

25.10.2019

Digital Image Processing (CSE/ECE 478)

Lecture-19: Image Restoration

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Many slides borrowed from Vineet Gandhi @CVIT!

degraded images



ideal image



Blurred image

- What caused the image to blur?

- Camera: translation, shake, out-of-focus ...
- Environment: scattered and reflected light
- Device noise: CCD/CMOS sensor and circuitry
- Quantization noise

degraded images



ideal image



Blurred image

- What caused the image to blur?
 - Camera: translation, shake, out-of-focus ...
 - Environment: scattered and reflected light
 - Device noise: CCD/CMOS sensor and circuitry
 - Quantization noise

- Can we improve the image, or “undo” the effects?

Degradations



- original



- optical blur



- motion blur

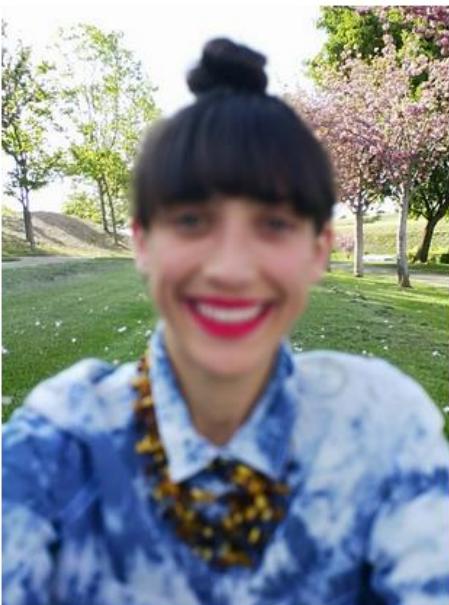


- spatial quantization (discrete pixels)



- additive intensity noise

Examples (Optical Blur)



Lens Blur selfie, background focus

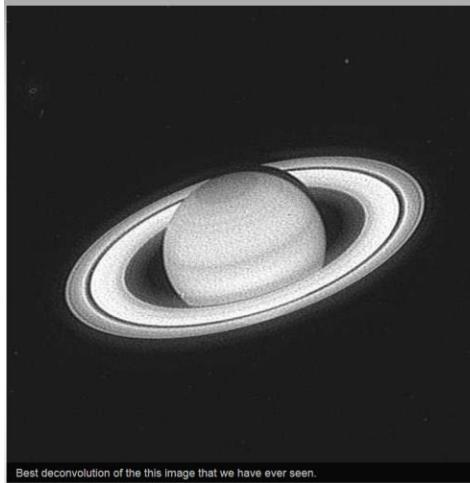
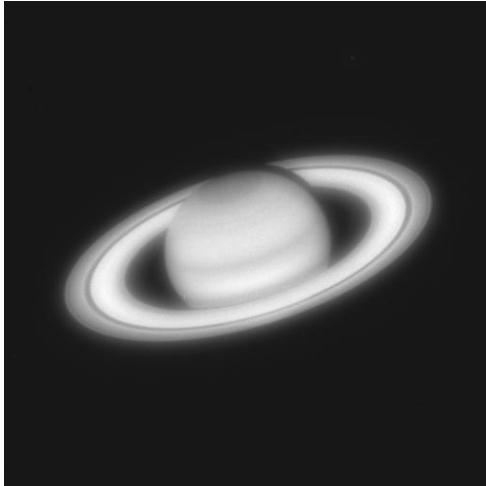


Lens Blur selfie, foreground focus

Photo by Rachel Been

Interesting read: Light Field Cameras

Examples



Best deconvolution of the this image that we have ever seen.

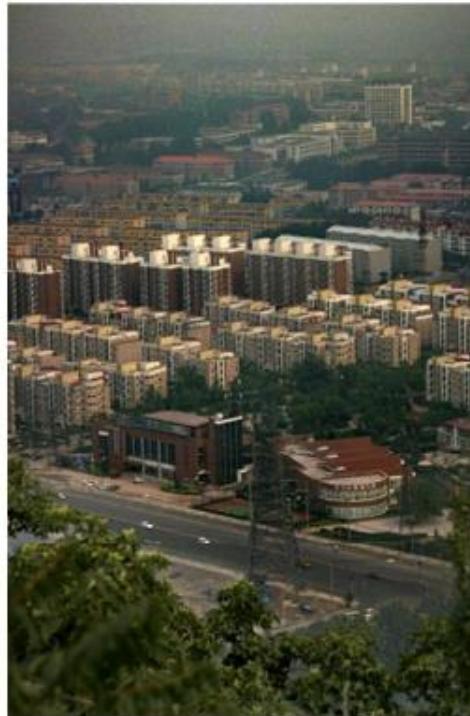
Courtesy: NASA

Examples (Restoration from camera shake)



Courtesy: Cho et al. ICCV 2007

Examples (Atmospheric conditions)

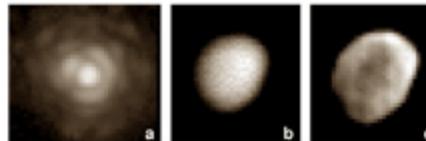


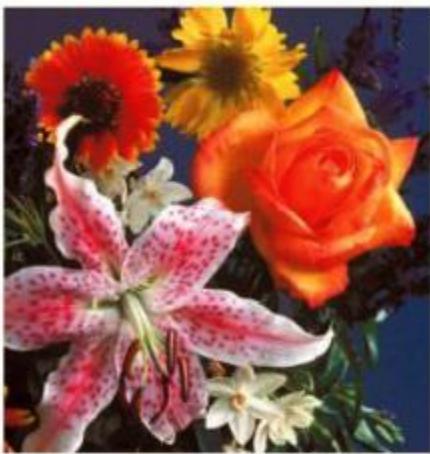
Single Image Haze Removal [He et al. CVPR 2009]

image restoration

- started from the 1950s
- application domains
 - Scientific explorations
 - Legal investigations
 - Film making and archival
 - Image and video (de-)coding
 - ...
 - Consumer photography
- related problem: image reconstruction in radio astronomy, radar imaging and tomography

Example of image restoration
Asteroid Vesta





Original image



Blurred image

- Image enhancement: “improve” an image subjectively.



Original image



Blurred image

- Image enhancement: “improve” an image subjectively.
- Image restoration: remove distortion from image in order to go back to the “original” → objective process.

a model for image distortion

- Image restoration

- Use a priori knowledge of the degradation
- Modeling the degradation and apply the inverse process
- Formulate and evaluate objective criteria of goodness

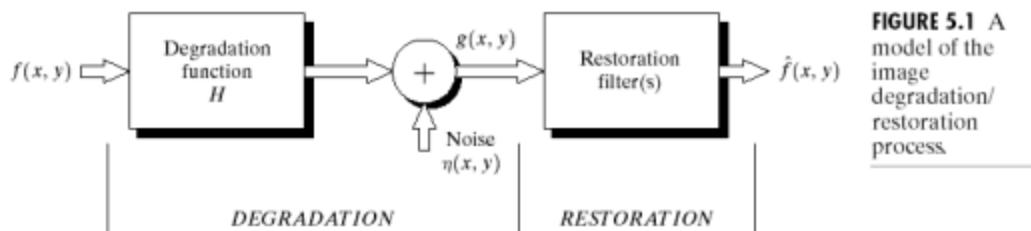
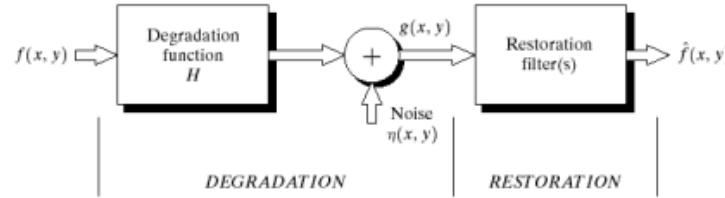


FIGURE 5.1 A model of the image degradation/ restoration process.

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

→ design restoration filters such that
 $\hat{f}(x, y)$ is as close to $f(x, y)$ as possible.



usual assumptions for the distortion model



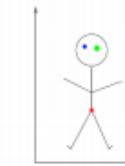
Noise

- Independent of spatial location
 - Exception: periodic noise ...
- Uncorrelated with image
- Degradation function H

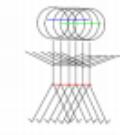
- Linear

- Position-invariant

Input Image

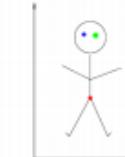


Blurred by Camera linear motion

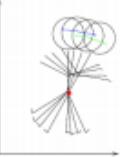


SPACE-INVARIANT RESPONSE - each point on image gives same response just shifted in position.

Input Image



Blurred by Camera Rotation



SPACE-VARIANT RESPONSE - each point on image gives a different response

Mathematical Model of Image Degradation/Restoration

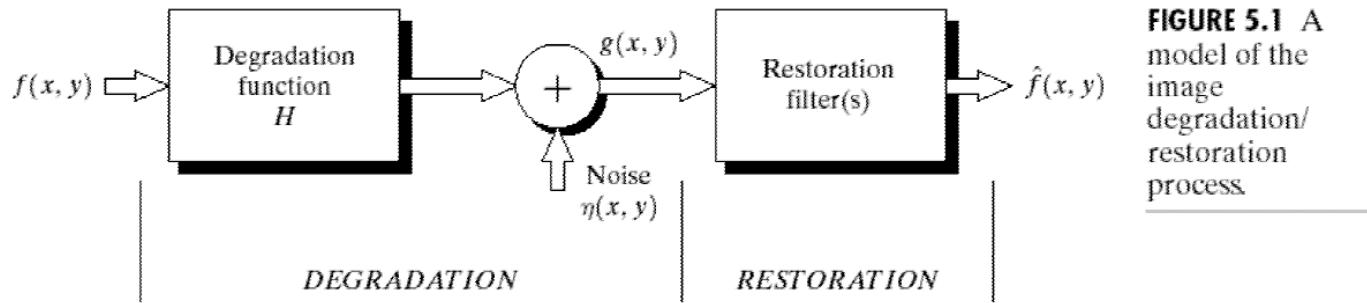


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

Mathematical Model of Image Degradation/Restoration

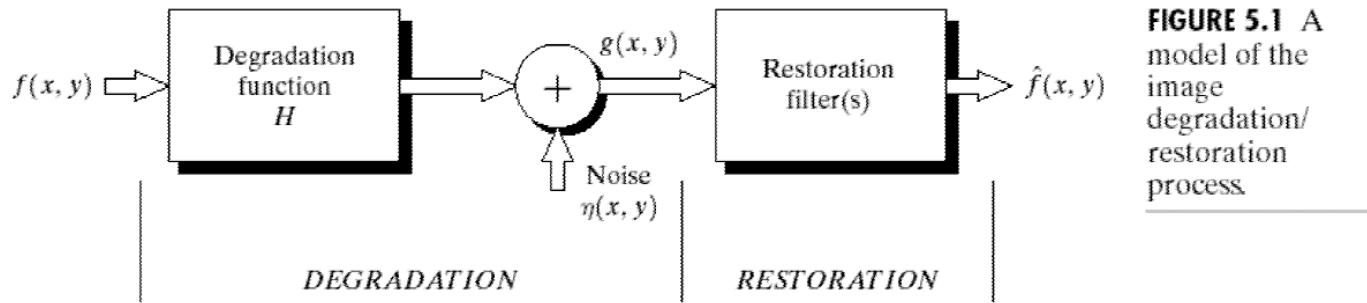


FIGURE 5.1 A model of the image degradation/restoration process.

