Student Information

Full Name : Muhammed Ömer
Id Number : 2683142

Answer 1

a)

Yes, there is an Euler path, because only two vertices (a and g) have odd degree. This is the path: $a - b - d - g - c - f - d - a - e - c - b - c - g$

b)

No, there is no Euler circuit in G, because two vertices (a and g) have odd degree. I can create an Euler circuit by adding edge between a and g. So this will be the path: $a - e - b - a - d - f - g - d - b - c - g - a$

c)

Yes, there is a Hamilton path such that: $a - d - f - g - c - b - e$

d)

Yes, there is a Hamilton circuit such that: $a - d - f - g - c - b - e - a$

Answer 2

Two graphs G and H with the following properties (from the diagram):

- G has 8 vertices, 16 edges, and each vertex has degree 4.
- H has 8 vertices, 16 edges, and each vertex has degree 4.

First, let's check the invariants: the number of vertices, the number of edges, and the degree of each vertex. These invariants are the same for both G and H, so they might be isomorphic. However, this is not sufficient to conclude isomorphism; we need to construct a one-to-one correspondence between the vertex sets and verify edge preservation.

We define the following bijection between the vertex sets of G and H:

$$\begin{aligned} &i(a) = z, && i(b) = t, && i(c) = u, && i(d) = v, \\ &i(e) = x, && i(f) = y, && i(g) = r, && i(h) = s. \end{aligned}$$

This function maps each vertex in G to a unique vertex in H.

To confirm that i is an isomorphism, we need to ensure that it preserves edges. We examine the adjacency matrices of G and H under the mapping i. Let the adjacency matrix of G be A_G, where the rows and columns are labeled a, b, c, d, e, f, g, h:

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

Now, relabel the rows and columns of A_G according to the mapping i to construct A_H for H:

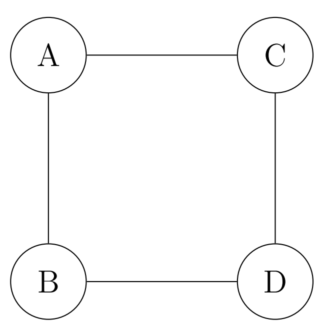
$$A_H = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

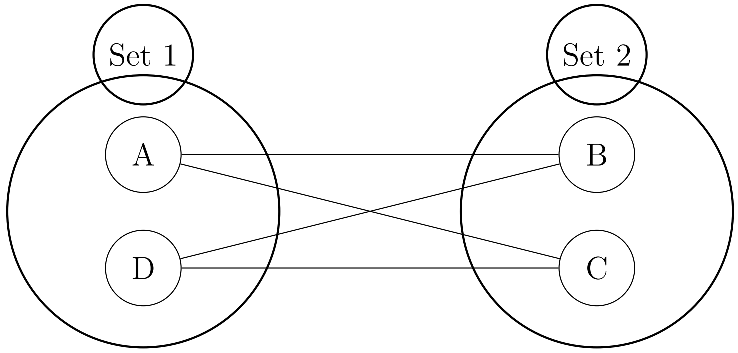
Since $A_G = A_H$, the mapping i preserves edges.
Since i is a bijection that preserves edges, the graphs G and H are isomorphic.

Answer 3

a)

A, D represents wives and B, C represents their children.





And this shows us it's bipartite.

b)

A graph $G = (V, E)$ is bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that no two vertices within the same set are adjacent. Formally:

$$\forall x, y \in V \mid (x \in V_1 \wedge y \in V_1) \vee (x \in V_2 \wedge y \in V_2) \implies (x, y) \notin E.$$

c)

A graph $G = (V, E)$ is regular if every vertex in V has the same degree k, which is constant. Formally:

$$\exists k \in N \ , \forall v \in V \mid \ \deg(v) = k.$$

where $\deg(v)$ denotes the degree of vertex v, defined as the number of edges incident to v.

d)

Suppose G is a tree with n vertices, and the graph is m-degree regular.
Sum of degrees should be equal, if the graph is a tree.

$$\text{Sum of degrees} = 2 \cdot (\text{number of edges}),$$

which implies:

$$mn = 2(n-1) \rightarrow m = 2 - 2/n$$

For $n > 2$, m is not an integer, leading to a contradiction. So, a regular graph can not be a tree if it has more than two vertices.

e)

For n-vertices, if we assume an m-regular bipartite graph, let V_1 and V_2 represent the partitions and $a + b = n$ such that:

$$|V_1| = a, \quad |V_2| = b.$$

Then:

$$ma = mb,$$

which implies:

$$a = b.$$

Also we can say that:
Let $G = (V, E)$ be a regular bipartite graph with partition $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$. Suppose the degree of each vertex is k. Since G is bipartite:

- Every edge connects a vertex in V_1 to a vertex in V_2.
- The total number of edges is $|E| = k|V_1| = k|V_2|$ (each vertex in V_1 contributes k edges, and similarly for V_2).

