20197164934874	0.22467493836861	-76.696528450557	31.910621831143	1.0617503149933	1.753853800355	-2.2225755306677e-12	1.5321077739827e-13			
62	-3.1187270058684	0.33351422511739	0.22098224878895	-77.758278765551	30.330706177715	0.93746971281627	1.7538538003549	1.4652474981724e-12	1.0103029524089e-13			
63	-2.8977447570795	0.19776597312314	0.22737670202626	-78.695748478367	32.779174272191	1.0645987135067	1.753853800355	-9.6711194738297e-13	6.6613381477509e-14			
64	-3.1251214591057	0.34838052693103	0.22160959120998	-79.760347191874	31.054004204794	0.9362438991582	1.7538538003549	6.3664153320235e-13	4.3964831775156e-14			
65	-2.9035118678957	0.19157111161155	0.23284731662975	-80.696591091032	33.59665873085	1.0618107822029	1.7538538003549	-4.2118778277996e-13	2.9087843245179e-14			
66	-3.1363591845255	0.37565482683174	0.22384864116749	-81.758401873235	31.912729240527	0.93790086966155	1.7538538003549	2.7620198832311e-13	1.9095836023553e-14			
67	-2.912510543358	0.18283785793825	0.24512870670603	-82.696302742896	34.450160737128	1.0629326732829	1.7538538003549	-1.8143473719754e-13	1.2434497875802e-14			
68	-3.157639250064	0.43096191884829	0.23138605132179	-83.759235416179	32.676217068078	0.93752913087994	1.7538538003549	1.1825656978056e-13	8.2156503822262e-15			
69	-2.9262531987422	0.17177321213615	0.2766468471544	-84.696764547059	35.278732444029	1.0614646273186	1.7538538003549	-7.8567721018583e-14	3.3538538003361			
70	-3.2029000458966	0.5611682020006	0.25868324708984	-85.758229174378	33.505060748584	0.93850598130892						
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
97	-2.9200470222546	0.17642345729761	0.26004128410359	-112.69844147542	47.016683477127	1.0570718284926						
98	-3.1800883063581	0.49374389850293	0.24318192748765	-113.75551330392	44.739682296172	0.94302804625227						
99	-2.9369063788705	0.16515446690999	2.9369063788705	-114.69854135017	47.85457538631	115.69854135017						

Figure 1: Método de Newton, raíz 5

c. Repita el ejercicio anterior utilizando el método de la secante con los siguientes pares de condiciones iniciales:

1. Los valores de β_i y γ_i.
2. Los valores de β_i y α_i.
3. Los valores de α_i y γ_i.
4. Los valores de β_i y b_i.
5. Los valores de b_i y γ_i.

Tablas Ejercicio 3 Método de la Secante

	Condición Inicial														
	$x_1 = \beta_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = \alpha_j$			$x_1 = \alpha_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = b_j$			$x_1 = b_j, x_0 = \gamma_j$		
Iteración	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$
0	-2	-3	1.5838531634529	-2.2	-3	1.6114988827447	-2	-2.2	1.5838531634529	-2.25	-3	1.6218263772773	-2	-2.25	1.5838531634529
1	1.7166186402113	-2	-1.8619247049594	1.035059517852	-2.2	-0.52458476325452	9.4582163461584	-2	-10.45765733544	0.88351088217189	-2.25	-0.24906963265532	8.4274368952346	-2	-8.96997470472
2	-0.2916547684024	1.7166186402113	1.2494241474649	0.24058551189949	1.035059517852	0.73061311874511	-0.49286912891373	9.4582163461584	1.3738480713974	0.46635118959649	0.88351088217189	0.42686365603961	-0.43511481146777	8.4274368952346	1.3419364650435
3	0.51480744796997	-0.2916547684024	0.35558009677894	0.70302505197121	0.24058551189949	0.059864846791221	0.66262876574455	-0.49286912891373	0.12574899744797	0.72979480822475	0.46635118959649	0.01551641459614	0.7182098354153	-0.43511481146777	0.034775095531029
4	0.83562617773179	0.51480744796997	-0.16491280999381	0.74429816943442	0.70302505197121	-0.0087346267926637	0.77904797062925	0.66262876574455	-0.067465212226929	0.7397321555496	0.72979480822475	-0.0010830190361806	0.74889233239623	0.7182098354153	-0.016448883369412
5	0.73397807160925	0.83562617773179	0.0085375863657623	0.7390429501335	0.74429816943442	7.0597455312682e-05	0.73839751133822	0.77904797062925	0.0011506374797408	0.73908379991529	0.7397321555496	2.2314260498257e-06	0.73903966524395	0.74889233239623	7.6094979584607e-05
6	0.73898140110951	0.73397807160925	0.00017360332329808	0.73908508479121	0.7390429501335	8.1042898614214e-08	0.73907918972928	0.73839751133822	9.9470764143295e-06	0.73908513302474	0.73908379991529	3.18687076728e-10	0.73908503525836	0.73903966524395	1.6394168078637e-07
7	0.73908525051049	0.73898140110951	-1.9630688463668e-07	0.73908513321561	0.73908508479121	-7.5484063444264e-13	0.73908513411789	0.73907918972928	-1.5108222450877e-09	0.73908513321516	0.73908513302474		0.73908513321614	0.73908503525836	-1.6459056340068e-12
8	0.73908513321247	0.73908525051049	4.4965142720343e-12	0.73908513321516	0.73908513321561		0.73908513321516	0.73908513411789	1.9984014443253e-15				0.73908513321516	0.73908513321614	
9	0.73908513321516	0.73908513321247													

Figure 1: Método Secante, raíz 1

	Condición Inicial														
	$x_1 = \beta_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = \alpha_j$			$x_1 = \alpha_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = b_j$			$x_1 = b_j, x_0 = \gamma_j$		
Iteración	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$
0	-1	-2	1.5403023058681	-1.6	-2	1.5708004776987	-1	-1.6	1.5403023058681	-1.65	-2	1.5708791111933	-1	-1.65	1.5403023058681
1	34.367898390332	-1	-35.349974170298	46.537234199474	-1.6	-47.370030919454	29.302845319879	-1	-29.818939737071	40.727483758875	-1.65	-41.721081135097	31.743659390458	-1	-30.796884826189
2	0.47673752132316	34.367898390332	0.41175922301768	-0.054991704942147	46.537234199474	1.0534800421436	0.48841424345216	29.302845319879	0.39466380356156	-0.11230330890471	40.727483758875	1.1060039171645	0.55966361130051	31.743659390458	0.28777013680727
3	0.86695904423175	0.47673752132316	-0.21981119157021	0.9586477262894	-0.054991704942147	-0.38402049371473	0.8648014211945	0.48841424345216	-0.21601018090968	0.94237889554626	-0.11230330890471	-0.3545136339185	0.84835340831606	0.55966361130051	-0.18713410561678
4	0.73114670099074	0.86695904423175	0.013262511527082	0.68785943430603	0.9586477262894	0.084747362373501	0.73166415904864	0.8648014211945	0.012399434818927	0.6863742890615	0.94237889554626	0.087174551393844	0.73459635554802	0.84835340831606	0.0075050161990802
5	0.73887478332614	0.73114670099074	0.00035202775237353	0.73681456894073	0.68785943430603	0.0037981372066431	0.73889164287712	0.73166415904864	0.00032381392134551	0.73690108028478	0.6863742890615	0.0036534933374669	0.73898267067223	0.73459635554802	0.00017147866459322
6	0.73908550350535	0.73887478332614	-6.1972217135775e-07	0.73911154359279	0.73681456894073	-4.4200983449816e-05	0.73908545153506	0.73889164287712	-5.3274404476777e-07	0.73911129288737	0.73690108028478	-4.3781394970521e-05	0.73908523501536	0.73898267067223	-1.7037403454712e-07
7	0.73908513319796	0.73908550350535	2.8787083827808e-11	0.73908511995865	0.73911154359279	2.2186259829304e-08	0.73908513320156	0.73908545153506	2.2763124718495e-11	0.73908512058535	0.73911129288737	2.113739638876e-08	0.73908513321286	0.73908523501536	3.8549163861035e-12
8	0.73908513321516	0.73908513319796		0.73908513321508	0.73908511995865	1.2945200467129e-13	0.73908513321516	0.73908513320156		0.73908513321509	0.73908512058535	1.220135104063e-13	0.73908513321516	0.73908513321286	
9				0.73908513321516	0.73908513321508					0.73908513321516	0.73908513321509				

Figure 1: Método Secante, raíz 2

	Condición Inicial														
	$x_1 = \beta_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = \alpha_j$			$x_1 = \alpha_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = b_j$			$x_1 = b_j, x_0 = \gamma_j$		
Iteración	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$
0	-1	-2	1.5403023058681	-1.4	-2	1.5699671429002	-1	-1.4	1.5403023058681	-1.45	-2	1.5705027693674	-1	-1.45	1.5403023058681
1	34.367898390332	-1	-35.349974170298	66.436590200252	-1.4	-67.331242286458	19.769401890883	-1	-19.163459194387	63.250451359019	-1.45	-62.336791752875	21.95117217848	-1	-22.950373228745
2	0.47673752132316	34.367898390332	0.41175922301768	0.14570897351193	66.436590200252	0.84369424237939	0.5451857684718	19.769401890883	0.30984520181767	0.13999436290161	63.250451359019	0.85022241991356	0.44347767700103	21.95117217848	0.45978723818811
3	0.86695904423175	0.47673752132316	-0.21981119157021	0.96608721896287	0.14570897351193	-0.39756442618276	0.85106765698148	0.5451857684718	-0.19188699686823	0.98918665615287	0.13999436290161	-0.43981700054741	0.86589953389504	0.44347767700103	-0.21794431153032
4	0.73114670099074	0.86695904423175	0.013262511527082	0.70332715559644	0.96608721896287	0.059367389775242	0.7340834228896	0.85106765698148	0.0083616636530288	0.69966898608641	0.98918665615287	0.065386404310085	0.73005748726998	0.86589953389504	0.015078577328838
5	0.73887478332614	0.73114670099074	0.00035202775237353	0.7374665579196	0.70332715559644	0.0027078984865123	0.73896826364527	0.7340834228896	0.00019558927043883	0.73714007050085	0.69966898608641	0.0032538814515526	0.73884763151106	0.73005748726998	0.00039746486262882
6	0.73908550350535	0.73887478332614	-6.1972217135775e-07	0.73909816515395	0.7374665579196	-2.181047227745e-05	0.73908526263338	0.73896826364527	-2.1659589921619e-07	0.7391024319331	0.73714007050085	-2.8951453010073e-05	0.73908560894508	0.73884763151106	-7.9618739112775e-07
7	0.73908513319796	0.73908550350535	2.8787083827808e-11	0.73908512855375	0.73909816515395	7.8013931981147e-09	0.73908513321182	0.73908526263338	5.5896398620803e-12	0.73908512577818	0.7391024319331	1.2446611963313e-08	0.73908513319021	0.73908560894508	4.1758485558319e-11
8	0.73908513321516	0.73908513319796		0.73908513321515	0.73908512855375	2.2426505097428e-14	0.73908513321516	0.73908513321182		0.73908513321513	0.73908512577818	4.7517545453957e-14	0.73908513321516	0.73908513319021	
9				0.73908513321516	0.73908513321515					0.73908513321516	0.73908513321513				

Figure 1: Método Secante, raíz 3

	Condición Inicial														
	$x_1 = \beta_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = \alpha_j$			$x_1 = \alpha_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = b_j$			$x_1 = b_j, x_0 = \gamma_j$		
Iteración	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$
0		-1	1	-0.76	-1	1.4848360107409		-0.76	1	-0.81	-1	1.4994984329517		-0.81	1
1	1.8508157176809		-2.1271900019096	5.664814236472	-0.76	-4.8499903948118	1.5675403294376		-1.5642843378333	6.1722955983762	-0.81	-5.178437564526	1.6216267090436		-1.672435205475
2	0.59184626343483	1.8508157176809	0.23806581102709	0.74592848641797	5.664814236472	-0.011470388418809	0.61129739253567	1.5675403294376	0.20760670172475	0.75784091853013	6.1722955983762	-0.031519160877545	0.60679739053034	1.6216267090436	0.21468109149008
3	0.71856303653994	0.59184626343483	0.034189427766473	0.73426757976339	0.74592848641797	0.0080541262246102	0.72333726279051	0.61129739253567	0.026263743956966	0.72468339499192	0.75784091853013	0.024025941458608	0.72224584811875	0.60679739053034	0.028077108451181
4	0.73981304054525	0.71856303653994	-0.0012184302222551	0.73907786139162	0.73426757976339	1.2170191805994e-05	0.73956390043647	0.72333726279051	-0.00080135527426139	0.73902562654674	0.72468339499192	9.9589767490582e-05	0.73961663952118	0.72224584811875	-0.00088963972587097
5	0.73908180040327	0.73981304054525	5.5778299714593e-06	0.7390851409701	0.73907786139162	-1.2978760843829e-08	0.73908345506473	0.73956390043647	2.8085716996529e-06	0.73908532388482	0.73902562654674	-3.1910704778593e-07	0.73908314000626	0.73961663952118	3.3358569319075e-06
6	0.7390851326797	0.73908180040327	8.9616158938099e-10	0.73908513321515	0.7390851409701	2.0983215165415e-14	0.7390851330378	0.73908345506473	2.9683222546595e-10	0.73908513321266	0.73908532388482	4.1930903194043e-12	0.7390851329813	0.73908314000626	3.9138592367038e-10
7	0.73908513321516	0.7390851326797	-5.5511151231258e-16	0.73908513321516	0.73908513321515		0.73908513321516	0.7390851330378	-2.2204460492503e-16	0.73908513321516	0.73908513321266		0.73908513321516	0.7390851329813	-2.2204460492503e-16
8	0.73908513321516	0.73908513321516					0.73908513321516	0.73908513321516					0.73908513321516	0.73908513321516	

Figure 1: Método Secante, raíz 4

	Condición Inicial														
	$x_1 = \beta_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = \alpha_j$			$x_1 = \alpha_j, x_0 = \gamma_j$			$x_1 = \beta_j, x_0 = b_j$			$x_1 = b_j, x_0 = \gamma_j$		
Iteración	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$	x_i	x_{i-1}	$f(x_i)$
0	2	1	-2.4161468365471	1.7	1	-1.8288444942955	2	1.7	-2.4161468365471	1.65	1	-1.7291208888067	2	1.65	-2.4161468365471
1	0.76503468239182	2	-0.043676344228605	0.7649715970166	1.7	-0.043569569754098	0.76580766188465	2	-0.044984875409925	0.76461474594196	1.65	-0.042965639356953	0.76911287619805	2	-0.0505849310223
2	0.74229940686494	0.76503468239182	-0.0053832612631972	0.74215226276963	0.7649715970166	-0.0051366580671285	0.74239298609618	0.76580766188465	-0.0055401017996455	0.74205386755889	0.76461474594196	-0.0049717634896531	0.74279171330295	0.76911287619805	-0.0062084483468557
3	0.73910327015894	0.74229940686494	-3.035432883447e-05	0.73910239971733	0.74215226276963	-2.8897535904959e-05	0.7391043454069	0.74239298609618	-3.2153891608822e-05	0.73910161936417	0.74205386755889	-2.7591517739012e-05	0.73910927678678	0.74279171330295	-4.0407187297253e-05
4	0.73908514606566	0.73910327015894	-2.150675060264e-08	0.73908514488979	0.73910239971733	-1.9538808238018e-08	0.73908514722311	0.7391043454069	-2.3443873176099e-08	0.73908514400512	0.73910161936417	-1.8058211814775e-08	0.73908515293641	0.73910927678678	-3.3005715938117e-08
5	0.73908513321521	0.73908514606566	-8.604228440845e-14	0.73908513321521	0.73908514488979	-7.4606987254811e-14	0.73908513321522	0.73908514722311	-9.9475983006414e-14	0.7390851332152	0.73908514400512	-6.5836225360272e-14	0.73908513321527	0.73908515293641	-1.7597034940309e-13
6	0.73908513321516	0.73908513321521		0.73908513321516	0.73908513321521	1.1102230246252e-16	0.73908513321516	0.73908513321522		0.73908513321516	0.7390851332152		0.73908513321516	0.73908513321527	
7				0.73908513321516	0.73908513321516										

Figure 1: Método Secante, raíz 5

d. Repita el ejercicio anterior utilizando el método de Muller con las siguientes tercias de condiciones iniciales:

1. Los valores de β_i, α_i γ_i.
2. Los valores de β_i, b_i γ_i.
3. Los valores de β_i, α_i b_i.
4. Los valores de α_i, b_i γ_i.

