NOTES ON THE SYNTHESIS

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TO MY DEAREST JAN

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PART TWO

6 / THE PROGRAM

Here is the problem. We wish to design clearly conceived forms which are well adapted to some given context. We have seen that for this to be feasible, the adaptation must take place independently within independent subsystems of variables. In the unselfconscious situation this occurs automatically, because the individual craftsman has too little control over the process to upset the pattern of adaptation implicit in the ensemble. Unfortunately this situation no longer exists; the number of variables has increased, the information confronting us is profuse and confusing, and our attempts to duplicate the natural organization of the unselfconscious process selfconsciously are thwarted, because the very thoughts we have, as we try to help ourselves, distort the problem and make it too unclear to solve.

The dilemma is simple. As time goes on the designer gets more and more control over the process of design. But as he does so, his efforts to deal with the increasing cognitive burden actually make it harder and harder for the real causal structure of the problem to express itself in this process.

What can we do to overcome this difficulty? On the face of it, it is hard to see how any systematic theory can ease it much. There are certain kinds of problems, like some of those that occur in economics, checkers, logic, or administration, which can be clarified and solved mechanically.[1] They can be solved mechanically, because they are well enough understood for us to turn them into selection problems.[2]

To solve a problem by selection, two things are necessary.

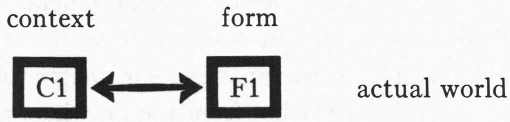
1. It must be possible to generate a wide enough range of possible alternative solutions symbolically.
2. It must be possible to express all the criteria for solution in terms of the same symbolism.

Whenever these two conditions are met, we may compare symbolically generated alternatives with one another by testing them against the criteria, until we find one which is satisfactory, or the one which is the best. It is at once obvious that wherever this kind of process is possible, we do not need to "design" a solution. Indeed, we might almost claim that a problem only calls for design (in the widest sense of that word) when selection cannot be used to solve it. Whether we accept this or not, the converse anyway is true. Those problems of creating form that are traditionally called "design problems" all demand invention.

Let us see why this is so. First of all, for physical forms, we know no general symbolic way of generating new alternatives — or rather, those alternatives which we can generate by varying the existing types do not exhibit the radically new organization that solutions to new design problems demand. These can only be created by invention. Second, what is perhaps more important, we do not know how to express the criteria for success in terms of any symbolic description of a form. In other words, given a new design, there is often no mechanical way of telling, purely from the drawings which describe it, whether or not it meets its requirements. Either we must put the real thing in the actual world, and see whether it works or not, or we must use our imagination and experience of the world to predict from the drawings whether it will work or not. But there is no general symbolic connection between the requirements and the form's description which provide criteria; and so there is no way of testing the form symbolically.[3] Third, even if these first two objections could be overcome somehow, there is a much more conclusive difficulty. This is the same difficulty, precisely, that we come across in trying to construct scientific hypotheses from a given body of data. The data alone are not enough to define a hypothesis; the construction of hypotheses demands the further introduction of principles like simplicity (Occam's razor), non-arbitrariness, and clear organization.[4] The construction of form, too, requires these principles. There is at present no prospect of introducing these principles mechanically, either into science or into design. Again, they require invention.

It is therefore not possible to replace the actions of a trained designer by mechanically computed decisions. Yet at the same time the individual designer's inventive capacity is too limited for him to solve design problems successfully entirely by himself. If theory cannot be expected to invent form, how is it likely to be useful to a designer?

Let us begin by stating rather more explicitly just what part the designer does play in the process of design. I shall contrast three possible kinds of design process, schematically.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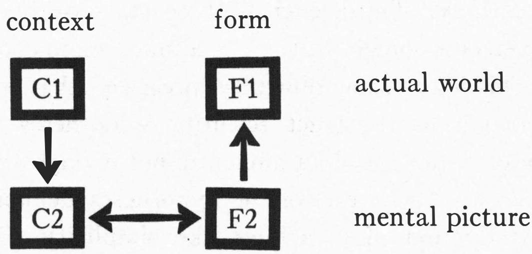
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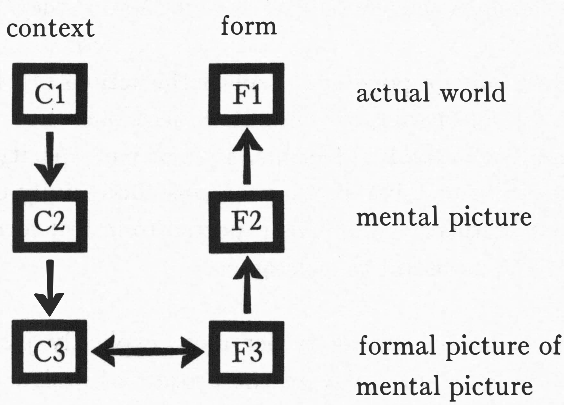
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The first scheme represents the unselfconscious situation described in Chapter 4. Here the process which shapes the form is a complex two-directional interaction between the context C1 and the form F1, in the world itself. The human being is only present as an agent in this process. He reacts to misfits by changing them; but is unlikely to impose any "designed" conception on the form.

The second scheme represents the selfconscious situation described in Chapter 5. Here the design process is remote from the ensemble itself; form is shaped not by interaction between the actual context's demands and the actual inadequacies of the form, but by a conceptual interaction between the conceptual picture of the context which the designer has learned and invented, on the one hand, and ideas and diagrams and drawings which stand for forms, on the other. This interaction contains both the probing in which the designer searches the problem for its major "issues," and the development of forms which satisfy them; but its exact nature is unclear.[5] In present design practice, this critical step, during which the problem is prepared and translated into design, always depends on some kind of intuition. Though design is by nature imaginative and intuitive, and we could easily trust it if the designer's intuition were reliable, as it is it inspires very little confidence.

In the unselfconscious process there is no possibility of misconstruing the situation: nobody makes a picture of the context, so the picture cannot be wrong. But the selfconscious designer works entirely from the picture in his mind, and this picture is almost always wrong.

The way to improve this is to make a further abstract picture of our first picture of the problem, which eradicates its bias and retains only its abstract structural features; this second picture may then be examined according to precisely defined operations, in a way not subject to the bias of language and experience.[6] The third scheme in the diagram represents a third process, based on the use of such a picture. The vague and unsatisfactory picture of the context's demands, C2, which first develops in the designer's mind, is followed by this mathematical picture, C3. Similarly, but in reverse, the design F2 is preceded by an orderly complex of diagrams F3. The derivation of these diagrams F3 from C3, though still intuitive, may be clearly understood. The form is actually shaped now by a process at the third level, remote from C2 or F2. It is out in the open, and therefore under control.

This third picture, C3, is built out of mathematical entities called "sets." A set, just as its name suggests, is any collection of things whatever, without regard to common properties, and has no internal structure until it is given one.[7] A collection of riddles in a book forms a set, a lemon and an orange and an apple form a set of three fruits, a collection of relationships like fatherhood, motherhood, brotherhood, sisterhood, forms a set (in this case a set of four elements). The elements of a set can be as abstract or as concrete as you like. It must only be possible to identify them uniquely, and to distinguish them from one another.[8]

The principal ideas of set theory are these:

1. An element x of a set S, is said to belong to that set. This is written $x \in S$. A set is uniquely defined by identifying its elements.
2. One set S_1 is said to be a subset of another set S_2, if and only if every element of S_1 belongs to S_2. This

is written $S_1 \sqsubset S_2$. If S_2 also contains elements which are not elements of S_1, so that S_2 is "larger" than S_1, then S_1 is called a proper subset of S_2, and we write $S_1 \subset S_2$.

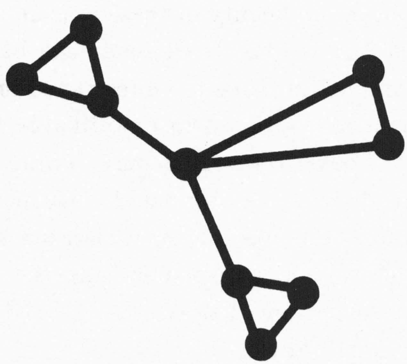
3. The union of two sets S_1 and S_2 is the set of those elements which belong to either S_1 or S_2 (or both, in the case where S_1 and S_2 have elements in common). We write it as $S_1 \cup S_2$.
4. The intersection of two sets S_1 and S_2 is the set of those elements which belong to both S_1 and S_2. We write it $S_1 \cap S_2$. If S_1 and S_2 have no elements in common, this intersection is empty, and we call the sets disjoint.

Let us be specific about the use of set theory to picture design problems. We already know, from Chapter 2, what the designer's conception of a problem looks like. The problem presents itself as a task of avoiding a number of specific potential misfits between the form and some given context. Let us suppose that there are m such misfit variables: $x_1 \cdots x_m$. These misfit variables form a set. We call the set of these m misfits M, so that we may write $x_i \in M$ (for all i, $i = 1 \cdots m$).[9]

The great power and beauty of the set, as an analytical tool for design problems, is that its elements can be as various as they need be, and do not have to be restricted only to requirements which can be expressed in quantifiable form. Thus in the design of a house, the set M may contain the need for individual solitude, the need for rapid construction, the need for family comfort, the need for easy maintenance, as well as such easily quantifiable requirements as the need for low capital cost and efficiency of operation. Indeed, M may contain any requirement at all.

These requirements are the individual conditions which must be met at the form-context boundary, in order to prevent misfit. The field structure of this form-context boundary, in so far as the designer is aware of it, is also not hard to describe. He knows that some of the misfits interfere with one another, as he tries to solve them, or conflict; that others have common physical implications, or concur; and that still others do not interact at all. It is the presence and absence of these interactions which give the set M the system character already referred to in Chapters 3, 4, and 5.[10] We represent the interactions by associating with M a second set L, of non-directed, signed, one-dimensional elements called links, where each link joins two elements of M, and contains no other elements of M. As we shall see in Chapter 8, the links bear a negative sign if they indicate conflict, and a positive sign if they indicate concurrence, and may also be weighted to indicate strength of interaction.

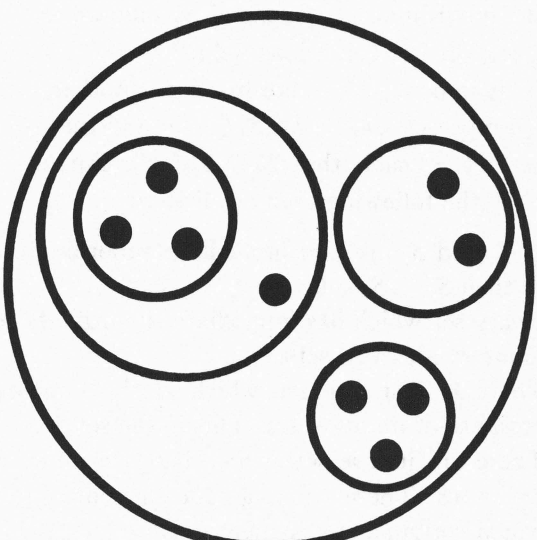
The two sets M and L together define a structure known as a linear graph or topological 1-complex, which we shall refer to as $G(M,L)$, or simply G for short.[11] A typical graph is shown below. Such a graph serves as a picture of a designer's view of



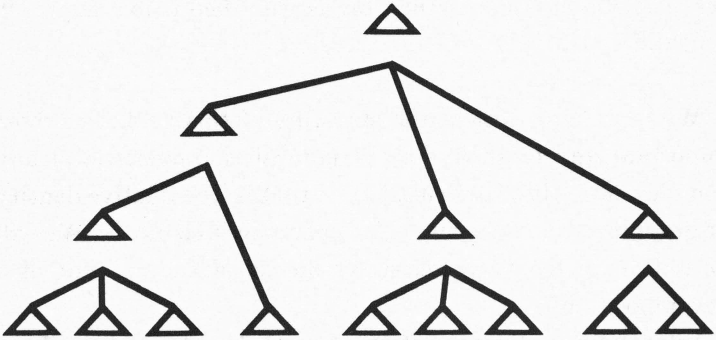
some specific problem. It is a fairly good picture, in the sense that its constituents, the sets M and L, are available to him introspectively without too much trouble; also because it keeps our attention, neatly and abstractly, on the fact that the set of misfits has a structure, or, as we called it in Chapter 2, a field.[12]

We must now explore the structure of this field. The most important and most obvious structural characteristic of any complex entity is its articulation — that is, the relative density or grouping and clustering of its component elements. We will be able to make this precise by means of the concept of a decomposition.

Informally, a decomposition of a set M into its subsidiary or subsystem sets is a hierarchical nesting of sets within sets, as is shown in the first of the two diagrams that follow. A more



usual diagram, which brings out the treelike character of the decomposition, is shown below. It refers to precisely the same structure as the other. Each element of the decomposition is a subset of those sets above it in the hierarchy.



Formally I define a decomposition of a set of misfits M as a tree (or partly ordered set) of sets in which a relation of immediate subordination is defined as follows, and in which the following further conditions hold:[13]

A set S_1 is immediately subordinate to another set S_2 if and only if S_2 properly includes S_1 ($S_1 \subset S_2$), and the tree contains no further set S_3 such that $S_1 \subset S_3 \subset S_2$. Further, the tree must satisfy the following four conditions:

1. If S_i and S_j are two immediate subordinates of a set S, then $S_i \cap S_j = 0$.
2. Every set which has immediate subordinate sets is the union of all these sets.
3. There is just one set which is the immediate subordinate of no other set. This is the set M.
4. There are just m sets which have no immediate subordinates. These are the one-element sets, each of which contains one element of M.

As it stands, such a decomposition deals only with the set M. L, the set of links, plays no part in it. But it is easy to see that the existence of these links makes some of the possible decompositions very much more sensible than others. Any graph of the type $G(M,L)$ tends to pull the elements of M together in natural clusters. Our task in the next chapters is to make this precise, and to decide which decomposition of M makes the most sense, once we have a given set L associated with it. Each subset of the set M which appears in the tree will then define a subproblem of the problem M. Each subproblem will have its own integrity, and be independent of the other subproblems, so that it can be solved independently.

It is very possible, and even likely, that the way the designer initially sees the problem already hinges on a conceptual hierarchy not too much unlike a decomposition in general outline.[14] In trying to show that the links of L favor a particular decomposition, I shall really be trying to show that for every problem there is one decomposition which is especially proper to it, and that this is usually different from the one in the designer's head. For this reason we shall refer to this special decomposition as the *program* for the problem represented by $G(M,L)$. We call it a program because it provides directions or instructions to the designer, as to which subsets of M are its significant "pieces," and so which major aspects of the problem he should apply himself to. This program is a reorganization of the way the designer thinks about the problem.[15]

7 / THE REALIZATION OF THE PROGRAM

Finding the right design program for a given problem is the first phase of the design process. It is, if we like, the analytical phase of the process. This first phase of the process must of course be followed by the synthetic phase, in which a form is derived from the program. We shall call this synthetic phase *the realization of the program*.[1] Although these notes are given principally to the analytical phase of the process, and to the invention of programs which can make the synthesis of form a reasonable task, we must now spend a little time thinking about the way this synthesis or realization will work. Until we do so, we cannot know how to develop the details of the program.

The starting point of analysis is the requirement. The end product of analysis is a program, which is a tree of sets of requirements. The starting point of synthesis is the diagram. The end product of synthesis is the realization of the problem, which is a tree of diagrams. The program is made by decomposing a set of requirements into successively smaller subsets. The realization is made by making small diagrams and putting them together as the program directs, to get more and more complex diagrams. To achieve this we must learn to match each set of requirements in the program with a corresponding diagram.

The invention of diagrams is familiar to every designer. Any pattern which, by being abstracted from a real situation, conveys the physical influence of certain demands or forces is a diagram.

The famous stroboscopic photograph of the splash of a milk drop is, for certain purposes, a diagram of the way the forces go at the moment of impact. If you want to study these forces, this photograph, by abstracting their *immediate* physical consequences from the confusion of what you usually see when a milk drop falls, tells you a great deal about them.[2]

Le Corbusier's *ville radieuse* is a diagram, which expresses the physical consequences of two very simple basic requirements: that people should be housed at high overall density, and that they should yet all have equal and maximum access to sunlight and air.[3]

The sphere is a diagram. It expresses, among other things, the physical implications of the need to enclose as large a volume as possible within as small a surface as possible. It also expresses the implication of the requirement that a number of things be equidistant from a single point.[4]

The texture of bathers on a crowded bathing beach is a diagram. The evenness of the texture tells you that there are forces tending to place family groups as far as possible (and hence at equal distances) from one another, instead of allowing them to place themselves randomly.

An arrow is a diagram, of course, which conveys direction. Many flow problems contain requirements which can be summarized by means of arrows.[5] Very occasionally the form called for turns out to be physically arrow-shaped itself; like the case where the aerodynamic needs of a fast aeroplane are embodied in a swept-wing design.

Kekulé's representation of the benzene molecule (as atoms, with linear bonds between them) is again a diagram. Given the valency forces represented by the bonds, the diagram expresses the physical arrangement of the atoms, relative to one another, which is thought to result from the interaction of these valencies.[6]

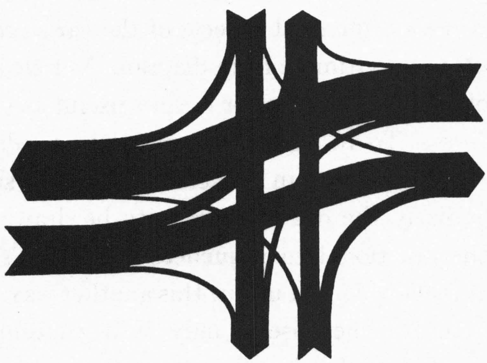
Van Doesburg's "de Stijl" drawings, though made for other reasons, could be interpreted as diagrams which present the rectilinear consequences of the need for machine tools and rapid prefabricated assembly.[7]

The engineer's preliminary sketch for a bridge structure is a diagram. After making the initial calculations, the engineer draws some pencil lines to show himself roughly how the bridge's major members might go under the influence of gravity, the given required span, the maximum tensile strength of available steel, and so on.[8]

We notice that these diagrams may have either or both of two distinct qualities, not always equally emphasized. On the one hand they may summarize aspects of a physical structure, by presenting one of the constituent patterns of its organization (as the photograph of the milk splash does, or the drawings for the *ville radieuse*). Although we can often infer a great deal about the demands responsible for the particular pattern such a diagram exhibits, it remains principally a description of formal characteristics. We shall call such a diagram a form diagram. On the other hand, the diagram may be intended to summarize a set of functional properties or constraints, like the arrow, or the population density map. This kind of diagram is principally a notation for the problem, rather than for the form. We shall call such a diagram a requirement diagram.

Let us consider extreme examples of a requirement diagram and a form diagram for a simple object. The mathematical statement $F = kv^2$ expresses the fact that under certain conditions the energy lost by a moving object because of friction depends on the square of its velocity. In the design of a racing car, it is obviously important to reduce this effect as far as possible; and in this sense the mathematical statement is a requirement diagram. At the other extreme, a water-color perspective view of a racing car is also a diagram. It summarizes certain physical aspects of the car's organization, and is therefore a legitimate form diagram. Yet clearly neither the equation nor the water color is very useful as such, in the search for form. To be useful, the equation needs to be interpreted, so that one can understand its physical consequences. Similarly the drawing needs to be drawn in such a way that the functional consequences of the car's shape are clearly comprehensible. Let us put this another way. A requirement diagram becomes useful only if it contains physical implications, that is, if it has the elements of a form diagram in it. A form diagram becomes useful only if its functional consequences are foreseeable, that is, if it has the elements of a requirement diagram in it. A diagram which expresses requirements alone or form alone is no help in effecting the translation of requirements into form, and will not play any constructive part in the search for form. We shall call a diagram constructive if and only if it is both at once — if and only if it is a requirement diagram and a form diagram at the same time. Let us consider an example.

Suppose that two streets of an existing town center are to be widened at and around their point of intersection, to lessen congestion. Suppose further that the only requirement is that today's traffic can flow without congestion. The requirement diagram, therefore, consists basically of information about how much traffic flows in various directions at different times of day. It is possible to present this information in a nonconstructive diagram by simply tabulating the flow numerically for each of the twelve possible paths, for different times of day. It is also possible, however, to present this same information in the condensed graphic form shown below.



Here we have a street map with arrows of various widths on it, representing the number of vehicles per hour flowing in various directions at peak hours. In this form the diagram indicates directly what form the new intersection must take. Clearly a thick arrow requires a wide street, so that the overall pattern called for emerges directly from the diagram.[9] It is both a requirement diagram and a form diagram. This diagram is a constructive one.

The constructive diagram is the bridge between requirements and form. But its great beauty is that it goes deeper still. The same duality between requirement and form which the constructive diagram is able to express and unify also appears at a second level: the duality is itself characteristic of our knowledge of form.

Every form can be described in two ways: from the point of view of what it is, and from the point of view of what it does. What it is is sometimes called the formal description. What it does, when it is put in contact with other things, is sometimes called the functional description.

Here are some formal descriptions. A raincoat is three feet long, made of polythene $\frac{1}{2}$ mm thick, its sleeves cut in such and such a way, and so on. A salt crystal is a cubical arrangement of alternating sodium and chloride ions. A human body contains a heart, of such and such a size, in this position in the chest, a pair of kidneys rather lower and further back, and so on again. These descriptions specify size, position, pattern, material.

The corresponding functional descriptions tell you what happens when these objects are put in various contexts in the world. The raincoat is impervious to rain, and melts when heated. The salt crystal is transparent, conducts electricity slightly, dissolves in water but not in oil, shatters when hit hard with a hammer, and so on. The heart beats faster at high altitudes, the kidneys work when the body is fed.

In many of these cases we should find it hard to relate the two descriptions to one another, because we do not understand the objects thoroughly enough, and do not know, say, how the arrangement of atoms in a crystal relates to the solubility of the crystal in different solutes. However, for some very simple objects, there is virtually no rift between formal and functional descriptions. Take a soap bubble for instance, or a soap film on a wire frame. The behavior of soap films is so thoroughly understood that we know the functional properties of any given physical arrangement, and we know what shapes and sizes of bubbles different external conditions lead to.[10] In this case, the formal descriptions and the functional descriptions are just different ways of saying the same things; we can say, if we like, that we have a unified description of a soap bubble. This unified description is the abstract equivalent of a constructive diagram.

It is the aim of science to give such a unified description for every object and phenomenon we know. The task of chemistry (and it has been remarkably successfül in this) is to relate functional and formal descriptions of chemical compounds to one another, so that we can go backwards and forwards between the two, without loss in understanding. The task of physiology has been to relate the functional behavior of the body to the organs we observe in anatomy. Again, it has been reasonably successful.

The solution of a design problem is really only another effort to find a unified description. The search for realization through constructive diagrams is an effort to understand the required form so fully that there is no longer a rift between its functional specification and the shape it takes.[11]

In other words, a constructive diagram, if it is a good one, actually contributes to our understanding of the functional specification which calls it into being.

We have already seen, in Chapter 2, that the designer never really understands the context fully. He may know, piecemeal, what the context demands of the form. But he does not see the context as a single pattern — a unitary field of forces. If he is a good designer the form he invents will penetrate the problem so deeply that it not only solves it but illuminates it.

A well-designed house not only fits its context well but also illuminates the problem of just what the context is, and thereby clarifies the life which it accommodates. Thus Le Corbusier's invention of new house forms in the 1920's really represented part of the modern attempt to understand the twentieth century's new way of life.[12]

The airfoil wing section which allows airplanes to fly was invented at a time when it had just been "proved" that no machine heavier than air could fly. Its aerodynamic properties were not understood until some time after it had been in use. Indeed the invention and use of the airfoil made a substantial contribution to the development of aerodynamic theory, rather than vice versa.[13]

At the time of its invention the geodesic dome could not be calculated on the basis of the structural calculations then in use. Its invention not only solved a specific problem, but drew attention to a different way of thinking about load-bearing structures.[14]

In all these cases, the invention is based on a hunch which actually makes it easier to understand the problem. Like such a hunch, a constructive diagram will often precede the precise knowledge which could prescribe its shape on rational grounds.

It is therefore quite reasonable to think of the realization as a way of probing the context's nature, beyond the program but parallel to it. This is borne out, perhaps, by the recent tendency among designers to think of their designs as hypotheses.[15] Each constructive diagram is a tentative assumption about the nature of the context. Like a hypothesis, it relates an unclear set of forces to one another conceptually; like a hypothesis, it is usually improved by clarity and economy of notation.[16] Like a hypothesis, it cannot be obtained by deductive methods, but only by abstraction and invention. Like a hypothesis, it is rejected when a discrepancy turns up and shows that it fails to account for some new force in the context.

The constructive diagram can describe the context, and it can describe the form. It offers us a way of probing the context, and a way of searching for form. Because it manages to do both simultaneously, it offers us a bridge between requirements and form, and therefore is a most important tool in the process of design.

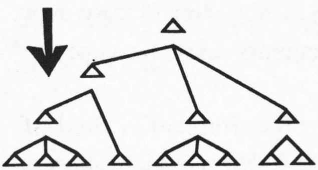
In all design tasks the designer has to translate sets of requirements into diagrams which capture their physical implications. In a literal sense these diagrams are no more than stages on the way to the specification of a form, like the circulation diagram of a building, or the expected population density map for some region under development. They specify only gross pattern aspects of the form. But the path from these diagrams to the final design is a matter of local detail. The form's basic organization is born precisely in the constructive diagrams which precede its design.

What we must now see is that the constructive diagram is not only useful in probing the more obvious, known aspects of a problem like circulation, but that it can also be used to create the newly discovered implications of a new problem. We have seen that the *extension* of any problem may be captured by a set of requirements; and that by the same token any new set of requirements may be regarded as the definition of a new problem. Going one step further, the *intension* (or physical meaning) of a known problem may be captured by a diagram; and by the same token the intension of any new, hitherto unconnected, set of requirements may be captured by a new diagram.[17]

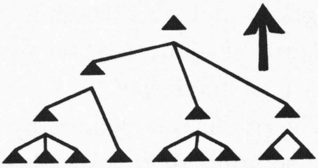
The problem is defined by a set of requirements called M. The solution to this problem will be a form which successfully satisfies all of these requirements. This form could be developed, in all its important details, as a single constructive diagram for the set M, if it were not for the complexity of M's internal interactions (represented by L), which makes it impossible to find such a diagram directly. Can we find it indirectly? Are there some simpler diagrams which the designer *can* construct, and which will contribute substantially to his ability to find a diagram for M? There are; and the program tells us how to find them.

The program is a hierarchy of the most significant subsets of M. Each subset is a subproblem with its own integrity. In the program the smallest sets fall together in larger sets; and these in turn again in larger sets. Each subset can be translated into a constructive diagram. And each of these subsets of M, because it contains fewer requirements than M itself, and less interaction between them, is simpler to diagram than M. It is therefore natural to begin by constructing diagrams for the smallest sets prescribed by the program. If we build up compound diagrams from these simplest diagrams according to the program's structure, and build up further compound diagrams from these in turn, we get a tree of diagrams. This tree of diagrams contains just one diagram for each set of requirements in the program's tree. We call it the realization of the program.

It is easy to bring out the contrast between the analytical nature of the program and the synthetic nature of its realiza-



Program, consisting of sets



Realization, consisting of diagrams

tion. As we see on the left, the tree of sets is obtained by successive division and partition. The tree of diagrams, on the right, is made by successive composition and fusion. At its apex is the last diagram, which captures the full implications of the whole problem, and is therefore the complete diagram for the form required. Examples of these two trees are given in Appendix 1.

8 / DEFINITIONS

We have seen roughly now how we shall try to represent a design problem by means of a graph, $G(M,L)$; that we shall then decompose the set M to give us a program; and how this program will be used as a basis for the construction of diagrams from which we can develop a form. We now come to the precise details of the analysis that defines the program. We begin, in this chapter, by establishing the exact character of the sets M and L which together provide us with the graph $G(M,L)$.

The problem presents itself, originally, when the ensemble is given, and when the proposed boundary between context and form, within that ensemble, is chosen. At this stage the problem is only defined within rather broad limits. Typical examples are these. We are to design a highway system for New York City; a kettle for use in the technical and cultural environment provided by metropolitan U.S.A. of 1965; a new town, for 30,000 people, forty miles from London. The context, in these cases, is fixed, and will remain constant for the duration of the problem; it may therefore be described in as much detail as possible. On the other hand, the nature of the required form is uncertain. It may be given a name, perhaps, like "kettle" or "town," to make the problem specific; but one of the designer's first tasks will be to strip the problem of the preconceptions which such names introduce.

Now, as we know already, the set M consists of all those possible kinds of misfit which might occur between the form and the context; in the case of the kettle–metropolitan U.S.A. ensemble, this set includes specific economic limitations, technical requirements of production, functional performance standards, matters of safety and appearance, and so on.[1] To be exact, each element of M is a variable which can be in one of two states: fit and misfit.[2] It is important to remember that the state of this variable depends on the entire ensemble. We cannot decide whether a misfit has occurred either by looking at the form alone, or by looking at the context alone. Misfit is a condition of the ensemble as a whole, which comes from the unsatisfactory interaction of the form and context.

