BE 521: Homework 4

HFOs

Spring 2021

58 points

Due: Tuesday, 2/23/2021 10:00pm

Objective: HFO detection and cross-validation

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HFO Dataset

High frequency oscillations (HFOs) are quasi-periodic intracranial EEG transients with durations on the order of tens of milliseconds and peak frequencies in the range of 80 to 500 Hz. There has been considerable interest among the epilepsy research community in the potential of these signals as biomarkers for epileptogenic networks.

In this homework exercise, you will explore a dataset of candidate HFOs detected using the algorithm of Staba et al. (see article on Canvas). The raw recordings from which this dataset arises come from a human subject with mesial temporal lobe epilepsy and were contributed by the laboratory of Dr. Greg Worrell at the Mayo Clinic in Rochester, MN.

The dataset `I521_A0004_D001` contains raw HFO clips that are normalized to zero mean and unit standard deviation but are otherwise unprocessed. The raw dataset contain two channels of data: `Test_raw_norm` and `Train_raw_norm`, storing raw testing and training sets of HFO clips respectively. The raw dataset also contains two annotation layers: `Testing windows` and `Training windows`, storing HFO clip start and stop times (in microseconds) for each of the two channels above. Annotations contain the classification by an "expert" reviewer (i.e., a doctor) of each candidate HFO as either an HFO (2) or an artifact (1). On ieeg.org and upon downloading the annotations, You can view this in the "description" field.

After loading the dataset in to a `session` variable as in prior assignments you will want to familiarize yourself with the `IEEGAnnotationLayer` class. Use the provided "getAnnotations.m" function to get all the annotations from a given dataset. The first output will be an array of annotation objects, which you will see also has multiple fields including a description field as well as start and stop times. Use You can use the information outputted by getAnnotations to pull each HFO clip.

1 Simulating the Staba Detector (12 pts)

Candidate HFO clips were detected with the Staba et al. algorithm and subsequently validated by an expert as a true HFO or not. In this first section, we will use the original iEEG clips containing HFOs and re-simulate a portion of the Staba detection.

1. How many samples exist for each class (HFO vs artifact) in the training set? (Show code to support your answer) (1 pt)

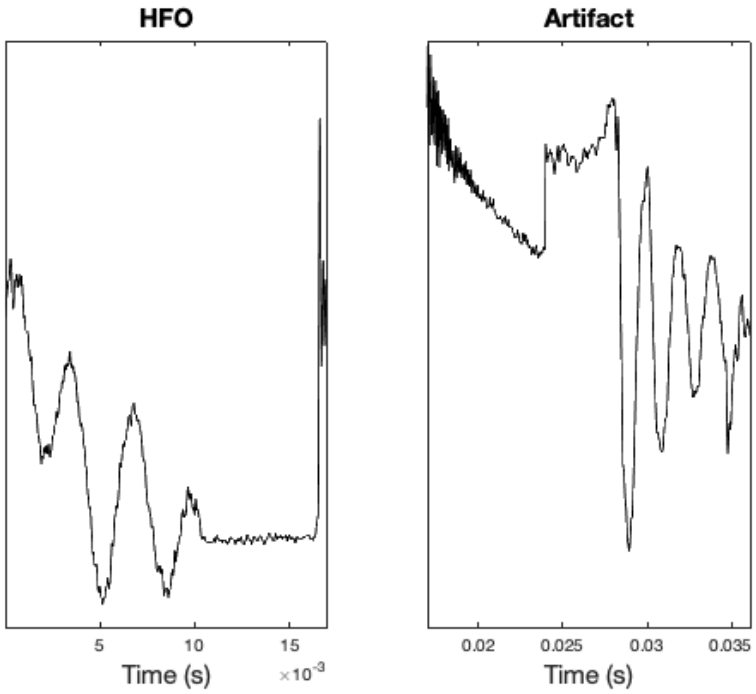
```
cd('/Users/sppatankar/Developer/BE-521')
addpath(genpath('Homework_4'));
addpath(genpath('ieeg-matlab-1.14.49'))

session = IEEGSession('I521_A0004_D001', 'spatank', 'spa_ieeglogin.bin');

% training
hfo_num = 0;
artifact_num = 0;
[all_train_events, ~, ~] = ...
    getAnnotations(session.data, session.data.annLayer(1, 1).name);
train_cell_array = cell(1, length(all_train_events));
for i = 1:length(all_train_events)
    if all_train_events(1, i).description == '2'
        hfo_num = hfo_num + 1;
        train_cell_array{1, i}.description = all_train_events(1, i).description;
    end
    if all_train_events(1, i).description == '1'
        artifact_num = artifact_num + 1;
        train_cell_array{1, i}.description = all_train_events(1, i).description;
    end
    start_time = all_train_events(1, i).start; % microseconds
    end_time = all_train_events(1, i).stop; % microseconds
    duration = end_time - start_time;
    channel = 1;
    train_cell_array{1, i}.start = start_time;
    train_cell_array{1, i}.end = end_time;
    train_cell_array{1, i}.signal = ...
        session.data.getvalues(start_time, duration, channel);
end

% testing
[all_test_events, ~, ~] = ...
    getAnnotations(session.data, session.data.annLayer(1, 2).name);
test_cell_array = cell(1, length(all_test_events));
for i = 1:length(all_test_events)
    test_cell_array{1, i}.description = all_test_events(1, i).description;
    start_time = all_test_events(1, i).start; % microseconds
    end_time = all_test_events(1, i).stop; % microseconds
    duration = end_time - start_time;
    channel = 1;
    test_cell_array{1, i}.start = start_time;
    test_cell_array{1, i}.end = end_time;
    test_cell_array{1, i}.signal = ...
        session.data.getvalues(start_time, duration, channel);
end

% print to console
hfo_num
artifact_num
```

```
IEEGSETUP: Found log4j on Java classpath.
URL: https://www.ieeg.org/services
Client user: spatank
Client password: ****

hfo_num =

   101
```

