13.09.2019

Digital Image Processing (CSE/ECE 478)

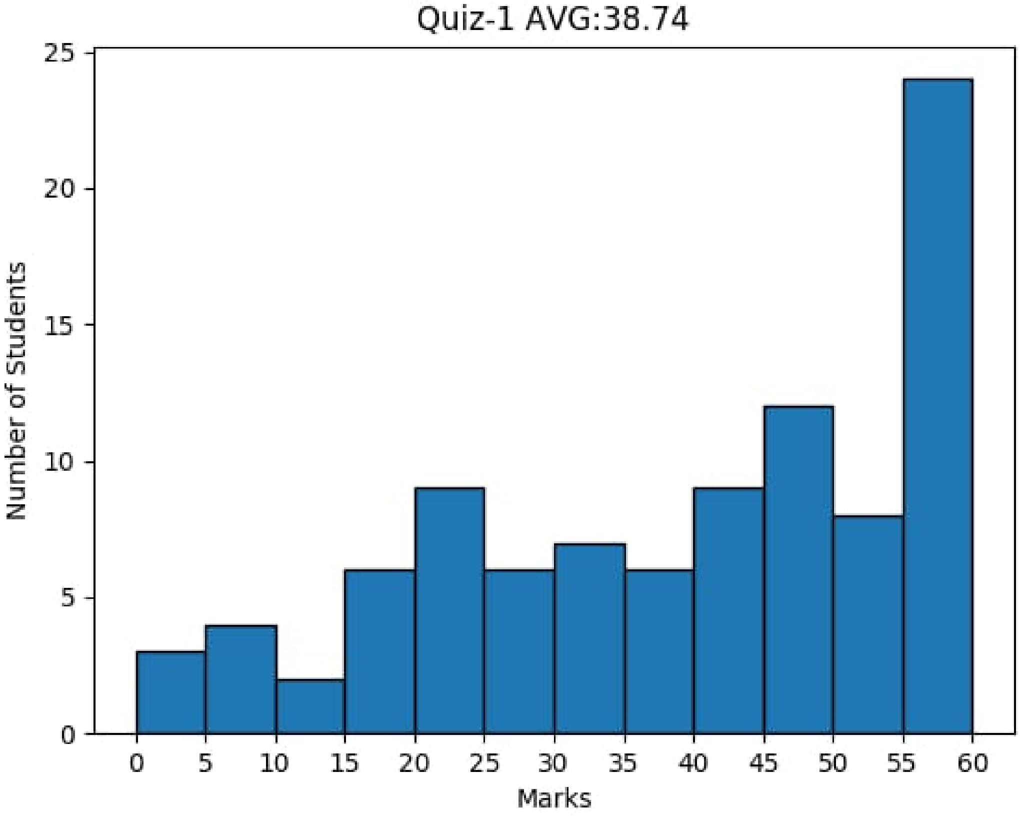
Lecture-12: Morphological Operations, Intro to Geometric Operations

Ravi Kiran

Center for Visual Information Technology (CVIT), IIIT Hyderabad

Slide credits: Volker Krüger, Rune Andersen, Roger S. Gaborski
CCL Image credits: aishack.in

Announcements



Announcements

- Projects
  - Form group(s) of >= 1 and <= 3
  - List will be announced at 9pm on 16th September
  - Read guidelines re: project preferences carefully
- Mid-1
  - Up to the Lecture of September 6
  - MCQ (negative marking), Numerical Questions (carry calculator)
  - No derivations ☺

Announcements

- No class next Tuesday
- Make-up class TBA (tentative: Wednesday afternoon after Mid-1 week)

Binary Images

Object = set of pixels (or coordinates of pixels)

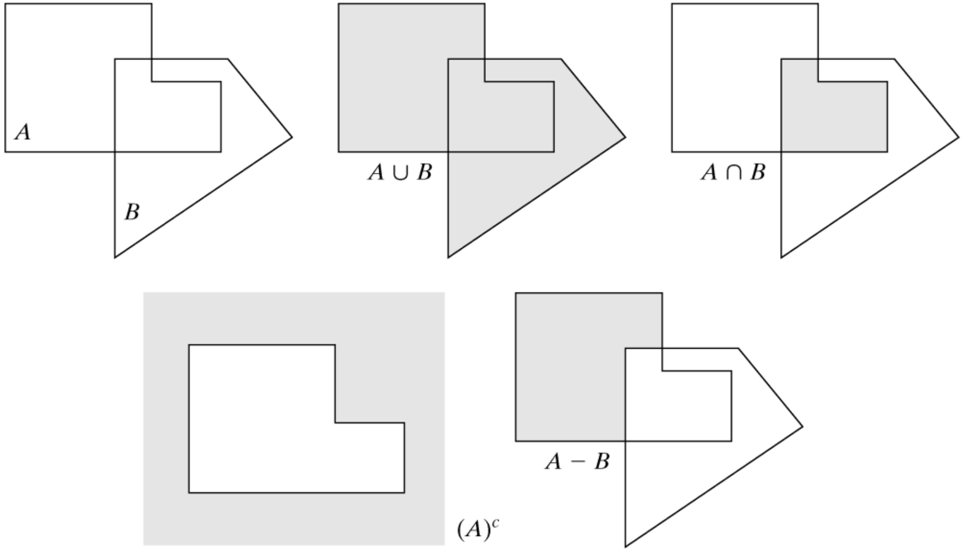
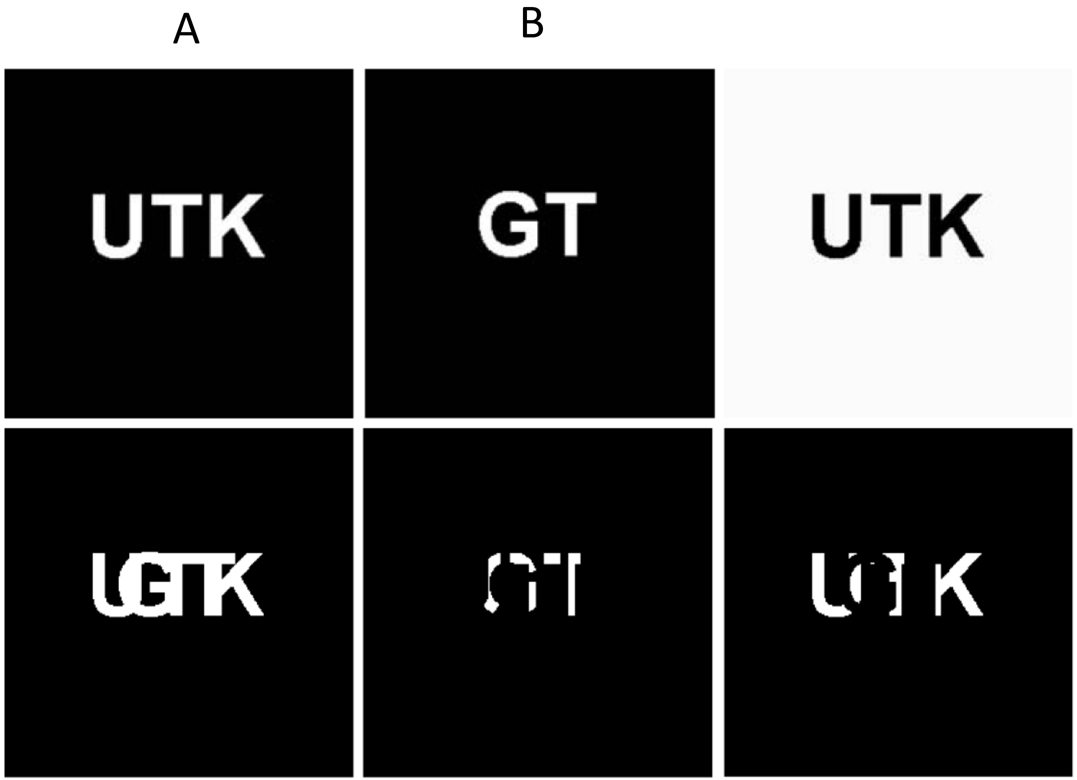


FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Basic operations on shapes

From: Digital Image Processing, Gonzalez, Woods And Eddins

Set Operations on Binary Images



a b c
d e f

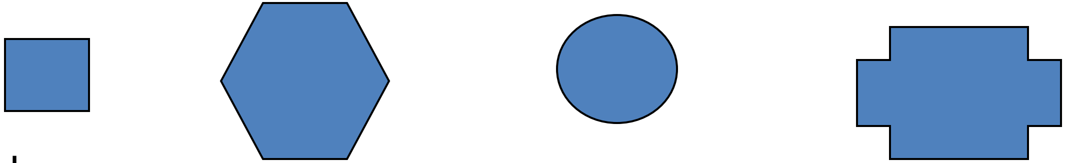
FIGURE 9.3 (a) Binary image A. (b) Binary image B. (c) Complement ~A. (d) Union A | B. (e) Intersection A & B. (f) Set difference A & ~B.

From: Digital Image Processing, Gonzalez,Woods And Eddins

Structuring Elements

A structuring element is a shape mask used in the basic morphological operations.

They can be any shape and size that is digitally representable, and each has an origin.



box

hexagon

disk

something

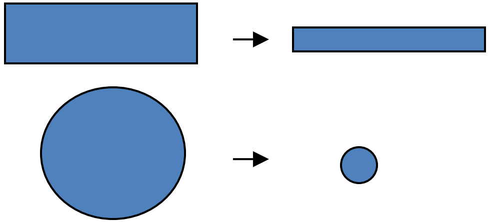
box(length,width)

disk(diameter)

Erosion

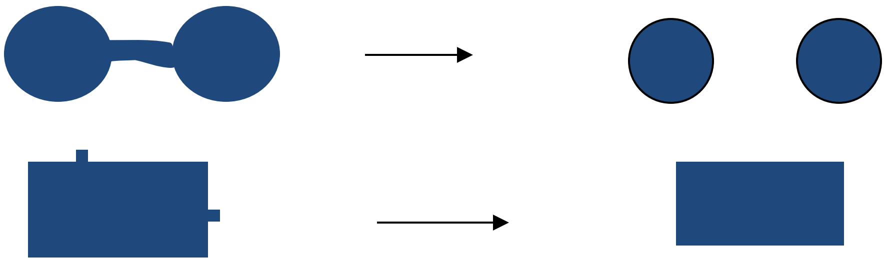
Erosion shrinks the connected sets of 1s of a binary image.

It can be used for



1. shrinking features

2. Removing bridges, branches and small protrusions

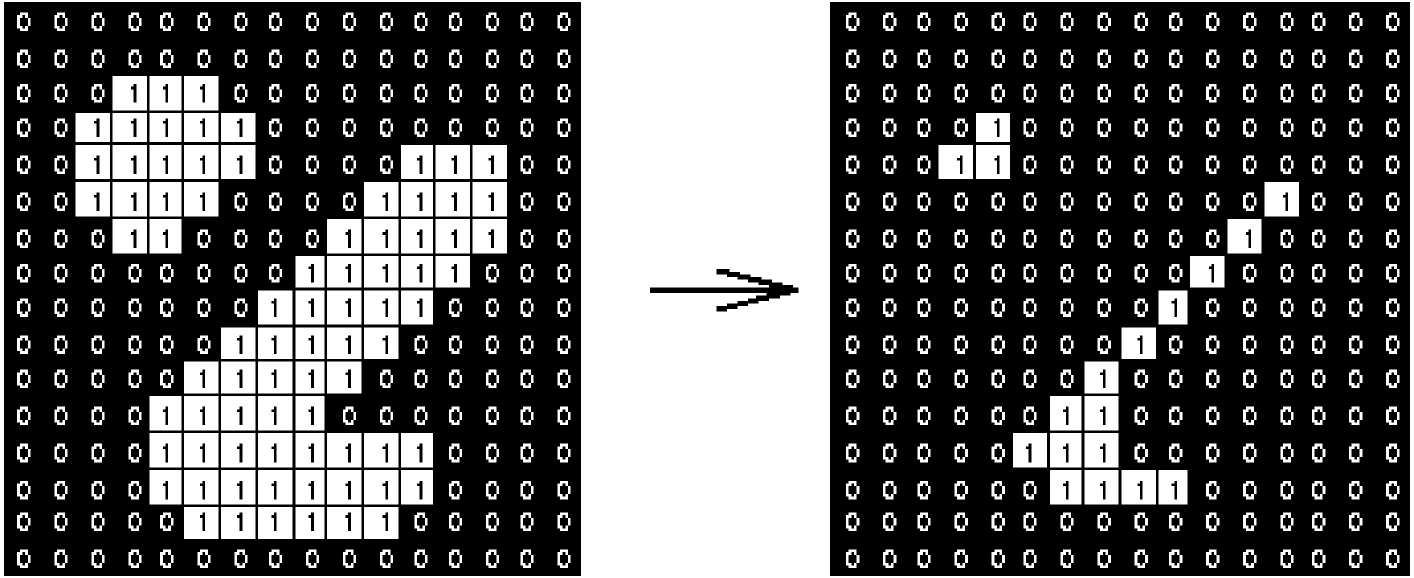


Erosion : Operation (min filter)

1	1	1
1	1	1
1	1	1

Set of coordinate points =

{ (-1, -1), (0, -1), (1, -1),
(-1, 0), (0, 0), (1, 0),
(-1, 1), (0, 1), (1, 1) }



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0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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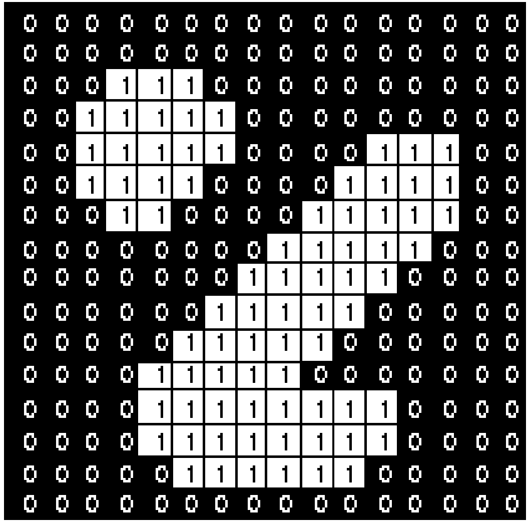
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Dilation (max filter)

1	1	1
1	1	1
1	1	1



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0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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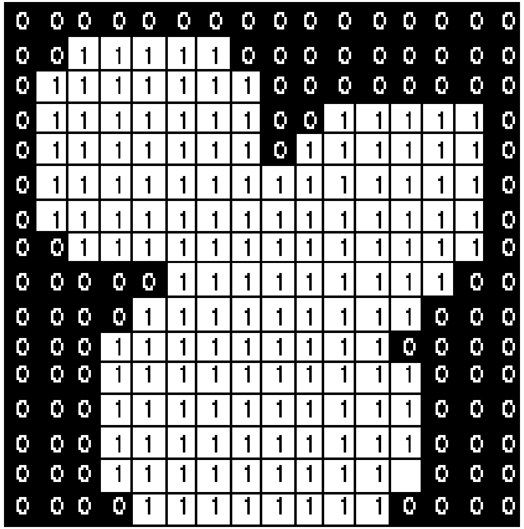
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→



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0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
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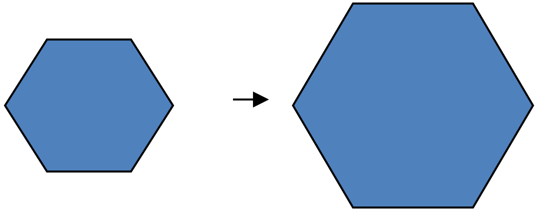
0	0	0	1	1	1	1	1	1	1	1	1		0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0

Dilation

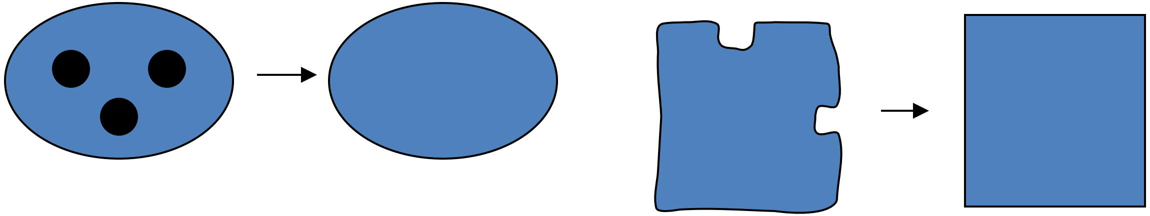
Dilation expands the connected sets of 1s of a binary image.