If H is a linear, position-invariant process

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

Mathematical Model of Image Degradation/Restoration

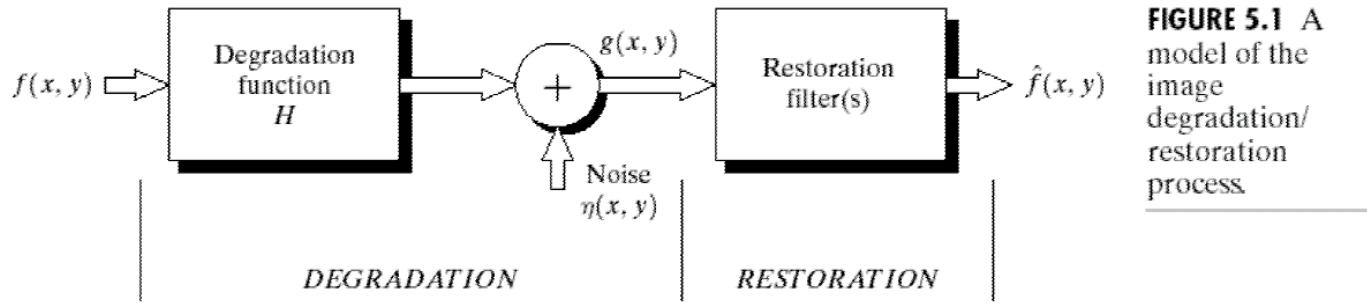


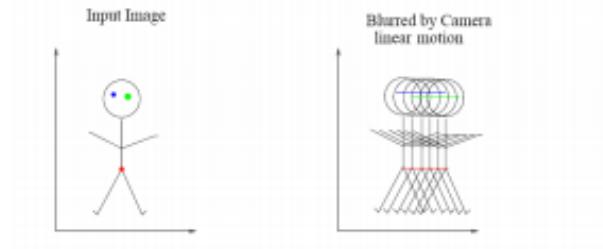
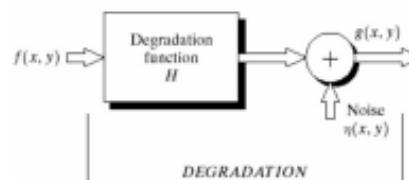
FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

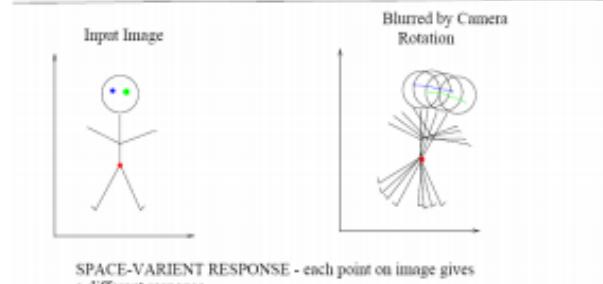
$$G(u, v) = H(u, v) F(u, v) + N(u, v)$$

usual assumptions for the distortion model

- Noise
 - Independent of spatial location
 - Exception: periodic noise ...
 - Uncorrelated with image
- Degradation function H
 - Linear
 - Position-invariant



SPACE-INVARIANT RESPONSE - each point on image gives same response just shifted in position.

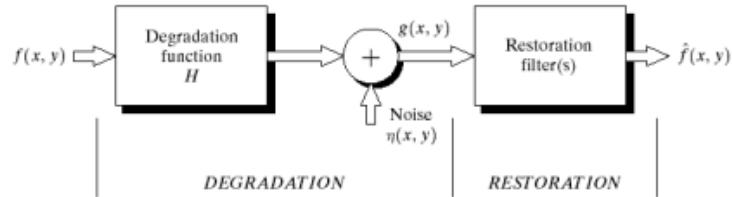


SPACE-VARIANT RESPONSE - each point on image gives a different response

divide-and-conquer step #1: image degraded only by noise.

Noise based Degradation

- Assuming H is identity, model reduces to:



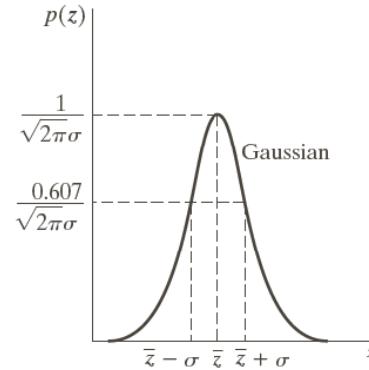
$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

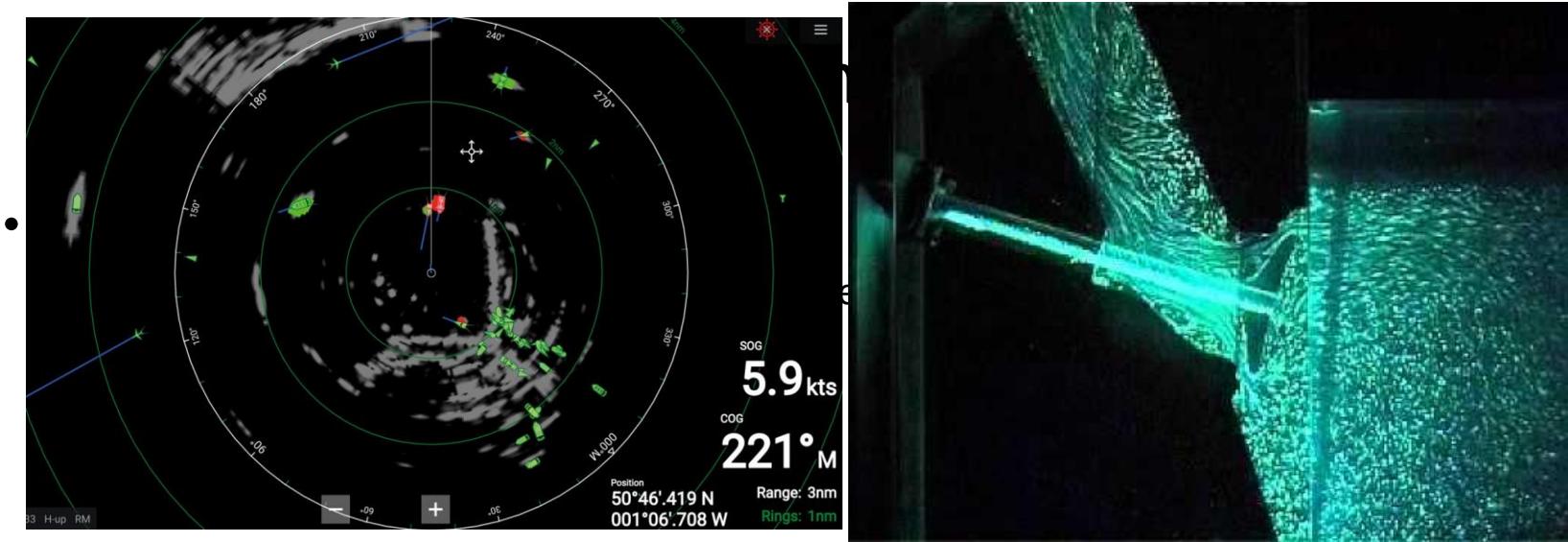
Noise Models

- Gaussian (normal) Noise
 - widely used due to mathematical convenience

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$



$$\begin{aligned} f(x,y) & \stackrel{\pi, y}{=} 0 \\ \gamma & \sim \text{Gaussian}(\bar{z}, \sigma) \\ g(x,y) & = f(x,y) + \gamma \end{aligned}$$

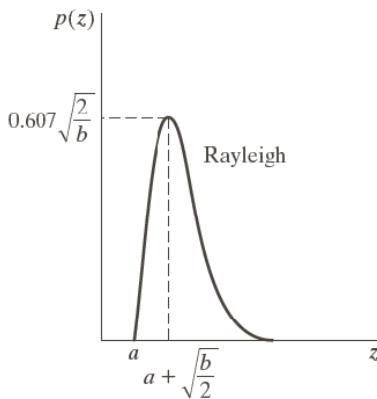


- Rayleigh Noise [Radar, Velocity images]

$$p(z) = \begin{cases} \frac{2}{b}(z - a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Mean: $\bar{z} = a + \sqrt{\pi b / 4}$ Variance: $\sigma^2 = \frac{b(4 - \pi)}{4}$

Useful for modelling skewed histograms

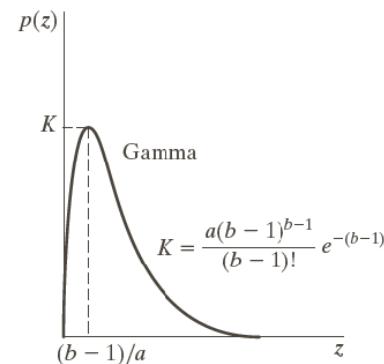


Noise Models

- Erlang (Gamma) Noise [Laser images]

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad \begin{matrix} a > 0 \\ b \text{ positive integer} \end{matrix}$$

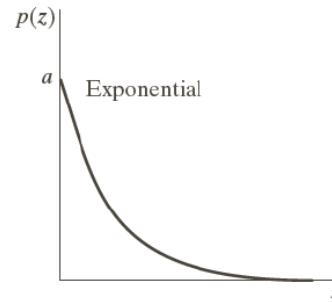
Mean: $\bar{z} = \frac{b}{a}$ Variance: $\sigma^2 = \frac{b}{a^2}$



- Exponential Noise

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$$a > 0$$



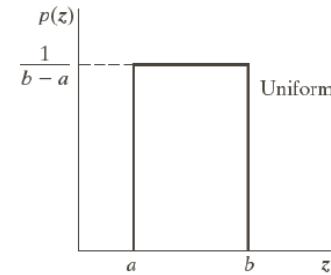
Noise Models

- Uniform Noise [quantization, most unbiased]

