Lesson 3

Data Structures and Algorithms
DSA

In this lesson we will talk about:

- data structures
- algorithm design
- algorithm efficiency
- searching and sorting algorithms

Data Structures

What is a data structure?

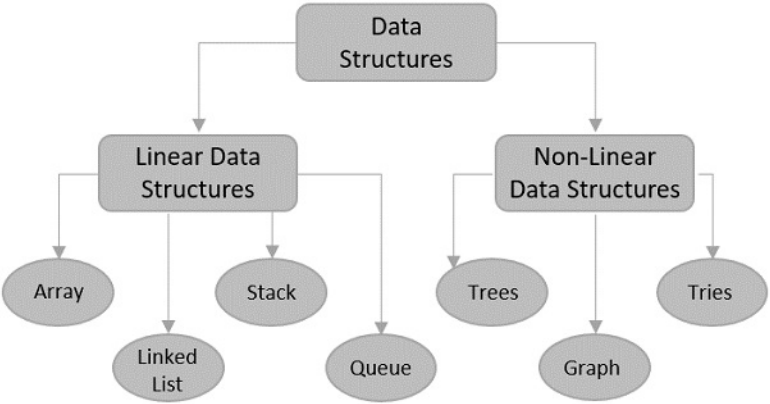
In computer science, a **data structure** is a data organization and storage format that is usually chosen for efficient access to data.[1][2][3] More precisely, a data structure is a collection of data values, the relationships among them, and the functions or operations that can be applied to the data,[4] i.e., it is an algebraic structure about data.

Data Structures

- How do we **organize** data
- For a very **efficient** access

It's a collection of data values, and the relationships among these values

Data Structures



Linear Data Structures: Arrays, Queues, Stacks

- Elements are arranged sequentially, one after the other
- The first element added will be the first one to be accessed or removed, and the last element added will be the last one to be accessed or removed
- Can have either fixed or dynamic sizes
- Offer very efficient data access

Non-linear Data Structures: Trees, Graphs

- Elements are arranged hierarchical
- We can't traverse all the elements in a single run only
- There are multiple levels which we must traverse
- It is more difficult to implement

Data structures

- Sets
- Arrays and Matrices
- Stacks
- Queues
- Linked lists
- Trees
- Graphs

Sets

Sets

A set is usually a **collection** of different things, fixed in size. Sets can also change size, usually when an algorithm will perform modifications against the set. We call these dynamic sets. These sets can change in size, grow or shrink, basically change over the time.

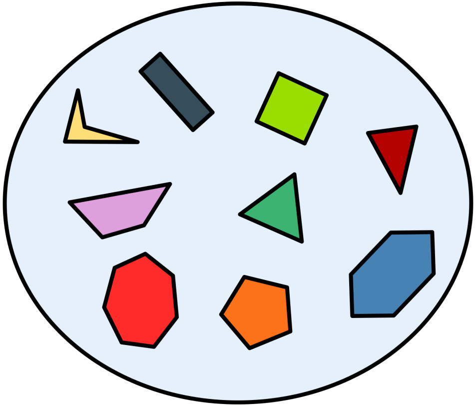
But what is a set?

Sets

The basic, fundamental data structure: $\{1,2,4,51,9\}$

- mathematical set
- unchanging, unique elements, no duplicates
- contains a fixed number of elements: finite set
- or it can contain an infinite number of elements

For example a set of polygons



Sets

A set is a **mathematical model** of a collection of different things. A set contains elements or members, which can be mathematical objects of any kind numbers, symbols, points in space, lines, other geometrical shapes, variables, or even other sets.

Sets, examples

- $\{$white, blue, red, yellow$\}$
- The empty set $\{\}$
- Natural numbers: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
- Natural numbers except 0: $\mathbb{N}^* = \{1, 2, 3, \ldots\}$
- Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Positive integers: $\mathbb{Z}_+ = \{0, 1, 2, 3, \ldots\}$

$\{1,2,3,4\}$

Defines a list of elements, using a simple enumeration notation (Roster notation) between curly brackets, separated by commas.

Basic Operations on Sets

- Insert - add a new element to a set
- Delete - remove an element from a set
- Test - if element X belongs to a set or not

A dynamic set which supports all these basic operations: **a dictionary**

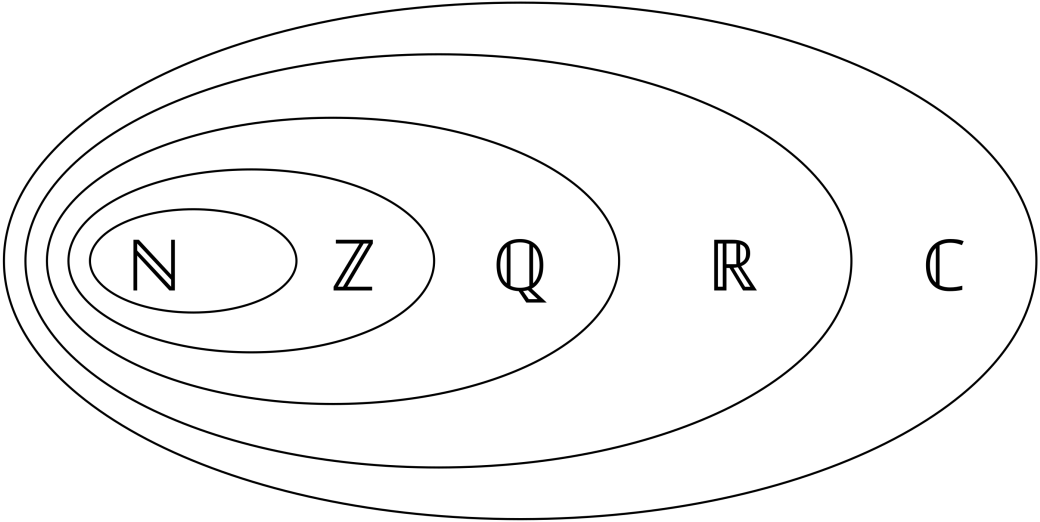
Lesson 3

Sets

Natural Numbers (Counting Numbers) ($\mathbb{N}$)	Numbers you use for counting: 1, 2, 3 ...	It's "natural" to count on your fingers: 1, 2, 3, ...
Whole Numbers	The natural numbers, plus 0: 0, 1, 2, 3 ...	The word "whole" has an "o" in it, so include 0.
Integers ($\mathbb{Z}$)	Whole numbers, their opposites (negatives), plus 0: ... $-2, -1, 0, 1, 2$ ...	Integers can be separated into negative, 0, and positive numbers.
Rationals ($\mathbb{Q}$)	Integers and all fractions, positive and negative, formed from integers. These include repeating fractions, such as $\frac{1}{3}$, or .33333. . or $.\overline{3}$.	The word "rational" is a derivation of "ratio", and rational numbers are numbers that can be written as a ratio of two integers. "Q" stands for quotient.
Irrationals	Numbers that cannot be expressed as a fraction, such as π, $\sqrt{2}$, e. (We'll learn about these later).	If something is "irrational", it's not easy to explain or understand.
Real Numbers ($\mathbb{R}$)	Rational numbers and Irrational Numbers. The real number system can be represented on a number line:  $-\infty$ $-.5$ 0 .5 1 $\sqrt{2}$ 2 ∞	If a number exists on a number line that you can see, it must be "real". Note that the "smallest" real number is negative ($-$) infinity ($-\infty$), and the largest real number is infinity (∞). We can never really get to these "numbers" ($-\infty$ and ∞), but we can indicate them as the "end" of the real numbers.
Complex Numbers ($\mathbb{C}$)	Real numbers, plus imaginary numbers (concept only, such as $\sqrt{-2}$).	"Imaginary" numbers are difficult to imagine, since they are so "complex".

Lesson 3

Sets



Advantages

Perform operations on a collection of elements in a very **efficient** and **organized** manner

Conclusions

Sets are basic, fundamental data structures, with:

- unique elements
- no duplicates
- unchanging
- fixed or infinite number of elements

Im confused. Does it mean a set is similar to a Python set? Or what is the difference?

Sets vs. Python Set

In computer science (CS), a set is an abstract data type that can store unique values, without any particular order. It is a computer implementation of the mathematical concept of a finite set.

Mathematical vs. Python Sets

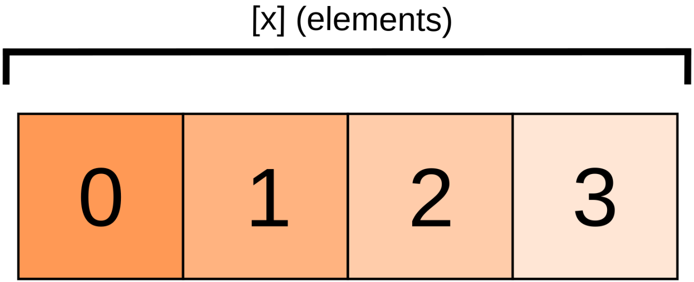
- Mathematical finite set: $\{1,2,3,4\}$
- Python set: S = {1,2,3,4}

Arrays

Arrays

In computer science, an **array** is a data structure consisting of a collection of elements, each identified by an **index** or a **key**. The simplest type of such data structure is a linear array, the one-dimensional array.

Typical "1 Dimensional" array



Element indexes are typically defined in the format *[x]*
[x] being the number of elements
For example: this array could be defined as *array[4]*

Arrays

Arrays are among the oldest and most important data structures, and are used by almost every program and programming language. They are also used to implement many other data structures, such as lists.

