Universität Rostock

Traditio et Innovatio

On the Connections of High-Frequency Approximated Ambisonics and Wave Field Synthesis

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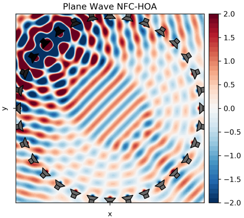
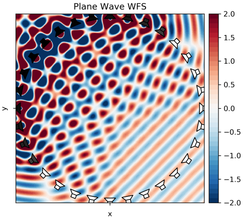
45th DAGA, Rostock, Virtual Acoustics

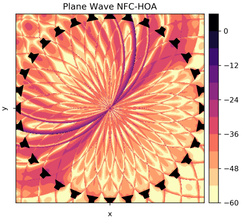
Introduction



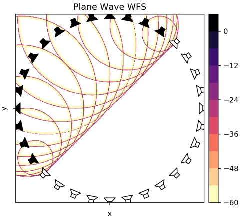
(a) 2.5D NFC-HOA



(b) 2.5D WFS monochromatic



(c) 2.5D NFC-HOA



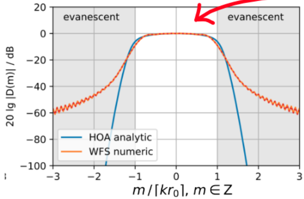
(d) 2.5D WFS broadband

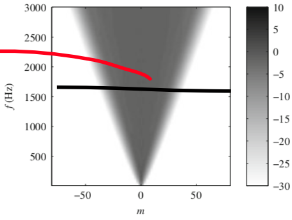
Near Field Compensated Higher Order Ambisonics (NFC-HOA), Wave Field Synthesis (WFS)

Motivation

Connections on Near Field Compensated **Infinite** Order Ambisonics (NFC-IOA) and WFS?

Fig. 4.15 $20 \log_{10} |D_m(\omega)|$ of the WFS driving function for a virtual plane wave





Obviously, WFS constitutes spatially fullband synthesis so that all previous discussions on fullband synthesis apply also here though keeping in mind that WFS is a high-frequency approximation. Similar results can be obtained for spherical secondary source distributions and other non-planar and non-linear geometries.

This important result may be summarized as:

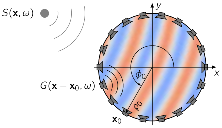
> Wave Field Synthesis constitutes a high-frequency approximation of Near-field Compensated *Infinite* Order Ambisonics.

Ahrens J (2012): *Analytic Methods of Sound Field Synthesis.* Springer

Motivation

$$P(\mathbf{x},\omega) = \int_C D(\mathbf{x}_0,\omega) G(\mathbf{x}-\mathbf{x}_0,\omega) \mathrm{d}C$$

$$P(\mathbf{x},\omega) = \sum_N D(\mathbf{x}_{0,n},\omega) G(\mathbf{x}-\mathbf{x}_{0,n},\omega) \Delta_n$$

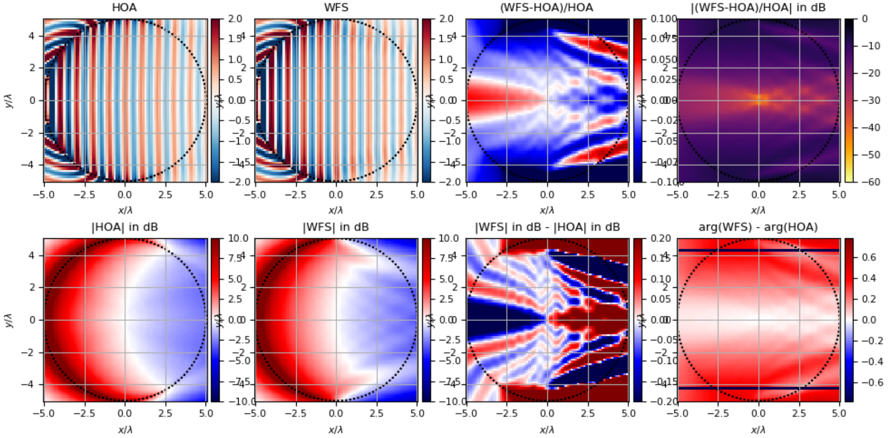


On the connections of different sound field production approaches:

- Fazi F (2010): Sound Field Reproduction, dissertation
- Ahrens J (2012): *Analytic Methods of Sound Field Synthesis.* Springer
- Zotter F, Frank M (2012): All-Round Ambisonic Panning and Decoding, JAES 60(10):807
- Franck A, Wang W, Fazi F (2017): Sparse ℓ_1-Optimal Multiloudspeaker Panning and Its Relation to Vector Base Amplitude Panning, IEEE TALSP 25(5):996
- Firtha G et al. (2018): On the General Relation of Wave Field Synthesis and Spectral Division Method for Linear Arrays, IEEE TASLP 26(12):2393
- Winter, Hahn, Spors: Local Wave Field Synthesis

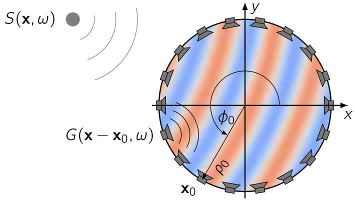
Plane Wave Synthesis WFS vs. NFC-IOA

Referencing scheme for WFS $\mathbf{x}_{\mathrm{Ref}} = \mathbf{0}$ (Firtha 2017)



How to Prove 2.5D NFC-IOA $\equiv$ 2.5D WFS?

Synthesis integral for circular secondary source distribution



$$P(\mathbf{x},\omega) = \int\limits_{\phi_0=0}^{2\pi} D(\mathbf{x}_0,\omega) G(\mathbf{x}-\mathbf{x}_0,\omega)\, r_0 \,\mathrm{d}\phi_0$$

We can search for a closed analytic description

- of the Fourier series of D_{IOA}
- of the Fourier coefficients for D_{WFS}
- of the Fourier series of P_{IOA}
- of the Fourier coefficients for P_{WFS}

in order to analytically prove one of

$$D_{\text{IOA,FS}}(m,\omega) \equiv D_{\text{WFS,FS}}(m,\omega) \quad D_{\text{IOA}}(\mathbf{x}_0,\omega) \equiv D_{\text{WFS}}(\mathbf{x}_0,\omega)$$
$$P_{\text{IOA,FS}}(m,\omega) \equiv P_{\text{WFS,FS}}(m,\omega) \quad P_{\text{IOA}}(\mathbf{x}_0,\omega) \equiv P_{\text{WFS}}(\mathbf{x}_0,\omega)$$

Stationary Phase Approximation of 2.5D SFS Integral

Synthesis integral for circular secondary source distribution

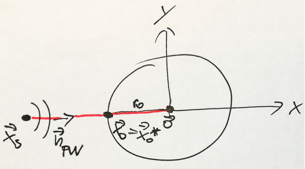
$$P(\mathbf{x},\omega) = \int\limits_{\phi_0=0}^{2\pi} D(\mathbf{x}_0,\omega) G(\mathbf{x}-\mathbf{x}_0,\omega)\, r_0 \,\mathrm{d}\phi_0$$

Example: unit amplitude plane wave driving filter for NFC-HOA

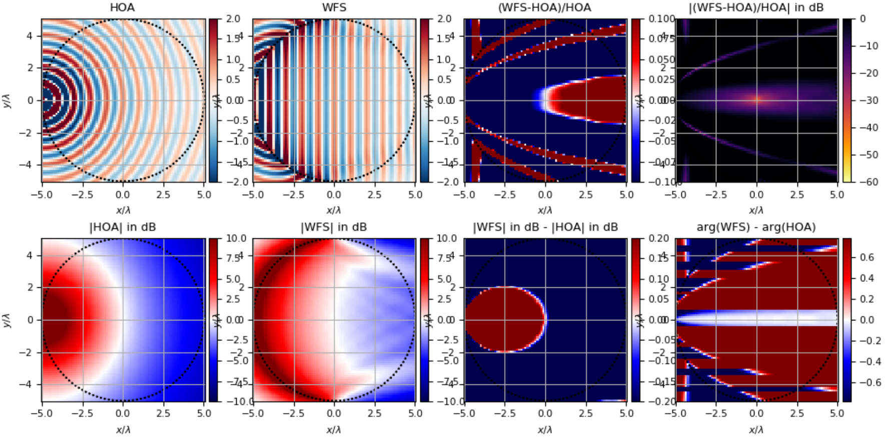
$$D(\mathbf{x}_0,\omega)_{\text{NFC-HOA}} = \sum_{m=-M}^{+M} \frac{2\mathrm{j}}{\frac{\omega}{c} r_0} \frac{(-\mathrm{j})^{|m|}}{h_{|m|}^{(2)}(\frac{\omega}{c} r_0)} \mathrm{e}^{\mathrm{j}m(\phi_0-\phi_{PW})}$$

