13.08.19

Digital Image Processing (CSE/ECE 478)

Lecture 5 : Using Histogram Statistics for Image Enhancement, Introduction to Spatial Filters

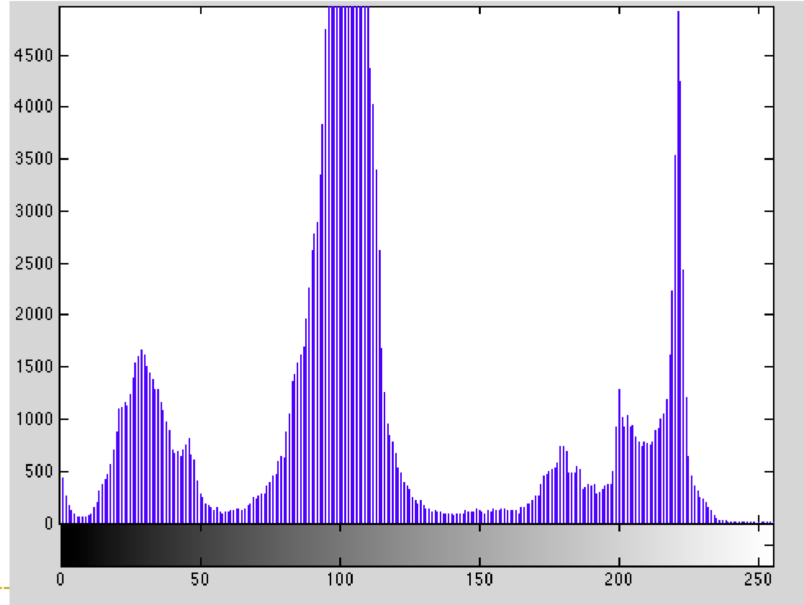
Ravi Kiran

Histogram

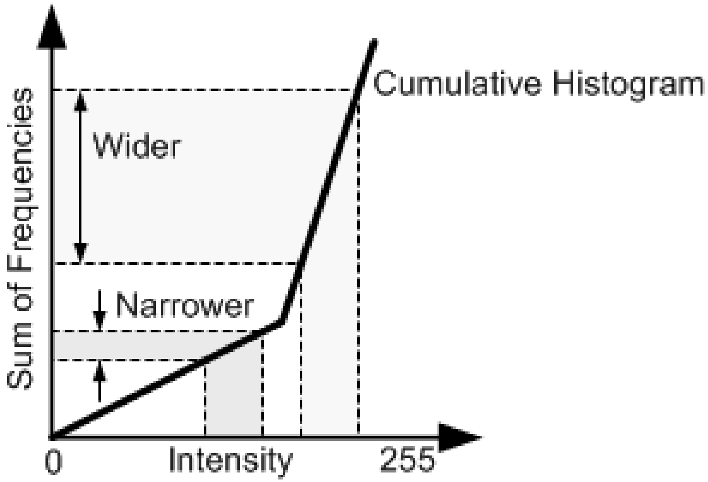
$$h_r(i) = n_i$$

i → intensity value, range [0 L-1]

n_i → number of pixels with intensity i



Histogram Equalization

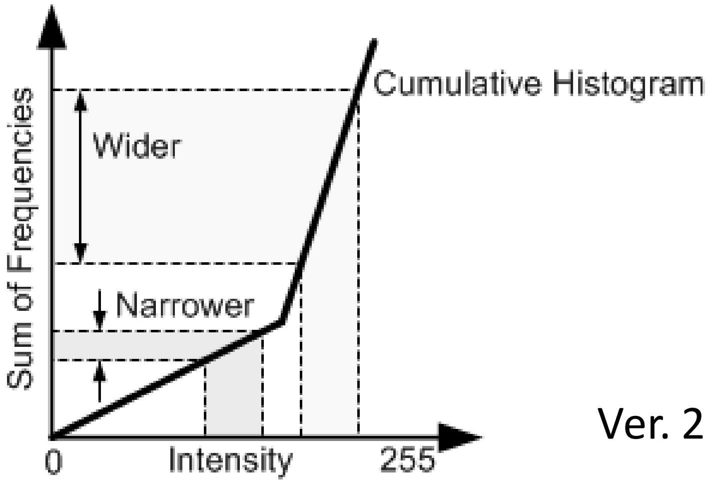


$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$s_k = T(r_k) = \texttt{round}\left((L-1)\sum_{j=0}^{j=k} p_r(r_j)\right)$$

Histogram Equalization

$$h[i] = \text{constant}, \qquad 0 \leq i \leq L-1$$



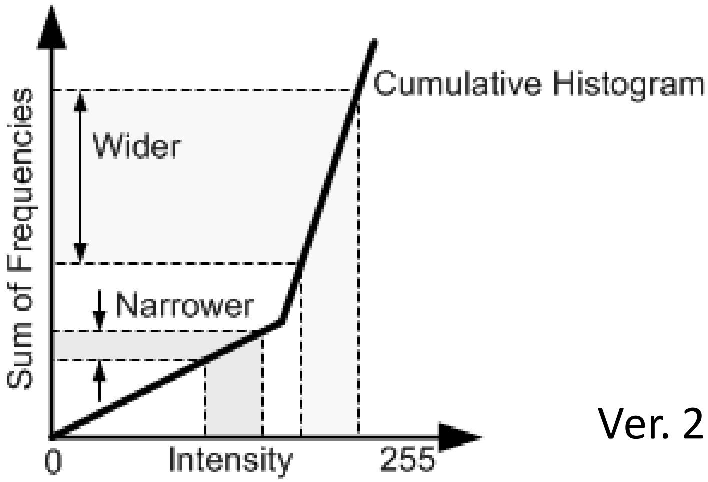
$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$s_k = T(r_k) = \texttt{round}\left((L-1)\sum_{j=0}^{j=k} p_r(r_j)\right)$$

Ver. 2 $$s_k = T(r_k) = \texttt{round}\left((L-1) * \frac{cdf(r_k) - cdf_{min}}{1 - cdf_{min}}\right)$$

Histogram Equalization

$$h[i] = \text{constant}, \qquad 0 \leq i \leq L-1$$



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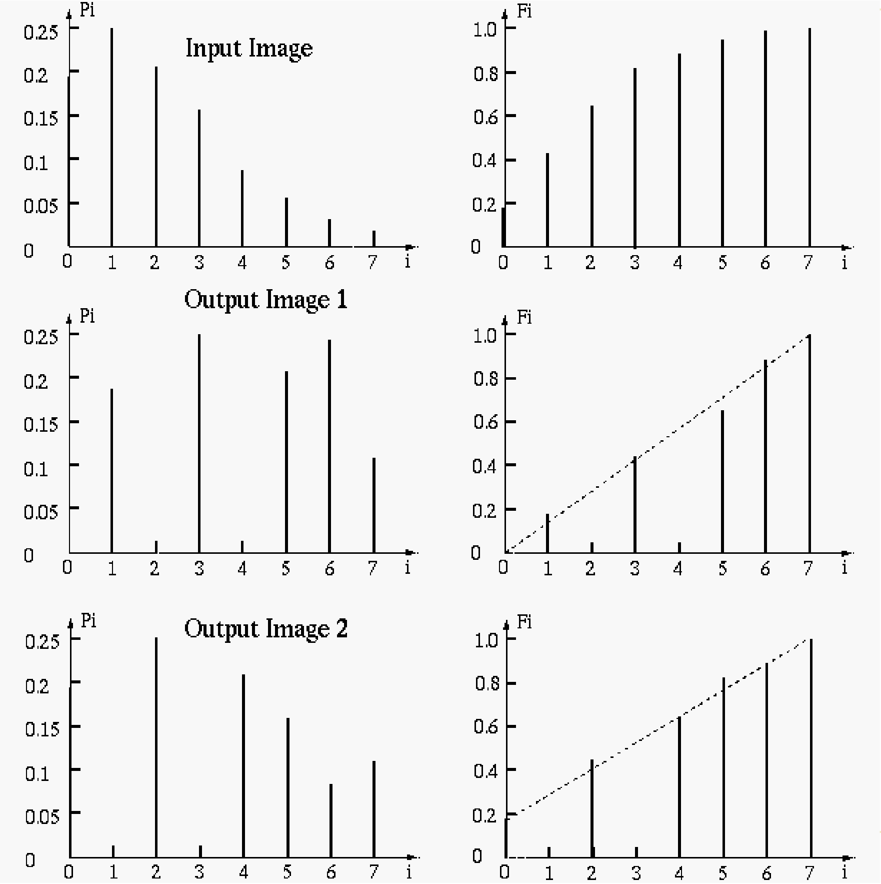
$$cdf_{min} = p_r(r_a) \text{ where } r_a = \min\{r_t | p_r(r_t) > 0\}; 0 \leqslant r_t \leqslant (L-1)$$