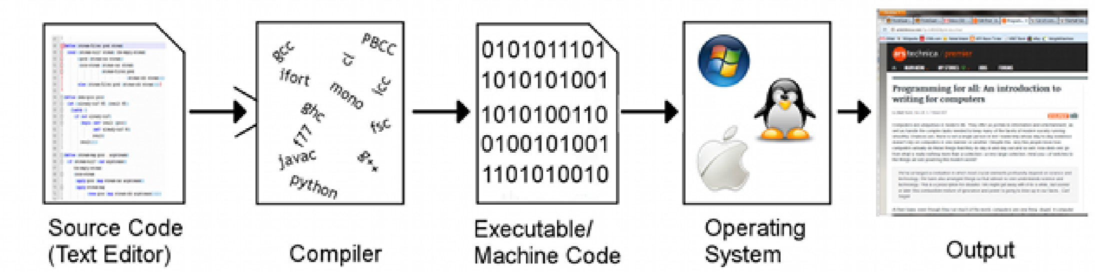
Arrays

Arrays are useful because the element indices can be computed at run time. Among other things, this feature allows a single iterative statement to process arbitrarily many elements of an array. For that reason, the elements of an array data structure are required to have the same size and should use the same data representation.

Run-time?

Runtime, run time, or execution time is the final phase of a computer program's life cycle, in which the code is being executed on the computer's central processing unit (CPU) as machine code. In other words, "runtime" is the running phase of a program.

Remember this? From source code to executable



Basic Array Operations

- traversal of an array
- access element X in an array
- searching element X in an array
- sorting an array

Example 1: Traversing the array A

```
procedure GETARRAY(A)                ▷ Returns the max value in A
    L ← length(A)
    for i=0 to L-1 do
        print A[i]
    end for
end procedure
```

Traversing the array

```
A = [1, 2, 3, 4, 5, 6, 7,8,9]

# Traversing the array
for element in A:
    print(element, end="-")}
```

Traversing the array, version 2

```
A = [1, 2, 3, 4, 5, 6, 7,8,9]

# Traversing the array
for i in range(len(A)):
    print(A[i], end="-")
```

Example 2: Find the max value in the array A

```
procedure MaxArray(A)                    ▷ Returns the max value in A
    N ← length(A)
    MAX ← A[0]
    for from i=1 to N-1 do
        if A[i] > MAX then
            MAX = A[i]                   ▷ The MAX is A[i]
        end if
    end for
    return MAX
end procedure
```

Example 3: Search element X in array A

```
procedure SEARCHARRAY(A)          ▷ Returns the max value in A
    X ← MyElement
    N ← length(A)
    for from i=0 to N-1 do
        if X = A[i] then
            return i              ▷ The index for my match
        end if
    end for
    return -1                     ▷ otherwise return -1
end procedure
```

Search element in array

```
A = [1, 2, 3, 4, 5, 6, 7,8,9]

def find_element(A, n, key):
    for i in range(n):
        if A[i] == key:
            return i
    return -1
```

There are numerous applications of arrays

- Storing data in databases. Storing a list of customer names.
- Traffic Management.Traffic management systems use arrays to track vehicles and their flow. By analyzing data stored in arrays, traffic control centers can implement efficient signal timings and manage congestion effectively.

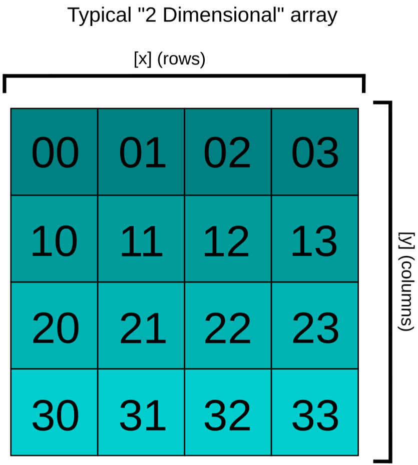
- Financial Analysis. It keeps track of various financial instruments, including stocks, bonds, and mutual funds. By organizing data in arrays, companies can perform analyses and make predictions easier
- Machine Learning. Machine learning algorithms often accept arrays as input, helping to train models and make predictions.

Matrices

What is a matrix?

In mathematics, a matrix (pl.: matrices) is a rectangular array or table of numbers, symbols, or expressions, with elements or entries arranged in rows and columns, which is used to represent a mathematical object or property of such an object.

Matrices



Element indexes are typically defined in the format *[x][y]*
[x] being the number of rows
[y] being the number of columns

Matrices

$$\begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}
\end{array}$$

Matrix multiplication

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrix multiplication M x N

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mp} \end{bmatrix}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in} + b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

Basic Matrix Operations

- access X element in a matrix
- traversal of a matrix
- searching a matrix
- sorting a matrix

Accessing the elements of a matrix

```
A = [[1, 2, 3], [4, 5, 6], [7,8,9]]

# Accessing certain elements in a matrix
print("1st element of 1st row:", A[0][0])
print("2nd element of the 2nd row:", A[1][2])
print("2nd element of 3rd row:", A[2][1])
```

Traversing the matrix

```
A = [[1, 2, 3], [4, 5, 6], [7,8,9]]

# Traversing the matrix
for row in A:
    # Traversing the matrix
    for x in row:
        print(x, end="-")
    print()
```

There are numerous applications of matrices

- Encryption: Matrices encrypt data into unreadable formats and decode it for secure communication.
- Computer Graphics: transformations like scaling, rotation, and translation of objects in 2D and 3D graphics
- Machine Learning: fundamental data structures for neural networks

- Economics and Business: optimize business operations like supply chains and financial forecasting
- Navigation Systems: GPS systems use matrices to calculate positions, distances, and directions in 2D and 3D space.
- Weather Prediction: Matrices solve systems of differential equations to model and predict climate and weather patterns

Stacks

What is a stack?

The stack is an analogy to a set of physical items stacked one atop another, such as a stack of plates.

Lesson 3

Stacks

Stacks

Stacks are collections of elements, which support two main operations:

- **PUSH**: which adds an element to the collection
- **POP**: which removes the most recent elements from the collection

Stacks

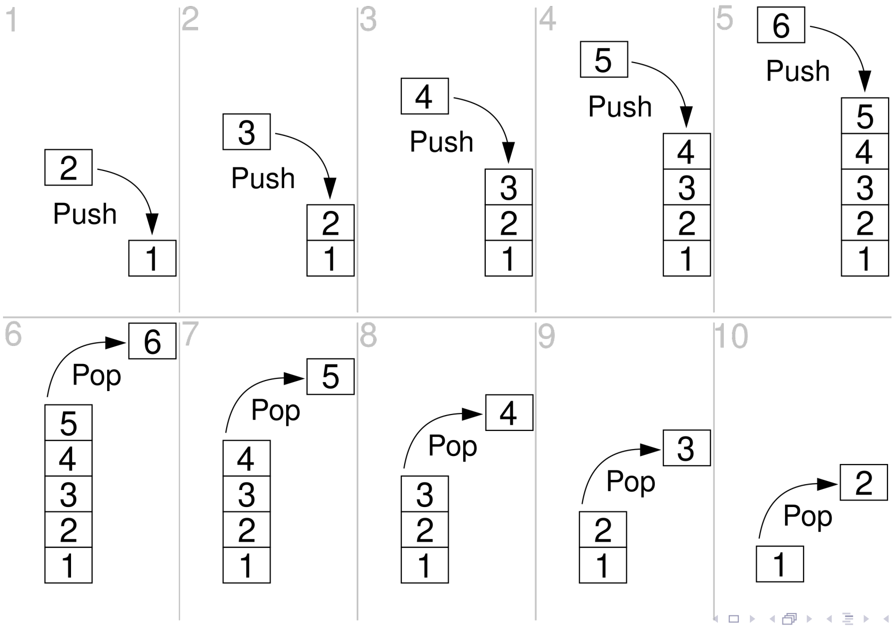
There might be, another operation called **PEEK**, which without modifying the stack, return the value of the last element added.

Stacks

- **PUSH**: which adds an element to the collection
- **POP**: which removes the most recent elements from the collection
- **PEEK**: returns the value of the last element added

Stacks

The stack supports few operations. And it operates in a certain, predefined order. The order in which an element added to or removed from a stack is described as last in, first out, referred to by the acronym **LIFO**


1
2
Push
1
2
3
Push
2
1
3
4
Push
3
2
1
4
5
Push
4
3
2
1
5
6
Push
5
4
3
2
1
6
6
Pop
5
4
3
2
1
7
5
Pop
4
3
2
1
8
4
Pop
3
2
1
9
3
Pop
2
1
10
2
Pop
1

Stacks

"Stacks entered the computer science literature in 1946, when Alan Turing used the terms "bury" and "unbury" as a means of calling and returning from subroutines."

Stacks

How can we implement a stack?

Stacks

A stack can be easily implemented using an array. The first element, usually at the zero offset, is the bottom, resulting in array[0] being the first element pushed onto the stack and the last element popped off.

Stacks

The program must keep track of the size (length) of the stack, using a variable top that records the number of items pushed so far, therefore pointing to the place in the array where the next element is to be inserted (assuming a zero-based index convention).

Stacks

The program must keep track of the size (length) of the stack, using a variable top that records the number of items pushed so far, therefore pointing to the place in the array where the next element is to be inserted (assuming a zero-based index convention).

Stack using arrays

```
# Create a stack. It initializes size of stack as 0
def createStack():
    stack = []
    return stack
```

```
# Stack is empty when stack size is 0
def isEmpty(stack):
    return len(stack) == 0

# Add an item to stack. It increases size by 1
def push(stack, item):
    stack.append(item)
    print(item + " pushed to stack ")
```

```
# Remove an item from stack. It decreases size by 1
def pop(stack):
    if (isEmpty(stack)):
        # return minus infinite
        return str(-maxsize -1)
    return stack.pop()

# Return the top from stack without removing it
def peek(stack):
    if (isEmpty(stack)):
        # return minus infinite
        return str(-maxsize -1)
    return stack[len(stack) - 1]
```

Stack using LiFoQueue

```
from queue import LifoQueue
stack = LifoQueue()
stack.put(1)
stack.put(2)
stack.put(3)
print(stack.get()) # Prints 3
print(stack.get()) # Prints 2
```

There are numerous applications of stacks

- Undo mechanism in text editors
- Fwd and back buttons on web browsers
- Memory management in computer programming (static memory allocation. It can be used to keep track of functions calls)
- Implementing recursion

But what is recursion?

