Q4 (b) Solve the following recurrence relation $a_r - 3a_{r-1} - 2 = 0$ where $r \geq 1$ with boundary condition $a_0 = 1$

A

$$\begin{cases} a_r = 3a_{r-1} + 2, & r \geq 1 \\ a_r = 1 & r = 0 \end{cases}$$

$a_r = 3a_{r-1} + 2$

$a_r = 3(3a_{r-2} + 2) + 2$

$a_r = 3^2 a_{r-2} + 3.2 + 2$

$a_r = 3^2(3a_{r-3} + 2) + 3.2 + 2$

$a_r = 3^3(a_{r-3}) + 2 \cancel{\times} 3^2 + 3.2 + 2$

$a_r = 3^4 a_{r-4} + 2.3^3 + 2.3^2 + 2.3 + 2$

$\vdots$

$a_r = 3^r a_{r-r} + 2.3^{r-1} + 2.3^{r-2} \cdots 2.3^3 + 2.3^2 + 2.3^1 + 2$

$a_r = 3^r + 2(3^{r-1} + 3^{r-2} + \cdots 3^1 + 1)$ $\quad a_0 = 1$

Solving

$S = 3^{r-1} + 3^{r-2} + \cdots 3^1 + 1$ using GP sum formula

$a = 1$, $r = 3$, $n = r$

$$S = \frac{1 \times (3^r - 1)}{3 - 1} = \frac{3^r - 1}{2}$$

$$a_r = 3^r + 2 \times \left(\frac{3^r - 1}{2}\right)$$

$$\boxed{a_r = 2 \times 3^r - 1}$$