Histogram Equalization



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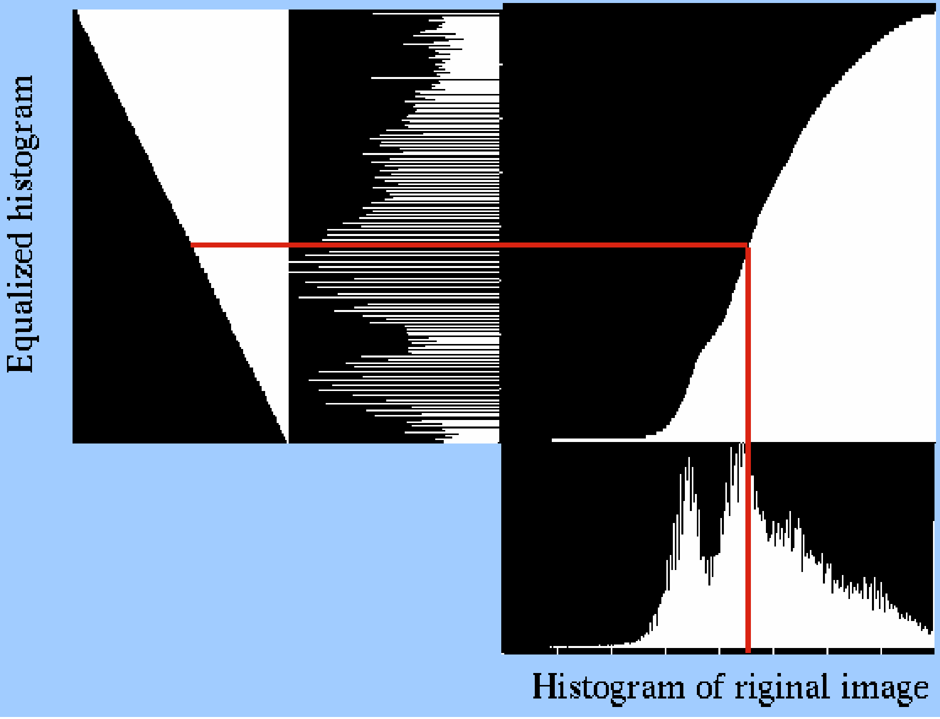
Histogram Equalization



Histogram Processing

- Histogram Equalization
- Histogram Specification
- Local Histogram Equalization

Histogram Specification / Matching

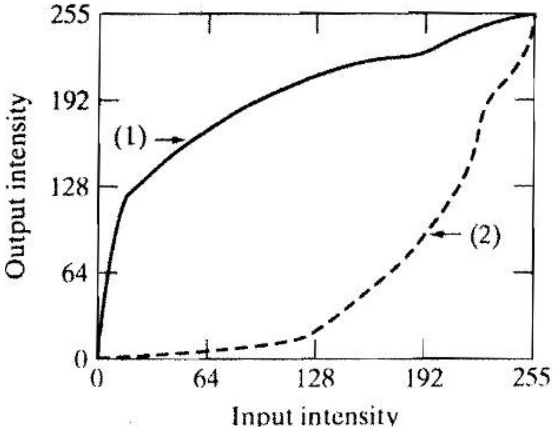
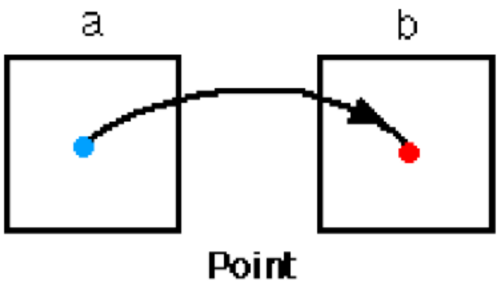
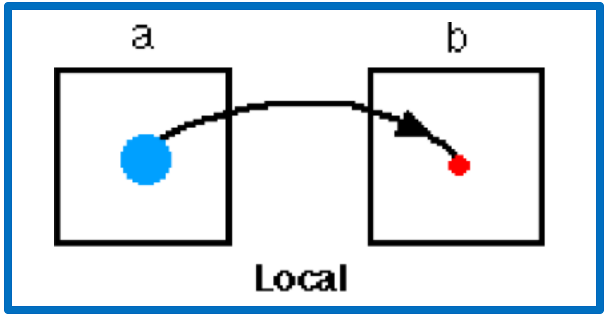


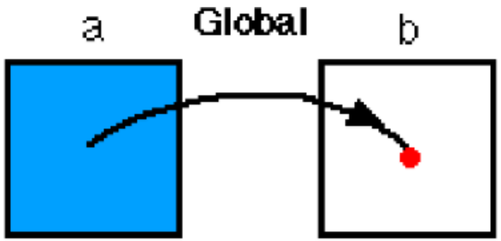
Image Courtesy: Gonzalez andWoods

- Manipulating Pixels Directly in Spatial Domain
- Point to Point

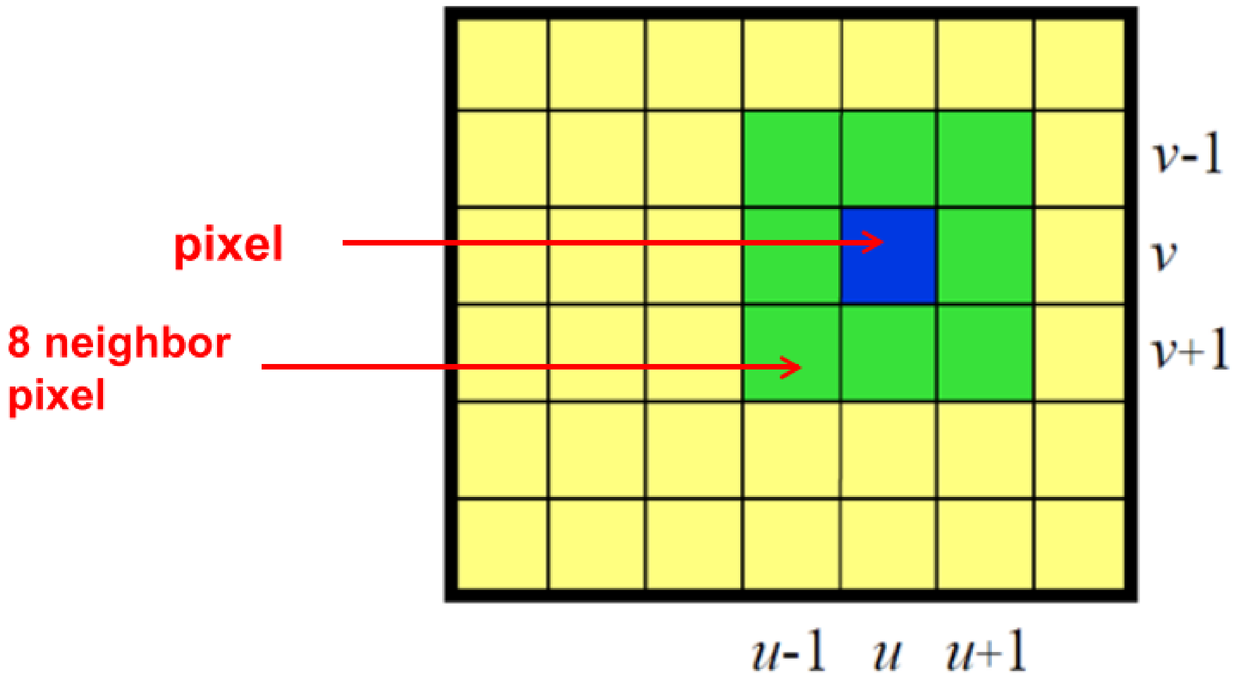




- **Neighborhood to Point**
- Global Attribute to Point



Neighborhood



Local Histogram Processing

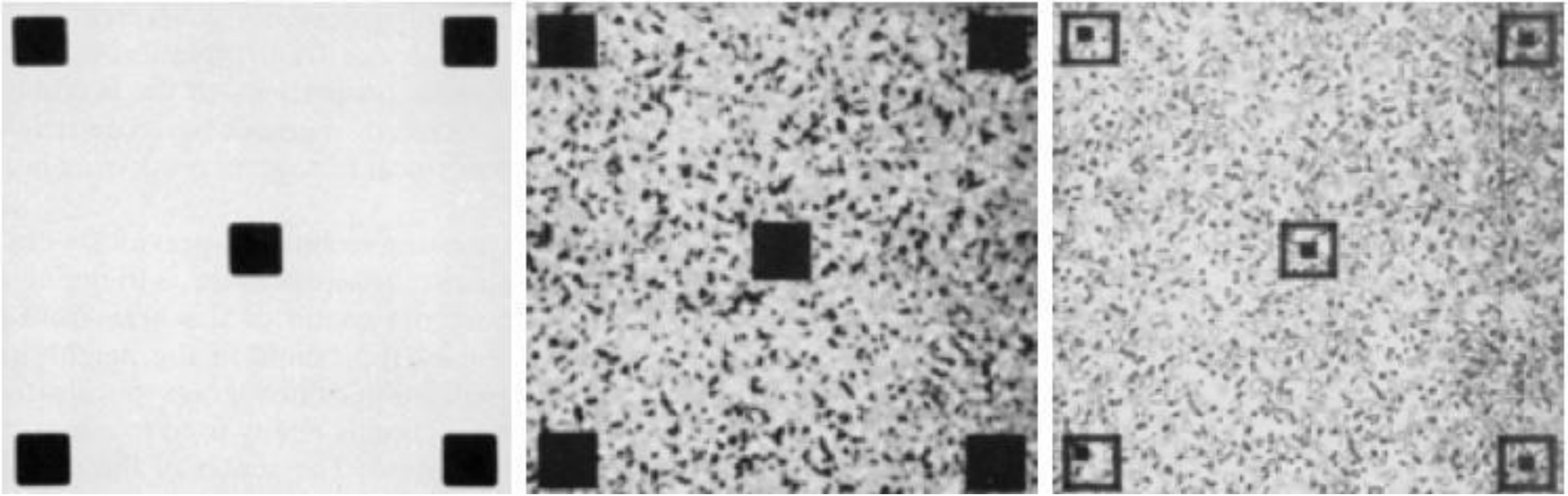
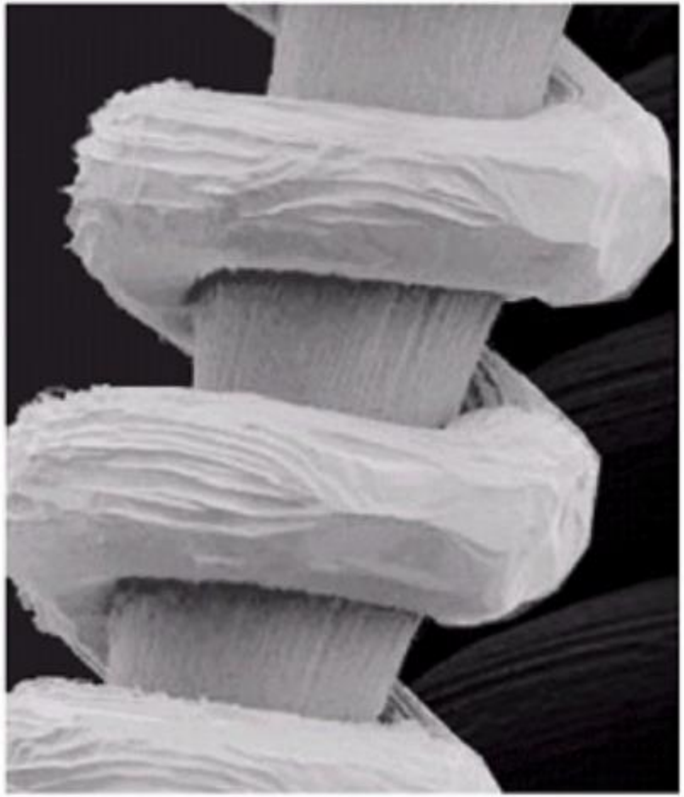