Tablas Ejercicio 3 Método de Muller

	Condición Inicial															
	$x_0 = \beta_j, x_1 = \alpha_j, x_2 = \gamma_j$				$x_0 = \beta_j, x_1 = b_j, x_2 = \gamma_j$				$x_0 = \beta_j, x_1 = \alpha_j, x_2 = b_j$				$x_0 = \alpha_j, x_1 = b_j, x_2 = \gamma_j$			
Iteración	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$
0	-3	-2.2	-2	-0.72727272727273	-3	-2.25	-2	-0.72727272727273	-3	-2.2	-2.25	0.18085106382979	-2.2	-2.25	-2	-0.72727272727273
1	-2.2	-2	-2.2037943812667	-0.0084209199318774	-2.25	-2	-2.0182914254968	-0.68259112810218	-2.2	-2.25	-2.2076286484839	0.007482459507042	-2.25	-2	-2.198609841354	-0.029939706041783
2	-2	-2.2037943812667	-2.2063121927627	0.0020237019382501	-2	-2.0182914254968	-2.3752256071923	0.60945185673965	-2.25	-2.2076286484839	-2.2057943891107	-0.00012386366529905	-2	-2.198609841354	-2.207477942	0.0068576375542019
3	-2.2037943812667	-2.2063121927627	-2.205824149415	-4.2761046623011e-07	-2.0182914254968	-2.3752256071923	-2.1955621407257	-0.04259207808789	-2.2076286484839	-2.2057943891107	-2.205824262709	4.229467549857e-08	-2.198609841354	-2.207477942	-2.2058227411456	-6.2686354345838e-06
4	-2.2063121927627	-2.205824149415	-2.2058242525185	2.8104922810372e-11	-2.3752256071923	-2.1955621407257	-2.2072899515838	0.0060782041325861	-2.2057943891107	-2.205824262709	-2.2058242525118	1.7798532593115e-13	-2.207477942	-2.2058227411456	-2.2058242528506	1.4053323560024e-09
5	-2.205824149415	-2.2058242525185	-2.2058242525118	-2.3219866684157e-15	-2.1955621407257	-2.2072899515838	-2.20582840921	1.7240567481806e-05	-2.205824262709	-2.2058242525118	-2.2058242525118	-1.0398462036818e-14	-2.2058227411456	-2.2058242528506	-2.2058242525118	-1.0095594210503e-15
6					-2.2072899515838	-2.20582840921	-2.2058242517106	-3.3229961358111e-09								
7					-2.20582840921	-2.2058242517106	-2.2058242525118	2.5238985526257e-15								
8																

Figure 1: Método Muller, raíz 1

	Condición Inicial															
	$x_0 = \beta_j, x_1 = \alpha_j, x_2 = \gamma_j$				$x_0 = \beta_j, x_1 = b_j, x_2 = \gamma_j$				$x_0 = \beta_j, x_1 = \alpha_j, x_2 = b_j$				$x_0 = \alpha_j, x_1 = b_j, x_2 = \gamma_j$			
Iteración	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$
0	-2	-1.6	-1	0.25	-2	-1.65	-1	0.25	-2	-1.6	-1.65	-0.17696082432197	-1.6	-1.65	-1	0.25
1	-1.6	-1	-1	0.25	-1.65	-1	-0.063383452705851	-0.37740959298393	-1.6	-1.65	-1.65	-0.17696082432197	-1.65	-1	-0.72005766984751	-0.11951332550145
2	-1	-1	nan	nan	-1	-0.063383452705851	-0.30643582824342	-0.34715024230078	-1.65	-1.65	nan	nan	-1	-0.72005766984751	-0.77875931559468	-0.048173547239076
3					-0.063383452705851	-0.30643582824342	nan	nan					-0.72005766984751	-0.77875931559468	-0.81959758795381	0.0058131184272754
4													-0.77875931559468	-0.81959758795381	-0.81509103693814	-0.00028128203982335
5													-0.81959758795381	-0.81509103693814	-0.81529861570175	-1.2158198329153e-06
6													-0.81509103693814	-0.81529861570175	-0.81529951691514	2.4103630458261e-10
7													-0.81529861570175	-0.81529951691514	-0.81529951673651	-4.8643986620899e-16
8																

Figure 1: Método Muller, raíz 2

	Condición Inicial															
	$x_0 = \beta_j, x_1 = \alpha_j, x_2 = \gamma_j$				$x_0 = \beta_j, x_1 = b_j, x_2 = \gamma_j$				$x_0 = \beta_j, x_1 = \alpha_j, x_2 = b_j$				$x_0 = \alpha_j, x_1 = b_j, x_2 = \gamma_j$			
Iteración	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$
0	-2	-1.4	-1	0.25	-2	-1.45	-1	0.25	-2	-1.4	-1.45	-0.080992937560207	-1.4	-1.45	-1	0.25
1	-1.4	-1	-0.25894251175844	-0.35120073419762	-1.45	-1	0.8732840735736	-0.78856066652166	-1.4	-1.45	-2.7085668841963	0.56312603615004	-1.45	-1	-0.67029643493888	-0.17271144873703
2	-1	-0.25894251175844	-0.41237750997926	-0.3290314659692	-1	0.8732840735736	0.37899640745977	-0.73617164858574	-1.45	-2.7085668841963	-2.2469093294821	0.16844268076348	-1	-0.67029643493888	-0.75022483798068	-0.083904502237005
3	-0.25894251175844	-0.41237750997926	nan	nan	0.8732840735736	0.37899640745977	nan	nan	-2.7085668841963	-2.2469093294821	-2.1219921884715	-0.34059615224113	-0.67029643493888	-0.75022483798068	-0.83210934436341	0.022877427070809
4									-2.2469093294821	-2.1219921884715	-2.2106401450646	0.019962331365798	-0.75022483798068	-0.83210934436341	-0.8136005664669	-0.0022903377034889
5									-2.1219921884715	-2.2106401450646	-2.2057268753649	-0.00040389163791288	-0.83210934436341	-0.8136005664669	-0.81527097183628	-3.8516746842751e-05
6									-2.2106401450646	-2.2057268753649	-2.2058242867382	1.4195986108274e-07	-0.8136005664669	-0.81527097183628	-0.81529956205955	6.1157106012116e-08
7									-2.2057268753649	-2.2058242867382	-2.2058242525122	1.9109950281044e-12	-0.81527097183628	-0.81529956205955	-0.81529951673527	-1.6783391483858e-12
8									-2.2058242867382	-2.2058242525122	-2.2058242525118	-4.6439733368314e-15	-0.81529956205955	-0.81529951673527	-0.81529951673651	1.2160996655225e-16
9																

Figure 1: Método Muller, raíz 3

	Condición Inicial															
	$x_0=\beta_j, x_1=\alpha_j, x_2=\gamma_j$				$x_0=\beta_j, x_1=b_j, x_2=\gamma_j$				$x_0=\beta_j, x_1=\alpha_j, x_2=b_j$				$x_0=\alpha_j, x_1=b_j, x_2=\gamma_j$			
Iteración	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$
0	-1	-0.76		-0.4	-1	-0.81		-0.4	-1	-0.76	-0.81	-0.0071296522431815	-0.76	-0.81		-0.4
1	-0.76		nan	nan	-0.81		nan	nan	-0.76	-0.81	-0.81554919091683	0.00033694663670063	-0.81		-0.64049472250976	-0.20100202403686
2									-0.81	-0.81554919091683	-0.81529807543648	-1.944831341696e-06		-0.64049472250976	nan	nan
3									-0.81554919091683	-0.81529807543648	-0.81529951638858	-4.6948609901962e-10				
4									-0.81529807543648	-0.81529951638858	-0.81529951673651	7.2965979931349e-16				

Figure 1: Método Muller, raíz 4

	Condición Inicial															
	$x_0=\beta_j, x_1=\alpha_j, x_2=\gamma_j$				$x_0=\beta_j, x_1=b_j, x_2=\gamma_j$				$x_0=\beta_j, x_1=\alpha_j, x_2=b_j$				$x_0=\alpha_j, x_1=b_j, x_2=\gamma_j$			
Iteración	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$	x_{i-2}	x_{i-1}	x_i	$f(x_i)$
0	1	1.7	2	-1.3333333333333	1	1.65	2	-1.3333333333333	1	1.7	1.65	-12.618932875964	1.7	1.65	2	-1.3333333333333
1	1.7	2	1.9188520942917	-1.278404981906	1.65	2	2.0427939831075	-1.3215817404791	1.7	1.65	-2.2422873561191	0.14979560008007	1.65	2	2.042917962412	-1.3215228712139
2	2	1.9188520942917	nan	nan	2	2.0427939831075	nan	nan	1.65	-2.2422873561191	-2.2071980601625	0.0056971964439259	2	2.042917962412	nan	nan
3									-2.2422873561191	-2.2071980601625	-2.2058115440726	-5.2710370495991e-05				
4									-2.2071980601625	-2.2058115440726	-2.2058242556364	1.2959758631339e-08				
5									-2.2058115440726	-2.2058242556364	-2.2058242525118	2.0998835957846e-14				
6																

Figure 1: Método Muller, raíz 5

Derivada analítica de la función $f(x)$

$$f'(x) = \frac{d}{dx}\frac{(\cos(2\pi x) - 6)x^3}{15x^2 - x - 40} - \frac{d}{dx}\frac{(2\cos(2\pi x) + 7)x^2}{15x^2 - x - 40} - \frac{d}{dx}\frac{2(\cos(2\pi x) - 10)x}{15x^2 - x - 40} - \frac{d}{dx}\frac{3\cos(2\pi x)}{15x^2 - x - 40}$$
$$+\frac{d}{dx}\frac{19}{15x^2 - x - 40}$$

$$\frac{d}{dx}\frac{(\cos(2\pi x) - 6)x^3}{15x^2 - x - 40} = \frac{d}{dx}\frac{(\cos(2\pi x) - 6)x^3}{15x^2 - x - 40} - \frac{d}{dx}\frac{-6x^3}{15x^2 - x - 40}$$

$$\frac{d}{dx}\frac{(\cos(2\pi x) - 6)x^3}{15x^2 - x - 40} = \frac{((-2\pi\sin(2\pi x)x^3 + 3\cos(2\pi x)x^2)(15x^2 - x - 40)) - (30x - 1)(\cos(2\pi x)x^3)}{(15x^2 - x - 40)^2}$$

$$= \frac{-30\pi\sin(2\pi x)x^5 + 2\pi\sin(2\pi x)x^4 + 15\pi\cos(2\pi x)x^4 + 80\pi\sin(2\pi x)x^3 - 2\cos(2\pi x)x^3 - 120\cos(2\pi x)x^2}{(15x^2 - x - 40)^2}$$

$$\frac{d}{dx}\frac{-6x^3}{15x^2 - x - 40} = \frac{-18x^2(15x^2 - x - 40) - (30x - 1)(-6x^3)}{(15x^2 - x - 40)^2}$$

$$= \frac{-270x^4 + 18x^3 + 720x^2 + 180x^4 - 6x^3}{(15x^2 - x - 40)^2}$$

$$= \frac{-90x^4 + 12x^3 + 720x^2}{(15x^2 - x - 40)^2}$$

$$\frac{d}{dx}\frac{(\cos(2\pi x) - 6)x^3}{15x^2 - x - 40} = \frac{-30\pi\sin(2\pi x)x^5 + 2\pi\sin(2\pi x)x^4 + 15\pi\cos(2\pi x)x^4 - 90x^4 + 80\pi\sin(2\pi x)x^3}{(15x^2 - x - 40)^2}$$
$$+\frac{-2\cos(2\pi x)x^3 + 12x^3 - 120\cos(2\pi x)x^2 + 720x^2}{(15x^2 - x - 40)^2}$$

$$\frac{d}{dx} - \frac{(2\cos(2\pi x)+7)x^2}{15x^2-x-40} = \frac{d}{dx}\frac{-2\cos(2\pi x)x^2}{15x^2-x-40} - \frac{d}{dx}\frac{7x^2}{15x^2-x-40}$$

$$\frac{d}{dx}\frac{-2\cos(2\pi x)x^2}{15x^2-x-40} = \frac{(4\sin(2\pi x)x^2 - 4\cos(2\pi x)x)(15x^2-x-40) - (30x-1)(-2\cos(2\pi x)x^2)}{(15x^2-x-40)^2}$$

$$= \frac{(60\sin(2\pi x)x^4 - 4\sin(2\pi x)x^3 - 160\sin(2\pi x)x^2 - 60\cos(2\pi x)x^3 + 4\cos(2\pi x)x^2 + 160\cos(2\pi x)x)}{(15x^2-x-40)^2}$$

$$+ \frac{(60\cos(2\pi x)x^3 - 2\cos(2\pi x)x^2)}{(15x^2-x-40)^2}$$

$$= \frac{60\sin(2\pi x)x^4 - 4\sin(2\pi x)x^3 - 160\sin(2\pi x)x^2 + 2\cos(2\pi x)x^2 + 160\cos(2\pi x)x}{(15x^2-x-40)^2}$$

$$\frac{d}{dx}\frac{7x^2}{15x^2-x-40} = \frac{14x(15x^2-x-40) - (30x-1)(7x^2)}{(15x^2-x-40)^2}$$

$$= \frac{-7x^2 - 560x}{(15x^2-x-40)^2}$$

$$\frac{d}{dx} - \frac{(2\cos(2\pi x)+7)x^2}{15x^2-x-40} = \frac{60\sin(2\pi x)x^4 - 4\sin(2\pi x)x^3 - 160\sin(2\pi x)x^2 + 2\cos(2\pi x)x^2}{(15x^2-x-40)^2}$$

$$+ \frac{7x^2 + 160\cos(2\pi x)x + 560x}{(15x^2-x-40)^2}$$

$$\frac{d}{dx}\frac{-2(\cos(2\pi x)-10)x}{15x^2-x-40} = \frac{d}{dx}\frac{-2\cos(2\pi x)x}{15x^2-x-40} + \frac{d}{dx}\frac{20x}{15x^2-x-40}$$

$$\frac{d}{dx}\frac{-2\cos(2\pi x)x}{15x^2-x-40} = \frac{(4\pi\sin(2\pi x)x-2\cos(2\pi x))(15x^2-x-40)+60\cos(2\pi x)x^2-2\cos(2\pi x)x}{(15x^2-x-40)^2}$$

$$= \frac{60\pi\sin(2\pi x)x^3-4\pi\sin(2\pi x)x^2+30\cos(2\pi x)x^2-160\pi\sin(2\pi x)x+80\cos(2\pi x)}{(15x^2-x-40)^2}$$

$$\frac{d}{dx}\frac{20x}{15x^2-x-40} = \frac{20(15x^2-x-40)-(30x-1)(20x)}{(15x^2-x-40)^2}$$

$$= \frac{-300x^2-800}{(15x^2-x-40)^2}$$

$$\frac{d}{dx}\frac{-2(\cos(2\pi x)-10)x}{15x^2-x-40} = \frac{60\pi\sin(2\pi x)x^3-4\pi\sin(2\pi x)x^2+30\cos(2\pi x)x^2-300x^2}{(15x^2-x-40)^2}$$

$$+\frac{-160\pi\sin(2\pi x)x+80\cos(2\pi x)-800}{(15x^2-x-40)^2}$$

$$\frac{d}{dx}\frac{-3\cos(2\pi x)}{15x^2-x-40} = \frac{(6\pi\sin(2\pi x))(15x^2-x-40)-(30x-1)(-3\cos(2\pi x))}{(15x^2-x-40)^2}$$

$$= \frac{90\pi\sin(2\pi x)x^2-6\pi\sin(2\pi x)x+90\cos(2\pi x)x-240\pi\sin(2\pi x)-3\cos(2\pi x)}{(15x^2-x-40)^2}$$

$$\frac{d}{dx}\frac{19}{15x^2-x-40} = \frac{-570x+19}{(15x^2-x-40)^2}$$

Ejercicio 4

4. Considere el siguiente polinomio de grado 9

$$P(x) = 756x^9 + 2448x^8 + 1605x^7 - 2583x^6 - 4705x^5 - 2069x^4 + 1643x^3 + 1773x^2 - 20x - 300$$

a. Tabule los valores de $P(x)$ en el intervalo $[-4, 4]$ para los puntos

$$x_k = -4 + \frac{k}{10}, k = 0, 1, 2, \ldots, 80$$

evaluando directamente el polinomio en esos puntos y luego con la regla de Horner.