It can be used for

1. growing features

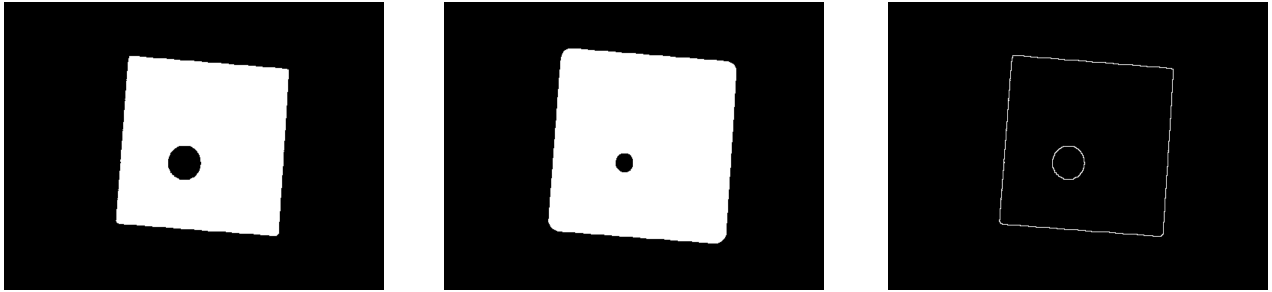


2. filling holes and gaps



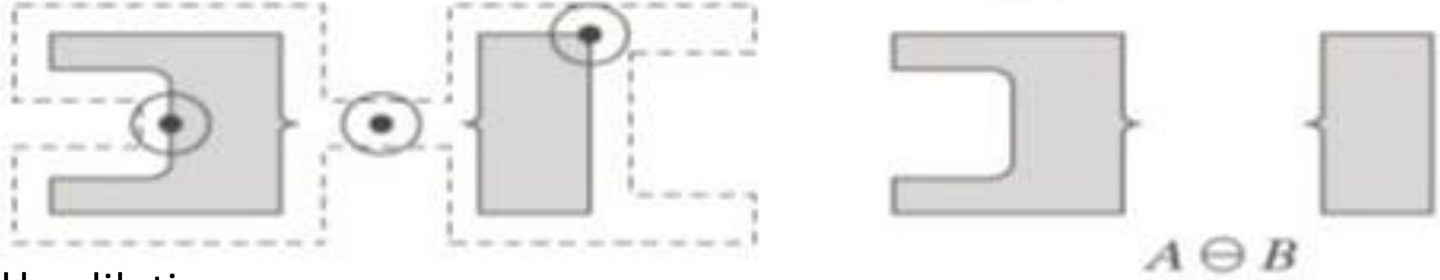
Boundary Detection

1. Dilate input image
2. Subtract input image from dilated image
3. Boundaries remain!



Opening

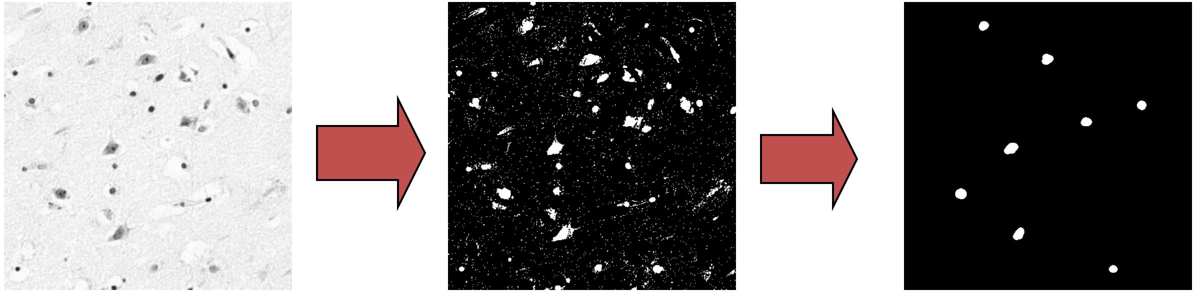
Erosion



Followed by dilation …

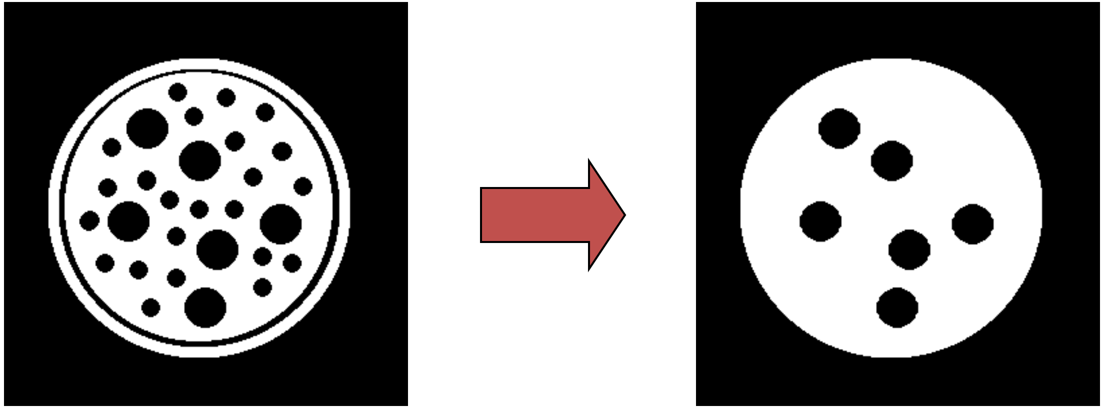
Use Opening for Separating Blobs

- Use large structuring element that fits into the big blobs
- Structuring Element: 11 pixel disc

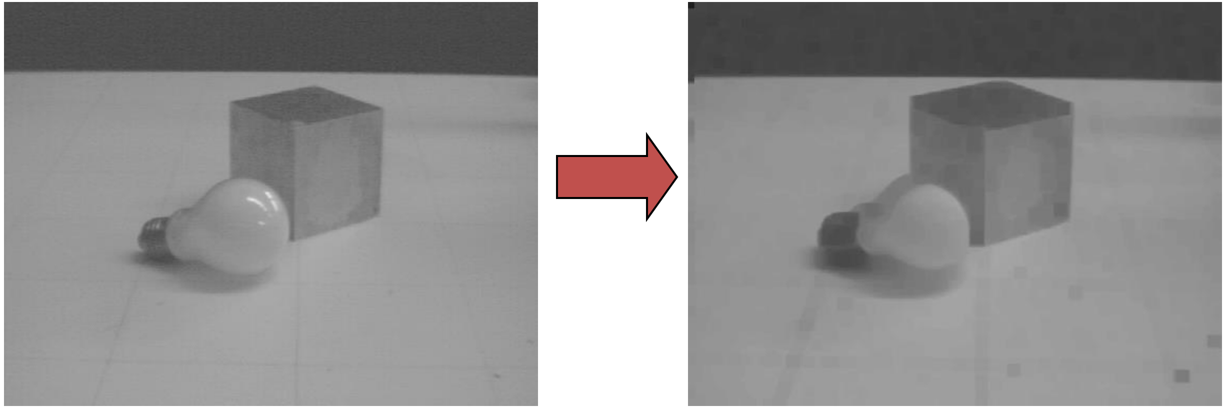


Closing: Example

- Closing operation with a 22 pixel disc
- Closes small holes in the foreground

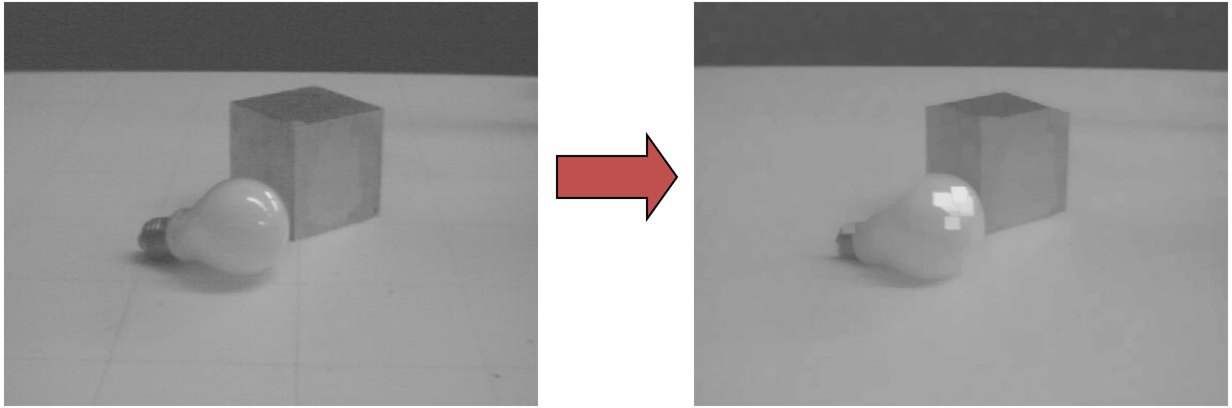


Erosion on Gray Value Images



- min filter
- Images get darker!

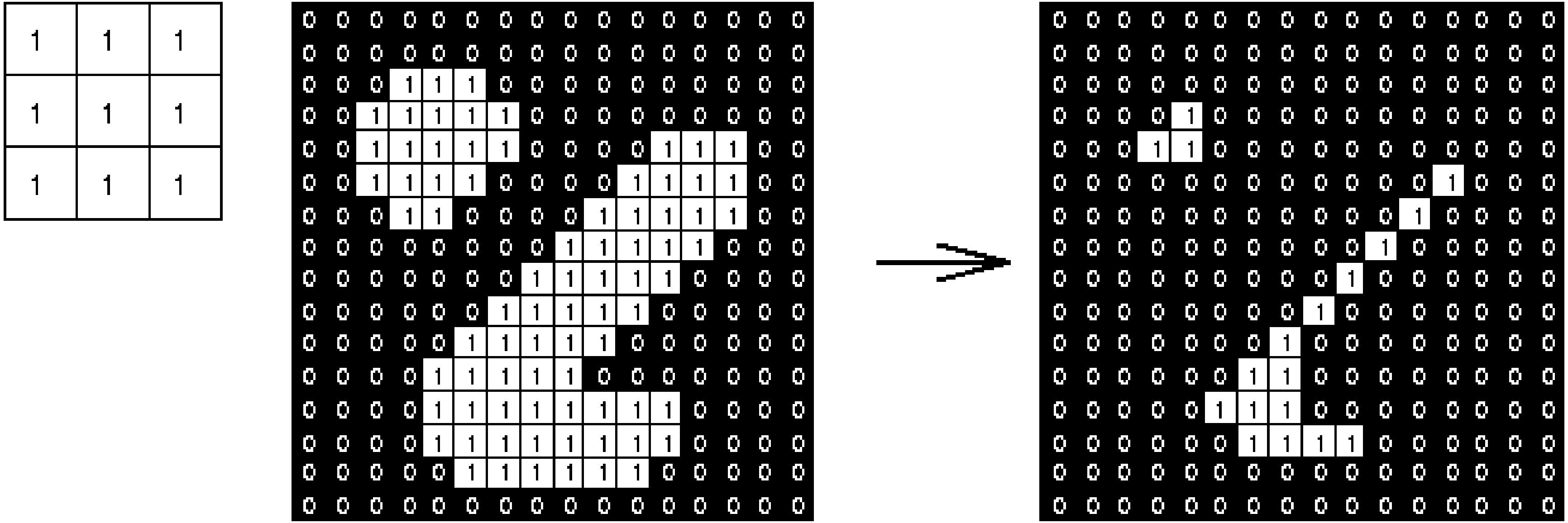
Dilation on Gray Value Images



- max filter
- More uniform intensity

Erosion

- Simple application of **pattern matching**



1	1	1
1	1	1
1	1	1

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

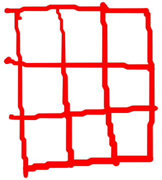
Hit-and-miss Transform

- Look for particular patterns of foreground and background pixels.

✖	1	✖
0	1	1
0	0	✖

- If matched, set pixel = 1

Example: Find right-angled convex corners



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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

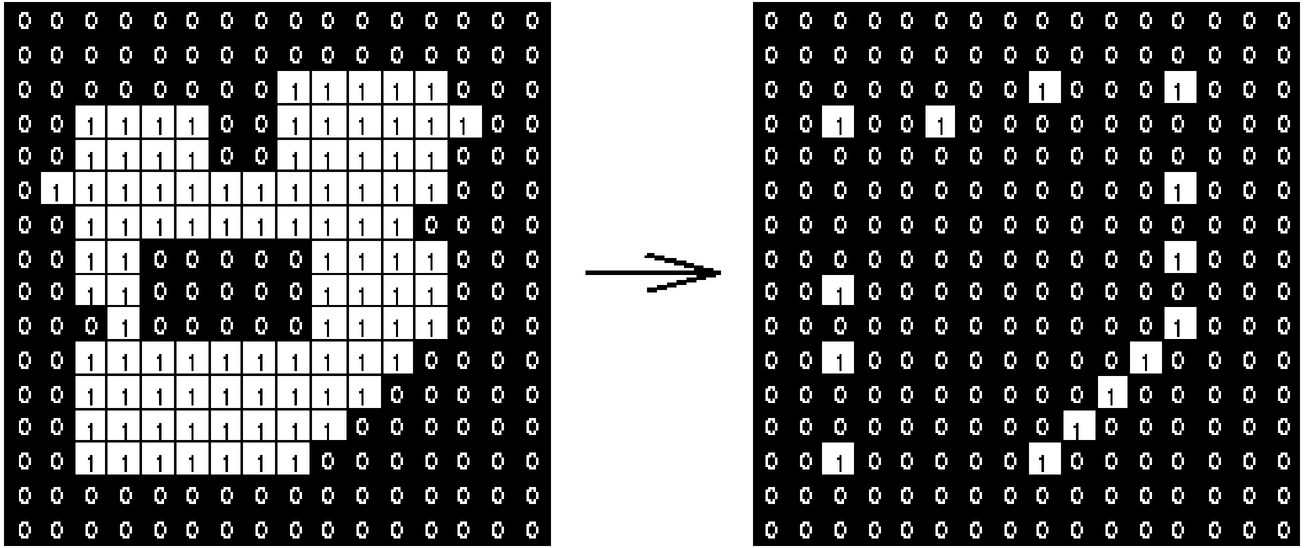
Example: Find right-angled convex corners

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	



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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Sample HAM transforms

1)

0	0	0
0	1	0
0	0	0

Locate isolated points

Sample HAM transforms

1)

0	0	0
0	1	0
0	0	0

Locate isolated points

2)

0	1	0
0	0	0

Locate end points on thin lines

Sample HAM transforms

1)

0	0	0
0	1	0
0	0	0

Locate isolated points

2)

0	1	0
0	0	0

Locate end points on thin/tapering structures

3a)

	1	
	1	
1		1

3b)

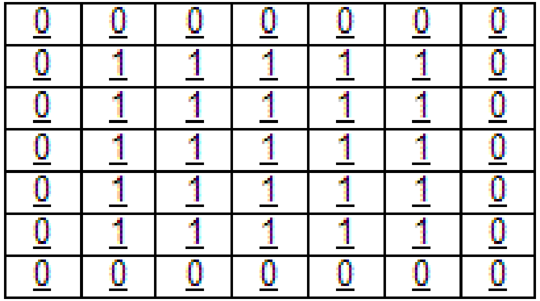
1		
	1	
1		1

3c)

*	0	1
1	1	0
*	1	*

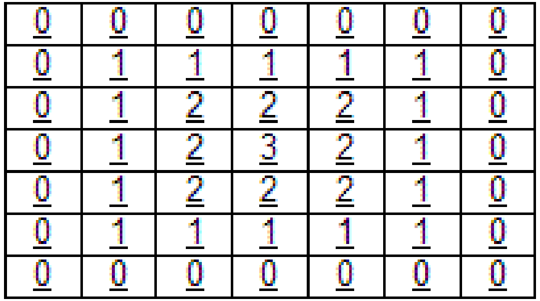
Locate triple junctions

Distance Transform



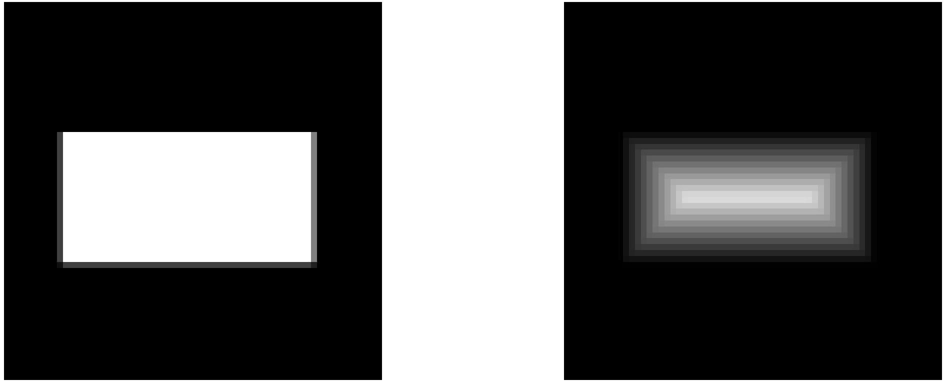
0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