Image Courtesy: Gonzalez andWoods

Using Histogram Statistics for Image Enhancement

- we use some statistical parameters
  - global:
    - $p(r_i) = \frac{n_i}{n}$
    - $m(r) = \sum_{i=0}^{L-1} p(r_i)\, r_i$
    - $\sigma^2(r) = \sum_{i=0}^{L-1} (r_i - m)^2\, p(r_i)$
  - local:
    - $p(r_{s,t})$: neighborhood normalized histogram at coordinates (s,t) using a mask centered at (x,y)
    - $m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} p(r_{s,t})\, r_{s,t}$
    - $\sigma^2(S_{xy}) = \sum_{(s,t)\in S_{xy}} [r_{s,t} - m_{S_{xy}}]^2\, p(r_{s,t})$

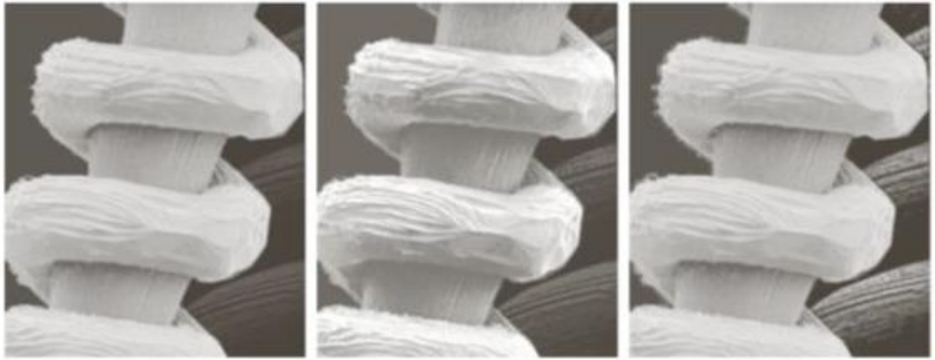
Using Histogram Statistics for Image Enhancement



- we use some statistical parameters
  - global:
    - $p(r_i) = \frac{n_i}{n}$
    - $m(r) = \sum_{i=0}^{L-1} p(r_i)\, r_i$
    - $\sigma^2(r) = \sum_{i=0}^{L-1} (r_i - m)^2\, p(r_i)$
  - local:
    - $p(r_{s,t})$: neighborhood normalized histogram at coordinates (s,t) using a mask centered at (x,y)
    - $m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} p(r_{s,t})\, r_{s,t}$
    - $\sigma^2(S_{xy}) = \sum_{(s,t)\in S_{xy}} [r_{s,t} - m_{S_{xy}}]^2\, p(r_{s,t})$

- Objective: Enhance dark areas while leaving light areas unchanged
- Can we use local statistic to obtain it ?

Using Histogram Statistics for Image Enhancement



a b c

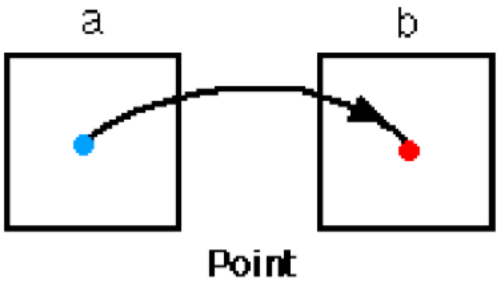
FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

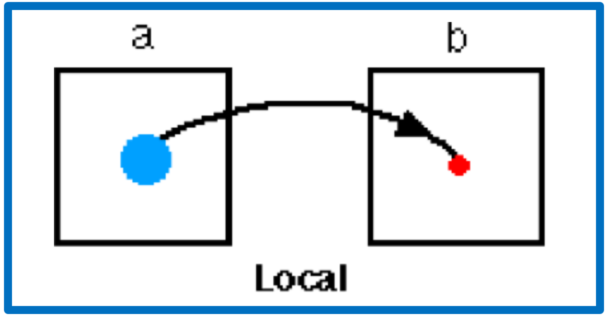
$$g(x,y) = \begin{cases} E \cdot f(x,y), & \text{if } m_{s_{xy}} \le k_0 m_G \text{ and } k_1\sigma_G \le \sigma_{s_{xy}} \le k_2\sigma_G \\ f(x,y), & \text{otherwise} \end{cases}$$

m_G : global mean; σ_G : global standard deviation

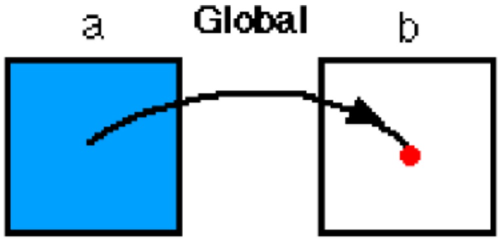
$k_0 = 0.4; \quad k_1 = 0.02; \; k_2 = 0.4; \; E = 4$

- Manipulating Pixels Directly in Spatial Domain
- Point to Point





- **Neighborhood to Point**



- Global Attribute to Point

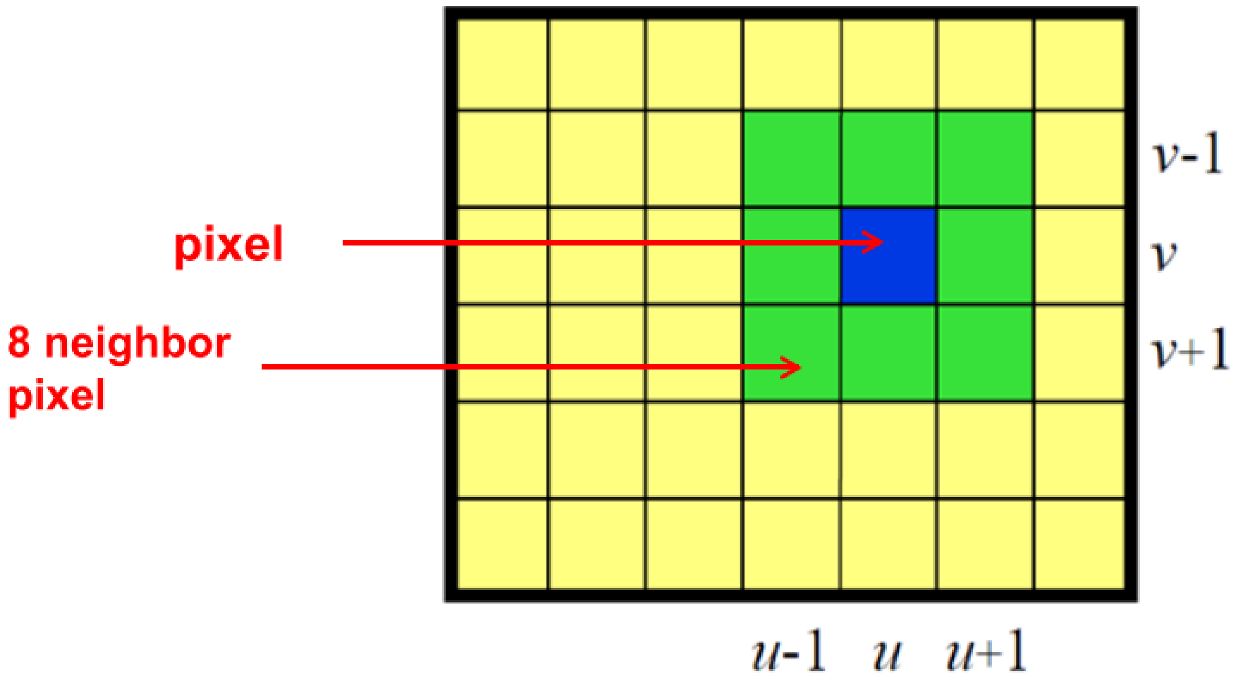
Spatial Domain Filtering

- What is a filter?
- What is filtering operation?