Take capital cost. The variable's two states are "too expensive," which represents misfit, and "OK," which represents fit. If a kettle is too expensive, this describes a property of the kettle plus its context — that is, of the ensemble. Out of context, the kettle's price either exceeds or does not exceed various figures we can name: nothing more. Only its relation to the rest of the ensemble makes it "too expensive" or "all right." In other words, it depends on how much we can afford. Again, take the kettle's capacity. If we look at the kettle by itself, all we can say is that it holds such and such a quantity of water. We cannot say whether this is enough, until we see what the context demands. Again, the fact that the kettle does not hold enough water, or that it does, is a property of the form plus context taken as a whole. This fact, that the variable describes the ensemble as a whole, and never the form alone, leads to the following important principle. In principle, to decide whether or not a form meets a given requirement, we must construct it, put it in contact with the context in question, and test the ensemble so formed to see whether misfit occurs in it or not. You can only tell whether a kettle is comfortable enough to hold by picking it up. In principle, you can only decide whether a road is wide enough to drive down by constructing it, and trying to drive a car down it under the conditions it is supposed to meet.

Of course we do not stick to this principle in practice; it would be impossibly inconvenient if we had to. If we know the maximum width of cars to be used on the highway, and also know that for comfortable driving and adequate room for braking at a certain speed you need an extra 2′6″ on either side, we can tell in advance whether or not a given roadway is going to cause this kind of misfit or not. We can do so because the measurable character of the property "width" allows us to establish a connection between the width of the roadway and the likelihood of malfunction in the ensemble. What we do in such a case, to simplify the design task, is to establish a performance standard — in this case specifying that all roadways must have a minimum lane width of 11′0″ perhaps, because large cars are 6′ wide. We can then say, with a reasonable amount of confidence, that every road which meets this standard will not cause this misfit in the ensemble.

We can set up such a performance standard for every misfit variable that exhibits continuous variation along a well-defined scale. Other typical examples are acoustic separation of rooms (noise reduction can be expressed in decibels), illumination for comfortable reading (expressed in lumens per sq. ft.), load-bearing capacity required to prevent danger of structural failure (safety factor times maximum expected load), reasonable maintenance costs (expressed in dollars per year). Once a scale like this has been found for a requirement, it is then almost always possible to find a connection between this scale and some intrinsic property of the form;[3] thus, given a house design on the drawing board, it is possible to calculate probable maintenance costs, the noise reduction between rooms, and so on; it is then, of course, no longer necessary to find out by trial and error whether the form fails to fit its context in these respects. A performance standard determined by the context can be decided for each of them in advance, and used as a criterion of fit. For this reason there is a growing tendency to look for suitable scales, and to set up performance standards, for as many requirements as possible.[4]

However, the existence of a performance standard, and the association of a numerical scale with a misfit variable, does not mean that the misfit is any more keenly felt in the ensemble when it occurs. There are of course many, many misfits for which we do not have such a scale. Some typical examples are "boredom in an exhibition," "comfort for a kettle handle," "security for a fastener or a lock," "human warmth in a living room," "lack of variety in a park." No one has yet invented a scale for unhappiness or discomfort or uneasiness, and it is therefore not possible to set up performance standards for them. Yet these misfits are among the most critical which occur in design problems.

The importance of these nonquantifiable variables is sometimes lost in the effort to be "scientific." A variable which exhibits continuous variation is easier to manipulate mathematically, and therefore seems more suitable for a scientific treatment. But although it is certainly true that the use of performance standards makes it less necessary for a designer to rely on personal experience, it also happens that the kind of mathematical optimization which quantifiable variables make possible is largely irrelevant to the design problem.

A design problem is not an optimization problem.[5] In other words, it is not a problem of meeting any one requirement or any function of a number of requirements in the *best possible* way (though we may sometimes speak loosely as though it were, and may actually try to optimize one or two things like cost or construction time). For most requirements it is important only to satisfy them at a level which suffices to prevent misfit between the form and the context, and to do this in the least arbitrary manner possible.[6] This is a strictly binary situation. The task is to bring each binary variable to the value 0 (for continuous variables the value 0 corresponds to the whole range of values on the "good" side of the required performance standard). It is therefore only important that each variable be specific enough and clearly enough defined, so that any actual design can be classified unambiguously as a fit or misfit.

For quantifiable variables this is easy. An obvious example, in the case of the kettle, is the need for adequate capacity. Since the capacity of a kettle can be described quantitatively, we can therefore very easily set up a standard capacity which we require of satisfactory kettles, and call smaller capacity a misfit for kettles. Then we say that this variable takes the value 0 for kettles with a capacity greater than or equal to the critical capacity, and the value 1 for kettles with smaller capacity. The natural scale of capacity measurement provides an objective basis for dividing kettles into those which fit the context in this respect, and those which don't.

For nonquantifiable variables, it is not quite so easy. Take the property "comfortable to hold" for kettles. There is no objectively measurable property that is known to correlate well enough with comfort to serve as a scale of "comfortableness." However, such a misfit variable can still be well enough defined. We can set up communicable limits which a group of experts can understand well enough to agree about classifying designs. We can certainly explain what we mean by comfort clearly enough, in commonsense language, for a group of people to learn to agree about which kettles are comfortable to hold, and which are not. This makes comfortableness an acceptable variable, for the purpose of the present analysis.

We shall treat a property of the ensemble (quantifiable or not), as an acceptable misfit variable, provided we can associate with it an unambiguous way of dividing all possible forms into two classes: those for which we agree that they fit or meet the requirement, which we describe by saying that the variable takes the value 0, and those for which we do not agree, which therefore fail to meet the requirement, and for which the variable is assigned the value 1.

This brings us to three questions, which may seem hard to answer.

1. How can we get an exhaustive set of variables M for a given problem; in other words, how can we be sure we haven't left out some important issue?
2. How do we know that all the variables we include in the list M are relevant to the problem?
3. For any specific variable, how do we decide at what point misfit occurs; or if it is a continuous variable, how do we know what value to set as a performance

standard? In other words, how do we recognize the condition so far described as misfit?

These questions have already been answered, substantially, in Chapter 2. Let us remind ourselves of the fundamental principle. *Any state of affairs in the ensemble which derives from the interaction between form and context, and causes stress in the ensemble, is a misfit.*

This concept of stress or misfit is a primitive one. We shall proceed without defining it. We may find precedents for this in the practice of common law, psychiatry, medicine, engineering, anthropology, where it also serves as a primitive undefined concept.[7] In all these cases, stress is said to occur wherever it can be shown, in a common-sense way, that some state of affairs is somehow detrimental to the unity and well-being of the whole ensemble. In design too, though it may seem hard to define the concept of stress in theory, it is easy in practice. In architecture, for example, when the context is defined by a client, this client will tell you in no uncertain terms what he won't put up with. Again, it is obvious that a kettle which is uncomfortable to hold causes stress, since the context demands that it should be comfortable to hold. The fact that the kettle is for use by human hands makes this no more than common sense. At the opposite extreme, if somebody suggests that the ensemble is stressed if the kettle will not reflect ultraviolet radiation, common sense tells us to reject this — unless some special reason can be given, which shows what damage the absorption of ultraviolet does to the ensemble.

This principle that stress or misfit is a primitive concept has the following consequences. First of all, it is clearly not possible to list all the types of stress which might occur in an ensemble exhaustively, and therefore impossible to hope that M could provide an exhaustive description of a problem. A moment's thought will convince us that we are never capable of stating a design problem except in terms of the errors we have observed in past solutions to past problems. Even if we try to design something for an entirely new purpose that has never been conceived before, the best we can do in stating the problem is to anticipate how it might possibly go wrong by scanning mentally all the ways in which other things have gone wrong in the past.

The best we can do therefore is to include in M all those kinds of stress which we can imagine. The set M can never be properly called complete. The process of design, even when it has become selfconscious, remains a process of error-reduction, and the set M remains a temporary catalogue of those errors which seem to need correction.

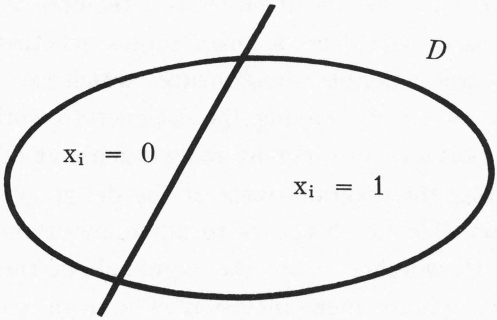
The fact that the design process must be viewed as an error-correcting process has a further consequence. The errors that seem most critical to one person will not be the same as those which seem most critical to another. Any list of errors or misfits, which are to be removed, therefore necessarily has something of a personal flavor.

For a problem like an urban dwelling, if we ask different designers to state the problem, we may find it hard even to get agreement about what the relevant issues are. Probably each designer has his private set of hunches about "where the issue really lies." The designer is free to look at a problem in any way he chooses; all we can hope to do is to put a fruitful structure on his view of it. It is for this reason that M cannot be thought of as objectively complete, and has been presented, instead, in Chapter 6, as a picture of a designer's view of a problem.

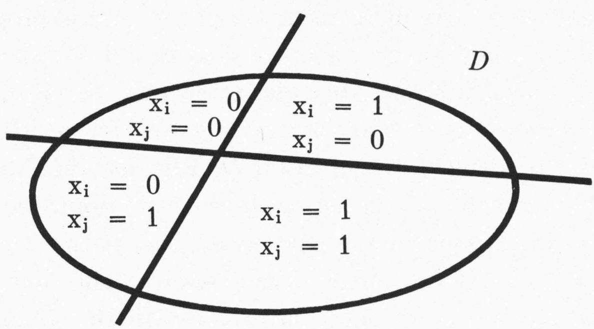
However, it should be pointed out that in spite of the natural bias which any one designer's statement of a problem is sure to carry, at the same time the use of the set M as a means of representation does have in it one great claim to neutrality. What designers disagree about is the relative importance of different requirements. In the present theory this would have to be expressed, if it were expressed at all, by assigning some sorts of weights or values to different variables. However, few designers will actually disagree about the variables themselves. While the relative importance of different requirements usually is a matter of personal opinion, the decision that a requirement either is a requirement or isn't, is less personal. The stress a misfit causes, whether slight or not, has simple tangible consequences which can be objectively determined. By leaving the designer to work out the relative importance of different requirements at his own discretion during the diagram phase of the design process, it is therefore possible for designers to agree about the contents of the set M, whether or not they agree about their relative importance, because mere inclusion of a requirement in M, as such, attaches no weight to it.

Before we say any more about the precise logical properties the misfit variables must have, we shall now define the interaction between variables. In order to do this, we must introduce a new concept: the domain of forms for which these variables are defined. Let us call it D. This domain D may be thought of roughly as the set of all those discriminable forms (good and bad) which might possibly be placed in contact with the given context to complete the ensemble. The contents of this domain cannot be specified precisely (if they could, the design problem would become a selection problem); the domain is imaginary, but serves to anchor the idea of inter-variable connections. We should think of it as the totality of possible forms within the cognitive reach of the designer. In other words, it is a shorthand way of talking about all those discriminable forms which a designer can imagine and design.[8]

Now, we know by postulate, that we can in principle decide, for each one of the forms in D, which requirements it meets, and which it fails to meet. This means that each misfit variable x_i cuts the domain D in two: into a set of those forms which fit, and a set of those which don't. Schematically we show this:



From two variables we get four sets, in which the forms take values as shown below.



If we superimpose all m variables, we get a division of the domain D into 2^m mutually exclusive classes, each labeled by a different pattern of values for $x_1 \cdots x_m$. We shall call the proportion of forms in D which do not satisfy requirement x_i the probability of the misfit x_i occurring. We write this $p(x_i = 1)$. (Naturally $0 \leqslant p(x_i = 1) \leqslant 1$.) In the same way we define the probability of avoiding the misfit x_i as $p(x_i = 0)$; and the probability of avoiding both x_i and x_j simultaneously as $p(x_i = 0, x_j = 0)$, and so forth.

If the variables $x_1 \cdots x_m$ are all pairwise independent then it is an axiom of probability theory that we may write $p(x_i = 0, x_j = 0) = p(x_i = 0) \cdot p(x_j = 0)$ for all i and j. And similarly if the variables are also three-way, four-way and n-way independent, then these independence relations hold for the conditional probabilities, and we write, for example, $p(x_i = 0, x_j = 0 \mid x_k = 1) = p(x_i = 0 \mid x_k = 1) \cdot p(x_j = 0 \mid x_k = 1)$ conditional on $x_k = 1$ and so on.[9]

Wherever the variables are not independent, the above relations break down. Essentially, then, we speak of a dependence among two variables wherever $p(x_i = 0, x_j = 0)$ is markedly unequal to $p(x_i = 0) \cdot p(x_j = 0)$, and similarly for more than two variables. Formally, we describe these dependences by means of the correlation coefficients.[10] The simplest correlation coefficient is that for two variables:[11]

$$c_{ij} = \frac{p(x_i{=}0,x_j{=}0) \cdot p(x_i{=}1,x_j{=}1) - p(x_i{=}0,x_j{=}1) \cdot p(x_i{=}1,x_j{=}0)}{[p(x_i{=}0)p(x_j{=}0)p(x_i{=}1)p(x_j{=}1)]^{\frac{1}{2}}}.$$

For any pair of variables x_i and x_j, then, we may distinguish the following three possibilities.

1. If c_{ij} is markedly less than 0, x_i and x_j conflict; like "The kettle's being too small" and "The kettle's oc-

cupying too much space." When we look for a form which avoids x_1 we weaken our chances of avoiding the other, x_2.

2. If c_{ij} is markedly greater than 0, x_i and x_j concur; like "the kettle's not being able to withstand the temperature of boiling water" and "the kettle's being liable to corrode in steamy kitchens." When we look for materials which avoid one of these difficulties, we improve our chances of avoiding the other.
3. If c_{ij} is not far from 0, x_i and x_j exhibit no noticeable interaction of either type.

In the first case we should write a negative link between the variables, in the second case we should write a positive link between them, and in the third case we should write no link at all between them. Roughly speaking, two requirements interact (and are therefore linked), if what you do about one of them in a design necessarily makes it more difficult or easier to do anything about the other.[12]

This at once suggests a simple way of estimating links, based on direct inspection of the known existing forms. Suppose we pick a sample of all the recently produced kettles we can find and examine it from the point of view of misfits x_i and x_j. Since we have defined each misfit variable in such a way that we can always decide which value it takes (0 or 1) in a given design, the proportions of kettles in our sample where x_i only has occurred $(x_i = 1, x_j = 0)$, where x_j only has occurred $(x_i = 0, x_j = 1)$, where both have occurred $(x_i = 1, x_j = 1)$, and where neither has occurred $(x_i = 0, x_j = 0)$, are easy to obtain. Provided the samples are carefully chosen, these sample proportions give us good estimates of the probability of x_i, of x_j, of both, of neither, occurring in a randomly selected contemporary kettle. From these joint two-variable probability estimates, we could compute the correlation c_{ij}, and write a link between any pair of variables whose correlation was statistically significant. We could use the same procedure to decide on the many-variable correlations.

However, such a method, being based on a sample of existing kettles, is not what we want at all. If we think carefully, we see that empirically found correlations have very different degrees of validity. Some are almost logically necessary — like the conflict between the need for sufficient capacity in the kettle and the need for economical storage space. The first calls for large volume, the second for small volume. This conflict exists almost by definition, at least until one is thinking of ways of heating water that are very much unlike kettles.[13]

Other correlations depend on physical laws — like the conflict between the need for a material which keeps the heat in after the kettle has boiled and the need for a material which allows the kettle water to be heated cheaply. It is hard to imagine a material whose thermal conductivity is different in opposite directions; so again, although there are ways round it, the conflict exists for most of the kettles one can imagine.

But other correlations will depend only on accidents of present taste and habit. If you look at kettles in the shops today, you might notice that the cheap ones have tin handles, and you might conclude that the need for safety when you pick up a hot kettle (that is, for a handle which doesn't burn you) conflicts with the economics of production and the need to keep down capital cost. However, this conclusion, being based on a sample of presently available kettles, will change as soon as we begin to think of other materials and designs. This conflict certainly does not exist for all imaginable kettles.

Clearly we want to avoid muddling this last kind of case with the other two. If we were to accept the linkage it suggests, then together with the essential logic of the ensemble we should also be freezing in its most temporary incidentals. We are interested in those links between variables which hold for all forms we can conceive (that is, for the whole of D). Any sample based on those possible solutions which happen to have been constructed is heavily biased toward the past. To avoid the bias we should need either to examine all the members of D exhaustively or to find a theory which offers us a way of sampling D unbiasedly. Neither of these is practicable today.

However, we may overcome the bias by another means. Instead of just looking for statistical connections between variables, we may try to find causal relations between them. Blind belief based only on observed regularity is not very strong, because it is not the result of a seen causal connection. But if we can invent an explanation for inter-variable correlation in terms of some conceptual model, we shall be much better inclined to believe in the regularity, because we shall then know which kinds of extraneous circumstances are likely to upset the regularity and which are not. We call a correlation "causal" in this second case, when we have some kind of understanding or model whose rules account for it.

For example, the molecular and crystalline structure of materials gives us good reason to believe that the thermal conductivity of a material is the same in any two opposite directions, and hence that the need to heat a kettle quickly conflicts with the need to keep the water hot once it has boiled. In this case, because we "understand" the connection between the two variables, we call it causal, and give it much greater weight — because we are convinced that it holds for almost all conceivable possibilities.

The search for causal relations of this sort cannot be mechanically experimental or statistical; it requires interpretation: to practice it we must adopt the same kind of common sense that we have to make use of all the time in the inductive part of science. The data of scientific method never go further than to display regularities. We put structure into them only by inference and interpretation.[14] In just the same way, the structural facts about a system of variables in an ensemble will come only from the thoughtful interpretation of observations.

We shall say that two variables interact if and only if the designer can find some reason (or conceptual model) which makes sense to him and tells him why they should do so.[15]

Again, as with the definition of the variables, this introduces a personal bias, and reminds us that L, like M, is a picture of the way the designer sees the problem, not an objective description of the problem itself. If the designer sees a conflict between the need to have sufficient capacity in a kettle and the need to conserve storage space, he does so because he has certain preconceptions in mind about the kinds of kettle which are possible. It is true that there are conceivable devices, not yet invented, for boiling water as it comes out of the faucet, and that these might take very little storage space. But until the designer understands this possibility, there is no point in telling him that the conflict is spurious; as far as he is concerned, there really is a conflict, which needs to be resolved, and therefore needs to be included in L and taken into account in the analysis of M. It is only after first including this link in L, and in the very act of asking himself whether two variables really do interact, and why they do, that the designer sees the possibility of avoiding the conflict and so sees further into the problem.

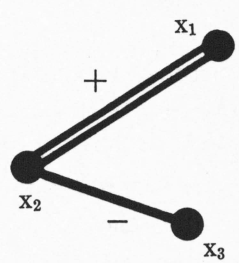
The reader may well ask how such a process, in which both the requirements and the links between requirements are defined by the designer from things already present in his mind, can possibly have any outcome which is not also already present in the designer's mind. In other words, how can all this process really be helpful? The answer is that, because it concentrates on structure, the process is able to make a coherent and therefore new whole out of incoherent pieces.

It is true that the designer must already have some physical ideas about the problem in his mind when he starts. In order to define requirements, he must be aware of the specific physical implications of each. In order to define links between requirements, he must be aware of the many specific ways in which these physical implications are likely to conflict and to concur. But the many piecemeal implications which the designer is aware of do not themselves amount to form. He is only able to define form at that moment when these physical implications coalesce in his mind, and take on organized shape. The process I am describing, as we shall see, helps precisely here, by forcing *organization* onto the specific but hitherto unorganized details in the designer's mind.

Undoubtedly the pattern of interactions in any real-world problem will have a great variety of different strengths. In one case two variables may conflict so strongly that they virtually exclude one another and can never take the same values at the same time. In another case, there may be no more than a barely discernible tendency for them to concur. But while an explicitly statistical test would give the interactions a continuous range of values, the *ad hoc* methods of practical common sense will hardly allow us to assign them a consistently scaled continuous range — particularly in view of the fact that different consultants may have incommensurable personal scales of evaluation, and that interactions which spring from different kinds of sources can be hard to compare. In practice we shall, at best, be able to distinguish two or three strengths of interaction.

In practice, then, we shall give each pair of variables (x_i,x_j) some small integral index, ν_{ij}, equal to 0 if there is no interaction, positive if there is concurrence, negative for conflict. It will usually be convenient to keep the absolute value of ν_{ij} less than or equal to some fixed integer ν. For the sake of consistent interpretation, assume that the link index ν_{ij} indicates a correlation of $\delta\nu_{ij}$, where δ is some arbitrary constant, such that $\delta\nu \leqslant 1$. We may display the values of the ν_{ij} in matrix form. The cell in the ith row and the jth column contains the value ν_{ij}. Thus the cell in the 1st row and the 2nd column $(i = 1, j = 2)$ contains ν_{12}. The matrix is symmetrical. Thus

$$\begin{array}{c|c|c|c|}
 & x_1 & x_2 & x_3 \\
\hline
x_1 & 0 & 2 & 0 \\
\hline
x_2 & 2 & 0 & -1 \\
\hline
x_3 & 0 & -1 & 0 \\
\hline
\end{array}$$



From this matrix we define the set L as a set of links associated with the variables of M, as follows.[16] For every pair of variables x_i and x_j, there are $|\nu_{ij}|$ distinct elements of L which join x_i to x_j. These elements bear the same sign as the index ν_{ij}, negative for conflict, and positive for concurrence.[17] The sets M and L together, completely define the graph $G(M,L)$.[18]

The definitions we have given so far still leave certain practical questions about the sets M and L unanswered. Does it matter, for instance, if two variables are very close in meaning, though slightly different? How specific or how general must they be? What do we do about three-variable interaction? The answers to these questions depend on three important formal properties of the system $G(M,L)$, which we shall now explore.

First of all, if the graph $G(M,L)$ is to give us an accurate picture of the variables' behavior, it is necessary that the set L describe *all* the interaction between variables which there is. Since the elements of L are links which represent two-variable correlation, this means that the variables must be chosen to be free from three-variable and higher-order correlations. The mathematics of Appendix 2 is also based on the assumption that the higher-order correlations vanish.[19] If this is not so, any analysis based on M and L alone is sure to give misleading results.

Second, even the two-variable correlation $\delta\nu_{ij}$ must be small, for each pair of variables. Specifically, as far as the mathematics of Appendix 2 is concerned, we must have $l\delta \leqslant 1$, where l is the total number of links in L.[20]

Third, the analysis in Appendix 2 is also based on the assumption of a certain simple symmetry among the variables of M. It demands that $p(x_i = 0)$ should be the same for all i.[21] Again, if this is not so, the analysis will be invalid.

Let us now consider the practical implications of these three formal properties which the system $G(M,L)$ must have. We take the last one first. It demands that $p(x_i = 0)$ should be the same for all i, or that the proportion of all thinkable forms which satisfy a requirement should be about the same for each requirement. What this amounts to, in common-sense language, is that all the variables should be roughly comparable in their scope and significance.

We cannot admit "economically satisfactory" as one requirement, and "maintenance costs low enough" as another. Plainly these have different degrees of significance, because the second is part of the first, while the first is not part of the second. Every design which is economically satisfactory must *a fortiori* have acceptable maintenance costs. But the reverse is not true. There are far more possible designs which meet the second than the first, because the first is much wider in scope and significance; their probabilities of occurrence are very unequal. In this case the inequality is especially clear because the second requirement is, as it were, contained in the first. But the difference would be just as great if we replaced the first by "functionally satisfactory." This is again wider in scope and significance than "maintenance costs low enough" even though it does not contain it. If we want to use "maintenance costs low enough" as one requirement, then we must break down "functionally satisfactory" into smaller, more specific requirements, comparable to it. The first step in constructing the set M is to make all its variables approximately equal in "size" or scope.[22]

Let us take the second of the three formal properties next.

In practice, of course, the preciseness of this mathematical expression is meaningless, since we judge the correlations "by eye," and do not obtain them numerically. What it does mean, in practice, though, is that we must be satisfied that all the variables are as independent as we can get them to be. An example should make this clear: Suppose the following two variables appear on our list, for the kettle problem.

1. "The kettle must heat water fast enough."
2. "The kettle must keep water hot once it is heated."

These two are clearly not at all independent. However, there are two fairly independent issues lurking behind them, if we can only find them. One way to bring this out would be by the following rearrangement, which covers more or less the same ground as the first pair, but consists of two more independent variables.

3. "The kettle must permit one-way heat transmission only."
4. "The kettle must have low thermal capacity."

A considerable amount of energy must be spent in the preliminary stages shuffling and reshuffling the variables in this fashion, until they are as independent as they can be made.[23]

The first formal property, that the three-variable or higher-order correlations among the elements of M should be negligible, is the hardest of all to achieve. It means that the two-variable correlation for any pair of variables must be independent of the states of all other variables. Since the state of one variable is most likely to affect the correlation between other variables, if that one variable is wide in scope the best we can do in satisfying this is to make all the individual variables as specific and minute as possible.

This policy of making all the variables highly specific is important for another reason. However much we may try to steer clear of existing categories, in practice we shall always have to generate the specific variables of M through intermediate stages. The brain is not made to think of such detailed lists amorphously. Whether we like it or not, if we think of one variable which has to do with acoustics, we shall inevitably then think of others which seem, to us, to fall under the same heading or to be in the same conceptual area. It is therefore a matter of practical psychology that we cannot avoid using superordinate concepts like "economics" and "acoustics" altogether, as intermediate steps in the task of listing misfit variables. At best we may treat these conceptual intermediates as key words, as loosely conceived labels for the principal issues in the problem, which we shall then break down further into finer pieces to get our set of variables M. The closer our variables are to these abstract and general key words, the more susceptible the problem remains to the kind of distortions discussed in Chapter 5. The more specific and detailed we make the variables, the less constrained $G(M,L)$ will be by previous conceptions, and the more open to detailed and unbiased examination of its causal structure.

Let us therefore sum up the properties the elements of M must have. They must be chosen (1) to be of equal scope, (2) to be as independent of one another as is reasonably possible, and (3) to be as small in scope and hence as specific and detailed and numerous as possible.[24] An example of a set M is given in Appendix 1, together with its associated set L.

9 / SOLUTION

We now have a graph $G(M,L)$ which represents the design problem. As we have seen in Chapter 6, to solve the problem, we shall try to decompose the set M in such a way that it gives us a helpful program for design. We shall now consider what criterion to use as a basis for decomposition.

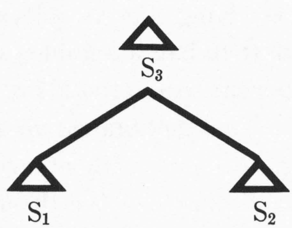
As we observed in Chapter 6, a program really gives us a series of simpler subproblems, and tells us in what order to solve them. Before we try to define a decomposition criterion we may want to question the assumption that such a decomposition can be of any use at all to a designer. The designer as a form-maker is looking for integrity (in the sense of singleness); he wishes to form a unit, to synthesize, to bring elements together. A design program's origin, on the other hand, is analytical, and its effect is to fragment the problem. The opposition between these two aims, analysis and synthesis, has sometimes led people to maintain that in design intellect and art are incompatible, and that no analytical process can help a designer form unified well-organized designs.

Let us look at this objection to analysis more closely. It is a common experience that attempts to solve one piece of a problem first, then others, and so on, lead to endless involutions. You no sooner solve one aspect of a thing than another is put out of joint. And when you go back to correct that one, something else goes wrong. You go round and round in circles, unable ever to produce a form which is thoroughly right, because there is no way of integrating the pieces you have tackled independently. This is the great argument against attempts to solve design problems piecemeal. And it is argued further that, since no amount of analyzable juggling can ever solve this difficulty, the designer has to rely on a subconscious creative force to juggle the pieces more successfully. His hand and eye must be secure enough, in other words, to take him to his answer more immediately than his intelligence can. If design problems were homogeneous, this recommendation would be important. For then any analytical subdivision would, so to speak, put cracks in them, which would destroy their unity. As it happens though, in practice problems are not homogeneous. They are full of knots and crevices which exhibit a well-defined structure. An analytical process fails only if it does not take this structure into account. If we can learn to draw the gross structural components of the problem out of the graph $G(M,L)$ which represents it, the difficulty will disappear.

The question is, how are these separable structural components of a problem to be recognized? We face this kind of task every day, constantly, even when we see nothing more complicated than a pair of oranges on a table side by side. In seeing two oranges lying side by side, and not one and a half oranges lying next to half an orange, we have recognized the structural components correctly. (Correctly, of course, because while we can pick either orange up and leave the other where it is, we cannot pick up $1\frac{1}{2}$ oranges, and leave $\frac{1}{2}$ an orange lying there.) Köhler and Wertheimer drew attention to the fact that even an apparently simple cognitive act like this, in fact demands a very complex perceptual operation.[1] It is not surprising to find, in the similar but more abstract task of recognizing the proper structural components of the system M, that our native perception and intuition fail us.

The task of replacing this intuition by some precisely defined mathematical operation has been tackled in a number of ways.[2] Many of them are worth examining, if for no better reason than that they will illustrate and deepen our conception of the task. One, which perhaps comes closest to what we want, simply divides M into those subsets which are connected by as few links of L as possible, thus leaving as many of the links as possible within the subsystems.[3] However, neither this nor any other of the existing methods is exactly suited to the conditions which confront us in this case. I shall now try to show that we can develop a well-defined criterion for decomposition, simply by thinking carefully about the relation between a design program and its realization.