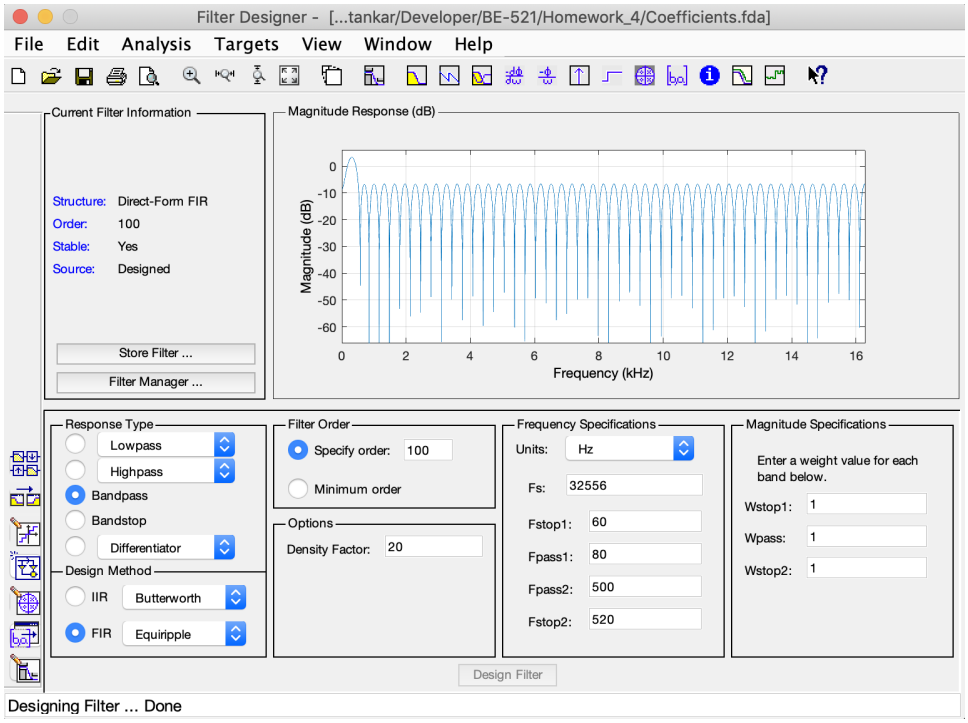
```
artifact_num =

    99
```

2. Using the training set, find the first occurrence of the first valid HFO and the first artifact. Using `subplot` with 2 plots, plot the valid HFO's (left) and artifact's (right) waveforms. Since the units are normalized, there's no need for a y-axis, so remove it with the command `set(gca,'YTick',[])`. (2 pts)

```
first_hfo = 0;
hfo_idx = 0;
while first_hfo ~= 1
    hfo_idx = hfo_idx + 1;
    label = train_cell_array{1, hfo_idx}.description;
    if label == '2' % HFO found
        first_hfo = 1;
    end
end

first_artifact = 0;
artifact_idx = 0;
while first_artifact ~= 1
    artifact_idx = artifact_idx + 1;
    label = train_cell_array{1, artifact_idx}.description;
    if label == '1' % artifact found
        first_artifact = 1;
    end
end

sampling_rate = session.data.sampleRate;
dt = 1/sampling_rate;

hfo_start_time = train_cell_array{1, hfo_idx}.start/1e6; % s
hfo_end_time = train_cell_array{1, hfo_idx}.end/1e6; % s
hfo_time = hfo_start_time:dt:hfo_end_time;
hfo_signal = train_cell_array{1, hfo_idx}.signal;

artifact_start_time = train_cell_array{1, artifact_idx}.start/1e6; % s
artifact_end_time = train_cell_array{1, artifact_idx}.end/1e6; % s
artifact_time = artifact_start_time:dt:artifact_end_time;
artifact_signal = train_cell_array{1, artifact_idx}.signal;

figure;
subplot(1, 2, 1);
plot(hfo_time, hfo_signal, 'LineWidth', 1, 'Color', [0, 0, 0]);
xlim([hfo_time(1), hfo_time(end)])
xlabel('Time (s)', 'FontSize', 15);
title('HFO', 'FontSize', 15);
set(gca, 'YTick', []);

subplot(1, 2, 2);
plot(artifact_time, artifact_signal, 'LineWidth', 1, 'Color', [0, 0, 0]);
xlim([artifact_time(1), artifact_time(end)])
xlabel('Time (s)', 'FontSize', 15);
title('Artifact', 'FontSize', 15);
set(gca, 'YTick', []);
```