With usual treatments $\frac{\omega}{c} r_0 \to \infty$, $M \to \infty$ SPA yields

$$P(\mathbf{x},\omega) = \frac{r_0}{\mathrm{e}^{-\mathrm{j}\frac{\omega}{c} r_0}} \frac{\mathrm{e}^{-\mathrm{j}\frac{\omega}{c}|\mathbf{x}-\mathbf{x}_0^*|}}{|\mathbf{x}-\mathbf{x}_0^*|} \overset{\text{for } \mathbf{x}=0}{=} 1$$



SPA of NFC Infinite Order Ambisonics



Fourier Series Coefficients of Driving Filter

$$D_{\text{PW,HOA}}(\mathbf{x}_0, \omega) = \sum_{m=-\infty}^{+\infty} \underbrace{\frac{2\mathrm{j}}{\frac{\omega}{c} r_0} \frac{(-\mathrm{j})^{|m|}}{h_{|m|}^{(2)}(\frac{\omega}{c} r_0)} \mathrm{e}^{-\mathrm{j} m \phi_{PW}}}_{D_{\text{PW,HOA}}(m,\omega)} \mathrm{e}^{\mathrm{j} m \phi_0}$$

$$D_{\text{PW,WFS}}(\mathbf{x}_0, \omega) = \sqrt{\frac{\mathrm{j}\omega}{c}} \sqrt{8\pi} \cdot w(\mathbf{x}_0) \cdot \sqrt{|\mathbf{x}_{\text{Ref}} - \mathbf{x}_0|} \cdot \langle \mathbf{n}_{\text{PW}}, \mathbf{n}(\mathbf{x}_0) \rangle \cdot \mathrm{e}^{-\mathrm{j} \frac{\omega}{c} \langle \mathbf{n}_{\text{PW}}, \mathbf{x}_0 \rangle} \circ\!\!-\!\!\bullet\, D_{\text{PW,FS}}(m, \omega)??$$

multiplication of secondary source selection window $w(\mathbf{x}_0)$ with 2.5D WFS Neumann-Rayleigh $\mathbf{x}_0$-part $\rightarrow$ convolution in Fourier series domain

Fourier Series Coefficients of Driving Filter

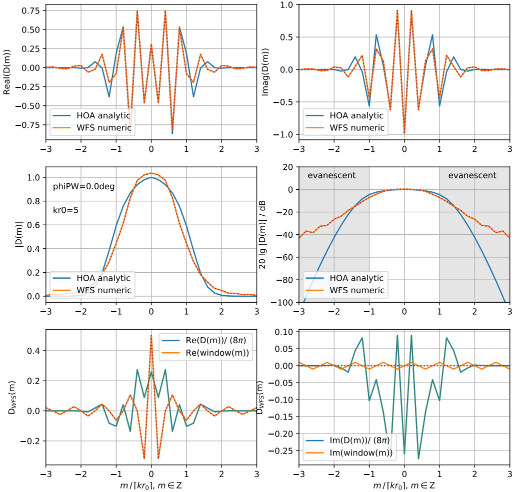
$$D_{\text{PW,HOA}}(m,\omega) = \frac{2\mathrm{j}}{\frac{\omega}{c} r_0} \frac{(-\mathrm{j})^{|m|}}{h^{(2)}_{|m|}(\frac{\omega}{c} r_0)} \mathrm{e}^{-\mathrm{j} m \phi_{PW}}$$

$$\begin{aligned} D_{\text{PW,WFS}}(m,\omega) = &- \sqrt{\frac{\mathrm{j}\omega}{c}} \sqrt{8\pi r_0} \cdot \\ &\left[\frac{1}{2\pi} \frac{-\mathrm{i}}{m} (\mathrm{e}^{-\mathrm{i}m(\phi_{PW}+\pi/2))} - \mathrm{e}^{-\mathrm{i}m(\phi_{PW}+3\pi/2))})\right] *_m \\ &\left[\frac{\mathrm{e}^{-\mathrm{j}m\phi_{PW}}}{2\,\mathrm{j}^{m-1}} (J_{m-1}(\frac{\omega}{c} r_0) - J_{m+1}(\frac{\omega}{c} r_0))\right] \end{aligned}$$

Hahn N, Winter F, Spors S (2016): Local Wave Field Synthesis by Spatial Band-limitation in the Circular/Spherical Harmonics Domain, Proc. 140th AES Conv Paris

