PH3205-Computational Physics

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Assignment 3

Aim

Use the non-linear equation, $\frac{d^2y}{dx^2} = y^2 + \cos(2\pi x) - \sin^4(\pi x)$ with boundary conditions $y(x = 0) = 0 = y(x = 1)$. Evaluate the solutions using roots problem approach in shooting method.

Solution

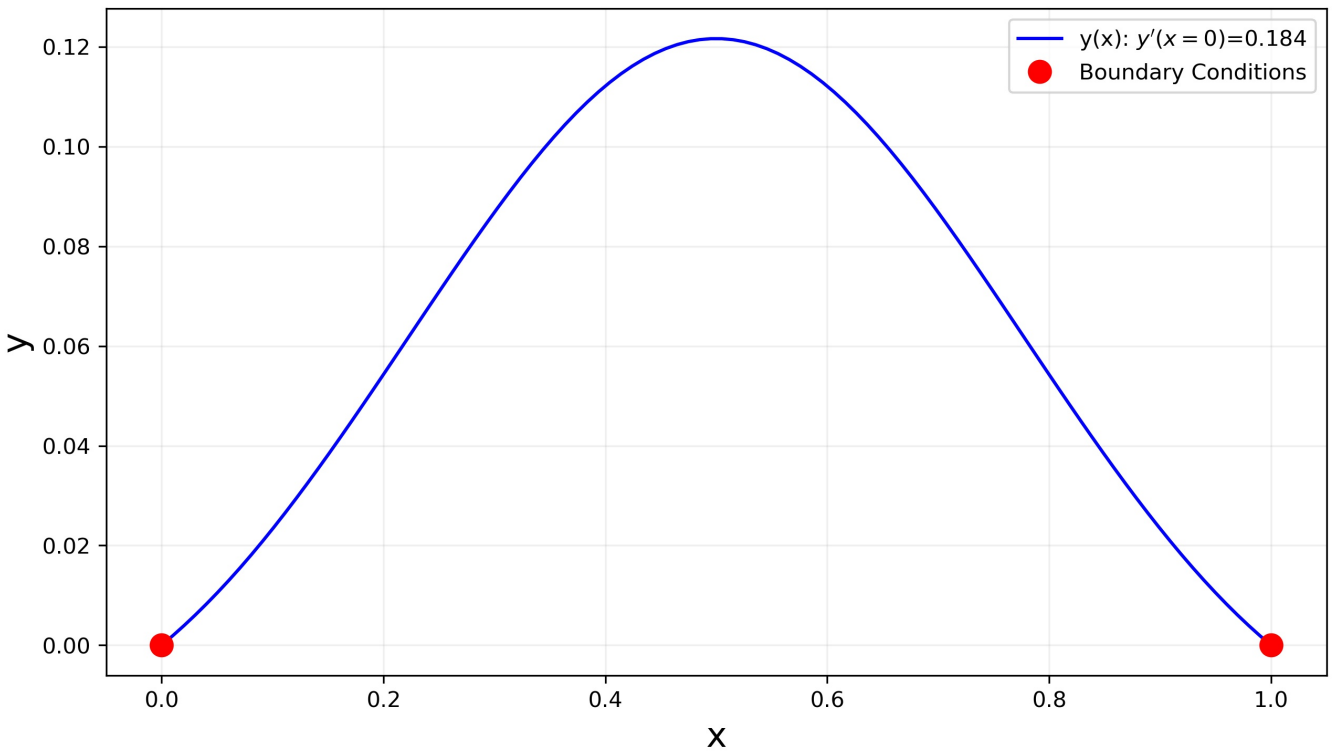
We used the shooting method to solve the problem and implemented this in Python. We defined the set of 1st order differential equation:

$$\frac{dy}{dx} = v$$
$$\frac{dv}{dx} = y^2 + \cos(2\pi x) - \sin^4(\pi x)$$

Then using *solve_ivp* from the *scipy.integrate* module, we obtain the solution for any given $\frac{dy}{dx}(x = 0)$. We then defined the objective function as a function of $\frac{dy}{dx}(x = 0)$ which returns the difference between the final position of the numerical solution and boundary value($y(x = 1) = 0$).
Then using *fsolve* function from *scipy.optimize* we get the optimal value of the $\frac{dy}{dx}(x = 0)$ and get $y(x)$.

The required python files are : *Assignment3.ipynb* (Jupyter Notebook), *Assignment3.py*.

The output of the code is in the next page(Assignment3.jpg)


Solution for $\frac{d^2y}{dt^2} = y^2 + \cos(2\pi x) - \sin^4(\pi x)$
y(x): $y'(x = 0)$=0.184
Boundary Conditions
0.12
0.10
0.08
0.06
0.04
0.02
0.00
y
0.0
0.2
0.4
0.6
0.8
1.0
x