Let us think just what the successful realization of the program demands. Fundamentally, it demands that the sets in the program have two kinds of property, which we may illustrate by taking the typical piece of a program shown below. S_1 and S_2 are two different sets of requirements. S_3 contains all the requirements in S_1 and S_2 together.



First we must be able to find constructive diagrams for S_1 and S_2 individually. This means that the misfits which S_1 contains must cohere somehow, and suggest a physical aspect or component of the form under consideration; and the same for S_2.

Secondly, if the decomposition is to serve any useful purpose, it must not be necessary to construct the diagram for S_3 from scratch. Instead, it must be possible to derive a constructive diagram for S_3 in some simple way from the diagrams already constructed for S_1 and S_2 in isolation.

To put it simply, the first of these conditions depends on the internal structure of the sets S_1 and S_2, while the second deals with the relations between these two sets.

Let us take the two conditions in order.

What is it about the internal structure of any problem that makes it hard to solve? In nine cases out of ten, we cannot solve it, because we cannot grasp it; we cannot see what the internal structure is "driving at." The subproblems we are considering here, because they are made up of sets of requirements that have been isolated from the rest of the design problem they belong to, show this acutely. Take two misfits at random. "The kettle must be comfortable for the hand to hold," and "The kettle must be economical to heat," which we should probably consider as noninteracting. These two define a two-element subset of M for the kettle problem. It is hard to see, however, what these two elements have in common, or indeed whether this set, taken by itself, means anything.

If the set M contains m misfits, there are 2^m possible subsets of M, and so 2^m subsidiary problems. Any design problem of practical interest and complexity will probably contain at least as many as 100 variables, and will therefore have 2^{100} or roughly 10^{30} (1,000,000,000,000,000,000,000,000,000,000) different subsets of variables. Almost each one of these subsets will be hard to grasp, because, as in the example of the two-element subset just given, it will not be clear what its rather disparate member-variables "have in common."

Our natural reaction to this is to look for those very rare sets of variables with integrity in which the variables do "have something in common," so that they do make sense.

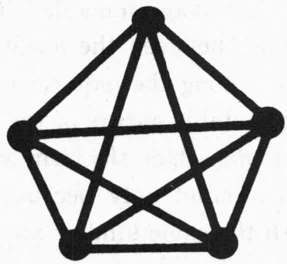
The use of verbal concepts is an efficient artificial way of finding sets which have something in common. Certain issues which appear in our analysis as subsets of *M*, happen to be tied together by familiar words; as a result everyone comes to be able to manipulate these sets, can understand what he is dealing with, and can therefore get to grips with the issues the set represents.

Unfortunately, the sets of misfits identified by verbal concepts do not have any special functional significance, and do not usually lend themselves particularly well to interpretation through constructive diagrams. A constructive diagram requires that the requirements it represents have some physical implications in common. From this point of view, it is easy to see that not all the possible subsets of *M* will be equally easy to diagram constructively. We may put this another way, perhaps, by saying that some subsets open up physical possibilities more readily than others. Some sets of misfits, in view of their interactions, seem naturally to belong together, and, taken as units, suggest physical form very strongly. Others will seem to have no special reason for being sets, and are not especially easy to diagram, and do not really "belong" to the problem.

If we are to make anything sensible of the subsets in this program, we must now ask just which sets of points to consider as being the most "diagrammable." This depends on the pattern of interactions between the misfits. Where, after all, does the interaction among the requirements spring from? It springs from the intractable nature of the available materials and the conditions under which the form has to be made. Two misfits are seen to interact only because, in some sense at least, they deal with the same kind of physical consideration. If they dealt with utterly different aspects, there could be no basis either for conflict or for concurrence.

In building, the need for acoustic insulation conflicts with the need to build with easily transportable prefabricated materials. These two needs conflict because the first calls for massive inert walls, while the second calls for light walls. The physical feature of the world their interaction depends on is mass. Again, in a highway, the need for safety on curves conflicts with the need to keep land costs down, because the wider the curves have to be for safety's sake, the larger the area eaten up by the transition curves at interchanges. In this case the interaction between the two requirements depends on the radius of the curve.

It is such a physical center of implication, if I may call it that, which the designer finds it easy to grasp. Because it refers to a distinguishable physical property or entity, it can be expressed diagrammatically, and provides a possible nonverbal point of entry into the problem. If we can find sets of variables in which there are specially dense interactions, we may assume, in these cases, that the density of the interaction resides in a particularly strong identifiable physical aspect of the problem. These sets will be the easiest of all to grasp constructively. Thus:

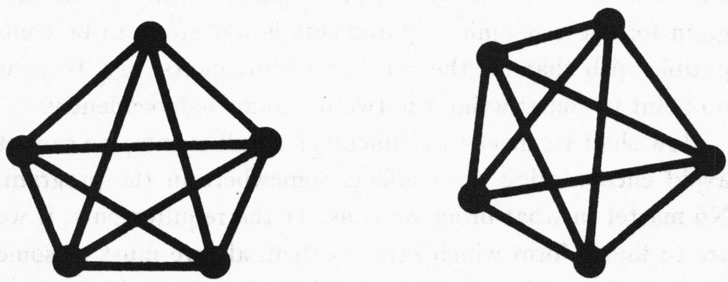


If, therefore, we break the problem apart in such a way that its clusters of variables are as richly connected, internally, as possible, we shall have clues to those physical aspects of the problem which play the most important functional part in the problem and are therefore most likely to furnish handles for the designer's comprehension. These are the sets which will be the easiest to diagram.

If we are to solve the problem M by working our way through the program, solving various subproblems separately, it must obviously be possible to put the resulting diagrams together somehow when we have them. This is the second condition a successful program must satisfy. But it will only be possible to fuse two diagrams under very special circumstances. Why, for instance, can we not simply make a diagram for each separate variable, so that we get m diagrams, and superimpose these m diagrams somehow? The reason is obvious. The physical characteristics demanded by one requirement conflict with the physical characteristics demanded by another. This is, in fact, exactly what we mean by saying that two misfit variables conflict. The same is true of more complex diagrams. We have already drawn attention to the fact that a subset which contained all the economic variables and no others, for example, would be comparatively useless, because its economic implications conflict too strongly with the other implications of the problem. Naturally if the diagram for the economic requirements is not going to be compatible with that for the comfort requirements, say, there is no point in constructing the two diagrams independently.

How shall we meet this difficulty? At all events, we cannot avoid encountering the conflicts somewhere in the program. No matter in what order we consider the requirements, if we are to find a form which satisfies them all, we must at some stage resolve each one of the conflicts. But if we think about it, we see that the difficulty of resolving them is different at different stages of the process of realization. At the beginning of the process, the sets of requirements we apply ourselves to are still small enough for their implications to be carried in the mind's eye; and these implications are therefore not yet frozen in any explicit diagrammatic form; they are still flexible enough to be successfully integrated with one another in spite of conflicts. The further along in the process we are, the more our thoughts about these implications have been forced by their complexity to become concrete, whether diagrammatically or conceptually, and the more their rigidity resists further modification. As a result, the later in the process conflicting diagrams have to be integrated, the more difficult the integration is.

Naturally, then, since the conflicts have to be resolved sooner or later, we should like to meet them as early in the process of realization as we can, while our ideas are still flexible. From this point of view, the fewer links there are between the major subsets of the decomposition, the better. Ideally, we should like to find a first partition of M like this, for instance, where no links are cut by the partition, though this will not in practice usually be possible.[4]



The need for subsets we can grasp diagrammatically calls for sets of variables whose *internal* interactions are very rich. The need to resolve the conflicts between the diagrams we get from them calls for as little interaction *between* subsets as possible. Clearly these two are compatible; indeed, they can be expressed jointly as follows.

Consider just one level of the decomposition, where some set S is to be partitioned into disjoint subsets $(S_1, S_2 \cdots S_\alpha \cdots S_\mu)$. We wish to choose these S_α in such a way that we can invent a constructive diagram for S_1 whose implications will not later turn out to be hopelessly contradicted by an independently conceived diagram for one of the other S_α; and the same for S_2, S_3, etcetera. Why is this difficult to do in terms of the variables' behavior?

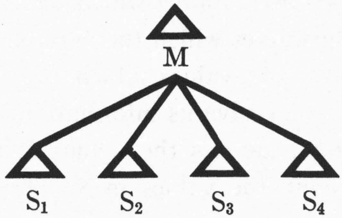
It is difficult because any variables which are linked exercise mutual constraint over one another's states. If we fix the values of the variables of S_1, the values which the variables of the other S_α can take are already constrained to some extent by the probabilistic links which bind them to this S_1. In other words, the values which the variables of S_1 take, tell us something about the values which the variables in the other S_α can take; they give us information. The sparser the links between the S_α, the less the values of the variables in S_1 can tell us about the values in S_2, etc.; the less information the links carry across the partition, the freer we are to construct a diagram for S_2 once we have fixed the solution of S_1 in our minds.

If we wish to construct a diagram for S_1 first, say, and then wish to construct a compatible diagram for S_2 independently, we want to be free to manipulate the values of the variables in S_2 without this manipulation being constrained by the fact that the variables of S_1 are now held constant in our minds, by the diagrammatic expression invented for them. To achieve this, we must choose the S_α in such a way that the variables in different subsets of the partition exercise as little informational constraint on one another as possible.

As shown in Appendix 2, the conditions specified in Chapter 8 define a unique probability distribution $p(\lambda)$ over the states λ of any set of variables S.[5] Appendix 2 then shows that, given any partition π of a set S into subsets, $\pi\{S_1 \cdots S_\mu\}$, we may establish a measure of information transfer or informational dependence among these subsets, called $R(\pi)$.[6] Since this $R(\pi)$ is defined for all possible partitions of any S, we may obtain the desired decomposition of the set M, by minimizing $R(\pi)$ for successive partitions of M and its descendants.

Thus, we first find that partition of M, $\pi(M)$, for which

$R(\pi)$ is minimum. This establishes the first level of the decomposition, thus, say:



We then apply the same method to the sets S_α: we look for that partition $\pi(S_1)$ of S_1, for which $R(\pi)$ is minimum, and similarly for $S_2 \cdots$, thus obtaining the second level of the decomposition. We continue with this procedure iteratively, until we reach a level of decomposition at which all the sets contain one variable only. (Condition 4 of Chapter 6, page 82.)

The tree of sets this decomposition gives is, within the terms of this book, a complete structural description of the design problem defined by M; and it therefore serves as a program for the synthesis of a form which solves this problem.

Let us remember the properties of the program.

1. The tree is, in its hierarchical form, the same as any other hierarchy of concepts — except that the concepts are here defined extensionally as sets of variables, rather than intensionally by meaning.
2. The particular tree arrived at by the method outlined gives an explicit description of the structure implicitly responsible for the success and stability of the unselfconscious form-making process.
3. The tree gives the strongest possible decomposition of the problem that does not interfere with the task

of synthesizing its parts in a unified way. Each subsidiary problem it defines has its own integrity, and is as independent as it can be of the rest of the problem.

4. We must remember that the hierarchy of sets which the tree defines will not always be easy to understand. Even in some of the smaller sets which contain only half a dozen variables, these variables will often seem disparate, and their juxtaposition may be startling. The relevance of each variable is only to be properly understood after careful examination of its functional relation to the other variables in the set. Since the potential coherence of such a set of variables comes from its physical implications, it can only be grasped graphically, by means of a constructive diagram that brings out these implications. Each diagram for a set *S* must do two things:

   As a requirement diagram:

   a. It must bring out just those features of the problem which are relevant to this set of requirements.
   b. It must include no information which is not explicitly called for by these requirements.

   As a form diagram:

   a. It must be so specific that it has all the physical characteristics called for by the requirements of *S*.
   b. Yet it must be so general that it contains no arbitrary characteristics, and so summarizes, abstractly, the nature of every form which might satisfy *S*.

   Above all, the designer must resist the temptation to summarize the contents of the tree in terms of well-known verbal concepts. He must not expect to be able to see for every *S* some verbal paradigm like "This one deals with the acoustic aspects of the form." If he tries to do that, he denies the whole purpose of the analysis, by allowing verbal preconceptions to interfere with the pattern which the program shows him. The effect of the design program is that each set of requirements draws his attention to just one major physical and functional issue, rather than to some verbal or preconceived issue. It thereby forces him to consolidate the physical ideas present in his mind as seedlings, and to make physical order out of them.

To finish this section I give an example of the way a set of requirements, when taken together, create a new idea about what one main feature of a physical form ought to be. Consider the design of the now familiar one-hole kettle. The single wide short spout embraces a number of requirements: all those which center round the problems of getting water in and out of the kettle, the problem of doing it safely without the lid's falling off, the problem of making manufacture as simple as possible, the problem of providing warning when the kettle boils, the need for internal maintenance. In the old kettles these requirements were met separately by three components: a spout for pouring, a hole in the top for filling and cleaning, and a top which kept the steam in and rattled when the kettle boiled. Suddenly, when it became possible to put non-corrosive metals on the market, and cheap, available descaler made it unnecessary to get into the kettle for descaling, it became apparent that all these requirements really had a single center of physical implication, not three. The wide spout can be used for filling and pouring, and as a whistle, and there is no top to fall open and let scalding water out over the pourer's hands. The set of requirements, once its unity is recognized, leads to a single physical component of the kettle.

The program, which represents a functional decomposition of the problem, is a way of identifying the problem's major functional aspects. But what kind of physical form, exactly, is the designer likely to realize with the help of such a program? Let us look at the form problem from the beginning.

The organization of any complex physical object is hierarchical. It is true that, if we wish, we may dismiss this observation as an hallucination caused by the way the human brain, being disposed to see in terms of articulations and hierarchies, perceives the world. On the whole, though, there are good reasons to believe in the hierarchical subdivision of the world as an objective feature of reality. Indeed, many scientists, trying to understand the physical world, find that they have first to identify its physical components, much as I have argued in these notes for isolating the abstract components of a problem. To understand the human body you need to know what to consider as its principal functional and structural divisions. You cannot understand it until you recognize the nervous system, the hormonal system, the vasomotor system, the heart, the arms, legs, trunk, head, and so on as entities.[7] You cannot understand chemistry without knowing the pieces of which molecules are made. You cannot claim to have much understanding of the universe until you recognize its galaxies as important pieces. You cannot understand the modern city until you know that although roads are physically intertwined with the distribution of services, the two remain functionally distinct.

Scientists try to identify the components of existing structure. Designers try to shape the components of new structures. The search for the right components, and the right way to build the form up from these components, is the greatest physical challenge faced by the designer. I believe that if the hierarchical program is intelligently used, it offers the key to this very basic problem — and will actually point to the major physical components of which the form should consist.

When we consider the kinds of constructive diagram which are likely to be suggested by sets of requirements, at first it seems that the nature of these diagrams is very various. Some diagrams seem to define overall pattern properties of the form, like being circular, being low rather than high, being homogeneous. Other diagrams seem to be piecelike rather than patternlike. They define pieces of which the whole form is made, like a diagram defining the street as a piece of the city, or the handle as a piece of the kettle, and so on.

Actually the distinction between patternlike and piecelike diagrams is more apparent than real. Take a simple example, a diagram which specifies a circular plan. Being circular is usually thought of as an overall property of a plan. But the plan's being circular may also be guaranteed by a surrounding wall or boundary of some sort. In other words, we can invest what is apparently a pattern property in a component which is much more of a piece: namely the boundary.

This is the general rule. Every aspect of a form, whether piecelike or patternlike, can be understood as a structure of components. Every object is a hierarchy of components, the large ones specifying the pattern of distribution of the smaller ones, the small ones themselves, though at first sight more clearly piecelike, in fact again patterns specifying the arrangement and distribution of still smaller components.

Every component has this twofold nature: it is first a unit, and second a pattern, both a pattern and a unit. Its nature as a unit makes it an entity distinct from its surroundings. Its nature as a pattern specifies the arrangement of its own component units. It is the culmination of the designer's task to make every diagram both a pattern and a unit. As a unit it will fit into the hierarchy of larger components that fall above it; as a pattern it will specify the hierarchy of smaller components which it itself is made of.

The hierarchical composition of these diagrams will then lead to a physical object whose structural hierarchy is the exact counterpart of the functional hierarchy established during the analysis of the problem; as the program clarifies the component *sources* of the form's structure, so its realization, in parallel, will actually begin to define the form's *physical* components and their hierarchical organization.[8]

EPILOGUE

My main task has been to show that there is a deep and important underlying structural correspondence between the pattern of a problem and the process of designing a physical form which answers that problem. I believe that the great architect has in the past always been aware of the patterned similarity of problem and process, and that it is only the sense of this similarity of structure that ever led him to the design of great forms.

The same pattern is implicit in the action of the unself-conscious form-producing system, and responsible for its success. But before we can ourselves turn a problem into form, because we are selfconscious, we need to make explicit maps of the problem's structure, and therefore need first to invent a conceptual framework for such maps. This is all that I have tried to do.

Since my effort may well meet with resistance, I like to see the few steps taken here reflected in a parable of an imaginary past society.

Suppose there was once a people who had no formalized arithmetic. When they wanted what we think of as arithmetical results, they got them by guessing. So if they wished to know the area of a corn patch they paced its two sides (six paces by ten paces, say), and then mulled the two numbers over. Eventually one of them came up with an answer — he would say some number, that is, which estimated the bags of corn needed to sow that patch. He might say 60, 61, 58, whatever occurred to him. (If we were in such a situation we should form what we call the prod-

uct of the two numbers, 60, and determine the amount of corn needed in terms of this area.)

It is easy to see that the people of this imaginary society might not have found formal arithmetic acceptable. Their own method was usually not too far off the mark (sowing corn is such a loose test, anyway, that what we call inaccuracy would not have been noticeable) — and besides, there was something rather noble about the seers (magicians?) who performed the tasks of "calculation." Some men were better at it than others, certainly; some had the power to produce appropriate answers, some produced answers rather wider of the mark. But that didn't seem to matter. Instead the power was regarded as a great human gift, the people who possessed it were honored for their capability. And both these seers themselves and their admirers opposed the introduction of a formalized arithmetic most rigidly, did not see the possible developments, were interested only in preserving their own limited capacities for calculation.

Such resistance was not altogether foolish either. There were wise men, too, among those who opposed arithmetic. They foresaw, correctly, the materialism which it would induce. Its very first achievement, once introduced, would be to make calculation more precise and easier, and thereby to save corn. And soon number and economy and size would dominate the human being. The immediate good done by the formulation of arithmetic would be small, and not worth taking risks for on its own account.

What neither the wise men nor the seers foresaw, however, was the miraculous developments that this formulation later led to. By first understanding the shape of the technique which produced the form of the result, man found further insight. He found that it is not only the result which is important, but the process too. Not only the form of the results, but the form of the path which led to them. It was only by questioning the foundations of geometry and the processes of geometrical proof that Riemann *invented the geometry which later became the basis for Einstein's theory of relativity. Other great theorems are possible today because multiplication and addition were once defined. It was only because man gave thought to the seemingly obvious processes which underlay arithmetic that he was able to refine mathematics, and able to proceed to forms of still higher order, mathematical shapes of greater elegance and fuller understanding.*

The shapes of mathematics are abstract, of course, and the shapes of architecture concrete and human. But that difference is inessential. The crucial quality of shape, no matter of what kind, lies in its organization, and when we think of it this way we call it form. Man's feeling for mathematical form was able to develop only from his feeling for the processes of proof. I believe that our feeling for architectural form can never reach a comparable order of development, until we too have first learned a comparable feeling for the process of design.

APPENDICES

APPENDIX 1 / A WORKED EXAMPLE

Here is a worked example, taken from a recent paper, "The Determination of Components for an Indian Village." * The problem treated is this. An agricultural village of six hundred people is to be reorganized to make it fit present and future conditions developing in rural India.

The set *M*, which follows, contains all the misfit variables that are pertinent to the organization of the village. All these misfit variables are stated here in their positive form; that is, as needs or requirements which must be satisfied positively in a properly functioning village. They are, however, all derived from statements about potential misfits: each one represents some aspect of the village which could go wrong, and is therefore a misfit variable in the terms of Chapter 2.

M includes variables which represent three different kinds of need:

(1) all those which are explicitly felt by villagers themselves as needs,

(2) all those which are called for by national and regional economy and social purpose, and

(3) all those already satisfied implicitly in the present village (which are required, though not felt as needs by anybody).

(The headings on the left are for convenience in the listing stage only, and play no part in the subsequent analysis.)

* In Christopher Jones, ed., *Conference on Design Method* (Oxford: Pergamon, 1963). My lists and diagrams are reproduced here by kind permission of Pergamon Press.

Religion and Caste

1. Harijans regarded as ritually impure, untouchable, etc.
2. Proper disposal of dead.
3. Rules about house door not facing south.
4. Certain water and certain trees are thought of as sacred.
5. Provision for festivals and religious meetings.
6. Wish for temples.
7. Cattle treated as sacred, and vegetarian attitude.
8. Members of castes maintain their caste profession as far as possible.
9. Members of one caste like to be together and separate from others, and will not eat or drink together.
10. Need for elaborate weddings.

Social Forces

11. Marriage is to person from another village.
12. Extended family is in one house.
13. Family solidarity and neighborliness even after separation.
14. Economic integration of village on payment-in-kind basis.
15. Modern move toward payment in cash.
16. Women gossip extensively while bathing, fetching water, on way to field latrines, etc.
17. Village has fixed men's social groups.
18. Need to divide land among sons of successive generations.
19. People want to own land personally.
20. People of different factions prefer to have no contact.
21. Eradication of untouchability.
22. Abolition of Zamindari and uneven land distribution.
23. Men's groups chatting, smoking, even late at night.
24. Place for village events — dancing, plays, singing, etc., wrestling.
25. Assistance for physically handicapped, aged, widows.
26. Sentimental system: wish not to destroy old way of life; love of present habits governing bathing, food, etc.

27. Family is authoritarian.
28. Proper boundaries of ownership and maintenance responsibility.
29. Provision for daily bath, segregated by sex, caste, and age.

Agriculture

30. Efficient and rapid distribution of seeds, fertilizer, etc., from block HQ.
31. Efficient distribution of fertilizer, manure, seed, from village storage to fields.
32. Reclamation and use of uncultivated land.
33. Fertile land to be used to best advantage.
34. Full collection of natural manure (animal and human).
35. Protection of crops from insects, weeds, disease.
36. Protection of crops from thieves, cattle, goats, monkeys, etc.
37. Provision of storage for distributing and marketing crops.
38. Provision of threshing floor and its protection from marauders.
39. Best cotton and cash crop.
40. Best food grain crop.
41. Good vegetable crop.
42. Efficient plowing, weeding, harvesting, leveling.
43. Consolidation of land.
44. Crops must be brought home from fields.
45. Development of horticulture.
46. Respect for traditional agricultural practices.
47. Need for new implements when old ones are damaged, etc.
48. Scarcity of land.
49. Cooperative farming.

Animal Husbandry

50. Protected storage of fodder.
51. Improve quality of fodder available.
52. Improve quantity of fodder available.
53. Upgrading of cattle.
54. Provision for feeding cattle.
55. Cattle access to water.

56. Sheltered accommodation for cattle (sleeping, milking, feeding).
57. Protection of cattle from disease.
58. Development of other animal industry.
59. Efficient use and marketing of dairy products.
60. Minimize the use of animal traction to take pressure off shortage.

Employment

61. Sufficient fluid employment for laborers temporarily (seasonally) out of work.
62. Provision of cottage industry and artisan workshops and training.
63. Development of village industry.
64. Simplify the mobility of labor, to and from villages, and to and from fields and industries and houses.
65. Diversification of villages' economic base — not all occupations agricultural.
66. Efficient provision and use of power.

Water

67. Drinking water to be good, sweet.
68. Easy access to drinking water.
69. Fullest possible irrigation benefit derived from available water.
70. Full collection of underground water for irrigation.
71. Full collection of monsoon water for use.
72. Prevent famine if monsoon fails.
73. Conservation of water resources for future.
74. Maintenance of irrigation facilities.
75. Drainage of land to prevent waterlogging, etc.
76. Flood control to protect houses, roads, etc.

Material Welfare

77. Village and individual houses must be protected from fire.
78. Shade for sitting and walking.
79. Provision of cool breeze.
80. Security for cattle.

81. Security for women and children.
82. Provision for children to play (under supervision).
83. In summer people sleep in open.
84. Accommodation for panchayat records, meetings, etc.
85. Everyone's accommodation for sitting and sleeping should be protected from rain.
86. No overcrowding.
87. Safe storage of goods.
88. Place to wash and dry clothes.
89. Provision of goods, for sale.
90. Better provision for preparing meals.
91. Provision and storage of fuel.
92. House has to be cleaned, washed, drained.
93. Lighting.

Transportation

94. Provision for animal traffic.
95. Access to bus as near as possible.
96. Access to railway station.
97. Minimize transportation costs for bulk produce (grain, potatoes, etc.).
98. Daily produce requires cheap and constant (monsoon) access to market.
99. Industry requires strong transportation support.
100. Provision for bicycle age in every village by 1965.
101. Pedestrian traffic within village.
102. Accommodation for processions.
103. Bullock cart access to house for bulk of grain, fodder.

Forests and Soils

104. Plant ecology to be kept healthy.
105. Insufficient forest land.
106. Young trees need protection from goats, etc.
107. Soil conservation.
108. Road and dwelling erosion.
109. Reclamation of eroded land, gullies, etc.
110. Prevent land erosion.

Education

111. Provision for primary education.
112. Access to a secondary school.
113. Good attendance in school.
114. Development of women's independent activities.
115. Opportunity for youth activities.
116. Improvement of adult literacy.
117. Spread of information about birth control, disease, etc.
118. Demonstration projects which spread by example.
119. Efficient use of school; no distraction of students.

Health

120. Curative measures for disease available to villagers.
121. Facilities for birth, pre- and post-natal care, birth control.
122. Disposal of human excreta.
123. Prevent breeding germs and disease starters.
124. Prevent spread of human disease by carriers, infection, contagion.
125. Prevent malnutrition.

Implementation

126. Close contact with village-level worker.
127. Contact with block development officer and extension officers.
128. Price assurance for crops.
129. Factions refuse to cooperate or agree.
130. Need for increased incentives and aspirations.
131. Panchayat must have more power and respect.
132. Need to develop projects which benefit from government subsidies.

Regional, Political, and National Development

133. Social integration with neighboring villages.
134. Wish to keep up with achievements of neighboring villages.
135. Spread of official information about taxes, elections, etc.

136. Accommodation of wandering caste groups, incoming labor, etc.
137. Radio communication.
138. Achieve economic independence so as not to strain national transportation and resources.
139. Proper connection with bridges, roads, hospitals, schools, proposed at the district level.
140. Develop rural community spirit: destroy selfishness, isolationism.
141. Prevent migration of young people and harijans to cities.

This defines the set M.

The links between these misfit variables are tabulated below. For the sake of simplicity, I allowed only one strength of link, so that $\nu = 1$, and for every pair of variables $\nu_{ij} = 0, 1$, or -1. Further, the signs of the links are not indicated: as we shall see in Appendix 2, the decomposition turns out to be independent of the link signs. The table below simply shows those linked pairs of variables for which $\nu_{ij} = 1$ or -1.

1 interacts with 8, 9, 12, 13, 14, 21, 28, 29, 48, 61, 67, 68, 70, 77, 86, 101, 106, 113, 124, 140, 141.

2 interacts with 3, 4, 6, 26, 29, 32, 52, 71, 98, 102, 105, 123, 133.

3 interacts with 2, 12, 13, 17, 26, 76, 78, 79, 88, 101, 103, 119.

4 interacts with 2, 5, 6, 17, 29, 32, 45, 56, 63, 71, 74, 78, 79, 88, 91, 105, 106, 110, 124.

5 interacts with 4, 6, 10, 14, 17, 21, 24, 46, 102, 113, 116, 118, 131, 133, 140.

6 interacts with 2, 4, 5, 20, 21, 53, 58, 61, 63, 82, 102, 111, 117, 130, 134, 135.

7 interacts with 20, 31, 34, 53, 57, 58, 59, 80, 85, 86, 94, 105, 106, 123, 124, 125.

8 interacts with 1, 9, 14, 15, 21, 22, 25, 27, 48, 58, 59, 61, 62, 63, 64, 65, 89, 95, 96, 99, 111, 112, 114, 115, 116, 121, 129, 136, 140, 141.

9 interacts with 1, 8, 11, 12, 13, 15, 17, 18, 20, 21, 28, 29, 36, 43, 49, 56, 62, 64, 80, 81, 101, 113, 118, 124, 129, 136, 140, 141.

10 interacts with 5, 13, 14, 15, 18, 24, 26, 65, 68, 93, 102, 134.

11 interacts with 9, 12, 64, 95, 96, 114, 133, 134.

12 interacts with 1, 3, 9, 11, 17, 18, 19, 25, 26, 28, 34, 36, 41, 43, 49, 56, 62, 63, 76, 80, 81, 85, 86, 87, 90, 91, 93, 121, 122, 129, 140, 141.

13 interacts with 1, 3, 9, 10, 17, 20, 25, 28, 33, 34, 36, 37, 41, 45, 56, 62, 68, 79, 80, 81, 83, 86, 91, 94, 101, 106, 108, 121, 122, 129, 137, 140, 141.

14 interacts with 1, 5, 8, 10, 15, 19, 20, 21, 28, 30, 40, 43, 44, 47, 54, 62, 63, 64, 65, 86, 97, 121, 129, 130, 133, 138, 141.