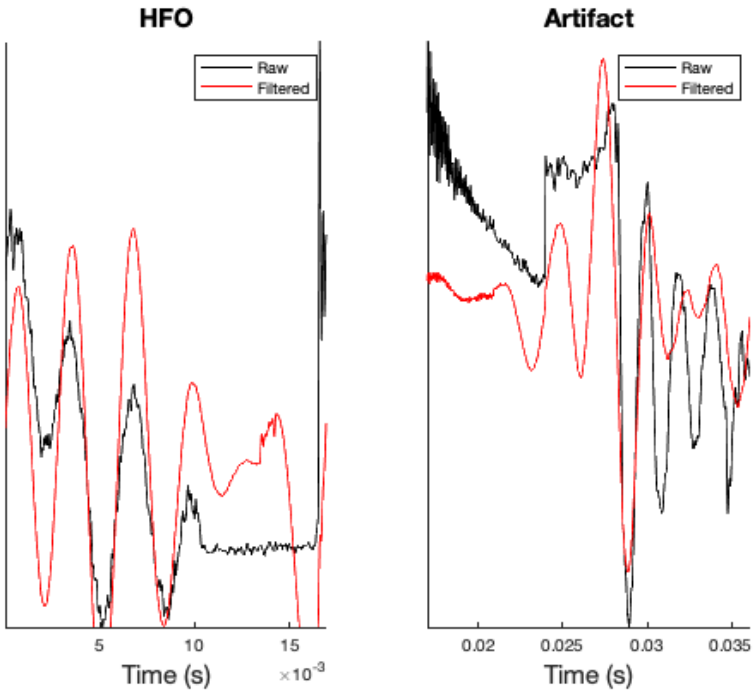


3. Using the `fdatool` in MATLAB, build an FIR bandpass filter of the equiripple type of order 100. Use the Staba et al. (2002) article to guide your choice of passband and stopband frequency. Once set, go to `File -> Export`, and export "Coefficients" as a MAT-File. Attach a screenshot of your filter's magnitude response. (Note: We will be flexible with the choice of frequency parameters within reason.) (3 pts)



4. Using the forward-backward filter function (`filtfilt`) and the numerator coefficients saved above, filter the valid HFO and artifact clips obtained earlier. You will need to make a decision about the input argument `a` in the `filtfilt` function. Plot these two filtered clips overlayed on their original signal in a two plot `subplot` as before. Remember to remove the y-axis. (3 pts)

```
load('Coefficients.mat');

hfo_signal_filt = filtfilt(Num, 1, hfo_signal);
artifact_signal_filt = filtfilt(Num, 1, artifact_signal);

figure;
subplot(1, 2, 1);
hold on
plot(hfo_time, hfo_signal, 'LineWidth', 1, 'Color', [0, 0, 0]);
plot(hfo_time, hfo_signal_filt, 'LineWidth', 1, 'Color', 'r');
hold off
xlim([hfo_time(1), hfo_time(end)])
ylim([min(hfo_signal), max(hfo_signal)])
legend('Raw', 'Filtered', 'Location', 'NorthEast');
xlabel('Time (s)', 'FontSize', 15);
title('HFO', 'FontSize', 15);
set(gca, 'YTick', []);

subplot(1, 2, 2);
hold on
plot(artifact_time, artifact_signal, 'LineWidth', 1, 'Color', [0, 0, 0]);
plot(artifact_time, artifact_signal_filt, 'LineWidth', 1, 'Color', 'r');
hold off
xlim([artifact_time(1), artifact_time(end)])
ylim([min(artifact_signal), max(artifact_signal)])
legend('Raw', 'Filtered', 'Location', 'NorthEast');
xlabel('Time (s)', 'FontSize', 15);
title('Artifact', 'FontSize', 15);
set(gca, 'YTick', []);
```



5. Speculate how processing the data using Staba's method may have erroneously led to a false HFO detection (3 pts)

   From the figure above, both the artifact and the HFO signals have high frequency components that end up being filtered out. As a result of the bandpass filtering process, at first glance it is difficult to distinguish the HFO from the artifact when they are plotted next to each other. Additionally, the artifact retains some of its higher frequency components in the earlier phase of the window, which may cause it to be mislabelled as an HFO. While this speculation is based on two examples from the data, it is possible that the same results repeat elsewhere leading to a large number of incorrect HFO labels.

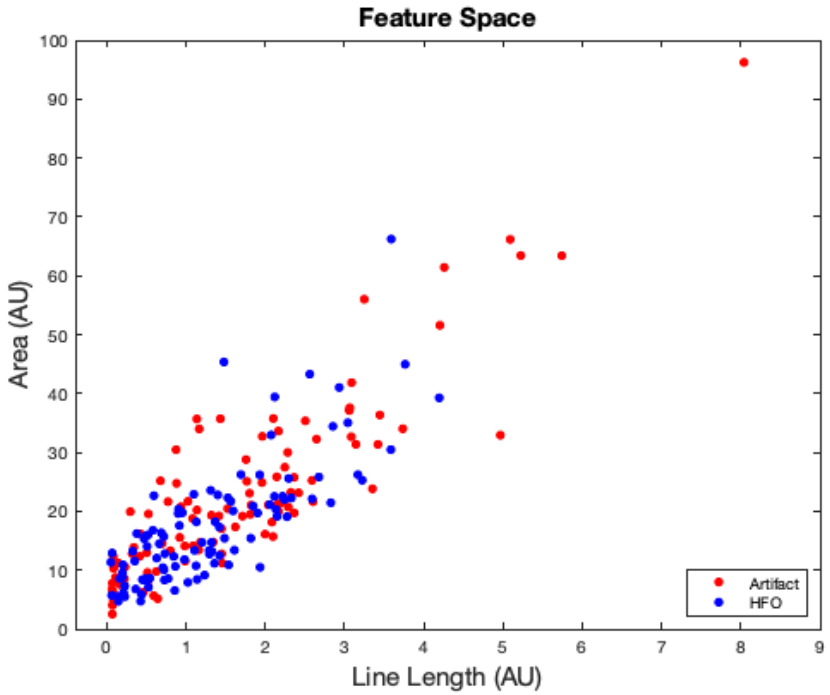
2 Defining Features for HFOs (9 pts)