Binary Image

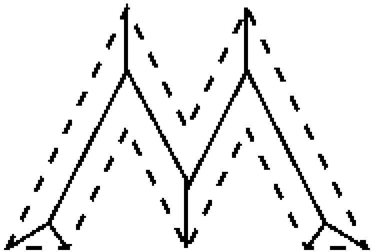


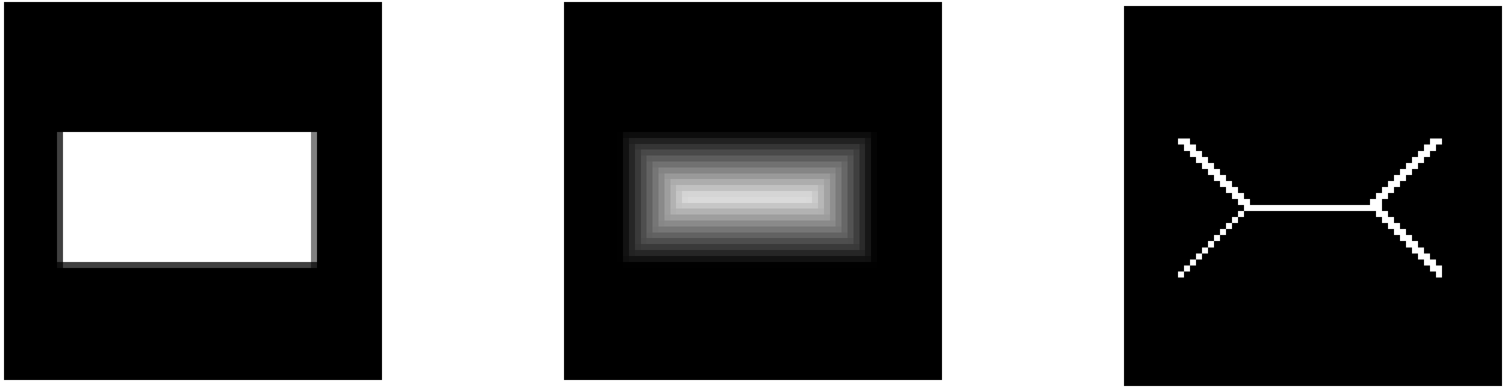
0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	2	2	2	1	0
0	1	2	3	2	1	0
0	1	2	2	2	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

Distance transformation

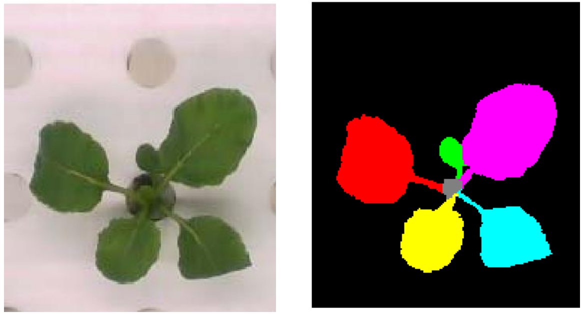


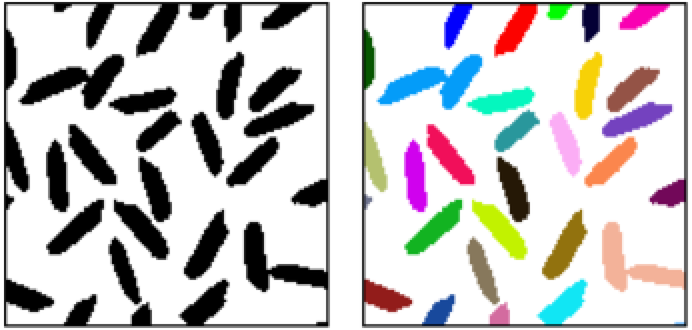
Skeletonization

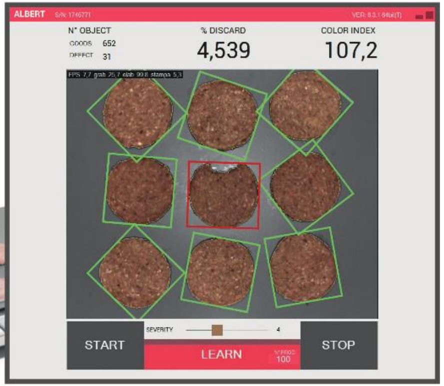




Components







ALBERT S/N: 1746771 VER: 8.3.1 64bit(T)

N° OBJECT
GOODS 652
DEFECT 31

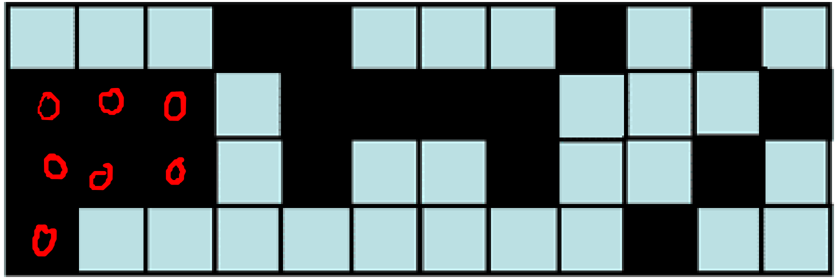
% DISCARD
4,539

COLOR INDEX
107,2

FPS 7,7 grab 25,7 elab 99,8 stampa 5,3

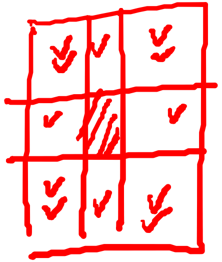
SEVERITY 4

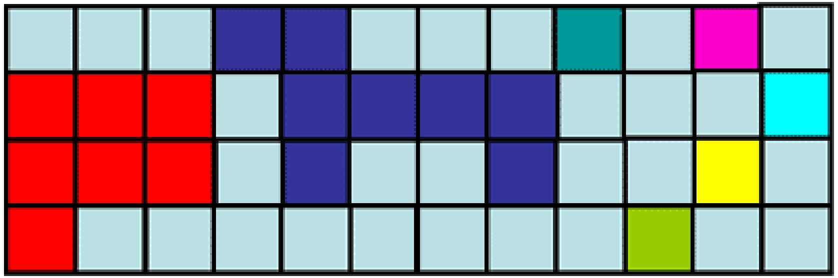
% PROD 100



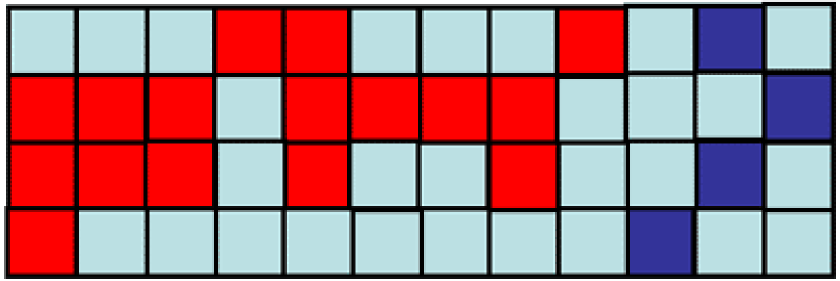
Binary image:
0 - objects;
1 -background

4- conn



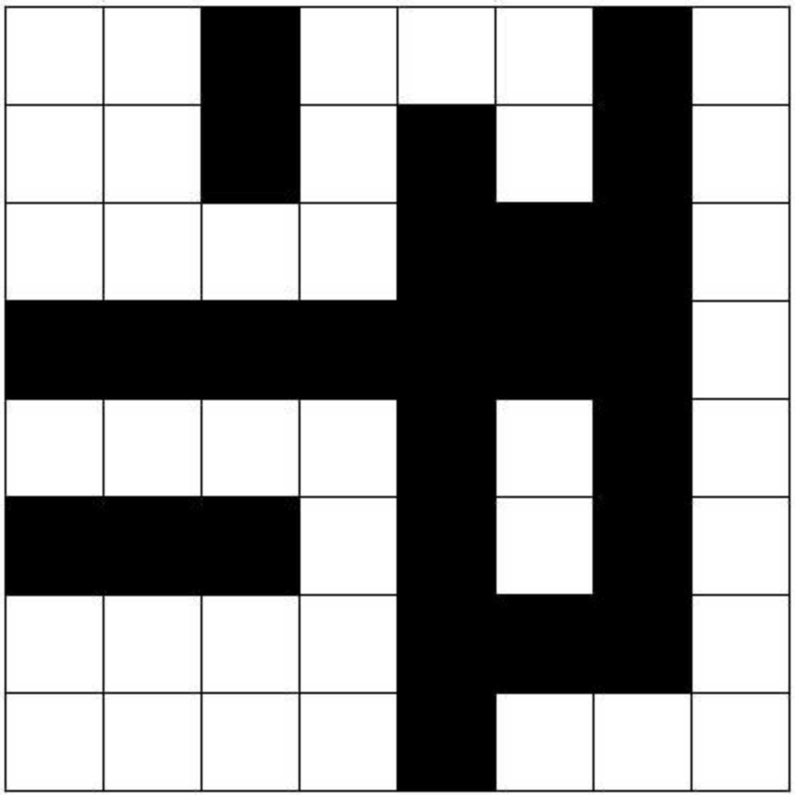


4-connected objects + 8-connected background

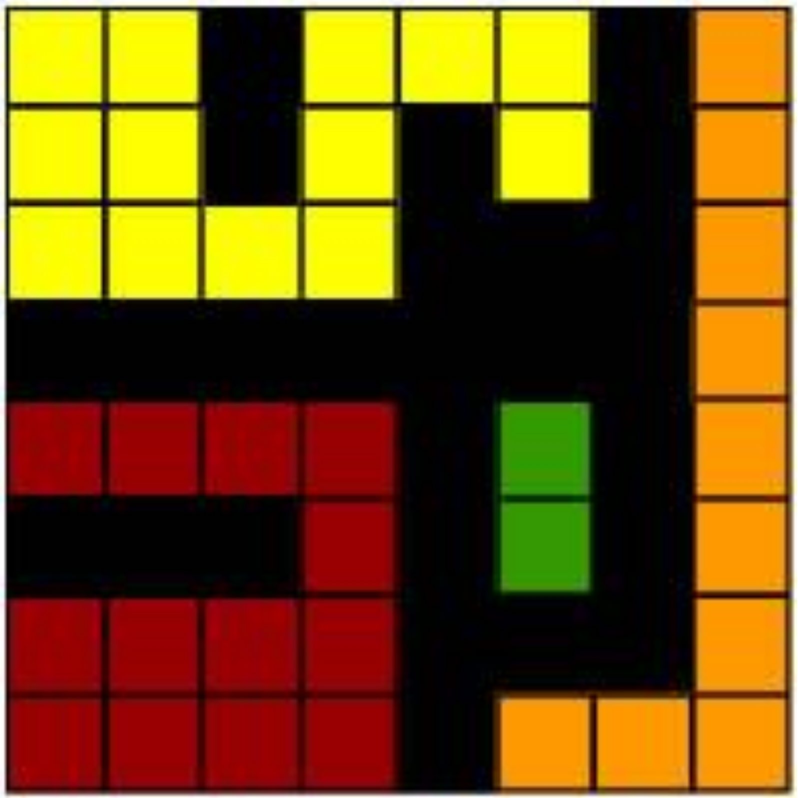


8-connected objects + 8-connected background

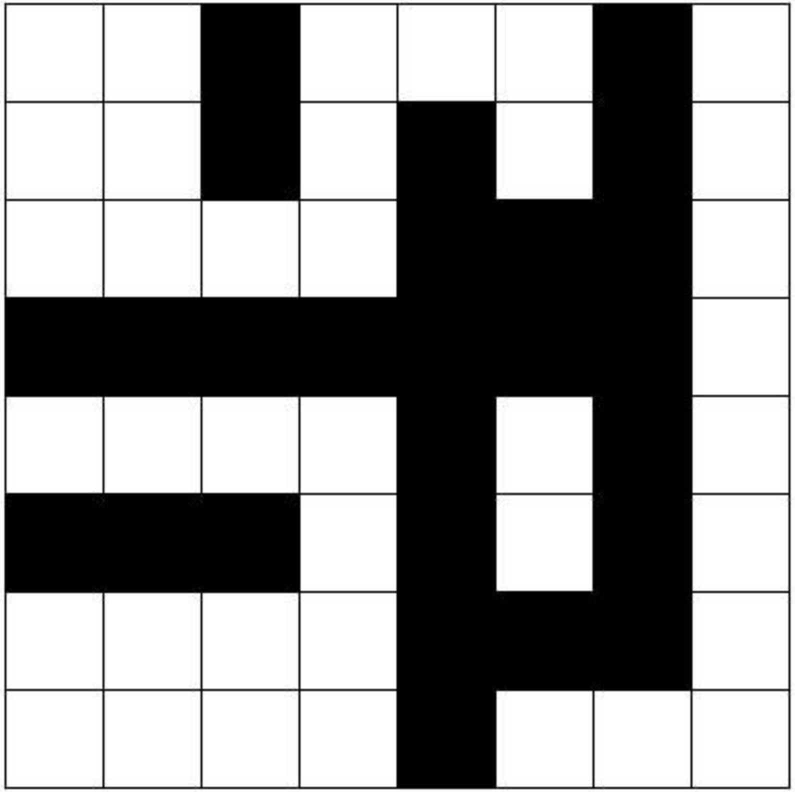
Two-Pass Algorithm for Connected Component Labelling