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\bar{z} = \frac{a+b}{2}$

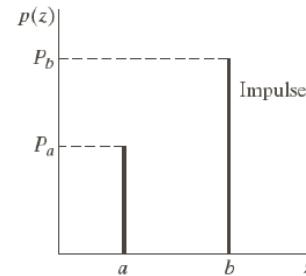
Variance: $\sigma^2 = \frac{(b-a)^2}{12}$



- Impulse (salt-and-pepper) Noise [sync errors in digitization or transmission, sensor malfunction]

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

$P_a = P_b \Rightarrow$ unipolar noise



Noise Models

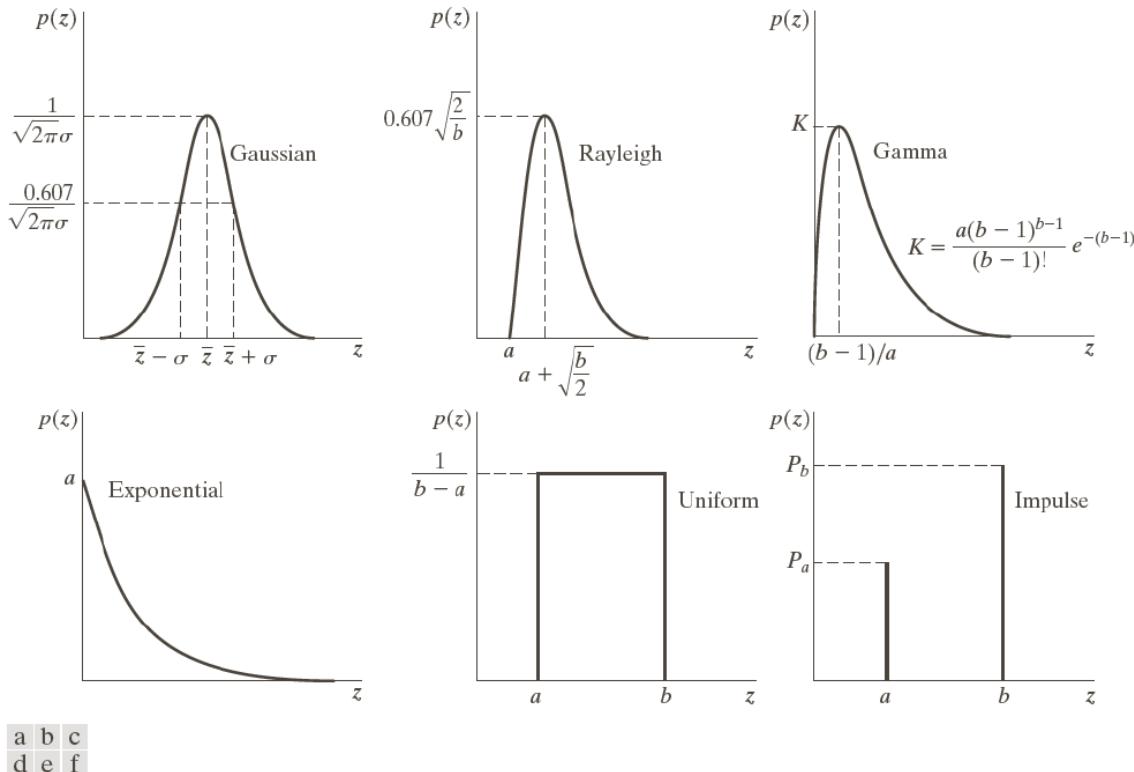


FIGURE 5.2 Some important probability density functions.

Illustration of Noise Models

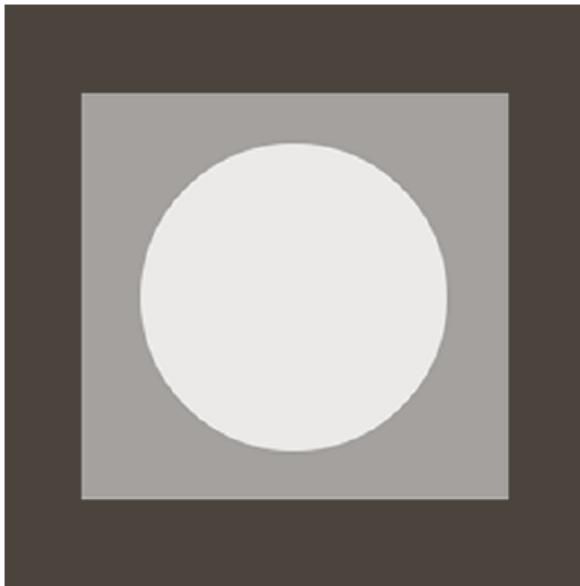


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Illustration of Noise Models