15 interacts with 8, 9, 10, 14, 18, 21, 22, 37, 39, 41, 44, 45, 46, 58, 59, 61, 62, 63, 64, 65, 66, 95, 96, 97, 98, 112, 116, 125, 127, 128, 129, 130, 132, 133, 135, 137, 138, 141.

16 interacts with 27, 29, 34, 68, 78, 79, 82, 88, 95, 101, 114, 117, 119, 122.

17 interacts with 3, 4, 5, 9, 12, 13, 20, 23, 27, 37, 38, 43, 49, 65, 69, 80, 81, 86, 89, 101, 110, 115, 116, 117, 118, 126, 129, 135.

18 interacts with 9, 10, 12, 15, 19, 26, 28, 31, 33, 42, 43, 44, 47, 48, 49, 60, 65, 69, 70, 74, 77, 79, 85, 97, 98, 103, 110, 140, 141.

19 interacts with 12, 14, 18, 22, 26, 28, 32, 33, 36, 37, 38, 41, 45, 49, 69, 71, 86, 104, 106, 107, 110, 118, 126, 140.

20 interacts with 6, 9, 13, 14, 17, 24, 29, 30, 36, 37, 43, 54, 64, 68, 80, 84, 89, 102, 116, 117, 129, 131, 133, 140.

21 interacts with 1, 5, 6, 8, 9, 14, 15, 24, 61, 63, 89, 95, 96, 111, 112, 113, 115, 116, 137, 139, 140, 141.

22 interacts with 8, 15, 19, 31, 32, 33, 36, 42, 44, 47, 49, 60, 61, 64, 69, 71, 74, 97, 98, 104, 107, 110, 127, 140.

23 interacts with 4, 17, 31, 34, 62, 63, 71, 76, 78, 79, 82, 83, 93, 95, 100, 101, 105, 115, 116, 119, 126, 132, 137.

24 interacts with 5, 10, 20, 21, 38, 82, 93, 100, 101, 102, 108, 115, 130, 133, 135, 140, 141.

25 interacts with 8, 12, 13, 26, 27, 36, 62, 81, 90, 92, 111, 114, 116, 120.

26 interacts with 2, 3, 10, 12, 18, 19, 25, 29, 31, 33, 34, 41, 53, 56, 58, 62, 67, 68, 76, 85, 90, 91, 92, 93, 108, 113, 122, 123, 124, 130.

27 interacts with 8, 16, 17, 25, 29, 62, 68, 81, 86, 88, 90, 92, 113, 114, 122, 130.

28 interacts with 1, 9, 12, 13, 14, 18, 19, 29, 31, 33, 34, 35, 36, 37, 38, 42, 45, 49, 50, 54, 55, 56, 62, 74, 92, 103, 106, 107, 108, 109, 110, 118, 127, 129, 131.

29 interacts with 1, 2, 4, 9, 16, 20, 26, 27, 28, 41, 67, 71, 81, 85, 88, 92, 101, 119, 122, 124.

30 interacts with 7, 14, 20, 31, 33, 35, 40, 47, 63, 95, 97, 98, 107, 126, 127, 129, 130, 131, 132, 133, 139.

31 interacts with 7, 18, 22, 23, 26, 28, 30, 33, 34, 35, 37, 40, 43, 44, 49, 50, 52, 54, 59, 60, 80, 89, 94, 98, 106, 107, 109, 128, 131, 132.

32 interacts with 2, 4, 19, 22, 34, 42, 43, 46, 48, 52, 54, 60, 61, 63, 65, 69, 70, 71, 73, 74, 75, 104, 105, 107, 109, 110, 122, 129.

33 interacts with 13, 18, 19, 22, 26, 28, 30, 31, 34, 35, 36, 41, 54, 56, 59, 74, 78, 80, 90, 91, 92, 94, 105, 107, 118, 122, 123, 124, 136.

34 interacts with 7, 12, 13, 16, 23, 26, 28, 31, 32, 33, 41, 54, 56, 59, 74, 78, 80, 90, 91, 92, 94, 105, 107, 118, 122, 123, 124, 136.

35 interacts with 28, 30, 31, 33, 39, 42, 43, 46, 61, 79, 104, 118, 137.

36 interacts with 9, 12, 13, 19, 20, 22, 25, 28, 33, 38, 40, 41, 43, 45, 52, 54, 61, 68, 80, 81, 86, 94, 106, 110, 136.

37 interacts with 13, 15, 17, 19, 20, 28, 31, 38, 43, 44, 49, 50, 72, 76, 97, 103, 128, 133, 140.

38 interacts with 17, 19, 24, 28, 36, 37, 40, 42, 43, 44, 50, 52, 58, 61, 68, 76, 78, 79, 94, 97, 106, 128.

39 interacts with 15, 33, 35, 44, 48, 62, 69, 70, 72, 75, 97, 104, 118, 127, 134, 137, 138.

40 interacts with 14, 30, 31, 33, 36, 38, 42, 44, 48, 69, 70, 97, 104, 107, 118, 125, 127, 134, 137, 138.

41 interacts with 12, 13, 15, 19, 26, 29, 33, 34, 36, 44, 48, 51, 65, 69, 70, 71, 72, 92, 98, 104, 107, 118, 122, 125, 127, 138.

42 interacts with 18, 22, 28, 32, 33, 35, 38, 40, 43, 48, 49, 50, 57, 69, 104, 105, 107, 110, 118, 137.

43 interacts with 9, 12, 14, 17, 18, 20, 31, 32, 33, 35, 36, 37, 38, 42, 48, 51, 60, 64, 69, 71, 86, 101, 104, 107, 109, 119, 129, 140.

44 interacts with 14, 15, 18, 22, 31, 37, 38, 39, 40, 41, 51, 52, 60, 62, 87, 97, 98, 110.

45 interacts with 4, 13, 15, 19, 28, 36, 48, 54, 65, 69, 70, 71, 73, 74, 78, 79, 91, 104, 105, 106, 110, 118, 125, 127, 130, 138.

46 interacts with 5, 15, 32, 33, 35, 47, 66, 106, 107, 118, 130.

47 interacts with 14, 18, 22, 30, 33, 46, 62, 107, 118, 130.

48 interacts with 1, 8, 18, 32, 33, 39, 40, 41, 42, 43, 45, 52, 63, 71, 75, 85, 86, 97, 99, 105, 107, 109, 110, 119, 129, 130, 141.

49 interacts with 9, 12, 17, 18, 19, 22, 28, 31, 37, 42, 51, 64, 68, 86, 97, 107, 110, 117, 118, 128, 129, 130, 132, 133, 138, 140.

50 interacts with 28, 31, 37, 38, 42, 52, 54, 60, 76, 77, 85, 87, 94, 103.

51 interacts with 33, 41, 43, 44, 49, 53, 54, 59, 69, 77, 104, 107, 118, 127, 136.

52 interacts with 2, 31, 32, 36, 38, 44, 48, 50, 53, 54, 59, 71, 91, 104, 106, 107, 136.

53 interacts with 6, 7, 26, 51, 52, 56, 57, 59, 60, 66, 72, 118, 126, 127, 137.

54 interacts with 14, 20, 28, 31, 32, 33, 34, 36, 45, 50, 51, 52, 56, 57, 59, 71, 80, 91, 94, 106, 107, 110, 115.

55 interacts with 28, 67, 68, 71, 80, 119, 123, 124.

56 interacts with 4, 9, 12, 13, 26, 28, 34, 53, 54, 57, 59, 76, 78, 80, 85, 86, 92, 102, 123, 124.

57 interacts with 7, 42, 53, 54, 56, 59, 60, 70, 86, 94, 117, 118, 123, 126, 127, 137.

58 interacts with 6, 7, 8, 15, 26, 38, 65, 72, 76, 78, 93, 96, 98, 99, 125, 127, 130, 138.

59 interacts with 7, 8, 15, 31, 34, 51, 52, 53, 54, 57, 58, 60, 65, 66, 72, 96, 98, 99, 125, 127, 130, 138.

60 interacts with 18, 22, 31, 32, 43, 44, 50, 53, 57, 59, 91, 94, 97, 98, 103, 131.

61 interacts with 1, 6, 8, 15, 21, 22, 32, 35, 36, 38, 63, 74, 86, 95, 96, 97, 98, 99, 105, 108, 109, 110, 119, 120, 127, 131, 139, 140, 141.

62 interacts with 8, 9, 12, 13, 14, 15, 23, 25, 26, 27, 28, 39, 44, 47, 65, 66, 72, 85, 86, 87, 89, 93, 114, 115, 116, 119, 127, 130, 132, 138, 141.

63 interacts with 4, 6, 8, 12, 14, 15, 21, 23, 30, 32, 48, 61, 64, 65, 66, 68, 70, 71, 72, 75, 86, 93, 96, 99, 100, 116, 119, 127, 129, 130, 132, 133, 134, 136, 138, 140, 141.

64 interacts with 8, 9, 11, 14, 15, 20, 22, 43, 49, 63, 81, 85, 86, 95, 99, 100, 101, 109, 112, 113, 127, 130, 133, 136, 139.

65 interacts with 8, 10, 14, 15, 17, 18, 32, 41, 45, 58, 59, 62, 63, 66, 72, 84, 99, 111, 114, 116, 127, 130, 133, 134, 138, 139, 141.

66 interacts with 15, 46, 53, 59, 62, 63, 65, 68, 70, 71, 75, 93, 130, 132, 133, 137, 139, 141.

67 interacts with 1, 26, 29, 55, 76, 86, 92, 122, 123.

68 interacts with 1, 10, 13, 16, 20, 26, 27, 36, 38, 49, 55, 63, 66, 71, 86, 94, 101, 109, 110, 114, 119, 124, 129, 131, 132, 141.

69 interacts with 17, 18, 19, 22, 32, 33, 39, 40, 41, 42, 43, 45, 51, 74, 75, 92, 104, 105, 107, 132.

70 interacts with 1, 18, 32, 33, 39, 40, 41, 45, 57, 63, 66, 71, 72, 73, 86, 104, 110, 131, 132.

71 interacts with 2, 4, 19, 22, 23, 29, 32, 33, 41, 43, 45, 48, 52, 54, 55, 63, 66, 68, 70, 73, 75, 76, 79, 88, 98, 104, 105, 107, 108, 109, 110, 120, 129, 131, 132, 133.

72 interacts with 33, 37, 39, 41, 53, 58, 59, 62, 63, 65, 70, 104, 128, 130, 131.

73 interacts with 32, 45, 70, 71, 78, 91, 104, 105, 108, 109, 110.

74 interacts with 4, 18, 22, 28, 32, 33, 34, 45, 61, 69, 105, 107, 109, 110, 127.

75 interacts with 32, 33, 39, 48, 63, 66, 69, 71, 98, 100, 104, 107, 123, 124, 133.

76 interacts with 3, 12, 23, 26, 37, 38, 50, 56, 58, 67, 71, 85, 87, 90, 91, 92, 95, 98, 101, 108, 113, 120, 122, 123, 124, 127.

77 interacts with 1, 18, 50, 51, 79, 83, 86, 90, 93, 103.

78 interacts with 3, 4, 16, 23, 34, 38, 45, 56, 58, 73, 79, 85, 86, 101, 105, 130.

79 interacts with 3, 4, 13, 16, 18, 23, 35, 38, 45, 71, 77, 78, 86, 88, 90, 104, 105, 111, 116, 124, 127, 130.

80 interacts with 7, 9, 12, 13, 17, 20, 31, 34, 36, 54, 55, 56, 86, 94, 103, 106, 123, 136.

81 interacts with 9, 12, 13, 17, 25, 27, 29, 36, 64, 82, 83, 85, 86, 92, 93, 113, 114, 119, 122, 133, 136.

82 interacts with 6, 16, 23, 24, 81, 111, 113, 115.

83 interacts with 13, 23, 77, 81, 85, 86, 101.

84 interacts with 20, 65, 120, 127, 131, 132, 134, 135.

85 interacts with 7, 12, 18, 26, 29, 48, 50, 56, 62, 64, 76, 78, 81, 83, 86, 87, 93, 108, 136.

86 interacts with 1, 3, 7, 12, 13, 14, 17, 19, 27, 36, 43, 48, 49, 56, 57, 61, 62, 63, 64, 67, 68, 70, 77, 78, 79, 80, 81, 83, 85, 103, 111, 117, 119, 120, 121, 123, 124, 125, 140, 141.

87 interacts with 12, 44, 50, 62, 76, 85, 90, 91, 93, 95, 100, 128.

88 interacts with 4, 16, 27, 29, 71, 79, 114, 123.

89 interacts with 8, 17, 20, 21, 31, 62, 100, 130, 138, 141.

90 interacts with 12, 25, 26, 27, 33, 34, 76, 77, 79, 87, 91, 93, 113, 114, 121, 124, 132.

91 interacts with 4, 12, 13, 26, 33, 34, 45, 52, 54, 60, 73, 76, 87, 90, 103, 105, 121, 132.

92 interacts with 25, 26, 27, 28, 29, 34, 41, 56, 67, 69, 76, 81, 114, 122, 123, 124, 132.

93 interacts with 10, 12, 23, 24, 26, 62, 63, 66, 77, 81, 87, 90, 116, 130, 132, 137, 141.

94 interacts with 13, 31, 34, 36, 38, 50, 54, 55, 57, 60, 68, 80, 103, 106, 119, 136.

95 interacts with 8, 11, 15, 16, 21, 23, 30, 61, 64, 76, 87, 102, 112, 117, 119, 121, 130, 132, 133, 135, 139, 141.

96 interacts with 8, 11, 15, 21, 58, 59, 61, 63, 97, 102, 119, 121, 130, 132, 133, 139, 141.

97 interacts with 14, 15, 18, 22, 30, 37, 38, 39, 40, 44, 48, 49, 60, 61, 96, 98, 119, 132, 133, 135.

98 interacts with 2, 15, 18, 22, 30, 31, 41, 44, 58, 59, 60, 61, 71, 75, 76, 97, 109, 110, 119, 120, 121, 132, 133, 139.

99 interacts with 8, 48, 58, 59, 61, 63, 64, 65, 131, 132, 133, 138.

100 interacts with 23, 24, 63, 64, 75, 87, 89, 101, 112, 113, 115, 121, 126, 130, 132, 133, 135, 141.

101 interacts with 1, 3, 9, 13, 16, 17, 23, 24, 29, 43, 64, 68, 76, 78, 83, 100, 102, 112, 113, 117, 119, 122, 133.

102 interacts with 2, 5, 6, 10, 20, 24, 56, 95, 96, 101, 115.

103 interacts with 3, 18, 28, 37, 50, 60, 77, 80, 86, 91, 94.

104 interacts with 19, 22, 32, 33, 35, 39, 40, 41, 42, 43, 45, 51, 52, 69, 70, 71, 72, 73, 75, 79, 105, 107, 109.

105 interacts with 2, 4, 7, 23, 32, 33, 34, 42, 45, 48, 61, 69, 71, 73, 74, 78, 79, 91, 104, 106, 110, 119, 137.

106 interacts with 1, 4, 7, 13, 19, 28, 31, 36, 38, 45, 46, 52, 54, 80, 94, 105, 129, 136.

107 interacts with 19, 22, 28, 30, 31, 32, 33, 34, 40, 41, 42, 43, 46, 47, 48, 49, 51, 52, 54, 69, 71, 74, 75, 104, 110, 122, 136.

108 interacts with 13, 24, 26, 28, 61, 73, 76, 85, 109, 110.

109 interacts with 28, 31, 32, 43, 48, 61, 64, 68, 71, 73, 74, 98, 104, 108, 110.

110 interacts with 4, 17, 18, 19, 22, 28, 32, 33, 36, 42, 43, 44, 45, 48, 49, 54, 61, 68, 70, 71, 73, 74, 98, 105, 107, 108, 109, 137.

111 interacts with 6, 8, 21, 25, 65, 79, 82, 86, 113, 115, 116, 117, 120, 130, 132, 134.

112 interacts with 8, 15, 21, 64, 95, 100, 101, 130, 133, 139, 141.

113 interacts with 1, 5, 9, 21, 26, 27, 64, 76, 81, 82, 90, 100, 101, 111, 114, 117, 119, 124.

114 interacts with 8, 11, 16, 25, 27, 62, 65, 68, 81, 88, 90, 92, 113, 117, 123, 127, 130, 132.

115 interacts with 8, 17, 21, 23, 24, 54, 62, 82, 100, 102, 111, 127, 132, 137, 140, 141.

116 interacts with 5, 8, 15, 17, 20, 21, 23, 25, 62, 63, 65, 79, 111, 117, 121, 127, 128, 131, 132, 135, 137.

117 interacts with 6, 16, 17, 20, 49, 57, 86, 95, 101, 111, 113, 114, 116, 121, 123, 124, 125, 133, 135, 137.

118 interacts with 5, 9, 17, 19, 28, 33, 34, 35, 39, 40, 41, 42, 45, 46, 47, 49, 51, 53, 57, 126, 127, 130, 131, 134.

119 interacts with 3, 16, 23, 29, 48, 55, 61, 62, 63, 68, 81, 86, 94, 95, 96, 97, 98, 101, 105, 113, 136.

120 interacts with 25, 61, 71, 76, 84, 86, 98, 111, 121, 126, 132, 133, 139.

121 interacts with 8, 12, 13, 14, 86, 90, 91, 95, 96, 98, 100, 116, 117, 120, 123, 124, 125, 127, 132, 133, 139.

122 interacts with 12, 13, 16, 26, 27, 29, 32, 33, 34, 41, 67, 76, 92, 101, 107, 123.

123 interacts with 2, 7, 26, 34, 55, 56, 57, 67, 75, 76, 80, 86, 88, 92, 114, 117, 121, 122, 127, 137.

124 interacts with 1, 4, 7, 9, 26, 29, 34, 55, 56, 68, 75, 76, 79, 86, 90, 92, 113, 117, 121, 137.

125 interacts with 7, 15, 40, 41, 45, 58, 59, 86, 117, 121.

126 interacts with 17, 19, 30, 33, 53, 57, 100, 118, 120, 133.

127 interacts with 15, 22, 28, 30, 33, 39, 40, 41, 45, 51, 53, 57, 58, 59, 61, 62, 63, 64, 65, 74, 76, 79, 84, 114, 115, 116, 118, 121, 123, 132, 135.

128 interacts with 15, 31, 33, 37, 38, 49, 72, 87, 116, 138, 140.

129 interacts with 8, 9, 12, 13, 14, 15, 17, 20, 28, 30, 32, 43, 48, 49, 63, 68, 71, 106, 131, 140.

130 interacts with 6, 10, 14, 15, 24, 26, 27, 30, 45, 46, 47, 48, 49, 58, 59, 62, 63, 64, 65, 66, 72, 78, 79, 89, 93, 95, 96, 100, 111, 112, 114, 118, 134, 137, 141.

131 interacts with 5, 20, 28, 30, 31, 60, 61, 68, 70, 71, 72, 84, 99, 116, 118, 129, 135.

132 interacts with 15, 23, 30, 31, 49, 62, 63, 66, 68, 69, 70, 71, 84, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 111, 114, 115, 116, 120, 121, 127.

133 interacts with 2, 5, 10, 11, 14, 15, 20, 24, 30, 37, 49, 63, 64, 65, 66, 71, 75, 81, 95, 96, 97, 98, 99, 100, 101, 112, 117, 120, 121, 126, 134, 136, 139, 140.

134 interacts with 6, 10, 11, 33, 39, 40, 63, 65, 84, 111, 118, 130, 133.

135 interacts with 6, 15, 17, 24, 84, 95, 97, 100, 116, 117, 127, 131, 137.

136 interacts with 8, 9, 34, 36, 51, 52, 63, 64, 80, 81, 85, 94, 106, 107, 119, 133, 140.

137 interacts with 13, 15, 21, 23, 33, 35, 39, 40, 42, 53, 57, 66, 93, 105, 110, 115, 116, 117, 123, 124, 130, 135, 140.

138 interacts with 14, 15, 33, 39, 40, 41, 45, 49, 58, 59, 62, 63, 65, 89, 128, 140, 141.

139 interacts with 21, 30, 61, 64, 65, 66, 95, 96, 98, 112, 120, 121, 133.

140 interacts with 1, 5, 8, 9, 12, 13, 18, 19, 20, 21, 22, 24, 37, 43, 49, 61, 63, 86, 115, 128, 129, 133, 136, 137, 138, 141.

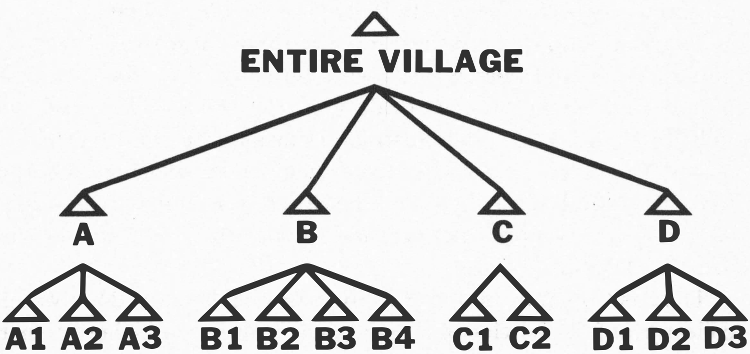
141 interacts with 1, 8, 9, 12, 13, 14, 15, 18, 21, 24, 48, 61, 62, 63, 65, 66, 68, 86, 89, 93, 95, 96, 100, 112, 115, 130, 138, 140.

Each link or absence of a link is a statement about the interaction between the two variables concerned. If what we can do physically about meeting one requirement in the form inevitably affects what we can do about the other (whether positively or negatively), we call the variables linked. If there is no such interaction, we call them independent.

Here is an example. Number 94 is the need for provision for animal traffic. This conflicts with 7, the need for cattle to be treated as sacred, because the sacredness of cattle allows the cattle great freedom, and hence more room for circulation, which makes 94 harder to meet adequately. On the other hand, 94 connects positively with 13, the need for family solidarity, because this latter requirement tends to group the houses of family members in compounds, and so reduces the number of access points required by cattle, making 94 easier to meet.

The complete list of interactions defines the set *L*. As we have seen before, the set *M* of misfit variables, together with the set *L* of links, define the graph *G*(*M*,*L*).

Analysis of the graph *G*(*M*,*L*), shows us the decomposition pictured below, where *M* itself falls into four major subsets A,B,C,D, and where these sets themselves break into twelve minor subsets, A1,A2,A3,B1,B2,B3,B4,C1,C2,D1,D2,D3, thus:



A1 contains requirements 7, 53, 57, 59, 60, 72, 125, 126, 128.
A2 contains requirements 31, 34, 36, 52, 54, 80, 94, 106, 136.
A3 contains requirements 37, 38, 50, 55, 77, 91, 103.
B1 contains requirements 39, 40, 41, 44, 51, 118, 127, 131, 138.
B2 contains requirements 30, 35, 46, 47, 61, 97, 98.

B3 contains requirements 18, 19, 22, 28, 33, 42, 43, 49, 69, 74, 107, 110.

B4 contains requirements 32, 45, 48, 70, 71, 73, 75, 104, 105, 108, 109.

C1 contains requirements 8, 10, 11, 14, 15, 58, 63, 64, 65, 66, 93, 95, 96, 99, 100, 112, 121, 130, 132, 133, 134, 139, 141.

C2 contains requirements 5, 6, 20, 21, 24, 84, 89, 102, 111, 115, 116, 117, 120, 129, 135, 137, 140.

D1 contains requirements 26, 29, 56, 67, 76, 85, 87, 90, 92, 122, 123, 124.

D2 contains requirements 1, 9, 12, 13, 25, 27, 62, 68, 81, 86, 113, 114.

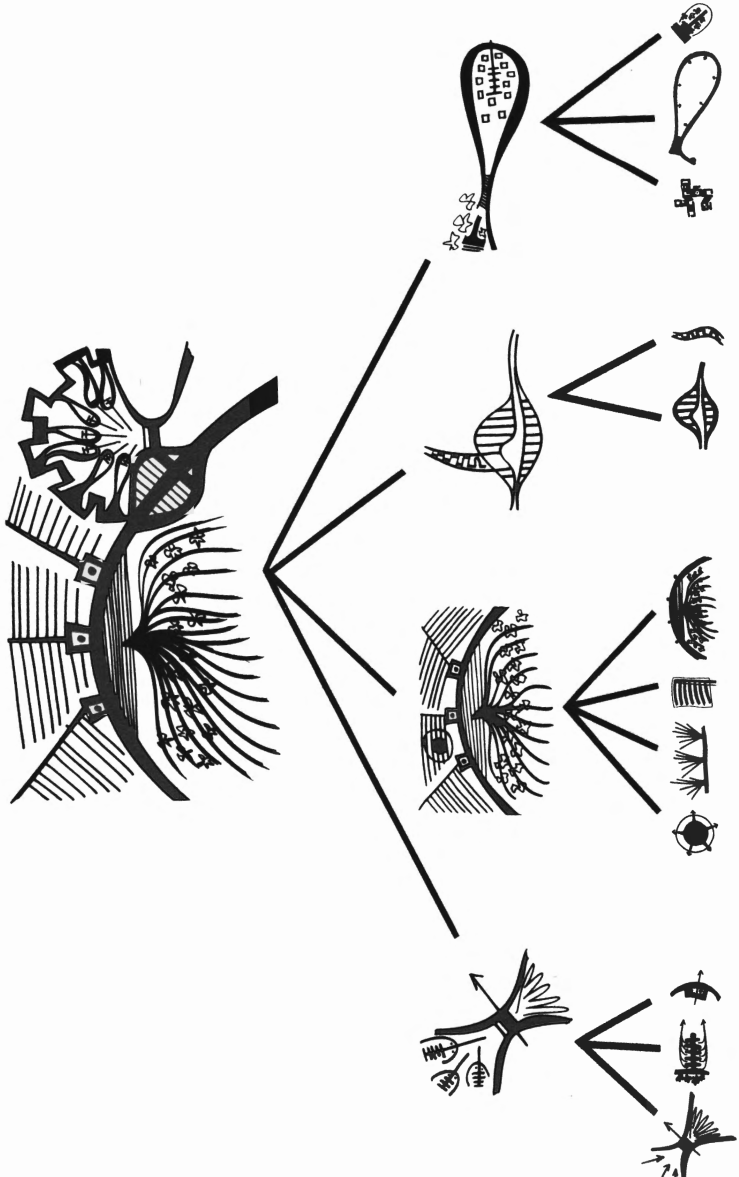
D3 contains requirements 2, 3, 4, 16, 17, 23, 78, 79, 82, 83, 88, 101, 119.

The tree of diagrams made during the realization of this program is illustrated on the next page.

I first give a summary of the diagrams, and the way they fit together, so that the more detailed account of each diagram and the functions which belong to it may be better understood.

The four main diagrams are roughly these: A deals with cattle, bullock carts, and fuel; B deals with agricultural production, irrigation, and distribution; C deals with the communal life of the village, both social and industrial; D deals with the private life of the villagers, their shelter, and small-scale activities. Of the four, B is the largest, being of the order of a mile across, while A, C, D, are all more compact, and fit together in an area of the order of 200 yards across.

The basic organization of B is given by the diagram B4, a water collector unit, consisting of a high bund, built in the highest corner of the village, at right angles to the slope of the terrain; within the curve of this bund, water gullies run together in a tank. This tank serves the rest of the village land, which lies lower, by means of sluices in the bund; the component B4 is intimately connected with B3, the distribution system for the fields. The principal element of this diagram is a road elevated from floods, which naturally



takes its place along the top of the bund defined by B4. At intervals along this road, distribution centers are placed providing storage for fertilizer, implements, and seeds; in view of the connection with B4, each one of these centers may be associated with a sluice, and with a well dug below the bund, so that it may also serve as a distribution center for irrigation water. Each distribution center serves one unit of type B2; this is a unit of cooperative farming, broken into contoured terraces, by anti-erosion bunds, and minor irrigation channels running along these bunds. B1 is a demonstration farm, surrounding the group of components ACD, just at those points of access which the farmers pass daily on their way to B2 and B3.

The smaller group of diagrams ACD is given its primary organization by the fact that several units of type D must function together. Each D copes with the small-scale activities of about fifty people. It is defined by D2, a compound wall carrying drinking water and gas along its top. At the entrance to the compound, where the walls come together, is a roofed area under which cottage industries take place. The compound contains the component D1, an assembly of storage huts, connected by roofed verandas which provide living space. Every third or fourth hut has a water tank on top, fed by the compound wall, and itself feeding simple bathing and washing-up spaces behind walls. D3 is a component attached to the entrance of the compound; it provides a line of open water at which women may wash clothes, trees with a sitting platform at their base for evening gossip, in such a way that the water and trees together form a climatic unit influencing the microclimate of the compound, and also, because of the water and trees, offering a suitable location for the household shrine.