Two-Pass Algorithm for CCL

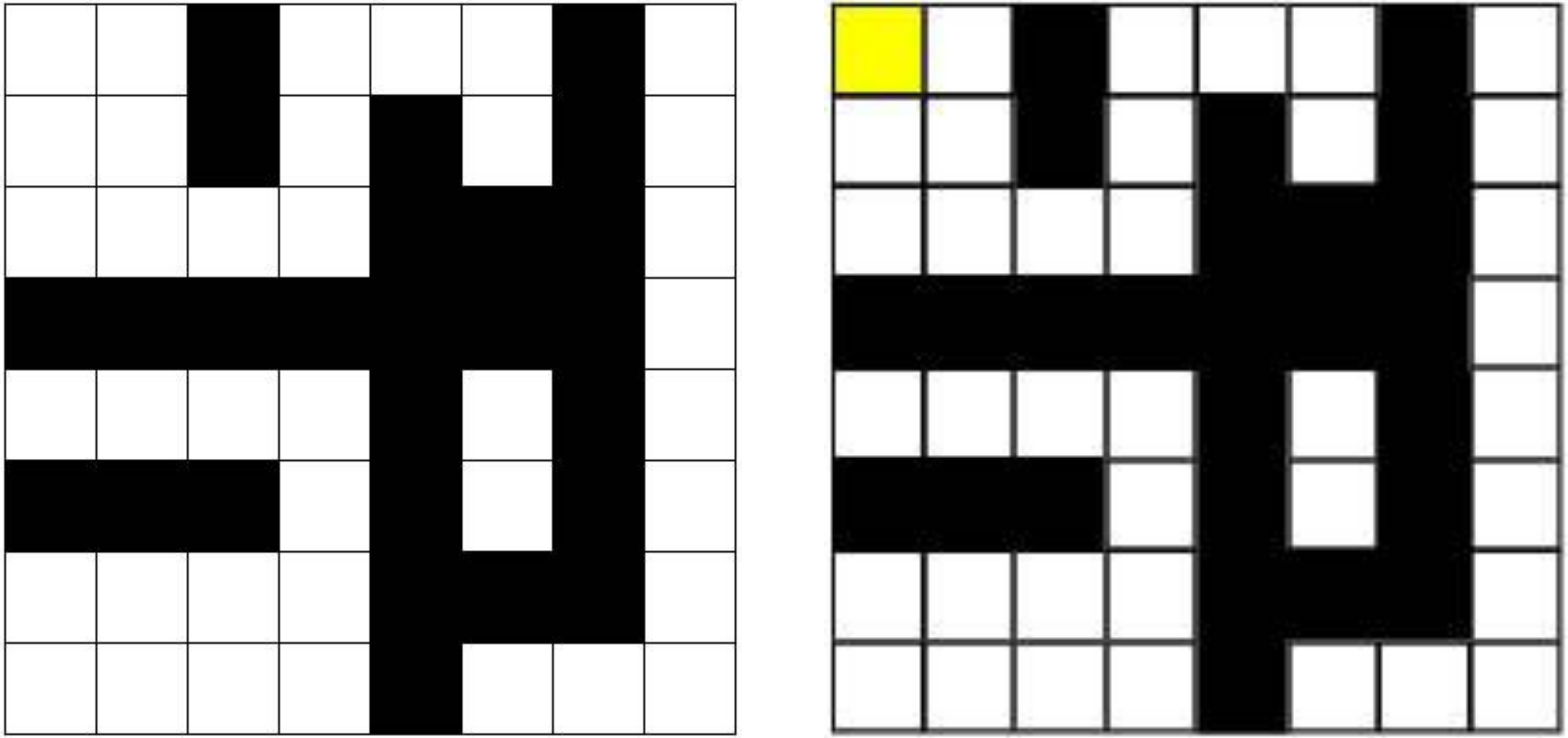


Two-Pass Algorithm for Connected Component Labelling

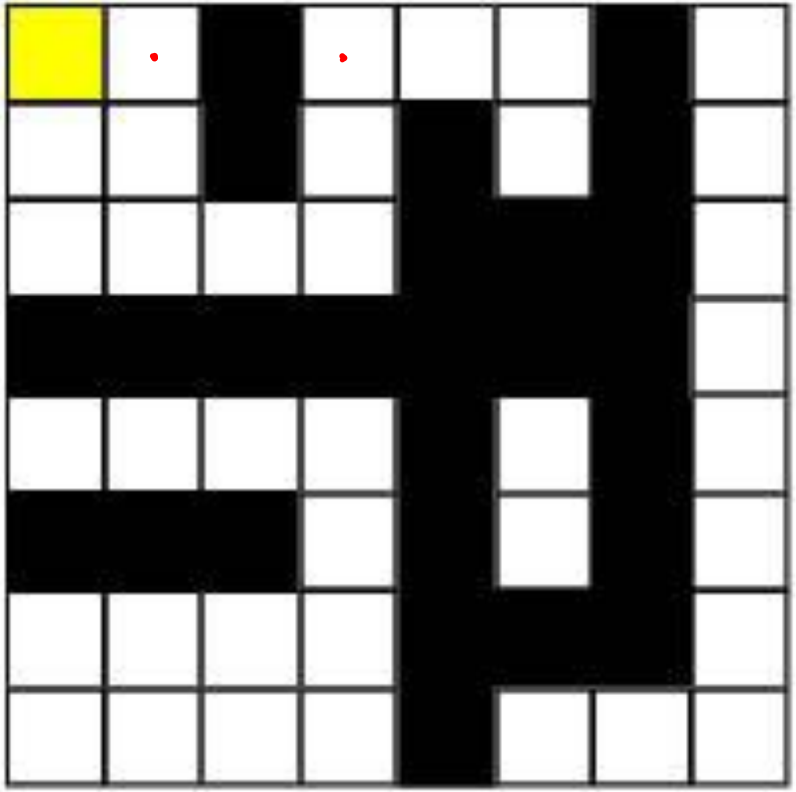


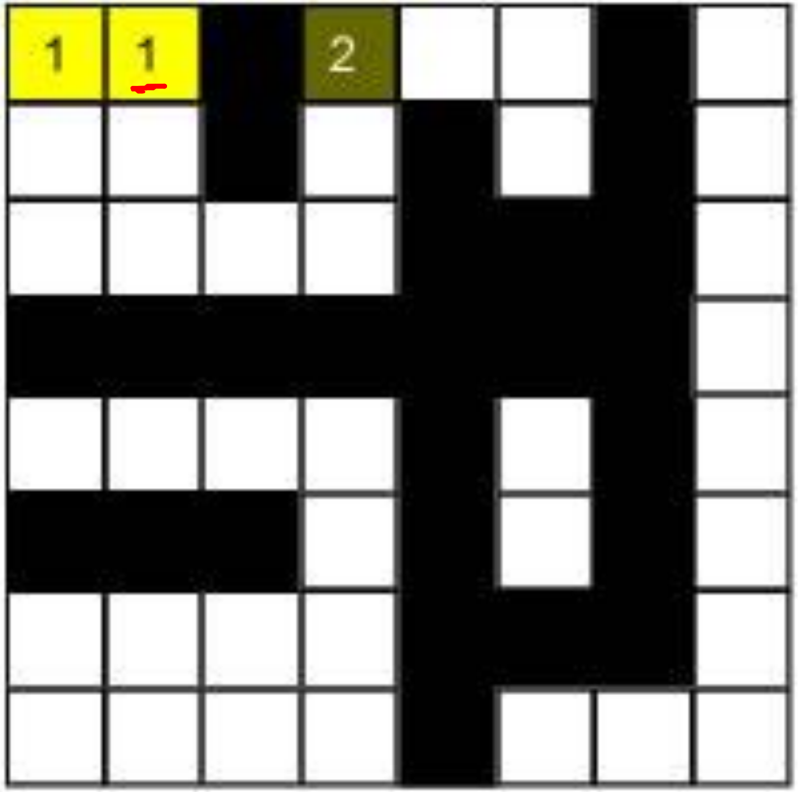
1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	1	1	0	0	0	1
1	1	1	1	0	1	1	1

2PA: No top, left pixels → Create new label

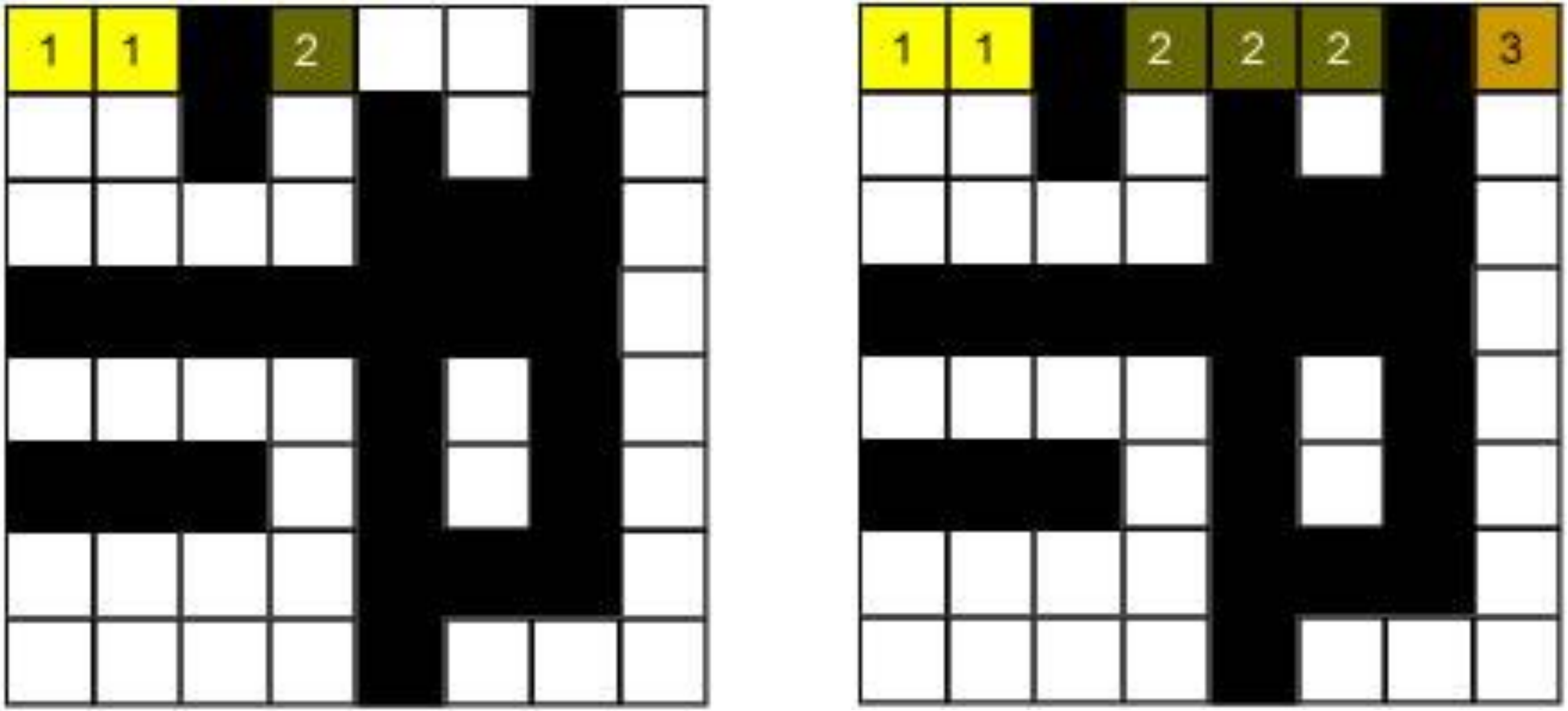


2PA: left pixel labeled→ Copy label ; left pixel BG → new label



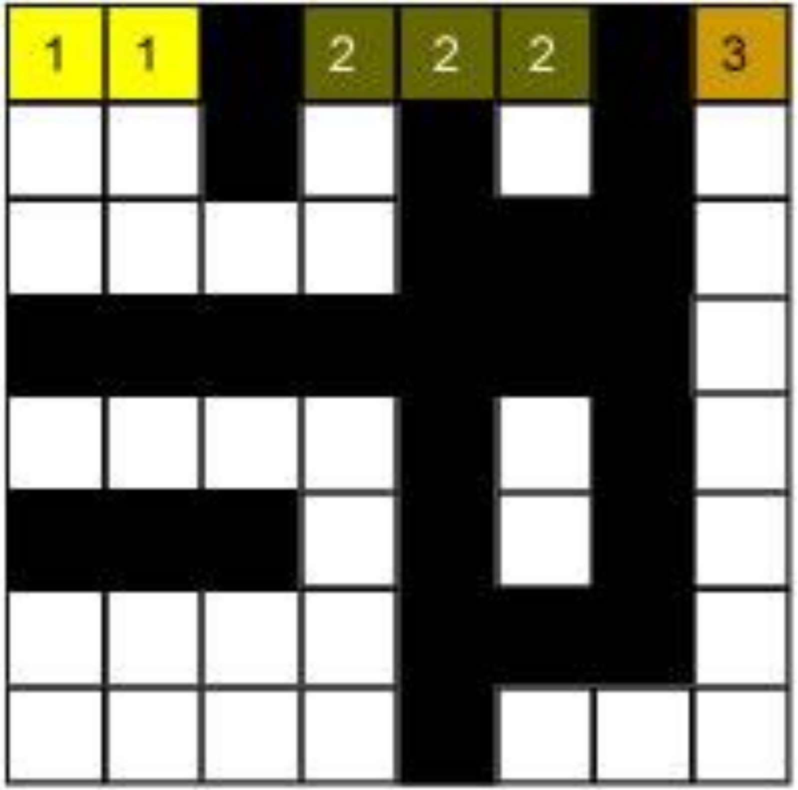


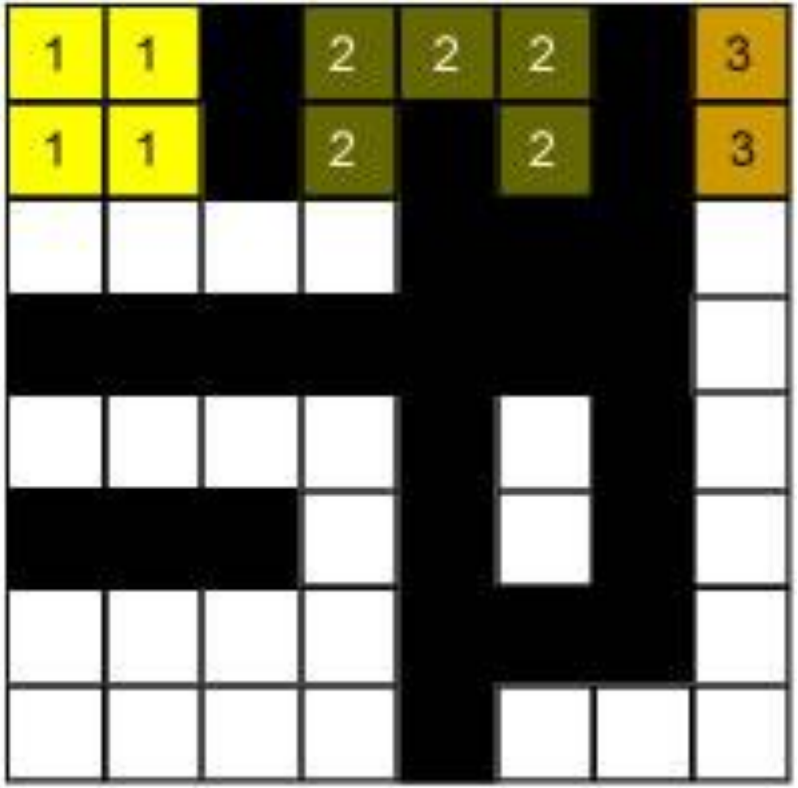
2PA: After row 1 is done



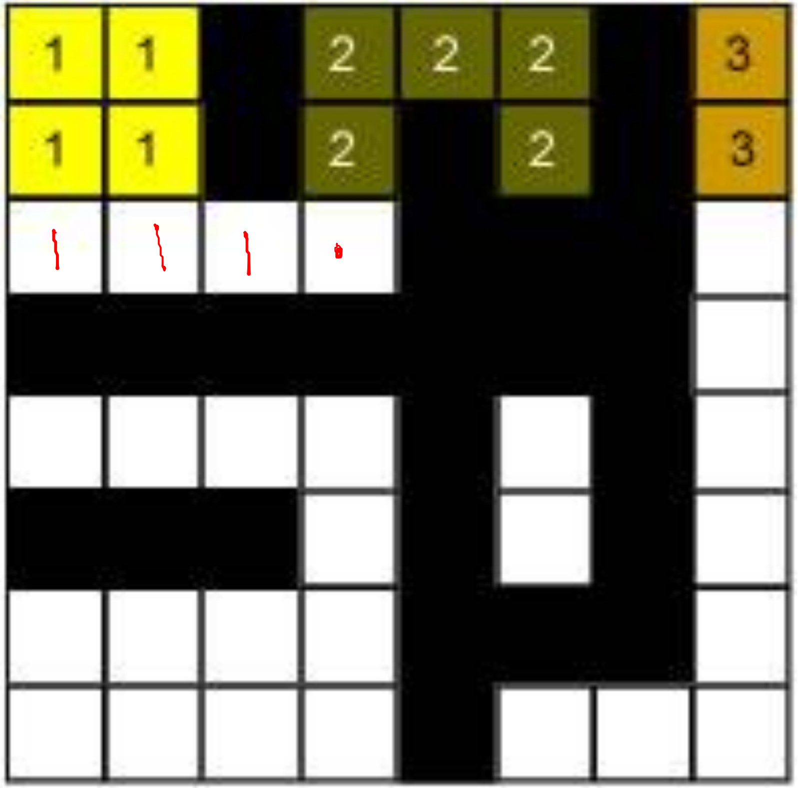
2PA: left/top pixel labeled→ Copy label

(overrides) left pixel BG → new label

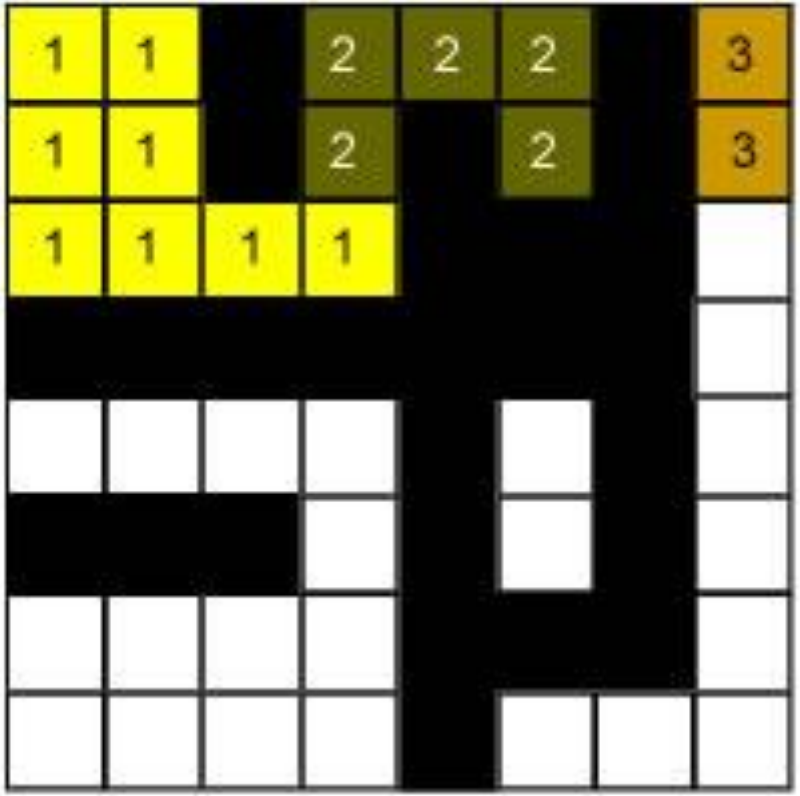




2PA: left/top pixel labeled, different labels →Copy smaller id label, record the assocation

