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Homework 6 - Numerical Analysis

Interpolation

We selected the function f(x) = e^x - 1, then we calculated the value for the given points.

Table 1: Points used with the interpolation method.

x	f(x)
-2.5	-0.9179150013761012
-1	-0.6321205588285577
0.5	0.6487212707001282
1.2	2.3201169227365472

With these points and their values we run the interpolation methods Vandermonde, Newton and Lagrange. For each method we obtained the following results:

Vandermonde Matrix:

```
([[-15.625,   6.25 ,  -2.5  ,   1.   ],
       [ -1.   ,   1.   ,  -1.   ,   1.   ],
       [  0.125,   0.25 ,   0.5  ,   1.   ],
       [  1.728,   1.44 ,   1.2  ,   1.   ]])
```

Polynomial:

0.12866657x^3 + 0.60712136 x^2 + 1.0609553x - 0.04962004

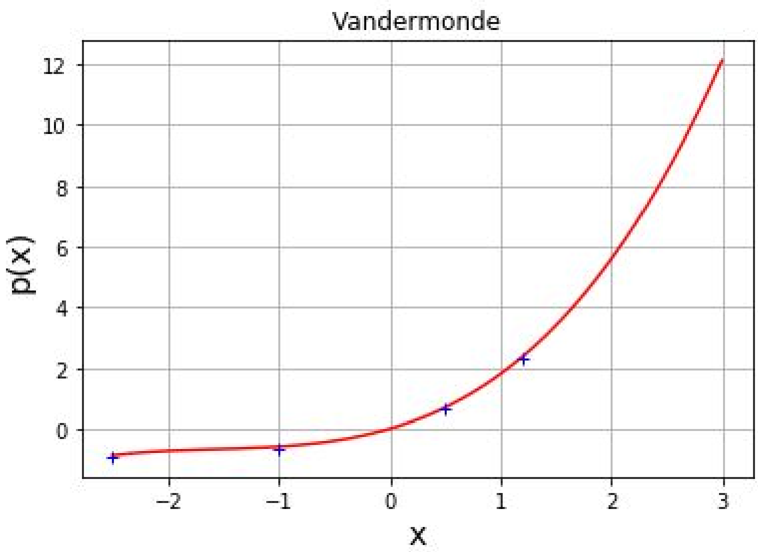

Fig 1: Approximation with Vandermonde method.

Divided Differences table:

```
([[-0.917915  ,  0.        ,  0.        ,  0.        ],
  [-0.63212056,  0.19052963,  0.        ,  0.        ],
  [ 0.64872127,  0.85389455,  0.22112164,  0.        ],
  [ 2.32011692,  2.38770807,  0.69718796,  0.12866657]])
```

Newton Polynomial:

-0.917915 + 0.19052963(x+2.5) + 0.22112164(x+2.5)(x+1) + 0.12866657(x+2.5)(x+1)(x-0.5)

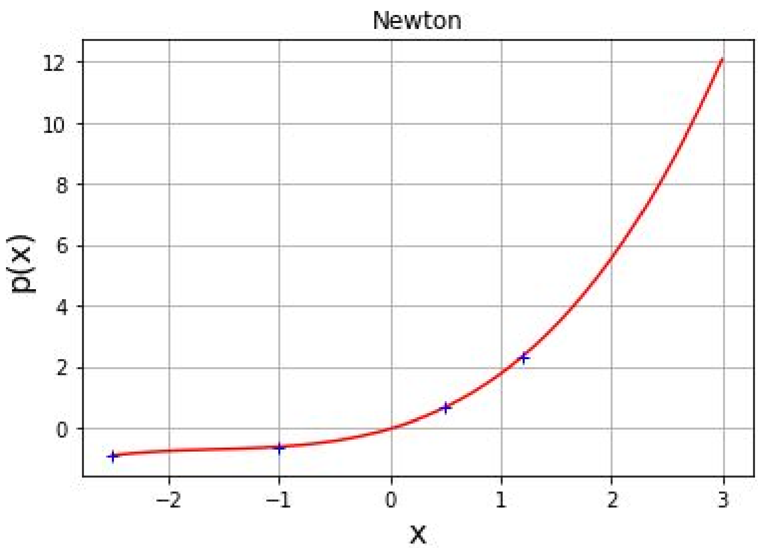

Fig 2: Approximation with Newton method.