it	x_k	$f(x_k)$	$h(x_k)$
0	-4	-7.04138e+07	-7.04138e+07
1	-3.9	-5.42003e+07	-5.42003e+07
2	-3.8	-4.13785e+07	-4.13785e+07
3	-3.7	-3.13137e+07	-3.13137e+07
4	-3.6	-2.34751e+07	-2.34751e+07
5	-3.5	-1.74217e+07	-1.74217e+07
6	-3.4	-1.27891e+07	-1.27891e+07
7	-3.3	-9.27834e+06	-9.27834e+06
8	-3.2	-6.64569e+06	-6.64569e+06
9	-3.1	-4.694e+06	-4.694e+06
10	-3	-3.26508e+06	-3.26508e+06
11	-2.9	-2.23309e+06	-2.23309e+06
12	-2.8	-1.49889e+06	-1.49889e+06
13	-2.7	-985170	-985170
14	-2.6	-632323	-632323
15	-2.5	-394977	-394977
16	-2.4	-239066	-239066
17	-2.3	-139404	-139404
18	-2.2	-77697.4	-77697.4
19	-2.1	-40916.8	-40916.8
20	-2	-19992	-19992
21	-1.9	-8773.96	-8773.96
22	-1.8	-3222.69	-3222.69
23	-1.7	-782.902	-782.902
24	-1.6	86.6898	86.6898
25	-1.5	259.875	259.875
26	-1.4	190.943	190.943
27	-1.3	86.0656	86.0656
28	-1.2	16.6393	16.6393

29	-1.1	-10.0124	-10.0124
30	-1	-10	-10
31	-0.9	-2.31724	-2.31724
32	-0.8	-0.697203	-0.697203
33	-0.7	-12.2276	-12.2276
34	-0.6	-38.8335	-38.8335
35	-0.5	-79.2188	-79.2188
36	-0.4	-130.063	-130.063
37	-0.3	-186.205	-186.205
38	-0.2	-240.209	-240.209
39	-0.1	-282.076	-282.076
40	0	-300	-300
41	0.1	-282.883	-282.883
42	0.2	-224.89	-224.89
43	0.3	-131.618	-131.618
44	0.4	-26.4614	-26.4614
45	0.5	45.5	45.5
46	0.6	20.3178	20.3178
47	0.7	-169.296	-169.296
48	0.8	-557.611	-557.611
49	0.9	-1078.22	-1078.22
50	1	-1452	-1452
51	1.1	-1014.65	-1014.65
52	1.2	1535.35	1535.35
53	1.3	8491.01	8491.01
54	1.4	23619.7	23619.7
55	1.5	52805.2	52805.2
56	1.6	104883	104883
57	1.7	192708	192708
58	1.8	334503	334503
59	1.9	555524	555524
60	2	890120	890120
61	2.1	1.38422e+06	1.38422e+06
62	2.2	2.09833e+06	2.09833e+06
63	2.3	3.11112e+06	3.11112e+06
64	2.4	4.52365e+06	4.52365e+06
65	2.5	6.46437e+06	6.46437e+06
66	2.6	9.09491e+06	9.09491e+06
67	2.7	1.26168e+07	1.26168e+07
68	2.8	1.72795e+07	1.72795e+07
69	2.9	2.33888e+07	2.33888e+07
70	3	3.13179e+07	3.13179e+07
71	3.1	4.1518e+07	4.1518e+07
72	3.2	5.45326e+07	5.45326e+07
73	3.3	7.10117e+07	7.10117e+07
74	3.4	9.17285e+07	9.17285e+07

75	3.5	1.17599e+08	1.17599e+08
76	3.6	1.49702e+08	1.49702e+08
77	3.7	1.89303e+08	1.89303e+08
78	3.8	2.3788e+08	2.3788e+08
79	3.9	2.97153e+08	2.97153e+08
80	4	3.69115e+08	3.69115e+08

b. Utilizando los métodos vistos en clase(regla de signos de Descartes y cotas para raíces) extraiga toda la información posible sobre las 9 raíces del polinomio $P(x) = 0$

Información obtenida por Regla de signos de Descartes: El polinomio $P(x)$ tiene 3 cambios de signo, por lo que tendrá 3 o 1 raíz real positiva, El polinomio $P(-x)$ tiene 6 cambios de signo, por lo que tendrá 6, 4, 2 o 0 raíces reales negativas y 0 o 4 raíces complejas según sea el caso.

Pos	Neg	Im	Total
3	6	0	9
3	4	2	9
3	2	4	9
3	0	6	9
1	6	2	9
1	4	4	9
1	2	6	9
1	0	8	9

Información obtenida por Cotas para raíces: Al calcular p_1 y p_2 para determinar R_1 se encontró que el mínimo fue el caso p_2 con un valor de $R1 = p_2 = 0.9024017729792827$ y en el caso para determinar el valor de R_2 el máximo de los casos fue $|\frac{a_5}{a_n}|$ con un valor de $R_2 = 7.223544973544974$

c) Utilizando tan solo la información proveniente de los incisos anteriores, encuentre todas las raíces del polinomio con 13 cifras significativas utilizando:

1. El método de Newton para encontrar una raíz y "desinflando" el polinomio para encontrar la siguiente y así sucesivamente hasta llegar a un polinomio en donde pueda estimar la raíz analíticamente. Si es necesario utilice números complejos para encontrar las raíces.
2. El método de Baristrow pero ahora encontrando un par de raíces y "desinflando" el polinomio para encontrar el siguiente par y así sucesivamente hasta llegar a un polinomio en donde pueda estimar la raíz analíticamente.

Raíces por método de Newton

Con $x_0 = 0.9$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	0.9	-1078.221308736
1	0.69220520726789	-147.41583925486
2	0.63814154242187	-29.479628343535
3	0.62029759744309	-2.9514324204214
4	0.61806854176472	-0.044366270043668
5	0.61803399701092	-1.0604663714275e-05
6	0.6180339887499	-7.389644451905e-13

Table 45: Raíz 1, método de Newton

ahora con la raíz 1 $r_1 = 0.6180339887499$, desinflando el polinomio obtengo:

$$Q_{n-1}(x) = 756.000000x^8 + 2915.233695x^7 + 3406.713509x^6 - 477.535262x^5$$
$$-5000.133022x^4 - 5159.252156x^3 - 1545.593189x^2 + 817.770876x + 485.410197$$

y su derivada

$$Q'_{n-1}(x) = 6048x^7 + 20406.635868464x^6 + 20440.281053789x^5 - 2387.6763076316x^4$$
$$-20000.532089799x^3 - 15477.756468434x^2 - 3091.1863780573x + 817.77087639997$$

Con $x_0 = 7.2$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	7.2	8833947864.5231
1	6.2491155490969	3032535388.7643
2	5.4186873991915	1040637595.2714
3	4.6939893278164	356917065.2372
4	4.0622280453408	122322283.73574
5	3.5123364179505	41874853.261848
6	3.0348092522332	14310450.049406
7	2.6215859865174	4877313.0982098
8	2.265990725388	1655019.4597158
9	1.9627505431414	557428.43949526
10	1.7081304746031	185241.99827423
11	1.5002444306906	59963.265306361
12	1.3395642302432	18332.373958553
13	1.2291466076913	4865.2385435735
14	1.1714989407442	872.07388053915
15	1.1557768478462	52.388554271875
16	1.154705292803	0.2303976019532
17	1.1547005384725	4.5190610080681e-06
18	1.1547005383793	-5.4569682106376e-12
98	1.1547005383793	-5.4569682106376e-12
99	1.1547005383793	5.7411853049416e-12

Table 46: Raíz 2, método de Newton

con $r_2 = 1.1547005383793$, y desinflando el polinomio nuevamente obtengo:

$$Q_{n-2}(x) = 756.000000x^7 3788.187303x^6 7780.935427x^5 8507.115065x^4 4823.037323x^3 409.911637x^2$$
$$-1072.268001x - 420.377562$$

Derivada de $Q_{n-2}(x)$

$$Q'_{n-2}(x) = 42336.000000x^6 + 122439.815211x^5 + 102201.405269x^4 - 9550.705231x^3 - 60001.596269x^2$$
$$-30955.512937x - 3091.186378$$

Con $x_0 = 7.2$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	7.2	1461292020.4543
1	6.0716307471977	496601829.8902
2	5.1049113924499	168746600.21953
3	4.2768572957628	57331459.65009
4	3.5678085249019	19473374.617062
5	2.9609783269826	6611612.6404186
6	2.4420822497512	2243154.661456
7	1.9990485942331	760055.8035643
8	1.6218206083417	256904.53144485
9	1.3022772212227	86422.405224078
10	1.0343263418813	28789.993522651
11	0.81426078551098	9390.4986054034
12	0.64145127626198	2916.3270086365
13	0.5189768841175	799.45004717264
14	0.45115845761014	154.90725742577
15	0.43036233052658	11.341381227103
16	0.42858370323706	0.077201517728781
17	0.42857142915253	3.6546327919496e-06
18	0.42857142857143	5.6843418860808e-14

Table 47: Raíz 3, método de Newton

con $r_3 = 0.42857142857143$ y desinflando el polinomio por tercera vez obtengo:

$$Q_{n-3}(x) = 756.000000x^6 + 4112.187303x^5 + 9543.301413x^4 + 12597.101385x^3 + 10221.795059x^2$$
$$+4790.680948x + 980.880977$$

Derivada de $Q_{n-3}(x)$

$$Q'_{n-3}(x) = 4536.000000x^5 + 20560.936513x^4 + 38173.205654x^3 + 37791.304154x^2 + 20443.590118x$$

$+4790.680948$

Con $x_0 = 7.2$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	7.2	215802619.05443
1	5.8460054083913	72296794.760267
2	4.7168862715742	24225213.086832
3	3.7749137286749	8119972.3033689
4	2.9885584086418	2723104.2733935
5	2.3314245728405	913971.36928463
6	1.7813517028358	307167.21684651
7	1.3196557575247	103448.05641561
8	0.93049302070294	34950.583050781
9	0.6003491782198	11862.604072821
10	0.31769862248211	4049.8313940485
11	0.07296856131745	1390.0484358065
12	-0.14091597975857	477.06596760218
13	-0.32717348264549	161.35423102412
14	-0.48397136862679	52.658353552034
15	-0.60681922354029	16.36292654539
98	-0.83333338461088	2.1600499167107e-12
99	-0.83333325631279	2.955857780762e-12

Table 48: Raíz 4, método de Newton

con $r_4 = -0.83333325631279$ y desinflando el polinomio por cuarta vez obtengo:

$$Q_{n-4}(x) = 756.000000x^5{+}3482.187272x^4{+}6641.478548x^3{+}7062.535664x^2{+}4336.348391x{+}1177.057116$$

Derivada de $Q_{n-4}(x)$

$$Q'_{n-4}(x) = 3780.000000x^4 + 13928.749089x^3 + 19924.435643x^2 + 14125.071327x + 4336.348391$$

Con $x_0 = 7.2$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	7.2	26863396.429485
1	5.5715319291962	8807447.8541488
2	4.2672940867799	2888813.2820935
3	3.2218435202267	948228.82847292
4	2.3825466641068	311669.23487045
5	1.7068778508339	102692.24617292
6	1.1601931332458	33984.77356062
7	0.71387291574441	11332.508717681
8	0.34380290869279	3824.8774418492
9	0.029425357729861	1310.9426658213
10	-0.24542884263391	451.98321797159
11	-0.48615241085565	148.99906867929
12	-0.67495549420086	42.28713173657
13	-0.78593638426771	9.1451875844693
14	-0.82704733560614	1.055449271365
15	-0.8331959706558	0.022555096039468
16	-0.83333322541581	1.1172034646734e-05
17	-0.83333329346852	2.0463630789891e-12
18	-0.83333329346853	0

Table 49: Raíz 5, método de Newton

con $r_5 = -0.83333329346853$ y desinflando el polinomio por quinta vez obtengo:

$$Q_{n-5}(x) = 756.000000x^4 + 2852.187303x^3 + 4264.655909x^2 + 3508.655909x + 1412.468607$$

Derivada de $Q_{n-5}(x)$

$$Q'_{n-5}(x) = 3024.000000x^3 + 8556.561908x^2 + 8529.311819x + 3508.655909$$

Con $x_0 = 7.2$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	7.2	3343991.2733766
1	5.1574860179343	1059132.6027454
2	3.6224106046245	335825.28507877
3	2.4660765854512	106737.18989656
4	1.5906946466955	34104.742688807
5	0.92063913613342	11025.992031673
6	0.39491791764972	3657.2751244424
7	-0.040587629098383	1276.8973349501
8	-0.44258720588394	476.69399095429
9	-0.85796584292579	149.72909203045
10	-1.1163368788279	16.440398563532
11	-1.1534170616183	0.53059636392572
12	-1.1546988242413	0.00070768584760117
13	-1.1547005383762	1.2701093510259e-09
14	-1.1547005383792	0

Table 50: Raíz 6, método de Newton

con $r_6 = -1.1547005383792$ y desinflando el polinomio por sexta vez obtengo:

$$Q_{n-6}(x) = 756.000000x^3 + 1979.233695x^2 + 1979.233695x + 1223.233695$$

Derivada de $Q_{n-6}(x)$

$$Q'_{n-6}(x) = 2268.000000x^2 + 3958.467391x + 1979.233695$$

Con $x_0 = 7.2$ (por el análisis de cotas)

it	x_i	$f(x_i)$
0	7.2	400252.67907751
1	4.4965639539689	118874.09682531
2	2.6854423932882	35452.747175461
3	1.4614727271793	10703.181878307
4	0.61259679477179	3352.2609807678
5	-0.025284973003326	1174.4419870668
6	-0.64979085544315	565.41777667011
7	-2.2002771939219	-1602.6550233931
8	-1.8231281634365	-387.73158557996
9	-1.6546085848338	-57.599117294015
10	-1.6194591000583	-2.1582037280832
11	-1.6180362550778	-0.0034266964448761
12	-1.6180339887556	-8.6827185441507e-09
13	-1.6180339887499	-6.821210263297e-13

Table 51: Raíz 7, método de Newton

con $r_7 = -1.6180339887499$ y desinflando el polinomio por séptima vez obtengo:

$$Q_{n-7}(x) = 756.000000x^2 + 1979.233695x + 1979.233695$$

De manera analítica para encontrar las 2 raíces restantes obtengo:

$$x = \frac{-1979.233695 \pm \sqrt{1979.233695^2 - 4 \times 756 \times 1979.233695}}{2 \times 756}$$

$$= \frac{-1979.233695 \pm \sqrt{-1714608}}{2 \times 756}$$

$$= \frac{-1979.233695 \pm 1309.430411i}{1512}$$

$$x_1 = \frac{-1979.233695 + 1309.430411i}{1512}$$

$$x_2 = \frac{-1979.233695 - 1309.430411i}{1512}$$

De aqui que r_8 y r_9 sean un par de raices complejas conjugadas donde:

$$r_8 = \frac{-1979.233695 + 1309.430411i}{1512}$$

$$r_9 = \frac{-1979.233695 - 1309.430411i}{1512}$$

raíz	valor
r_1	0.6180339887499
r_2	1.1547005383793
r_3	0.42857142857143
r_4	-0.83333325631279
r_5	-0.83333329346853
r_6	-1.1547005383792
r_7	-1.6180339887499
r_8	$\frac{-1979.233695+1309.430411i}{1512}$
r_9	$\frac{-1979.233695-1309.430411i}{1512}$