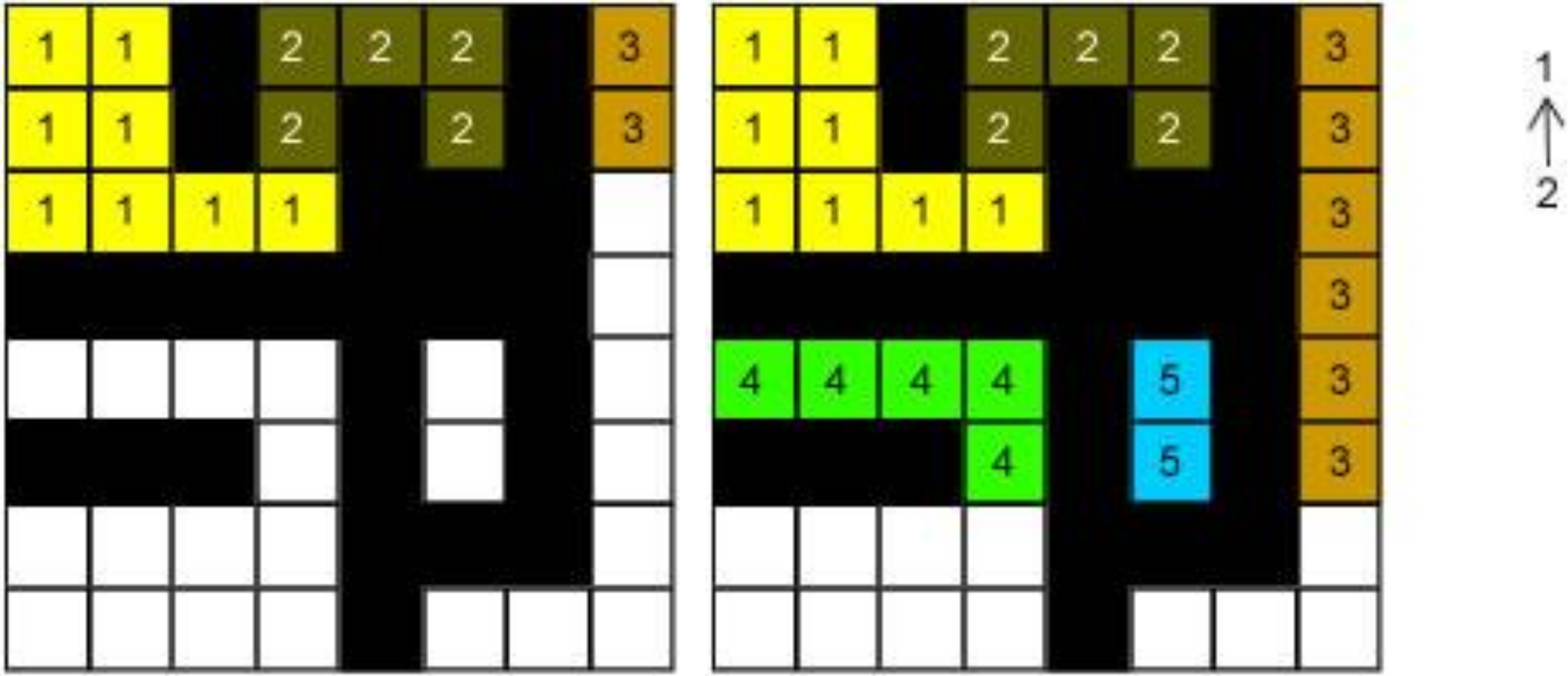


1	1	■	2	2	2	■	3
1	1	■	2	■	2	■	3
1	1	1		■	■	■	
■	■	■	■	■	■	■	
				■		■	
■	■	■		■		■	
				■	■	■	
				■			



1	1	■	2	2	2	■	3
1	1	■	2	■	2	■	3
1	1	1	1	■	■	■	
■	■	■	■	■	■	■	
				■		■	
■	■	■		■		■	
				■	■	■	
				■			

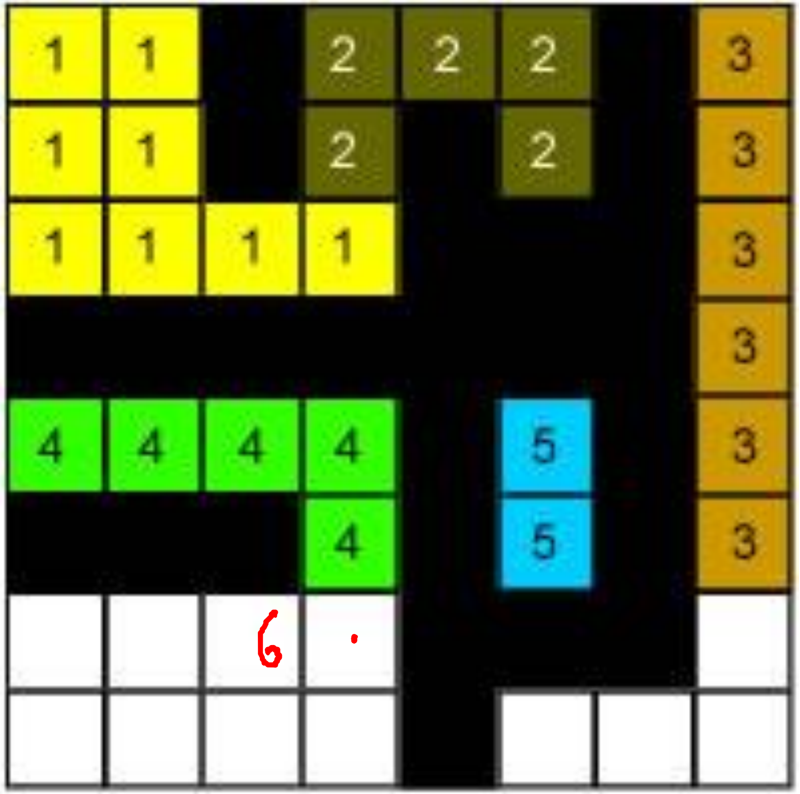
2PA: left/top pixel labeled, different labels →Copy smaller id label, record the association



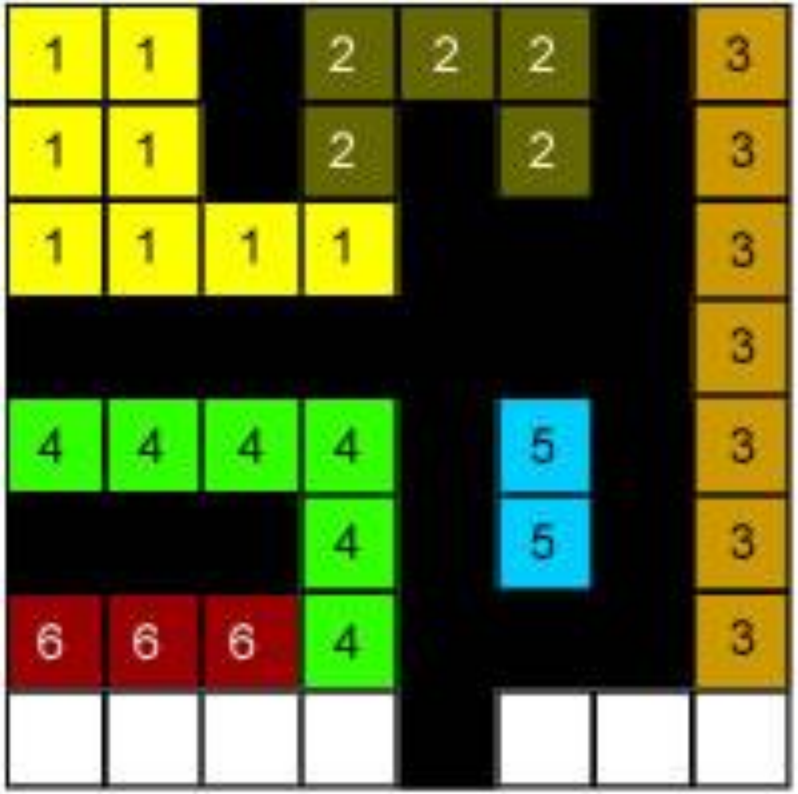
1	1	■	2	2	2	■	3
1	1	■	2	■	2	■	3
1	1	1	1	■	■	■	
■	■	■	■	■	■	■	
				■		■	
■	■	■		■		■	
				■	■	■	
				■			

1	1	■	2	2	2	■	3
1	1	■	2	■	2	■	3
1	1	1	1	■	■	■	3
■	■	■	■	■	■	■	3
4	4	4	4	■	5	■	3
■	■	■	4	■	5	■	3
				■	■	■	
				■			

2PA



1	1	■	2	2	2	■	3
1	1	■	2	■	2	■	3
1	1	1	1	■	■	■	3
■	■	■	■	■	■	■	3
4	4	4	4	■	5	■	3
■	■	■	4	■	5	■	3
		6	·	■	■	■	
				■			

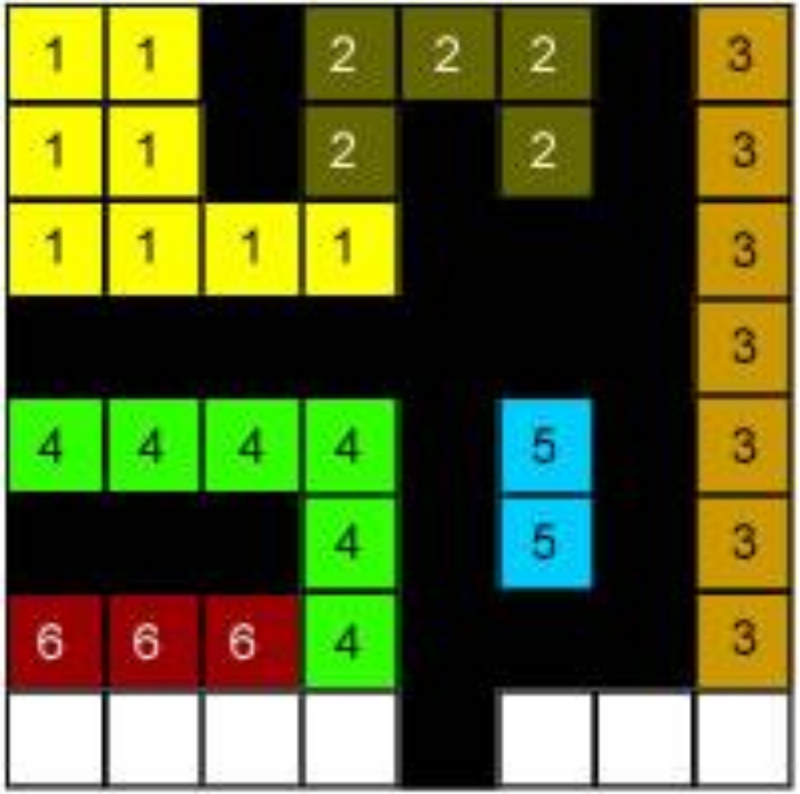


1	1	■	2	2	2	■	3
1	1	■	2	■	2	■	3
1	1	1	1	■	■	■	3
■	■	■	■	■	■	■	3
4	4	4	4	■	5	■	3
■	■	■	4	■	5	■	3
6	6	6	4	■	■	■	3
				■			

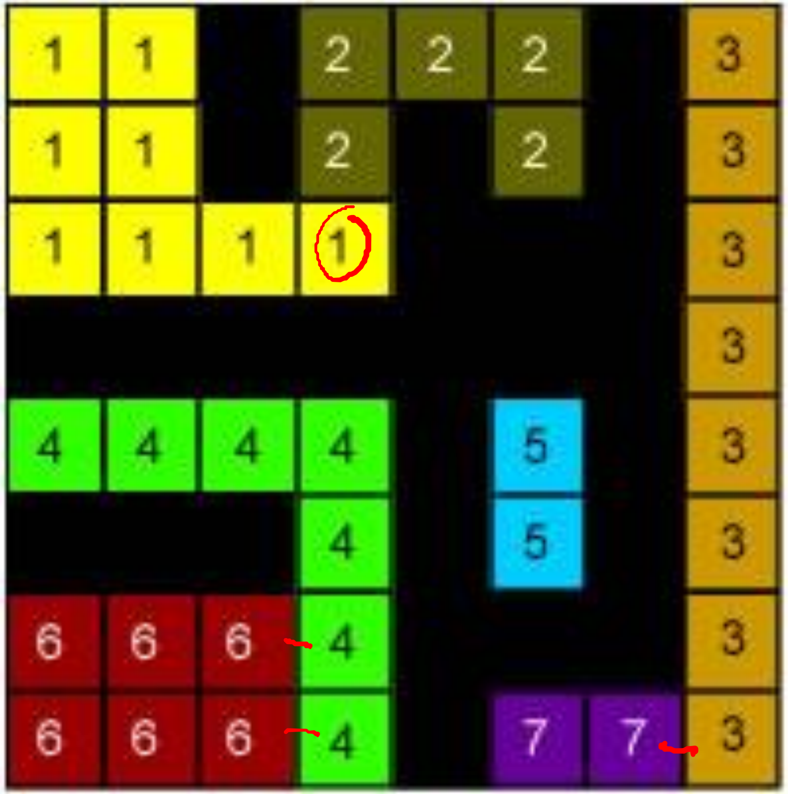
1
↑
2

④
↑
⑥

2PA: After first pass is complete



1	1		2	2	2		3
1	1		2		2		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3



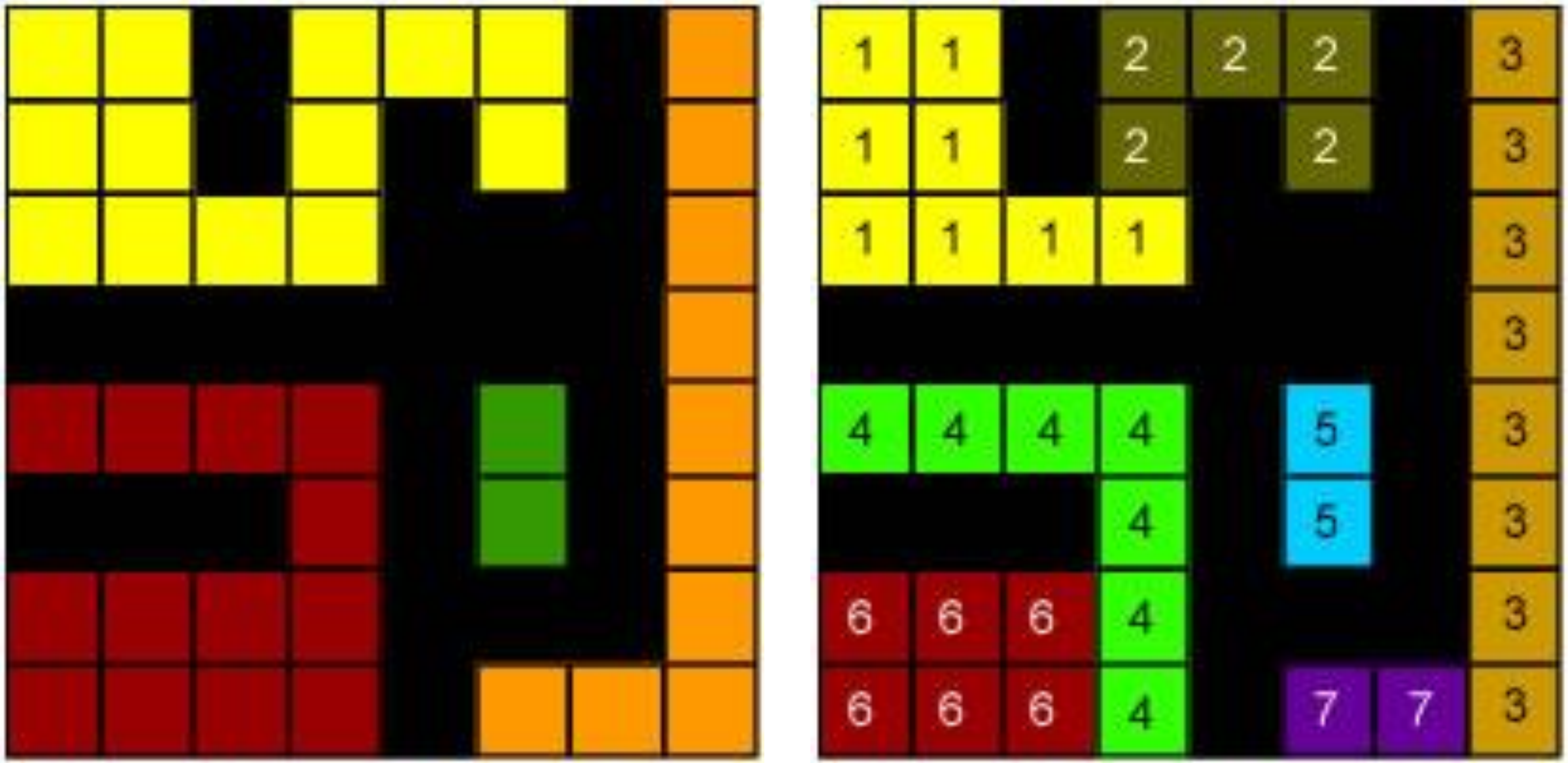
1	1		2	2	2		3
1	1		2		2		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