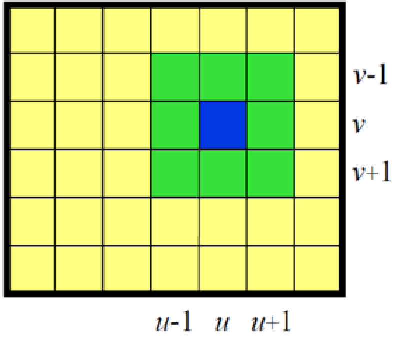
Spatial Domain Filtering - Approaches

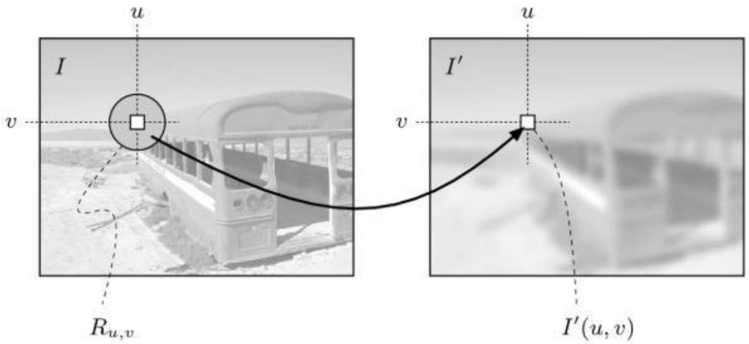
- Linear
- Non-linear

Neighborhood Operation



Smoothing Operation





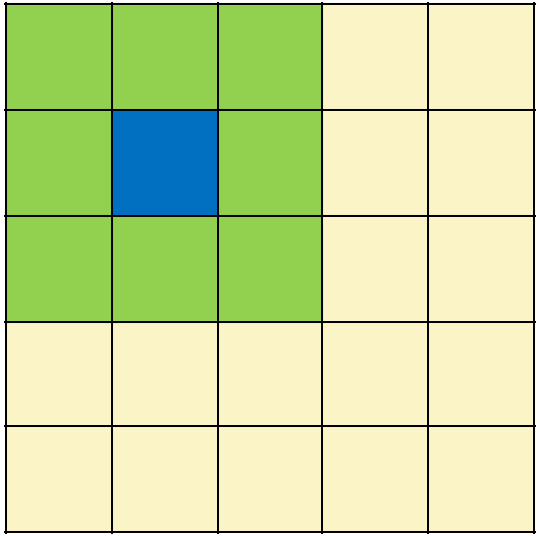
$$I'(u,v) \leftarrow \frac{p_0+p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8}{9}$$

$$\begin{aligned} I'(u,v) \leftarrow \tfrac{1}{9} \cdot [\, & I(u-1,v-1) && +I(u,v-1) && +I(u+1,v-1) && + \\ & I(u-1,v) && +I(u,v) && +I(u+1,v) && + \\ & I(u-1,v+1) && +I(u,v+1) && +I(u+1,v+1)\,] && \end{aligned}$$

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

Smoothing as Averaging

I



H → Weight Mask

Kernel

Filter

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$$I'(u,v) \leftarrow \frac{1}{9} \cdot \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

$$I'(u,v) \leftarrow \sum_{j=-1}^{1} \sum_{i=-1}^{1} I(u+i, v+j)$$

- $H(i,j)$

Effect of Mask Size

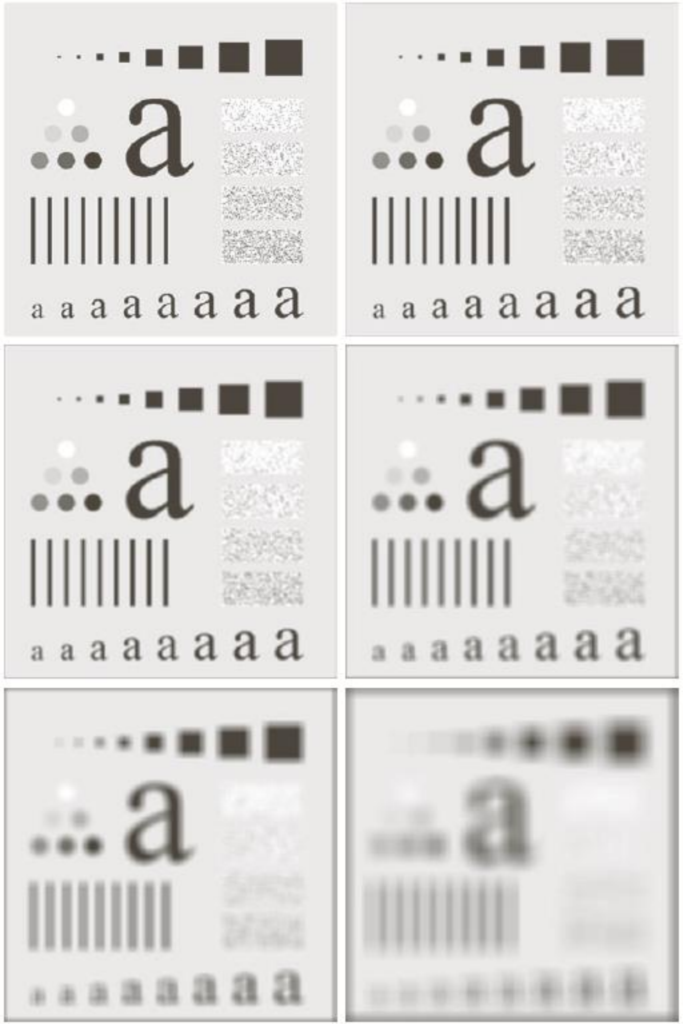
Original Image

[3x3]

[5x5]

[7x7]