In this section we will be defining a feature space for the iEEG containing HFOs and artifacts. These features will describe certain attributes about the waveforms upon which a variety of classification tools will be applied to better segregate HFOs and artifacts

1. Create two new matrices, `trainFeats` and `testFeats`, such that the number of rows correspond to observations (i.e. number of training and testing clips) and the number of columns is two. Extract the line-length and area features (seen previously in lecture and Homework 3) from the normalized raw signals (note: use the raw signal from ieeg.org, do not filter the signal). Store the line-length value in the first column and area value for each sample in the second column of your features matrices. Make a scatter plot of the training data in the 2-dimensional feature space, coloring the valid detections blue and the artifacts red. (Note: Since we only want one value for each feature of each clip, you will effectively treat the entire clip as the one and only "window".) (4 pts)

```
LLFn = @(x) sum(abs(diff(x)));
areaFn = @(x) sum(abs(x));
```

```
trainFeats = zeros(length(train_cell_array), 2);
trainLabels = zeros(length(train_cell_array), 1);

for i = 1:length(train_cell_array)
    signal = train_cell_array{1, i}.signal;
    trainFeats(i, 1) = LLFn(signal);
    trainFeats(i, 2) = areaFn(signal);
    trainLabels(i, 1) = str2double(train_cell_array{1, i}.description);
end

figure;
gscatter(trainFeats(:, 1), trainFeats(:, 2), trainLabels, 'rb');
xlabel('Line Length (AU)', 'FontSize', 15);
ylabel('Area (AU)', 'FontSize', 15);
legend('Artifact', 'HFO', 'Location', 'SouthEast');
title('Feature Space', 'FontSize', 15);

testFeats = zeros(length(test_cell_array), 2);
testLabels = zeros(length(test_cell_array), 1);

for j = 1:length(test_cell_array)
    signal = test_cell_array{1, j}.signal;
    testFeats(j, 1) = LLFn(signal);
    testFeats(j, 2) = areaFn(signal);
    testLabels(j, 1) = str2double(test_cell_array{1, j}.description);
end
```



2. Feature normalization is often important. One simple normalization method is to subtract each feature by its mean and then divide by its standard deviation (creating features with zero mean and unit variance). Using the means and standard deviations calculated in your *training* set features, normalize both the training and testing sets. You should use these normalized features for the remainder of the assignment.

```
trainFeatsNorm = (trainFeats - mean(trainFeats))./std(trainFeats);
testFeatsNorm = (testFeats - mean(trainFeats))./std(trainFeats);
```

(a) What is the statistical term for the normalized value, which you have just computed? (1 pt)
The normalized value is termed the z-score.

(b) Explain why such a feature normalization might be critical to the performance of a k-NN classifier. (2 pts)
k-NN uses a distance metric (often, the Euclidean distance) to iteratively cluster data points into groups. Large feature values can have a large contribution to the distance between points in feature space. By normalizing, every feature is assigned equal importance in determining distances.

(c) Explain why (philosophically) you use the training feature means and standard deviations to normalize the testing set. (2 pts)
The test data is unseen during training time. As a result, any operations performed on this data should not be used when constructing the algorithm that will be tested on it. We also typically assume that descriptive statistics such as the mean and standard deviation hold for data points across the training and testing data sets, since all data comes from an underlying unseen distribution whose parameters we want to infer.

3 Comparing Classifiers (20 pts)

In this section, you will explore how well a few standard classifiers perform on this dataset. Note, the logistic regression, k-NN, and SVM classifiers are functions built into some of Matlabs statistics packages. If you don't have these (i.e., Matlab doesn't recognize the functions), and you are experiencing any difficulty downloading the necessary packages, please let us know.