1
↑
2

4
↑
6

3
↑
7
↑

2PA: After first pass is complete



1	1		2	2	2		3
1	1		2		2		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

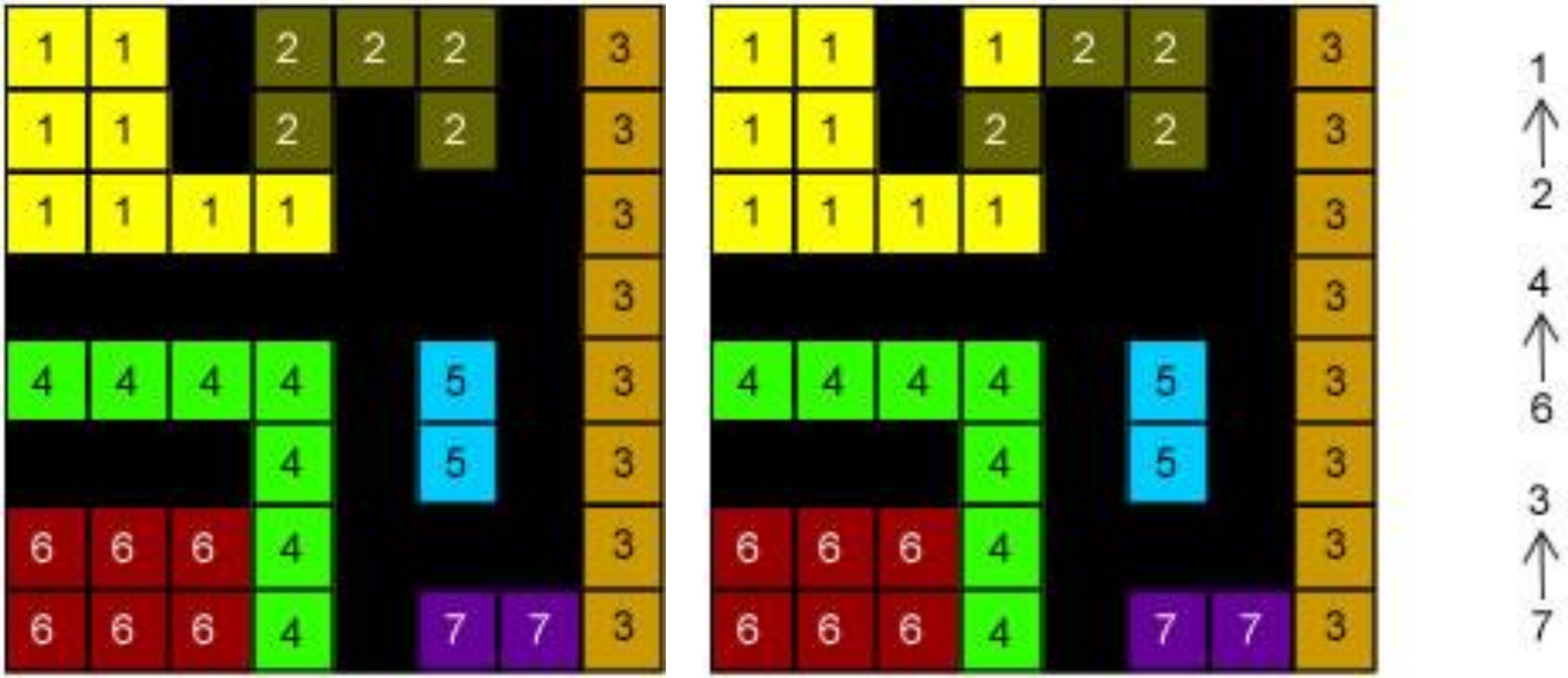
1
↑
2

4
↑
6

3
↑
7

2PA: Second pass: Replace child label with root label.

Union-Find data structure ensures 'find'-ing root is O(1).

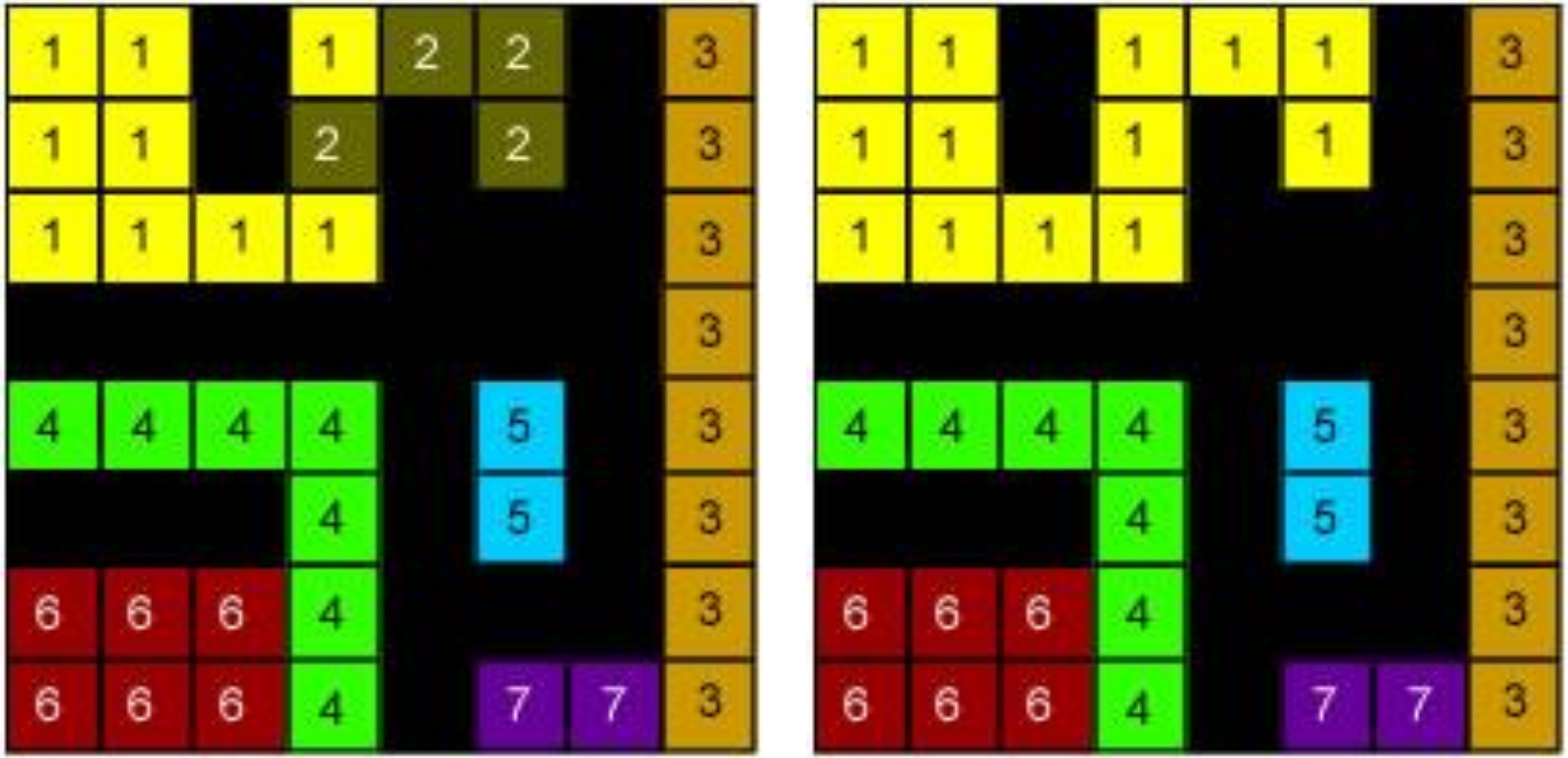


1	1		2	2	2		3
1	1		2		2		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

1	1		1	2	2		3
1	1		2		2		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

2PA: Second pass: Replace child label with root label.

Union-Find data structure ensures ‘find’-ing root is O(1).

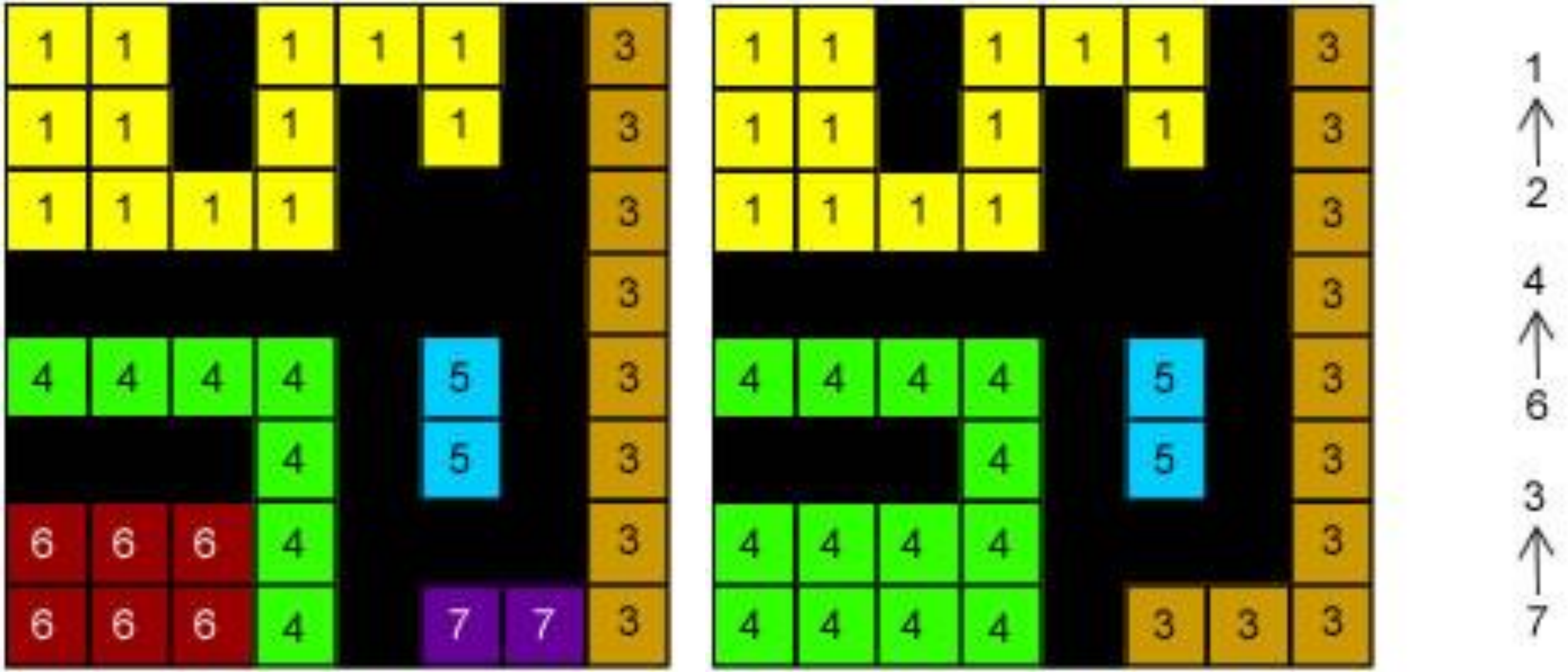


1	1		1	2	2		3
1	1		2		2		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

1	1		1	1	1		3
1	1		1		1		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

2PA: Second pass: Replace child label with root label.

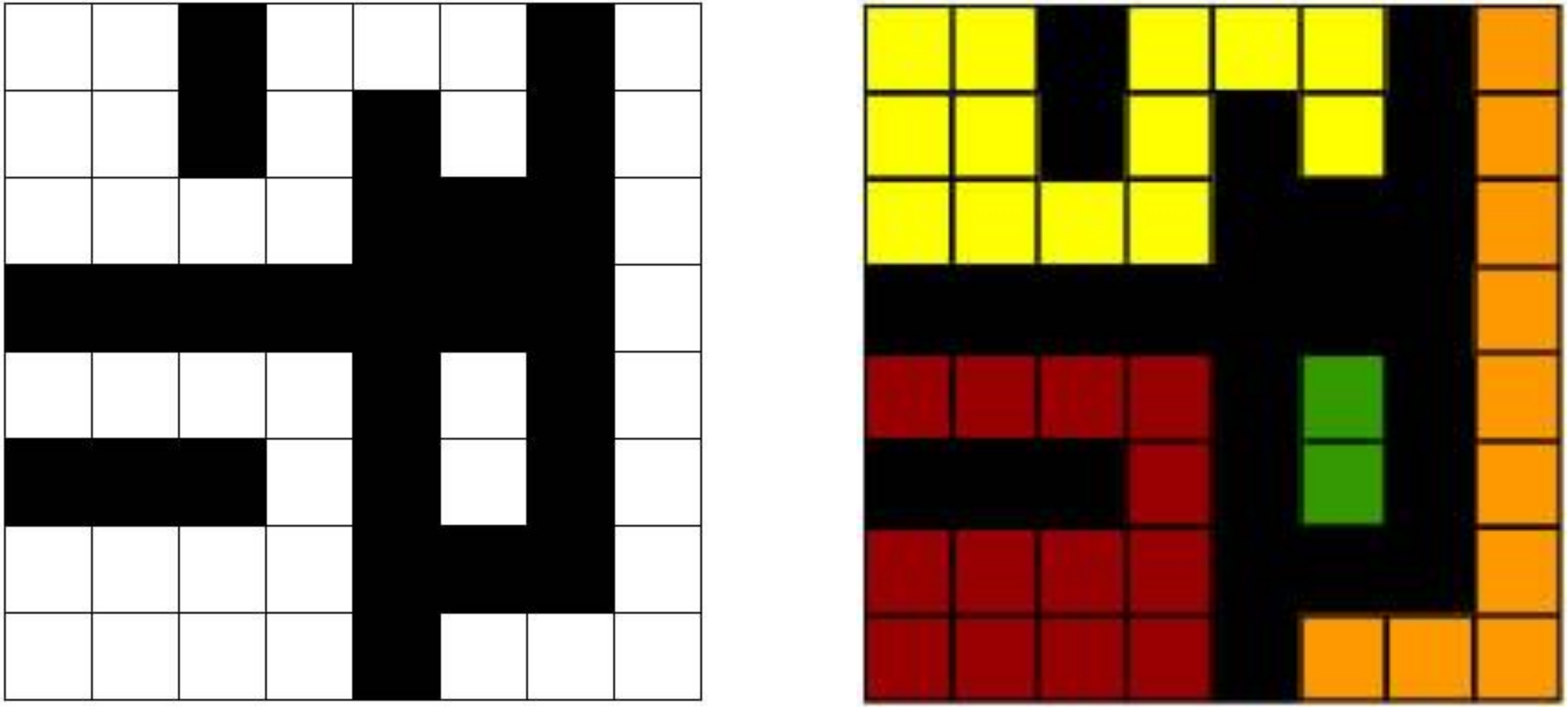
Union-Find data structure ensures ‘find’-ing root is O(1).



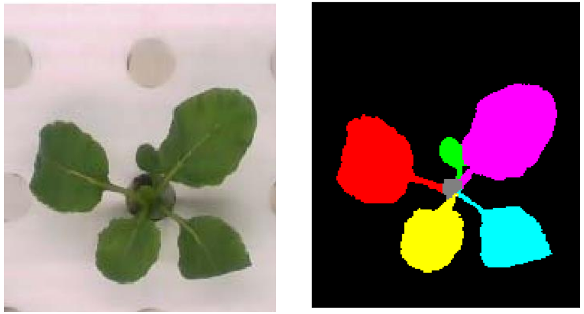
1	1		1	1	1		3
1	1		1		1		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
6	6	6	4				3
6	6	6	4		7	7	3

1	1		1	1	1		3
1	1		1		1		3
1	1	1	1				3
							3
4	4	4	4		5		3
			4		5		3
4	4	4	4				3
4	4	4	4		3	3	3

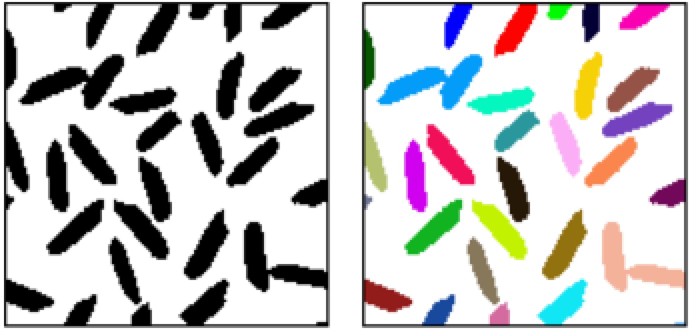
2PA-CCL (Rosenfeld&PFaltz 1968): Requires only two rows of image at a time