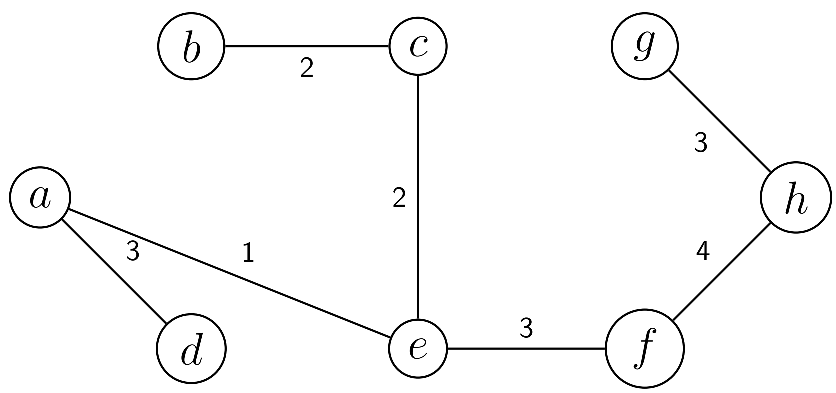
From $k|V_1| = k|V_2|$, it follows that $|V_1| = |V_2|$.

Answer 4

a)

Prim's Algorithm: Traverse by Vertex Starting from vertex a, the traversal by Prim's algorithm is as follows:

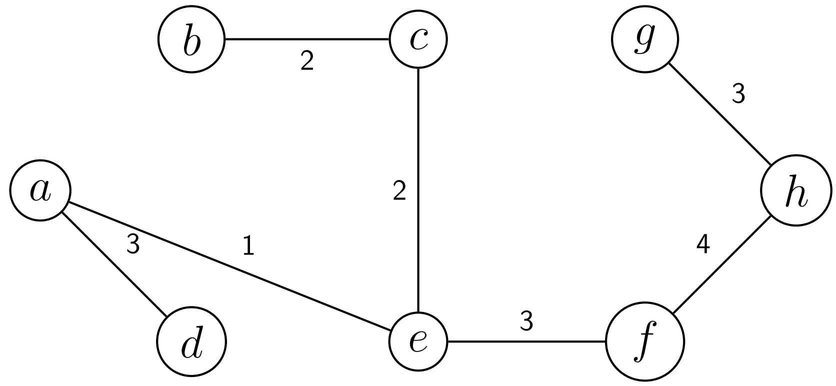
$$a \to e \to c \to b \to d \to f \to h \to g$$



b)

Kruskal's Algorithm: Traverse by Edges The traversal by Kruskal's algorithm is as follows:

$$a - e,\ c - e,\ c - b,\ a - d,\ e - f,\ g - h,\ f - h$$



c)

Dijkstra's Algorithm Table

The table below represents the steps in Dijkstra's algorithm. Boxed values indicate updated shortest paths.

Step	b	c	d	e	f	g	h
a	7	–	3	$\boxed{1}$	–	–	–
e	4	$\boxed{3}$	3	$\boxed{1}$	4	–	–
c	4	$\boxed{3}$	$\boxed{3}$	–	4	8	11
d	$\boxed{4}$	–	$\boxed{3}$	–	4	8	11
b	$\boxed{4}$	–	–	–	$\boxed{4}$	8	11
f	–	–	–	–	$\boxed{4}$	$\boxed{8}$	8
g	–	–	–	–	–	$\boxed{8}$	$\boxed{8}$
h	–	–	–	–	–	–	$\boxed{8}$

1) a $\rightarrow$ e is the shortest one among adjacent vertices.
2) e $\rightarrow$ c is the shortest, choose c.
3) c $\rightarrow$ b is the shortest, choose b.
3) There is no unvisited vertex from b so, let us get back.
4) From c there is no other shorter path to unvisited than from e. So again from e to the shortest unvisited one is f, choose f.
5) f $\rightarrow$ h is the shortest one among adjacent vertices. So, choose h.

Thus, we reached the vertex h from the shortest path and distance is 8. There is no other shorter path or a path that has similar distance so, it is unique.

So, we obtained a path such that a $\rightarrow$ e $\rightarrow$ f $\rightarrow$ h.