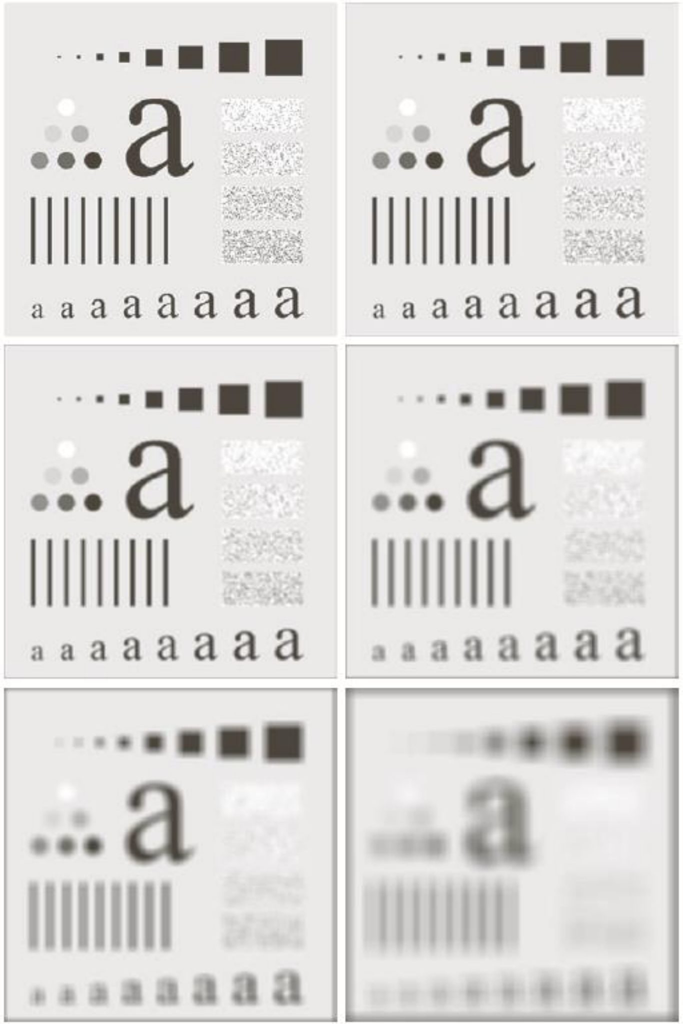
Square averaging filter



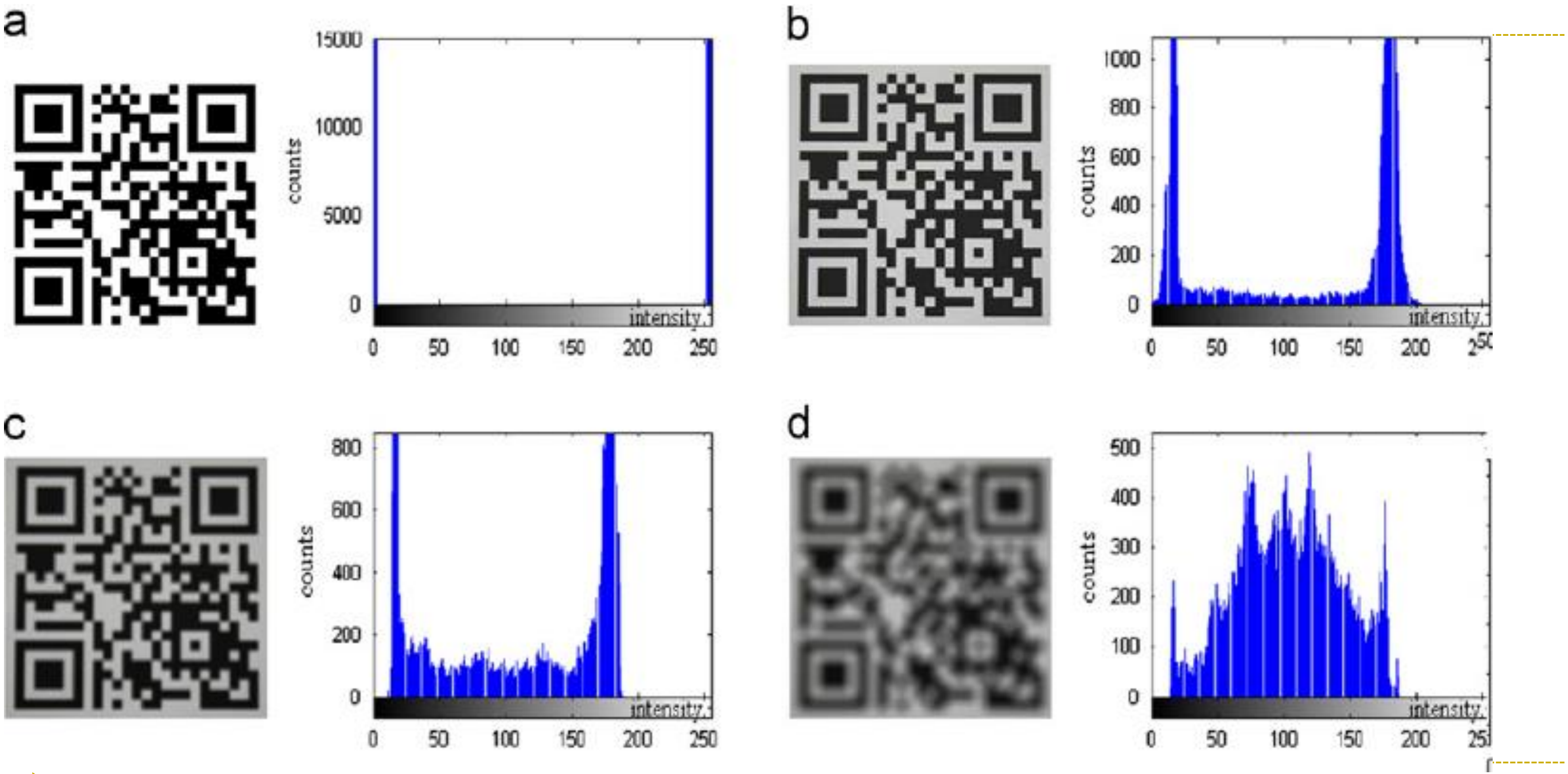
Square averaging filter

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a b
c d
e f

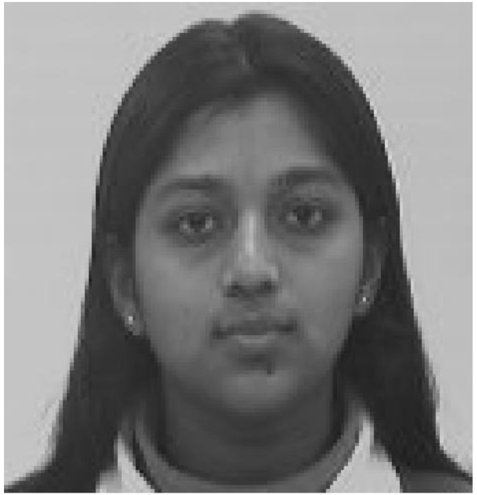


Smoothing – a histogram perspective



https://www.researchgate.net/figure/Histograms-of-barcode-images-with-different-levels-of-blur-a-a-synthetic-QR-code-image_fig3_224201499

Effect of Repeated Smoothing



Before

After

After repeated averaging

NOTE: Can get the effect of larger filters by smoothing repeatedly with smaller filters

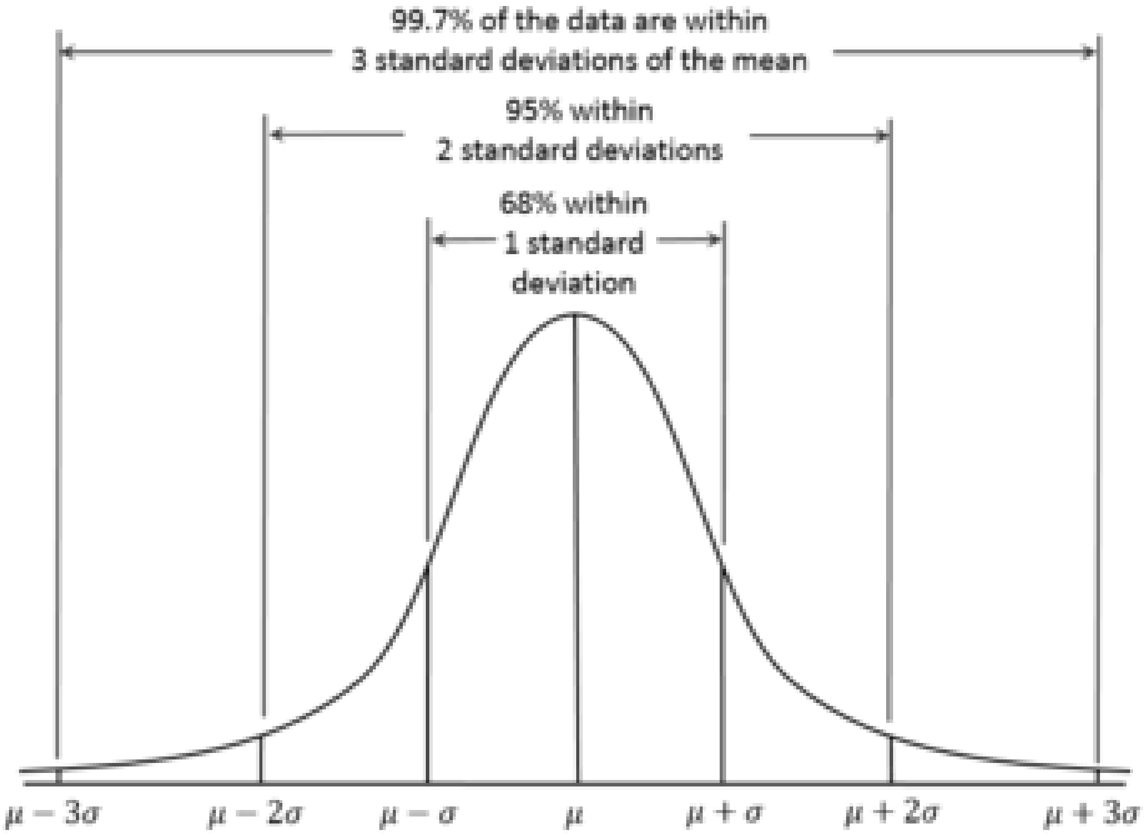
Weighted Averaging

$$I'(u,v)=\frac{\sum_{(j=-a)}^{a}\sum_{(i=-b)}^{b} I(u+i,v+j)\cdot H(i,j)}{\sum_{(j=-a)}^{a}\sum_{(i=-b)}^{b} H(i,j)}$$

$$\frac{1}{9}\times\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} \qquad \frac{1}{16}\times\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$

Standard average   Weighted average

Gaussian Function



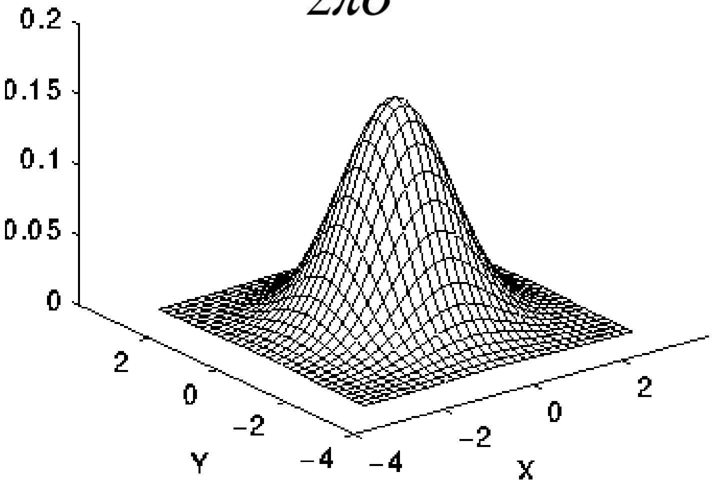
$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu =$ Mean
$\sigma =$ Standard Deviation
$\pi \approx 3.14159\cdots$
$e \approx 2.71828\cdots$

Gaussian Smoothing

- Mask weights are samples of a Gaussian Function

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2+y^2)/2\sigma^2\}$$



$\frac{1}{265}$

1	4	6	4	1
4	16	26	16	4
6	26	43	26	6
4	16	26	16	4
1	4	6	4	1

5×5 Gaussian filter, $\sigma=1$

Gaussian Smoothing – RGB images

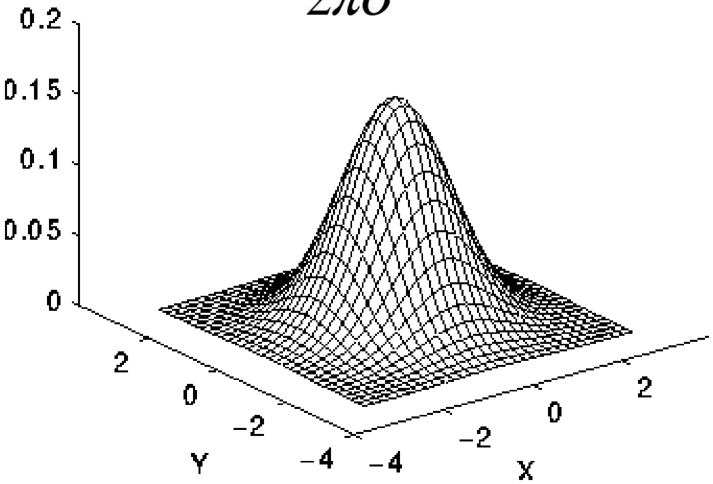
$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$