Table 52: Tabla de Raíces, por método de Newton

Ejercicio 5

5. Utilizar el método de Newton para encontrar las raíces de un polinomio puede generar estructuras geométricas interesantes. Un ejemplo clásico esta dado por las tres raíces de la unidad $\alpha_1 = 1, \alpha_2 = -\frac{1}{2}(1+\sqrt{3}i), \alpha_3 = -\frac{1}{2}(1-\sqrt{3}i)$ tomando a los puntos dentro de un cuadrado en el plano complejo $|Rez| < L/2$, $|Imz| < L/2$ como los puntos de partida del método de Newton. De esta manera, utilizamos como punto de partida a los $(n+1)\times(n+1)$ puntos definidos por

$$z_0 = \alpha_{j,k} = -\frac{L}{2} + jh + \left(-\frac{L}{2} + kh\right) i$$

$$j = 0, 1, 2, \ldots, n$$

$$k = 0, 1, 2, \ldots, n$$

y

$$h = \frac{L}{n}$$

para encontrar las raices del polinomio

$$P(z) = z^3 - 1$$

a. Genere un código que grafique de blanco al punto z_0 si se llega a una de las raíces y de negro en caso contrario. Se espera que como entrada del código se puedan dar los valores de la longitud del intervalo L, la densidad de la malla n, y los criterios de término para le método de Newton: el máximo de iteraciones (maxit) y el "cero" de $P(z)$ (es decir ϵ tal que $|P(z)| < \epsilon$).

i Resuelva con $L = 2$,$n = 256$, maxit= 16 fijos y diferentes valores de $\epsilon = 10^{-6}, 10^{-8}, 10^{-10}, 10^{-12}$ y 10^{-14}

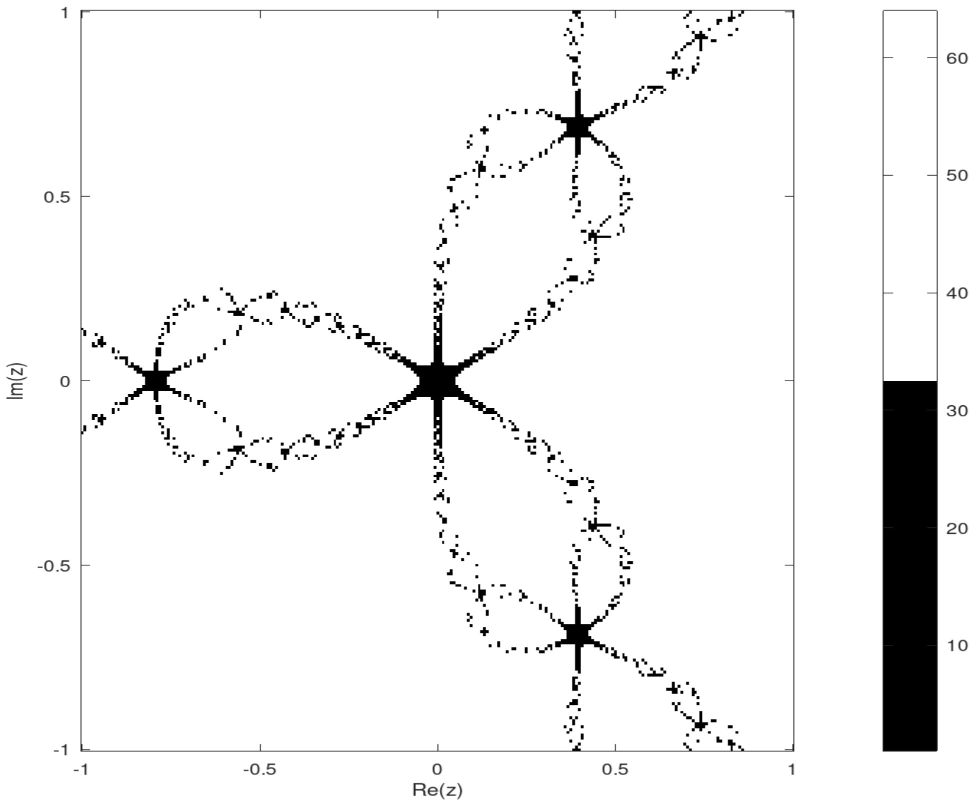


Figure 1: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-6}$

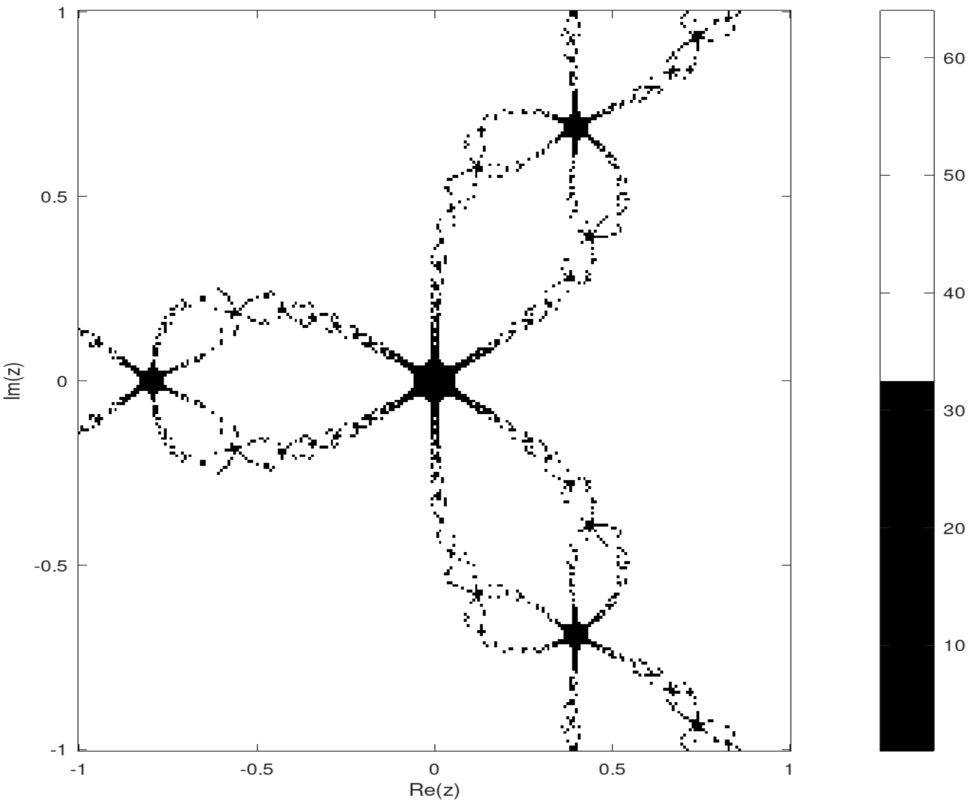


Figure 2: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-8}$

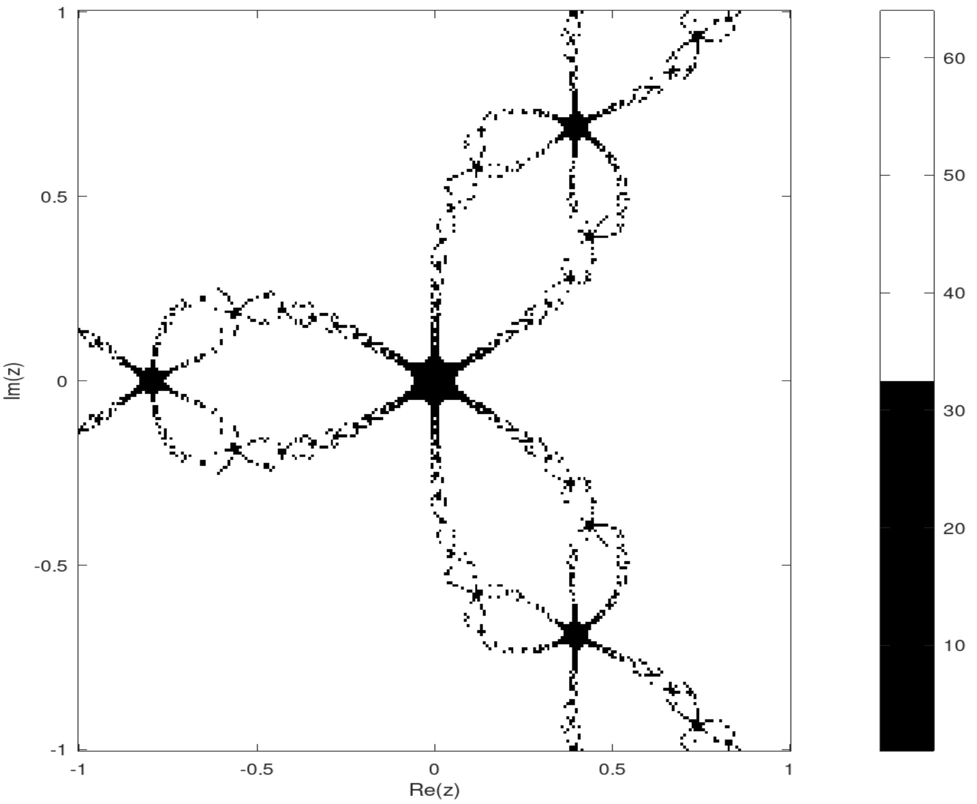


Figure 3: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-10}$

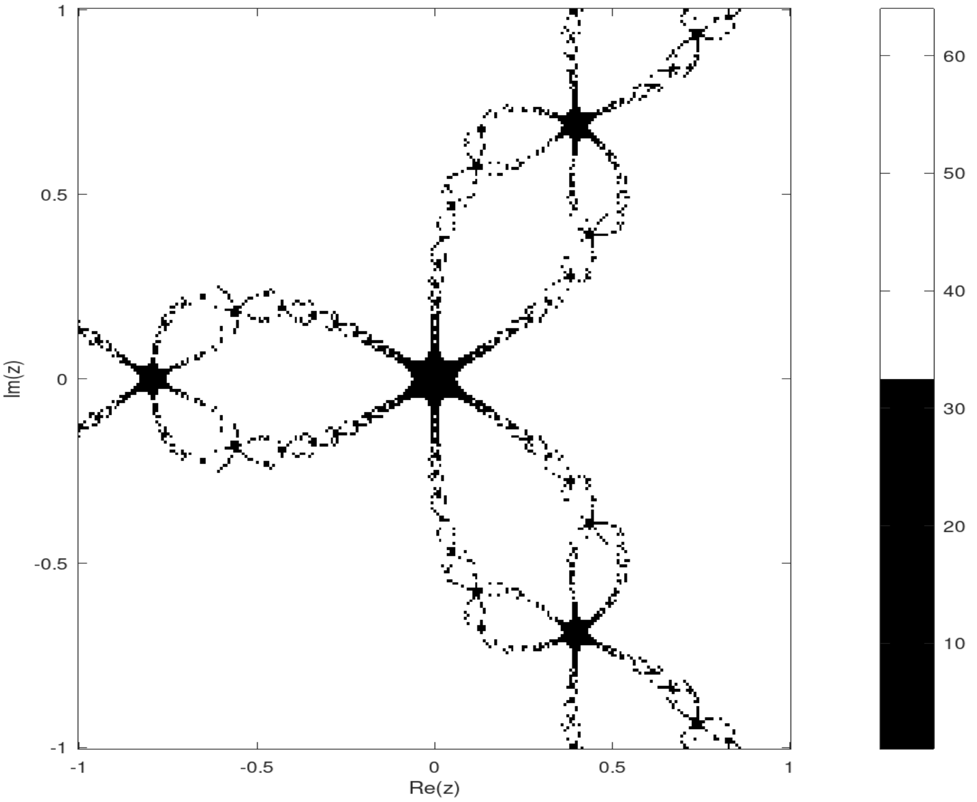


Figure 4: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-12}$

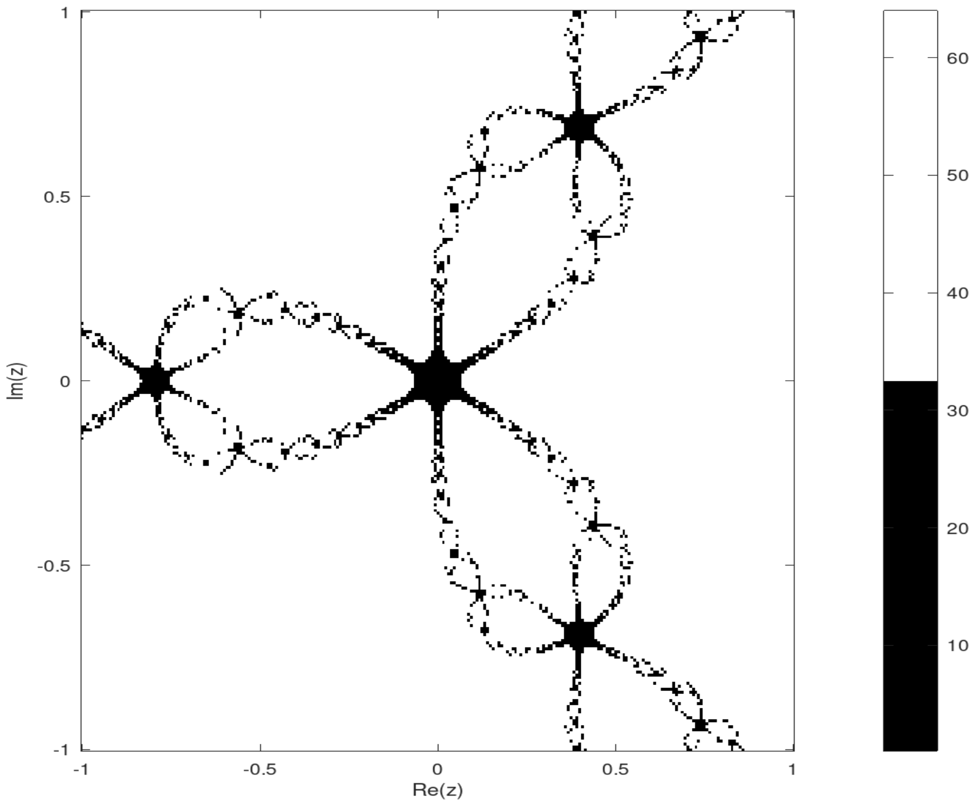


Figure 5: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-14}$

ii. Fije ahora $L = 2$, $n = 256$, $\epsilon = 10^{-10}$ y tome maxit $= 8,16,32$;