1. Using Matlab's logistic regression classifier function, (`mnrfit`), and its default parameters, train a model on the training set. Using Matlab's `mnrval` function, calculate the training error (as a percentage) on the data. For extracting labels from the matrix of class probabilities, you may find the command `[~,Ypred] = max(X,[],2)` useful[1], which gets the column-index of the maximum value in each row (i.e., the class with the highest predicted probability). (3 pts)

```
Mdl_logistic = mnrfit(trainFeatsNorm, trainLabels);
[~, trainLabelsPred] = max(mnrval(Mdl_logistic, trainFeatsNorm), [], 2);

train_error_logistic = sum(trainLabels ~= trainLabelsPred)/length(trainLabels)
```

```
train_error_logistic =

    0.4150
```

2. Using the model trained on the training data, predict the labels of the test samples and calculate the testing error. Is the testing error larger or smaller than the training error? Give one sentence explaining why this might be so. (2 pts)

```
[~, testLabelsPred] = max(mnrval(Mdl_logistic, testFeatsNorm), [], 2);

test_error_logistic = sum(testLabels ~= testLabelsPred)/length(testLabels)
```

[1]Note: some earlier versions of Matlab don't like the ~, which discards an argument, so just use something like `[trash,Ypred] = max(X,[],2)` instead.

```
test_error_logistic =

    0.3714
```

The testing error is smaller than the training error, which at first glance seems odd. However, it is important to note that the training error gives information only about how well the model trains and performs with seen data, and not about how well it would generalize to unseen data. Therefore, training and testing error values cannot be directly compared to each other.

3. (a) Use Matlab's k-nearest neighbors function, `fitcknn`, and its default parameters ($k = 1$, among other things), calculate the training and testing errors. (3 pts)

```
Mdl_kNN = fitcknn(trainFeatsNorm, trainLabels);

trainLabelsPred = predict(Mdl_kNN, trainFeatsNorm);
train_error_knn = sum(trainLabels ~= trainLabelsPred)/length(trainLabels)

testLabelsPred = predict(Mdl_kNN, testFeatsNorm);
test_error_knn = sum(testLabels ~= testLabelsPred)/length(testLabels)
```

```
train_error_knn =

     0


test_error_knn =

    0.4476
```

(b) Why is the training error zero? (2 pts)

Matlab sets the number of neighbors used in the k-NN algorithm to 1 by default. As a result, each training point ends up being closest to itself and is therefore accurately classified resulting in 0 training error.

4. Now, train Matlab's implementation of a support vector machine (SVM), `fitcsvm`. Report the training and testing errors for an SVM model with a radial basis function (RBF) kernel function, while keeping other parameters at their default values. (3 pts)

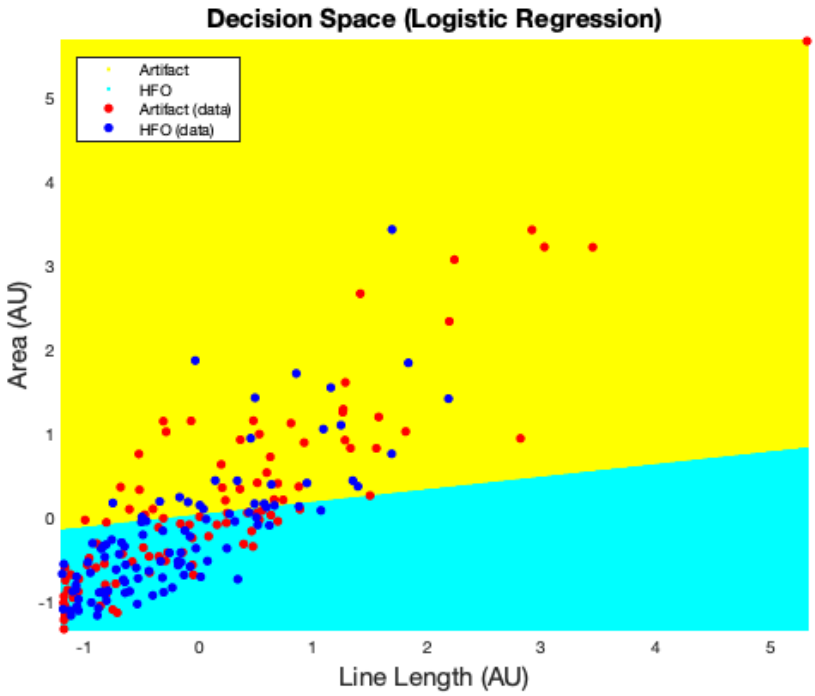
```
Mdl_SVM = fitcsvm(trainFeatsNorm, trainLabels, 'KernelFunction', 'rbf');

trainLabelsPred = predict(Mdl_SVM, trainFeatsNorm);
train_error_svm = sum(trainLabels ~= trainLabelsPred)/length(trainLabels)

testLabelsPred = predict(Mdl_SVM, testFeatsNorm);
test_error_svm = sum(testLabels ~= testLabelsPred)/length(testLabels)
```