Gaussian Smoothing – animation

Gaussian Smoothing

- Mask weights are samples of a Gaussian Function

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2 + y^2)/2\sigma^2\}$$



$$\frac{1}{265}$$

1	4	6	4	1
4	16	26	16	4
6	26	43	26	6
4	16	26	16	4
1	4	6	4	1

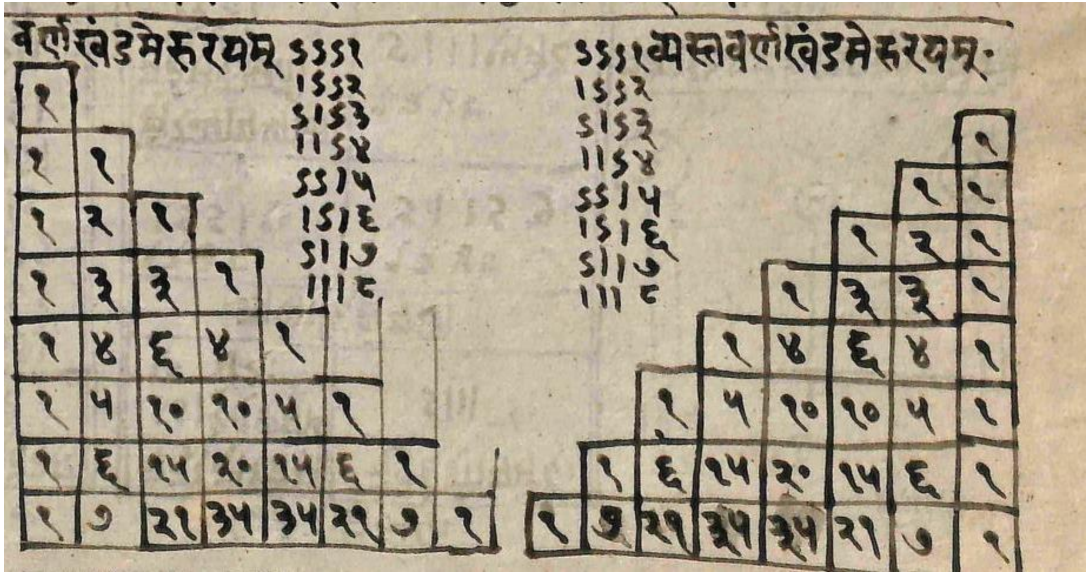
5×5 Gaussian filter, $\sigma=1$

- Relation between σ and smoothing?

Gaussian Smoothing – Effect of sigma

$$G(x,y)=\frac{1}{2\pi\sigma^2}\exp\{-(x^2+y^2)/2\sigma^2\}$$

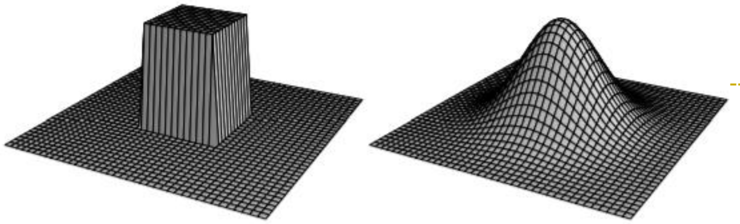
Detour: Generating the coefficients

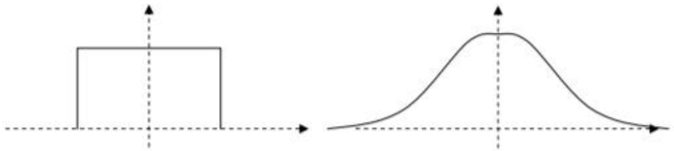


$$\frac{1}{265}$$

1	4	6	4	1
4	16	26	16	4
6	26	43	26	6
4	16	26	16	4
1	4	6	4	1

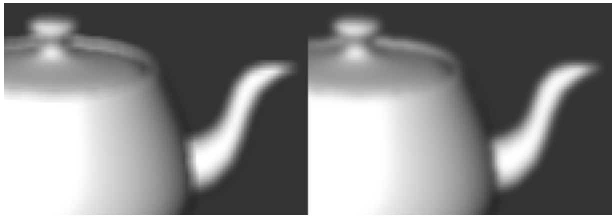
- *Meru Prastaara*, derived from Pingala's formulae (2 BCE), Manuscript from Raghunath Temple Library, Jammu





0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

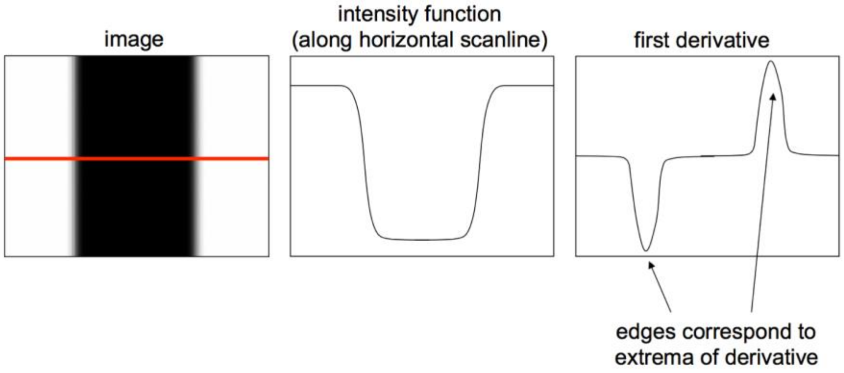
0	1	2	1	0
1	3	5	3	1
2	5	9	5	2
1	3	5	3	1
0	1	2	1	0



Out of focus blur

- # First Derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

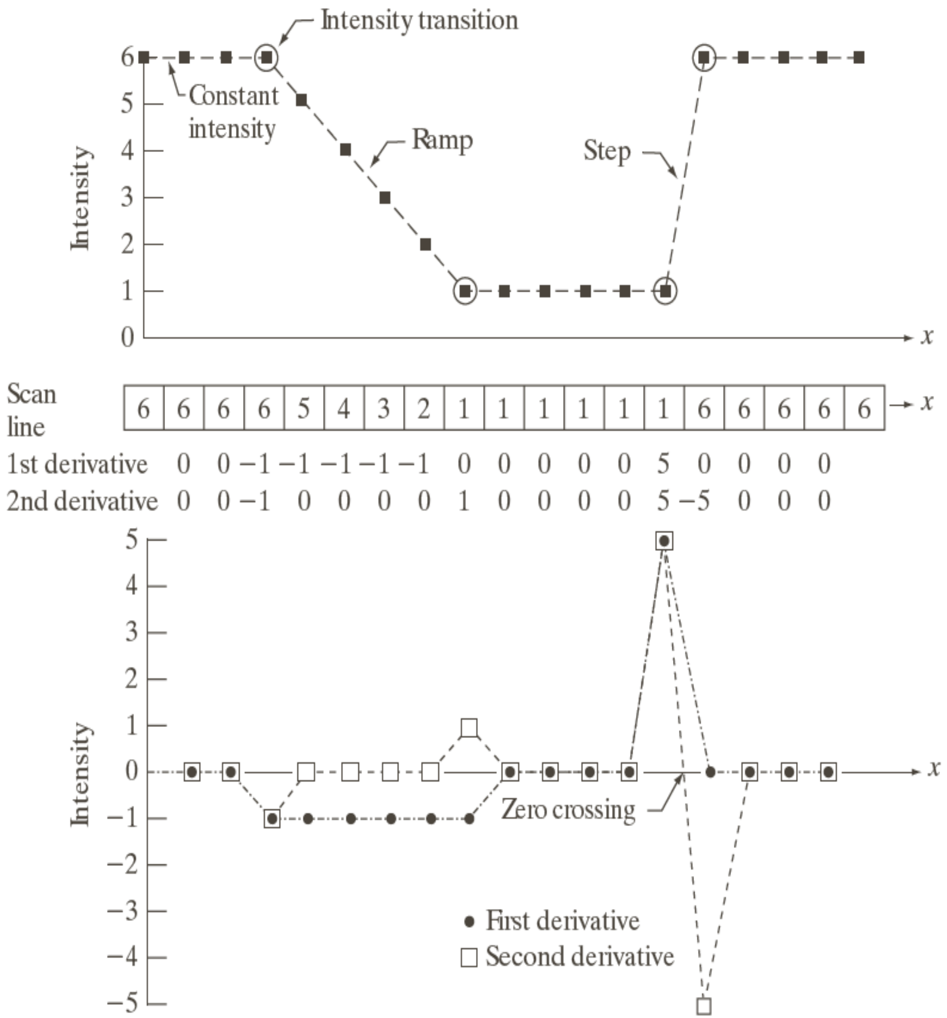


- First Derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- Second Derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



1-D Derivatives

- First Derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

  - Zero in flat segments
  - Nonzero at the onset of a step or ramp
  - Nonzero along ramps

- Second Derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$

  - Zero in flat areas;
  - Nonzero at the onset and end of a gray-level step or ramp;
  - Zero along ramps of constant slope