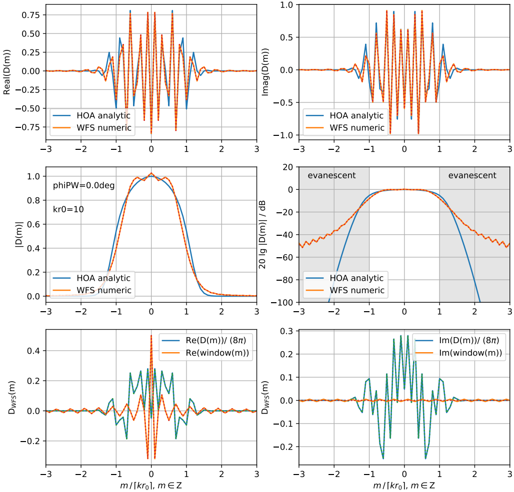
Fourier Series Coefficients of Driving Filter

$k\,r_0 = 5$



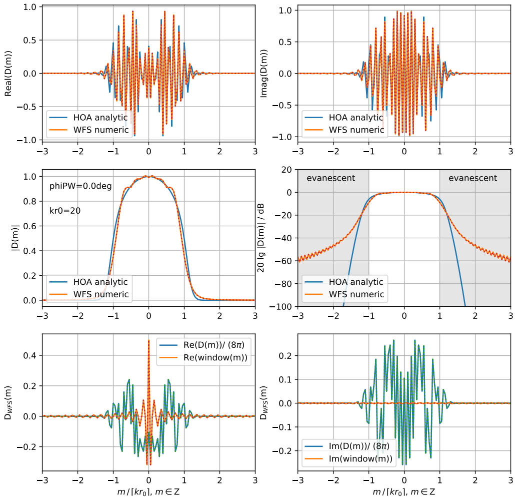
Fourier Series Coefficients of Driving Filter

$k\, r_0 = 10$



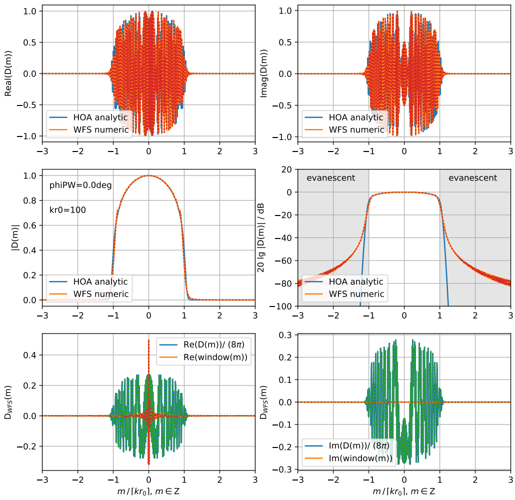
Fourier Series Coefficients of Driving Filter

$k\,r_0 = 20$



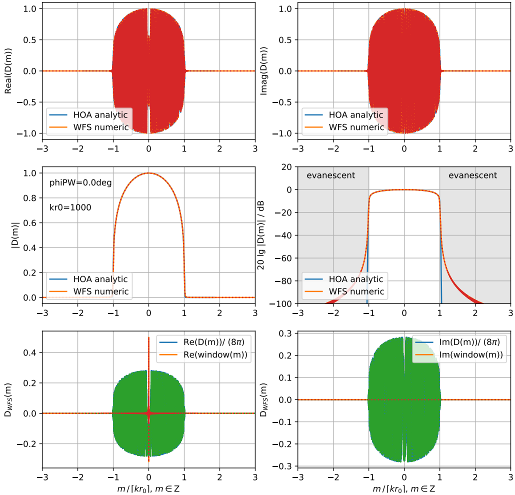
Fourier Series Coefficients of Driving Filter

$k\,r_0 = 100$



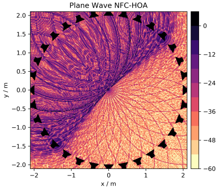
Fourier Series Coefficients of Driving Filter

$k\,r_0 = 1000$



Conclusion

- WFS and NFC-HOA are precisely identical in the single stationary phase point, when number of modes $M \to \infty$ and $k\,r_0 \to \infty$
- NFC-HOA for $M \to \infty$, $k\,r_0 \to \infty$ merges to WFS for sources that fulfill WFS's high frequency / farfield approximation with **referencing to the origin**



Open Source:

- `https://github.com/spatialaudio/sfs-with-local-wavenumber-vector-concept`
- `https://sfs-python.readthedocs.io/en/0.5.0/`