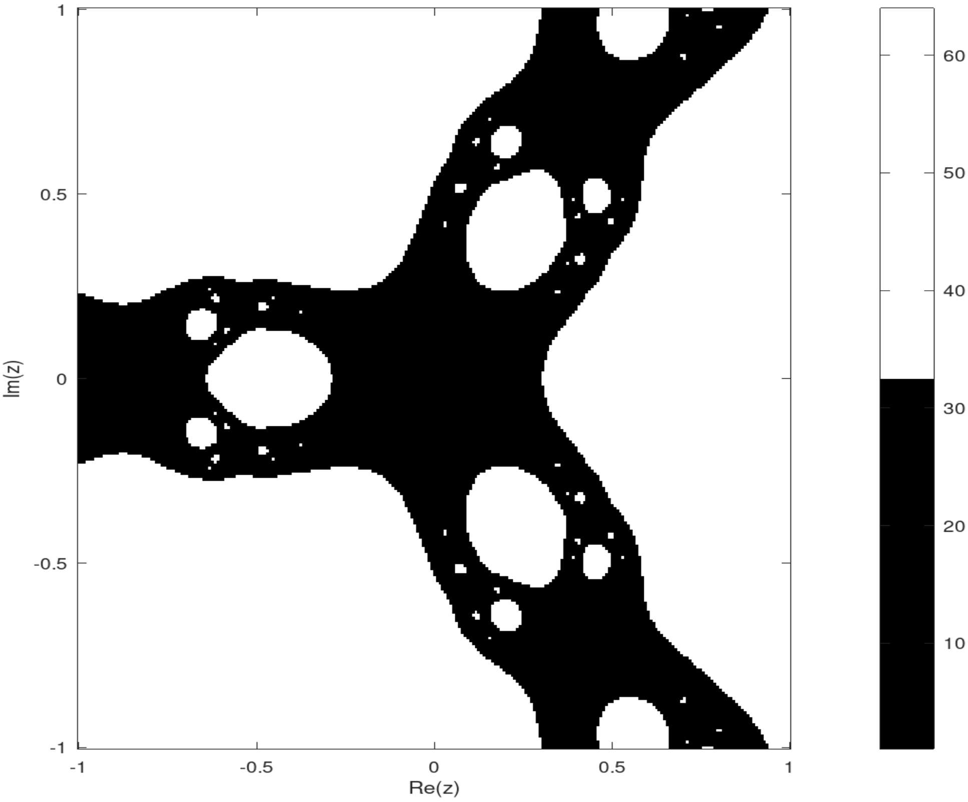


Figure 6: $L = 2$, $n = 256$, maxit $= 8$, $\epsilon = 10^{-10}$

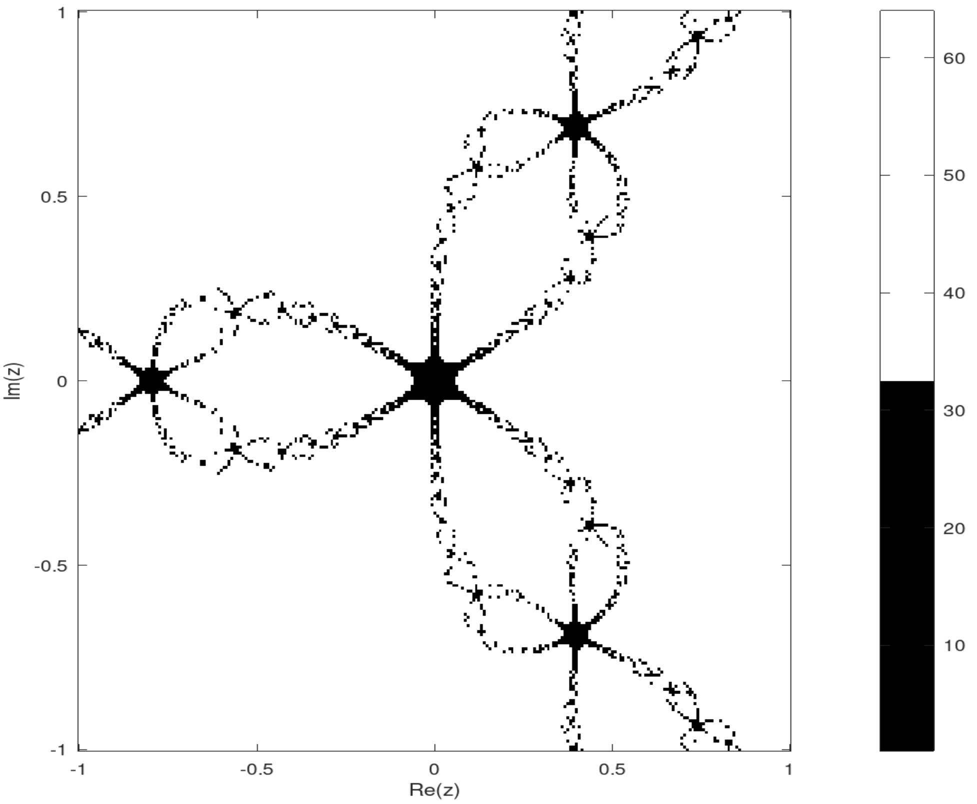


Figure 7: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-10}$

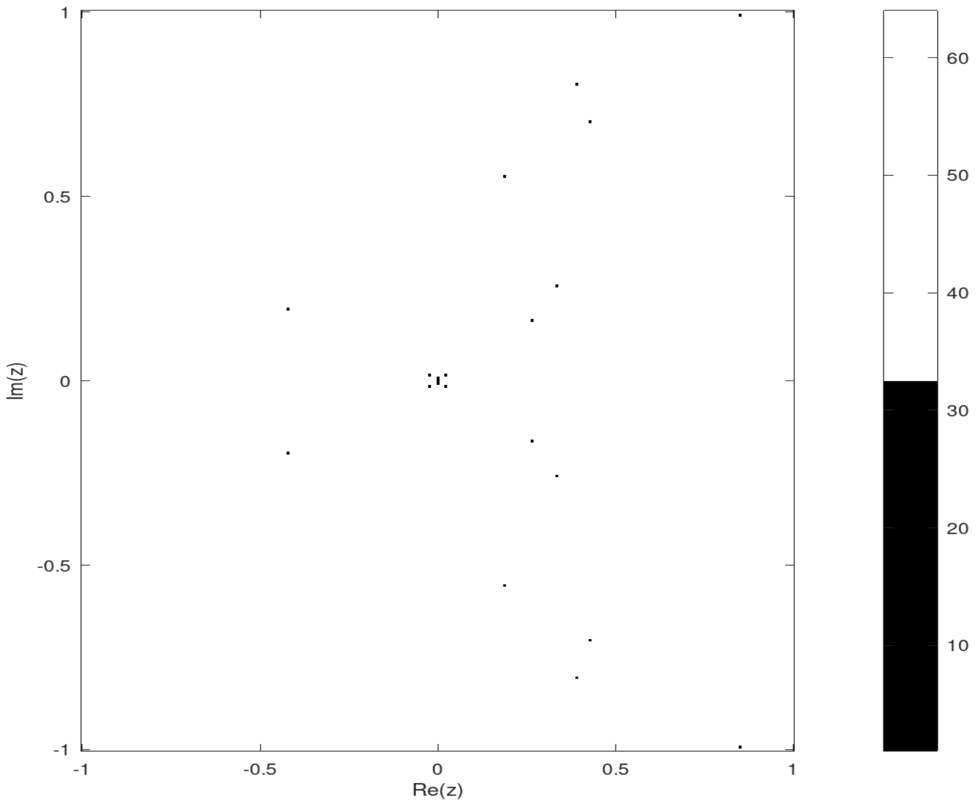


Figure 8: $L = 2$, $n = 256$, $\text{maxit} = 32$, $\epsilon = 10^{-10}$

iii. Fije ahora $L = 2$, $\text{maxit} = 16$, $\epsilon = 10^{-10}$ y tome $n = 32, 64, 128, 256, 512$.

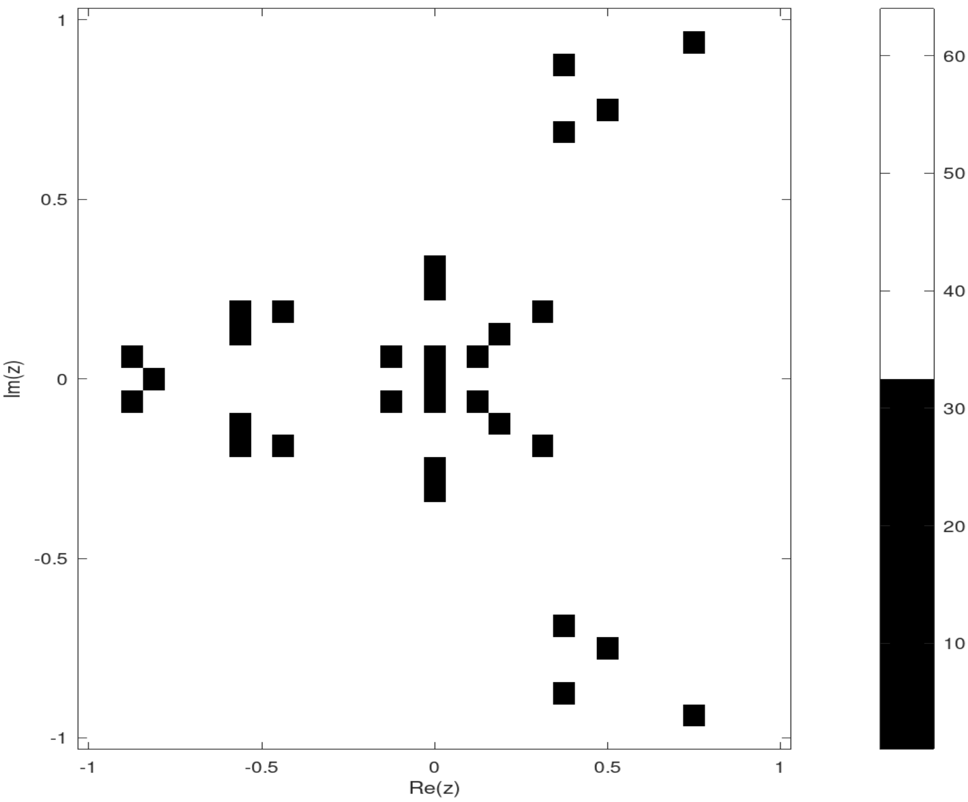


Figure 9: $L = 2$, $n = 32$, maxit $= 16$, $\epsilon = 10^{-10}$

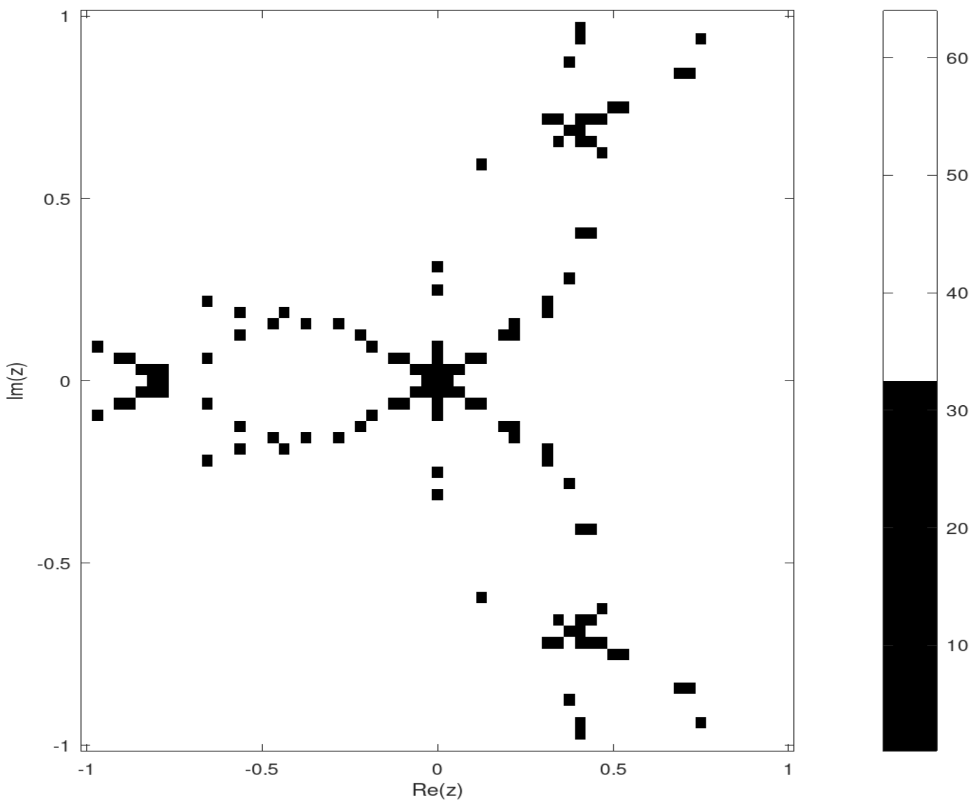


Figure 10: $L = 2$, $n = 64$, maxit $= 16$, $\epsilon = 10^{-10}$

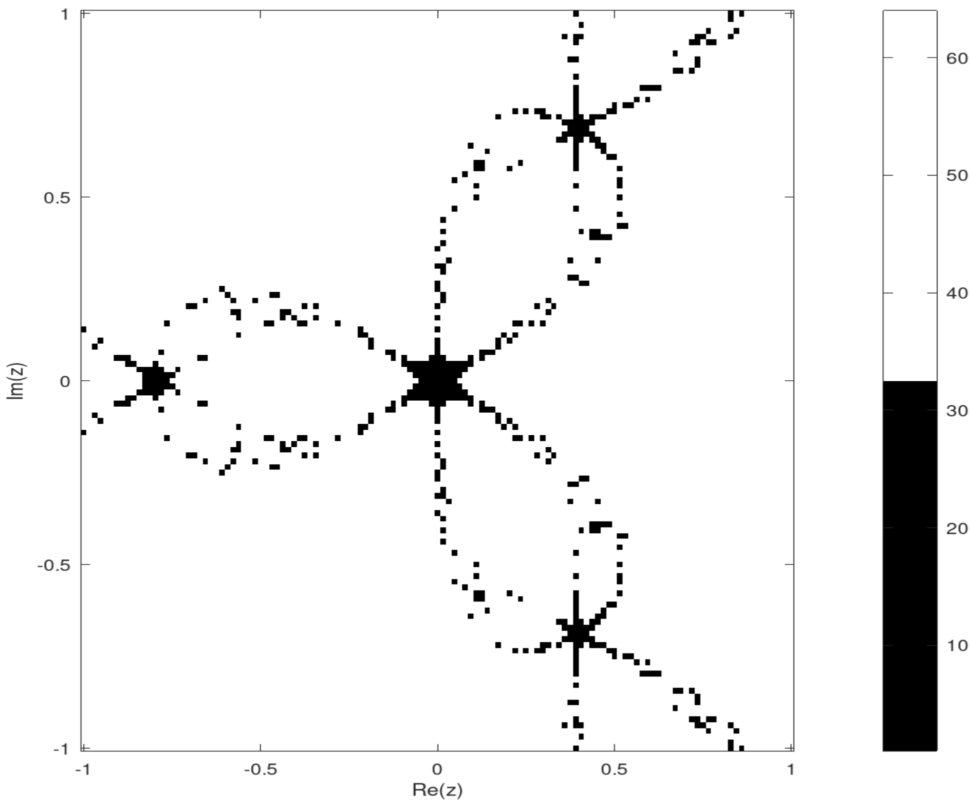


Figure 11: $L = 2$, $n = 128$, $\text{maxit} = 16$, $\epsilon = 10^{-10}$

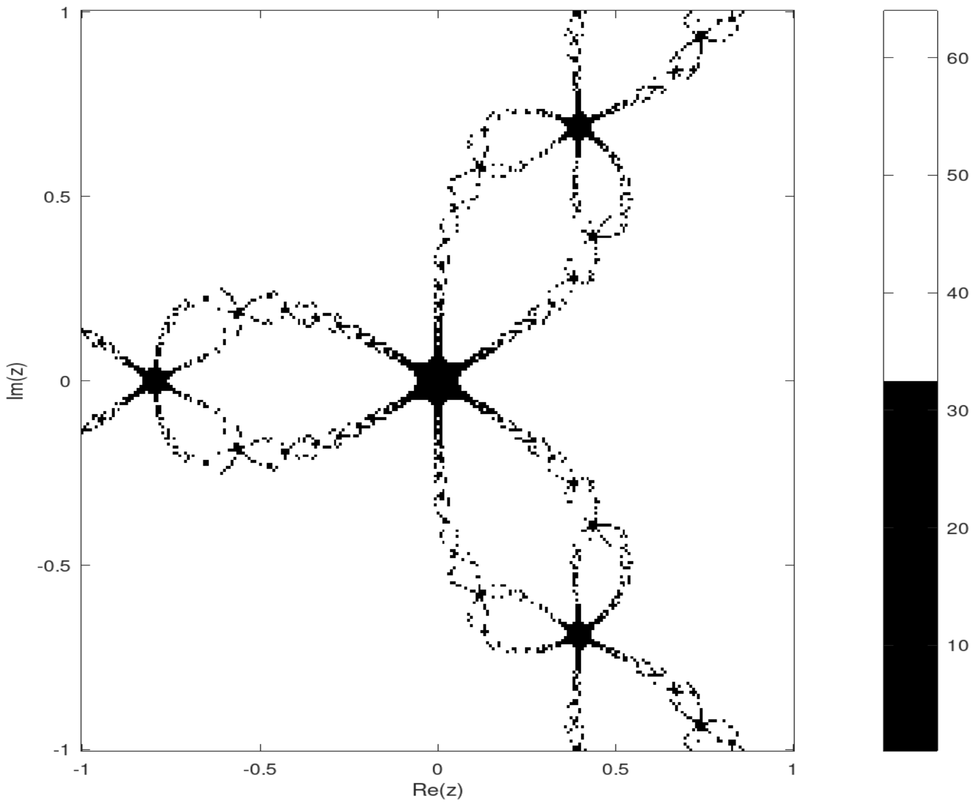


Figure 12: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-10}$

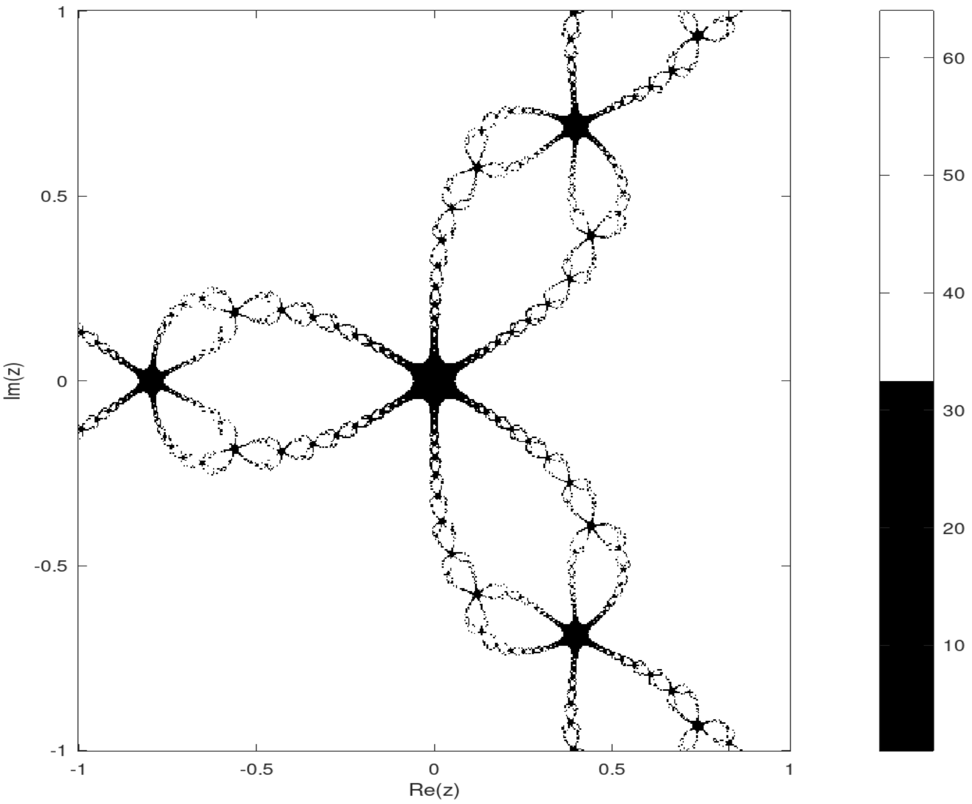


Figure 13: $L = 2$, $n = 512$, maxit $= 16$, $\epsilon = 10^{-10}$

b. Repita el inciso a, pero ahora pinte de rojo si el método de Newton llega a la raíz α_1, de verde si llega a α_2, de azul si llega a α_3 y de blanco si no llega a ninguna raíz.