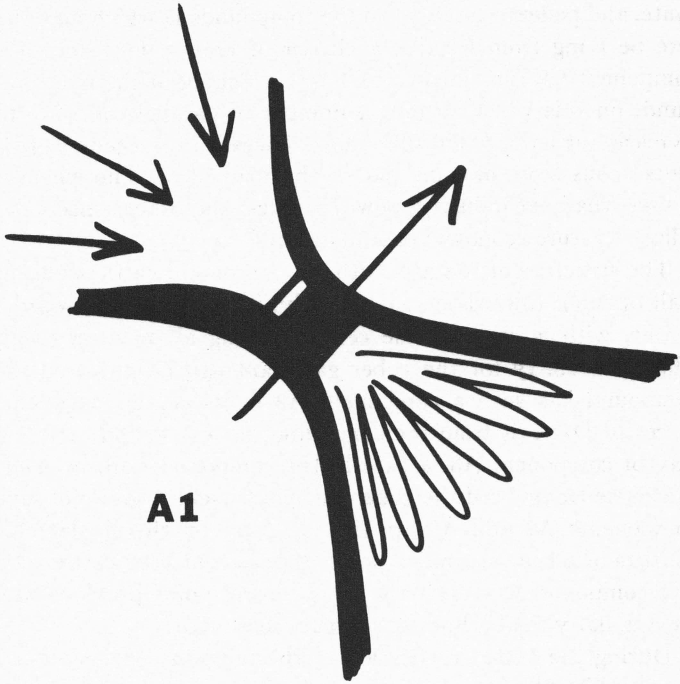
C is made of two diagrams; C2 is a series of communal buildings (school, temple, panchayat office, village meeting place, etc.), each with a court, the courts opening in alternatingly opposite directions. The cross walls are all pierced by gates, in such a way that there is a continuous path down the middle. This path serves as a connecting link between different centers, a processional route, and pedestrian access to the compounds D which may therefore be hung from C2 like a cluster of grapes. One end of this component C2 runs into C1; C1 is a widening of the road on the bund; on this widening out, a number of parallel walls are built to mark out narrow, city-like plots. There is in the center of these plots a bus stop, opening out of the road itself. The whole unit houses whatever industry, power sources, and other aspects of the village's future economic base, develop.

The structure of A starts with A2, a group of cattle stalls, each stall opening toward the outside only, its floor falling toward the center, with a drain in the center leading all manure to a pit where the slurry for the gober gas plant can be prepared. Each compound has such a component A2 in its center, between the pieces of D1; exit from the compound, for cattle and carts, is by way of component A3, a gate in the compound wall, marked by the cattle trough and the gober gas plant itself. A group of several components A2 and A3 are tied together by the single A1. A1 consists of a central control point through which all cattle leaving any compound have to pass. This control point provides a hoof bath, a dairy, and a link to the main road via C1.

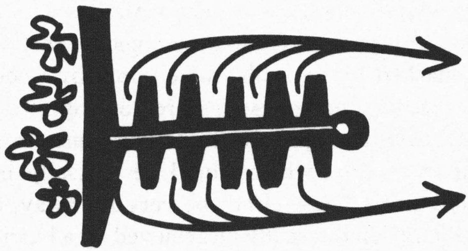
During the actual realization of the program, that stage came last in which the four diagrams A, B, C, D, were combined to give the diagram labeled "Entire Village."

There now follows a more detailed account of the reasons behind the organization of each of the twelve minor diagrams.

A1: 7 Cattle treated as sacred and vegetarian attitude.
53 Upgrading of cattle.
57 Protection of cattle from disease.
59 Efficient use and marketing of dairy products.
60 Minimize the use of animal traction to take pressure off shortage.
72 Prevent famine if monsoon fails.
125 Prevent malnutrition.
126 Close contact with village-level worker.
128 Price assurance for crops.


A1

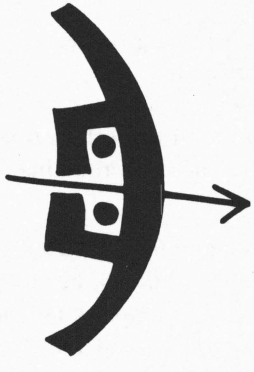
The sacredness of cattle (7) tends to make people unwilling to control them, so they wander everywhere eating and destroying crops, unless they are carefully controlled. Similarly, the need to upgrade cattle (53) calls for a control which keeps cows out of contact with roaming scrub bulls; and further calls for some sort of center where a pedigree bull might be kept (even if only for visits); and a center where scrub bulls can be castrated. Cattle diseases (57) are mainly transferred from foot to foot, through the dirt — this can be prevented if the cattle regularly pass through a hoof bath of disinfecting permanganate. If milk (59) is to be sold cooperatively, provision must be made for central milking (besides processing); if cows are milked at home, and the milk then pooled, individual farmers will adulterate the milk. Famine prevention (72), the prevention of malnutrition (125), and price assurance for crops (128) all suggest some kind of center offering both storage, and production of nourishing foods (milk, eggs, groundnuts). If the village-level worker (126) is to come often to the village and help, quarters must be provided for him here. Animal traction (60) calls for access to and from the cattle stalls (A2) on the one hand, and the road on the other.



A2

A2: 31 Efficient distribution of fertilizer, manure, seed, from village storage to fields.
34 Full collection of natural manure (animal and human).
36 Protection of crops from thieves, cattle, goats, monkeys, etc.
52 Improve quantity of fodder available.
54 Provision for feeding cattle.
80 Security for cattle.
94 Provision for animal traffic.
106 Young trees need protection from goats, etc.
136 Accommodation of wandering caste groups, incoming labor, etc.

Here 31, 34, 54, 80, and 94 form a subset connected with cattle movement and manure, while 36, 52, 106, and 136 form a subset mainly concerned with the protection of crops and trees from wandering cattle. 31 and 34 call for the collection of urine and dung, which suggests cattle should be in one place as much of the time as possible, where there is a pucca floor draining toward a central manure collector. This is of course closely connected with feeding stalls, the most permanent standing place for cattle. 80 calls for psychological security — cattle owners want their cattle as near to them as possible, if not actually in the house, and are therefore absolutely opposed to the idea of a central communal cattle shed. In view of disease and germ-breeding difficulties the closest arrangement possible seems to be one where individual stalls are immediately opposite owners' verandas with nothing but a path between; this path serves to accommodate cattle traffic (94). Each stall is marked by its walls, roofed only by wood purlins at 2′ centers, so that the fodder itself, stored on top, provides shade. Rains are not heavy enough to warrant permanent roofing. Vegetables, young trees, etc., which would be specially benefited by protection from cattle, must either be very far away, or else very close so that separation can really be achieved by a barrier (36, 106). To make this work, 52 must be assured by other means — stall feeding perhaps, which then connects with 54. To prevent the cattle of wandering shepherds from causing trouble (136), the proper grazing ground must abut the road, and access to it must be the normal road-village access. This grazing ground should be on the good land side of the bund, so that when green silage is introduced, land can be irrigated and cultivated.

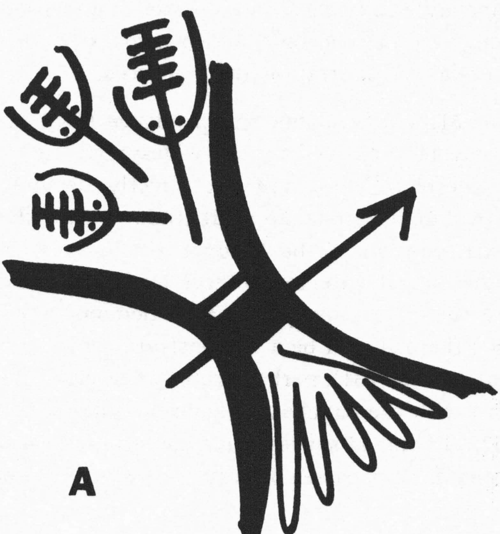


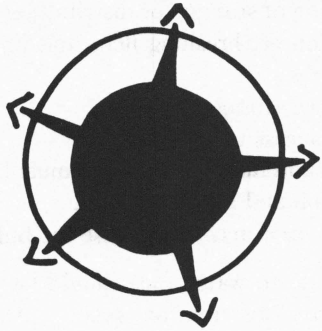
A3

A3: 37 Provision of storage for distributing and marketing crops.
38 Provision of threshing floor and its protection from marauders.
50 Protected storage of fodder.
55 Cattle access to water.
77 Village and individual houses must be protected from fire.
91 Provision and storage of fuel.
103 Bullock cart access to house for bulk of grain, fodder.

Access for cattle to water (55) should be to good water, hence to drinking water distribution system, feeding off compound wall D2. 77 and 91 are best achieved by a controlled fuel supply, like gas, supplied by a gober gas plant using manure from A2, the gas distributed to individual kitchens by the same artery that distributes water, i.e., the compound wall.

At the point on the compound wall indicated by these previous items, there must be an opening to allow passage of bullock carts (103), and at this point there should also be a store for supplies and fodder — or at least an easy unloading and access point to the roofs of the cattle bays (37, 38, 50).

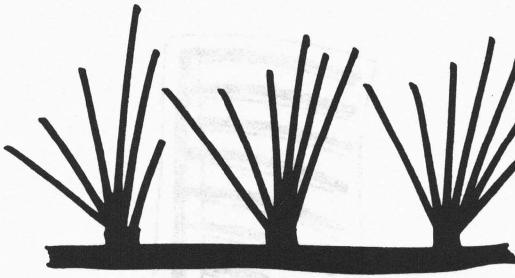




B1

B1: 39 Best cotton and cash crop.
40 Best food grain crop.
41 Good vegetable crop.
44 Crops must be brought home from fields.
51 Improve quality of fodder available.
118 Demonstration projects which spread by example.
127 Contact with block development officer.
131 Panchayat must have more power and respect.
138 Achieve economic independence so as not to strain national transportation and resources.

39, 40, 41, 51, and economic independence (138) are all items which can only be improved by the widespread use of improved agricultural methods; these are not directly dependent on the physical plan, but on a change of attitude in the villagers. This change of attitude cannot be brought about by sporadic visits from the agricultural extension officer and village-level worker, but only by the continuing presence of demonstration methods, on site (118); there should be a demonstration farm, government- or panchayat-owned (131), perhaps run by the village-level worker in association with the panchayat (hence accommodation for such officers, 127). 118 and 44 suggest that the farm be placed in such a way that every farmer passes it daily, on his way to and from the fields.



B2

2: 30 Efficient and rapid distribution of seeds, fertilizer, etc., from block HQ.

35 Protection of crops from insects, weeds, disease.

46 Respect for traditional agricultural practices.

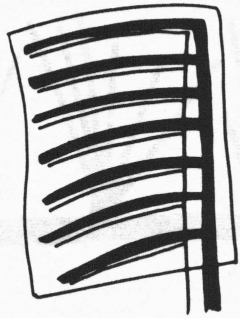
47 Need for new implements when old ones are damaged, etc.

61 Sufficient fluid employment for laborers temporarily (seasonally) out of work.

97 Minimize transportation costs for bulk produce (grain, potatoes, etc.).

98 Daily produce requires cheap and constant (monsoon) access to market.

97 and 98 are critical, and call for access to and from the fields on a road which is not closed in the monsoon — i.e., on an embankment. 30 and 35 call for efficient distribution within the plots, of seeds, fertilizers, insecticides, etc., which must themselves be stored at some point where delivery is easy — i.e., on the road. Hence the idea of distribution centers located at intervals along the main road, serving wedge-shaped or quasi-circular units of agricultural land. 46, 47, 61, have little discernible physical implication.



B3

B3: 18 Need to divide land among sons of successive generations.

19 People want to own land personally.

22 Abolition of Zamindari and uneven land distribution.

28 Proper boundaries of ownership and maintenance responsibility.

33 Fertile land to be used to best advantage.

42 Efficient plowing, weeding, harvesting, leveling.

43 Consolidation of land.

49 Cooperative farming.

69 Fullest possible irrigation benefit derived from available water.

74 Maintenance of irrigation facilities.

107 Soil conservation.

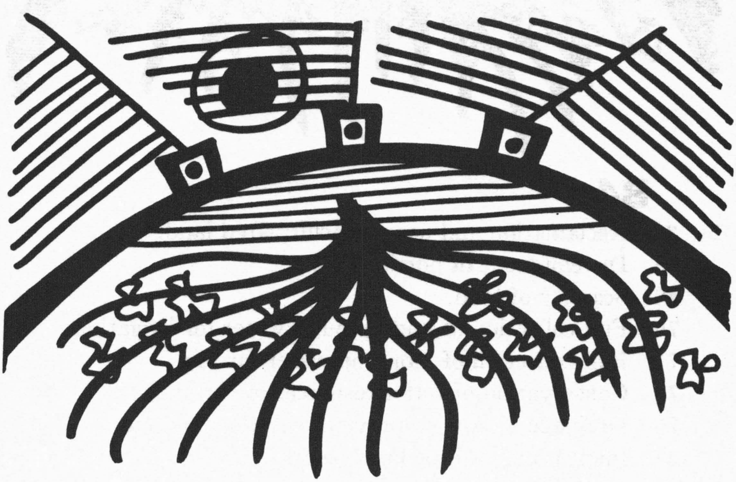
110 Prevent land erosion.

18–49 all point to the development of cooperative farms of some sort, from the point of view of increasing efficiency of resources, manpower, machines, better crops, rotation of crops, etc. 69 cannot be implemented unless water is distributed from the HQ of such cooperatives because otherwise faction and personal rivalries, etc., prevent full use of wells — i.e., warring neighbors adjacent to the source of water (well) will not agree to cooperate about sharing its use. Irrigation (74) requires consolidated ownership of channels, otherwise neglect at one place holds up the efficient use somewhere else. Soil conservation (107) depends on rotation of crops, which is only feasible if large plots are under single ownership control, so that they can carry the full pattern of rotation. Erosion (110) is prevented by long continuous contour bunds, which can only be put across land of integrated ownership. Bund and irrigation divisions on contours suggest terraced strips of land as units of co-op farm, fed from single uphill source.

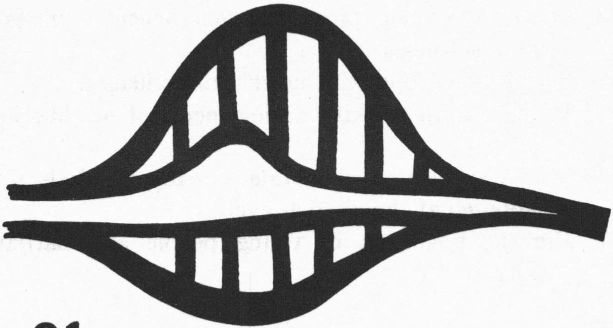
B4

B4: 32 Reclamation and use of uncultivated land.
45 Development of horticulture.
48 Scarcity of land.
70 Full collection of underground water for irrigation.
71 Full collection of monsoon water for use.
73 Conservation of water resources for future.
75 Drainage of land to prevent waterlogging, etc.
104 Plant ecology to be kept healthy.
105 Insufficient forest land.
108 Road and dwelling erosion.
109 Reclamation of eroded land, gullies, etc.

32 and 48 call for use of wasteland, which often contains river bed area. 48 calls for irrigation of this area. 71, 73, 75, suggest the use of monsoon water instead of and as well as well water for irrigation, since well irrigation is temporary in the long run, because it causes a drop in the water table. Apart from actually using monsoon water for irrigation, the water table in the wells can be preserved if the wells are backed up by a tank. Hence a curved bund collects water above wells placed under the bund (70). Rainfall in the catchment area (again a water resource issue, 73) will be improved by tree planting (104, 105), which suggests putting fruit trees (45) inside the curve of the bund. (Incidentally, placing the trees within the bund offers us a way of protecting young trees from cattle, by keeping the cattle on the other side of the bund, which then forms a natural barrier.) Further, if water is to flow toward the tank, horizontal contour bunds cannot be used to check erosion as they are in B3, so erosion of gullies, streams, etc., can only be controlled by tree planting (109). Road erosion is controlled if the road is on top of the bund itself (108).



B



C1

C1: 8 Members of castes maintain their caste profession as far as possible.
10 Need for elaborate weddings.
11 Marriage is to person from another village.
14 Economic integration of village on payment-in-kind basis.
15 Modern move toward payment in cash.
58 Development of other animal industry.
63 Development of village industry.
64 Simplify the mobility of labor, to and from villages, and to and from fields and industries and houses.
65 Diversification of village's economic base — not all occupations agricultural.
66 Efficient provision and use of power.
93 Lighting.
95 Access to bus as near as possible.
96 Access to railway station.
99 Industry requires strong transportation support.
100 Provision for bicycle age in every village by 1965.
112 Access to a secondary school.
121 Facilities for birth, pre- and post-natal care (birth control).
130 Need for increased incentives and aspirations.

132 Need to develop projects which benefit from government subsidies.
133 Social integration with neighboring villages.
134 Wish to keep up with achievements of neighboring villages.
139 Proper connection with bridges, roads, hospitals, schools, proposed at the district level.
141 Prevent migration of young people and harijans to cities.

This is composed of two major functional sets: 11, 64, 95, 100, 112, 121, 133, 134, 139, which concerns the integration of the village with neighboring villages and with the region, and 8, 10, 14, 15, 58, 63, 65, 66, 93, 96, 99, 130, 132, 141, which concerns the future economic base of the village, and all the aspects of "modern" life and society.

These two are almost inseparable. They call for a center, away from the heart of the village, on the road, able, because of being on the road, to sustain connections between the village and other villages (11) and capable of acting as a meeting place for villagers of different villages (112, 121). This function is promoted by the need to provide a bus stop (95), village industries with optimum access to the road (63–66, 99), the social gathering place connected with the bus and with jobs made available by the industries (61, 133, 134); the development of a modern and almost urban atmosphere to combat migration of the best people to cities (141), and to develop incentives (14, 15, 130, 132). A center of industry promotes 8, 63, 64. The road satisfies 64, 95, 96, 99, 100, 139. The center will be the natural physical location for sources of power and electricity transformer (66, 93); also the most efficient place for the poultry and dairy farming which require road access (58); the bus stop is the natural arrival place for incoming wedding processions (10).

C2

C2: 5 Provision for festivals and religious meetings.

6 Wish for temples.

20 People of different factions prefer to have no contact.

21 Eradication of untouchability.

24 Place for village events — dancing, plays, singing, etc., wrestling.

84 Accommodation for panchayat records, meetings, etc.

89 Provision of goods, for sale.

102 Accommodation for processions.

111 Provision for primary education.

115 Opportunity for youth activities.

116 Improvement of adult literacy.

117 Spread of information about birth control, disease, etc.

120 Curative measures for disease available to villagers.

129 Factions refuse to cooperate or agree.

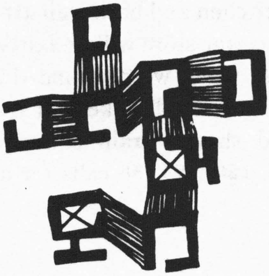
135 Spread of official information about taxes, elections, etc.

137 Radio communication.

140 Develop rural community spirit: destroy selfishness, isolationism.

The major fact about the communal social life of the village is the presence of factions, political parties, etc.; these can be a great hindrance to development (20, 129). If the various communal facilities of the village (5, 6, 24, 84, 89, 111, 115, 120, 137) are provided in a central place, this place will very likely get associated with one party, or certain families, and may actually not contribute to social life at all. On the other hand, it is important from the point of view of social integration (21, 140) to provide a single structure rather than isolated buildings. What is more, isolated buildings also have the possible connection with the single family nearest them, which can again discourage other families from going there. What is required is a community center which somehow manages to pull all the communal functions together so that none are left isolated, but at the same time does not have a location more in favor of some families than others. To achieve this, a linear center, containing some buildings facing in, some out, zigzagging between the different compounds, is necessary. This also meets (102) the need for processions with important stopping places; and adult literacy calls for a series of walls along the major pedestrian paths, with the alphabet and messages written in such a way that their continuing presence forces people to absorb them (116, 117, 135).

C



D1

D1: 26 Sentimental system: wish not to destroy old way of life; love of present habits governing bathing, food, etc.

29 Provision for daily bath, segregated by sex, caste, and age.

56 Sheltered accommodation for cattle (sleeping, milking, feeding).

67 Drinking water to be good, sweet.

76 Flood control to protect houses, roads, etc.

85 Everyone's accommodation for sitting and sleeping should be protected from rain.

87 Safe storage of goods.

90 Better provision for preparing meals.

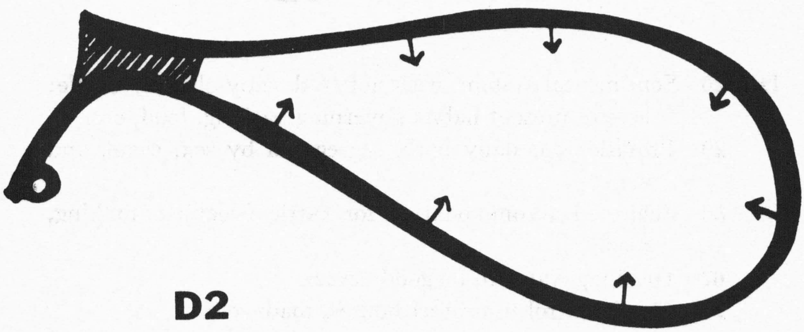
92 House has to be cleaned, washed, drained.

122 Disposal of human excreta.

123 Prevent breeding germs and disease starters.

124 Prevent spread of human disease by carriers, infection, contagion.

Houses, as they are used at present, are chiefly storerooms; people actually live on their verandas most of the time. The one thing which inner rooms provide, namely privacy and psychological security, appears among the needs to be met by D2, not here. Hence, we solve 87 by providing storerooms, which in a column-like manner support veranda roofs stretching between them (85). 26 is mainly concerned with bathing and food, connected with (29, 67, 90). These suggest a water store on top of occasional store-houses, with kitchen and bath wall attached to this store (also 122); probably this water store will be fairly close to the source of water, as we shall see when we combine this with D2. The floor of the veranda must be raised to keep it out of flood water (76) — also the compound should drain toward the center, to remove the dangers of 92, 123, 124. 56 calls for a space to house A2.



D2

D2:
1 Harijans regarded as ritually impure, untouchable, etc.
9 Members of one caste like to be together and separate from others, and will not eat or drink together.
12 Extended family is in one house.
13 Family solidarity and neighborliness even after separation.
25 Assistance for physically handicapped, aged, widows.
27 Family is authoritarian.
62 Provision of cottage industry and artisan workshops and training.
68 Easy access to drinking water.
81 Security for women and children.
86 No overcrowding.
113 Good attendance in school.
114 Development of women's independent activities.

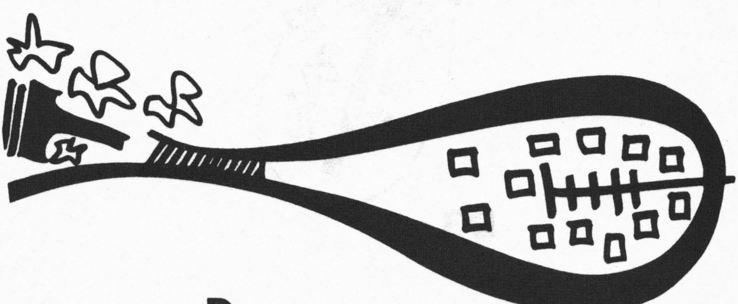
1, 9, 12, 13, suggest group compounds, as they are found at present, each of about 5 to 10 families, i.e., 25 to 50 persons. To provide security (81), especially for women, surround it by a wall, whose top serves as a distribution channel for water (68). The fact that the space within the wall is all protected, allows women more freedom within the compound for women's communal activities (114), gives more freedom to widows (25), and allows cottage industries, which are likely to be run largely by women, to flourish (62). The space for cottage industry (62) should go at the entrance to the compound, where women going to and from washing activities pass it constantly; this may to some extent combat the effects of purdah (27); it encourages women to come out from their houses (which the usual house discourages, because it allows women to shut themselves up in seclusion), and may even help girls' attendance in school by making the women more bold (113). Since containing walls are moved outward, overcrowding is less likely to take place (86) — adjustment and expansion can take place more easily within the compound walls than within individual house walls.

D3

D3: 2 Proper disposal of dead.
3 Rules about house door not facing south.
4 Certain water and certain trees are thought of as sacred.

16 Women gossip extensively while bathing, fetching water, on way to field latrines, etc.
17 Village has fixed men's social groups.
23 Men's groups chatting, smoking, even late at night.
78 Shade for sitting and walking.
79 Provision of cool breeze.
82 Provision for children to play (under supervision).
83 In summer people sleep in open.
88 Place to wash and dry clothes.
101 Pedestrian traffic within village.
119 Efficient use of school; no distraction of students.

Here there are several overlapping functions. 23, 78, 79, 82, 83, all require the control of climate — in particular getting cool conditions — which can be best achieved by the juxtaposition of water and trees. 16, 17, 23, 88, 101, require a unit for gossip, washing clothes, meeting purposes, at the compound level. 2, 3, 4, demand the construction of a place with certain qualities of sacredness, perhaps quiet, water, neem trees. Pedestrian traffic and quiet are called for again by 101, 119. All these functions call for a unit in which water, trees, washing facilities, pedestrian movement, sitting under the trees, are juxtaposed; the unit fits directly onto the compound, just outside the entrance. Washing may be either on ghats, etc., or on steps fed from the water wall unit D2.



D

ENTIRE VILLAGE

APPENDIX 2 / MATHEMATICAL TREATMENT OF DECOMPOSITION

We face the following specific, purely mathematical problem. Given a system of binary stochastic variables, some of them pairwise dependent, which satisfy certain conditions, how should this system be decomposed into a set of subsystems, so that the information transfer between the subsystems is a minimum?

We begin by restating the conditions on the graph which represents the system, and the further conditions on the system.

We have a finite signed graph G which consists of two finite disjoint sets $M(G)$, and $L(G)$, where the elements of M are points called the vertices of G, and the elements of L are line-segments called the links of G, each one of which passes through two and only two vertices, and carries either a positive or a negative sign.[1] The link is said to join these two vertices. The vertices are called the end-points of the link. Where two vertices are joined by more than one link, the links are regarded as distinct and identifiable. Two links are said to meet if they have a common end-point. The degree of a vertex is the number of links for which it is an end-point. Let m be the number of vertices in M, and l^+ the number of positive links in L, and l^- the number of negative links in L, and l the total number of links ($l = l^+ + l^-$). It will also be convenient later to refer to the set of positive links and the set of negative links separately. We shall call them L^+ and L^- respectively (where $L^+ \cup L^- = L$).

The graph G fully defines the system on the set M. We shall refer to it as the system M, for short. Let us further define the subsystems of M as follows. Given any subset of S of M, construct that graph whose vertices are the points of S, and whose links are just those elements of L for which both end-points belong to S. We call such a graph a full subgraph of G. It is clear that once L is given, each subset S of M has a uniquely defined associated full subgraph of G. It fully defines a subsystem on S, which we may again call S for short.

Associated with the ith vertex of G is a binary random variable x_i, taking the values 0 and 1 with respective probabilities p and $1 - p$ (p being the same for all variables). We must at this point insert a brief note about this p. It is possible in practice that there might be a different p_i for each variable. However, it is clear that the decomposition of the system into subsystems cannot be invariant for any pattern of p_i's. In other words, if variable x_1 has a large probability of being 0, but all the other variables have a large probability of being 1, we cannot expect to get the same decomposition into subsystems as in the case where these probabilities are relatively very different.

If we allowed the p_i to be different for different variables x_i, we should have to bring this into the following analysis, which would lead to very complicated equations, and make it impossible to find a simple and general basis for decomposition. It is for this reason, to avoid an intolerably difficult mathematical problem, that we have arranged, as described in Chapter 8, to make all the variables in M have roughly equal scope or significance. And we write $p_i = p$ for all p_i, so that $p(x_i = 0) = p$ for all i, and $p(x_i = 1) = 1 - p$, for all i.

We shall now make a further assumption, to simplify the mathematics still further. The decomposition of M depends on the relative amounts of information transmitted from one subsystem to another. While the absolute amounts of information must of course depend on the absolute values of the state probabilities, the relative amounts should depend only on the relative values of state probabilities. We should expect, therefore, that the decomposition of the system into subsystems should be the same, no matter what the absolute value of p. In other words, on grounds of symmetry alone, it would be very strange if, by changing the probability p to some new value p^* for all variables simultaneously, we could alter the system's subsystems. We shall not try to prove this intuition. The reader is invited to reconsider it after reading the proof which follows. We shall assume that it is so, and that we may therefore base our decomposition on the most convenient possible value for p. The value we choose, for convenience of computation, is the one which satisfies $p = 1 - p$; i.e., $p = \frac{1}{2}$.

We therefore redefine the system, for the purpose of computation, so that there is, associated with the ith vertex of G, a binary stochastic variable x_i, taking the values 0 and 1 with *equal* probabilities, and we write $p(x_i = 0) = p(x_i = 1) = \frac{1}{2}$ for all x_i.

Since there are m variables in M, there are clearly 2^m ways of assigning them values. Each of these 2^m ways is called a state of the system M. (From an abstract point of view, we may also think of each vertex of the set M as being in one of two conditions, black or white, say, in which case we refer to the states of the system conveniently as colorings of the set M.) Each state of the m-variable system is completely defined by a row of m 1's and 0's (in the lexicographic order of the variables); we may call it σ for short. And similarly the state of any s-variable subsystem is defined by a row of s 1's and 0's, which we shall call λ for short.

In what follows we shall associate with each system a probability distribution over its states. We shall adopt the natural notation that $p(01100\ldots)$, for instance, is the probability of the state defined by the row of 1's and 0's in the bracket. For the extreme case of a one-variable system, we have, as observed above, $p(0) = p(1) = \frac{1}{2}$ for all variables. If there is ever any ambiguity about which variables are referred to, we shall label the 1's and 0's with subscripts. Thus $p(0_j)$ is, specifically, the probability of x_j taking the value 0.