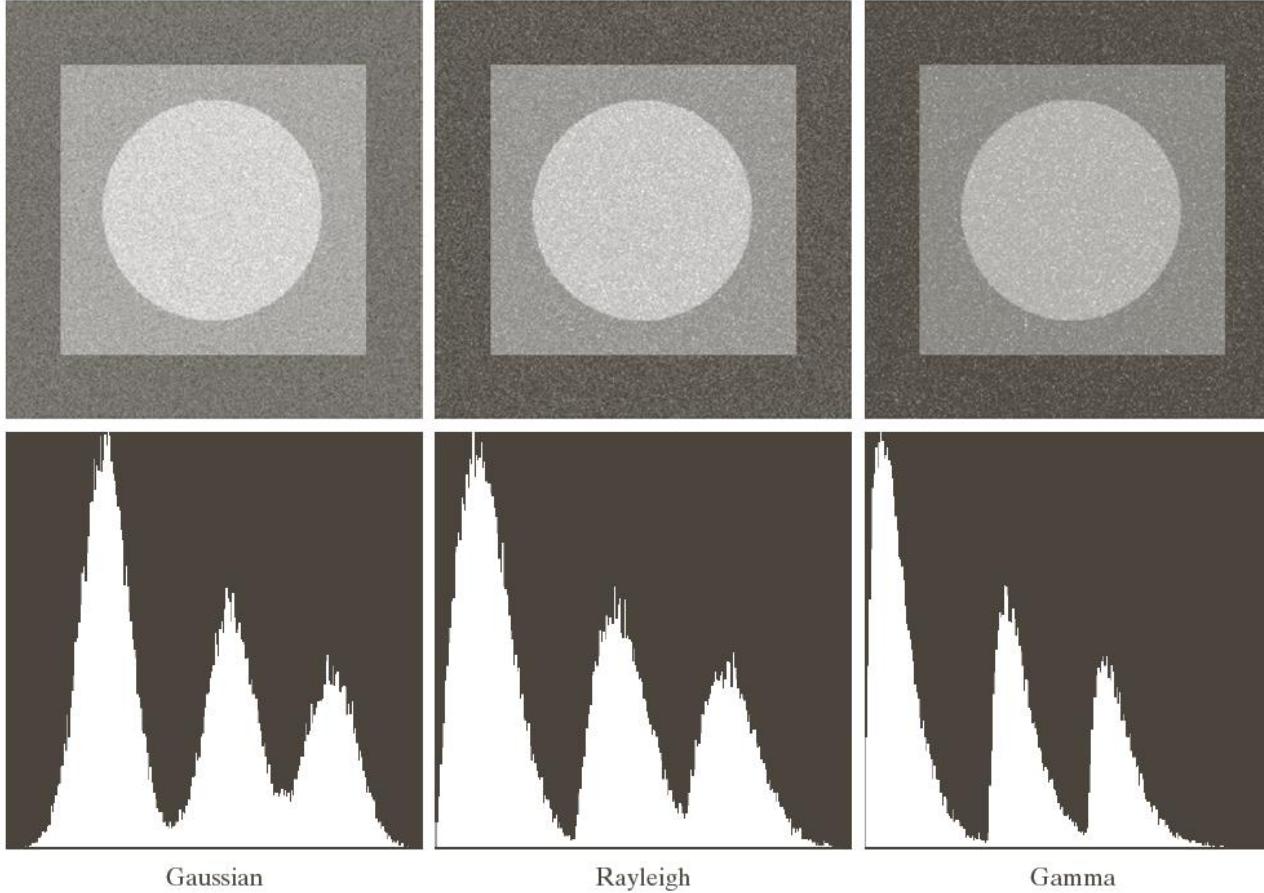


Illustration of Noise Models

Visually similar.
Not easy to determine
noise model from
appearance

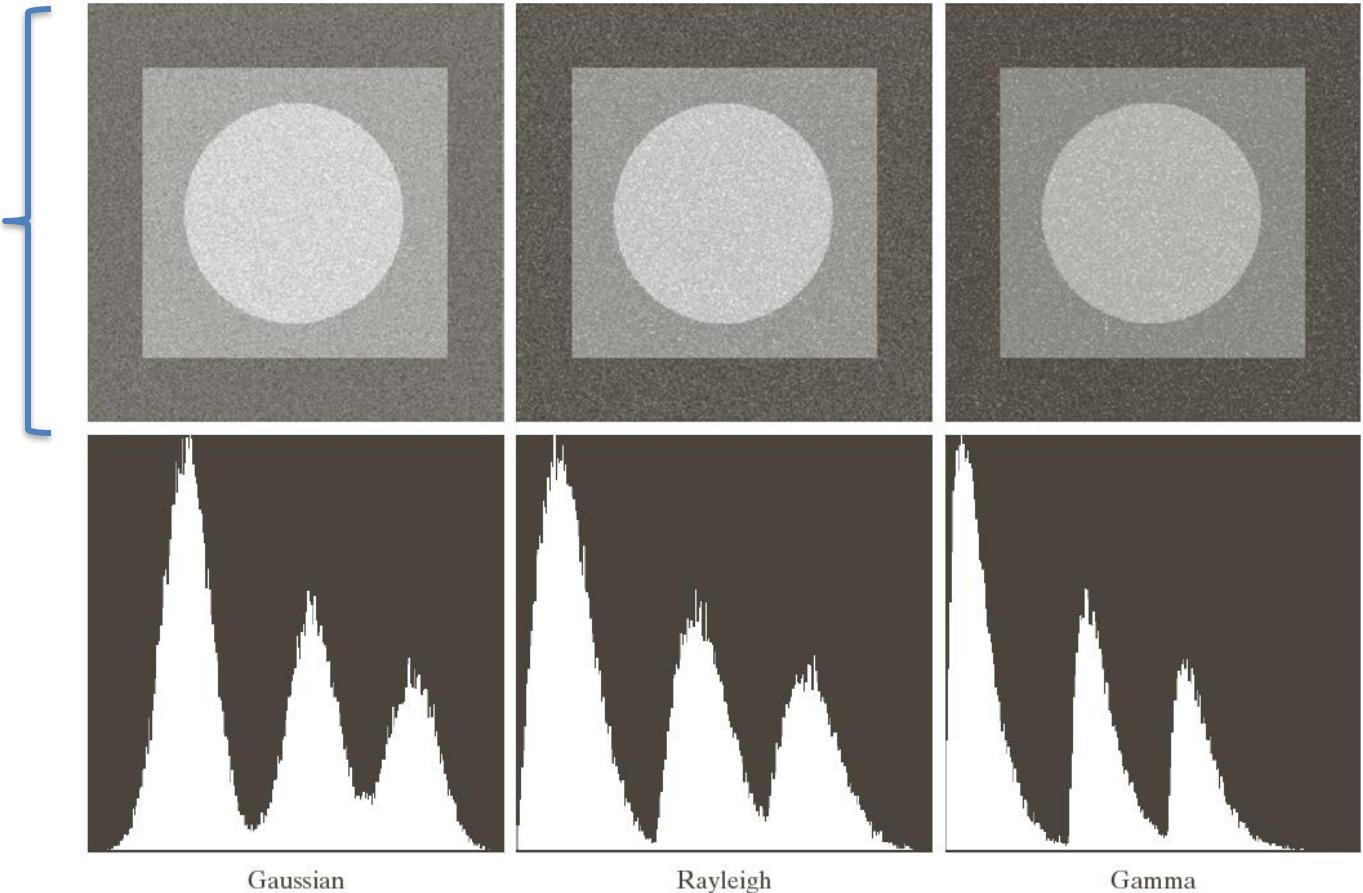
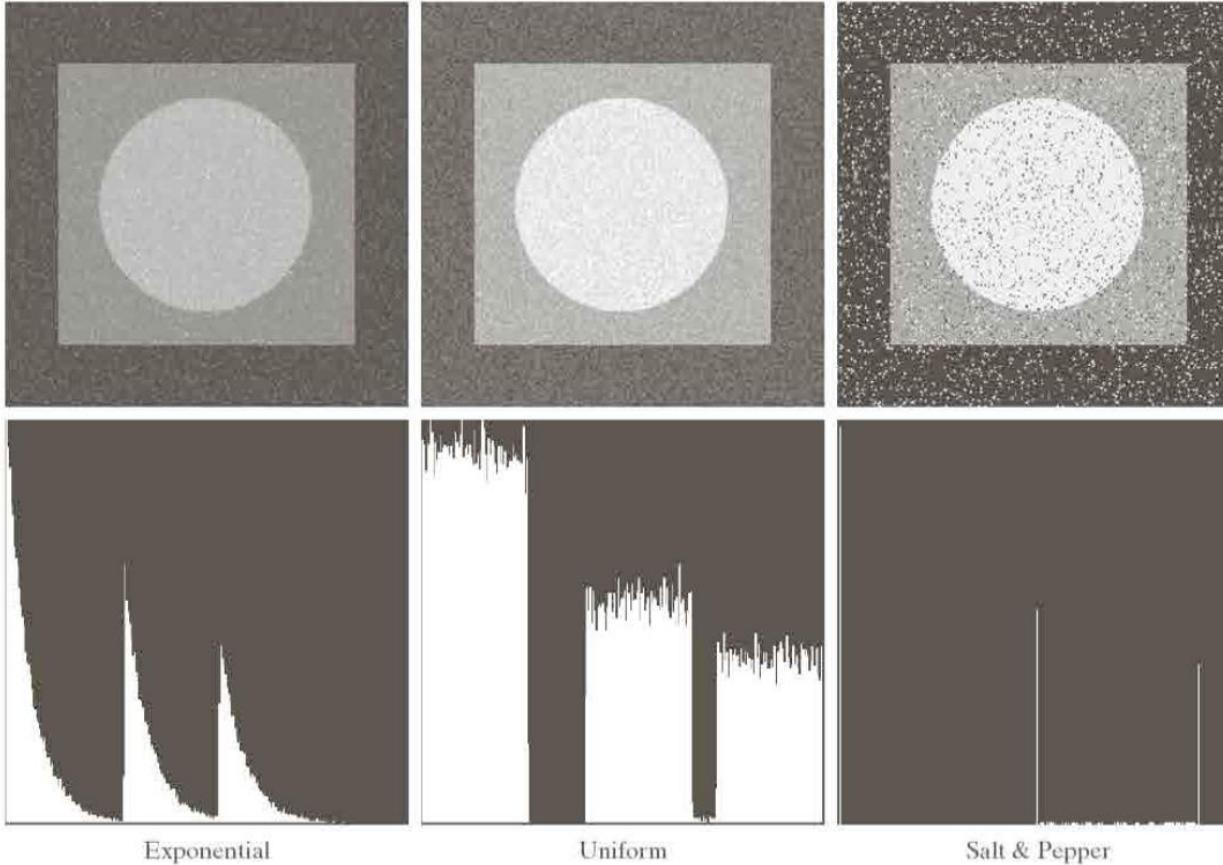


Illustration of Noise Models



How to study system noise

- Imaging system available
 - Noise Calibration: Capture a set of ‘flat environments’ (e.g. solid gray board, object at fixed location)
- Only images available
 - Estimate from patches of constant intensity

Estimating the noise parameters

- Consider a strip from the image

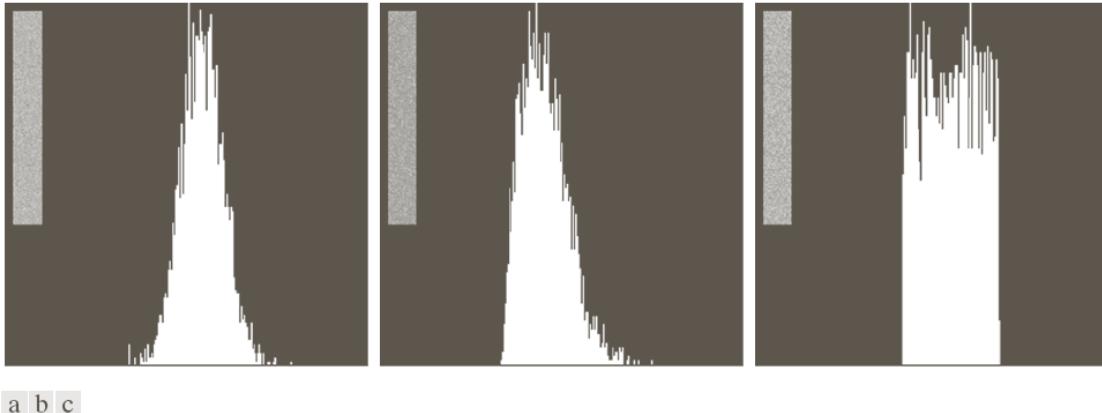
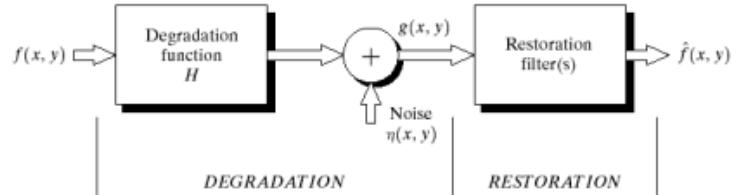


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in presence of noise only

- Assuming H is identity, model reduces to:



$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

Restoration (in presence of noise only)

- mean filters

Arithmetic mean filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

Geometric mean filter

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$



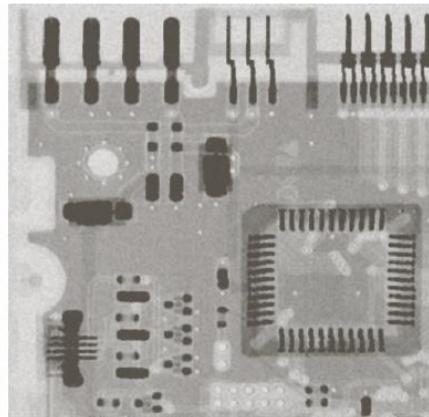
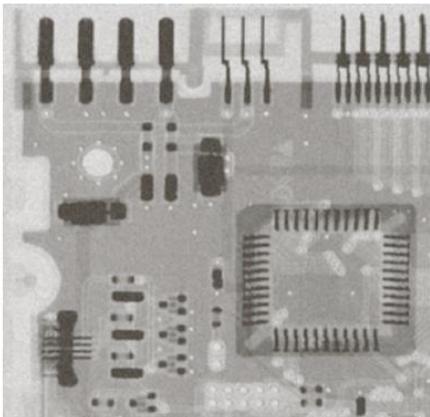
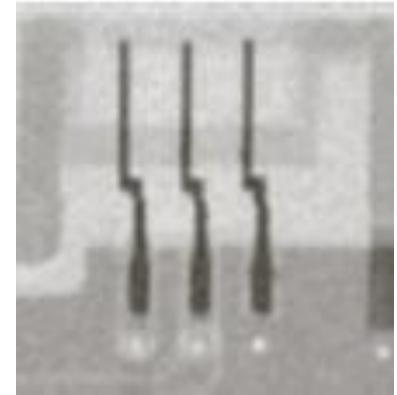
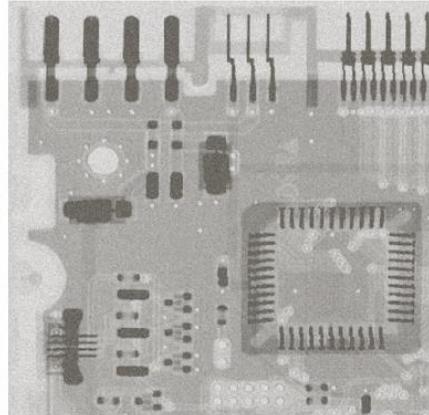
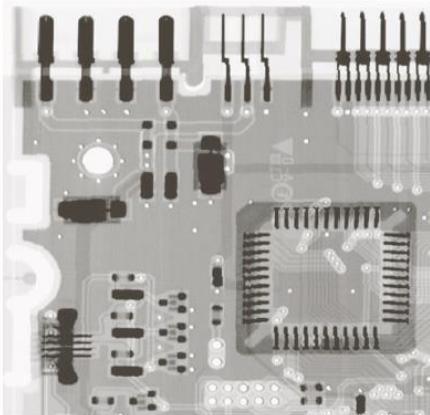
Restoration (in presence of noise only)

a
b
c
d

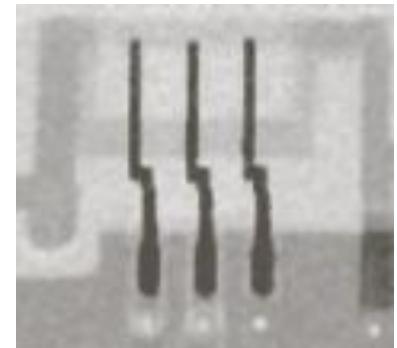
FIGURE 5.7

(a) X-ray image.
(b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3 . (d) Result of filtering with a geometric mean filter of the same size.

(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



?



