Taller 1 Santiago Passos - Alejandro Rojas

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1. P.F. $0.1\ldots\ldots\ldots \times 2^{k}$ $k \in \mathbb{Z}$

Sea x un número de la forma $0.d_1d_2d_3\ldots d_k d_{k+1}\ldots * 2^{n}$

Sea $\hat{x}$ un número en base 2 normalizada con k cifras significativas. $\hat{x}$ es una aproximación de x.

Sea $\varepsilon = \left| \frac{x-\hat{x}}{x} \right|$ el valor del error relativo.

Caso (i) $d_{k+1} = 0$.

$$\varepsilon = \left| \frac{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots * 2^{n} - 0.d_1d_2\ldots d_k * 2^{n}}{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots * 2^{n}} \right|$$

$$\varepsilon = \left| \frac{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots - 0.d_1d_2\ldots d_k}{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots} \right|$$

Como $d_{k+1} = 0$, $0.d_{k+1}d_{k+2}\ldots < 0.1$.

$$\varepsilon = \left| \frac{0.d_{k+1}d_{k+2}\ldots * 2^{-k}}{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots} \right| < \left| \frac{0.1 * 2^{-k}}{0.1} \right| = 2^{-k}$$

Caso (ii) $d_{k+1} = 1$

$$\varepsilon = \left| \frac{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots * 2^{n} - 0.d_1d_2\ldots(d_k+1) * 2^{n}}{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots * 2^{n}} \right|$$

$$\varepsilon = \left| \frac{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots - 0.d_1d_2\ldots d_k - 2^{-k}}{0.d_1d_2d_3\ldots d_k d_{k+1}\ldots} \right|$$

Como $d_{k+1} = 1$, $|0.d_{k+1}d_{k+2}\ldots - 1| = |1 - 0.d_{k+1}d_{k+2}| < 0.1$

$$\varepsilon = \left| \frac{(0.d_{k+1}d_{k+2}\ldots - 1) * 2^{-k}}{0.d_1 d_2 d_3 \ldots d_k d_{k+1} \ldots} \right| \leq \left| \frac{0.1 \; * 2^{-k}}{0.1} \right| = 2^{-k}$$

Ahora usando redondeo por corte.

$$\varepsilon = \left| \frac{0.d_1 d_2 d_3 \ldots d_k d_{k+1} \ldots * 2^n - 0.d_1 d_2 \ldots d_k * 2^n}{0.d_1 d_2 d_3 \ldots d_k d_{k+1} \ldots * 2^n} \right|$$

$$\varepsilon = \left| \frac{0.d_1 d_2 d_3 \ldots d_k d_{k+1} \ldots \quad - 0.d_1 d_2 \ldots d_k}{0.d_1 d_2 d_3 \ldots d_k d_{k+1} \ldots} \right|$$

$$\varepsilon = \left| \frac{0.d_{k+1}d_{k+2}\ldots * 2^{-k}}{0.d_1 d_2 d_3 \ldots d_k d_{k+1} \ldots} \right| < \left| \frac{1 * 2^{-k}}{0.1} \right| = \left| \frac{1 * 2^{-k}}{1 * 2^{-1}} \right| = 2^{-k+1}$$