PH3205-Computational Physics

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MidSem Practicals

A notebook is provided containing solution to both the problems: *MS_notebook.ipynb*. Also individual *.py* files are provided separately for this.

Problem 1: Required Python file: *Problem1.py*

Part A

Given probability distribution:

$$P(y) = (1 - 1/5) \times y^{-1/5}$$

we want to generate random numbers following this distribution. We first generate x_i's from a uniform distribution, then we calculate y_i's from the formula above. We use the *inverse transform method* to generate y_i's.

$$y(x) = \int_0^x (1 - 1/5) \times t^{-1/5} dt = x^{4/5}$$
$$\implies x_i = y_i^{5/4}$$

Required Plots : *Problem_1a.jpg*

Part B

Using the above given distribution, we integrate the given function:

$$I = \int_0^1 y^{-1/5} e^{-y}\, dy$$

Since we have already mapped the uniform random numbers in $(0, 1)$ to the given weight function, we can use the importance sampling approach. The estimated integral will be

$$\hat{I} = \frac{1}{N} \sum_{i=1}^{N} \frac{x_i^{-1/5} e^{-x_i}}{(4/5) \times x_i^{-1/5}}$$

where N is the number of samples and

$$x_i = y_i^{5/4} \qquad y_i \in Uniform(0, 1)$$

Using this, Approximated value of integral up-to 3 decimal places: 0.837

Problem 2: Required Python file: *Problem2.py*

Given Hamiltonian:

$$H = \frac{1}{2}[p_x^2 + p_y^2] + \frac{1}{2}[x^2 + y^2] + x^2 y - \frac{1}{3}y^3$$

The system of equations for the given Hamiltonian are:

$$\frac{dx}{dt} = p_x \qquad \frac{dp_x}{dt} = -x - 2xy$$
$$\frac{dy}{dt} = p_y \qquad \frac{dp_y}{dt} = y^2 - y - x^2$$

Since the Hamiltonian doesn't depend on time, the energy is constant.

Required Plots : *Problem_2.jpg* .

Note that the y axis have mean value subtracted to observe the deviations better. It is clearly seen that the energy is more or less constant throughout the time.

For given $[x(0), y(0), p_x(0), p_y(0)] = [0.2, 0.2, 0.1, 0.1]$ the average energy upto 2 decimal places is 0.05