Image Derivatives

f(x-1,y)	f(x,y)	f(x+1,y)

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

f(x,y-1)
f(x,y)
f(x,y+1)

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x,y)$$

Laplacian Filter

$$\nabla^2 f = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

0 response in ‘flat’ areas

Laplacian Filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Sharpening Filter

- Objective of sharpening: Highlight fine detail / Enhance detail that has been blurred.
- Smoothing → Averaging → Summation
- Sharpening → Difference

Implementation

$$I'(u,v) = \begin{cases} I(u,v) - \nabla^2 I(u,v) \\ I(u,v) + \nabla^2 I(u,v) \end{cases}$$

If the center coefficient is negative

If the center coefficient is positive

Where $I(u,v)$ is the original image

$\nabla^2 I(u,v)$ is Laplacian filtered image

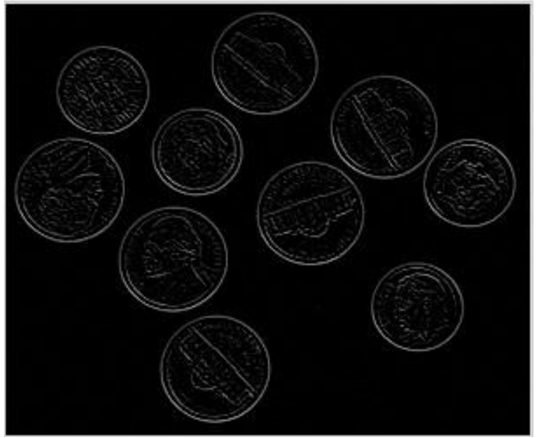
$I'(u,v)$ is the sharpened image

0	1	0
1	-4	1
0	1	0

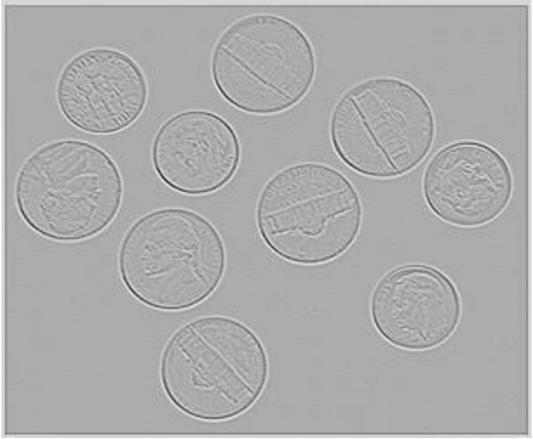
sharpened = range_normalize(input + filtered)

Sharpening with Laplacian Filters

$I(u,v)$



$\nabla^2 I(u,v)$

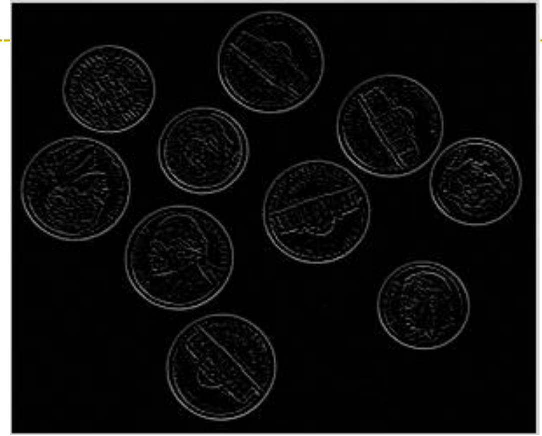


$\nabla^2 I(u,v) + 128$

(ForVisualization)

Laplacian Filters

$\nabla^2 I(u,v)$

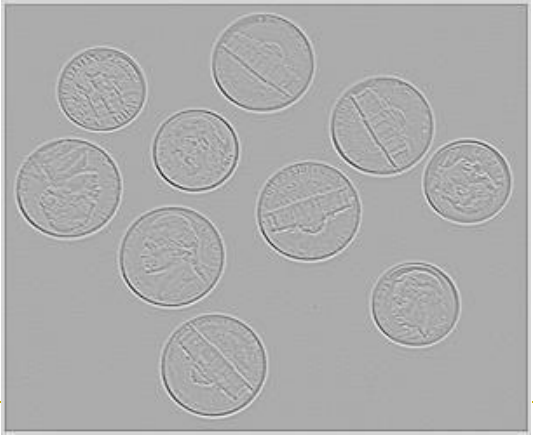


$I(u,v)$

$I'(u,v)$

$\nabla^2 I(u,v) + 128$

(ForVisualization)



Sharpening with Laplacian Filters



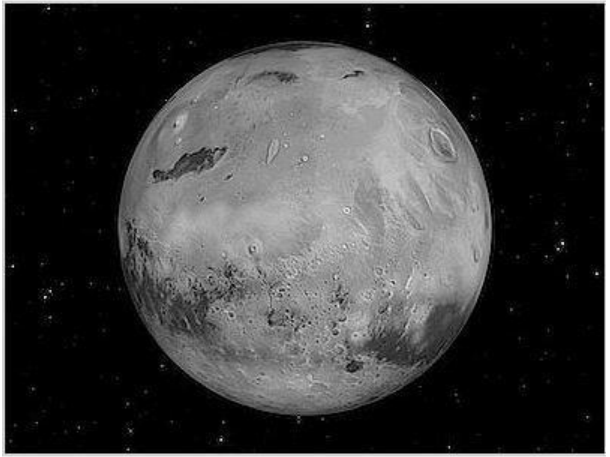
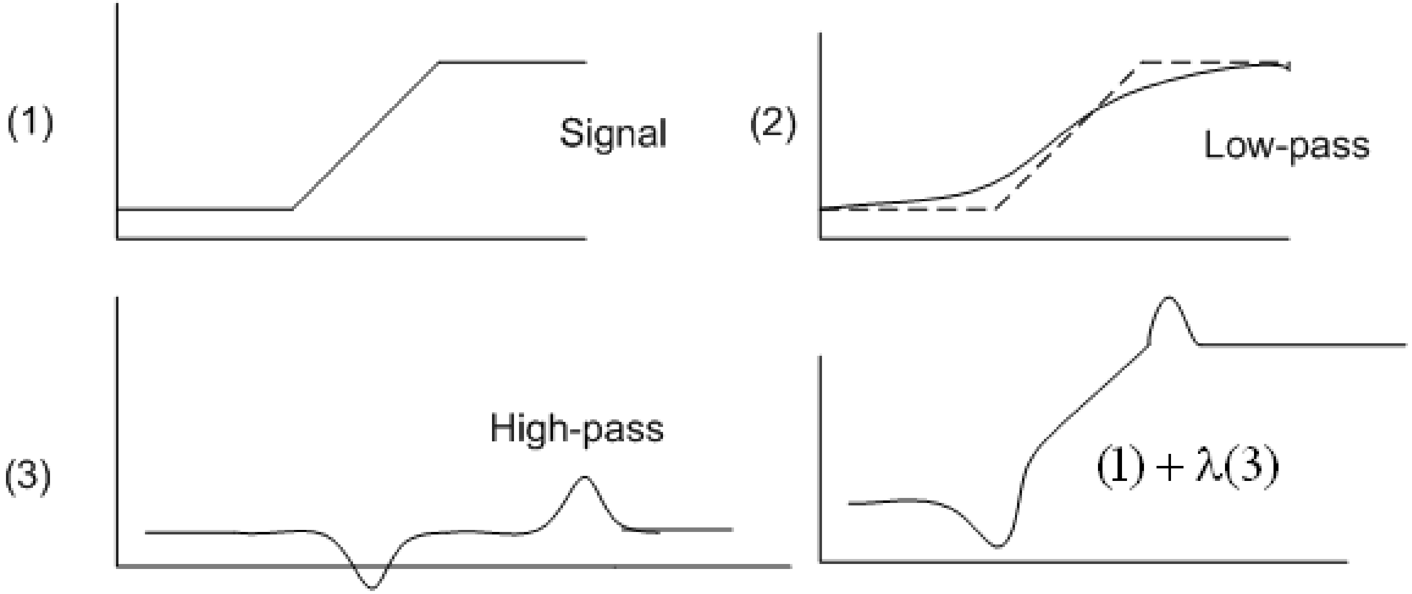


Image Courtesy:NASA

Unsharp Masking (and Highboost Filtering)



$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y), \quad g(x,y) = f(x,y) + k * g_{mask}(x,y).$$

Unsharp Masking (and Highboost Filtering)

- **High boost filter**: amplify input image, then subtract a lowpass image

$$Highboost = A\ Original - Lowpass$$

$$= (A-1)\ Original + Original - Lowpass$$

$$= (A-1)\ Original + Highpass$$

(A-1) 