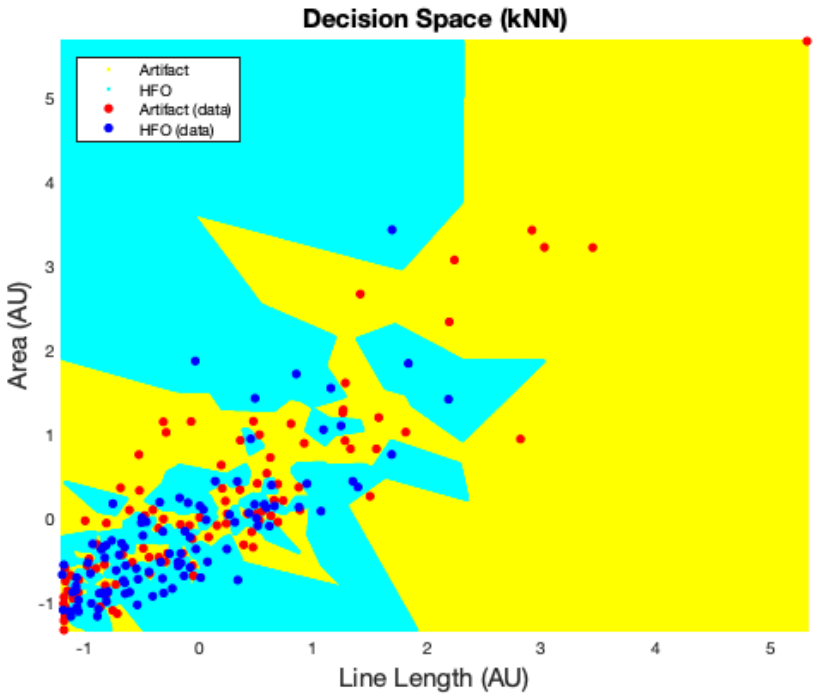
```
train_error_svm =

    0.4050


test_error_svm =
```

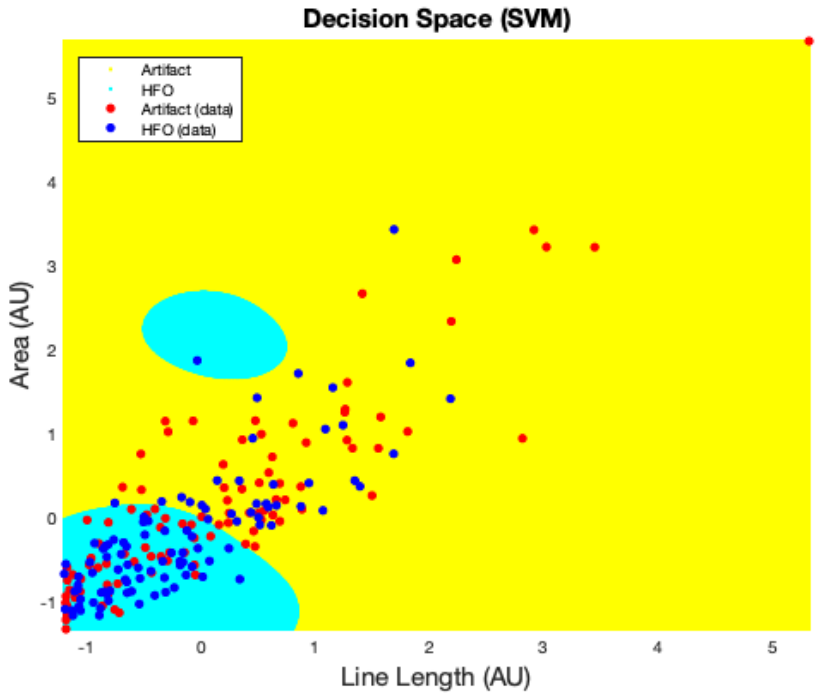
```
    0.6214
```

5. It is sometimes useful to visualize the decision boundary of a classifier. To do this, we'll plot the classifier's prediction value at every point in the "decision" space. Use the `meshgrid` function to generate points in the line-length and area 2D feature space and a scatter plot (with the `'.'` point marker) to visualize the classifier decisions at each point (use yellow and cyan for your colors). In the same plot, show the training samples (plotted with the '*' marker to make them more visible) as well. As before use blue for the valid detections and red for the artifacts. Use ranges of the features that encompass all the training points and a density that yields that is sufficiently high to make the decision boundaries clear. Make such a plot for the logistic regression, k-NN, and SVM classifiers. (4 pts)

```
x = linspace(min(trainFeatsNorm(:, 1)), max(trainFeatsNorm(:, 1)), 1000);
y = linspace(min(trainFeatsNorm(:, 2)), max(trainFeatsNorm(:, 2)), 1000);
[X, Y] = meshgrid(x, y);
feat_space = [X(:), Y(:)];

[~, labels_logistic] = max(mnrval(Mdl_logistic, feat_space), [], 2);
labels_kNN = predict(Mdl_kNN, feat_space);
labels_SVM = predict(Mdl_SVM, feat_space);

figure;
hold on
gscatter(X(:), Y(:), labels_logistic, 'yc');
gscatter(trainFeatsNorm(:, 1), trainFeatsNorm(:, 2), trainLabels, 'rb');
hold off
xlim([min(X(:)), max(X(:))])
ylim([min(Y(:)), max(Y(:))])
xlabel('Line Length (AU)', 'FontSize', 15);
ylabel('Area (AU)', 'FontSize', 15);
legend('Artifact', 'HFO', 'Artifact (data)', 'HFO (data)', 'Location', 'NW');
title('Decision Space (Logistic Regression)', 'FontSize', 15);

figure;
hold on
gscatter(X(:), Y(:), labels_kNN, 'yc');
gscatter(trainFeatsNorm(:, 1), trainFeatsNorm(:, 2), trainLabels, 'rb');
hold off
xlim([min(X(:)), max(X(:))])
ylim([min(Y(:)), max(Y(:))])
xlabel('Line Length (AU)', 'FontSize', 15);
ylabel('Area (AU)', 'FontSize', 15);
legend('Artifact', 'HFO', 'Artifact (data)', 'HFO (data)', 'Location', 'NW');
title('Decision Space (kNN)', 'FontSize', 15);

figure;
hold on
gscatter(X(:), Y(:), labels_SVM, 'yc');
gscatter(trainFeatsNorm(:, 1), trainFeatsNorm(:, 2), trainLabels, 'rb');
hold off
xlim([min(X(:)), max(X(:))])
ylim([min(Y(:)), max(Y(:))])
xlabel('Line Length (AU)', 'FontSize', 15);
ylabel('Area (AU)', 'FontSize', 15);
legend('Artifact', 'HFO', 'Artifact (data)', 'HFO (data)', 'Location', 'NW');
title('Decision Space (SVM)', 'FontSize', 15);
```


