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Answer 1

a)

p	q	$\neg q$	$\neg p$	$\neg p \wedge q$	$p \vee q$	$p \wedge (p \vee q)$	$(p \wedge (p \vee q)) \wedge \neg q$	$(\neg p \vee q) \vee ((p \wedge (p \vee q)) \wedge \neg q)$
T	T	F	F	F	T	T	F	F
T	F	T	F	F	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	T	F	F	F	F	F

Neither.

b)

$$\begin{aligned}
 p \vee (\neg q \rightarrow (p \wedge r)) &\equiv p \vee (q \vee (p \wedge r)) && \text{(Implication Elimination Law)} \\
 &\equiv (p \vee q) \vee (p \wedge r) && \text{(Association Laws)} \\
 &\equiv (q \vee p) \vee (p \wedge r) && \text{(Commutative Laws)} \\
 &\equiv q \vee (p \vee (p \wedge r)) && \text{(Association Laws)} \\
 &\equiv q \vee ((p \wedge T) \vee (p \wedge r)) && \text{(Identity Laws)} \\
 &\equiv q \vee (p \wedge (T \vee r)) && \text{(Distributive Laws)} \\
 &\equiv q \vee (p \wedge T) && \text{(Domination Laws)} \\
 &\equiv q \vee p && \text{(Identity Laws)} \\
 &\equiv p \vee q && \text{(Commutative Laws)}
 \end{aligned}$$

Answer 2

a)

$$\forall x \forall y (S(x) \wedge C(y) \wedge E(x, y) \rightarrow \forall z (C(z) \wedge R(z, y) \rightarrow P(x, z)))$$

b)

$$\exists x (S(x) \wedge \exists y (C(y) \wedge E(x, y) \wedge \forall z (C(z) \wedge E(x, z) \rightarrow (y = z))))$$

c)

$$\forall y ((C(y) \rightarrow \forall x (S(x) \wedge P(x, y) \rightarrow \forall z (C(z) \wedge R(z, y) \rightarrow P(x, z))))$$

d)

$$\exists x(S(x) \wedge \exists y(C(y) \wedge E(x, y) \wedge \neg P(x, y)))$$

e)

$$\exists y(C(y) \wedge \forall x(S(x) \rightarrow \neg P(x, y)))$$

f)

$$\forall y(C(y) \rightarrow \exists x(S(x) \wedge (E(x, y) \vee P(x, y))))$$

Answer 3

- | | |
|-------------------------------|---------|
| 1. $p \rightarrow (q \vee r)$ | Premise |
| 2. $\neg r \wedge \neg s$ | Premise |
| 3. $q \rightarrow s$ | Premise |

- | | |
|---------------|-----------------------|
| 4. p | Assumption |
| 5. $q \vee r$ | $\rightarrow e, 1, 4$ |

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|-------------|-----------------------|
| 6. q | Assumption |
| 7. s | $\rightarrow e, 3, 6$ |
| 8. $\neg s$ | $\wedge e, 2$ |
| 9. \perp | $\neg e, 7, 8$ |

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|--------------|------------------|
| 10. r | Assumption |
| 11. $\neg r$ | $\wedge e, 2$ |
| 12. \perp | $\neg e, 10, 11$ |

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|-------------|----------------------|
| 13. \perp | $\vee e, 6-9, 10-12$ |
|-------------|----------------------|

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|--------------|----------------|
| 14. $\neg p$ | $\neg i, 4-13$ |
|--------------|----------------|

Answer 4

a)

1. $\exists x(S(x) \wedge P(x))$
2. $\forall x(P(x) \rightarrow K(x))$
3. $\exists x(S(x) \wedge K(x))$

b)

1. $\exists x(S(x) \wedge P(x))$	Premise
2. $\forall x(P(x) \rightarrow K(x))$	Premise
3. $P(c) \wedge S(c)$	Assumption (c, Skolem constant)
4. $P(c)$	$\wedge e, 3$
5. $S(c)$	$\wedge e, 3$
6. $P(c) \rightarrow K(c)$	$\forall e, 2$
7. $K(c)$	$\rightarrow e, 4, 6$
8. $S(c) \wedge K(c)$	$\wedge i, 5, 7$
9. $\exists x(S(x) \wedge K(x))$	$\exists i, 8$
10. $\exists x(S(x) \wedge K(x))$	$\exists e, 1, 2, 3-9$