i Resuelva con $L = 2$,$n = 256$, maxit$= 16$ fijos y diferentes valores de $\epsilon = 10^{-6}, 10^{-8}, 10^{-10}, 10^{-12}$ y 10^{-14}

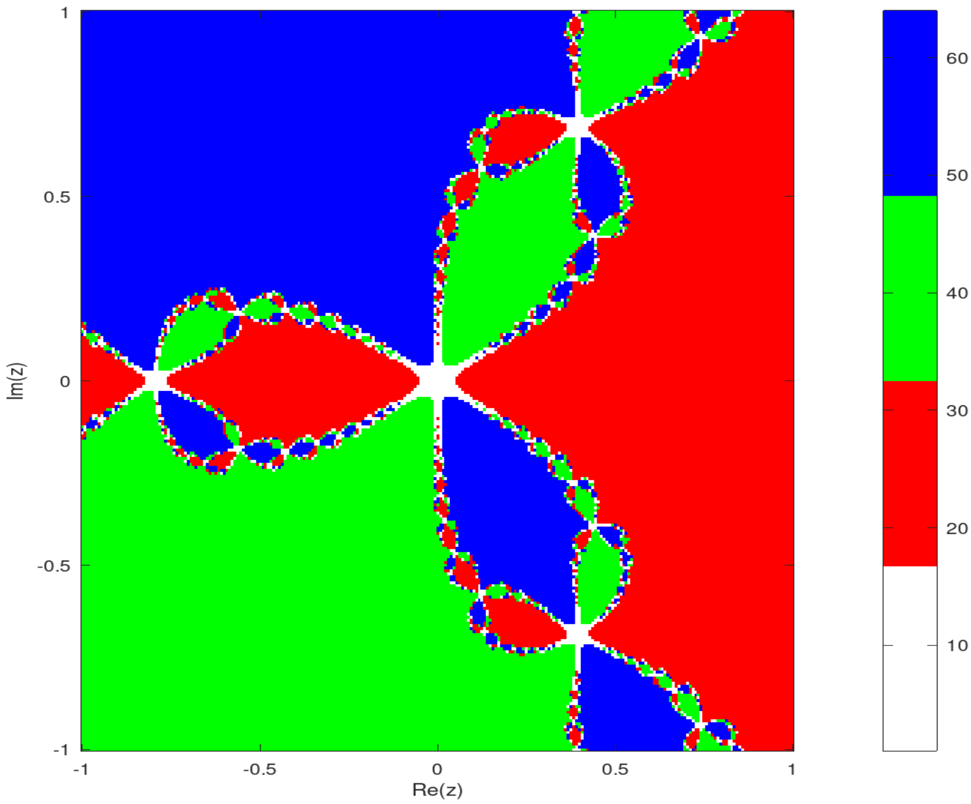


Figure 14: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-6}$ con colores

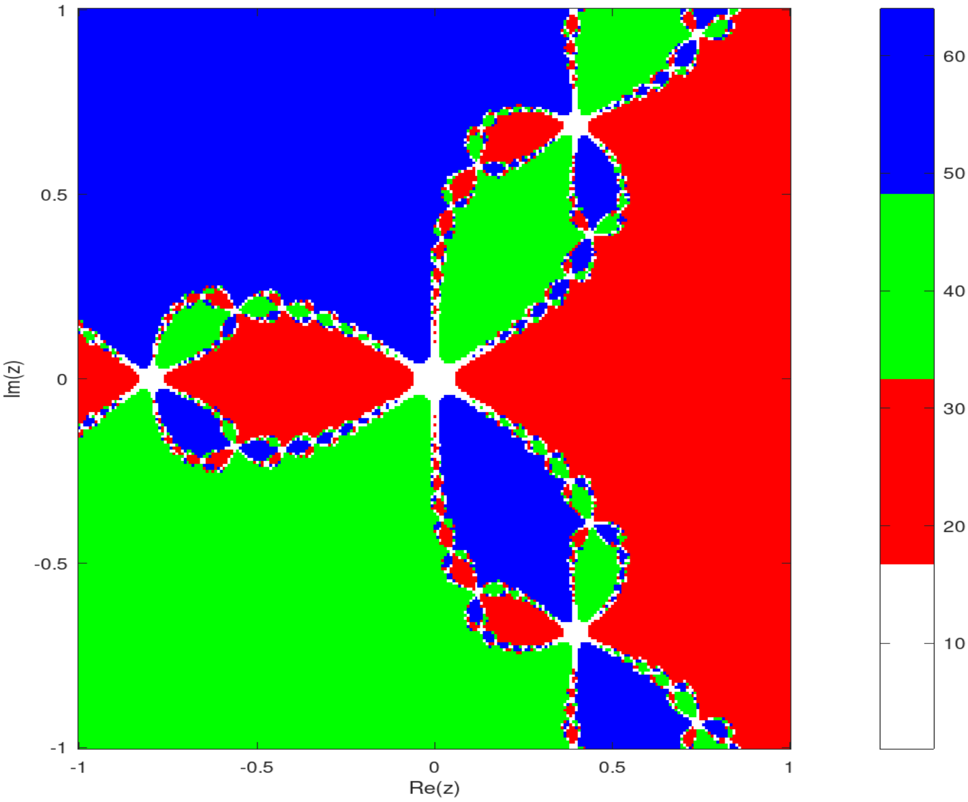


Figure 15: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-8}$ con colores

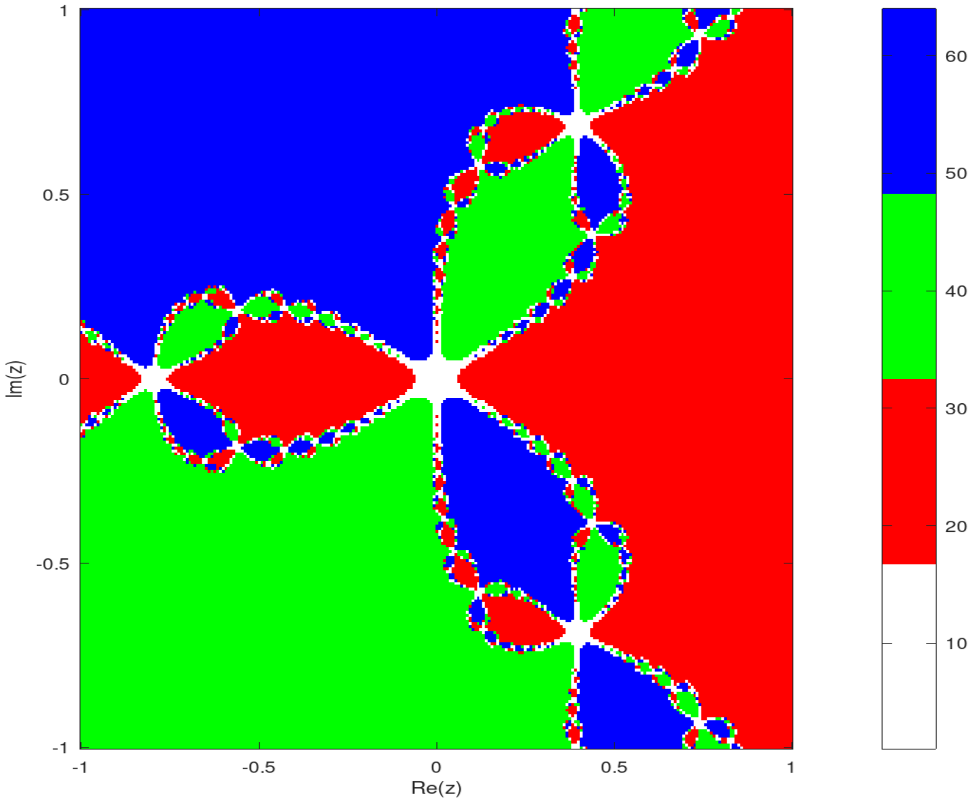


Figure 16: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

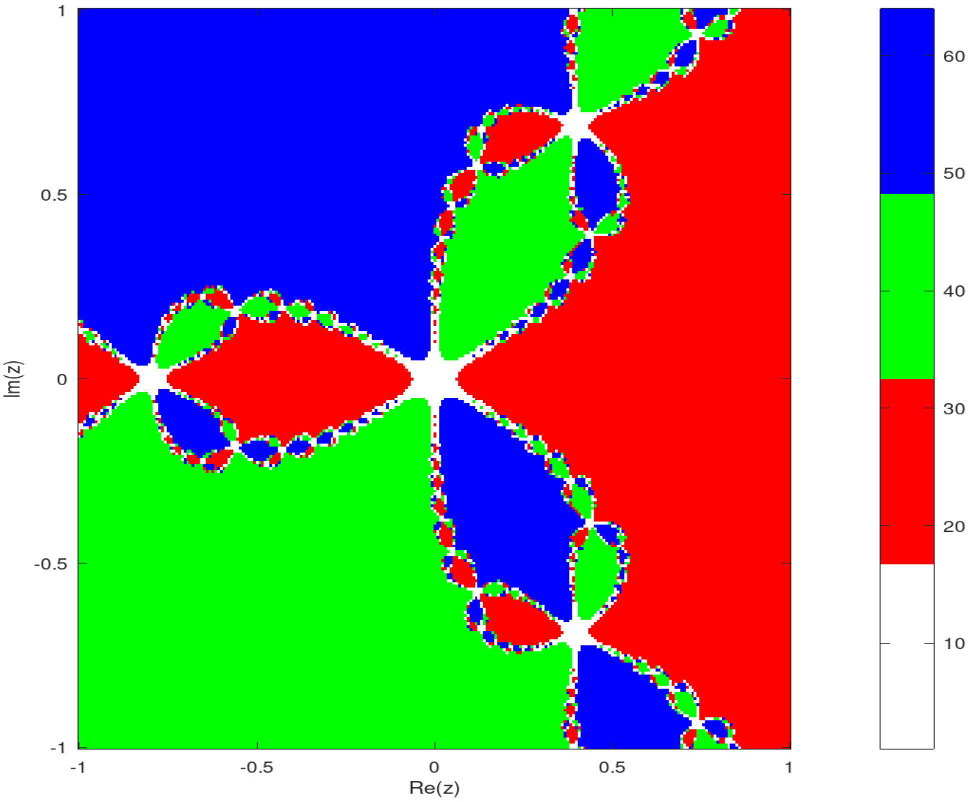


Figure 17: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-12}$ con colores

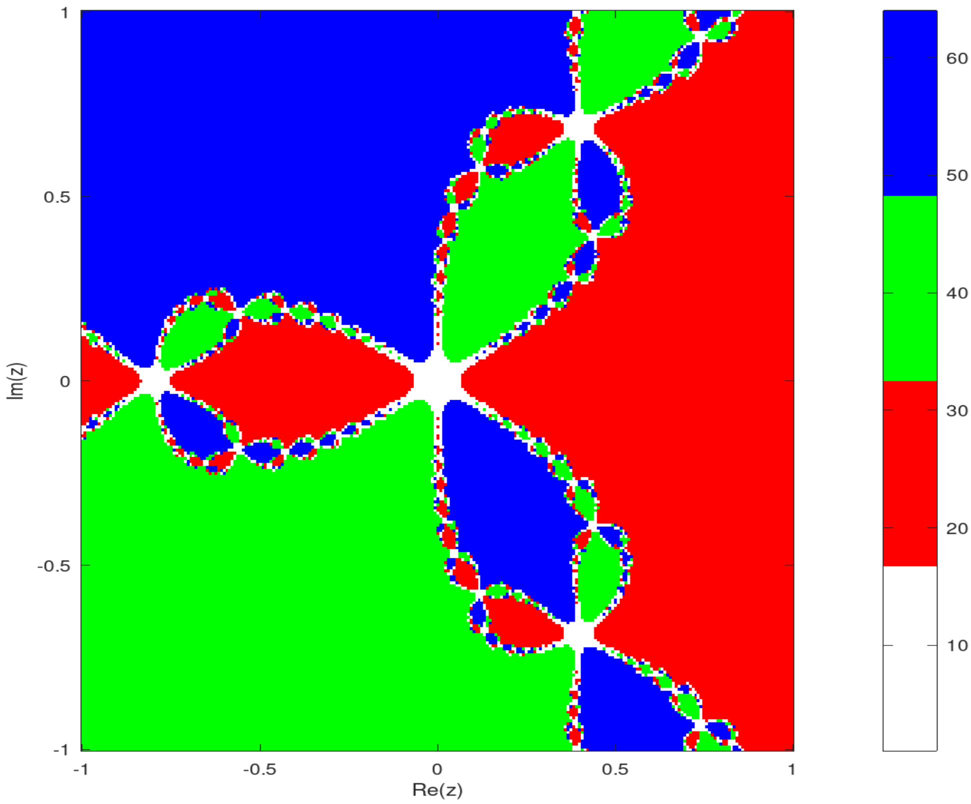


Figure 18: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-14}$ con colores

ii. Fije ahora $L = 2$, $n = 256$, $\epsilon = 10^{-10}$ y tome maxit $= 8,16,32$;

