

Paper can be made from new wood pulp, recycled office paper, or recycled newsprint. New pulp cost \$200 per ton, recycled office paper, \$100 per ton, and recycled newsprint, \$40 per ton. One available process uses 3 tons of pulp to make 1 ton of paper; a second uses 1 ton of pulp and 4 tons of recycled office paper; a third uses 1 ton of pulp and 12 tons of recycled newsprint; a fourth 8 tons of recycled office paper. Currently, only 80 tons of pulp are available. We wish to produce 100 tons of new paper at minimum total cost. The problem can be modeled as the following LP.

$$\min 200x_1 + 100x_2 + 40x_3$$

$$\text{s.t. } x_1 = 3y_1 + y_2 + y_3$$

$$x_2 = 4y_2 + 8y_4$$

$$x_3 = 12y_3$$

$$x_1 \leq 80$$

$$\sum_{j=1}^4 y_j \geq 100$$

$$x_1, x_2, x_3, y_1, y_2, y_3, y_4 \geq 0$$

The LP was implemented in Excel and run using Solver. Find below Solver's sensitivity report along with a screen capture of the Excel implementation showing the optimal solution.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$16	Pulp purchased (tons)	80.00	0.00	200.00	200.00	1E+30
\$B\$17	Recycled office paper purchased (tons)	480.00	0.00	100.00	20.00	50.00
\$B\$18	Recycled newsprint purchased (tons)	0.00	0.00	40.00	1E+30	6.67
\$B\$21	Amount produced (tons) - Process 1	0.00	400.00	0.00	1E+30	400.00
\$C\$21	Amount produced (tons) - Process 2	80.00	0.00	0.00	80.00	1E+30
\$D\$21	Amount produced (tons) - Process 3	0.00	80.00	0.00	1E+30	80.00
\$E\$21	Amount produced (tons) - Process 4	20.00	0.00	0.00	1E+30	200.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$24	Balance for pulp	80.00	400.00	0.00	80.00	20.00
\$B\$25	Balance for recycled office paper	480.00	100.00	0.00	1E+30	480.00
\$B\$26	Balance for recycled newsprint	0.00	40.00	0.00	1E+30	0.00
\$B\$30	Pulp availability (tons)	80.00	-200.00	80.00	20.00	80.00
\$B\$34	Paper required (tons)	100.00	800.00	100.00	1E+30	20.00

	A	B	C	D	E
1					
2	Cost of Processing Material				
3		Cost (per ton)			
4	Pulp	\$200			
5	Recycled office paper	\$100			
6	Recycled newsprint	\$40			
7					
8	Requirements for the Various Processes				
9		Process 1	Process 2	Process 3	Process 4
10	Pulp (ton)	3	1	1	
11	Recycled office paper (ton)		4		8
12	Recycled newsprint (ton)			12	
13					
14	Decision Variables				
15		Purchased			
16	Pulp (tons)	80.00			
17	Recycled office paper (tons)	480.00			
18	Recycled newsprint (tons)	0.00			
19					
20		Process 1	Process 2	Process 3	Process 4
21	Amount produced (tons) -	0.00	80.00	0.00	20.00
22					
23	Balance Constraints for Resource Usage				
24	Balance for pulp	80.00	=	80.00	
25	Balance for recycled office paper	480.00	=	480.00	
26	Balance for recycled newsprint	0.00	=	0.00	
27					
28	Limit on Pulp Availability				
29		Used		Available	
30	Pulp availability (tons)	80.00	<=	80.00	
31					
32	Demand Requirement				
33		Produced		Required	
34	Paper required (tons)	100.00	>=	100.00	
35					
36	Objective				
37		Cost			
38	Pulp	\$16,000.00			
39	Recycled office paper	\$48,000.00			
40	Recycled newsprint	\$0.00			
41	Total	\$64,000.00			

Based on the stated model, the Excel screen capture and the sensitivity results shown, answer the following (1.5 points per question):

- Explain why this problem can be modeled using the given LP. Make sure to explain the meaning of the decision variables and constraints.
- What is the marginal cost of production at optimality?
- How much extra amount (beyond \$200) should we be willing to pay to obtain an additional ton of pulp?
- How much will the optimal cost change if the price of pulp is increased to \$300 per pulp?

- e) How much will the optimal cost change if the price of recycled office paper is decreased to \$40 per ton? If you cannot determine the exact value, try to find a lower bound for it.
- f) How much will the optimal cost change if the price of recycled office paper is increased to \$150 per ton? If you cannot determine the exact value, try to find a lower and an upper bound for it.
- g) How much will the optimal cost change if the number of tons of paper required decreased to 60? If you cannot determine the exact value, try to find a lower and an upper bound for it.
- h) How much will the optimal cost change if the optimal cost would change if the number of tons of new paper needed increased to 200?
- i) How cheap recycled newsprint would have to become before the primal solution could change.
- j) An experimental new process will use 6 tons of newsprint and an undetermined number α tons of recycled office paper. How low would α have to be for the new process to be competitive with existing one?
- k) Would a limit of 400 tons on recycled office paper change the optimal solution?
- l) Would a limit of 400 tons on recycled newsprint change the optimal solution?