Lagrange Polynomial:

0.128667x^3 + 0.607121x^2 + 1.06096x - 0.04962

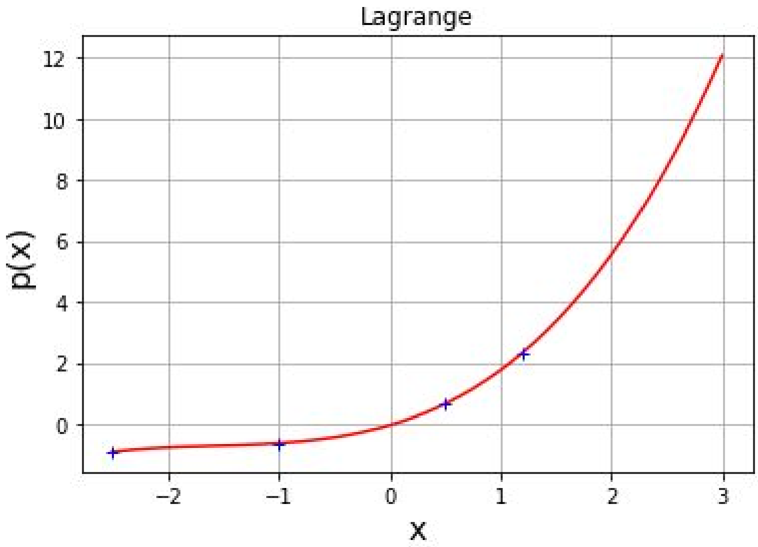

Fig 2: Approximation with Lagrange method.

Root finding methods

For this section we used the vandermonde polynomial which had the following coefficients:
0.12866657 X ^3 + 0.60712136X ^2 + 1.0609553 X - 0.04962004

We used the incremental search method starting from **-3** and with a step of **0.001**. The interval found was **[-2.1938700855983484e-13 , 0.000999999999780613]**.

Using the previous interval we run the methods bisection, regula falsi, newton and secant. For each method we obtained the following results using a tolerance of **1e-7** and an iteration limit of **10000**:

Result	Bisection	Regula falsi	Newton	Secant
Iterations	14	2	5	6
Error	0.0000000648	0.0000000000	0.0000000000	0.0000000005
x	0.0000000610	0.0000000000	0.0000000000	0.0000000000

The approximate polynomial obtained using the Vandermonde method was good enough to obtain a very close value to the real root of the original function.

X0 for newton: -0.1
x0 and x1 for secant: -0.12, -0.1

Iterations:

Bisection

Iteration	a	x	b	f(x)	Error
1	-0.0000000000	0.0005000000	0.0010000000	0.0005306294	-
2	-0.0000000000	0.0002500000	0.0005000000	0.0002652768	0.0002500000
3	-0.0000000000	0.0001250000	0.0002500000	0.0001326289	0.0001250000
4	-0.0000000000	0.0000625000	0.0001250000	0.0000663121	0.0000625000
5	-0.0000000000	0.0000312500	0.0000625000	0.0000331554	0.0000312500
6	-0.0000000000	0.0000156250	0.0000312500	0.0000165776	0.0000156250
7	-0.0000000000	0.0000078125	0.0000156250	0.0000082888	0.0000078125
8	-0.0000000000	0.0000039062	0.0000078125	0.0000041444	0.0000039063
9	-0.0000000000	0.0000019531	0.0000039062	0.0000020722	0.0000019531
10	-0.0000000000	0.0000009766	0.0000019531	0.0000010361	0.0000009766
11	-0.0000000000	0.0000004883	0.0000009766	0.0000005180	0.0000004883
12	-0.0000000000	0.0000002441	0.0000004883	0.0000002590	0.0000002441
13	-0.0000000000	0.0000001221	0.0000002441	0.0000001295	0.0000001221
14	-0.0000000000	0.0000000610	0.0000001221	0.0000000648	0.0000000610

Regula Falsi

Iteration	a	x	b	f(x)	Error
1	-0.0000000000	-0.0000000000	0.0010000000	-0.0000000000	-
2	-0.0000000000	-0.0000000000	0.0010000000	-0.0000000000	0.0000000000

Newton(root)

Iteration	x	f(x)	Error
0	-0.1000000000	-0.1001529834	-
1	0.0207118034	0.0222358831	0.1207118034
2	0.0007110158	0.0007546629	0.0200007877
3	0.0000008668	0.0000009196	0.0007101490

4	0.0000000000	0.0000000000	0.0000008668
5	-0.0000000000	-0.0000000000	0.0000000000

Secant

Iteration	x	f(x)	Error
0	-0.1200000000	-0.1187944247	-
1	-0.1000000000	-0.1187944247	-
2	0.0074519738	0.0079399790	0.1074519738
3	-0.0004409213	-0.0004676798	0.0078928951
4	-0.0000018755	-0.0000019898	0.0004390458
5	0.0000000005	0.0000000005	0.0000018760
6	-0.0000000000	-0.0000000000	0.0000000005