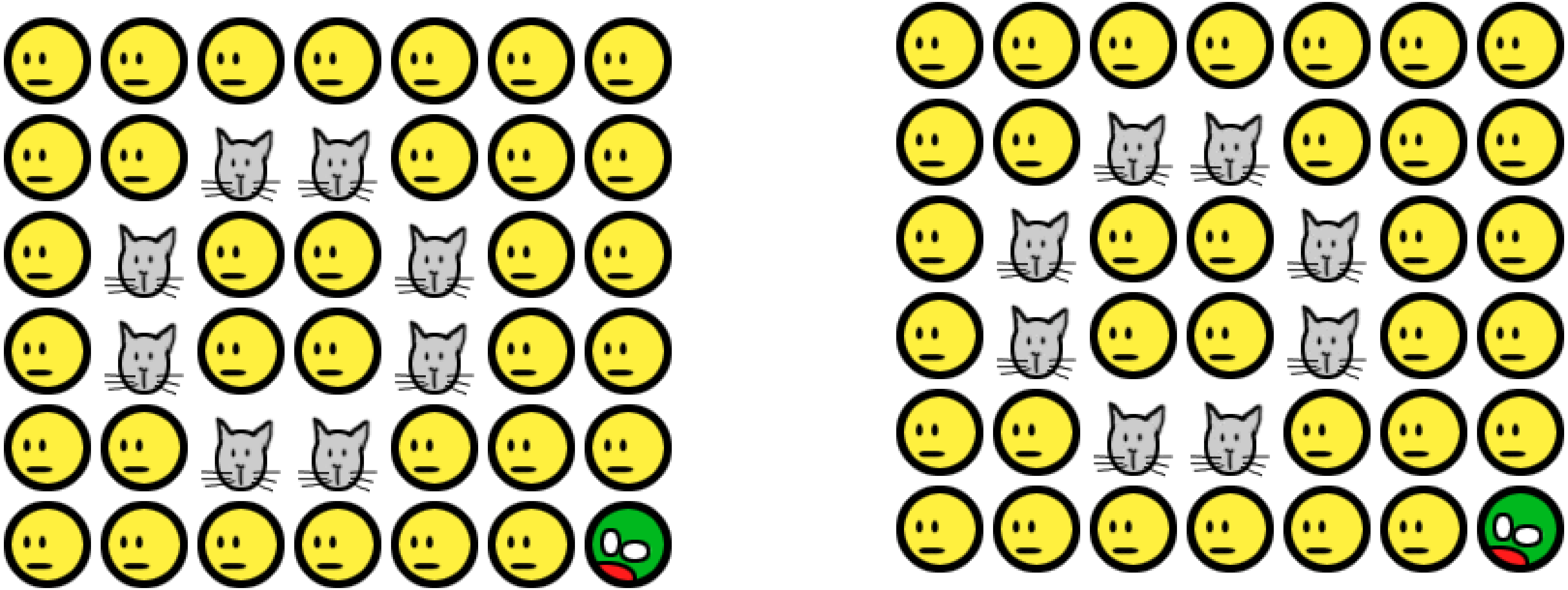


Connected Component Labeling



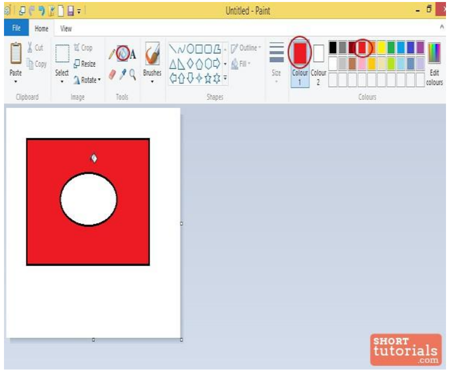
MATLAB: `bwlabel, label2rgb`

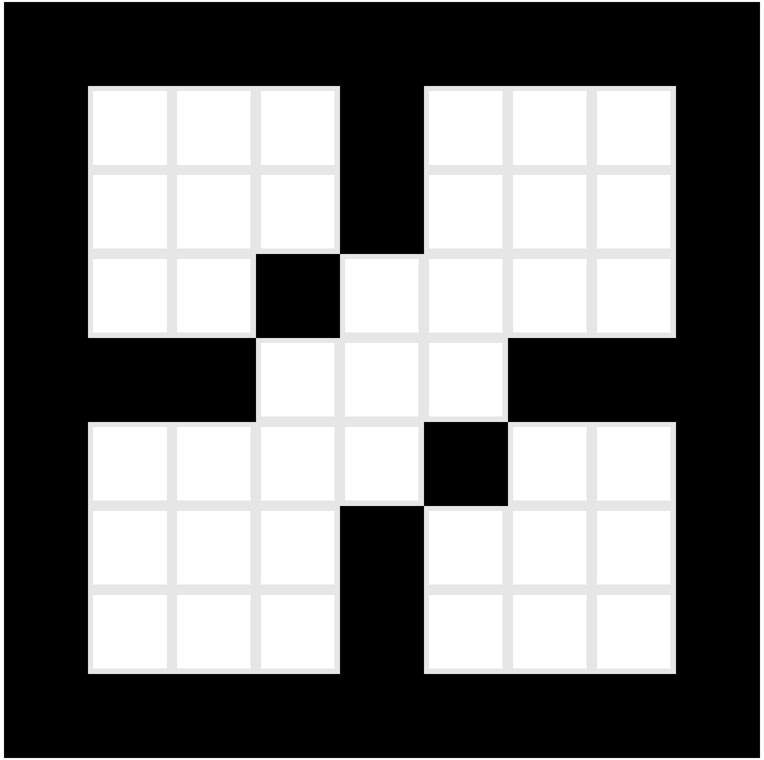




http://inventwithpython.com/blogstatic/floodfill/zombie2.gif

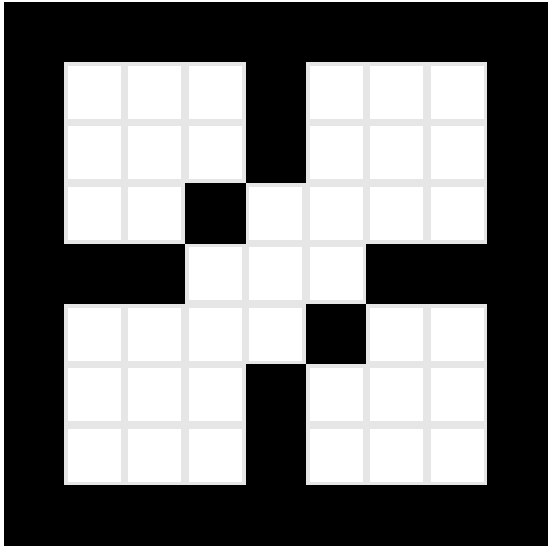
Flood-Fill





https://upload.wikimedia.org/wikipedia/commons/7/7e/Recursive_Flood_Fill_4_%28aka%29.gif

Flood-Fill Algorithm (4-conn)

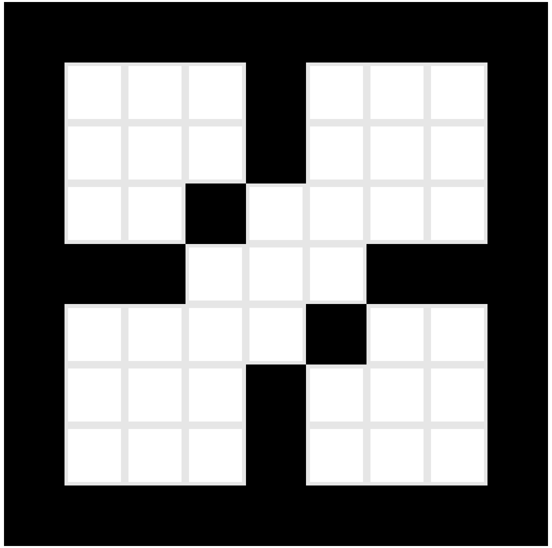


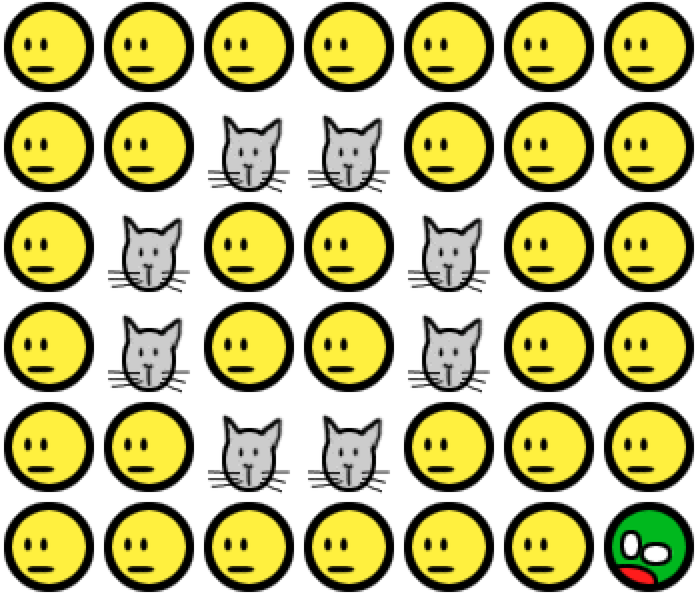
```
void floodFill(int x, int y, int fill, int old)
{
    if ((x < 0) || (x >= width)) return;
    if ((y < 0) || (y >= height)) return;
    if (getPixel(x, y) == old) {
           setPixel(fill, x, y);
           floodFill(x+1, y, fill, old);
           floodFill(x, y+1, fill, old);
           floodFill(x-1, y, fill, old);
           floodFill(x, y-1, fill, old);
    }
}
```

Many/More efficient versions exist !

If 8-conn, all white pixels filled

(all humans eventually become zombie !)





Results of Morphological Operations

Originalimage

Dilated image

Eroded image

Internal Boundary

External Boundary

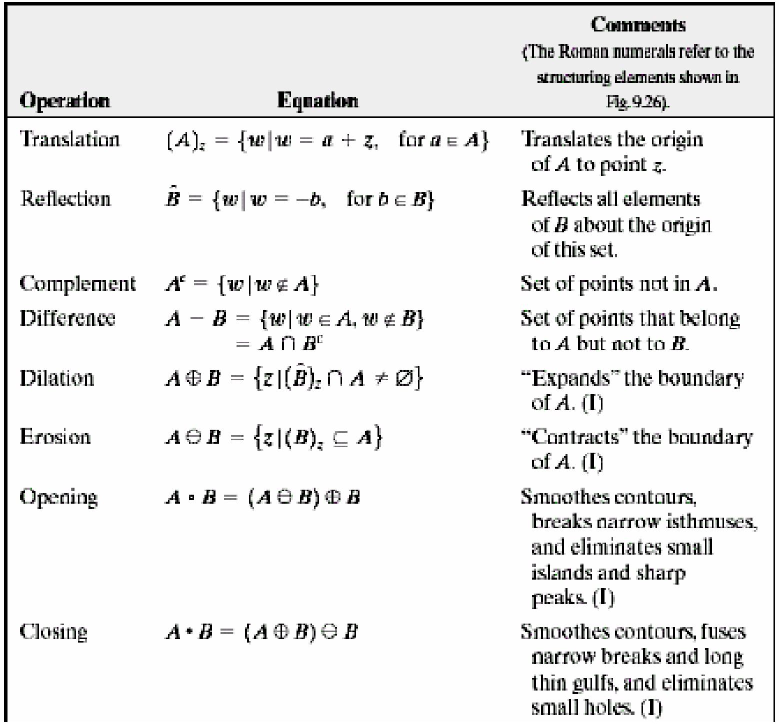
Morphological Gradient

Thinning of the Image

Thickening of the Image

Skeletonization - 9 iterations

Summary of Morphological Filtering



Operation	Equation	**Comments** (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

MATLAB codes

circshift(A,z)

fliplr(flipud(B))

~A or 1-A

A &~B

imdilate(A,B)

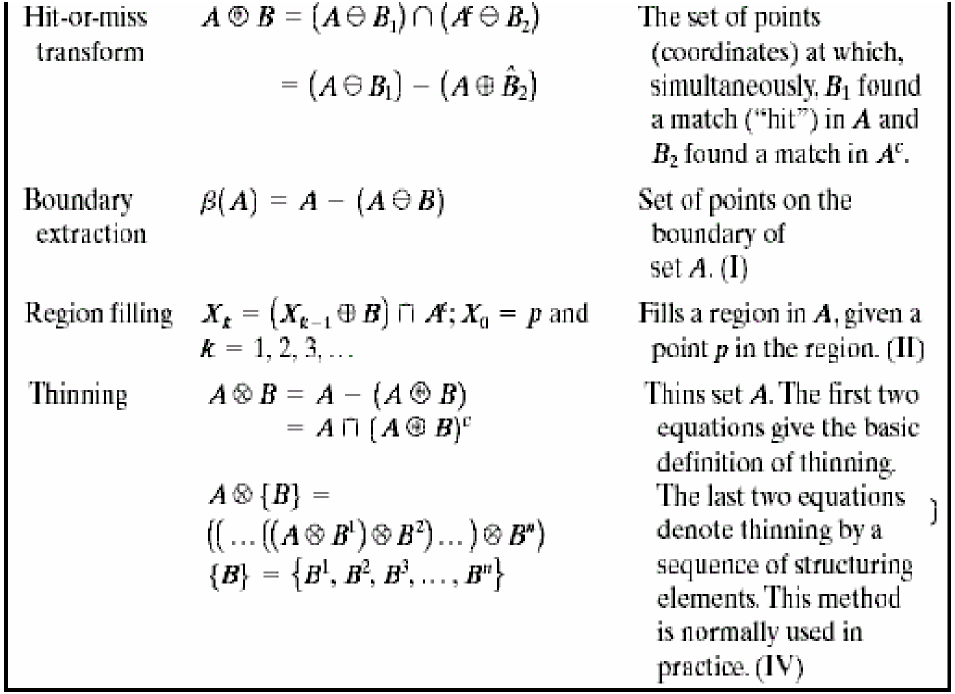
imerode(A,B)

imopen(A,B)

imclose(A,B)

Slide courtesy: EE465: Introduction to Digital Image Processing

Summary (Con’d)



Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match (“hit”) in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)

bwhitmiss(A,B)

A&~(imerode(A,B))

region_fill.m

bwmorph(A,’thin’);

Slide courtesy: EE465: Introduction to Digital Image Processing

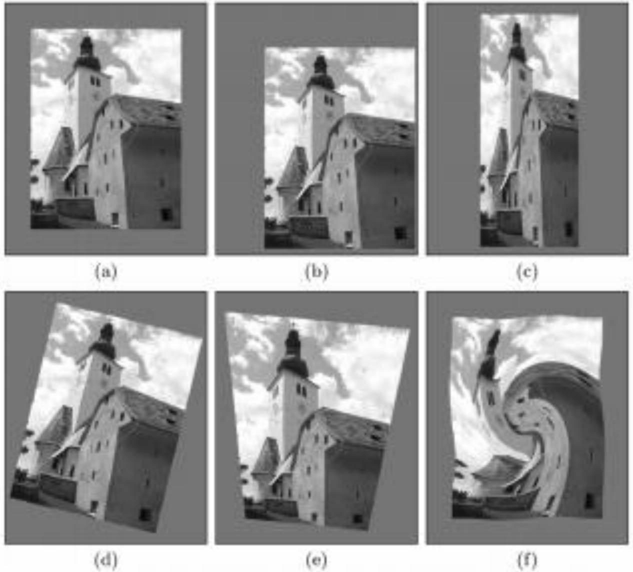
Morphological Filtering using MATLAB

- https://in.mathworks.com/help/images/morphological-filtering.html

GEOMETRIC OPERATIONS

Geometric Operations

- Filters, point operations change intensity
- Pixel position (and geometry) unchanged
- Geometric operations: change image geometry
- **Examples:** translating, rotating, scaling an image



Examples of Geometric operations

https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

Geometric Operations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} \leftarrow T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

$I(x,y)$

- Example applications of geometric operations:
  - Zooming images, windows to arbitrary size
  - Computer graphics: deform textures and map to arbitrary surfaces
- **Definition:** Geometric operation transforms image *I* to new image *I'* by modifying **coordinates of image pixels**

$$I(x,y) \rightarrow I'(x',y')$$

- Intensity value originally at (x,y) moved to new position (x',y')

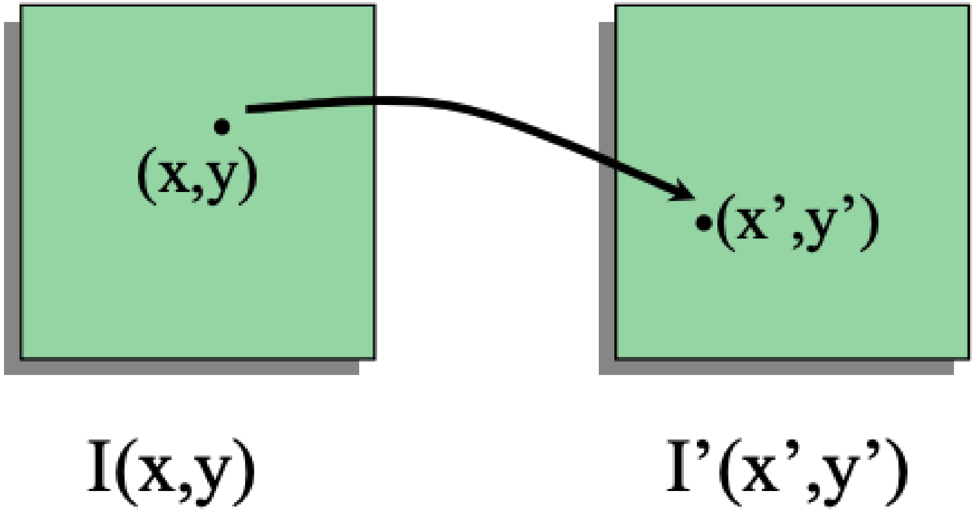
Example: Translation geometric operation moves value at (x,y) to $(x + d_x, y + d_y)$

https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

$$x \rightarrow f_x(x, y) = x'$$

$$y \rightarrow f_y(x, y) = y'$$

$$I(x, y) = I'\left(f_x(x, y), f_y(x, y)\right)$$

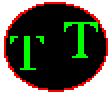


Common Geometric Operations

- **Scale** - change image content size

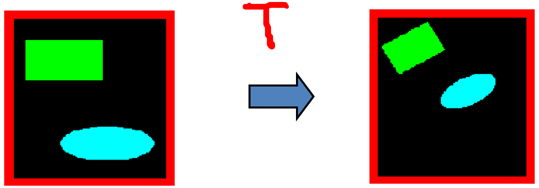
- **Rotate** - change image content orientation

