Systems prognostics via degradation models

This primer on prognostics and health monitoring (PHM) is based off Gebraeel's journal paper entitled "Residual-life distributions from component degradation signals: a Bayesian approach." This article was published in 2007, so more sophisticated methods must have been developed since then. However, the general procedure followed in the paper to use degradation models to build Residual Life Distributions (RLDs) is enlightening and could help guide the reading of more recent papers on the same topic.

The approach taken by the paper comprises 3 steps. The first 3 steps might be performed by a manufacturer who wants to get an idea of the reliability of a device that it manufactures. The subsequent steps are followed by a customer who bought a device from that manufacturer. The customer wants to use that device's real-time data together with reliability data computed by the manufacturer to compute the device's RLD.

1. Collect reliability data about a population of identical devices.
2. Fit a parametric model (*aka* "degradation model") to the reliability data of each tested device. (The model parameters are stochastic because the fit of the model to each device's data yields different parameter values.)
3. Compute the probability distributions of the model's stochastic parameters – these are called "prior distributions" because they embed our prior knowledge of the devices' behavior.
4. Collect real-time data from an operating device.
5. Use the collected real-time data to update the prior distributions into a "posterior distribution" of the degradation model's stochastic parameters. (Posterior meaning after our state of knowledge is modified through observing new data.) This is where Bayesian updating comes into play: you update the prior distributions into a posterior distribution using Bayes theorem.
6. Compute the RLD using the posterior distribution.

Note: Bayesian updating is a method among many others. However, it's popular, not only in PHM but also in other fields.

Steps 1 and 2 details

In his paper, Gebraeel applies this method to bearings. To collect reliability data, he ran 50 bearings continuously until failure. The data collected from each device is vibration (voltage) signals from accelerometers sampled every 2 minutes. He then observed that the degradation portion of the signal could be fitted as an exponential function (without measurement noise) – see fig. 1. The model chosen is as follows:

$$S(t) = \phi + \theta \exp(\beta t) \exp\left(\epsilon(t) - \frac{\sigma^2 t}{2}\right)$$

Where θ and β are the stochastic parameters mentioned above and $\epsilon(t)$ is a Brownian motion process modeling measurement errors. ϕ is a constant common to all bearings (he sets it to 0 in his paper so not very important) while θ and β are specific to each bearing.

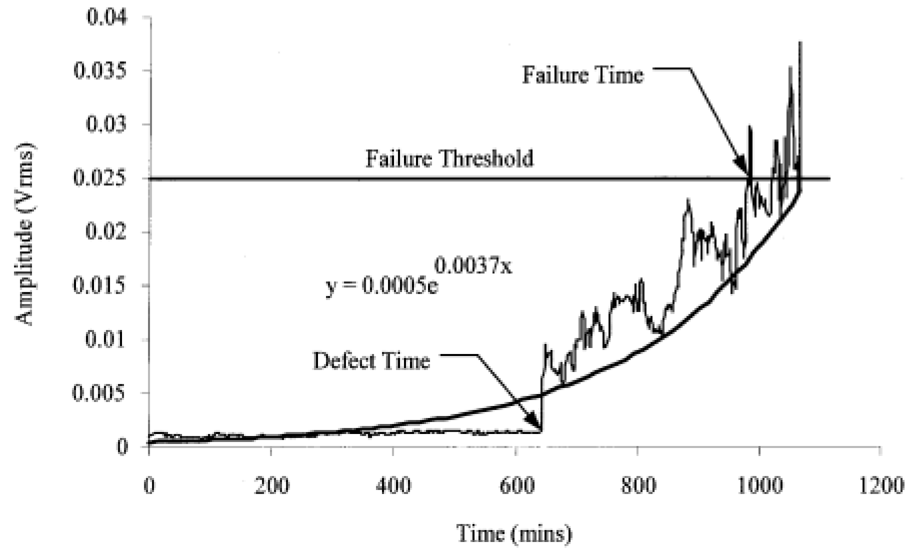

Figure 3.24 Degradation signal with exponential trend line

Figure 1. Degradation signal with exponential trend line (from Gebraeel's PhD thesis (2003))

Step 3 details

For each tested bearing, Gebraeel fitted the above exponential model to the collected vibration data. Thus, for each bearing, he computed a value for θ and a value for β. Since he tested 50 bearings, he ended up with 50 values of θ and 50 values of β. He then estimated the mean and variance of θ and β using the sample mean and sample variance. Finally, by plotting the histogram of $\ln(\theta)$ and β, he formulated the hypothesis that they followed normal distributions. He then used the chi-square goodness-of-fit test to confirm that hypothesis. As a result of this step, he ended up with a prior distribution for each stochastic parameter: θ is distributed as a lognormal (equivalent to saying that $\ln(\theta)$ is distributed as a normal), β is distributed as a normal.

Step 4 details

Steps 1 to 3 would be performed by a device's manufacturer. It would indicate in the device's datasheet that the degradation signal can be modeled as a growing exponential with stochastic parameters following the prior distributions determined as a result of step

3. A customer would buy one of those devices and integrate it with sensors to collect real-time data while operating.

Step 5 details

The prior distributions calculated in step 3 give the device's buyer prior knowledge of the device's behavior. These models now need to be specialized to the operating device using real-time operational data. This is done by updating the prior distributions of the stochastic parameters into a joint posterior distribution. For this, Bayes' theorem is used:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

Where M is the stochastic degradation model, D is the collected real-time data, $P(M)$ is the model's prior distribution, $P(D|M)$ is the likelihood (because it represents the likelihood to observe the collected data given the current model), $P(D)$ is called the marginal probability and is a constant which is typically hard to compute but not always necessary in analyses, and $P(M|D)$ is the posterior distribution of the degradation model. $P(M|D)$ represents the updated degradation model after observing the real-time data.

The above equation is applied whenever a new data point is sampled from the device.

Step 6 details

Once the posterior distribution is computed, one can deduce the RLD of the operating device. Let random variable T be the time until failure from the last sampled datapoint. Failure time is defined as being the first time the degradation signal reaches some user-defined threshold F. If t_k is the time at which the last datapoint was recorded, then the device is deemed to have failed when $S(t_k + T) = F$. Let $d_1, d_2, \dots, d_k$ be the k datapoints obtained so far, then, the cumulative distribution function (CDF) of T given $d_1, d_2, \dots, d_k$ is what we look for: $P(T \leq t|d_1, d_2, \dots, d_k)$.

Gebraeel, in his 2007 paper, approximates this as follows:

$$P(T \leq t|d_1, d_2, \dots, d_k) = P(S(t_k + t) \geq F|d_1, d_2, \dots, d_k)$$

Using the posterior distribution calculated in Step 5, the right-hand side of the above equation can be computed in closed form under the paper's assumptions. (However, Gebraeel notes that the method is more general than that and numerical techniques can be used to compute $P(T \leq t|d_1, d_2, \dots, d_k)$ if no closed form is available.)

One then deduces the probability density function (PDF) of T by taking the derivative of the expression of $P(T \leq t|d_1, d_2, \dots, d_k)$ with respect to t. That's the RLD we are looking for at time t_k. Of course, the above procedure must be done each time a new data point is recorded, at t_{k+1}, t_{k+2}, etc.