Restoration (in presence of noise only)

- mean filters

Harmonic mean filter $\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$

Works well for salt noise or Gaussian noise, but fails for pepper noise

Contraharmonic mean filter $\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$

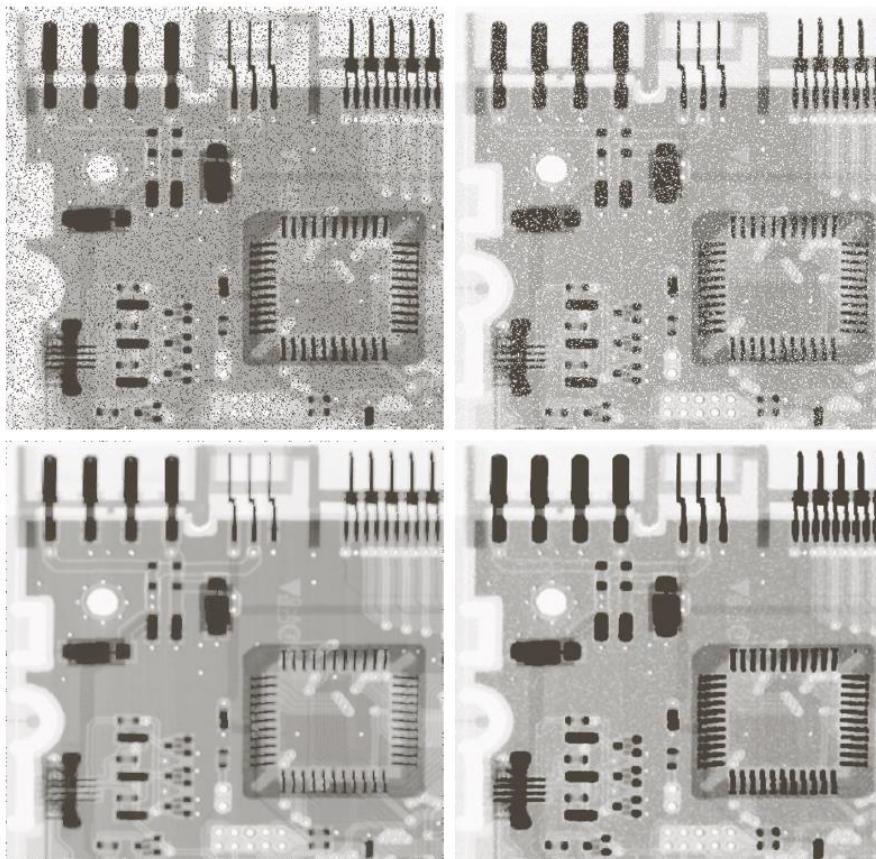
Q = *order* of the filter

Good for salt-and-pepper noise.

Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$

NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$

Restoration (in presence of noise only)



a
b
c
d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Restoration (in presence of noise only)

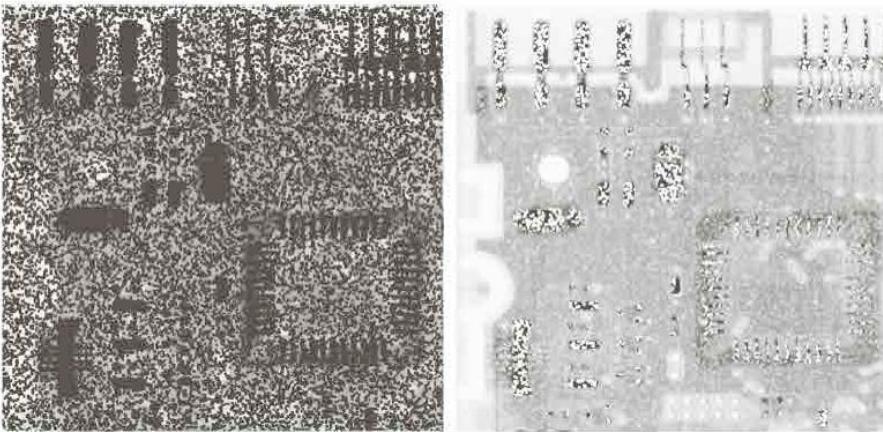
a b

FIGURE 5.9

Results of selecting the wrong sign in contraharmonic filtering.

(a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$.

(b) Result of filtering 5.8(b) with $Q = 1.5$.



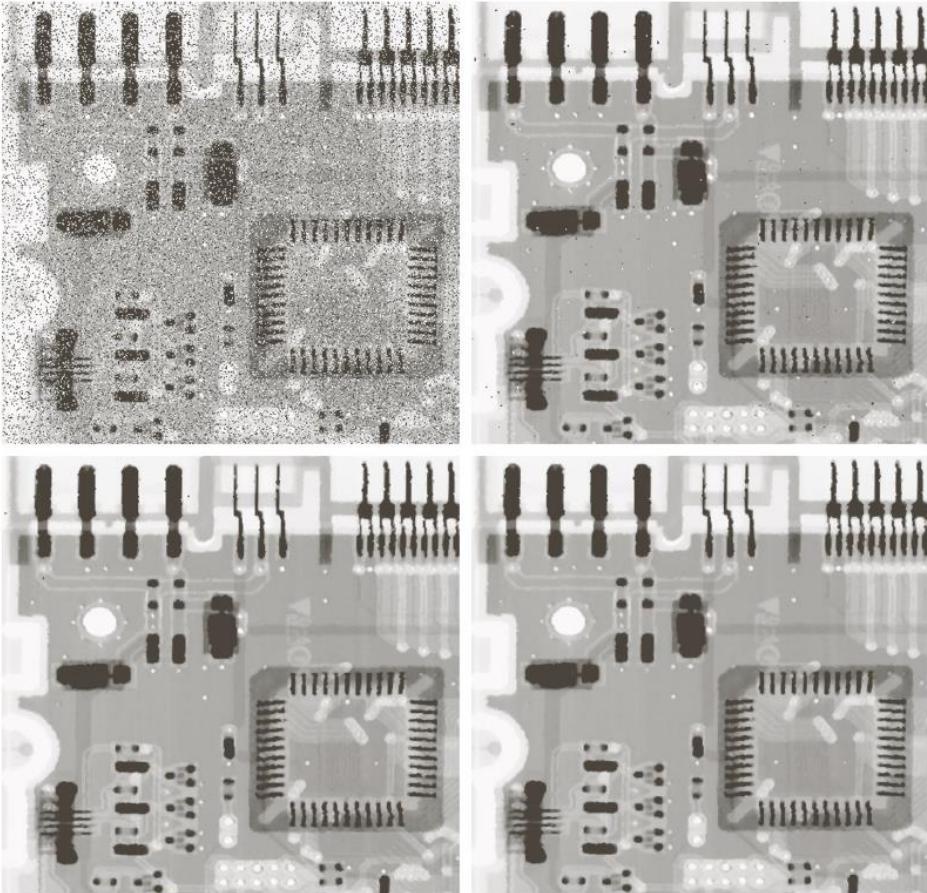
Restoration (in presence of noise only)

- Median filter

a
b
c
d

FIGURE 5.10

- (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3 .
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



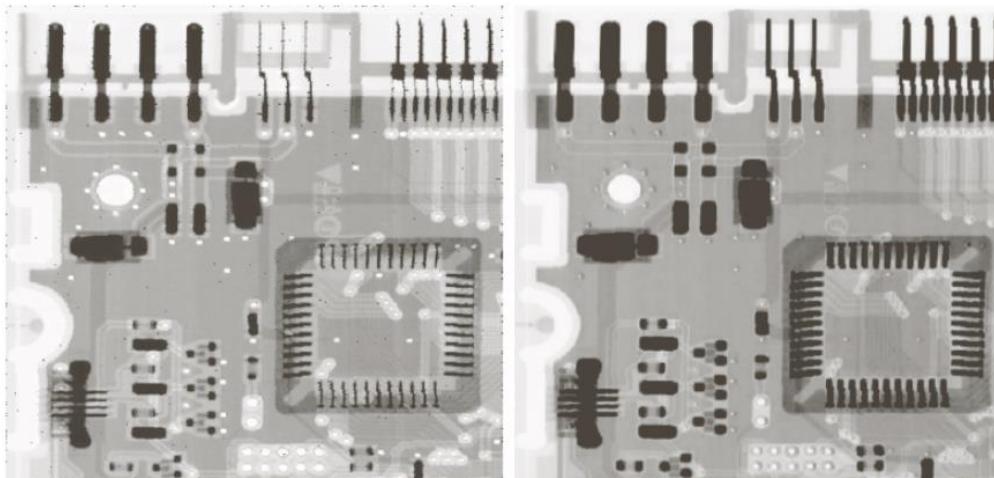
Restoration (in presence of noise only)

- Max, Min filters

a b

FIGURE 5.11

(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3 . (b) Result of filtering 5.8(b) with a min filter of the same size.



Max filter

Min filter

Restoration (in presence of noise only)

- Midpoint filter

$$\hat{f}(x, y) = \frac{1}{2} \left[\max\{g(s, t)\}_{(s, t) \in S_{xy}} + \min\{g(s, t)\}_{(s, t) \in S_{xy}} \right]$$

Best for
Uniform
or
Gaussian
noise

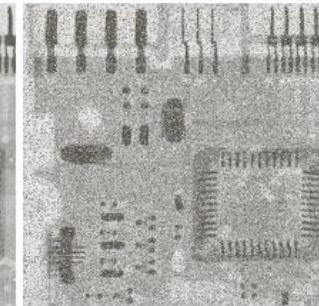
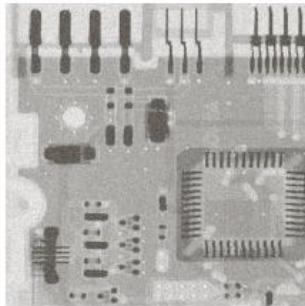
- Alpha trimmed filter

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s, t) \in S_{xy}} g_r(s, t)$$

Where g_r represents the image g in which the $d/2$ lowest and $d/2$ highest intensity values in the neighbourhood S_{xy} were deleted

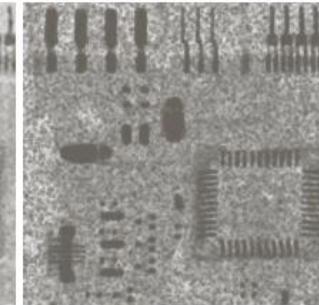
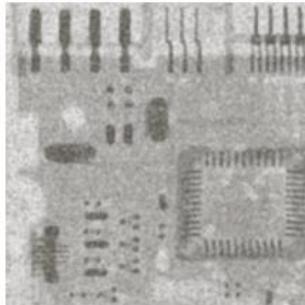
Restoration (in presence of noise only)

original



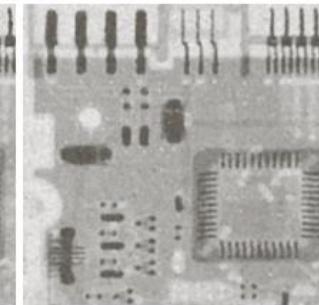
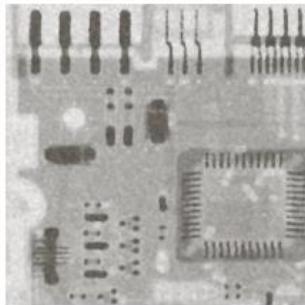
Original + salt and
pepper noise

Arithmetic mean filter



Geometric mean filter

Median filter



Alpha Trimmed filter

Adaptive, Local Noise Reduction Filter

Given the corrupted image $g(x, y)$, find $f(x, y)$.