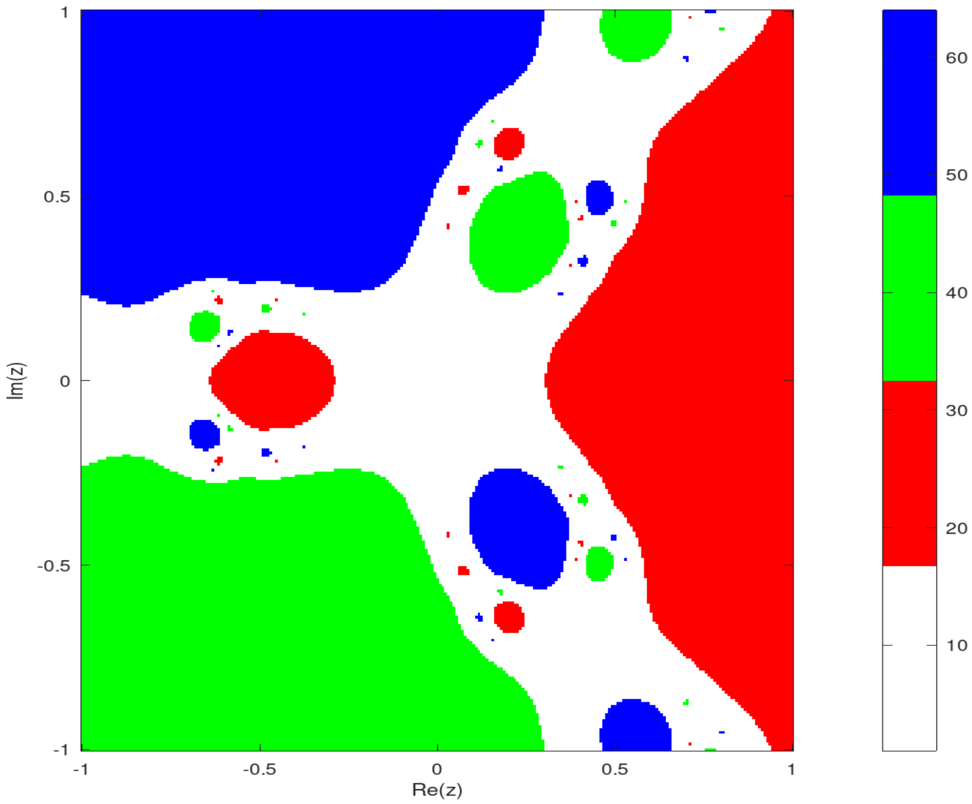


Figure 19: $L = 2$, $n = 256$, maxit $= 8$, $\epsilon = 10^{-10}$ con colores

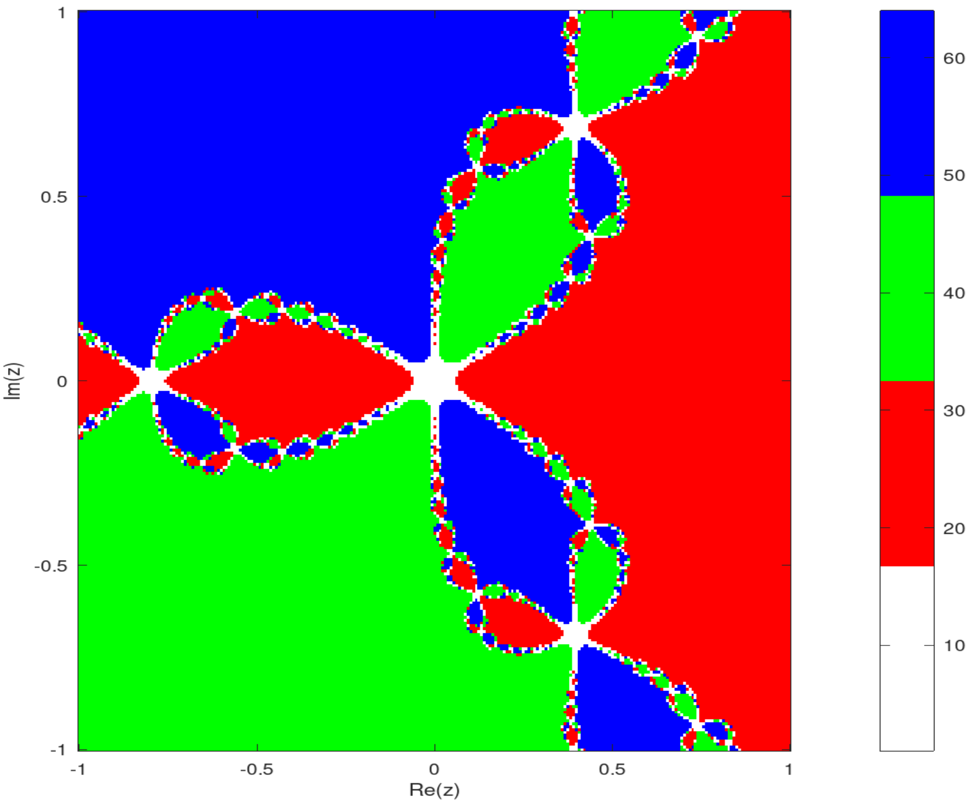


Figure 20: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

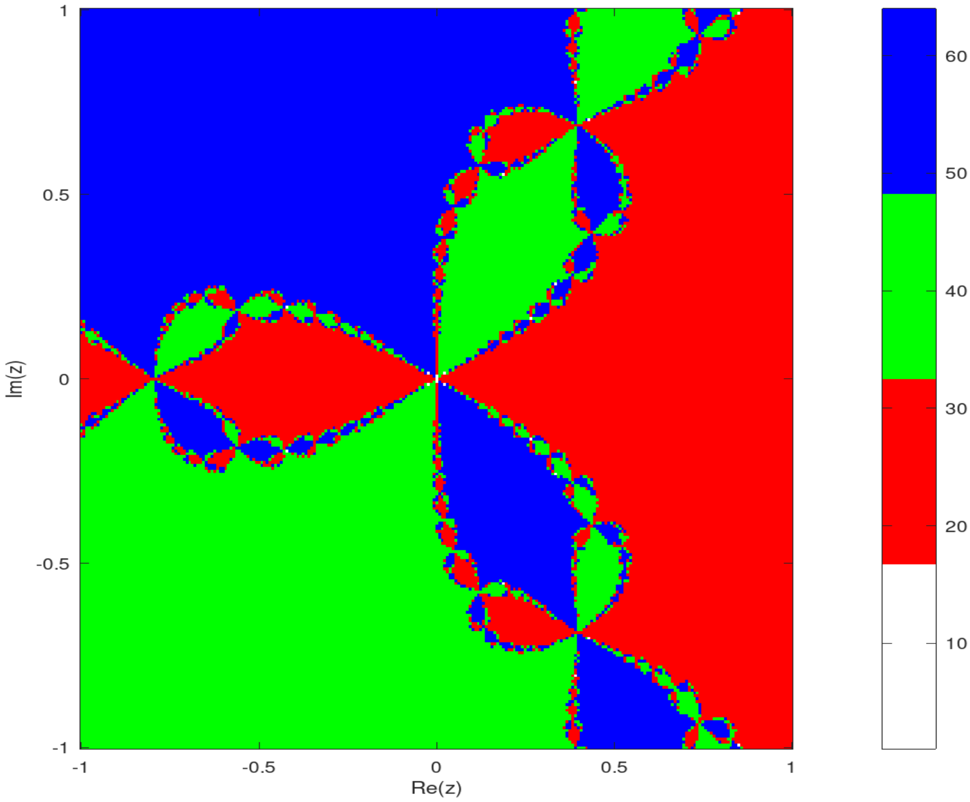


Figure 21: $L = 2$, $n = 256$, maxit $= 32$, $\epsilon = 10^{-10}$ con colores

iii. Fije ahora $L = 2$, maxit $= 16$, $\epsilon = 10^{-10}$ y tome $n = 32, 64, 128, 256, 512$.

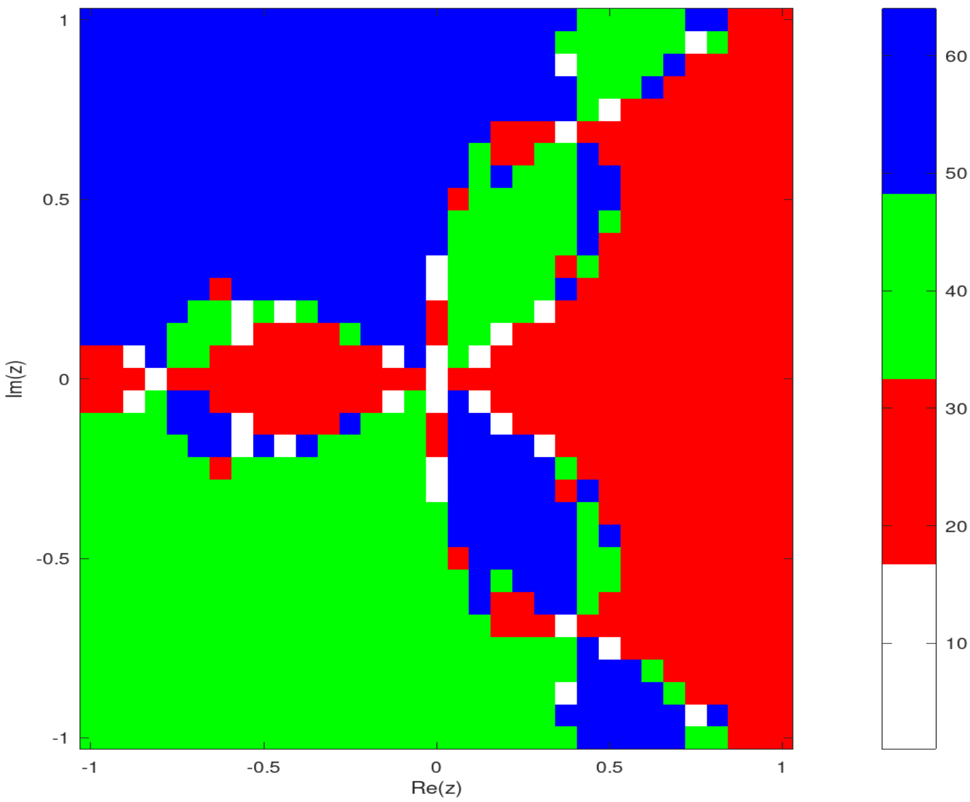


Figure 22: $L = 2$, $n = 32$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

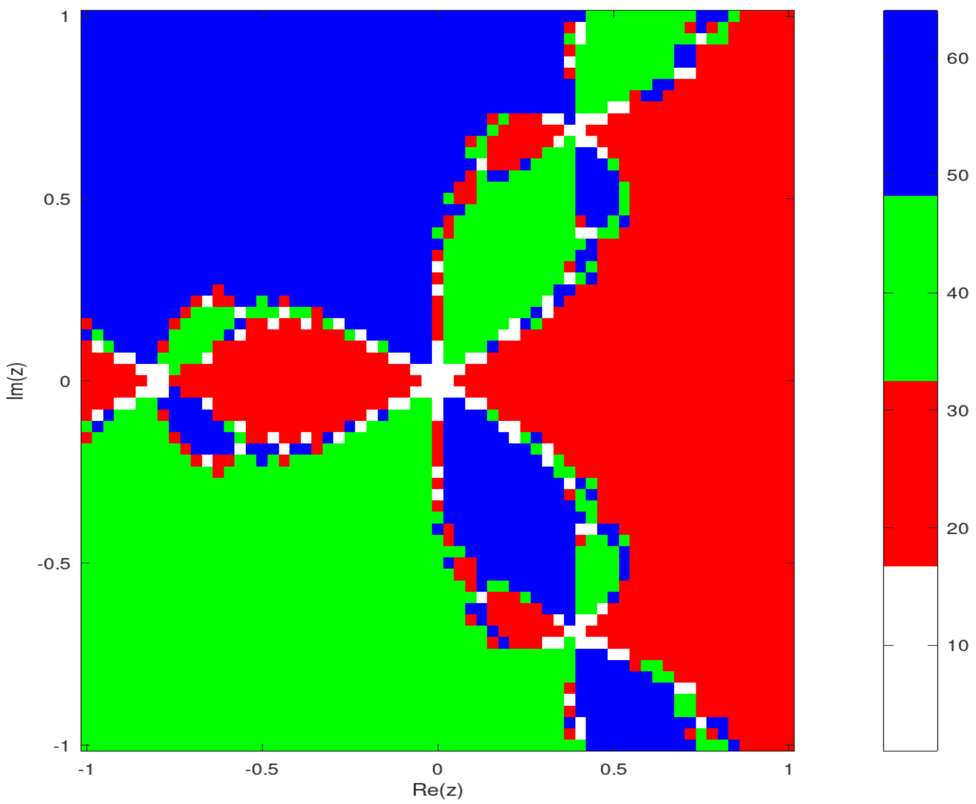


Figure 23: $L = 2$, $n = 64$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

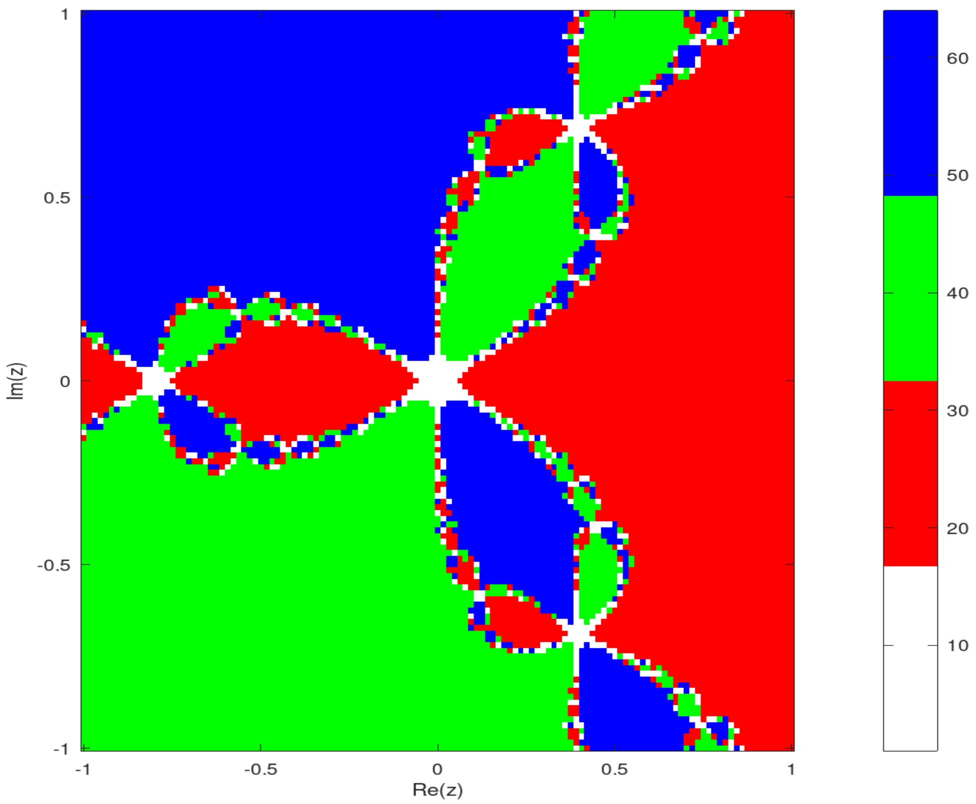


Figure 24: $L = 2$, $n = 128$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

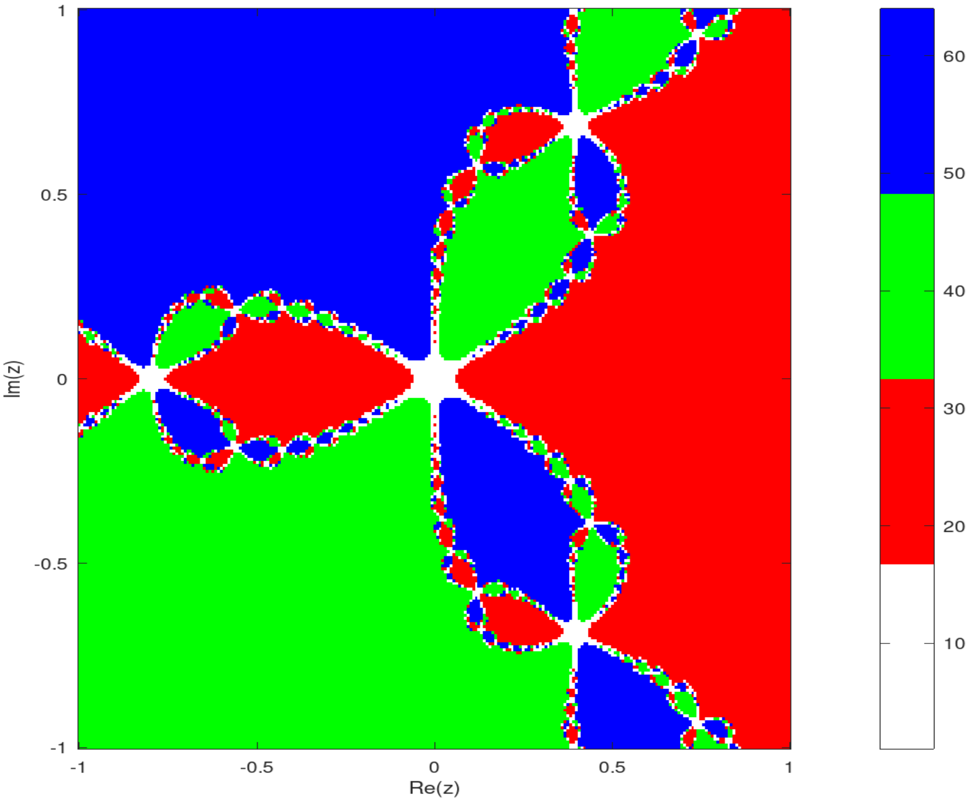


Figure 25: $L = 2$, $n = 256$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