Recursion

- It is a programming technique, a way to program and develop a program where a function calls itself many times
- You take a big problem, into smaller problems, trying to apply a solution to solve the small problems
- You define a problem in terms of itself

Recursion

You are standing in a long queue of people. You must answer, how many people are behind you in the line?

Note: One person can see only the person standing directly in front and behind. One cannot look back and count. Each person is allowed to ask questions from the person standing in front or behind.

Recursion

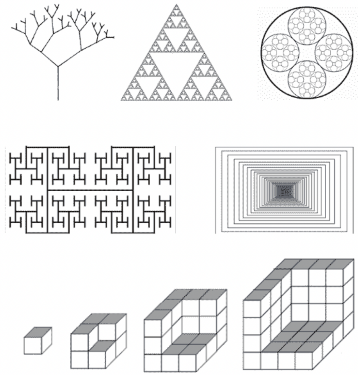
You look behind and see if there is a person there. If not, then you can return the answer "0". If there is a person, repeat this step and wait for a response from the person standing behind. Once a person receives a response, they add 1 and respond to the person that asked them or the person standing in front of them.

Recursion example

```
procedure PERSONCOUNT(currPerson)
    if noOneBehind(currPerson) == TRUE then
        return 0
    else personBehind == currPerson.checkBehind
        return 1 + PERSONCOUNT()
    end if
end procedure
```

Thinking recursively

- Learning to look for big things
- That are made from smaller things



Solving a problem recursively

Recursion is a function calling itself until a generic, base condition is true to produce the correct output. In other words, to solve a problem, we solve a problem that is a smaller instance of the same problem, and then use the solution to that smaller instance to solve the original problem.

Factorial number

In mathematics, the factorial of a non-negative integer $n!$ is the product of all positive integers less than or equal to n

$$n! = n \times (n-1)! \tag{1}$$

$$n! = n \times (n-1) \times (n-2) \times \ldots 3 \times 2 \times 1 \tag{2}$$

$$5! = 5 \times 4! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

n	$n!$
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5 040
8	40 320
9	362 880
10	3 628 800
11	39 916 800
12	479 001 600
13	6 227 020 800
14	87 178 291 200
15	1 307 674 368 000
16	20 922 789 888 000
17	355 687 428 096 000
18	6 402 373 705 728 000
19	121 645 100 408 832 000
20	2 432 902 008 176 640 000
25	$1.551\ 121\ 004 \times 10^{25}$
50	$3.041\ 409\ 320 \times 10^{64}$

Recursion example

procedure $\textsc{Factorial}(N)$
    if $N == 0$ **then**
        return 1
    else
        return $N * \textsc{Factorial}(N - 1)$
    end if
end procedure

Factorial, using recursion

```
def factorial(n):
    if n == 1:
        return 1
    else:
        return n * factorial(n-1)

print(factorial(5))
```

Solving a problem recursively

For a recursive algorithm to work, smaller subproblems must be found and arrive at the base case. In simple words, any recursive algorithm has two parts: the base case and the recursive structure.

The Base Case

The base case is a terminating condition where a function immediately returns the result. This is the smallest version of the problem for which we already know the solution.

The Recursive structure

The recursive structure is an idea to design a solution to a problem via the solution of its smaller sub-problems, i.e., the same problem but for a smaller input size. We continue calling the same problem for smaller input sizes until we reach the base case of recursion.

How recursion works?

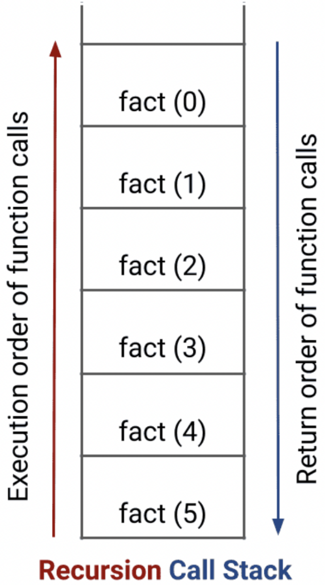
If we draw the flow of recursion for the factorial program, one can find this pattern: we are calling fact(0) last, but it is returning the value first. Similarly, we are calling fact(n) first, but it is returning the value last. Its a Last In First Out (LIFO) order for all recursive calls and return values.

Recursion uses Stacks behind

Order of recursive calls: larger problem to smaller problem
$fact(n) \rightarrow fact(n-1) \rightarrow \ldots \rightarrow fact(1) \rightarrow fact(0)$

Order of return values: smaller problem to larger problem
$fact(0) \rightarrow fact(1) \rightarrow \ldots \rightarrow fact(n-1) \rightarrow fact(n)$

Execution call stack!



Recursion is important!

- the basis for Dynamic Programming and Divide and Conquer algorithms
- helps in solving complex problems by breaking them into smaller subproblems
- fundamental to sorting, like quicksort, mergesort
- used in traversing trees and other complex data structures

Recursion vs Iteration?

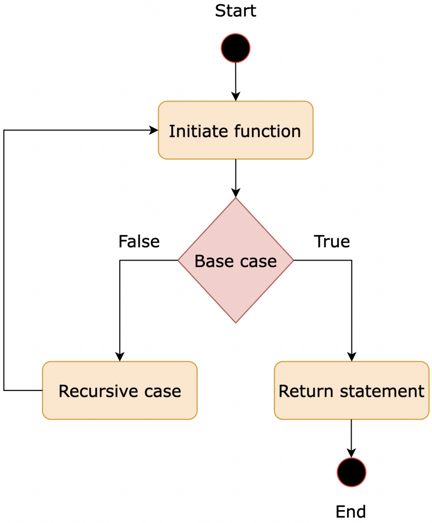
A program is called **recursive** when an entity calls itself. A program is called **iterative** when there is a loop (or repetition) of some sort.

Recursion

- each function call creates a smaller problem to solve
- and these calls continue until reaching a basic case which is trivial to solve

Recursive function

- **Base case** The condition under which the function stops calling itself. This prevents infinite recursion and provides a direct answer for the simplest instance of the problem.
- **Recursive case** The part of the function that reduces the problem into smaller instances and calls itself with small data.



Recursive Factorial

```
def factorial(n):
    # Base case: if n is 1 or 0, factorial is 1
    if n == 0 or n == 1:
        return 1
    # Recursive case: n * factorial of (n-1)
    else:
        return n * factorial(n - 1)
print(factorial(5))
```

Recursion Types

- direct recursion
- indirect recursion
- tail recursion

Direct Recursion - the simplest form of recursion. A function directly calls itself within its definition.

```
def direct_recursion(n):
    if n <= 0:
        return
    print(n)
    direct_recursion(n - 1)
```

Indirect Recursion - creates a cylces of function calls. A function calls another function, which calls the original function.

```
def functionA(n):
    if n <= 0:
        return
    print(n)
    functionB(n - 1)

def functionB(n):
    if n <= 0:
        return
    print(n)
    functionA(n - 2)
```

Tail Recursion - the recursive call is the last operation in the function.

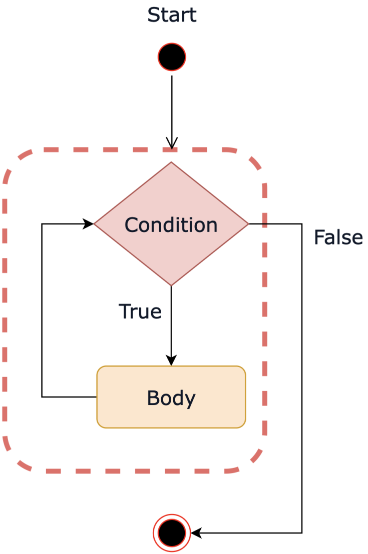
```
def tail_recursion(n, aggregate=1):
    if n == 0:
        return aggregate
    return tail_recursion(n - 1, n * aggregate)
```

Iterative

- a repetitive process
- runs a number of times until a specified condition is met
- or certain number of iterations have been reached

Iterative function

- **Initialization** Start with initializing variables or setting initial conditions required for the iterative process
- **Condition** Check a condition that determines whether the iteration should continue or stop
- **Body** Execute a set of instructions or operations that represent the core logic of the iteration.



How to build an iterative function?

- loops: while, do-while, for loops
- for loop: when the number of iterations is known beforehand
- while loop: when the number of iterations is unknown, and the loop continues as long as a condition is true
- do-while loop: same as while loop but ensures that the loop is executed at least once before the condition is tested

Iterative Factorial - the iterative function to calculate the factorial

```
def factorial(n):
    result = 1
    for i in range(1, n + 1):
        result *= i
    return result
print(factorial(5))
```

When to use recursion vs. iteration

The nature of the problem

- naturally fit a recursive structure or require breaking down into smaller subproblems
- for loop: when the number of iterations is known beforehand
- use iteration for problems with a straightforward repetitive process or when the number of iterations is known beforehand

When to use recursion vs. iteration

Complexity and readability

- Use recursion when it simplifies the code and makes it more readable
- Use iteration when recursion would make the code unnecessarily complex or when avoiding the risk of stack overflow is essential

When to use recursion vs. iteration

Performance consideration

- Use recursion if the problem's recursive nature allows for more elegant and maintainable code and manageable stack usage
- Use iteration if memory usage is a concern and the problem can be solved efficiently with loops.