Consider M, or any of its subsystems S. Since each separate variable takes the values 0 and 1 with equal probability, then if the variables were all independent of one another, the 2^m states of M would be equiprobable, and for any S its 2^s states would be equiprobable. We should have:

$$p(\sigma) = \frac{1}{2^m} \text{ for all } \sigma, \text{ and } p(\lambda) = \frac{1}{2^s} \text{ for all } \lambda.$$

In general, however, since there is some kind of interaction between the variables, represented by the links, the various states of a system will not be equiprobable; and we face the problem of determining the $p(\sigma)$ or $p(\lambda)$ for different σ and λ. What are the conditions these distributions must satisfy?

Condition 1

The two-variable product moment correlation for each pair of variables (x_i,x_j) is $\nu_{ij}\delta$, where $\nu_{ij} = (| l_{ij}^+ | - | l_{ij}^- |)$ is the signed number of links between the vertices i and j of G, and where δ is a constant, satisfying $l\delta \leqslant 1$. Since at most one of l_{ij}^+, l_{ij}^- is non zero, this makes ν_{ij} an integer lying between $-\nu$ and $+\nu$. It means also, that each individual link makes an equal contribution of δ to the correlation, positive or negative according to its sign. We get from this,[2] the fact that in every two-variable system (x_i,x_j), the $p(\lambda)$ must satisfy

$$\frac{p(00)p(11) - p(01)p(10)}{[p(0_i)p(1_i)p(0_j)p(1_j)]^{\frac{1}{2}}} = \nu_{ij}\delta.$$

Condition 2

We also know from Chapter 8 that all three variable and higher correlations vanish. What this means is that the value of the correlation function for any pair of variables is not dependent on the state of any other variable or set of variables in M,[3] i.e., formally we write

$$\frac{p(00\lambda)p(11\lambda) - p(01\lambda)p(10\lambda)}{[p(0_i\lambda)p(1_i\lambda)p(0_j\lambda)p(1_j\lambda)]^{\frac{1}{2}}} = \nu_{ij}\delta,$$

where λ represents any fixed pattern of values taken by any set of variables which does not include x_i and x_j. The simplest case, where λ is the state of a single variable x_k, say $x_k = 0$, gives the condition:

$$\frac{p(000_k)p(110_k) - p(010_k)p(100_k)}{[p(0_i0_k)p(1_i0_k)p(0_j0_k)p(1_j0_k)]^{\frac{1}{2}}} = \nu_{ij}\delta.$$

Among m variables, there are $\frac{1}{2}m(m-1) \cdot 3^{m-2}$ such conditions to be met, of which $2^m - (m+1)$ are independent.[4]

We now show that all the probability distributions for all subsystems are uniquely determined by the conditions stated, when we introduce the following further conditions which must be satisfied, by definition, by any probability distribution.

Condition 3

In any state of M, each of the m variables takes a fixed value. Take any subsystem S. Suppose, without loss of generality, we renumber the variables so that $x_1 \cdots x_s$ are in S, and $x_{s+1} \cdots x_m$ are not in S. Then in any state λ of S, each of the s variables $x_1 \cdots x_s$ takes a fixed value, and the remaining variables $x_{s+1} \cdots x_m$ are free. There are 2^{m-s} states of M in which the variables $x_1 \cdots x_s$ take the fixed pattern of values λ, one for each possible pattern of values taken by the set of $m - s$ free variables $x_{s+1} \cdots x_m$. We may therefore write the probability of λ as the sum of the probabilities of these 2^{m-s} states of M, thus:[5] $p(\lambda) = \sum p(\sigma)$ summed over all combinations of values for variables not in S.

Condition 4

Finally, we must have $p(\sigma) \geqslant 0$ for all σ.[6]

Condition 5

And we must have $\sum_{\sigma} p(\sigma) = 1$.[7]

We may use these facts as a way of building up the probabilities of the larger systems' states from the smaller, as follows.

Let us begin by considering the states of the 1-variable subsystems. We know by postulate, of course, that these probabilities $p(0)$ and $p(1)$ are $\frac{1}{2}$ and $\frac{1}{2}$. Let us now consider any 2-variable subsystem. We know 4 equations of the form: $p(00) + p(01) = p(0)$, of which 3 are independent, and we have 1 further equation from the fact that the graph G tells us the value of the correlation coefficient:

$$\frac{p(00)p(11) - p(01)p(10)}{[p(0)p(1)p(0)p(1)]^{\frac{1}{2}}}.$$

The probabilities of the 2-variable subsystem's states are therefore determinate.

Let us now consider any 3-variable subsystem. Again its state probabilities are determined to within one degree of freedom, by the probabilities of the constituent 2-variable subsystems' states, which we know. As before, the one degree of freedom is resolved by the fact that we know the value taken by one of the partial correlation functions of the form:

$$\frac{p(000)p(110) - p(010)p(100)}{[p(00)p(10)p(00)p(10)]^{\frac{1}{2}}}.$$

We thus see easily that at each stage of this process, the probabilities of the states of an s-variable subsystem, are determined to within 1 degree of freedom, by its constituent $(s - 1)$-variable-subsystems' state probabilities. And we can supply the one further constraint required to determine the probabilities uniquely, by looking at the appropriate partial correlation, whose value we know:

$$\frac{p(00\lambda)p(11\lambda) - p(01\lambda)p(10\lambda)}{[p(0\lambda)p(1\lambda)p(0\lambda)p(1\lambda)]^{\frac{1}{2}}},$$

where λ refers to some fixed state of $s - 2$ variables.

We shall now define a probability distribution which meets conditions 1–5, and must therefore be the unique distribution whose construction we have just described.[8]

In the state σ, call the links of L^+ satisfied or dissatisfied according as their end-points take the same or different values, and call the links of L^- satisfied or dissatisfied according as their end-points take different values or the same values. Then define the following:

$$e_{\sigma i} = +1 \text{ if vertex } x_i \text{ is 0 in state } \sigma,$$
$$e_{\sigma i} = -1 \text{ if vertex } x_i \text{ is 1 in state } \sigma,$$

so that $e_{\sigma i}e_{\sigma j}$ is 1 if the link ij is satisfied in σ,
and -1 if the link ij is dissatisfied in σ.

Then we define $\quad k_\sigma = \sum_{ij} \nu_{ij}e_{\sigma i}e_{\sigma j} \quad (i = 1 \cdots m, j = 1 \cdots m).$

In other words, the integer k_σ is the number of satisfied links in σ, less the number of dissatisfied links in σ. Hence, for all σ, $-l \leqslant k_\sigma \leqslant l$. Let us now consider the measure

$$p(\sigma) = \frac{1 + k_\sigma \delta}{2^m}.$$

Take condition 4 first:

We know that $$k_\sigma \geqslant -l.$$

Hence $$p(\sigma) \geqslant \frac{1 - l\delta}{2^m}.$$

Therefore $p(\sigma) \geqslant 0$ provided that $\delta < 1/l$, and this is so by postulate.[9]

Take next condition 5.

$$\sum_\sigma p(\sigma) = \sum_\sigma \frac{1 + k_\sigma \delta}{2^m} = 1 + \frac{\delta}{2^m} \sum_\sigma k_\sigma = 1 + \frac{\delta}{2^m} \sum_{ij} \nu_{ij} \sum_\sigma e_{\sigma i} e_{\sigma j}.$$

Now, if i and j are different, then in 2^{m-1} cases $e_{\sigma i}$ and $e_{\sigma j}$ will take the same sign so that their product is $+1$, and in 2^{m-1} cases they will take different signs so that their product is -1. Thus, for i and j different, the sum over all 2^m possible σ, vanishes. For i and j the same, ν_{ij} vanishes. Hence the last right hand side term is identical to 0.

$$\therefore \quad \sum_\sigma p(\sigma) = 1.$$

We next prove condition 3, namely that if the measure is defined for all subsystems S in the same way as it is for M, then all the relations of the form $p(\lambda) = \sum_{\substack{\text{variables} \\ \text{not in } S}} p(\sigma)$ hold identically.

Since we get any subset S of M, by removing $m - s$ variables from M, one at a time, it is sufficient to prove the result for a single step of removing one variable, and the general result follows by induction. Consider, therefore, any variable x_k of M, and define S as the subsystem obtained from M by removing x_k. Pick an arbitrary λ of this subsystem S. Suppose σ_1 and σ_2 are the two states of M in which the variables of S are in the same condition as in λ, and for which x_k takes the value 0 in σ_1 and the value 1 in σ_2.

We wish to prove that $p(\sigma_1) + p(\sigma_2) = p(\lambda)$.
To see that this is so we note that

$$[p(\sigma_1) + p(\sigma_2)] - [p(\lambda)] = \frac{1 + k_{\sigma_1}\delta}{2^m} + \frac{1 + k_{\sigma_2}\delta}{2^m} - \frac{1 + k_\lambda\delta}{2^{m-1}}$$
$$= \frac{\delta}{2^m}\left(\sum_{ij} \nu_{ij} e_{\sigma_1 i} e_{\sigma_1 j} + \sum_{ij} \nu_{ij} e_{\sigma_2 i} e_{\sigma_2 j} - 2 \sum_{ij} \nu_{ij} e_{\lambda i} e_{\lambda j}\right).$$

For $i, j \neq k$, $e_{\sigma_1 i}$, $e_{\sigma_2 i}$ and $e_{\lambda i}$ are identical. For i or $j = k$, the terms from σ_1 cancel with those from σ_2, which makes the right-hand side equal to 0, and proves the point.

We now return to the correlation coefficients. Let us first take the total correlation for a pair of variables, i and j. The above result allows us to write the state probabilities of the two variable subsystem, (x_i, x_j), as:

$$p(00) = \frac{1 + \nu_{ij}\delta}{4} \qquad p(10) = \frac{1 - \nu_{ij}\delta}{4}$$

$$p(01) = \frac{1 - \nu_{ij}\delta}{4} \qquad p(11) = \frac{1 + \nu_{ij}\delta}{4}$$

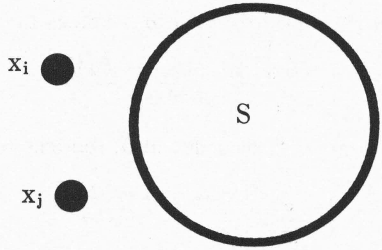
where ν_{ij} is the number of links between x_i and x_j in G. This gives a product moment correlation coefficient

$$\frac{p(00)p(11) - p(01)p(10)}{[p(0)p(1)p(0)p(1)]^{\frac{1}{2}}} = \frac{4\nu_{ij}\delta}{16} \Big/ \frac{1}{4} = \nu_{ij}\delta,$$

and thus satisfies condition 1.

Consider finally, the partial correlation coefficient for any two variables x_i, x_j in any subsystem $(S + x_i + x_j)$ while the variables of S are held constant.

Let us picture this situation as below:



Suppose the variables in S are held constant in some fixed state λ, we may then write

$$p(00\lambda) = \frac{1 + (k_{00} + k_\lambda + k_i + k_j)\delta}{2^{s+2}},$$

where k_{00} is the term coming from the links between x_i and x_j, k_λ is the term coming from the links inside S, and k_i and k_j are the terms coming from the links between S and x_i, x_j, respectively. It is then easy to see that similarly

$$p(11\lambda) = \frac{1 + (k_{11} + k_\lambda - k_i - k_j)\delta}{2^{s+2}},$$

$$p(01\lambda) = \frac{1 + (k_{01} + k_\lambda + k_i - k_j)\delta}{2^{s+2}},$$

$$p(10\lambda) = \frac{1 + (k_{10} + k_\lambda - k_i + k_j)\delta}{2^{s+2}}.$$

Also

$$p(0_i\lambda) = \frac{1 + (k_\lambda + k_i)\delta}{2^{s+1}},$$

$$p(1_i\lambda) = \frac{1 + (k_\lambda - k_i)\delta}{2^{s+1}},$$

$$p(0_j\lambda) = \frac{1 + (k_\lambda + k_j)\delta}{2^{s+1}},$$

$$p(1_j\lambda) = \frac{1 + (k_\lambda - k_j)\delta}{2^{s+1}}.$$

The partial correlation is given by

$$\frac{p(00\lambda)p(11\lambda) - p(01\lambda)p(10\lambda)}{[p(0\lambda)p(1\lambda)p(0\lambda)p(1\lambda)]^{\frac{1}{2}}}.$$

The numerator, to the first order in δ, reduces to

$$\frac{(k_{00} + k_{11} - k_{01} - k_{10})\delta}{2^{2s+4}}.$$

The denominator, to the first order in δ, reduces to

$$\left[\frac{1 + 4k_\lambda\delta}{2^{4s+4}}\right]^{\frac{1}{2}} = \frac{1 + 2k_\lambda\delta}{2^{2s+2}}.$$

Since $\qquad k_{00} = k_{11} = \nu_{ij}$ and $k_{01} = k_{10} = -\nu_{ij}$,

this makes the partial correlation equal to

$$\frac{4\nu_{ij}\delta}{4(1 + 2k_\lambda\delta)} = \nu_{ij}\delta$$

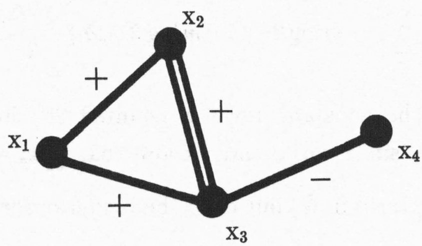
to the first order in δ, which is very small. Hence the partial correlation is $\nu_{ij}\delta$ for all λ, and satisfies condition 2.

Thus the measure

$$p(\sigma) = \frac{1 + k_\sigma\delta}{2^m}$$

has been shown to satisfy conditions 1–5, and is therefore, to within the stated approximations, that distribution uniquely determined by these conditions.

The probability distribution generated by this function for a specific graph is illustrated below.



$$p(0000) = \frac{1 + 3\delta}{16} \qquad p(0011) = \frac{1 - 3\delta}{16} \qquad p(0111) = \frac{1 - \delta}{16}$$

$$p(0001) = \frac{1 + 5\delta}{16} \qquad p(0101) = \frac{1 - \delta}{16} \qquad p(1011) = \frac{1 - 3\delta}{16}$$

$$p(0010) = \frac{1 - \delta}{16} \qquad p(1001) = \frac{1 + \delta}{16} \qquad p(1101) = \frac{1 - \delta}{16}$$

$$p(0100) = \frac{1 - 3\delta}{16} \qquad p(0110) = \frac{1 + \delta}{16} \qquad p(1110) = \frac{1 + 5\delta}{16}$$

$$p(1000) = \frac{1 - \delta}{16} \qquad p(1010) = \frac{1 - \delta}{16} \qquad p(1111) = \frac{1 + 3\delta}{16}$$

$$p(1100) = \frac{1 - 3\delta}{16}$$

Since we now have a workable probability distribution defined over the states of M, we can write down an expression for the average information carried by the system M. We use the Shannon-Wiener measure, and define $H(M)$, the average information carried by M, as

$$-\sum_{\sigma} p(\sigma) \log p(\sigma).$$ [10]

We may rewrite this now, as

$$
\begin{aligned}
H(M) &= -\sum_{\sigma} \left(\frac{1 + k_\sigma \delta}{2^m}\right) \log \left(\frac{1 + k_\sigma \delta}{2^m}\right) \\
&= -\frac{1}{2^m} \sum_{\sigma} \{(1 + k_\sigma \delta)[\log (1 + k_\sigma \delta) - m \log 2]\} \\
&= -\frac{1}{2^m} \sum_{\sigma} \left\{ (1 + k_\sigma \delta)(+ k_\sigma \delta - \frac{k_\sigma{}^2 \delta^2}{2} + \cdot \cdot \cdot - m \log 2) \right\} \\
&= -\frac{1}{2^m} \sum_{\sigma} \left\{ -m \log 2 + (1 - m \log 2) k_\sigma \delta + \frac{k_\sigma{}^2 \delta^2}{2} + \begin{matrix}\text{terms in } \delta^3 \\ \text{and above}\end{matrix} \right\}.
\end{aligned}
$$

In the sum, the constant term is counted 2^m times. The term in δ vanishes, since we already know that $\sum_{\sigma} k_\sigma = 0$. We therefore retain the term in δ^2, but drop the higher order terms, leaving

$$H(M) = m \log 2 - \frac{\delta^2}{2^{m+1}} \sum_{\sigma} k_\sigma{}^2.$$

Similarly we obtain, for any S,

$$H(S) = s \log 2 - \frac{\delta^2}{2^{s+1}} \sum_{\lambda} k_\lambda{}^2.$$

Even now, this expression for $H(S)$ is computationally impracticable. To compute it directly we should first have to compute the index k_λ for each of the 2^s states of the set S, as described above. For large s, even a high speed electronic computer will not be able to calculate and sum the powers of the 2^s values of k_λ in any reasonable time. It is therefore necessary, for computational purposes, to express $\sum_{\lambda} k_{\lambda}^2$ as a function of simpler structural parameters of the graph $G(S,L)$.

For the sake of notational simplicity, let us continue to work with the graph $G(M,L)$ and the function $\sum_{\sigma} k_{\sigma}^2$; by keeping the argument general, we may then again apply it to any of its subgraphs $G(S,L)$ and their associated functions $\sum_{\lambda} k_{\lambda}^2$.

We have defined
$$k_{\sigma} = \sum_{ij\epsilon L} \nu_{ij} e_{\sigma i} e_{\sigma j}.$$

Since we specified earlier that where there are several links between a pair of vertices these links are individually identifiable, we may now rewrite this expression as

$$k_{\sigma} = \Big(\sum_{L^+} e_{\sigma i} e_{\sigma j} - \sum_{L^-} e_{\sigma i} e_{\sigma j}\Big),$$

where each sum is taken over all the links belonging to L^+ and L^- respectively, so that the total contains l terms. It must be understood, of course, that this expression could be reduced, since each of its l terms is either 1 or -1. But, for the sake of clarity in the following proof, we shall leave it in its expanded form. We may write, then,

$$\begin{aligned}\sum_{\sigma} (k_{\sigma})^2 &= \sum_{\sigma} \Big(\sum_{L^+} e_{\sigma i} e_{\sigma j} - \sum_{L^-} e_{\sigma i} e_{\sigma j}\Big)^2 \\ &= \sum_{\sigma} \Big\{ \Big(\sum_{L^+} e_{\sigma i} e_{\sigma j}\Big)^2 + \Big(\sum_{L^-} e_{\sigma i} e_{\sigma j}\Big)^2 - 2\Big(\sum_{L^+} e_{\sigma i} e_{\sigma j} \sum_{L^-} e_{\sigma k} e_{\sigma l}\Big)\Big\}.\end{aligned}$$

Let us first look at the last bracket in this expansion. Since no vertex pair can be connected by a link from L^+ and a link from L^- simultaneously, every term in this last bracket will be of the form $e_{\sigma i} e_{\sigma j}^2 e_{\sigma k}$ or of the form $e_{\sigma i} e_{\sigma j} e_{\sigma k} e_{\sigma l}$, i,j,k,l all different. Since $e_{\sigma i}$, for any given i, takes the value $+1$ for half the σ, and -1 for the other half of the σ, and is evenly distributed over the values taken by the $e_{\sigma j}$, $e_{\sigma k}$, and $e_{\sigma l}$, we see that either of the above forms, since they both contain an $e_{\sigma i}$ raised to an odd power, will vanish when summed over σ. Let us now look at the first and second brackets in their expanded form. Again, all terms of the form $e_{\sigma i}e_{\sigma j}^2 e_{\sigma k}$ or $e_{\sigma i}e_{\sigma j}e_{\sigma k}e_{\sigma l}$ will vanish when summed over σ. There are therefore only two kinds of term left, both of the form $e_{\sigma i}^2 e_{\sigma j}^2$: those which represent the same link taken twice, and those which represent different links between the same vertex pair. We are therefore left with

$$\sum_{\sigma} k_{\sigma}^2 = \sum_{\sigma} \sum_{\substack{\text{links } ij \text{ of} \\ \text{either } L^+ \text{ or} \\ L^- \text{ alone}}} (e_{\sigma i}e_{\sigma j})^2 + 2\sum_{\sigma} \sum_{\substack{\text{over different} \\ \text{links between} \\ \text{the same vertex} \\ \text{pair}}} (e_{\sigma i}e_{\sigma j})^2$$

$$= 2^m\left\{\sum \nu_{ij} + 2\sum \tfrac{1}{2}\nu_{ij}(\nu_{ij} - 1)\right\}$$

$$= 2^m \sum_{M} \nu_{ij}^2,$$

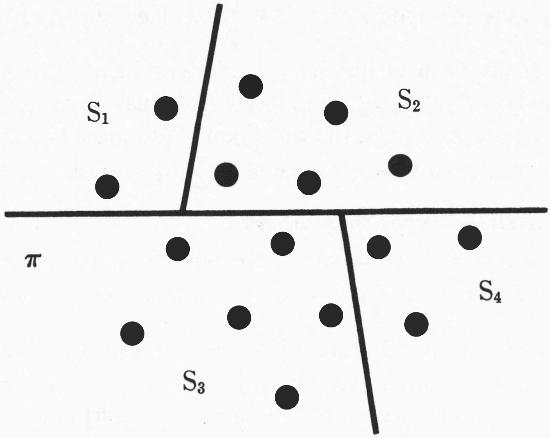
where the sum is taken over all pairs of variables i,j in M.

We therefore have $\quad H(M) = m \log 2 + \dfrac{\delta^2}{2} \displaystyle\sum_{M} \nu_{ij}^2,$

and similarly $\quad H(S) = s \log 2 + \dfrac{\delta^2}{2} \displaystyle\sum_{S} \nu_{ij}^2.$

The fact that ν_{ij} appears in this function, squared, means that the distinction between L^+ and L^- will not affect the result. As noted above in Chapter 8 then, we shall proceed without making the distinction between L^+ and L^-, using L alone and assuming that ν_{ij} takes positive values only. It also means, of course, that it is not worth making the distinction between negative and positive interaction, when stating the problem.[11]

Let us now consider an arbitrary partition of M into subsets $S_1, S_2 \cdots S_{\mu}$, such that $S_{\alpha} \cap S_{\beta} = 0$, and $\bigcup_{\mu} S_{\alpha} = M$. We shall refer to such a partition, typically, as π.



The information contained in M is $H(M)$. The information contained in the S_α taken separately is $\sum_\pi H(S_\alpha)$. Except in the case where there is no interaction at all between the different subsystems, the second of these two expressions will be larger than the first, because some information will, as it were, be counted more than once. As a result, we may use the difference between the two expressions, $\{[\sum_\pi H(S_\alpha)] - H(M)\}$ as a measure of the strength of the connections severed by the partition π.[12] The larger it is, the stronger the connections severed are. The smaller it is, the weaker the connections are, and the less information transfer there is across the partition. The value of this difference is given by

$$\left\{ (s_1 + \cdot\cdot\cdot + s_\mu) \log 2 + \frac{\delta^2}{2} \sum_{S_1, S_2, \cdots} \nu_{ij}{}^2 - m \log 2 - \frac{\delta^2}{2} \sum_M \nu_{ij}{}^2 \right\},$$

where the sum $\sum_{S_1, S_2, \cdots}$ is taken only over pairs i,j, which are wholly contained in one of the S_α. The difference, or redundancy, of the partition is therefore $\frac{1}{2}\delta^2 \sum_\pi \nu_{ij}{}^2$, where the sum is taken over all links ij cut by the partition π.

As it stands the redundancy $\frac{1}{2}\delta^2 \sum_{\pi} \nu_{ij}^2$ does not give us a fair basis for comparison of different π. Each π belongs to a certain "partition-type." That is, the subsets it defines have $s_1, s_2, \cdots s_\mu$ variables respectively, and the collection of numbers $\{s_1, s_2, \cdots s_\mu\}$ defines the partition-type. The value of $\sum_{\pi} \nu_{ij}^2$ will tend to be lower for some partition-types than others.

To normalize the redundancy, we now compute the expected value and variance of $\sum_{\pi} \nu_{ij}^2$ as a function of the partition-type, given a random distribution of l links among the $\frac{1}{2}m(m-1)$ possible spaces for links provided by m vertices. (For the sake of simplicity we shall assume that no space can hold more than one link, i.e., $\nu = 1$, so that $\nu_{ij} = 0$ or 1).[13] If all distinguishable distributions of the l links are equiprobable, the expected value and variance of $\sum_{\pi} \nu_{ij}^2$ will depend on four parameters. Two of them are constant. The first, l, is the number of links in L. The second, l_0, is the number of possible spaces to which links might be assigned. It is given by $l_0 = \frac{m(m-1)}{2}\cdot$ The other two parameters depend on the partition π. The first, l_0^π, is the number of the l_0 potential spaces which are cut by the partition π, i.e., the number of vertex pairs in which vertices come from different subsets of the partition. This depends on the partition-type of π, and is given by $l_0^\pi = \sum_{\pi} s_\alpha s_\beta$, where s_α is the number of variables in S_α. We note that $l_0^\pi \leqslant l_0$. The second of these parameters, l^π, is the number of actual links cut by the partition π. This is given by $l^\pi = \sum_{\pi} | \nu_{ij} |$. Of course $l^\pi \leqslant l$.

We consider first the expected value of $\sum_{\pi} \nu_{ij}^2 = \mathrm{E}\left(\sum_{\pi} \nu_{ij}^2\right)$. Since the ν_{ij} are independent we may write

$$\mathrm{E}\left(\sum_{\pi} \nu_{ij}^2\right) = \sum_{\pi} \mathrm{E}(\nu_{ij}^2) = l_0^\pi \mathrm{E}(\nu_{ij}^2),$$

where $E(\nu_{ij}^2)$ is the expected value of ν_{ij}^2 for some one fixed space spanning two points i,j.

Clearly $$E(\nu_{ij}^2) = \frac{l}{l_0},$$

so this reduces to

$$E\Big(\sum_{\pi} \nu_{ij}^2\Big) = \frac{l l_0^{\pi}}{l_0},$$

which depends on the value of l_0^{π} and so on the partition-type of π.

Let us now consider the variance of $\sum_{\pi} \nu_{ij}^2$.[14]

$$\mathrm{Var}\Big(\sum_{\pi} \nu_{ij}^2\Big) = E\Big[\Big(\sum_{\pi} \nu_{ij}^2\Big)^2\Big] - \Big[E\Big(\sum_{\pi} \nu_{ij}^2\Big)\Big]^2.$$

We already know the value of the second term. As for the first:

$$E\Big[\Big(\sum_{\pi} \nu_{ij}^2\Big)^2\Big] = E\Big[\sum_{\pi} \nu_{ij}^4 + 2\sum_{\pi} \nu_{ij}^2\nu_{kl}^2\Big].$$

Since we have arranged to take ν_{ij} as positive, $=0$ or 1, we have $\nu_{ij}^4 = \nu_{ij}^2 = \nu_{ij}$ and hence:

$$\mathrm{Var}\Big(\sum_{\pi} \nu_{ij}^2\Big) = E\Big(\sum_{\pi} \nu_{ij}\Big) + 2E\Big(\sum_{\pi} \nu_{ij}\nu_{kl}\Big) - \Big[E\Big(\sum_{\pi} \nu_{ij}\Big)\Big]^2.$$

Let us consider two fixed spaces ij and kl.
Now

$$\begin{aligned} E(\nu_{ij}\nu_{kl}) &= 0\cdot p(\nu_{ij}\nu_{kl} = 0) + 1\cdot p(\nu_{ij}\nu_{kl} = 1) \\ &= p(\nu_{ij}\nu_{kl} = 1) \\ &= \frac{l}{l_0}\cdot\frac{l-1}{l_0-1} = \frac{l(l-1)}{l_0(l_0-1)}. \end{aligned}$$

$$\begin{aligned} \therefore\quad E\Big(\sum_{\pi} \nu_{ij}\nu_{kl}\Big) &= \tfrac{1}{2} l_0^{\pi}(l_0^{\pi} - 1)\cdot E(\nu_{ij}\nu_{kl}) \\ &= \tfrac{1}{2} l_0^{\pi}(l_0^{\pi} - 1)\cdot\frac{l(l-1)}{l_0(l_0-1)}. \end{aligned}$$

This gives us

$$\mathrm{Var}\Big(\sum_{\pi} \nu_{ij}^2\Big) = \frac{l\cdot l_0^{\pi}}{l_0} + l_0(l_0^{\pi} - 1)\,\frac{l(l-1)}{l_0(l_0-1)} - \left(\frac{l\cdot l_0^{\pi}}{l_0}\right)^2$$

$$= \frac{ll_0^\pi}{l_0^2 \cdot (l_0 - 1)} [l_0^2 - l_0 + l_0(l_0^\pi - 1)(l - 1) - ll_0^\pi(l_0 - 1)]$$

$$= \frac{ll_0^\pi}{l_0^2(l_0 - 1)} [l_0^2 - l_0 l_0^\pi] = \frac{ll_0^\pi}{l_0(l_0 - 1)} (l_0 - l_0^\pi).$$

Again the variance depends on the value of l_0^π and hence on the partition-type of π.

In the case we are considering, where $\nu = 1$, the straightforward redundancy of a partition π, is

$$\tfrac{1}{2}\delta^2 \sum_\pi \nu_{ij}^2 = \tfrac{1}{2}\delta^2 l^\pi.$$

To normalize this for different partition-types, we now replace it by [15]

$$R(\pi) = \frac{\text{constant} \cdot [l^\pi - \mathrm{E}(l^\pi)]}{[\mathrm{Var}\ (l^\pi)]^{\frac{1}{2}}} = \frac{\text{constant}\ [l^\pi - ll_0^\pi/l_0]}{[ll_0^\pi(l_0 - l_0^\pi)/l_0(l_0 - 1)]^{\frac{1}{2}}},$$

and choose the constant to make this

$$\frac{l_0 l^\pi - ll_0^\pi}{[l_0^\pi(l_0 - l_0^\pi)]^{\frac{1}{2}}}.$$

This function has the same expected value and variance for all partition-types, and may therefore be used to compare partitions of all types with one another.