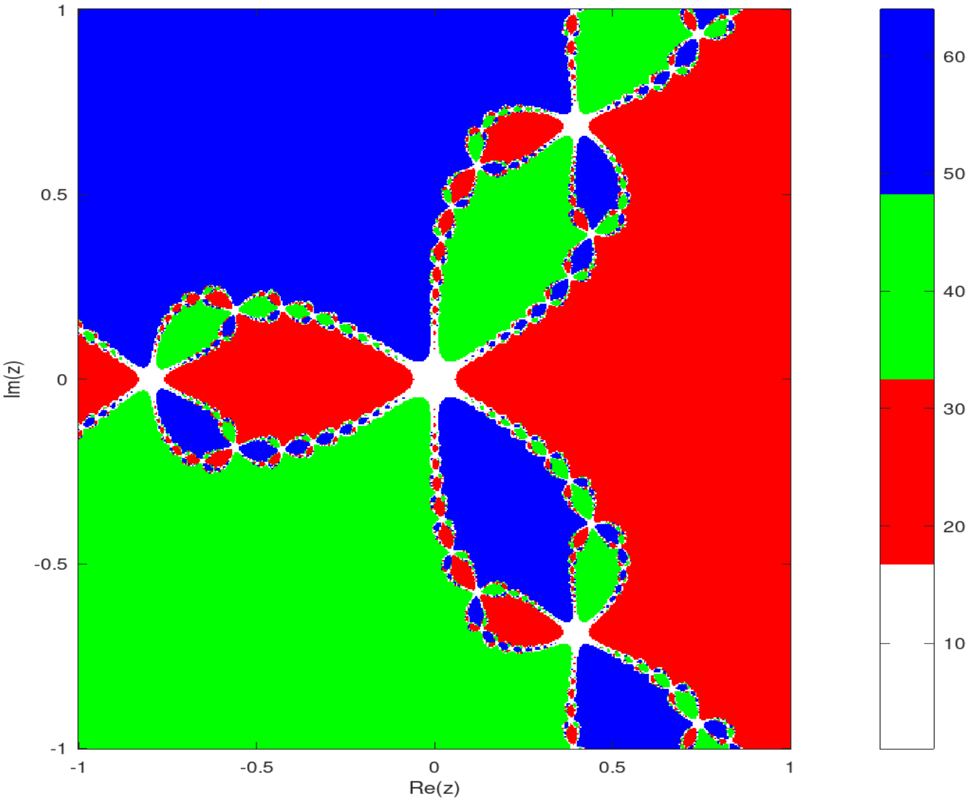


Figure 26: $L = 2$, $n = 512$, maxit $= 16$, $\epsilon = 10^{-10}$ con colores

c. Tome ahora $L = 2$, $n = 256$, $\epsilon = 10^{-10}$ y tome maxit $= 32$. Como el caso anterior, pinte de un color de acuerdo a la raíz a la que llegue pero ahora, en el caso en que llegue a una raíz pinte tonalidades del color correspondiente de acuerdo al numero de iteraciones del método de Newton.

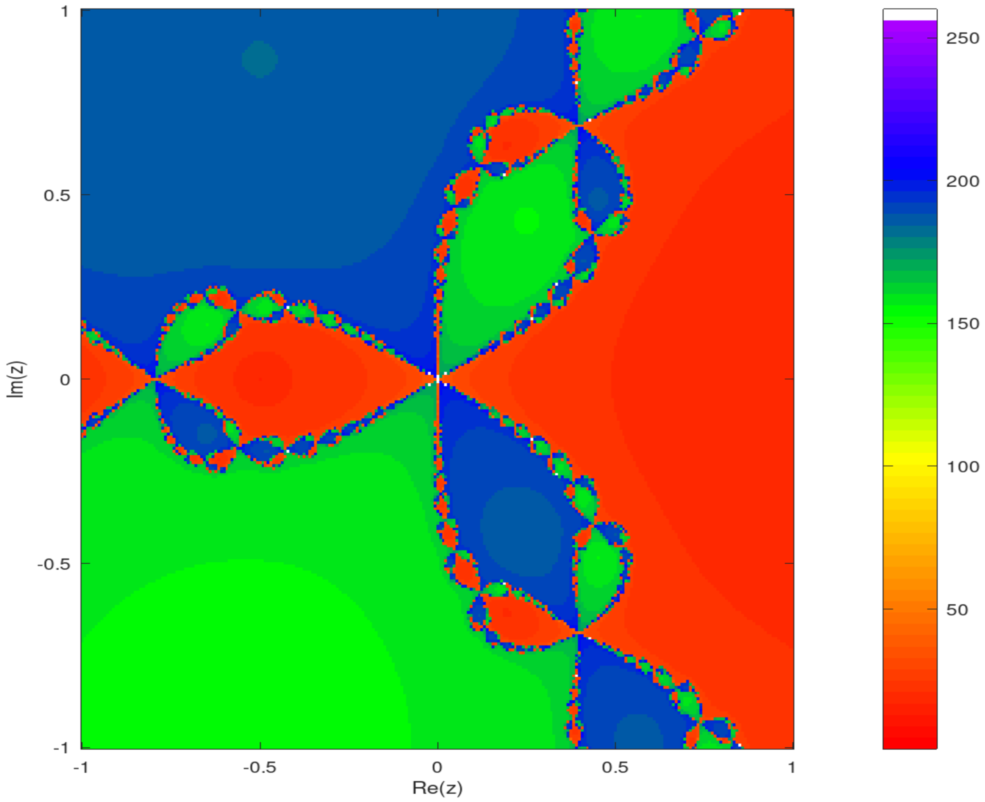


Figure 27: $L = 2$, $n = 256$, maxit $= 32$, $\epsilon = 10^{-10}$ con colores según iteraciones

d. Repita el inciso anterior pero ahora tome $n = 512$ y $n = 1024$ con maxit $= 32$ y 64.

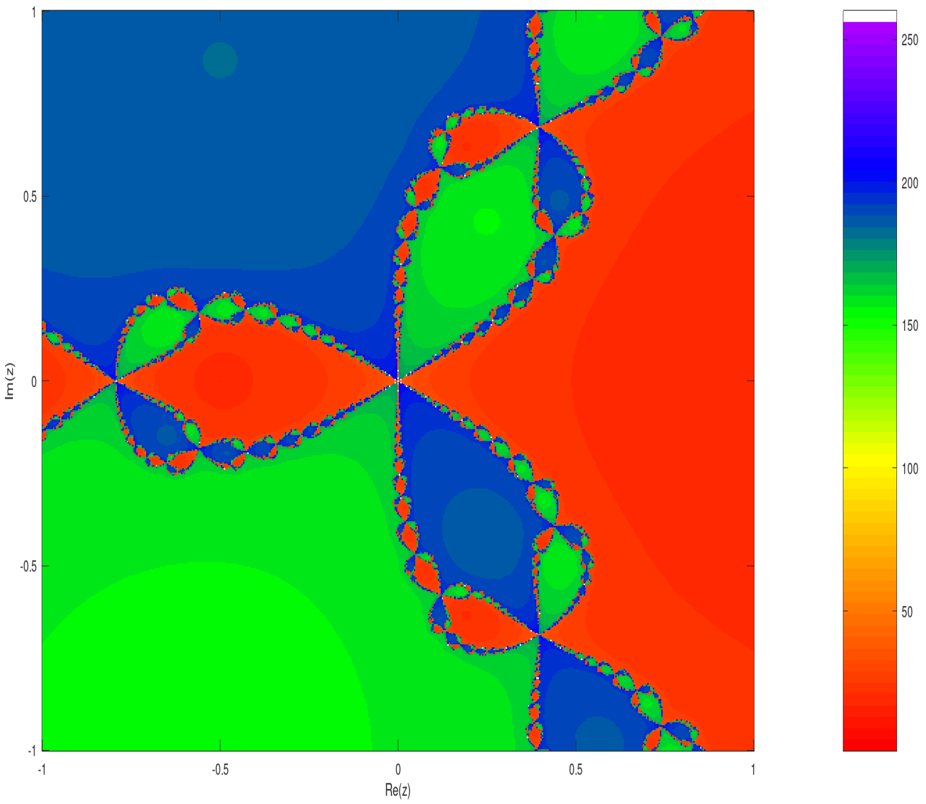


Figure 28: $L = 2$, $n = 512$, maxit $= 32$, $\epsilon = 10^{-10}$ con colores según iteraciones

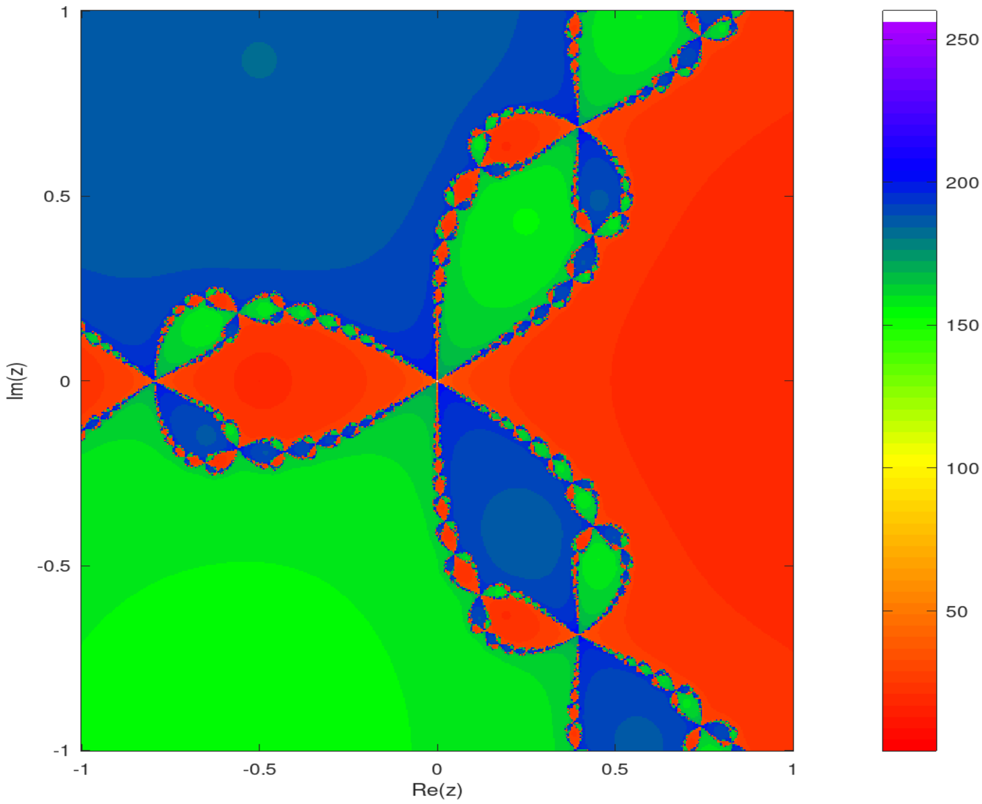


Figure 29: $L = 2$, $n = 512$, maxit $= 64$, $\epsilon = 10^{-10}$ con colores según iteraciones

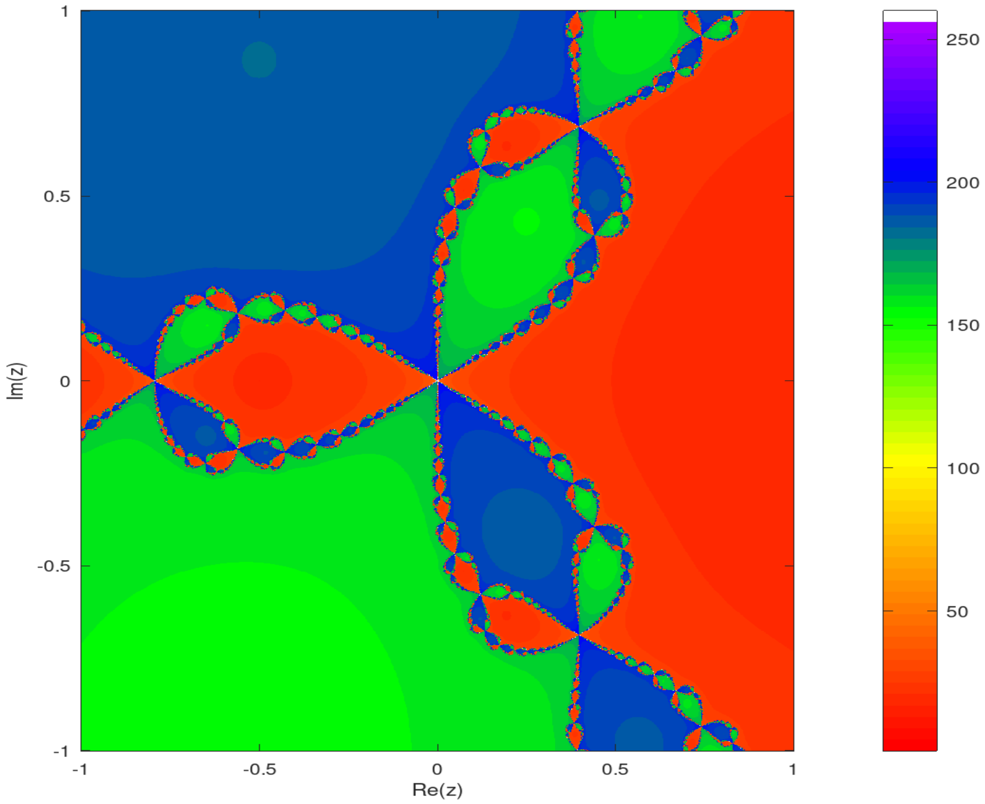


Figure 30: $L = 2$, $n = 1024$, maxit $= 32$, $\epsilon = 10^{-10}$ con colores según iteraciones

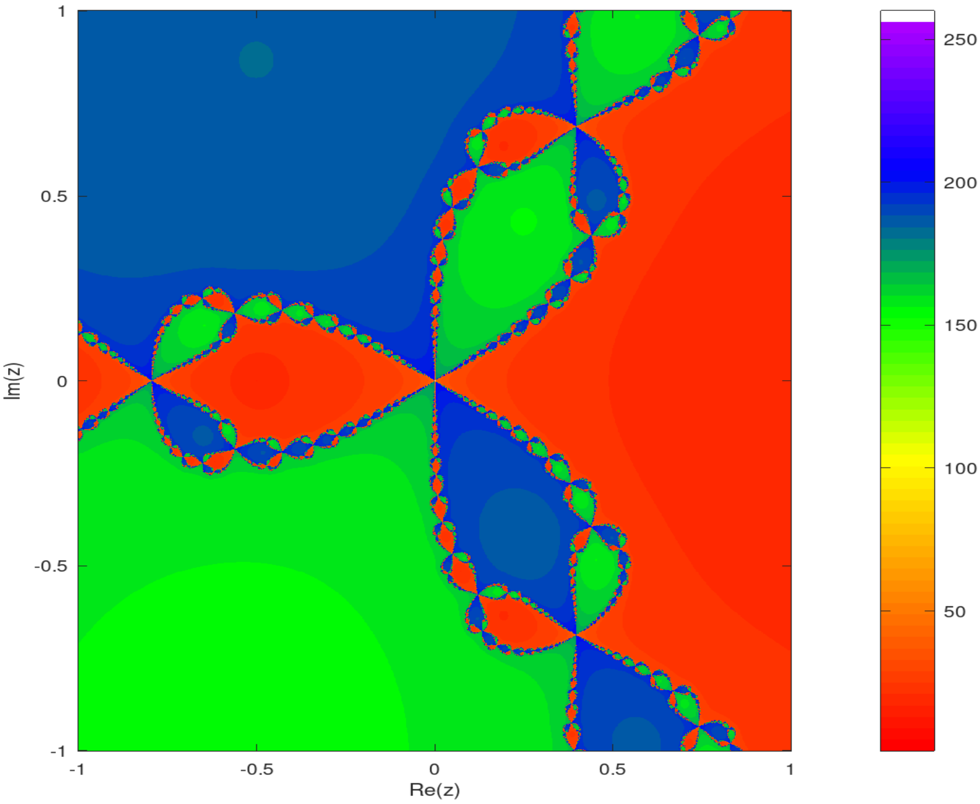


Figure 31: $L = 2$, $n = 1024$, maxit $= 64$, $\epsilon = 10^{-10}$ con colores según iteraciones