Use recursion

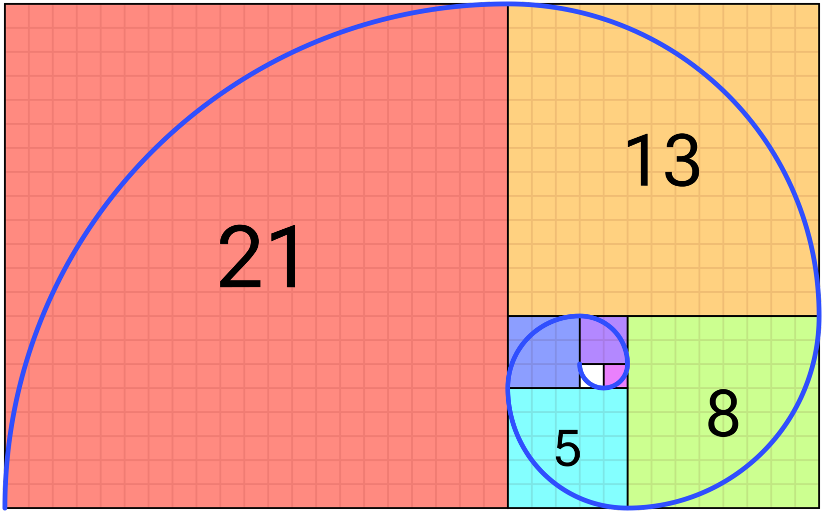
- **Divide and conquer** problems like merge sort, quicksort, and binary search where the problem is divided into smaller subproblems that are solved recursively
- **Tree and graph problems** Traversals (e.g., in-order, pre-order, post-order for trees) and pathfinding in graphs that naturally fit a recursive approach.
- **Dynamic programming** Problems like the Fibonacci sequence, where recursion with memoization simplifies the solution
- **Combinatorial problems** Generating permutations and combinations and solving puzzles like the Tower of Hanoi

Use iteration

- **Simple loops** Problems like summing a list of numbers, iterating through arrays or lists, and simple counting problems
- **Linear problems** Iterative solutions for problems requiring a linear scan, such as finding an array's maximum or minimum value
- **Repetitive tasks** Problems that require repeating a task a known number of times, such as printing numbers from 1 to N or iterating through data structures like arrays and linked lists.

Lesson 3

Stacks



Fibonacci sequence

Fibonacci sequence

In mathematics, the **Fibonacci sequence** is a sequence in which each element is the sum of the two elements that precede it. Numbers that are part of the Fibonacci sequence are known as **Fibonacci numbers**, commonly denoted F_n. Many writers begin the sequence with 0 and 1, although some authors start it from 1 and 1[1][2] and some (as did Fibonacci) from 1 and 2. Starting from 0 and 1, the sequence begins

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... (sequence A000045 in the OEIS)

Fibonacci sequence

Definition [edit]

The Fibonacci numbers may be defined by the recurrence relation[7]

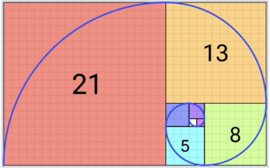
$$F_0 = 0, \quad F_1 = 1,$$

and

$$F_n = F_{n-1} + F_{n-2}$$

for $n > 1$.

Under some older definitions, the value $F_0 = 0$ is omitted, so that the sequence starts with $F_1 = F_2 = 1$, and the recurrence $F_n = F_{n-1} + F_{n-2}$ is valid for $n > 2$.[8][9]



The Fibonacci spiral: an approximation of the golden spiral created by drawing circular arcs connecting the opposite corners of squares in the Fibonacci tiling (see preceding image)

The first 20 Fibonacci numbers F_n are:

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181

Fibonacci Recursion

```
def fibonacci_recursive(n):
    if n <= 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fibonacci_recursive(n-1) + fibonacci_recursive(n-2)

# Main Code
n = 10
print(f"Recursive method: Fibonacci({n}) = { fibonacci_recursive(n)}
```

Fibonacci Iterative

```
def fibonacci_iterative(n):
    if n <= 0:
        return 0
    elif n == 1:
        return 1
    a, b = 0, 1
    for _ in range(2, n + 1):
        a, b = b, a + b
    return b

# Main code
n = 10
print(f"Iterative method: Fibonacci({n}) = {fibonacci_iterative(n)}
```

Advantages of Stacks

- Simplicity: Stacks are a simple and easy-to-understand data structure
- Efficiency: Push and pop operations on a stack are very fast, providing efficient access to data.
- LIFO: Stacks follow the LIFO principle, ensuring that the last element added to the stack is the first one removed.
- Limited memory usage: Stacks only need to store the elements that have been pushed onto them, making them memory-efficient compared to other data structures.

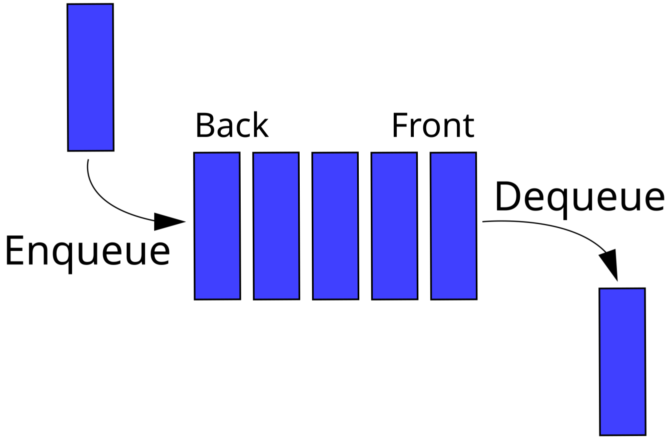
Disadvantages of Stacks

- Limited access: Elements can only be accessed from the top, making it difficult to retrieve or modify elements in the middle of the stack.
- Overflow: If more elements are pushed onto a stack than it can hold, an overflow error will occur, resulting in a loss of data.
- No random access: Stacks do not allow for random access to elements, making them unsuitable for applications where elements need to be accessed in a specific order.
- Limited capacity: Fixed capacity, a limitation if the number of elements that need to be stored is unknown or highly variable.

Queues

Queues

A queue is a collection of entities that are maintained in a sequence and can be modified by the addition of entities at one end of the sequence and the removal of entities from the other end of the sequence.


Back
Front
Dequeue
Enqueue

By convention, the end of the sequence at which elements are added is called **the back, tail, or rear** of the queue, and the end at which elements are removed is called the **head or front** of the queue, analogously to the words used when people line up to wait for goods or services.

Queues

The operations of a queue make it a first-in-first-out **FIFO** data structure. In a FIFO data structure, the first element added to the queue will be the first one to be removed. A queue is an example of a linear data structure, or more abstractly a sequential collection.

Queue Properties

- **Front:** Position of the entry in a queue ready to be served, that is, the first entry that will be removed from the queue, is called the front of the queue (head of the queue)
- **Rear:** Position of the last entry in the queue, that is, the one most recently added, is called the rear of the queue. (tail of the queue)
- **Size:** the current number of elements in the queue
- **Capacity:** the maximum number of elements the queue can hold

Queue Operations

- **Enqueue** — Add an element to the end of the queue
- **Dequeue** — Remove an element from the front of the queue
- **Is Empty** — Check if the queue is empty
- **Is Full** — Check if the queue is full
- **Peek** — Get the value of the front element without removing it

Queue implements a basic FIFO queue

```
from queue import Queue
q = Queue()
q.put(1)  # Add 1 to queue
q.put(2)
q.put(3)
print(q.qsize())  # Prints 3
print(q.get())  # Prints 1
print(q.get())  # Prints 2
```

Python Queue

```python
from queue import Queue
q = Queue()
# The key methods available are:
# qsize() - Get the size of the queue
# empty() - Check if queue is empty
# full() - Check if queue is full
# put(item) - Put an item into the queue
# get() - Remove and return an item from the queue
# join() - Block until all tasks are processed
```

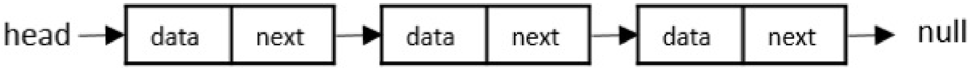
Linked lists

Linked Lists

A linked list is a linear collection of data elements where each element points to the next. It is a data structure consisting of a collection of nodes which together represent a sequence.

Lesson 3

Linked lists

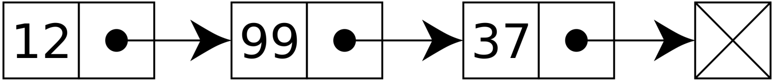


Linked Lists

Each node stores both data and **a reference (a pointer)** to the next node in the sequence.

Lesson 3

Linked lists



Linked Lists

Each node contains data, and a reference (in other words, a link) to the next node in the sequence. This structure allows for efficient insertion or removal of elements from any position in the sequence during iteration.

Linked Lists

A drawback of linked lists is that data access time is linear in respect to the number of nodes in the list. Because nodes are serially linked, accessing any node requires that the prior node be accessed beforehand.

Linked Lists

Linked lists are among the simplest and most common data structures. They can be used to implement several other common abstract data types, including stacks, queues, associative and dynamic arrays.

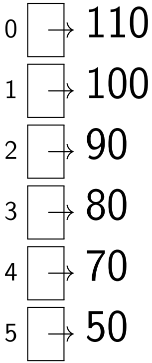
Dynamic Arrays

Variable-size list data structure that allows elements to be added or removed. Dynamic arrays overcome a limit of static arrays, which have a fixed capacity that needs to be specified at allocation.

Associative Arrays

A map, symbol table, or dictionary is an abstract data type that stores a collection of (key, value) pairs, such that each possible key appears at most once in the collection. It supports lookup, remove, and insert operations.

