

Name : Muhammed Ömer

ID: 2683142

## Answer 1

a)

$p$	$q$	$\neg q$	$\neg p$	$\neg p \wedge q$	$p \vee q$	$p \wedge (p \vee q)$	$(p \wedge (p \vee q)) \wedge \neg q$	$(\neg p \vee q) \vee ((p \wedge (p \vee q)) \wedge \neg q)$
$T$	$T$	$F$	$F$	$F$	$T$	$T$	$F$	$F$
$T$	$F$	$T$	$F$	$F$	$T$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$	$T$	$F$	$F$	$T$
$F$	$F$	$T$	$T$	$F$	$F$	$F$	$F$	$F$

Neither.

b)

$$\begin{aligned} p \vee (\neg q \rightarrow (p \wedge r)) &\equiv p \vee (q \vee (p \wedge r)) && \text{(Implication Elimination Law)} \\ &\equiv (p \vee q) \vee (p \wedge r) && \text{(Association Laws)} \\ &\equiv (q \vee p) \vee (p \wedge r) && \text{(Commutative Laws)} \\ &\equiv q \vee (p \vee (p \wedge r)) && \text{(Association Laws)} \\ &\equiv q \vee ((p \wedge T) \vee (p \wedge r)) && \text{(Identity Laws)} \\ &\equiv q \vee (p \wedge (T \vee r)) && \text{(Distributive Laws)} \\ &\equiv q \vee (p \wedge T) && \text{(Domination Laws)} \\ &\equiv q \vee p && \text{(Identity Laws)} \\ &\equiv p \vee q && \text{(Commutative Laws)} \end{aligned}$$

## Answer 2

a)

$$\forall x \forall y (S(x) \wedge C(y) \wedge E(x, y) \rightarrow \forall z (C(z) \wedge R(z, y) \rightarrow P(x, z)))$$

b)

$$\exists x (S(x) \wedge \exists y (C(y) \wedge E(x, y) \wedge \forall z (C(z) \wedge E(x, z) \rightarrow (y = z))))$$

c)

$$\forall y ((C(y) \rightarrow \forall x (S(x) \wedge P(x, y) \rightarrow \forall z (C(z) \wedge R(z, y) \rightarrow P(x, z))))$$

d)

$$\exists x(S(x) \wedge \exists y(C(y) \wedge E(x, y) \wedge \neg P(x, y)))$$

e)

$$\exists y(C(y) \wedge \forall x(S(x) \rightarrow \neg P(x, y)))$$

f)

$$\forall y(C(y) \rightarrow \exists x(S(x) \wedge (E(x, y) \vee P(x, y))))$$

### Answer 3

- |                               |         |
|-------------------------------|---------|
| 1. $p \rightarrow (q \vee r)$ | Premise |
| 2. $\neg r \wedge \neg s$     | Premise |
| 3. $q \rightarrow s$          | Premise |

4. $p$	Assumption								
5. $q \vee r$	$\rightarrow$ e, 1, 4								
<table><tr><td>6. <math>q</math></td><td>Assumption</td></tr><tr><td>7. <math>s</math></td><td><math>\rightarrow</math> e, 3, 6</td></tr><tr><td>8. <math>\neg s</math></td><td><math>\wedge</math> e, 2</td></tr><tr><td>9. <math>\perp</math></td><td><math>\neg</math> e, 7, 8</td></tr></table>		6. $q$	Assumption	7. $s$	$\rightarrow$ e, 3, 6	8. $\neg s$	$\wedge$ e, 2	9. $\perp$	$\neg$ e, 7, 8
6. $q$	Assumption								
7. $s$	$\rightarrow$ e, 3, 6								
8. $\neg s$	$\wedge$ e, 2								
9. $\perp$	$\neg$ e, 7, 8								
<table><tr><td>10. <math>r</math></td><td>Assumption</td></tr><tr><td>11. <math>\neg r</math></td><td><math>\wedge</math> e, 2</td></tr><tr><td>12. <math>\perp</math></td><td><math>\neg</math> e, 10, 11</td></tr></table>		10. $r$	Assumption	11. $\neg r$	$\wedge$ e, 2	12. $\perp$	$\neg$ e, 10, 11		
10. $r$	Assumption								
11. $\neg r$	$\wedge$ e, 2								
12. $\perp$	$\neg$ e, 10, 11								
13. $\perp$	$\vee$ e, 6–9, 10–12								

- |              |                |
|--------------|----------------|
| 14. $\neg p$ | $\neg$ i, 4–13 |
|--------------|----------------|

## Answer 4

a)

1.  $\exists x(S(x) \wedge P(x))$
2.  $\forall x(P(x) \rightarrow K(x))$
3.  $\exists x(S(x) \wedge K(x))$

b)

1. $\exists x(S(x) \wedge P(x))$	Premise
2. $\forall x(P(x) \rightarrow K(x))$	Premise
<div><div>3. <math>P(c) \wedge S(c)</math>      Assumption (c, Skolem constant)</div><div>4. <math>P(c)</math>      <math>\wedge</math>e, 3</div><div>5. <math>S(c)</math>      <math>\wedge</math>e, 3</div><div>6. <math>P(c) \rightarrow K(c)</math>      <math>\forall</math>e, 2</div><div>7. <math>K(c)</math>      <math>\rightarrow</math> e, 4, 6</div><div>8. <math>S(c) \wedge K(c)</math>      <math>\wedge</math>i, 5, 7</div><div>9. <math>\exists x(S(x) \wedge K(x))</math>      <math>\exists</math>i, 8</div></div>	
10. $\exists x(S(x) \wedge K(x))$	$\exists$ e, 1, 2, 3-9