Expressed in terms of the earlier notation, this function is [16]

$$R(\pi) = \frac{\tfrac{1}{2}m(m - 1)\sum_\pi \nu_{ij} - l\sum_\pi s_\alpha s_\beta}{\left[\left(\sum_\pi s_\alpha s_\beta\right)\left(\tfrac{1}{2}m(m - 1) - \sum_\pi s_\alpha s_\beta\right)\right]^{\frac{1}{2}}}.$$

Let us consider, lastly, the practical problem of finding that partition π, of the set M, for which this function $R(\pi)$ takes the smallest (algebraic) value.

To find the best partition of a set S, we use a hill-climbing procedure which consists essentially of taking the partition into one-element subsets, computing the value of $R(\pi)$ for this partition, and then comparing with it all those partitions which can be obtained from it by combining two of its sets. Whichever of these partitions has the lowest value of $R(\pi)$ is then substituted for the original partition; and the procedure continues. It continues until it comes to a partition whose value of $R(\pi)$ is lower than that of any partition which can be obtained from it by combining two sets.

Another hill-climbing procedure, which finds a tree of partitions directly, goes in the opposite direction. It starts with the whole set S, and breaks it into its two most independent disjoint subsets, by computing $R(\pi)$ for a random two-way partition, and improving the partition by moving one variable at a time from side to side, until no further improvement is possible. It then repeats this process for each of the two subsets obtained, breaking each of them into two smaller subsets, and so on iteratively, until the entire set S is decomposed.

These and other methods have been programed for the IBM 7090, and are described in full elsewhere.[17] It is important, and rather surprising, that the techniques do not suffer from the sampling difficulties often found in hill-climbing procedures, but gives extremely stable optima even for short computation times.

NOTES

Chapter One. The Need for Rationality

1. D. Bullivant, "Information for the Architect," *Architect's Journal*, 129:504–21 (April 1959); Serge Chermayeff and René d'Harnancourt, "Design for Use," in *Art in Progress* (New York, 1944), pp. 190–201.

2. For some practical suggestions as to how this might be improved, see Christopher Alexander, "Information and an Organized Process of Design," in National Academy of Sciences, *Proceedings of the Building Research Institute* (Washington, D.C.), Spring 1961, pp. 115–24.

3. T. W. Cook, "The Relation between Amount of Material and Difficulty of Problem-Solving," *Journal of Experimental Psychology*, 20 (1937):178–83, 288–96; E. J. Archer, L. E. Bourne, Jr., and F. G. Brown, "Concept Identification as a Functicn of Irrelevant Information and Instructions," *ibid.*, 49 (1955):153–64.

4. This feeling has been expressed in many quarters, ever since the beginning of the Modern Movement. See, for instance, L. Moholy-Nagy, *The New Vision: From Material to Architecture*, revised trans. by Daphne Hoffman (New York, 1947), p. 54; Walter Gropius, *The New Architecture and the Bauhaus*, trans. P. Morton Shand (London, 1935), pp. 17–20.

5. Karl Duncker, "A Qualitative (Experimental and Theoretical) Study of Productive Thinking (Solving of Comprehensible Problems)," *Journal of Genetic Psychology*, 33 (1926): 642–708, and *On Problem Solving*, trans. Lynnes Lees, American Psychological Association, *Psychological Monographs*, No. 270 (Washington, D.C., 1945); Max Wertheimer, *Productive Thinking* (New York, 1945).

6. George A. Miller, "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information," *Psychological Review*, 63 (1956):81–97; D. B. Yntema and G. E. Mueser, "Remembering the Present States of a Number of Variables," *Journal of Experimental Psychology*, 60:18–22 (July 1960).

7. Alex Bavelas and Howard Perlmutter, classified work done at the Center for International Studies, M.I.T., quoted in "The Relation of

Knowledge to Action," by Max Millikan, in *The Human Meaning of the Social Sciences*, ed. Daniel Lerner (New York, 1959), p. 164.

8. In fact there are cases where a form has been uniquely determined by its requirements, but such cases are very rare. One striking example is the crane hook. See L. Bruce Archer, *Design*, No. 90 (June 1956), pp. 12–19, esp. p. 16; H. J. Gough, H. L. Cox, D. G. Sopwith, "The Design of Crane Hooks," *Proceedings of the Institute of Mechanical Engineers* (England), 1935; also Annual Report of the British Iron and Steel Research Association, 1954.

9. A typical collection of paintings based on such a kind of "logical" formalism is to be found in Karl Gerstner, *Kalte Kunst*, published by Arthur Niggli (Teufen AR, Switzerland, 1957).

10. Jacomo Barozzio Vignola, *Regola delli cinque ordini d'architettura* (Rome, 1562; Jacques-François Blondel, *Cours d'architecture* (Paris, 1771), Book IV.

11. Another example of this "logically" inspired formalism is to be found in Ludwig Hilbersheimer, *The New City* (Chicago, 1944), pp. 106–21.

12. Whether we like it or not, however rational we should like to be, there is a factor of judgment in the choice and use of a logical system which we cannot avoid. Logical pictures, like any others, are made by simplification and selection. It is up to us to see which simplifications we wish to make, which aspects to select as significant, which picture to adopt. And this decision is logically arbitrary. However reasonable and sound the picture is internally, the choice of a picture must be, in the end, irrational. For even if we can give reasons for choosing one logical scheme rather than another, these reasons only imply that there is another decision scheme behind the first (very likely not explicit). Perhaps there is still another behind this second one. But somewhere there are decisions made that are not rational in any sense, that are subject to nothing more than the personal bias of the decision maker. *Logical methods, at best, rearrange the way in which personal bias is to be introduced into a problem.* Of course, this "at best" is rather important. Present intuitive methods unhappily introduce personal bias in such a way that it makes problems impossible to solve correctly. Our purpose must be to repattern the bias, so that it no longer interferes in this destructive way with the process of design, and no longer inhibits clarity of form.

13. The relevant part of William Morris' thinking is to be found in volumes 22 and 23 of the 1915 London edition of his complete works. See also Nikolaus Pevsner, *Pioneers of Modern Design* (New York, 1949), pp. 24–30.

14. *Ibid.*, pp. 18–19.

15. Their work and ideas are fully discussed by Emil Kaufmann in

Architecture and the Age of Reason (Cambridge, Mass., 1955), pp. 95–99 and 134. No writings of Lodoli's remain, but see F. Algarotti, *Saggio sopra l'architettura*, in *Opere*, vol. II (Livorno, 1764); Marc-Antoine Laugier, *Essai sur l'architecture*, 2nd ed. (Paris, 1775), and *Observations sur l'architecture* (The Hague, 1765).

16. Nicolaus Pevsner, *An Outline of European Architecture*, Penguin Books (London, 1953), pp. 242–62.

17. In denying the possibility of understanding reasonably the processes of form production, the fetish of intuition is closely parallel to other famous attempts to shelter from the loss of innocence under the wings of magic and taboo; see, for comments, Sigmund Freud, *Civilization and Its Discontents*, trans. James Strachey (New York, 1962), or K. R. Popper in *The Open Society and Its Enemies* (Princeton, 1950).

18. For some recent protests against the willful nature of modern intuition in design, see Serge Chermayeff, "The Shape of Quality," *Architecture Plus* (Division of Architecture, A. & M. College of Texas), 2 (1959–60):16–23.

19. The possibility of amplifying intelligence has already been hinted at in W. Ross Ashby, "Design for an Intelligence Amplifier," in *Automata Studies*, ed. C. E. Shannon and J. McCarthy (Princeton, 1956), pp. 215–34. See also M. Minsky, "Steps towards Artificial Intelligence," *Proceedings of the Institute of Radio Engineers*, 49:8–30 (January 1961).

Chapter Two. Goodness of Fit

1. The source of form actually lies in the fact that the world tries to compensate for its irregularities as economically as possible. This principle, sometimes called the principle of least action, has been noted in various fields: notably by Le Chatelier, who observed that chemical systems tend to react to external forces in such a way as to neutralize the forces; also in mechanics as Newton's law, as Lenz's law in electricity, again as Volterra's theory of populations. See Adolph Mayer, *Geschichte des Prinzips der kleinsten Action* (Leipzig, 1877).

2. D'Arcy Wentworth Thompson, *On Growth and Form*, 2nd ed. (Cambridge, 1959), p. 16.

3. This old idea is at least as old as Plato: see, e.g., *Gorgias* 474–75.

4. The symmetry of this situation (i.e., the fact that adaptation is a mutual phenomenon referring to the context's adaptation to the form as much as to the form's adaptation to its context) is very important. See L. J. Henderson, *The Fitness of the Environment* (New York, 1913), page v: "Darwinian fitness is compounded of a mutual relationship between the organism and the environment." Also E. H. Starling's remark, "Organism and environment form a whole, and must be viewed

as such." For a beautifully concise description of the concept "form," see Albert M. Dalcq, "Form and Modern Embryology," in *Aspects of Form*, ed. Lancelot Whyte (London, 1951), pp. 91–116, and other articles in the same symposium.

5. At later points in the text where I use the word "system," this always refers to the whole ensemble. However, some care is required here, since many writers refer to that part of the ensemble which is held constant as the environment, and call only the part under adjustment the "system." For these writers my form, not my ensemble, would be the system.

6. In essence this is a very old idea. It was the first clearly formulated by Darwin in *The Origin of Species*, and has since been highly developed by such writers as W. B. Cannon, *The Wisdom of the Body* (London, 1932), and W. Ross Ashby, *Design for a Brain*, 2nd ed. (New York, 1960).

7. Wolfgang Köhler, *The Place of Value in a World of Facts* (New York, 1938), p. 96.

8. A. D. de Groot, "Über das Denken des Schachspielers," *Rivista di psicologia*, 50:90–91 (October–December 1956). Ludwig Wittgenstein, *Philosophical Investigations* (Oxford, 1953), p. 15.

9. See Max Wertheimer, "Zu dem Problem der Unterscheidung von Einzelinhalt und Teil," *Zeitschrift für Psychologie*, 129 (1933):356, and "On Truth," *Social Research*, 1:144 (May 1934).

10. K. Lönberg Holm and C. Theodore Larsen, *Development Index* (Ann Arbor, 1953), p. Ib.

11. Again, this idea is not a new one. It was certainly present in Frank Lloyd Wright's use of the phrase "organic architecture," for example, though on his tongue the phrase contained so many other intentions that it is hard to understand it clearly. For a good discussion see Peter Collins, "Biological Analogy," *Architectural Review*, 126:303–6 (December 1959).

12. This observation appears with beautiful clarity in Ozenfant's *Foundations of Modern Art* (New York, 1952), pp. 340–41. Also Kurt Koffka, *Principles of Gestalt Psychology* (London, 1935), pp. 638–44.

13. The idea that the residual patterns of adaptive processes are intrinsically well organized is expressed by W. Ross Ashby in *Design for a Brain*, p. 233, and by Norbert Wiener in *The Human Use of Human Beings* (New York, 1954), p. 37.

14. See note 2.

15. The concept of an image, comparable to the ideal field statement of a problem, is discussed at great length in G. A. Miller, Eugene Galanter, and Karl H. Pribram, *Plans and the Structure of Behavior* (New York, 1960). The "image" is presented there as something present in

every problem solver's mind, and used by him as a criterion for the problem's solution and hence as the chief guide in problem planning and solving. It seems worth making a brief comment. In the majority of interesting cases I do not believe that such an image exists psychologically, so that the testing paradigm described by Miller et al. in *Plans* is therefore an incorrect description of complex problem-solving behavior. In interesting cases the solution of the problem cannot be tested against an image, because the search for the image or criterion for success is actually going on at the same time as the search for a solution.

Miller does make a brief comment acknowledging this possibility on pp. 171–72. He also agreed to this point in personal discussions at Harvard in 1961.

16. It is not hard to see why, if this is so, the concept of good fit is relatively hard to grasp. It has been shown by a number of investigators, for example, Jerome Bruner et al., *A Study of Thinking* (New York, 1958), that people are very unwilling and slow to accept disjunctive concepts. To be told what something is not is of very little use if you are trying to find out what it is. See pp. 156–81. See also C. L. Hovland and W. Weiss, "Transmission of Information Concerning Concepts through Positive and Negative Instances," *Journal of Experimental Psychology*, 45 (1953): 175–82.

17. The near identity of "force" on the one hand, and the "requiredness" generated by the context on the other, is discussed fully in Köhler, *The Place of Value in a World of Facts*, p. 345, and throughout pp. 329–60. There is, to my mind, a striking similarity between the difficulty of dealing with good fit directly, in spite of its primary importance, and the difficulty of the concept zero. Zero and the concept of emptiness, too, are comparatively late inventions (clearly because they too leave one nothing to hold onto in explaining them). Even now we find it hard to conceive of emptiness as such: we only manage to think of it as the absence of something positive. Yet in many metaphysical systems, notably those of the East, emptiness and absence are regarded as more fundamental and ultimately more substantial than presence.

This is also connected with the fact, now acknowledged by most biologists, that symmetry, being the natural condition of an unstressed situation, does not require explanation, but that on the contrary it is asymmetry which needs to be explained. See D'Arcy Thompson, *On Growth and Form*, p. 357; Wilhelm Ludwig, *Recht-links-problem im Tierreich und beim Menschen* (Berlin, 1932); Hermann Weyl, *Symmetry* (Princeton, 1952), pp. 25–26; Ernst Mach, "Über die physikalische Bedeutung der Gesetze der Symmetrie," *Lotos*, 21 (1871): 139–47.

18. The logical equivalence of these two views is expressed by De

Morgan's law, which says essentially that if A, B, C, etc., are propositions, then [(Not A) and (Not B) and (Not C) . . .] is always the same as Not [(A or B or C or . . .)].

19. For the idea that departures from closure force themselves on the attention more strikingly than closure itself, and are actually the primary data of a certain kind of evaluative experience, and for a number of specific examples (not only ethical), see Max Wertheimer, "Some Problems in Ethics," *Social Research*, 2:352ff (August 1935). In particular, what I have been describing as misfits are described there as *Leerstellen* or emptinesses. The feeling that something is missing, and the need to fill whatever is incomplete (*Lückenfüllung*), are discussed in some detail.

20. Any psychological theory which treats perception or cognition as information processing is bound to come to the same kind of conclusion. For a typical discussion of such information-reducing processes, see Bruner et al., *A Study of Thinking*, p. 166.

21. It is perhaps instructive to note that both the concept of organic health in medicine and the concept of psychological normality in psychiatry are subject to the same kind of difficulties as my conception of a well-fitting form or coherent ensemble. In their respective professions they are considered to be well defined. Yet the only definitions that can be given are of a negative kind. See, for instance, Sir Geoffrey Vickers, "The Concept of Stress in Relation to the Disorganization of Human Behavior," in *Stress and Psychiatric Disorder*, ed. J. M. Tanner (Oxford, 1960).

22. In case it seems doubtful whether all the relevant properties of an ensemble can be expressed as variables, let us be quite clear about the fact that these variables are not necessarily capable of continuous variation. Indeed, it is quite obvious that most of the issues which occur in a design problem cannot be treated numerically, as this would require. A binary variable is simply a formal shorthand way of classifying situations; it is an indicator which distinguishes between forms that work and those that do not, in a given context.

Chapter Three: The Source of Good Fit

1. Alan Houghton Brodrick, "Grass Roots," *Architectural Review*, 115:101–11 (February 1954); W. G. Sumner, *Folkways* (Boston, 1908), p. 2. The same point is made by Adolf Loos in his famous story of the saddle-maker, *Trotzdem*, 2nd ed. (Innsbruck, 1931), pp. 13–14; to be found translated by Eduard Sekler in *Journal of Architectural Education*, vol. 12, no. 2 (Summer 1957), p. 31.

2. Ludwig Hilbersheimer, *Mies van der Rohe* (Chicago, 1956), p. 63.

3. Robert W. Marks, *The Dymaxion World of Buckminster Fuller* (New York, 1960), pp. 110–33.

4. Peter Collins, "Not with Steel and Cement," *Manchester Guardian Weekly*, January 14, 1960.

5. Office de la Recherche Scientifique Outre-Mer, *L'Habitat aux Cameroun* (Paris, 1952), p. 35.

6. *Ibid.*, p. 38.

7. *Ibid.*, p. 34.

8. See this chapter, p. 28.

9. Brodrick, "Grass Roots," p. 101.

10. In case this needs justification as a procedure, it is worth pointing out perhaps that the concept of "economic man," which underlay more than a century of economic theory, was admitted to be no more than a useful explanatory fiction. More recently, Robert Redfield has made much the same suggestion in "The Folk Society," *American Journal of Sociology*, 52:293–308 (January 1947), where he puts forward the "ideal" primitive society as a mental construct which serves a useful basis for comparison.

11. A. R. Radcliffe-Brown, "The Mother's Brother in South Africa," *South African Journal of Science*, 21 (1925):544–45.

12. Redfield, "The Folk Society," p. 293.

13. K. R. Popper, *The Open Society and Its Enemies* (Princeton, 1950), p. 169.

14. Sybil Moholy-Nagy, *Native Genius in Anonymous Architecture* (New York, 1957), throughout.

15. Of course, although selfconsciousness, as I shall define it, does tend to affect many aspects of culture at once, we certainly know of cases where cultures are highly selfconscious in some respects, yet quite unselfconscious in others. It is especially important to avoid any suggestion of evolution here (to the effect that all cultures are at first unselfconscious, and become uniformly less so as they grow more mature). The fact is that selfconsciousness is differently directed in different cultures; some peoples give their closest attention to one sort of thing, some to another. This is excellently demonstrated by Marcel Mauss in "Les Techniques du corps," *Journal de psychologie*, 32 (1935):271–93.

16. Sumner, *Folkways*, pp. 3–4; Lucien Lévy-Bruhl, *How Natives Think* (New York, 1925), pp. 109–16, 127; Roger Brown, *Words and Things* (Glencoe, Ill., 1958), pp. 272–73; B. L. Whorf, "Linguistic Factors in the Terminology of Hopi Architecture," *International Journal of American Linguistics*, 19 (1953):141.

17. Redfield, "The Folk Society," pp. 297, 299–300, 303. For further specific examples, see, for instance, Margaret Mead, "Art and Reality,"

College Art Journal, 2:119 (May 1943); A. I. Richards, *Land, Labour and Diet in Northern Rhodesia* (Oxford, 1939), pp. 230–34, and "Huts and Hut-Building among the Bemba," *Man*, 50 (1950):89; Raymond Firth, *We the Tikopia* (London, 1936), pp. 75–80; Clyde Kluckhohn and Dorothea Leighton, *The Navaho* (Cambridge, Mass., 1946), p. 46.

18. For a rather extreme description of this kind of education, see B. F. Skinner, *The Behavior of Organisms* (New York, 1938). A more balanced discussion of the growth of feeling for a skill is to be found in J. L. Gillin and J. P. Gillin, *Cultural Sociology* (New York, 1948), p. 80.

19. *Ibid.*, pp. 400–3.

20. *Ibid.*, pp. 403–4.

21. Jerome Bruner, *The Process of Education* (Cambridge, Mass., 1960), p. 24.

22. The distinction between implicit rules and explicit rules is explored at some length by E. T. Hall in *The Silent Language* (New York, 1959), pp. 69–74 and 91–95.

23. It has been common, ever since the great Paris exhibition of primitive art at the turn of the century, to claim all sorts of things for the primitive artists—that they are more sensitive than we, more highly developed as artists, etc. The same thought appears in Barbara Hutton, *The Unsophisticated Arts* (London, 1945). I am profoundly skeptical. The secret of the primitive form-builders' success lies not in the men themselves, but in the process of design they are accustomed to. Willy-nilly they are caught up in a process of design which produces good form *on account of the organization of the process*. Similar skepticism is to be found in Ralph Linton, "Primitive Art," *The Kenyon Review*, 3:34–51 (Winter 1941).

24. See, typically, Sumner, *Folkways*, p. 54; A. R. Radcliffe-Brown, *Structure and Function in Primitive Society* (Glencoe, Ill., 1952), pp. 7–9.

25. The archeological evidence is so thin that any pseudo-Darwinian accounts based on it cannot be more than highly general and rather doubtful fictions. Radcliffe-Brown, *Structure and Function in Primitive Society*, pp. 202–3.

26. To see that this kind of assumption, implicit throughout the writings of Lewis Morgan, for example, is unjustified, see Radcliffe-Brown, *Structure and Function in Primitive Society*, p. 203.

27. The concept of homeostasis was first used extensively by W. B. Cannon in *The Wisdom of the Body* (London, 1932). For a precise definition see W. Ross Ashby, *Design for a Brain*, 2nd ed. (New York, 1960), chapter 5. And for a number of discussions see *Self-Organizing Systems*, ed. Marshall Yovits and Scott Cameron (New York, 1960). For a de-

tailed descriptive discussion see also H. von Foerster, "Basic Concepts of Homeostasis," *Homeostatic Mechanisms*, Brookhaven Symposia in Biology, No. 10 (Upton, N.Y., 1957), pp. 216–42.

28. This example is based on one given in Ashby, *Design for a Brain*, p. 151.

29. *Ibid.*

30. See Chapter 9, note 4.

31. Ashby, pp. 192–204.

32. As Ashby puts it, "For the accumulation of adaptations to be possible, the system must not be fully joined" (p. 155).

33. This behavior of the misfits may be represented in step-function form. See Ashby, pp. 87–90.

34. This would correspond to what Ashby calls ultrastability, *ibid.*, pp. 122–37.

Chapter Four: The Unselfconscious Process

1. By the definition of Chapter 3, p. 36.

2. Alexander Scharff, *Archeologische Beiträge zur Frage der Entstehung der Hieroglyphenschrift* (Munich, 1942), and "Ägypten," in *Handbuch der Archäologie*, ed. Walter Otto (Munich, 1937), pp. 431–642, especially pp. 437–38.

3. L. G. Bark, "Beehive Dwellings of Apulia," *Antiquity*, 6 (1932):410.

4. Werner Kissling, "House Traditions in the Outer Hebrides," *Man*, 44 (1944):137; H. A. and B. H. Huscher, "The Hogan Builders of Colorado," *Southwestern Lore*, 9 (1943):1–92.

5. In the *Song of Songs* i. 5 we find, "I am black, but comely, O ye daughters of Jerusalem, as the tents of Kedar . . .," and *Exodus* contains many colorful descriptions of the tabernacle (the legendary form of the tent): xxvi.14, "And thou shalt make a covering for the tent of rams' skins dyed red, and a covering above of badgers' skins," and xxvi.36, "And thou shalt make an hanging for the door of the tent, of blue, and purple, and scarlet, and fine twined linen, wrought with needlework." C. G. Peilberg, "La Tente noire," *Nationalmuseets Skrifter*, Etnografisk Raekke, Vol. 2 (Copenhagen, 1944), pp. 205–9.

6. All houses in county Kerry have two doors, but you must always leave by the door you entered by, since a man who comes in through one and goes out through the other takes the house's luck away with him. Åke Campbell, "Notes on the Irish House," *Folk-Liv* (Stockholm), 2 (1938):192; E. E. Evans, "Donegal Survivals," *Antiquity*, 13 (1939):212.

7. Thomas Whiffen, *The North-West Amazons* (London, 1915), p. 225.

And the same is true of many other peoples. For instance: Gunnar Landtman, "The Folk Tales of the Kiwai Papuans," *Acta Societatis Scientiarum Fennicae* (Helsinki), 47 (1917): 116, and "Papuan Magic in the Building of Houses," *Acta Academiae Aboensis, Humaniora*, 1 (1920): 5.

8. Margaret Mead, *An Inquiry into the Question of Cultural Stability in Polynesia*, Columbia University Contributions to Anthropology, Vol. 9 (New York, 1928), pp. 45, 50, 57, 68–69.

9. The blessing way rite, a collection of legends and prayers, makes a positive link between their world view and the shape of the dwelling by relating the parts of the hogan, fourfold, to the four points of the compass, and by referring to them, always, in the order of the sun's path—east, south, west, north. Thus one song describes the hogan's structure: "A white bead pole in the east, a turquoise pole in the south, an abalone pole in the west, a jet pole in the north." The ritual involved in the hogan's use goes further still, so far that it even gives details of how ashes should be taken from the hogan fire. Berard Haile, "Some Cultural Aspects of the Navaho Hogan," mimeographed, Dept. of Anthropology, University of Chicago, 1937, pp. 5–6, and "Why the Navaho Hogan," *Primitive Man*, Vol. 15, Nos. 3–4 (1942), pp. 41–42.

10. Hiroa Te Rangi (P. H. Buck), *Samoan Material Culture*, Bernice P. Bishop Museum Bulletin No. 75 (Honolulu, 1930), p. 19.

11. L. G. Bark, "Beehive Dwellings of Apulia," p. 409.

12. William Edwards, "To Build a Hut," *The South Rhodesia Native Affairs Department Annual* (Salisbury, Rhodesia), No. 6 (1928): 73–74.

13. Iowerth C. Peate, *The Welsh House*, Honorary Society of Cymmrodorion (London, 1940), pp. 183–90.

14. H. Frobenius, *Oceanische Bautypen* (Berlin, 1899), p. 12.

15. Campbell, "Notes on the Irish House," p. 223.

16. Clark Wissler, "Material Culture of the Blackfoot Indians," *Anthropological Papers of the American Museum of History*, Vol. 5, part 1 (New York, 1910), p. 99.

17. L. G. Bark, "Beehive Dwellings of Apulia," p. 408.

18. A. I. Richards, "Huts and Hut-Building among the Bemba," *Man*, 50 (1950): 89.

19. It is true that craftsmen do appear in certain cultures which we should want to call unselfconscious (e.g., carpenters in the Marquesas, thatchers in South Wales), but their effect is never more than partial. They have no monopoly on skill, but simply do what they do rather better than most other men. And while thatchers or carpenters may be employed during the *construction* of the house, repairs are still undertaken by the owner. The skills needed are universal, and at some level or other practiced by everyone. Ralph Linton, *Material Culture of the Marquesas*,

Bernice P. Bishop Museum Memoirs, Vol. 8., No. 5 (Honolulu, 1923), p. 268. Peate, *The Welsh House*, pp. 201–5.

20. Barr Ferree, "Climatic Influence in Primitive Architecture," *The American Anthropologist*, 3 (1890): 149.

21. Richard King, "On the Industrial Arts of the Esquimaux," *Journal of the Ethnological Society of London*, 1 (1848): 281–82. Diamond Jenness, *Report of the Canadian Arctic Expedition (1913–1918)*, vol. 12: *The Life of the Copper Eskimos* (Ottawa, 1922), p. 63; J. Gabus, "La Construction des iglous chez les Padleirmiut," *Bulletin de la Société Neuchateloise de Géographie*, 47 (1939–40): 43–51. D. B. Marsh, "Life in a Snowhouse," *Natural History*, 60.2: 66 (February 1951).

22. W. G. Sumner, *Folkways*, p. 2.

23. Jenness, *Copper Eskimos*, p. 60.

24. W. McClintock, "The Blackfoot Tipi," *Southwestern Museum Leaflets*, No. 5 (Los Angeles, 1936), pp. 6–7.

25. Not only are the walls themselves daubed whenever they need to be, but whole rooms are added and subtracted whenever the accommodation is felt to be inadequate or superfluous. Meyer Fortes, *The Web of Kinship among the Tallensi* (London, 1949), pp. 47–50. Jack Goody, "The Fission of Domestic Groups among the LoDagoba," in *The Development Cycle in Domestic Groups*, ed. J. Goody (Cambridge, 1958), p. 80.

26. Whiffen, *The North-West Amazons*, p. 41.

27. Norbert Wiener, *Cybernetics* (New York, 1948), pp. 113–36.

28. *Ibid.*, pp. 121–22; Ross Ashby, *Design for a Brain* (New York, 1960), pp. 100–4.

29. Strictly speaking, what we have shown concerns only the *reaction* of the unselfconscious culture to misfit. We have not yet explained the occurrence of good fit in the first place. But all we need to explain it, now, is the inductive argument. We must assume that there was once a very simple situation in which forms fitted well. Once this had occurred, the tradition and directness of the unselfconscious system would have maintained the fit over all later changes in culture.

Since the moment of accidental fit may have been in the remotest prehistoric past, when the culture was in its infancy (and good fit an easy matter on account of the culture's simplicity), the assumption is not a taxing one.

30. This is an obvious point. In another context Pericles put it nicely: "Although only a few may originate a policy, we are all able to judge it." Thucydides ii.41.

31. I am indebted to E. H. Gombrich for drawing my attention to this phenomenon. The interpretation is mine.

Chapter Five: The Selfconscious Process

1. Thus selfconsciousness can arise as a natural outcome of scientific and technological development, by imposition from a conquering culture, by infiltration as in the underdeveloped countries today. See Bruno Snell, *The Discovery of the Mind*, trans. T. G. Rosenmeyer (Cambridge, Mass., 1953), chapter 10, "The Origin of Scientific Thought."

2. Hiroa Te Rangi (P. H. Buck), *Samoan Material Culture*, Bernice P. Bishop Museum Bulletin No. 75 (Honolulu, 1930), pp. 85–86.