Associative Arrays



Linked lists vs. Arrays

Linked lists vs. Arrays

Linked lists

- Dynamic in size
- Can contain any number of nodes
- Store various data types

Arrays

- Fixed in size
- Size is given at the time of creation
- Similar data types

Linked Lists

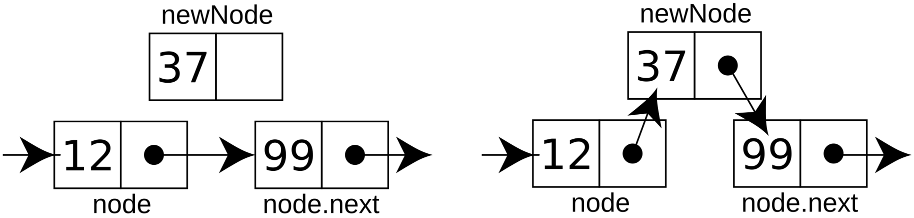
The principal benefit of a linked list over a conventional array is that the list elements can be easily inserted or removed without reallocation or reorganization of the entire structure because the data items do not need to be stored contiguously in memory or on disk, while restructuring an array at run-time is a much more expensive operation.

Linked Lists

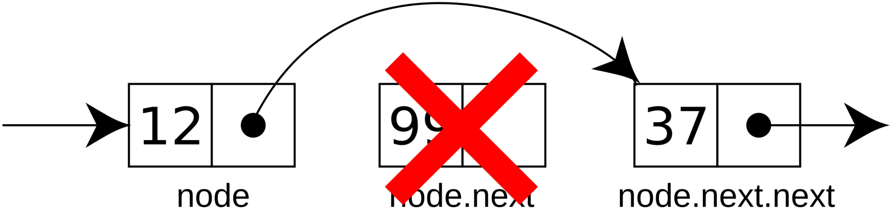
Linked lists allow insertion and removal of nodes at any point in the list, and allow doing so with a constant number of operations by keeping the link previous to the link being added or removed in memory during list traversal.

Linked list Operations

- **INSERT** — Add an element to the end of the queue
- **DELETE** — Remove an element from the front of the queue
- **TRAVERSAL** — Check if the queue is empty



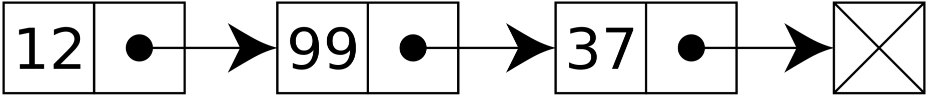
newNode
37
12
99
node
node.next
newNode
37
12
99
node
node.next


12
99
37
node
node.next
node.next.next
12
99
37
node
node.next
node.next.next

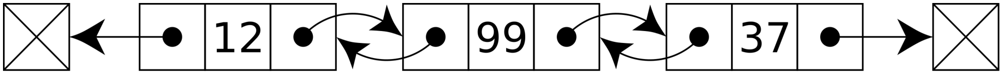
Linked lists

- **Single linked list** lists contain nodes which have a value and a next pointer
- **Double linked list** contains nodes which includes the next and previous pointer links
- **circular linked list** a list where the last node has a pointer to the first node

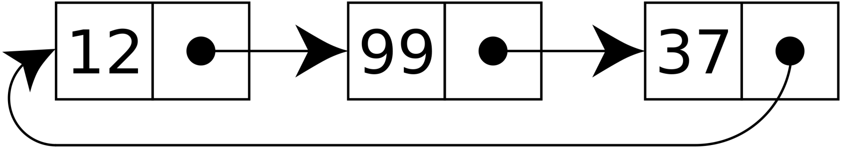
Single Linked list



Double Linked list



Circular Linked list



Linked List Example

```
# importing module
import collections

# initialising a deque() of arbitrary length
linked_list = collections.deque()
# filling deque() with elements
linked_list.append('subaru')
linked_list.append('toyota')
linked_list.append('mercedes')

print("Elements in the linked_list:")
print(linked_list)
```

Hash Table

Linked Lists

A hash table is a data structure that implements an associative array, also called a dictionary or simply map; an associative array is an abstract data type that maps keys to values.

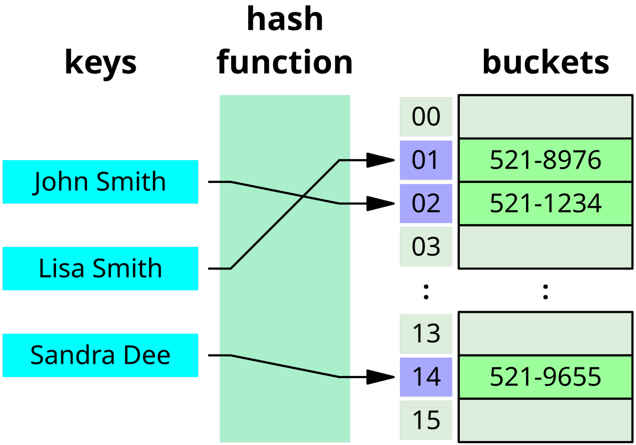
Remember Associative Arrays

A map, symbol table, or dictionary is an abstract data type that stores a collection of (key, value) pairs, such that each possible key appears at most once in the collection. It supports lookup, remove, and insert operations.

Hash Table

A hash table uses a hash function to compute an index, also called a hash code, into an array of buckets or slots, from which the desired value can be found. During lookup, the key is hashed and the resulting hash indicates where the corresponding value is stored. A map implemented by a hash table is called a hash map.

Hash Table


keys
hash function
buckets
John Smith
Lisa Smith
Sandra Dee
00
01
02
03
:
13
14
15
521-8976
521-1234
:
521-9655

Hash Table

Hashing concept - each key is translated by a hash function into a distinct index in an array. The index functions as a storage location for the matching value. In simple words, it maps the keys with the value.

Hash Functions: cyclic redundancy checks

Name	Length	Type [hide]
cksum (Unix)	32 bits	CRC with length appended
CRC-8	8 bits	CRC
CRC-16	16 bits	CRC
CRC-32	32 bits	CRC
CRC-64	64 bits	CRC

Hash Functions: checksums

Name	Length	Type [hide]
BSD checksum (Unix)	16 bits	sum with circular rotation
SYSV checksum (Unix)	16 bits	sum with circular rotation
sum8	8 bits	sum
Internet Checksum	16 bits	sum (ones' complement)
sum24	24 bits	sum
sum32	32 bits	sum
fletcher-4	4 bits	sum
fletcher-8	8 bits	sum
fletcher-16	16 bits	sum
fletcher-32	32 bits	sum
Adler-32	32 bits	sum
xor8	8 bits	sum
Luhn algorithm	1 decimal digit	sum
Verhoeff algorithm	1 decimal digit	sum
Damm algorithm	1 decimal digit	Quasigroup operation

Hash Functions: universal

Name	Length	Type [hide]
Rabin fingerprint	variable	multiply
tabulation hashing	variable	XOR
universal one-way hash function		
Zobrist hashing	variable	XOR

Hash Functions: keyed cryptographic

Name	Tag Length	Type [hide]
BLAKE2		keyed hash function (prefix-MAC)
BLAKE3	256 bits	keyed hash function (supplied IV)
HMAC		
KMAC	arbitrary	based on Keccak
MD6	512 bits	Merkle tree NLFSR
One-key MAC (OMAC; CMAC)		
PMAC (cryptography)		
Poly1305-AES	128 bits	nonce-based
SipHash	32, 64 or 128 bits	non-collision-resistant PRF
HighwayHash[16]	64, 128 or 256 bits	non-collision-resistant PRF
UMAC		
VMAC		

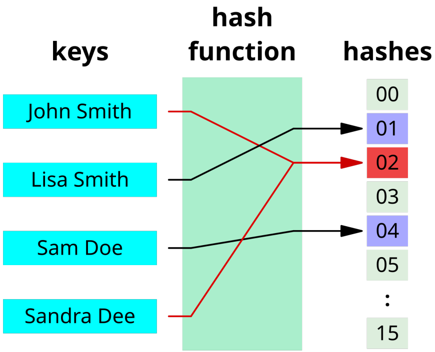
Hash Functions: unkeyed cryptographic

HAVAL	128 to 256 bits	hash
JH	224 to 512 bits	hash
LSH[19]	256 to 512 bits	wide-pipe Merkle–Damgård construction
MD2	128 bits	hash
MD4	128 bits	hash
MD5	128 bits	Merkle–Damgård construction
MD6	up to 512 bits	Merkle tree NLFSR (it is also a keyed hash function)
RadioGatún	arbitrary	ideal mangling function
RIPEMD	128 bits	hash
RIPEMD-128	128 bits	hash
RIPEMD-160	160 bits	hash
RIPEMD-256	256 bits	hash
RIPEMD-320	320 bits	hash
SHA-1	160 bits	Merkle–Damgård construction
SHA-224	224 bits	Merkle–Damgård construction
SHA-256	256 bits	Merkle–Damgård construction
SHA-384	384 bits	Merkle–Damgård construction
SHA-512	512 bits	Merkle–Damgård construction
SHA-3 (subset of Keccak)	arbitrary	sponge function

Hash Collisions

A hash collision or hash clash is when two distinct pieces of data in a hash table share the same hash value. The hash value in this case is derived from a hash function which takes a data input and returns a fixed length of bits.

Hash Collisions



Hash Table

Hash tables are very much used as table lookup structures, in many kinds of computer software, particularly database indexing, caches, and sets.

Hash Table

In hash tables, since hash collisions are inevitable, hash tables have mechanisms of dealing with them, known as collision resolutions.

- **open addressing** cells in the hash table are assigned one of three states in this method – occupied, empty, or deleted
- **separate chaining** allows more than one record to be chained to the cells of a hash table.

Hash Table

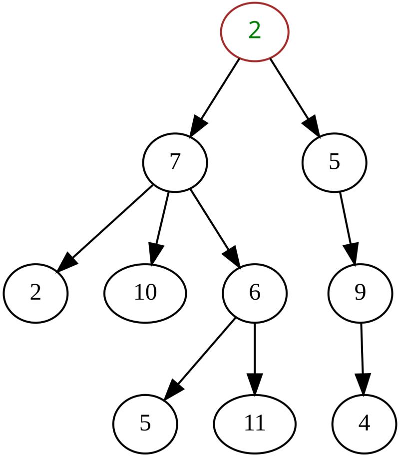
```
hash_table = {"Alice": "January", "Bob": "May", "Charlie": "January

my_dictionary = {}
my_dictionary["Alice"] = "January"
# Hash function determines the location for "January"

# print the key value
print(my_dictionary["Alice"]) # "January"
```

Trees

What is a tree?