General expression:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_n^2}{\sigma_L^2} [g(x, y) - m_L]$$

σ_L^2 - Local variance of the local region

m_L - Local Mean

σ_n^2 - Variance of overall noise

$g(x, y)$ - Pixel value at the position (x, y)

Adaptive, Local Noise Reduction Filter

Given the corrupted image $g(x, y)$, find $f(x, y)$.

Conditions:

- (a) σ_{η}^2 is zero (Zero-noise case)
 - Simply return the value of $g(x, y)$.

General expression:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_{\eta}^2}{\sigma_L^2} [g(x, y) - m_L]$$

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Adaptive, Local Noise Reduction Filter

Given the corrupted image $g(x, y)$, find $f(x, y)$.

Conditions:

- (a) σ^2_η is zero (Zero-noise case)
 - Simply return the value of $g(x, y)$.
- (b) If σ^2_L is higher than σ^2_η
 - Could be edge and should be preserved.

General expression:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2} [g(x, y) - m_L]$$

σ_L^2 - Local variance of the local region

m_L - Local Mean

σ_η^2 - Variance of overall noise

$g(x, y)$ - Pixel value at the position (x, y)

Adaptive, Local Noise Reduction Filter

Given the corrupted image $g(x, y)$, find $f(x, y)$.

Conditions:

- (a) σ_η^2 is zero (Zero-noise case)
 - Simply return the value of $g(x, y)$.
- (b) If σ_L^2 is higher than σ_η^2
 - Could be edge and should be preserved.
 - Return value close to $g(x, y)$.
- (c) If $\sigma_L^2 = \sigma_\eta^2$
 - when the local area has similar properties with the overall image.
 - Return arithmetic mean value of the pixels in S_{xy} .

General expression:

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2} [g(x, y) - m_L]$$



Assumed < 1

σ_L^2 - Local variance of the local region

m_L - Local Mean

σ_η^2 - Variance of overall noise

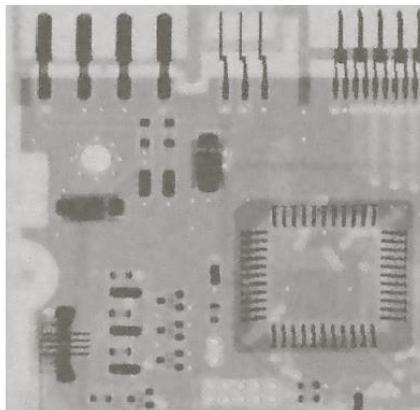
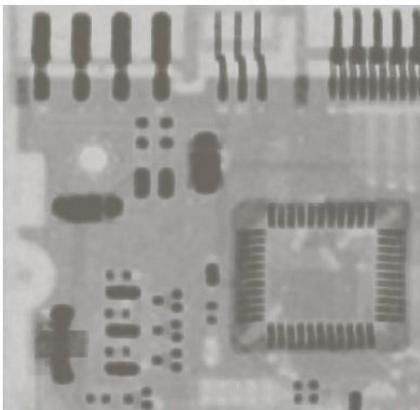
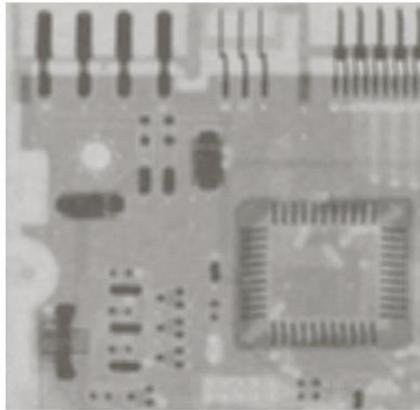
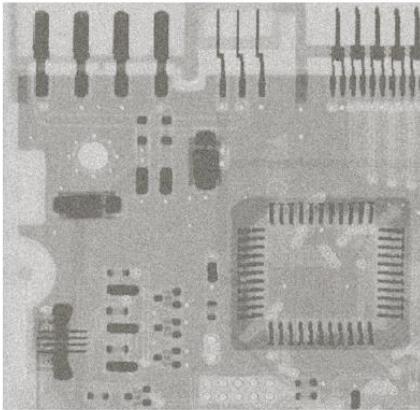
$g(x, y)$ - Pixel value at the position (x, y)

Adaptive mean filtering

a
b
c
d

FIGURE 5.13

- (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7 .



Restoration (in presence of noise only)

- Band pass/reject

a
b
c
d

FIGURE 5.16
(a) Image corrupted by sinusoidal noise.
(b) Spectrum of (a).
(c) Butterworth bandreject filter (white represents 1).
(d) Result of filtering.
(Original image courtesy of NASA.)

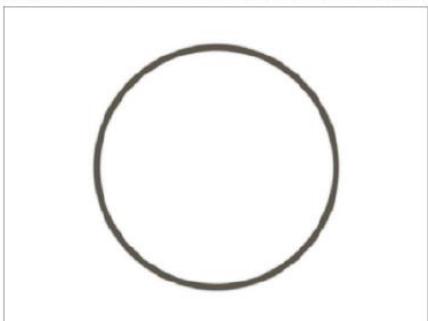
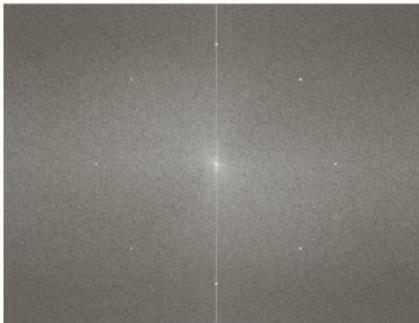
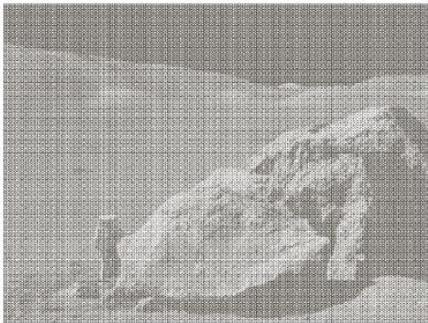
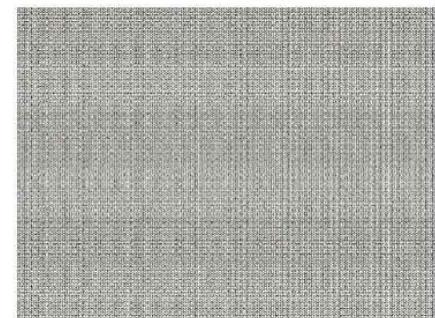


FIGURE 5.17
Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Very difficult to get result of this quality via spatial domain filtering using small convolutional masks

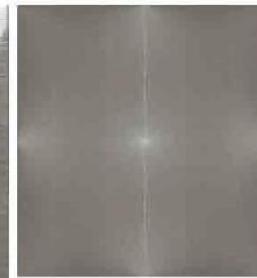
Restoration (in presence of noise only)

- Notch pass/reject

Degraded image



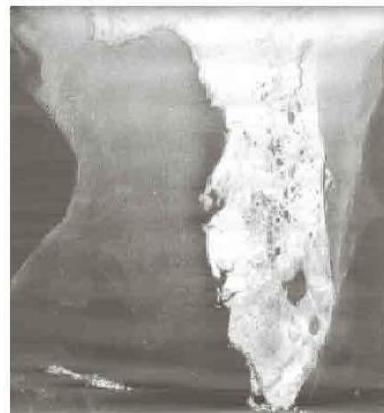
spectrum



a
b
c
d

FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines.
(b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering.
(Original image courtesy of NOAA.)

Filtered image



Notch pass filter



Spatial noise pattern

Estimation of degradation function

- Three main ways:
 - Observation → look, find, iterate
 - Experimentation → important idea for calibration
 - Mathematical modeling

a b

FIGURE 5.24
Degradation
estimation by
impulse
characterization.
(a) An impulse of
light (shown
magnified).
(b) Imaged
(degraded)
impulse.



Motion Blur

- Exposure
- If amount of light hitting the sensor changes significantly over exposure period → Motion Blur
- Causes (one or more of)
 - Camera motion
 - Subject motion
 - (Drastic) change in Lighting condition



Motion blur effect

```
#define filterWidth 9
#define filterHeight 9

double filter[filterHeight][filterWidth] =
{
    1, 0, 0, 0, 0, 0, 0, 0, 0,
    0, 1, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 1, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 1, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 1, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 1, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 1, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 1, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 1,
};

double factor = 1.0 / 9.0;
double bias = 0.0;
```



Estimation by Modeling (uniform motion blurring)



$$g(x, y) = \int_0^T f [x - x_0(t), y - y_0(t)] dt$$

$$G(u, v) = F(u, v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

Estimation by Modeling (uniform motion blurring)

$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

putting, $x_0(t) = at/T$ and $y_0(t) = bt/T$

$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin [\pi(ua + vb)] e^{-j\pi(ua + vb)}$$



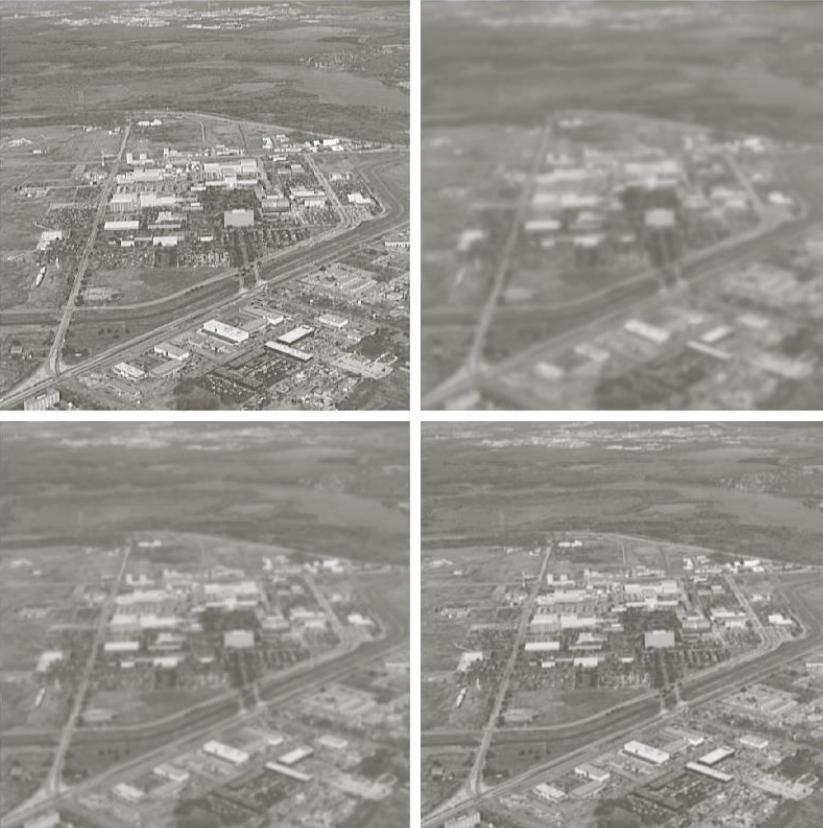
a b

FIGURE 5.26
(a) Original image.
(b) Result of
blurring using the
function in Eq.
(5.6-11) with
 $a = b = 0.1$ and
 $T = 1$.