3. *Ibid.*, p. 86.

4. For discussion of this development in present-day architecture see Serge Chermayeff, "The Shape of Quality," *Architecture Plus* (Division of Architecture, A. & M. College of Texas), 2 (1959–60): 16–23. For an astute and comparatively early comment of this kind, see J. M. Richards, "The Condition of Architecture, and the Principle of Anonymity," in *Circle*, ed. J. L. Martin, Ben Nicholson, and Naum Gabo (London, 1937), pp. 184–89.

5. In Chapter 3, an architecturally selfconscious culture was defined as one in which the rules and precepts of design have been made explicit. In Western Europe technical training of a formal kind began roundabout the mid-fifth century B.C. And the architectural academies themselves were introduced in the late Renaissance. Werner Jaeger, *Paideia*, Vol. I (New York, 1945), pp. 314–16; H. M. Colvin, *A Biographical Dictionary of English Architects, 1660–1840* (Cambridge, Mass., 1954), p. 16. It is of course no accident that the first of these two periods coincided with the prime of Plato's academy (the first establishment where intellectual self-criticism was welcomed and invited), and also with the first extensive recognition of the architect as an individual with a name, and the second with the first widespread crop of architectural treatises. F. M. Cornford, *Before and After Socrates* (Cambridge, 1932); Eduard Sekler, "Der Architekt im Wandel der Zeiten," *Der Aufbau*, 14: 486, 489 (December 1959).

6. For a detailed account of the origin and growth of the academies, see the monograph by Nicolaus Pevsner, *Academies of Art* (Cambridge, 1940), esp. pp. 1–24, 243–95.

7. Margaret Mead, "Art and Reality," *College Art Journal*, 2: 119 (May 1943); Ralph Linton, "Primitive Art," *Kenyon Review*, 3: 42 (Winter 1941).

8. Ralph Linton, *The Study of Man* (New York, 1936), p. 311.

9. See Chapter 3, pp. 41–42.

10. The invention and use of concepts seems to be common to most human problem-solving behavior. Jerome Bruner et al., *A Study of Thinking* (New York, 1956), pp. 10–17. For a description of this process

as re-encoding, see George A. Miller, "The Magical Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information," *Psychological Review*, 63 (1956): 108.

11. See, for instance, American Association of State Highway Officials, *A Policy on Geometric Design of Rural Highways* (Washington, D.C., 1954), Contents; or F. R. S. Yorke, *Specification* (London, 1959), p. 3; or E. E. Seelye, *Specification and Costs*, vol. II (New York, 1957), pp. xv–xviii.

12. John Summerson, "The Case for a Theory of Modern Architecture," *Royal Institute of British Architects Journal* 64: 307–11 (June 1957).

13. Serge Chermayeff and Christopher Alexander, *Community and Privacy* (New York, 1963), pp. 159–175.

14. Reginald R. Isaacs, "The Neighborhood Theory: An Analysis of Its Adequacy," *Journal of the American Institute of Planners*, 14.2: 15–23 (Spring 1948).

15. For a complete treatment of this subject, see Rudolph Carnap, *Meaning and Necessity* (Chicago, 1956). See esp. pp. 23–42, and for a summary see pp. 202–4.

16. *Ibid.*, p. 45.

17. It could be argued possibly that the word "acoustics" is not arbitrary but corresponds to a clearly objective collection of requirements — namely those which deal with auditory phenomena. But this only serves to emphasize its arbitrariness. After all, what has the fact that we happen to have ears got to do with the problem's causal structure?

18. For the fullest treatment of the arbitrariness of language, as far as its descriptions of the world are concerned, and the dependence of such descriptions on the internal structure of the language, see B. L. Whorf, "The Relation of Habitual Thought and Behavior to Language," in *Language, Culture and Personality: Essays in Memory of Edward Sapir*, ed. Leslie Spier (Menasha, Wis., 1941), pp. 75–93.

19. L. Carmichael, H. P. Hogan, and A. A. Walter, "An Experimental Study of the Effect of Language on the Reproduction of Visually Perceived Form," *Journal of Experimental Psychology*, 15 (1932): 73–86.

20. Whorf, "Relation of Habitual Thought and Behavior to Language," p. 76. Whorf, who worked for a time as a fire insurance agent, found that certain fires were started because workmen, though careful with matches and cigarettes when they were near full gasoline drums, became careless near empty ones. Actually the empty drums, containing vapor, are more dangerous then the relatively inert full drums. But the word "empty" carries with it the idea of safety, while the word "full" seems to suggest pregnant danger. Thus the concepts "full" and "empty" actually reverse the real structure of the situation, and hence lead to fire.

The effect of concepts on the structure of architectural problems is much the same. *Ibid.*, pp. 75–76. See also Ludwig Wittgenstein, *The Blue and Brown Books* (Oxford, 1958), pp. 17–20.

21. Vitruvius, *De architectura* 3.1, 3, 4. E. R. De Zurko, *Origins of Functionalist Theory* (New York, 1957), pp. 26–28.

22. Werner Sombart, quoted in *Intellectual and Cultural History of the Western World*, by Harry Elmer Barnes (New York, 1937), p. 509: "Ideas of profit seeking and economic rationalism first became possible with the invention of double entry book-keeping. Through this system can be grasped but one thing — the increase in the amount of values considered purely quantitatively. Whoever becomes immersed in double entry book-keeping must forget all qualities of goods and services, abandon the limitations of the need-covering principle, and be filled with the single idea of profit; he may not think of boots and cargoes, of meal and cotton, but only of amounts of values, increasing or diminishing." What is more, these concepts even shut out requirements very close to the center of the intended meaning! Thus in the case of "economics" even such obvious misfit variables as the cost of maintenance and depreciation have only recently been made the subject of architectural consideration. See J. C. Weston, "Economics of Building," *Royal Institute of British Architects Journal*, 62:256–57 (April 1955), 63:268–78 (May 1956), 63:316–29 (June 1956). As for the cost of social overheads — the milkman's rounds; the laundries and TB sanatoria which have to cope with the effects of smoke from open fireplaces — even the economists are only just beginning to consider these. See Benjamin Higgins, *Economic Development* (New York, 1959), pp. 254–56, 660–61. Yet the cost of the form is found in all these things. The true cost of a form is much more complicated than the concept "economics" at first suggests.

Chapter Six: The Program

1. John von Neumann and Oscar Morgenstern, *Theory of Games and Economic Behavior* (Princeton, 1944); Allen Newell, J. C. Shaw, and H. A. Simon, "Chess-Playing Programs and the Problem of Complexity," *IBM Journal of Research and Development*, 2:320–35 (October 1958); Hao Wang, "Toward Mechanical Mathematics," *IBM Journal of Research and Development*, 4:2–22 (January 1960); A. S. Luchins, *Mechanization in Problem Solving*, American Psychological Association, *Psychological Monographs*, No. 248 (Washington, D.C., 1942); Allen Newell, J. C. Shaw, and H. A. Simon, "Elements of a Theory of Human Problem Solving," *Psychological Review*, 65 (1958):151–66.

2. Marvin Minsky, "Heuristic Aspects of the Artificial Intelligence

Problem," Group Reports 34–55, Lincoln Laboratory, M.I.T., 1956, and "Steps towards Artificial Intelligence," *Proceedings of the Institute of Radio Engineers*, 49:8–30 (January 1961). For further references, see Donald T. Campbell, "Blind Variation and Selective Retention in Creative Thought as in Other Knowledge Processes," *Psychological Review*, vol. 67 (1960), esp. pp. 392–95.

3. See Chapter 7, p. 90. Also Chapter 2, p. 20.

4. See, for instance, Karl R. Popper, *The Logic of Scientific Discovery* (New York, 1959), pp. 53–54, 136–45, 278–81; George Polya, *Patterns of Plausible Inference* (Princeton, 1953); Nelson Goodman, *Fact, Fiction, and Forecast* (Cambridge, Mass., 1955), pp. 82-120; W. Pitts and W. S. McCulloch, "How We Know Universals," *Bulletin of Mathematical Biophysics*, 9 (1947):124–47.

5. There are many speculations about the nature of this process in the literature. See such books as Brewster Ghiselin, *The Creative Process* (Berkeley, 1952), and Paul Souriau, *Théorie de l'invention* (Paris, 1881).

6. From the failure of selfconsciousness we might argue first that we should dispense with the designer altogether, and should therefore make the self-organizing character of the unselfconscious ensemble our point of departure. With this end in mind, we might concentrate on giving the ensemble inself properites which would enhance its power to effect internal adaptations. In a trivial sense we already do this when we fit a steam engine with a governor. The regulation of a series of dams or a production line by means of automatic electronic control is a more elaborate example of the same thing. Providing a city with a governmental structure which lets the administration get things done fast is another example. In the future it may even be possible to give cities a physical organization that encourages them to grow and to adapt to new conditions better than they do at present. Cf. Lancelot Whyte, "Some Thoughts on the Design of Nature and Their Implication for Education," *Arts and Architecture*, 73:16–17 (January 1956). All these devices take the burden off selfconscious control and design, because, like the unselfconscious process, they tend to make the ensemble self-organizing.

The drawback of such devices is that they are only useful in very special and limited situations. Their application demands even greater grasp of the ensemble's condition than the selfconscious designer requires. When we come across unfamiliar circumstances where they cannot be applied, there is no alternative to inventiveness; and we must acknowledge something which has so far perhaps not been brought out strongly enough: the human brain is, in spite of its drawbacks, potentially capable of much deeper insight and resolution than anything an external self-organizing process can achieve. Its great potential strength lies in the fact that it derives forms from a conceptual picture of the ensemble, rather than from the ensemble itself. This allows a much wider range of more flexible and intricate forms to develop than does the unselfconscious process, whose forms must always be of a type which can emerge from the everyday events of the real-world ensemble.

7. For a quick introduction to set theory, see Paul R. Halmos, *Naive Set Theory* (New York, 1960). More complete discussion of the theory is to be found in Felix Hausdorff, *Set Theory*, trans. J. R. Aumann (New York, 1957).

8. See the axiom of specification, Halmos, *Naive Set Theory*, p. 6. For the ideas which follow, see *ibid.*, pp. 2, 3, 12, 14.

9. It is commonly understood among designers that the first task in dealing with a design problem is to strip the definition of the problem down to practical terms, to decide just what conditions a successful form must meet. As one designer, Louis Kahn, puts it, when he wants to know what the form really has to do, he asks himself, "what the form wants to be." The set M is just a precise way of summarizing the elements of what the form wants to be.

10. See pp. 38–45, 64–66.

11. The main works on graph theory are Denes König, *Theorie der endlichen and unendlichen Graphen* (New York, 1950), Claude Berge, *Théorie des graphes et ses applications* (Paris, 1958), which has now been translated (London, 1962), and Oystein Ore, *Theory of Graphs*, American Mathematical Society Colloquium Publications, vol. 38 (Providence, 1962). See also, as a brief introduction, Frank Harary and Robert Z. Norman, *Graph Theory as a Mathematical Model in Social Science* (Ann Arbor, 1955).

12. In a sense the web of this graph might be regarded as an explicit version of what designers and artists have often talked about as the "internal logic" of a problem.

13. A decomposition is a special case of a partly ordered system; for which see Garrett Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25 (New York, 1948), pp. 1–2.

14. For a discussion of the part played by conceptual hierarchies in cognitive behavior, see George A. Miller, Eugene Galanter, and Karl H. Pribram, *Plans and the Structure of Behavior* (New York, 1960), p. 16.

15. The word "program" has occurred a great deal in the recent literature on the psychology of problem solving — the implication throughout being that man's natural way of solving complex problems is to make them easier for himself by means of heuristics which lead him to a solution stepwise. A. D. de Groot, "Über das Denken des Schachspielers," *Revista di psicologia*, 50:89–90 (October-December 1956); Newell, Shaw, and Simon, "Elements of a Theory of Human Problem Solving," pp. 151–66; Miller et al., *Plans and the Structure of Behavior*, throughout;

James G. March and Herbert A. Simon, *Organizations* (New York, 1958), pp. 190–91. It is interesting that John Summerson recently singled out the fact of programs' being used as a source of architectural unity as the distinguishing feature of modern architecture. "The Case for a Theory of Modern Architecture," *Royal Institute of British Architects Journal*, 64:307–11 (June 1957).

Chapter Seven: The Realization of the Program

1. I owe the word "realization" to Louis Kahn, who has used it extensively, and often with a rather wider meaning; his whole teaching revolves about the point discussed in this chapter. See Louis Kahn, "Concluding Talk," in Oscar Newman, ed., *New Frontiers in Architecture: CIAM '59 in Otterlo* (New York, 1961), pp. 205–16.

2. For this photograph, taken by Professor H. Edgerton, Massachusetts Institute of Technology, see, for instance, Gyorgy Kepes, *The New Landscape* (Chicago, 1956), p. 288.

3. See Le Corbusier and Pierre Jeanneret, *Oeuvres complètes, 1934–1938* (Zurich, 1939), pp. 142–47, and Le Corbusier, *La Ville radieuse* (Boulogne, 1935).

4. For the eleven properties of the sphere, see David Hilbert and Stephan Cohn-Vossen, *Geometry and the Imagination* (New York, 1952), pp. 215–32.

5. For a full discussion of the arrow as a diagrammatic symbol, see Paul Klee, *Pedagogical Sketchbook* (New York, 1953), pp. 54-57.

6. See any elementary textbook on organic chemistry. Also, for a graphic presentation, see Max Bill, *Form* (Basel, 1952), p. 19.

7. Theo van Doesburg, *Grundbegriffe der neuen gestaltenden Kunst*, Bauhausbücher No. 6 (Munich, 1924), illustrations 3, 4, 11, 31. Though van Doesburg did not intend his drawings in this way, but only as an exploration of formal possibilities, it can hardly be a coincidence that these drawings coincide, in time, with the birth of an architecture based on rectilinear components.

8. For actual bridges which have this diagrammatic quality very strongly, see Maillart's bridges in Max Bill, *Maillart* (Zurich, 1955), esp. p. 40. The engineer Nervi also has a good deal to say about the use of diagrams; see Pier Luigi Nervi, *Structures* (New York, 1956), pp. 17–26, 97.

9. Of course, the required street widths will not be in exact proportion to the flow densities; flow viscosity, parked cars, etc., mean that the number of vehicles per hour in a given direction is not related linearly to the width required to accommodate them. But the basic organization of the new form will still be that given by the pattern of the diagram.

10. The problem of soap films was first solved by Joseph Plateau, *Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires* (Paris, 1873). For recent discussions see D'Arcy Wentworth Thompson, *On Growth and Form*, 2nd ed. (Cambridge, 1959), pp. 365–77; and a beautiful little book by C. V. Boys, *Soap Bubbles and the Forces Which Mold Them*, Doubleday Anchor Science Study Series (New York, 1959).

11. This does not mean, in any sense, that function is capable of defining form; for any one functional program there will usually be many possible forms.

12. François de Pierrefeu and Le Corbusier, *La Maison des hommes* (Paris, 1942).

13. *Encyclopaedia Britannica*, 14th edition, article on "Aeronautics."

14. Robert W. Marks, *The Dymaxion World of Buckminster Fuller* (New York, 1960).

15. Many "projects," which remain unbuilt, but indicate certain extreme possibilities, are really "hypotheses" about particular aspects of some problem. See, for instance, the projects exhibited in 1960 at the Museum of Modern Art under the title "Visionary Architecture," described in Arthur Drexler, "Visionary Architecture," *Arts and Architecture*, 78:10–13 (January 1961).

16. The vital part played by lucid notation in the invention of new mathematics is a striking instance of this. See Ludwig Wittgenstein, *Remarks on the Foundations of Mathematics* (Oxford, 1956), pp. 47, 73, 78, 82.

17. See Appendix 1, pp. 154–173.

Chapter Eight: Definitions

1. In some cases where a designer has explicitly broken his intentions down into a specific list of requirements the list he produces has almost exactly the character of a set of misfit variables. See, for example, A. and P. Smithson, "Criteria for Mass Housing," in Oscar Newman, ed., *New Frontiers in Architecture: CIAM '59 in Otterlo* (New York, 1961), p. 79.

2. In the text which follows, we shall speak interchangeably of meeting the requirement x, of avoiding the misfit x, and of the variable x taking the value 0; and similarly of failing to meet the requirement x, of the misfit x occurring, and of the variable x taking the value 1.

3. Naturally enough, there is always a time lag between the introduction of some new scale and the time when its value can be established predictively for any given form. Thus the sabin, a measure of acoustic

absorption, was introduced in the 1920's. Even now, 1963, the absorption of an auditorium of complicated shape can still not be predicted, and needs to be determined experimentally. See Wallace C. Sabine, *Collected Papers* (Cambridge, Mass., 1922); V. O. Knudsen, *Architectural Acoustics* (New York, 1932), pp. 119–239.

4. See any typical handbook. For instance, the Dodge Corporation's *Time-Saver Standards: A Manual of Essential Architectural Data* (New York, 1946).

5. Herbert Simon has introduced the concept of "satisficing," as a more accurate picture than "optimization" of what we actually do in complex decision situations. See his three papers, "Rationality and Administrative Decision Making," "A Behavioral Model of Rational Choice," and "Rational Choice and the Structure of the Environment," all published in *Models of Man* (New York, 1957), esp. pp. 204–5, 247–52, and 261–71. Also see James G. March and Herbert A. Simon, *Organizations* (New York, 1958), pp. 140–41.

6. *Ibid.*, pp. 162–63.

7. Karl R. Popper, *The Open Society and Its Enemies* (Princeton, 1950), p. 155. "The piecemeal engineer will, accordingly, adopt the method of searching for, and fighting against, the greatest and most urgent evils of society, rather than searching for, and fighting for, its greatest ultimate good." Also called "social engineering" by Roscoe Pound, *Introduction to the Philosophy of Law* (New Haven, 1922), p. 99. For an economic example see C. G. F. Simkin, "Budgetary Reform," *Economic Record*, 17 (1941): 192ff, and 18 (1942): 16ff.

8. To convince ourselves that this domain D is in principle finite (though of course very large), we must first put arbitrary limits on the actual physical size of the form to be designed. It doesn't matter what size we choose, we can make these limits wide enough to cover anything imaginable. In the case of a drinking-water heater, which must go inside a house, it isn't unreasonable for instance, to expect that even taking its possibly very complex relation with other fitments in the house into consideration, it should not occupy a space larger than 10 meters by 10 meters by 10 meters. Suppose we consider a cubical volume, 10 meters on an edge. It isn't unreasonable to assume that any kettle will fit into it. Divide the cube, by means of a three-dimensional grid, into small cubical cells. Let us say, for the sake of argument, that we choose cells which are 1 micron (1/1000 mm) on an edge. There are then $(10^7)^3$ or 10^{21} of these in the cube. Now let us consider the possibility of filling each one of these cells, independently, cell by cell, with one of 1,000,000 materials (including air, copper, water, silica, etc.). There are then $(10^6)^{10^{21}}$, or roughly $10^{10^{22}}$, different possible ways of arranging our materials, distributing our materials among the cells. (Writing three zeros per second, it would

take 10^{12} centuries to write this number down in full.) Let us call each one of these ways a possible configuration. And let us call the set of all $10^{10^{22}}$ possible configurations, the domain D of possible configurations. *Most* of the configurations, like the distribution of air and water in alternating cells, are clearly absurd. But it is also evident that any conceivable kind of kettle corresponds to one of the $10^{10^{22}}$ configurations in the domain D. For the discussion of such domains (what statisticians often call "sample spaces") see William Feller, *An Introduction to Probability Theory and Its Applications*, I (New York, 1957), 7–25.

9. *Ibid.*, I, 114.

10. G. U. Yule and M. G. Kendall, *An Introduction to the Theory of Statistics*, 14th ed. (London, 1950), pp. 19–29. We can also compare $p(x_i = 1)$ with $p(x_i = 1/x_j = 0)$ — the probability of x_i occurring given that x_j does *not* occur. Or $p(x_i = 0)$ with $p(x_i = 0/x_j = 1)$. There are eight such tests. While they are the same in the case of independence, in the case of dependence they are four slightly different cases, and it is therefore more usual to estimate the common difference which is symmetrical; cf. p. 29.

11. Yule and Kendall, p. 271. This function (the product moment correlation coefficient is also equal to χ^2/N; *ibid.*, p. 272.

12. Requirements are *not* connected simply because they seem in some sense similar. In particular, for instance, the kind of connection we see on account of the fact that two variables have both "to do with acoustics" has no physical implications, and is therefore irrelevant. Here again the language would have become unjustifiably compulsive; for it is to some large extent accidental that we have a concept called "acoustics."

We must be careful too, not to think requirements connected because of what seem like good design ideas. It seems sensible perhaps, to give a house a service core containing kitchen, laundry, plumbing, bathrooms. But the fact that the service core simultaneously meets several requirements does not, per se, make these requirements connected.

13. See p. 109.

14. R. B. Braithwaite, *Scientific Explanation* (Cambridge, 1953), pp. 257–64, 367–68.

15. This is rather like the idea of interpreting the probability of an event as a property of the situation governing that event, rather than the limiting frequency of its occurrence over a number of trials. See Karl R. Popper, "The Propensity Interpretation of the Calculus of Probability, and the Quantum Theory," in *Observation and Interpretation*, ed. by S. Körner, Proceedings of the Ninth Symposium of the Colston Research Society, Bristol (London, 1957), pp. 65–70, and the comment by D. Bohm on page 82 of the same volume. See also W. Kneale *Probability and Induction* (Oxford, 1949), p. 198.

16. For the isomorphism between dyadic relations and graphs see Denes König, *Theorie der endlichen und unendlichen Graphen* (New York, 1950), pp. 107–9, and Claude Berge. *Théorie des graphes et ses applications* (Paris, 1958), p. 6. Also for the isomorphism of dyadic relations and square matrices see Irving M. Copilowish, "Matrix Developments of the Calculus of Relations," *Journal of Symbolic Logic*, 13: 193–203 (December 1948). For the extensional definition of a relation as the set of pairs related under it, see Alfred Tarski, "On the Calculus of Relations," *Journal of Symbolic Logic*, 6: 73–89 (March 1941).

17. In fact, as we shall see in Appendix 2, p. 187, the distinction between positive and negative links is irrelevant, and we only need to establish L, not L^+ and L^- separately. We shall also find it convenient in practice to put $\nu = 1$, so that ν_{ij} can only be 0 or 1.

18. It is sometimes quite hard to draw the graph in a simple way, so that the links are not all tangled. For a way to draw graphs, given the matrix of links, see a recent paper published in the *Journal of the Acoustical Society of America*, 33 (1961): 1183, on "Realization of a Linear Graph Given Its Algebraic Specification."

19. See Appendix 2, p. 177.

20. See Appendix 2, p. 177.

21. See Appendix 2, p. 175.

22. Let us note that this condition of equal "size" only refers to the purely formal character of the system of variables. It does not imply that the different variables have equal importance in the solution of the problem. If it is more important to meet one requirement than another, this still has no place in an analysis of the problem's causal structure, but must be handled as it arises during the realization of the program.

23. We know that we shall never find requirements which are *totally* independent. If we could, we could satisfy them one after the other, without ever running into conflicts. The very problem of design springs from the fact that this is not possible because of the field character of the form-context interaction.

24. See the list of variables given in the worked example, Appendix 1, pp. 137–142.

Chapter Nine: Solution

1. For a general discussion see Max Wertheimer, "Untersuchungen zur Lehre von Gestalt, II," *Psychologische Forschung*, 4 (1923): 301–50, translated in shortened form in *Readings in Perception*, ed. by David C. Beardslee and Michael Wertheimer (New York, 1958), pp. 115–35, for a specific reference to this point, see Wolfgang Kohler, *Gestalt Psychology* (New York, 1929), pp. 148–86.

2. L. S. Pontryagin, *Foundations of Combinatorial Topology* (New York, 1952), p. 13. The practical aspects of this method have been developed chiefly by writers on sociometry: Frank Harary and Ian C. Ross, "A Procedure for Clique Detection Using the Group Matrix," *Sociometry*, 20:205–15 (September 1957); R. Duncan Luce and A. D. Perry, "A Method of Matrix Analysis of Group Structure," *Psychometrika*, 14 (1949):95–116; R. D. Luce, "Connectivity and Generalized Cliques in Sociometric Group Structure," *Psychometrika*, 15 (1950):169–90; Denes König, *Theorie der endlichen und undendlichen Graphen* (New York, 1950), pp. 224–37; Claude Berge, *Théorie des graphes et ses applications* (Paris, 1958), pp. 195, 201; G. A. Dirac, "Some Theorems on Abstract Graphs," *Proceedings of the London Mathematical Society*, 3.2 (1952), 69. See also W. Ross Ashby, *Design for a Brain* (New York, 1960), p. 169; R. Duncan Luce, "Two Decomposition Theorems for a Class of Finite Oriented Graphs," *American Journal of Mathematics*, 74:701–22, esp. 703 (July 1952); H. Whitney, "Non-separable and Planar Graphs," *Transactions of the American Mathematical Society*, 34 (1932):339–62, and "Congruent Graphs and the Connectivity of Graphs," *American Journal of* Mathematics, 54 (1932):150; A. Shimbel, "Structural Parameters of Communications Networks," *Bulletin of Mathematical Biophysics*, 15 (1953):501–7, and "Structure in Communication Nets," *Proceedings of the Symposium on Information Networks*, April 1954, Polytechnic Institute, Brooklyn (1955); Satosi Watanabe, "Concept Formation and Classification by Information — Theoretical Correlation Analysis," letter to the editor, *IBM Journal of Research and Development*, January 30, 1961.

Perhaps the broadest discussion is to be found in Kurt Lewin, *Field Theory in Social Science* (New York, 1951), in the appendix called "Analysis of the Concepts Whole, Differentiation, and Unity," pp. 305–38, esp. pp. 305–11.

3. Luce, "Two Decomposition Theorems," p. 703.

4. In practice G will usually be connected; that is, there is a path of links connecting every two vertices. It is then, of course, impossible to find a partition which cuts no links, and we are reduced to finding one across which there is the least, rather than no, interaction. It is worth pointing out right away that it is only possible to look for such minimum interaction partitions because the interactions are probabilistic. As Ashby has pointed out, in a connected system with deterministic linkages, even when every variable is not immediately linked to every other, the system behaves as if it were, so that no one part is less connected to the rest than any other, and it means nothing to compare degrees of independence. Ross Ashby, *Design for a Brain*, 1st. ed. (London, 1952), pp. 161–62, 251–52.

5. See Appendix 2, pp. 176–184.

6. See Appendix 2, p. 190.

7. Ludwig von Bertalanffy, *Problems of Life* (New York, 1960), pp. 37–47.

8. The following note must be appended to this conjecture. If it is true that the causal structure of the problem actually defines the physical constituents of a successful form, we naturally wish to know whether the result of the analysis is independent of the particular set of variables which have been chosen to describe the problem. It is clear that the same problem might have been stated in terms of an altogether different set of variables, which as a whole covers the same ground, but breaks it up differently. This new set would then be clustered in different sets and systems. But would the content of these new systems, or to put it more concretely, the physical components they implied, have been the same. Intuition suggests strongly that this would be so. Indeed, I feel that some sort of invariance theorem of this sort is necessary as a secure foundation for the whole method (like showing that the properties of a vector space are invariant for different bases); but I have not yet succeeded in finding a proof of such a theorem.

Appendix Two: Mathematical Treatment of Decomposition

1. See the previous references to graph theory given in Chapter 6, note 11.

2. G. U. Yule and M. G. Kendall, *An Introduction to the Theory of Statistics*, 14th ed. (London, 1950), p. 272.

3. *Ibid.*, pp. 35, 281.

4. *Ibid.*, pp. 35–36.

5. William Feller, *An Introduction to Probability Theory and Its Applications*, I (New York, 1957), p. 22.

6. *Ibid.*, p. 22.

7. *Ibid.*

8. Because we have artificially made $p(x_i = 0) = \frac{1}{2}$, this probability distribution must not be confused with the proportions of misfits in the domain of solutions D. In that case, $p(x_i = 0)$ is small compared with $p(x_i = 1)$. The present distribution is designed solely to give us the decomposition of the system: it only reflects the actual behavior of the variables as far as their correlations are concerned.

9. See Chapter 8, p. 112.

10. C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication* (Urbana, Ill., 1949), pp. 18–22.

11. See note 17 to Chapter 8.

12. Satosi Watanabe, "Information Theoretical Analysis of Multi-

variate Correlation," *IBM Journal of Research and Development*, 4:69 (January 1960).

13. See note 17 to Chapter 8.

14. Feller, *Probability Theory*, p. 213.

15. To normalize a random variable X, we replace it by $(X-\mu)/\sigma$, where μ is the mean and σ^2 the variance. See Feller, p. 215.

16. We remember that $l_0 = \frac{1}{2}m(m-1)$, $l^\pi = \sum_\pi v_{ij}$, $l_0^\pi = \sum_\pi s_\alpha s_\beta$.

17. Christopher Alexander and Marvin Manheim, *HIDECS 2: A Computer Program for the Hierarchical Decomposition of a Set with an Associated Graph*, M.I.T. Civil Engineering Systems Laboratory Publication No. 160 (Cambridge, Mass., 1962); and Christopher Alexander, *HIDECS 3: Four Computer Programs for the Hierarchical Decomposition of Systems Which Have an Associated Linear Graph*, M.I.T. Civil Engineering Systems Laboratory Research Report R63–27 (Cambridge, Mass., 1963).