A tree is a very widely used abstract data type that represents a hierarchical structure with a set of connected nodes. Each node i the tree can be connected to many children (depending on the type of tree), but must be connected to exactly one parent, except for the root node, which has no parent

Unsorted tree



Tree

There should be no cycles or "loops" (no node can be its own ancestor), and also that each child can be treated like the root node of its own subtree, making recursion a useful technique for tree traversal.

Tree Node

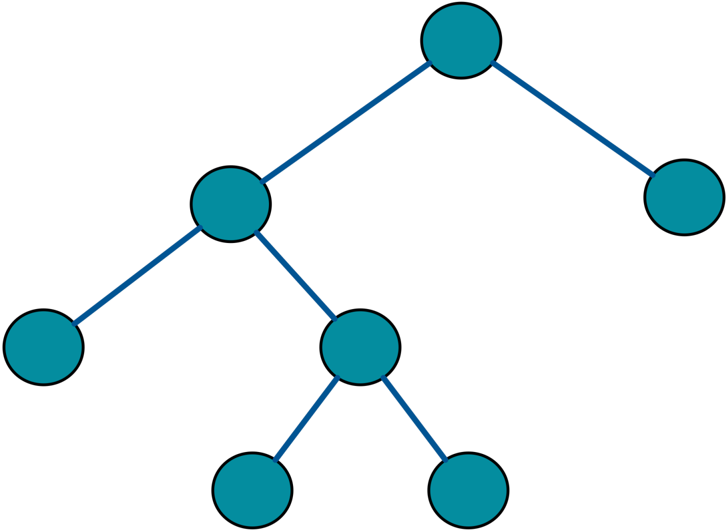
A node is a structure which may contain data and connections to other nodes, sometimes called edges or links. Each node in a tree has zero or more child nodes, which are below it in the tree (by convention, trees are drawn with descendants going downwards).

Tree Node

A node that has a child is called the child's parent node (or superior). All nodes have exactly one parent, except the topmost root node, which has none. A node might have many ancestor nodes, such as the parent's parent.

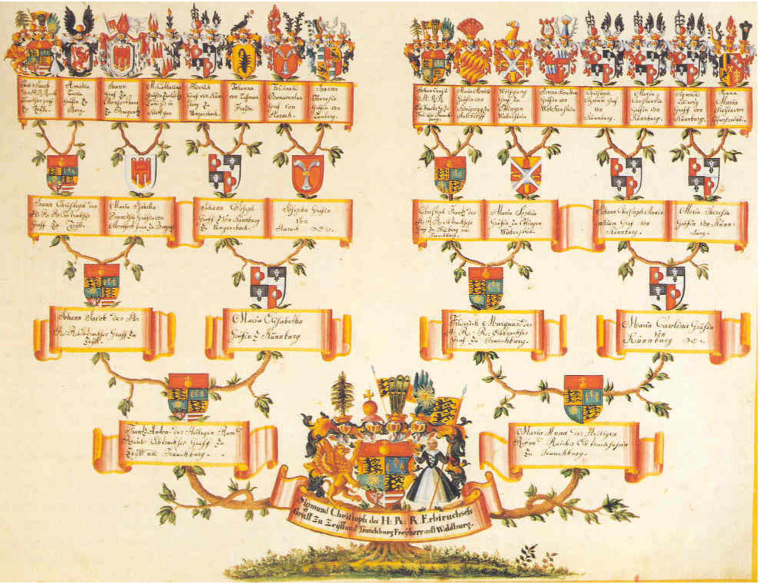
Lesson 3

Trees



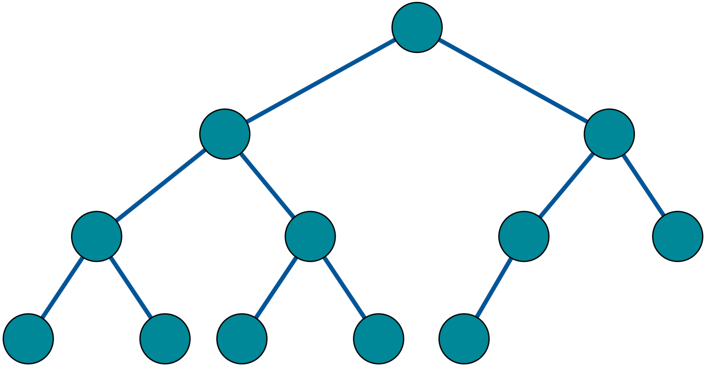
Lesson 3

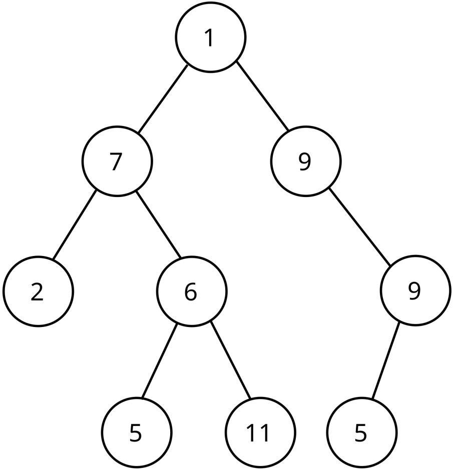
Trees



Lesson 3

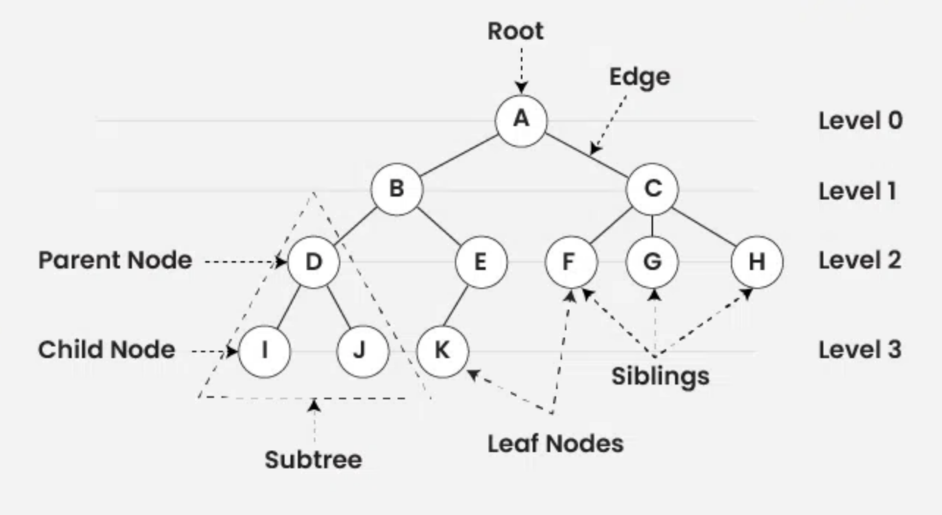
Trees



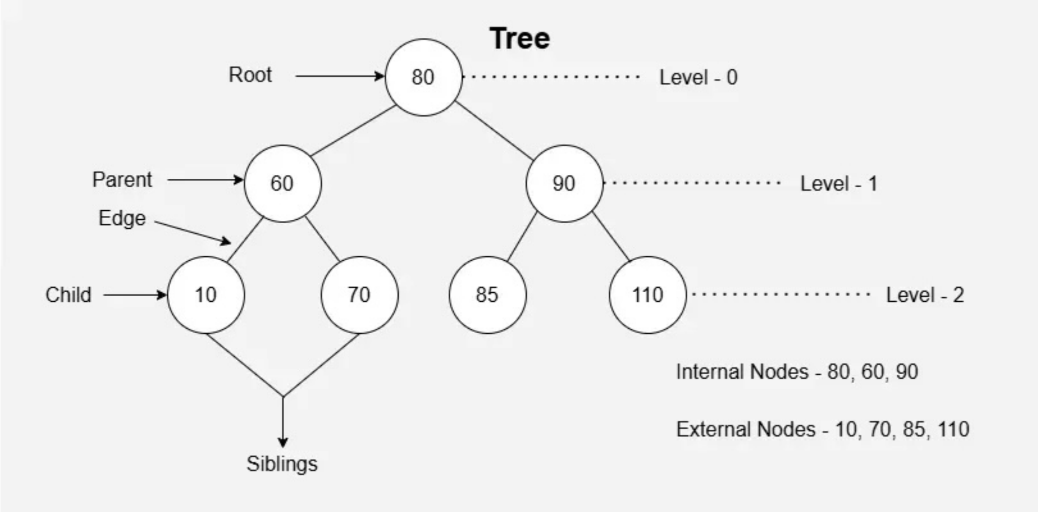


1
7
9
2
6
9
5
11
5

Structure


Root
Edge
A
B
C
D
E
F
G
H
I
J
K
Level 0
Level 1
Level 2
Level 3
Parent Node
Child Node
Siblings
Subtree
Leaf Nodes

Organization



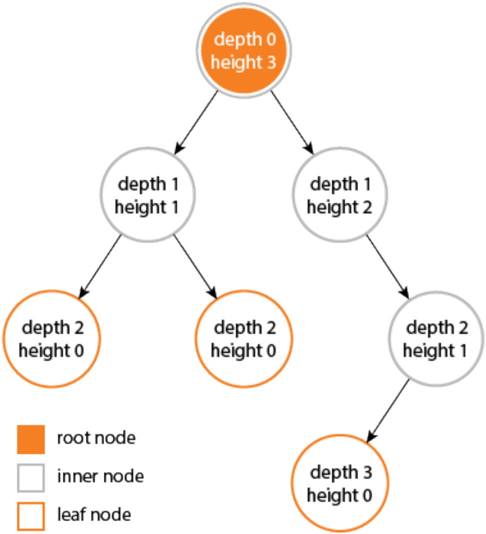
Tree - Properties

- **Number of edges** as the connection between two nodes. If a tree has N nodes then it will have (N-1) edges. There is only one path from each node to any other node of the tree
- **Depth of a node** the length of the path from the root to that node. Each edge adds 1 unit of length to the path. So, it can also be defined as the number of edges in the path from the root of the tree to the node.