- **Reflect** - flip over image contents



- **Translate** - change image content position

- **Affine Transformation**
  - general image content linear geometric transformation



Simple Mappings

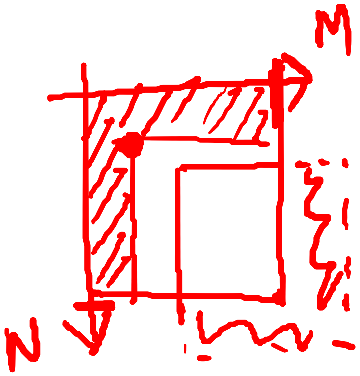
- **Translation:** (shift) by a vector (d_x, d_y)

$$T_x : x' = x + d_x \qquad T_y : y' = y + \underline{d_y} \quad \text{or} \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} d_x \\ d_y \end{pmatrix}$$

$$I2(x',y') \leftarrow I(x,y)$$

N

M



https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

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$$T_y : y' = y + d_y$$

$\alpha f(x)$

- **Scaling:** (contracting or stretching) along x or y axis by a factor s_x or s_y

$f(x)$

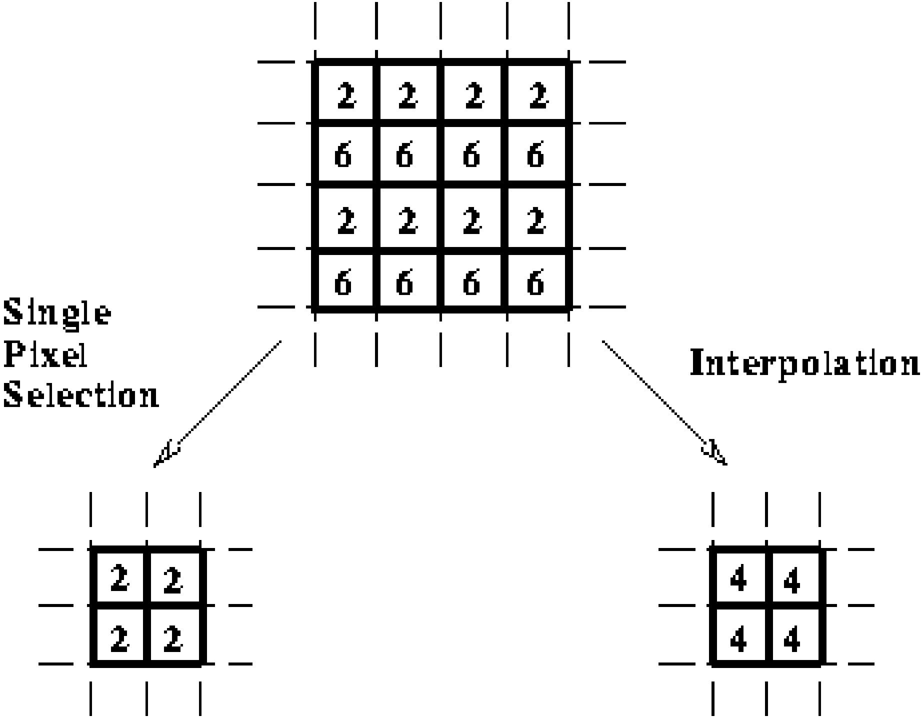
$g(x) = f(sx)$

$$T_x : x' = s_x \cdot x \qquad \text{or} \qquad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$
$$T_y : y' = s_y \cdot y$$

$x' = s_x x$

https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

Scaling (Shrink)

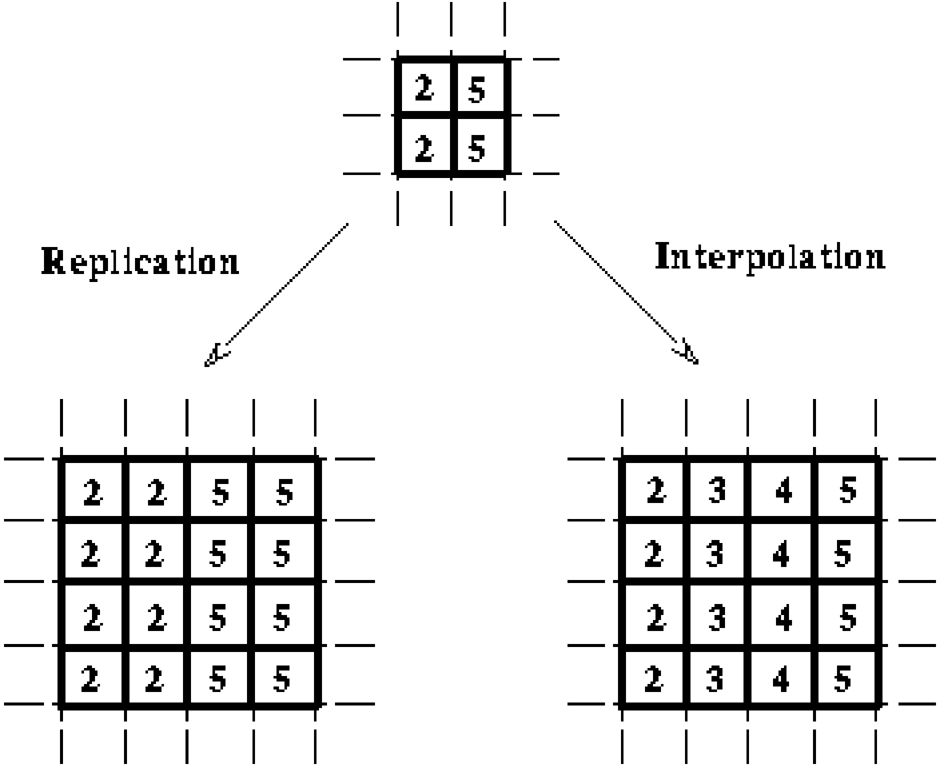


2	2	2	2
6	6	6	6
2	2	2	2
6	6	6	6

2	2
2	2

4	4
4	4

Scaling (Stretch)



2	5
2	5

Replication

2	2	5	5
2	2	5	5
2	2	5	5
2	2	5	5

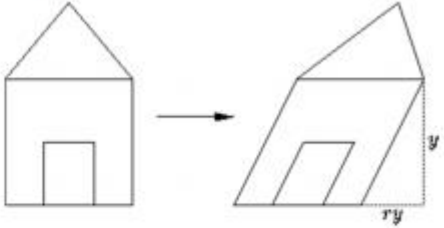
Interpolation

2	3	4	5
2	3	4	5
2	3	4	5
2	3	4	5

Simple Mappings

- **Shearing:** along x and y axis by factor b_x and b_y

$$T_x : x' = x + b_x \cdot y \qquad \text{or} \qquad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & b_x \\ b_y & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$
$$T_y : y' = y + b_y \cdot x$$



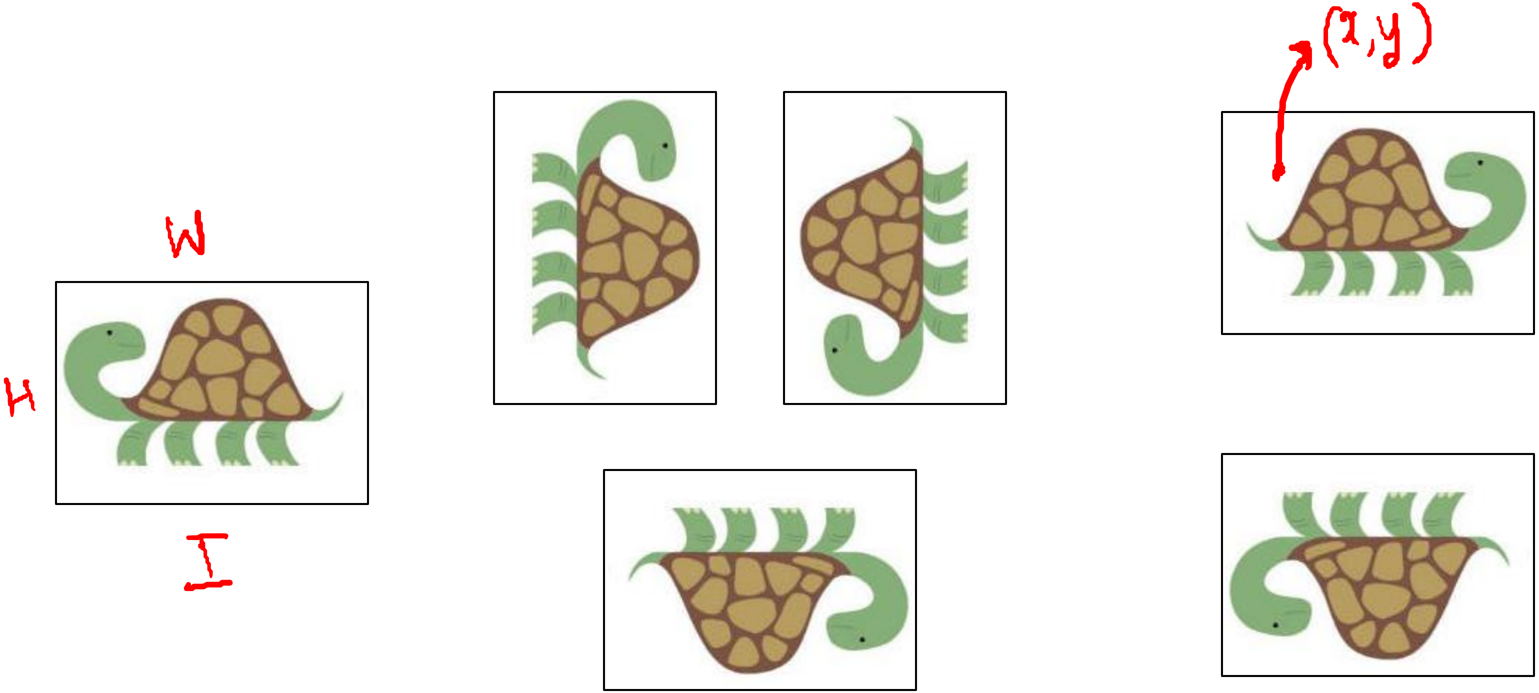
- **Rotation:** the image by an angle α

$$T_x : x' = x \cdot \underline{\cos\alpha} - y \cdot \underline{\sin\alpha}$$
$$T_y : y' = x \cdot \underline{\sin\alpha} + y \cdot \underline{\cos\alpha}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \cdot \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}$$

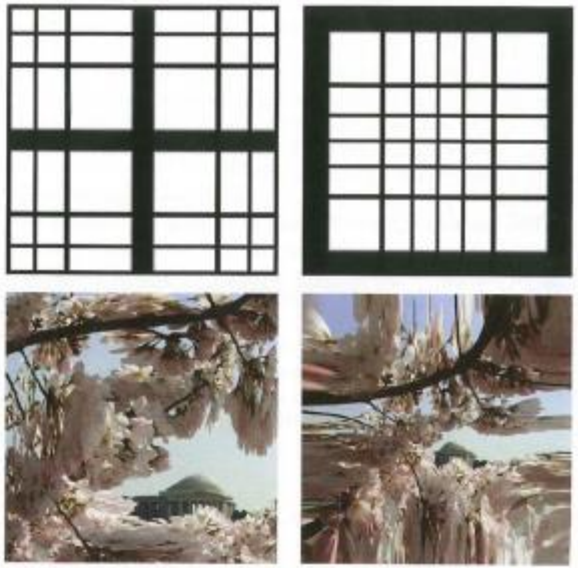
https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

90, 180 rotations , Flipping



- **Image warping:** we can use a function to select which pixel somewhere else in the image to look up
- For example: apply function on both texel coordinates (x, y)

$$x' = x + y * \sin(\pi * x)$$



https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

Homogeneous Coordinates

- Notation useful for converting scaling, translation, rotating into point-matrix multiplication
- To convert ordinary coordinates into homogeneous coordinates

$$\boldsymbol{x} = \underline{\begin{pmatrix} x \\ y \end{pmatrix}} \quad \text{converts to} \quad \hat{\boldsymbol{x}} = \underline{\begin{pmatrix} \hat{x} \\ \hat{y} \\ h \end{pmatrix}} = \begin{pmatrix} h\,x \\ h\,y \\ h \end{pmatrix}$$

$$\begin{matrix} x \\ y \end{matrix} \qquad \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

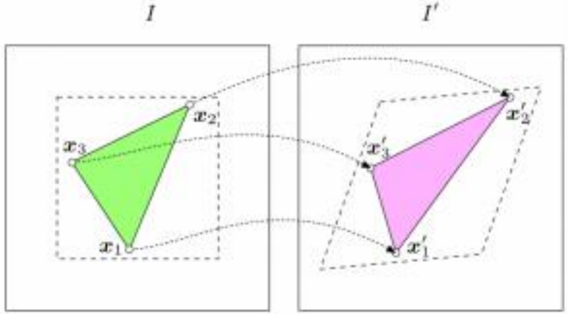
Affine (3-Point) Mapping

- Can use homogeneous coordinates to rewrite translation, rotation, scaling, etc as vector-matrix multiplication

$$\begin{pmatrix}x'\\y'\\1\end{pmatrix} = \begin{pmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}x\\y\\1\end{pmatrix}$$

$$\begin{matrix}x'\\y'\end{matrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}t_x\\t_y\end{bmatrix}$$

- **Affine mapping:** Can then derive values of matrix that achieve desired transformation (or combination of transformations)



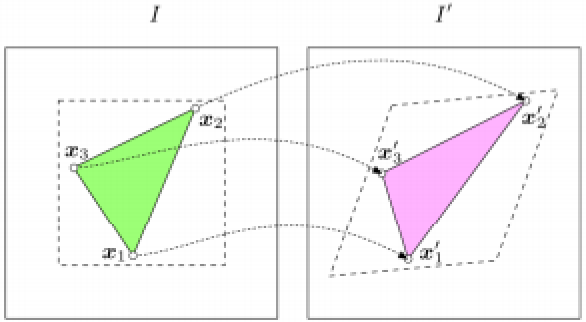
$$\underset{\substack{\uparrow\\ \text{rot}}}{T_1}\,\overbrace{\underset{\substack{\uparrow\\ \text{translation}}}{T_2}\,P}^{p'}$$

- Inverse of transform matrix is **inverse mapping**

https://web.cs.wpi.edu/~emmanuel/courses/cs545/S14/slides/lecture11.pdf

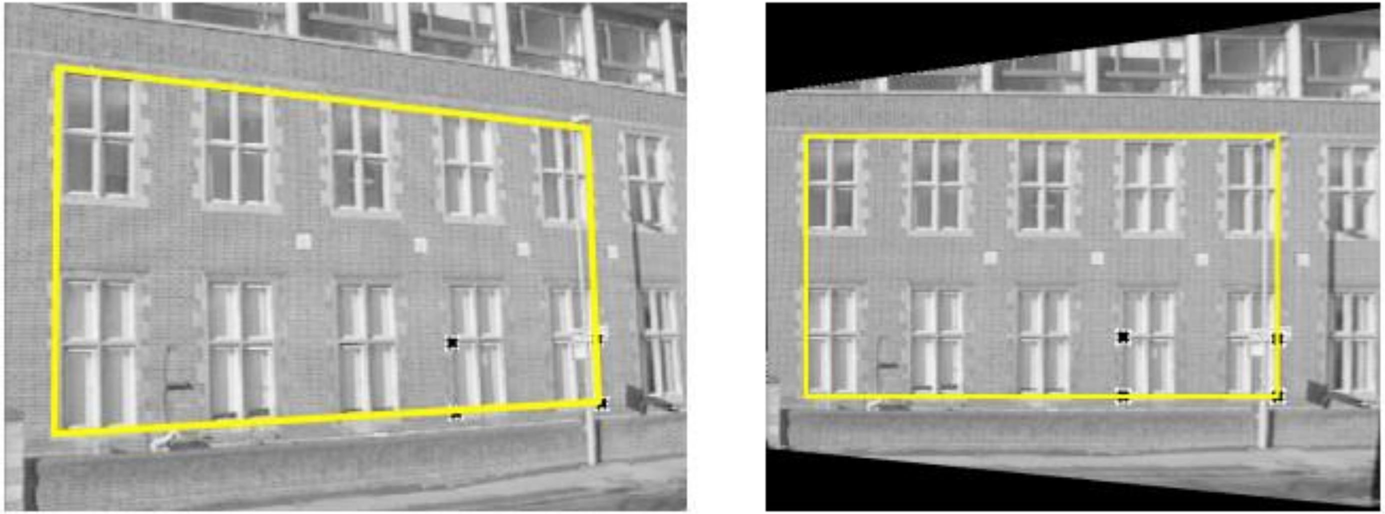
Affine (3-Point) Mapping

- What's so special about affine mapping?



- Maps
  - straight lines -> straight lines,
  - triangles -> triangles
  - rectangles -> parallelograms
  - Parallel lines -> parallel lines
- Distance ratio on lines do not change

Homography



from Hartley & Zisserman

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

References

- G&W, 3rd Ed., 9.1-9.3, 9.6