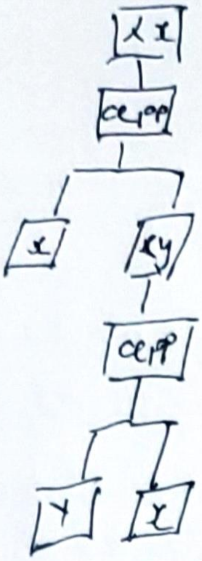
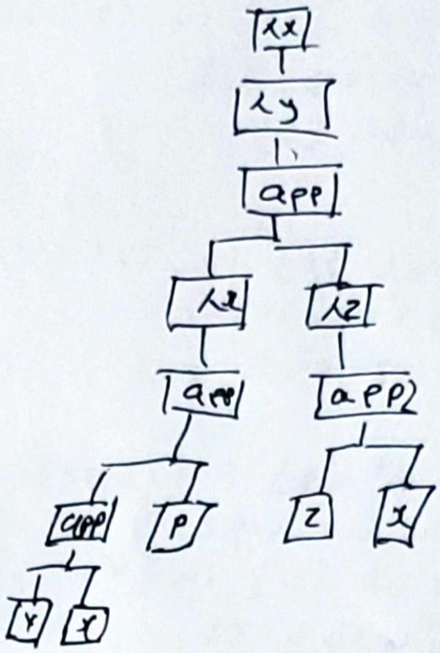
1. a) $(\lambda x.(x(\lambda y.(yx))))$

Tree



b) $(\lambda x.(\lambda y.((\lambda x.((xx)p))(\lambda z.(zx)))))$



2. a) $((\lambda p.(pz))(\lambda q.(w(\lambda w.((((wq)z)p))))))$

b) $(\lambda p.((pq)(\lambda p.(qp))))$

3. a) λ is bounded to variable s q q, Hence closed variables are s and q. Free variable is z

So,
s bound at {s} in λs.sz λq.sq
s free at {s} in λs.sz λq.sq
in λq.{s}q
in {s}
z free at {z} in λs.s{z}λq.sq
in λs.s{z}
in {z}
q bound at {q} in λs.sz λq.sq
in λq.sq

∴ λs.sz λq.sq → (λs.((sz)(λq.(sq))))

b b free b b free

3. b) s bound at $\{s\}$ in $(\lambda s.sz)\lambda q.w\lambda w.wqzs$

s free at $\{s\}$ in $(\lambda s.sz)\lambda q.w\lambda w.wqzs$

in $\lambda q.w\lambda w.wqz\{s\}$

in $\lambda w.wqz\{s\}$

in $\{s\}$

z free at $\{z\}$ in $(\lambda s.sz)\lambda q.w\lambda w.wqzs$

in $(\lambda s\, s\{z\}\}$ in $\{z\}$

in $\lambda w.wq\{z\}s$

in $\{z\}$

q bound at $\{q\}$ in $\{\lambda s.sz)\lambda q.w\lambda w.wqzs$

in $\lambda q.w\lambda w.wqzs$

q free at $\{q\}$ in $\{\lambda s.sz\}\lambda q.w\lambda w.w\{q\}zs$

in $\lambda w.w\{q\}zs$

in $\{q\}$

w bound at $\{w\}$ in $(\lambda s.sz)\lambda q.w\lambda w.wqzs$

in $\lambda w.wqzs$

w free at $\{w\}$ in $(\lambda s.sz)\lambda q.\{w\}\lambda w.wqzs$

in $\lambda q.\{w\}\lambda w.wqzs$

in $\lambda q.\{w\}$

in $\{w\}$

free, free, b

$\therefore (\lambda s.sz)\lambda q.w\lambda w.wqzs \rightarrow ((\lambda s.(sz))(\lambda q.(\underline{w}(\lambda w.(((wq)\underline{z})\underline{s})))))$

b, b

free free

4. a) $(\lambda z.z)(\lambda z.zz)(\lambda z.zq)$

B reduction: $(\lambda z.zz)(\lambda z.zq)$

$(\lambda z.zq)(\lambda z.zq)$

$(\lambda z.zq)\, q$

$q\, q$

4. b) $(\lambda s.\lambda q.sqq)(\lambda q.q)q$

$(\lambda q.(\lambda q.q)qq)q$

$(\lambda q.(\lambda a.a)qq)q$

$(\lambda a.a)qq$

qq

c) $(\lambda s.ss)(\lambda q.q)(\lambda q.q)$

$(\lambda q.q)(\lambda q.q)(q.q)$

$(\lambda q.q)(\lambda q.q)$

$(\lambda q.q)$

5. a) AND FALSE TRUE

= $(\lambda b1.\lambda b2.b1\ b2$ False) FALSE TRUE

= FALSE TRUE FALSE

= FALSE

$(\lambda b2(\lambda x.\lambda y.y)b2$ False$)(\lambda x.\lambda y.x)$

$(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)$ False

$(\lambda y.y)$ False

False

b) AND TRUE TRUE

= $(\lambda b1.\lambda b2.b1 b2$ FALSE) TRUE TRUE

= TRUE TRUE FALSE

= TRUE

$(\lambda b2.(\lambda x.\lambda y.x)b2.(\lambda x.\lambda y.x))$

$(\lambda x.\lambda y.x)$

$(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)$

$(\lambda x.\lambda y.x)$

$(\lambda x.\lambda y.x)$

True