Tree Properties

- **Height of a node** as the length of the longest path from the node to a leaf node of the tree.
- **Height of the Tree** is the length of the longest path from the root of the tree to a leaf node of the tree
- **Degree of a Node** the total count of subtrees attached to that node

Height vs. Depth



Tree - Non-linear data structure

- **not linear** data in a tree are not stored in a sequential manner
- **hierarchical structure** data arranged on multiple levels

Types of Trees

- **Binary tree**
- **Ternary Tree**
- **Generic Tree**

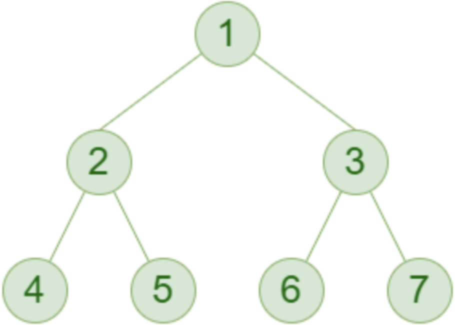
Binary Trees

Binary Tree

A binary tree is a tree data structure in which each node has at most two children, referred to as the left child and the right child.

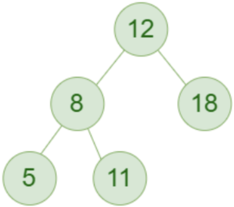
Lesson 3

Trees



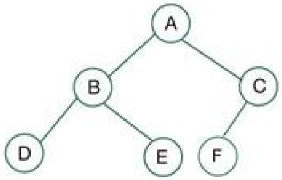
Full Binary Tree

A full binary tree is a binary tree with either zero or two child nodes for each node.



Complete Binary Tree

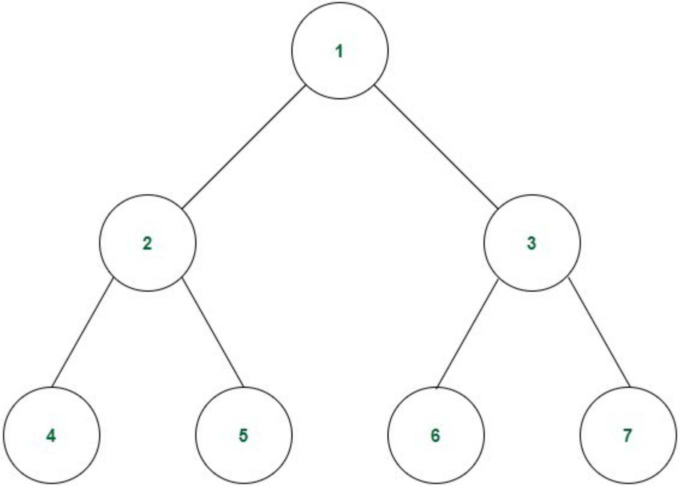
A complete binary tree is a special type of binary tree where all the levels of the tree are filled completely except the lowest level nodes which are filled from as left as possible.



Perfect Binary Tree

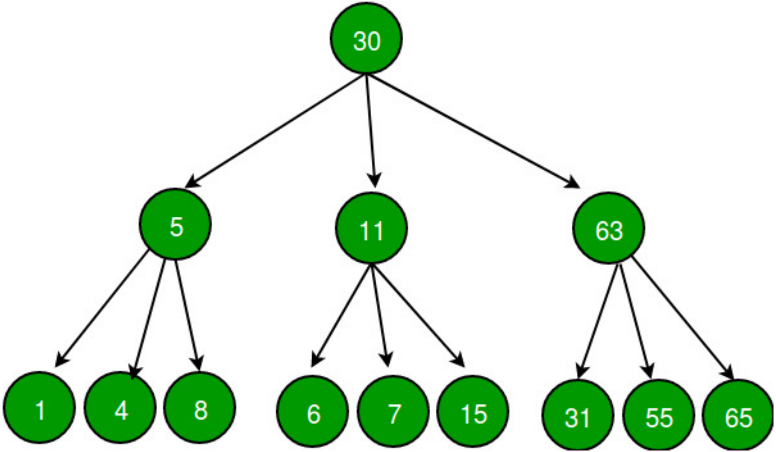
A perfect binary tree is a special type of binary tree in which all the leaf nodes are at the same depth, and all non-leaf nodes have two children. In simple terms, this means that all leaf nodes are at the maximum depth of the tree, and the tree is completely filled with no gaps.

Perfect Binary Tree



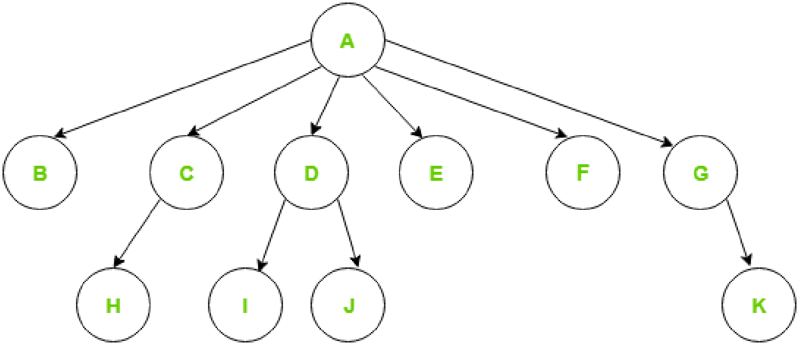
Ternary Trees

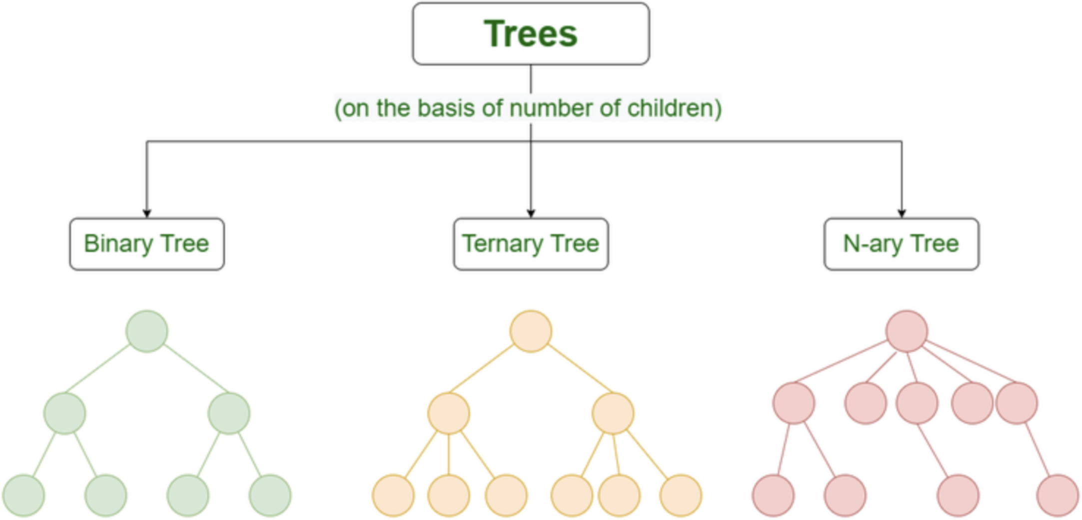
Ternary Tree



Generic Trees

Generic Tree

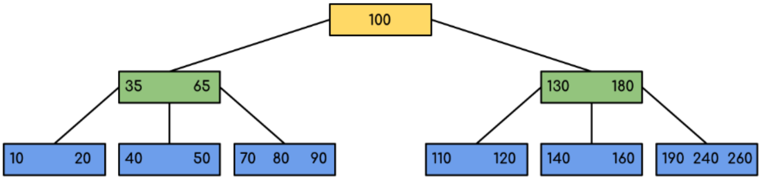




Trees
(on the basis of number of children)
Binary Tree
Ternary Tree
N-ary Tree

Binary search tree (BST)

A B-Tree is a specialized N-way tree designed to optimize data access, especially on disk-based storage systems.



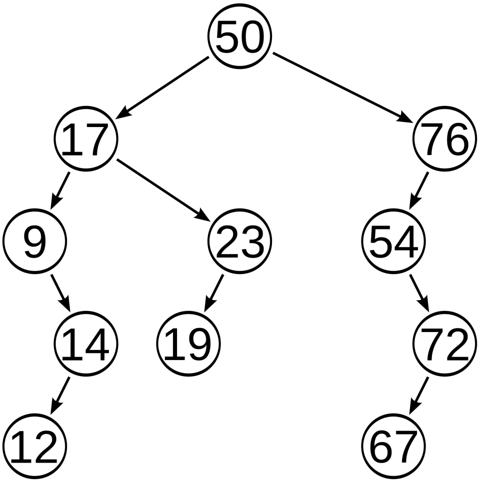
Binary Search Trees (BST)

A B-tree is a self-balancing tree data structure that maintains sorted data and allows searches, sequential access, insertions, and deletions very fast (logarithmic time). The B-tree generalizes the binary search tree, allowing for nodes with more than two children.