Estimation by Modeling (atmospheric turbulence)

a
b
c
d

FIGURE 5.25
Illustration of the atmospheric turbulence model.
(a) Negligible turbulence.
(b) Severe turbulence,
 $k = 0.0025$.
(c) Mild turbulence,
 $k = 0.001$.
(d) Low turbulence,
 $k = 0.00025$.
(Original image courtesy of NASA.)



Degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence:

$$H(u, v) = e^{-k(u^2 + v^2)^{5/6}}$$

Model of Image Degradation/Restoration

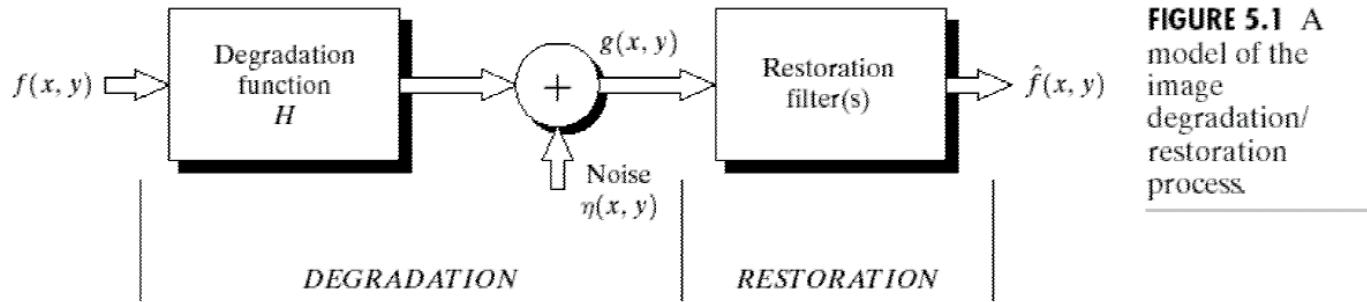


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v) F(u, v) + N(u, v)$$

Recovering image (in presence of both Noise and degradation)

- Even if we know the degradation function we cannot recover the undegraded image!!

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v) \Rightarrow \hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

Two problems:

1. $N(u, v)$ is a random function whose fourier transform is not known
2. If degradation has zero or small values $\rightarrow N(u, v)/H(u, v)$ will dominate

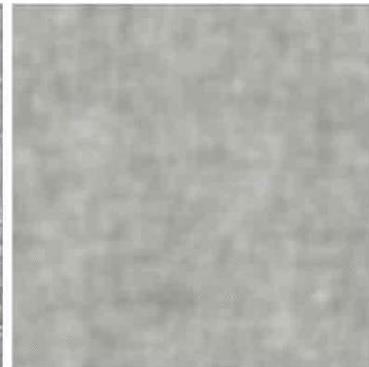
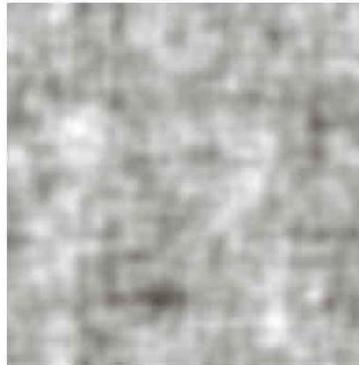
Recovering image (in presence of both Noise and degradation)



Degraded Image
(with known model)

a b
c d

FIGURE 5.27
Restoring
Fig. 5.25(b) with
Eq. (5.7-1).
(a) Result of
using the full
filter. (b) Result
with H cut off
outside a radius of
40; (c) outside a
radius of 70; and
(d) outside a
radius of 85.



No explicit provision for handling noise!

Weiner filter

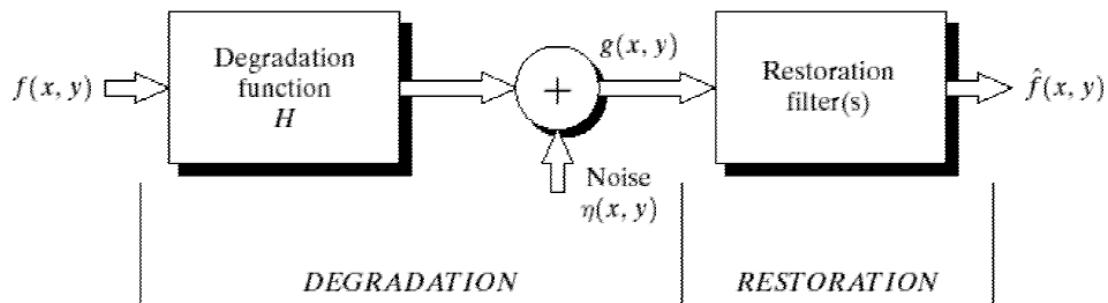


FIGURE 5.1 A model of the image degradation/restoration process

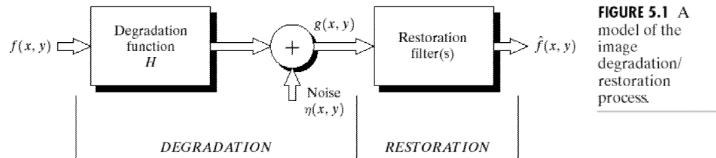
Consider image and noise as random variables

$$e^2 = E\{(f - \hat{f})^2\}$$

Assumption:

- Noise and image are uncorrelated

Weiner filter



$$e^2 = E\{(f - \hat{f})^2\}$$

The minimum of the error function e is given by:

$$H^*(u, v)$$

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)} \right] G(u, v)$$

$$S_\eta(u, v) = |N(u, v)|^2 = \text{Power spectrum of the noise (autocorrelation of noise)}$$

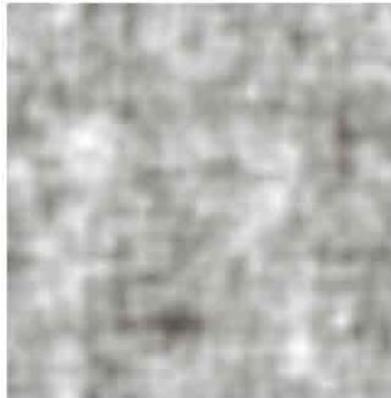
$$S_f(u, v) = |F(u, v)|^2 = \text{Power spectrum of the undegraded image}$$

Weiner filter

- When two spectrums are not known or cannot be estimated, the equation is approximated as:

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$

Weiner filter



Weiner filter

blur + add. noise



Inverse filtering



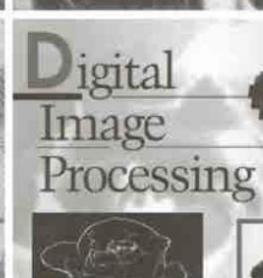
Wiener filtering



Reduced noise variance

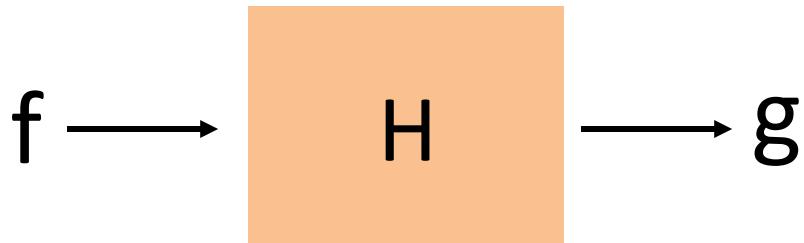


Reduced noise variance



a	b	c
d	e	f
g	h	i

Image Restoration



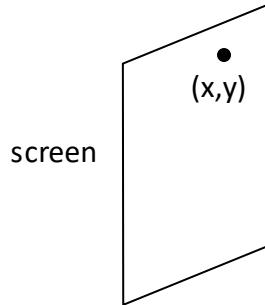
Inverse
problems



Known	Problem type
H, g	Recovery
g	Blind recovery
g, H partially	Semi blind recovery
f, g	System identification

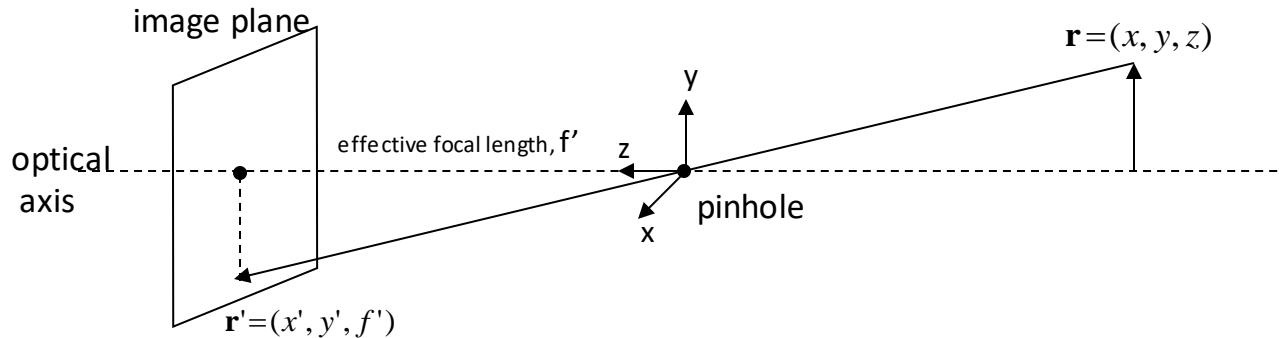
Geometric Distortion

Pinhole and the Perspective Projection



Is an image being formed on the screen?