+ 

= 

Unsharp Masking / Highboost Filtering

A>=1
w = 9A-1

-1	-1	-1
-1	w	-1
-1	-1	-1

A=2
w = 17

-1	-1	-1
-1	17	-1
-1	-1	-1

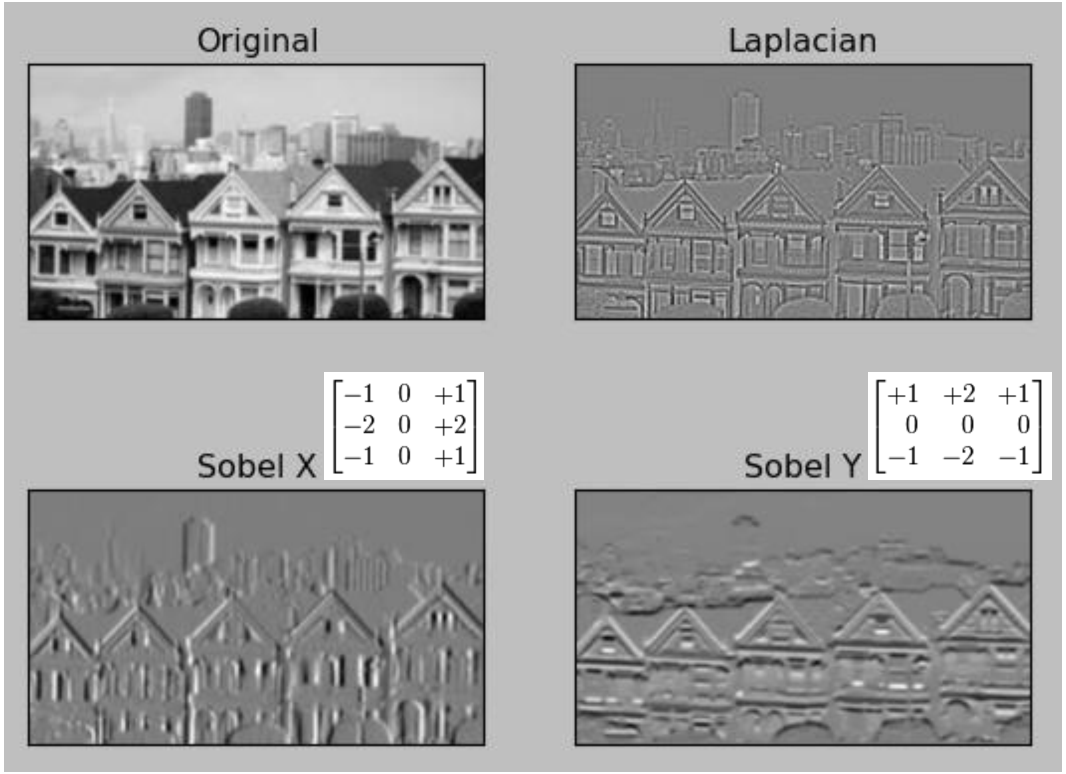
- If **A=1**, we get unsharp masking.
- If **A>1**, part of the original image is added back to the high pass filtered image (highboost filtering).

Unsharp Masking (and Highboost Filtering)

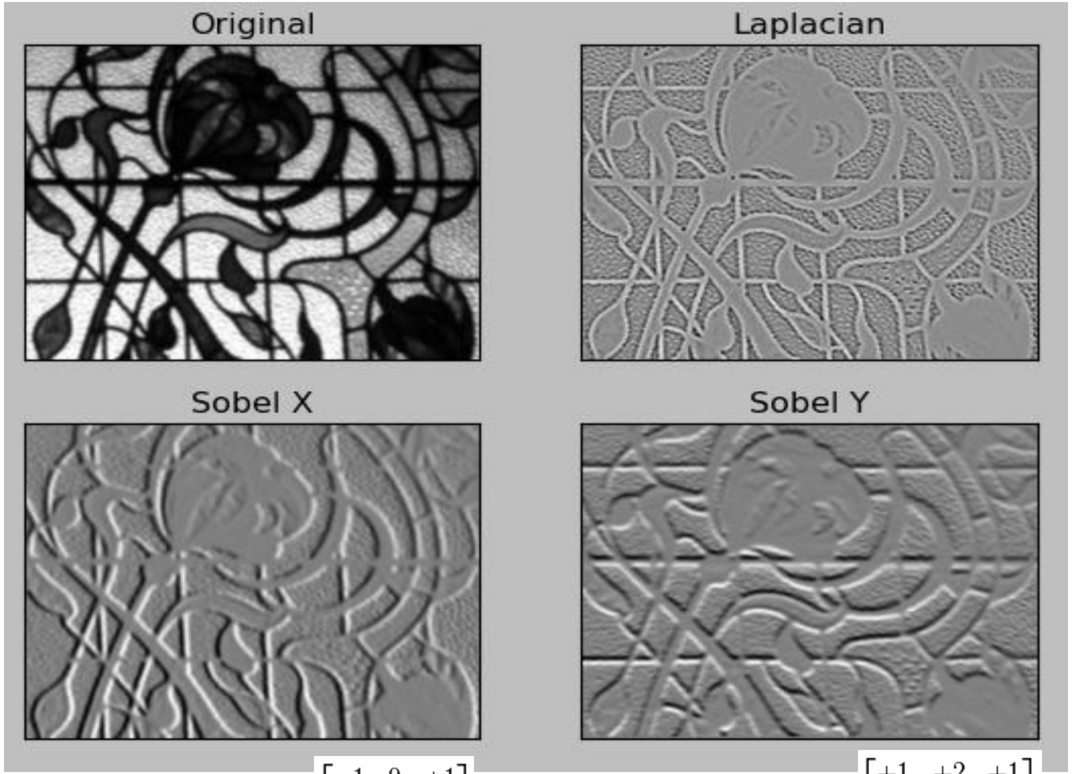
Unsharp Masking (and Highboost Filtering)

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Sobel Edge Masks



0	−1	0
−1	4	−1
0	−1	0



0	−1	0
−1	4	−1
0	−1	0

$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

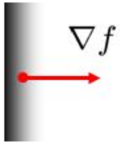
Edge Magnitude and Gradient

Image gradient

The gradient of an image:

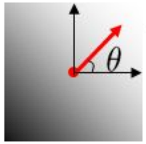
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient direction is given by:

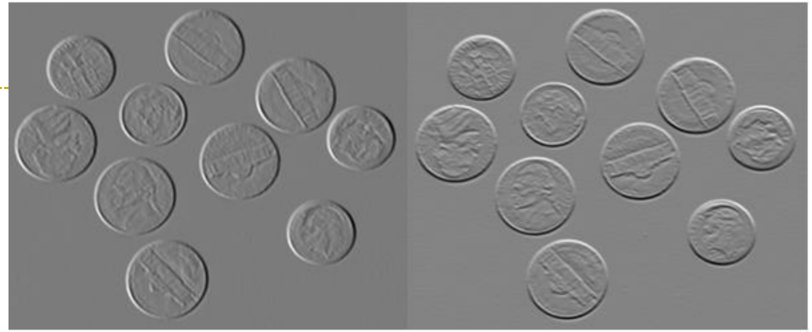
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

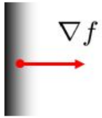
- how does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

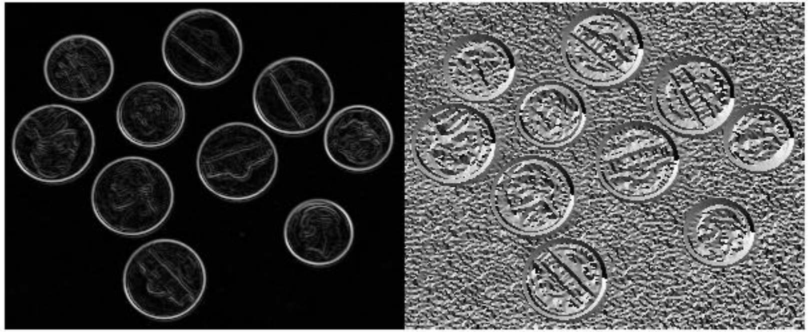
Edge Magnitude and Gradient

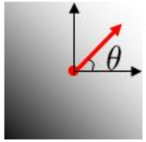




$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$

$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

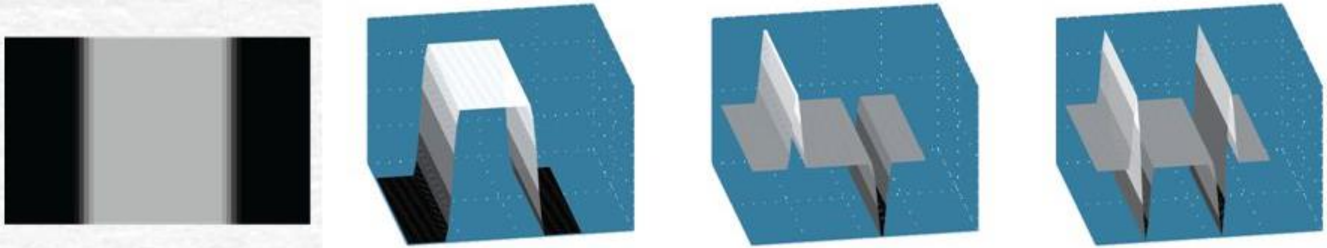




$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

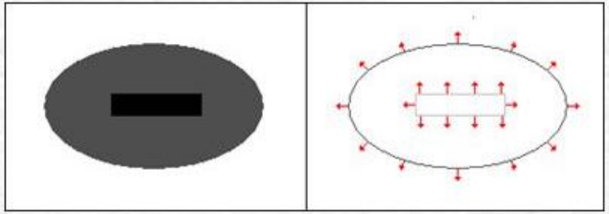
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$



- Calculus...
  - Rate of change in two directions
  - Vector variable:
    - Gradient Magnitude
    - Orientation (0° is East)

$$\nabla f(i,j) = \sqrt{\left(\frac{\delta f(i,j)}{\delta i}\right)^2 + \left(\frac{\delta f(i,j)}{\delta j}\right)^2}$$

$$\phi(i,j) = \arctan\left(\frac{\delta f(i,j)}{\delta j}, \frac{\delta f(i,j)}{\delta i}\right)$$



References

- GW Chapter – 3.4.1, 3.5.1, 3.6