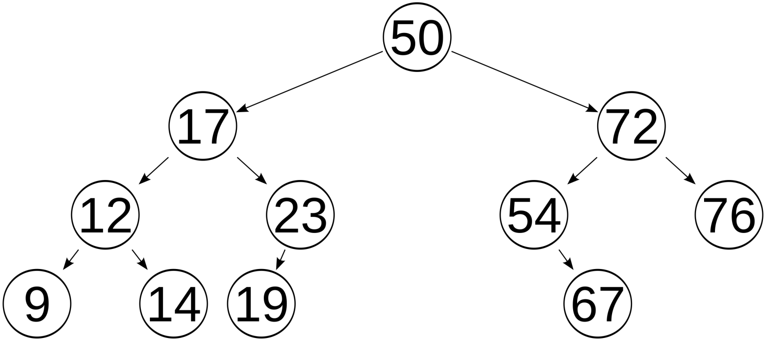
Self-Balancing?

A self-balancing tree is a tree that automatically keeps its height (maximal number of levels below the root) small in the face of arbitrary item insertions and deletions.

Un-balanced tree



Self-balanced tree



Binary Search Trees (BST)

Binary search trees allow very fast lookup, addition, and removal of data items using binary search. Binary search trees are the basis and very important for sorting and search algorithms.

Tree Operations

- **CREATE** — create a tree in a data structure
- **INSERT** — insert data in a tree
- **SEARCH** — search data in a tree
- **TRAVERSAL** — Depth-First, Breadth-First

There are numerous applications of trees

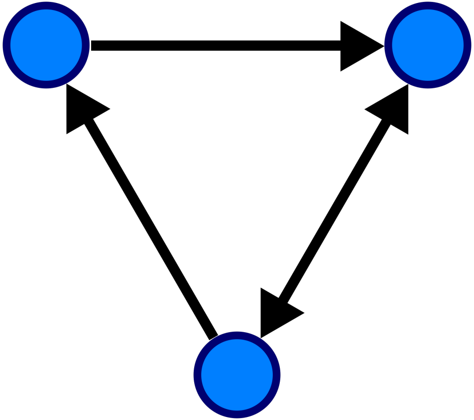
- File system implementation: directory structure
- Natural language processing (NLP)
- AI, Genetic programming
- Database indexing (BST B-Trees, B+ Trees)

Graphs

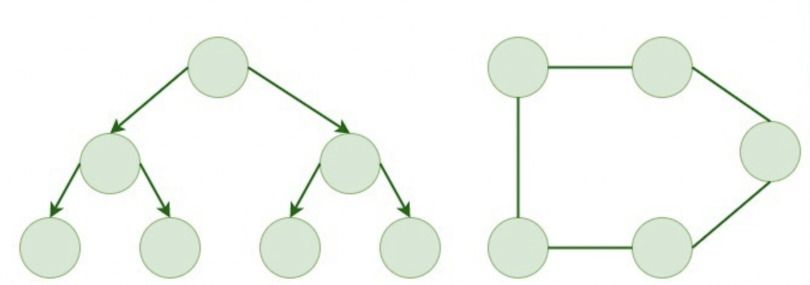
What is a graph?

A graph is a non-linear data structure consisting of vertices and edges. It is an abstract data type that consists of a finite set of vertices (nodes or points), together with a set of edges (also called links or lines).

Graphs



Graphs vs Trees



Graphs

A graph data structure is a collection of nodes (also called vertices) and edges that connect them. Nodes can represent entities, such as people, places, or things, while edges represent relationships between those entities.

Trees

A tree data structure is a hierarchical data structure that consists of nodes connected by edges. Each node can have multiple child nodes, but only one parent node. The topmost node in the tree is called the root node.

Feature	Graph	Tree
Definition	A collection of nodes (vertices) and edges, where edges connect nodes.	A hierarchical data structure consisting of nodes connected by edges with a single root node.
Structure	Can have cycles (loops) and disconnected components.	No cycles; connected structure with exactly one path between any two nodes.
Root Node	No root node; nodes may have multiple parents or no parents at all.	Has a designated root node that has no parent.
Node Relationship	Relationships between nodes are arbitrary.	Parent-child relationship; each node (except the root) has exactly one parent.
Edges	Each node can have any number of edges.	If there is **n** nodes then there would be **n-1** number of edges
Traversal Complexity	Traversal can be complex due to cycles and disconnected components.	Traversal is straightforward and can be done in linear time.

Graphs vs Trees

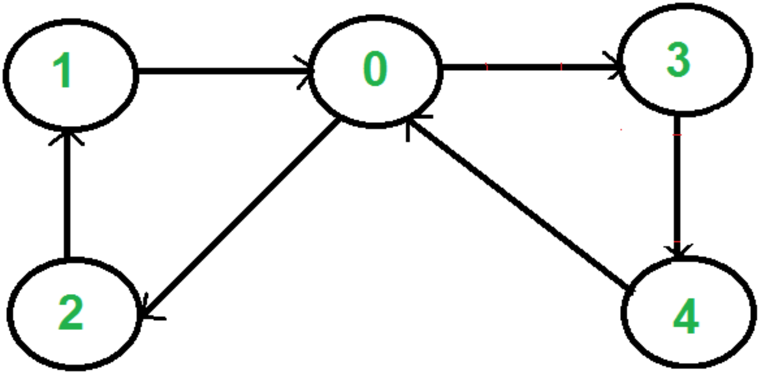
- **Cycles** Graphs can contain cycles, while trees cannot
- **Connectivity** Graphs can be disconnected, while trees are always connected
- **Hierarchy** Trees have a hierarchical structure, with one vertex designated as the root. Graphs do not have this hierarchical structure
- **Applications** Trees used in hierarchical data structures, such as file systems and XML documents. Graphs better for modelling transportation networks, social networks, computer networks

Types of graphs

- Directed and Undirected graphs
- Weighted Vs Unweighted graphs

Directed graph

A directed graph is defined as a type of graph where the edges have a direction associated with them.



Directed graphs

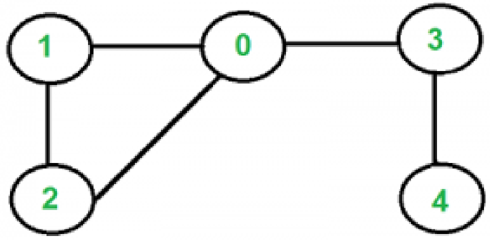
- edges have a direction associated with them, indicating a one-way relationship between vertices
- each vertex in a directed graph has two different degree measures: indegree and outdegree. Indegree is the number of incoming edges to a vertex, while outdegree is the number of outgoing edges from a vertex
- A directed graph can contain cycles, which are paths that start and end at the same vertex and contain at least one edge

There are numerous applications of directed graphs

- social networks
- transportation networks: flight routes, railroads, and road networks. Cities, towns, and intersections are represented by nodes, and the links that connect these locations—such as highways, train tracks, and flight routes—are represented by edges.
- computer networks

Undirected graph

An undirected graph is a type of graph where the edges have no specified direction assigned to the them.



Undirected graphs

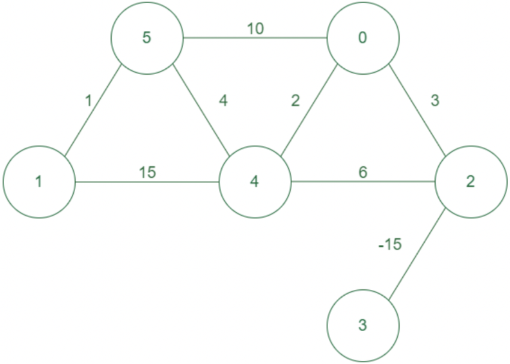
- edges in an undirected graph are bidirectional
- there is no concept of a "parent" or "child" vertex as there is no direction to the edges.
- may contain loops, which are edges that connect a vertex to itself

Applications of un-directed graphs

- traffic flow optimization: Undirected graphs are used in traffic flow optimization to model the flow of vehicles on road networks. The vertices of the graph represent intersections or road segments, and the edges represent the connections between them. The graph can be used to optimize traffic flow and plan transportation infrastructure
- website analysis: analyze the links between web pages on the internet

Weighted graph

A weighted graph is defined as a special type of graph in which the edges are assigned some weights which represent cost, distance, and many other relative measuring units.

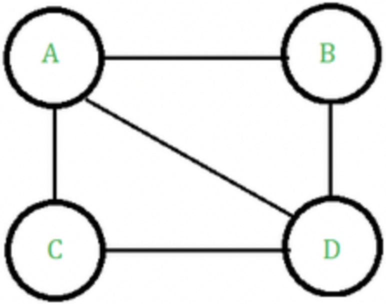


Applications of weighted graphs

- Transportation networks: solve the path that takes the least time, or the path with the least overall distance. This is a simplification of how weighted graphs can be used for more complex things like a GPS system. Graphs are used to study traffic patterns, traffic light timings and much more by many big tech companies such as UBER etc. Graph networks are used by many map programs such as Google Maps, Bing Maps
- Epidemiology: Weighted graphs can be used to find the maximum distance transmission from an infectious to a healthy person

Unweighted graph

An unweighted graph is a graph in which the edges do not have weights or costs associated with them. Instead, they simply represent the presence of a connection between two vertices.

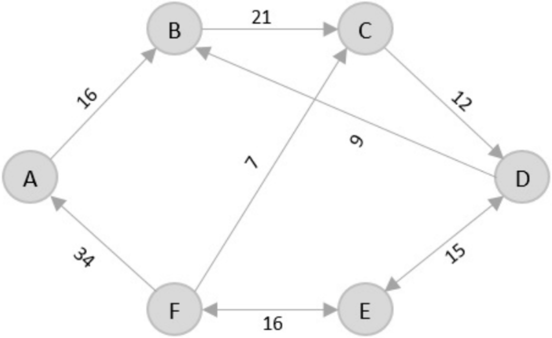


Applications of unweighted graphs

- represent circuit diagrams
- solve puzzles
- used in social media sites to find whether two users are connected or not
- can modelate the possible moves in a game

A real problem

Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?



How we plan to solve the problem?

- think how we represent the problem
- what data structure we plan to use
- and what algorithm