Decision Space (Logistic Regression)
Artifact
HFO
Artifact (data)
HFO (data)
Area (AU)
Line Length (AU)
5
4
3
2
1
0
-1
-1
0
1
2
3
4
5


Decision Space (kNN)
Artifact
HFO
Artifact (data)
HFO (data)
Area (AU)
Line Length (AU)
5
4
3
2
1
0
-1
-1
0
1
2
3
4
5



6. In a few sentences, report some observations about the three plots, especially similarities and differences between them. Which of these has overfit the data the most? Which has underfit the data the most? (3 pts)

   Low area and low line length data points are generally classified as HFOs across the three models. Logistic regression has a linear boundary that divides the decision space into exactly two regions corresponding to each class. kNN and the SVM, on the other hand, have nonlinear pockets of space that separate HFOs and artifacts. kNN overfits the most since by default it assigns the nearest data point's label to a point in the decision space. Logistic regression underfits the most.

4 Cross-Validation (17 pts)

In this section, you will investigate the importance of cross-validation, which is essential for choosing the tunable parameters of a model (as opposed to the internal parameters the the classifier "learns" by itself on the training data).

1. Since you cannot do any validation on the testing set, you'll have to split up the training set. One way of doing this is to randomly split it into k unique "folds," with roughly the same number of samples (n/k for n total training samples) in each fold, training on $k-1$ of the folds and doing predictions on the remaining one. In this section, you will do 10-fold cross-validation, so create a cell array[2] `folds` that contains 10 elements, each of which is itself an array of the indices of training samples in that fold. You may find the `randperm` function useful for this. Using the command `length(unique([folds{:}]))`, show that you have 200 unique sample indices (i.e. there are no repeats between folds). (2 pts)

```
num_folds = 10; % number of CV folds
```

[2] A cell array is slightly different from a normal Matlab numeric array in that it can hold elements of variable size (and type), for example `folds{1} = [1 3 6]; folds{2} = [2 5]; folds{3} = [4 7];`.

```
rand_idx = randperm(length(trainFeatsNorm));
folds = mat2cell(reshape(rand_idx, num_folds, []), ones(1, 10));

length(unique([folds{:}]))
```

```
ans =

   200
```

2. Train a new k-NN model (still using the default parameters) on the folds you just created. Predict the labels for each fold using a classifier trained on all the other folds. After running through all the folds, you will have label predictions for each training sample.

   (a) Compute the error (called the validation error) of these predictions. (3 pts)

```
train_errors_CV = zeros(1, num_folds);
val_errors_CV = zeros(1, num_folds);

for i = 1:num_folds
    fold_train_data = trainFeatsNorm;
    fold_train_labels = trainLabels;
    fold_train_data(folds{i, 1}, :) = []; % remove one fold for validation
    fold_train_labels(folds{i, 1}, :) = []; % remove one fold for validation

    fold_val_data = trainFeatsNorm(folds{i, 1}, :);
    fold_val_labels = trainLabels(folds{i, 1}, :);

    Mdl_kNN = fitcknn(fold_train_data, fold_train_labels);

    fold_train_labels_pred = predict(Mdl_kNN, fold_train_data);
    train_errors_CV(i) = sum(fold_train_labels ~= fold_train_labels_pred)/length(fold_train_labels);

    fold_val_labels_pred = predict(Mdl_kNN, fold_val_data);
    val_errors_CV(i) = sum(fold_val_labels ~= fold_val_labels_pred)/length(fold_val_labels);
end

val_error = mean(val_errors_CV)
```

```
val_error =

    0.5850
```