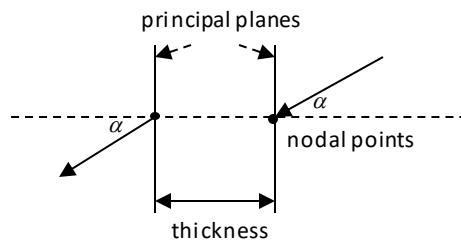
YES! But, not a “clear” one.



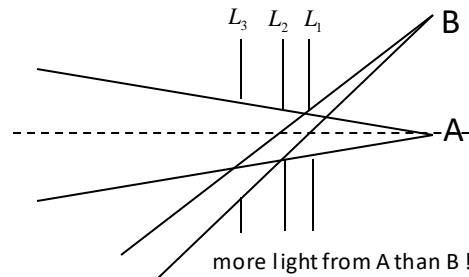
$$\frac{\mathbf{r}'}{f'} = \frac{\mathbf{r}}{z} \quad \Rightarrow \quad \frac{x'}{f'} = \frac{x}{z} \quad \frac{y'}{f'} = \frac{y}{z}$$

Common Lens Related Issues - Summary

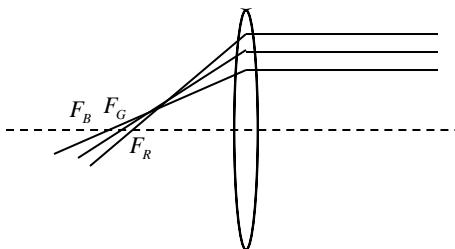
Compound (Thick) Lens



Vignetting

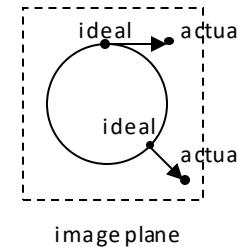


Chromatic Abberation



Lens has different refractive indices
for different wavelengths.

Radial and Tangential Distortion



Lens Glare



- Stray interreflections of light within the optical lens system.
- Happens when very bright sources are present in the scene.

Vignetting

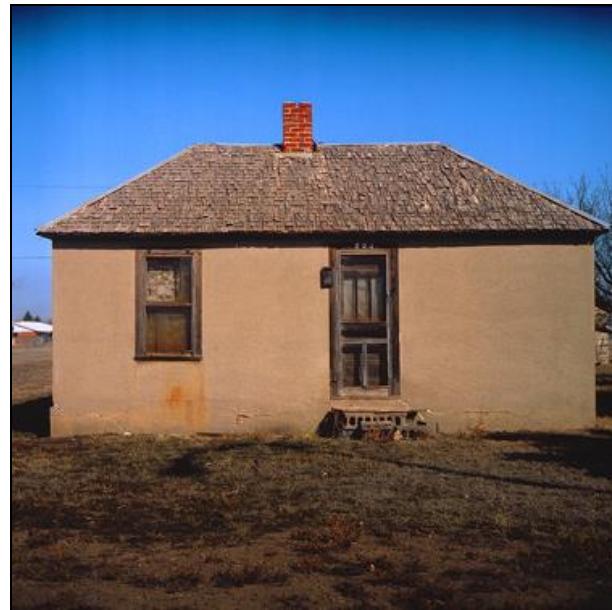
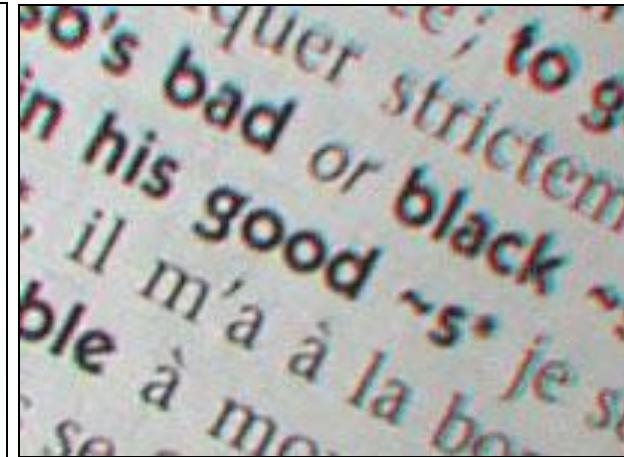


photo by Robert Johnes

Chromatic Aberrations

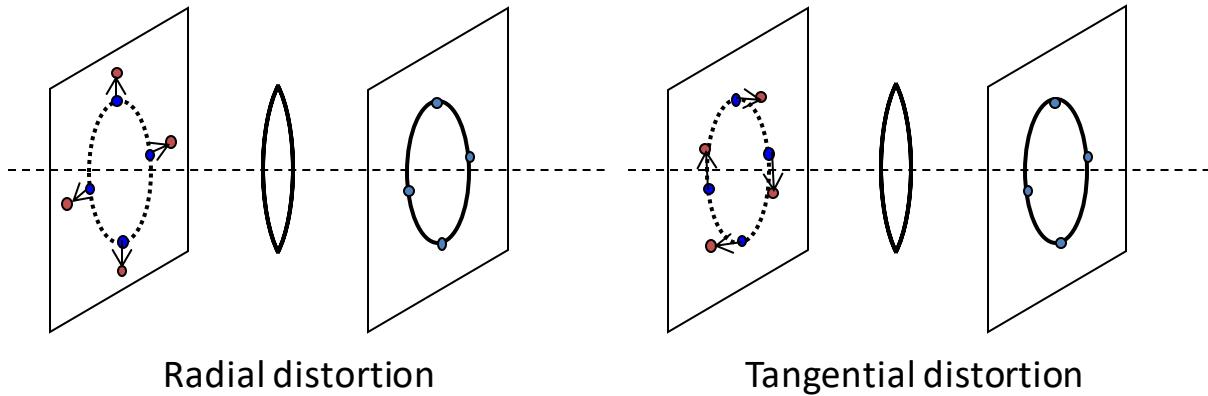


longitudinal chromatic aberration
(axial)



transverse chromatic aberration
(lateral)

Geometric Lens Distortions



Radial distortion

Tangential distortion



Photo by Helmut Dersch

Both due to lens imperfection
Rectify with geometric camera calibration

who said distortion is a bad thing?



blur ...

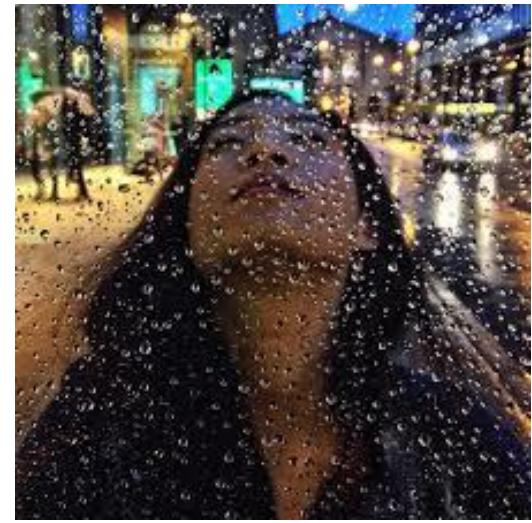


noise ...

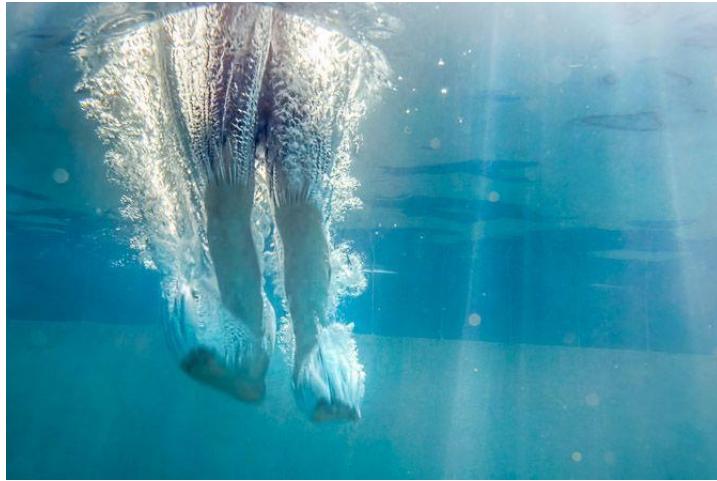


geometric ...

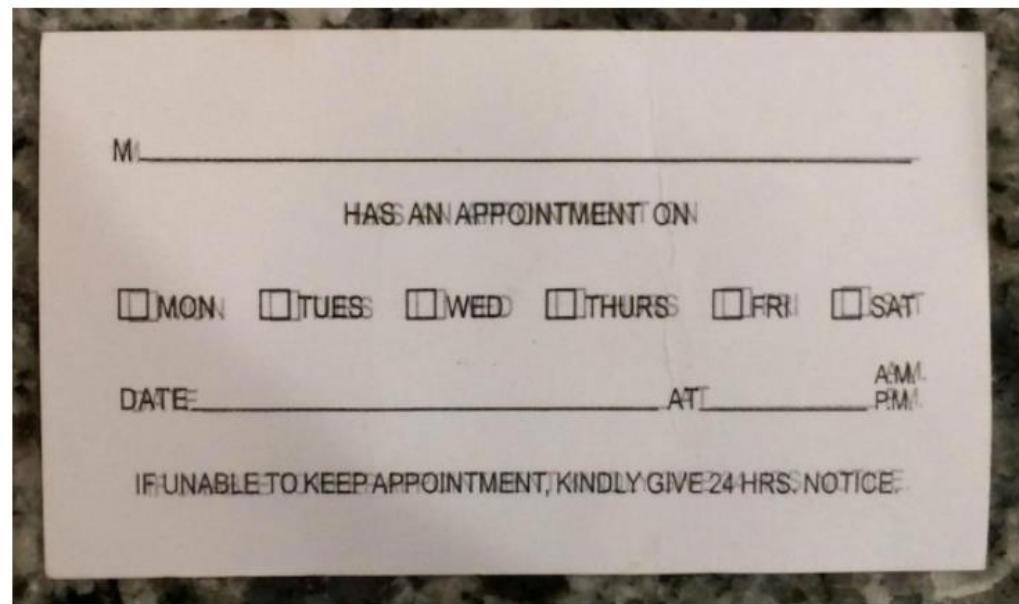
© Declan McCullagh Photography, mccullagh.org







An eye doctor business card with a printing fail...







DNsqNJaXkAADito.jpg



References

- <http://www.robots.ox.ac.uk/~az/lectures/ia/lect3.pdf>
- https://www.ece.iastate.edu/~namrata/EE528_Spring07/ImageRestoration1.pdf
- <http://www.ee.columbia.edu/~xlx/ee4830/notes/lec7.pdf>
- DIP Ch. 5 (G&W)