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CS 558 - Computer Vision
Professor Xinchao Wang
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Homework Set 3

Problem 1:

Covariance Matrix: [[1.00130548 0.12901392 0.14138122 -0.08260275 -0.07309413 0.01754089
-0.03397176 0.54472834]
[0.12901392 1.00130548 0.15269695 0.05645473 0.33281735 0.22124355
0.13708168 0.26275075]
[0.14138122 0.15269695 1.00130548 0.20757846 0.08921407 0.2821451
0.0412338 0.23988377]
[-0.08260275 0.05645473 0.20757846 1.00130548 0.43854604 0.3930657
0.18373769 -0.11602413]
[-0.07309413 0.33281735 0.08921407 0.43854604 1.00130548 0.19836937
0.18582106 -0.0409953]
[0.01754089 0.22124355 0.2821451 0.3930657 0.19836937 1.00130548
0.14072974 0.03595806]
[-0.03397176 0.13708168 0.0412338 0.18373769 0.18582106 0.14072974
1.00130548 0.03278114]
[0.54472834 0.26275075 0.23988377 -0.11602413 -0.0409953 0.03595806
0.03278114 1.00130548]]

Eigenvectors
[[-0.12631788 -0.59404237 -0.59537627 0.07536675 -0.19542947 0.47535915
-0.08158945 0.01373196]
[-0.39243328 -0.17528237 -0.09339313 0.4446406 -0.09300894 -0.46613059
0.40414775 -0.46859457]
[-0.35947089 -0.18550652 -0.19162077 -0.02616264 0.63396883 -0.32772515
-0.05653763 0.53520861]
[-0.44054292 0.33140493 0.23945437 0.58572021 -0.0089089 0.48726778
-0.03677036 0.23837958]
[-0.43683384 0.24788907 -0.09150076 -0.55762163 0.2695852 0.34781321
0.34804809 -0.33708345]
[-0.45215652 0.09918102 -0.00863174 -0.34219401 -0.68569968 -0.2542284
-0.05305254 0.36182269]

[-0.27060956 0.12154879 -0.08449476 -0.01341205 0.08464217 -0.11966157
 -0.83449461 -0.43221831]
[-0.1955599 -0.6213597   0.72605898 -0.15698229 0.03370896 0.10953212
 -0.07159535 -0.0748592]]

Eigenvalues
[2.0969295 1.73390801 0.42017396 0.40400274 0.68343308 0.763857
 0.87674237 1.03139721]

Since the first, second, and last eigenvalues are the greatest we will pick them for 3 dimensions.
Selected eigenvalues: 2.0969295026211467 1.7339080083010403 1.0313972060910501

List: [0.7057291666666666, 0.7161458333333334, 0.7135416666666666, 0.75, 0.703125, 0.75, 0.71875, 0.703125, 0.7265625, 0.734375]

Mean Accuracy= 0.7221354166666667 , Standard Deviation: 0.017749560532944315

Problem 2:
CLASS 1: [[-2 1]
 [-5 -4]
 [-3 1]
 [0 -3]
 [-8 -1]]
CLASS 2: [[2 5]
 [1 0]
 [5 -1]
 [-1 -3]
 [6 1]]

MEAN 1: [[-3.6]
 [-1.2]]
MEAN 2: [[2.6]
 [0.4]]

S_1: [[37.2 1.4]

[1.4 20.8]]
S_2: [[33.2 8.8]
 [8.8 35.2]]

Within class scatter: [[70.4 10.2]
 [10.2 56.]]

Inverse within class scatter: [[0.01458956 -0.00265738]
 [-0.00265738 0.01834117]]

Optimal Line Direction: [[-0.08620348]
 [-0.01287008]]

Class 1 Projection: [[0.15953689 0.48249773 0.24574037 0.03861024 0.70249794]]
Class 2 Projection: [[-0.23675737 -0.08620348 -0.41814733 0.12481372 -0.53009098]]

Problem 3:

CLASS 1: [[1.00e+00 8.90e+01 6.60e+01 ... 2.81e+01 1.67e-01 2.10e+01]
 [5.00e+00 1.16e+02 7.40e+01 ... 2.56e+01 2.01e-01 3.00e+01]
 [1.00e+01 1.15e+02 0.00e+00 ... 3.53e+01 1.34e-01 2.90e+01]
 ...
 [5.00e+00 1.21e+02 7.20e+01 ... 2.62e+01 2.45e-01 3.00e+01]
 [1.00e+00 9.30e+01 7.00e+01 ... 3.04e+01 3.15e-01 2.30e+01]
 [6.00e+00 1.48e+02 7.20e+01 ... 3.36e+01 6.27e-01 5.00e+01]]
CLASS 2: [[0.000e+00 1.370e+02 4.000e+01 ... 4.310e+01 2.288e+00 3.300e+01]
 [3.000e+00 7.800e+01 5.000e+01 ... 3.100e+01 2.480e-01 2.600e+01]
 [2.000e+00 1.970e+02 7.000e+01 ... 3.050e+01 1.580e-01 5.300e+01]
 ...
 [6.000e+00 1.900e+02 9.200e+01 ... 3.550e+01 2.780e-01 6.600e+01]
 [9.000e+00 1.700e+02 7.400e+01 ... 4.400e+01 4.030e-01 4.300e+01]
 [1.000e+00 1.260e+02 6.000e+01 ... 3.010e+01 3.490e-01 4.700e+01]]

MEAN 1: [[3.308   ]
 [110.106   ]

[68.196]
[19.676]
[68.792]
[30.3182]
[0.430286]
[31.228]]
MEAN 2: [[4.84962406]
[141.07518797]
[70.84586466]
[22.19924812]
[101.09022556]
[35.19285714]
[0.54975564]
[37.03759398]]

List: [0.7890625, 0.7630208333333334, 0.7864583333333334, 0.796875, 0.7838541666666666, 0.7734375, 0.7786458333333334, 0.7890625, 0.7473958333333334, 0.7864583333333334]

Mean Accuracy= 0.7794270833333333 , Standard Deviation: 0.01468271773339595

Problem 4:
Augment samples by adding an extra feature equal to 1. Replace all items from w2 to negative values.

ω2 -1 -1 -1 1 0 -2
ω1 1 0 0 1 2 0
ω2 -1 1 1 -1 -1 0
ω1 1 4 0 1 2 1
ω1 1 -1 1 1 1 0
ω1 1 -1 -1 -1 1 0
ω2 -1 1 -1 -1 -2 -1

Initial vector weight = [3 1 1 − 1 2 − 7]

Start checking each row:

1: [3 1 1 − 1 2 − 7] ^t * [-1 -1 -1 1 0 -2] = -3 -1 -1 -1 + 0 + 14 = 8 > 0 so not misclassified. Keep the weight the same.
2: [3 1 1 − 1 2 − 7] ^t * [1 0 0 1 2 0] = 3 + 0 + 0 -1 + 4 + 0 = 6 > 0 so not misclassified. Keep the weight the same.

3: [3 1 1 − 1 2 − 7] ^t * [-1 1 1 -1 -1 0] = -3 + 1 +1 +1 -2 + 0 = -2 < 0 so misclassified. Need to modify the weight. New weight W = [3 1 1 − 1 2 − 7] + [-1 1 1 -1 -1 0] = [2 2 2 -2 1 -7]
4: [2 2 2 -2 1 -7]^t * [1 4 0 1 2 1] = 2 + 8 + 0 -2 + 2 -7 = 3 > 0 so not misclassified.
5: [2 2 2 -2 1 -7]^t * [1 -1 1 1 1 0] = 2 + -2 +2 -2 +1 = 1 > 0 so not misclassified.
6: [2 2 2 -2 1 -7]^t * [1 -1 -1 -1 1 0] = 2 + -2 +2 + 2 +1 = 5 > 0 so not misclassified.
7: [2 2 2 -2 1 -7]^t * [-1 1 -1 -1 -2 -1] = -2 + 2 -2 +2 -2 +7 = 5 > 0 so not misclassified.

[2 2 2 -2 1 -7]
g(y) = 2y0 + 2y1 + 2y2 - 2y3 + 1y4 - 7y5

Solution vector:
g(x) = 2x(1) + 2x(2) - 2x(3) + 1x(4) - 7x(5) > -2 ---> w1
g(x) = 2x(1) + 2x(2) - 2x(3) + 1x(4) - 7x(5) < -2 ---> w2