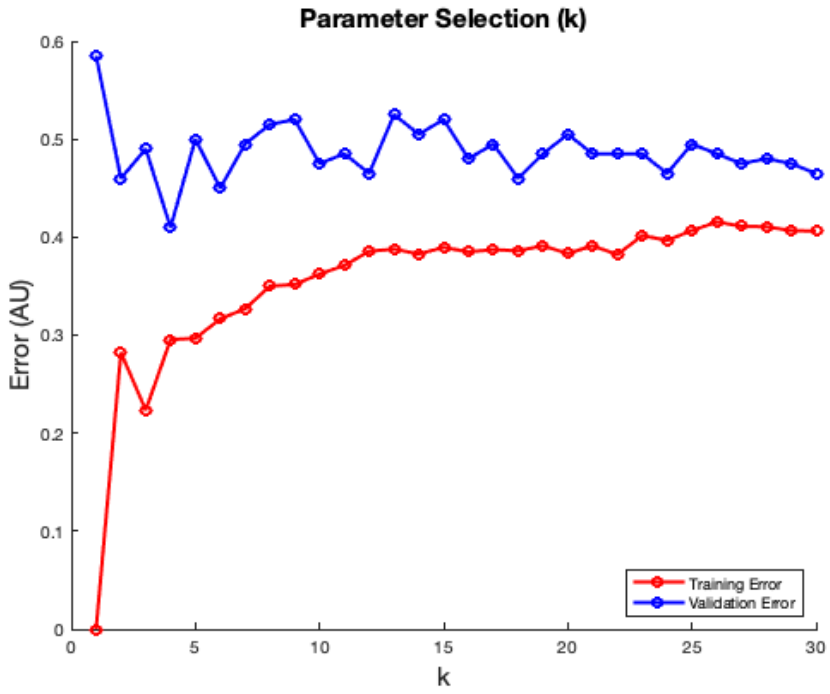
   (b) How does this error compare (lower, higher, the same?) to the error you found in question 3.3? Does it make sense? (2 pts)

   The test error in 3.3 is 0.44, which is lower than the cross-validation error of 0.59. This makes sense since the 3.3 model is trained on the full training data, and as such has seen more examples that might help it generalize better. By contrast, each of the CV models trains only on 90% of the training data.

3. Create a parameter space for your k-NN model by setting a vector of possible k values from 1 to 30. For each values of k, calculate the validation error and average training error over the 10 folds.

   (a) Plot the training and validation error values over the values of k, using the formatting string `'b-o'` for the validation error and `'r-o'` for the training error. (4 pts)

```
ks = 1:30;
train_errors_ks = zeros(1, length(ks));
val_errors_ks = zeros(1, length(ks));

for i = 1:length(ks)
    k = ks(i);
    train_errors_CV = zeros(1, num_folds);
    val_errors_CV = zeros(1, num_folds);
    for j = 1:num_folds
        fold_train_data = trainFeatsNorm;
        fold_train_labels = trainLabels;
        fold_train_data(folds{j, 1}, :) = []; % remove one fold for validation
        fold_train_labels(folds{j, 1}, :) = []; % remove one fold for validation

        fold_val_data = trainFeatsNorm(folds{j, 1}, :);
        fold_val_labels = trainLabels(folds{j, 1}, :);

        Mdl_kNN = fitcknn(fold_train_data, fold_train_labels, ...
            'NumNeighbors', k);

        fold_train_labels_pred = predict(Mdl_kNN, fold_train_data);
        train_errors_CV(j) = sum(fold_train_labels ~= ...
            fold_train_labels_pred)/length(fold_train_labels);

        fold_val_labels_pred = predict(Mdl_kNN, fold_val_data);
        val_errors_CV(j) = sum(fold_val_labels ~= ...
            fold_val_labels_pred)/length(fold_val_labels);
    end
    train_errors_ks(i) = mean(train_errors_CV);
    val_errors_ks(i) = mean(val_errors_CV);
end

figure;
hold on
plot(ks, train_errors_ks, 'r-o', 'LineWidth', 2);
plot(ks, val_errors_ks, 'b-o', 'LineWidth', 2);
legend('Training Error', 'Validation Error', 'Location', 'SouthEast');
hold off
xlabel('k', 'FontSize', 15);
ylabel('Error (AU)', 'FontSize', 15);
title('Parameter Selection (k)', 'FontSize', 15);
```



(b) What is the optimal k value and its training and testing error? (1 pts)

```
[~, k_opt] = min(val_errors_ks)
k_opt_train_error = train_errors_ks(k_opt)
k_opt_val_error = val_errors_ks(k_opt)
```

```
k_opt = 4

    0.2956

k_opt_train_error =

    0.2956


k_opt_val_error =

    0.4100
```

(c) Explain why k-NN generally overfits less with higher values of k. (2 pts)
When $k = 1$, every training point is assigned the label of its nearest neighbor, which is itself. This leads to a training error of 0. Higher values of k have the effect of distributing the responsibility for a point's label amongst multiple training points. This leads to less over-fitting.

4. (a) Using your optimal value for k from CV, calculate the k-NN model's *testing* error. (1 pts)

```
Mdl_kNN_opt = fitcknn(trainFeatsNorm, trainLabels, 'NumNeighbors', k_opt);

trainLabelsPred = predict(Mdl_kNN_opt, trainFeatsNorm);
```

```
testLabelsPred = predict(Mdl_kNN_opt, testFeatsNorm);
test_error_knn_k_opt = sum(testLabels ~= testLabelsPred)/length(testLabels)
```

```
test_error_knn_k_opt =

    0.5476
```

(b) How does this model's testing error compare to the k-NN model you trained in question 3.3? Is it the best of the three models you trained in Section 3? (2 pts)

The test error for the model with k_{opt} is 0.55 which is greater than the error for the logistic regression (0.37) and kNN (0.44) models from 3, and lower than the error for the SVM (0.62). In principle, cross-validation should lead to hyperparameters that help models generalize better. However, it is possible that in this particular instance, there is not enough data to train or cross-validate any of the models being considered. It is also possible that 10 folds are not sufficient to average over when considering such a small data set. By increasing the number of folds, chances are that the plot in 4.3b becomes smoother and yields a different hyperparameter value for k.