Siddhanth Patel

CS-559 HW 1

Problem 1:

Initial: \$B

After win: \$W

After loss: \$L

probability: p_w

Expected Value $= p_w \cdot (W-B) - (1-p_w)\cdot(B-L)$

We want this to be greater than or equal 0 to accept the bet, so:

$$p_w(W-B) - (1-p_w)\cdot(B-L) \geq 0$$

$$p_w W - \cancel{p_w B} - B + L + \cancel{p_w B} - p_w L \geq 0$$

$$p_w W - B + L - p_w L \geq 0$$

$$p_w(W-L) - B + L \geq 0$$

$$p_w(W-L) \geq B-L$$

$$\boxed{p_w \geq \frac{B-L}{W-L}}$$

Problem 2:

G = Wallet is green

B = Wallet is black

Green Wallet: 6 pennies, 4 dimes

Black Wallet: 8 pennies, 2 dimes

$P(W=\text{green}) = 4/5 = .8$

$P(W=\text{black}) = 1/5 = .2$

S = 1 dime followed by 2 pennies

$P(G) = .8$

$P(S|G) = \frac{4}{10}\cdot\frac{6}{9}\cdot\frac{5}{8} = \frac{1}{6}$

$P(S|B) = \frac{2}{10}\cdot\frac{8}{9}\cdot\frac{7}{8} = \frac{7}{45}$

$$P(G|S) = \frac{P(S|G)\cdot P(G)}{P(S|G)\cdot P(G) + P(S|B)\cdot P(B)}$$

$$= \frac{\frac{1}{6}\cdot\frac{4}{5}}{\frac{1}{6}\cdot\frac{4}{5} + \frac{7}{45}\cdot\frac{1}{5}}$$

$$= \frac{30}{37} = \boxed{0.811}$$

So green wallet is more likely

$P(B|S) = 1 - P(G|S) = 1 - \frac{30}{37} = \frac{7}{37} = 0.189 \rightarrow$ There is an 18.9% chance that this answer is wrong.

Problem 3:

Part 1: See attached. Type: python part3.py to run.

Enter mean, variance, n.

(Requires numpy)

Part 2:

Theoretically: $\text{Mean}_{\text{combined}} = \sum_{i=1}^{n} \frac{N_i}{N_{3000}} \text{mean}_i = \frac{2000}{3000} \cdot 1 + \frac{1000}{3000} \cdot 4 = \boxed{2}$

$$\text{Variance}_{\text{combined}} = \frac{\sum_{i=1}^{n} N_i \left[\text{Var}_i + (\text{mean}_i - \text{mean}_{\text{combined}})^2 \right]}{\sum_{i=1}^{n} N_i}$$

$$= \frac{\left(4 + (1-2)^2\right) \cdot 2000 + \left(9 + (4-2)^2\right) \cdot 1000}{1000 + 2000}$$

Experimentally:

Theoretical estimates are approximately correct ✓

$$= \frac{10{,}000 + 13000}{3000} = \frac{23000}{3000} = \boxed{7.\overline{6}}$$