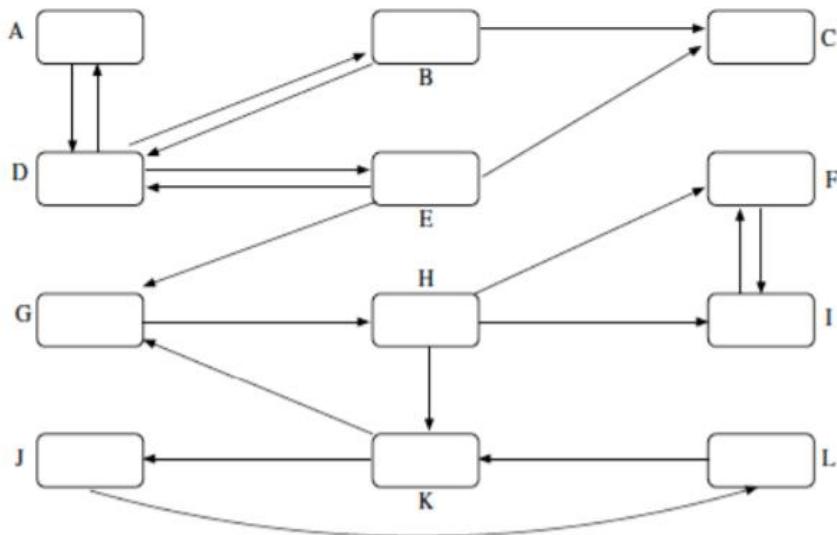


# CS590-HW6

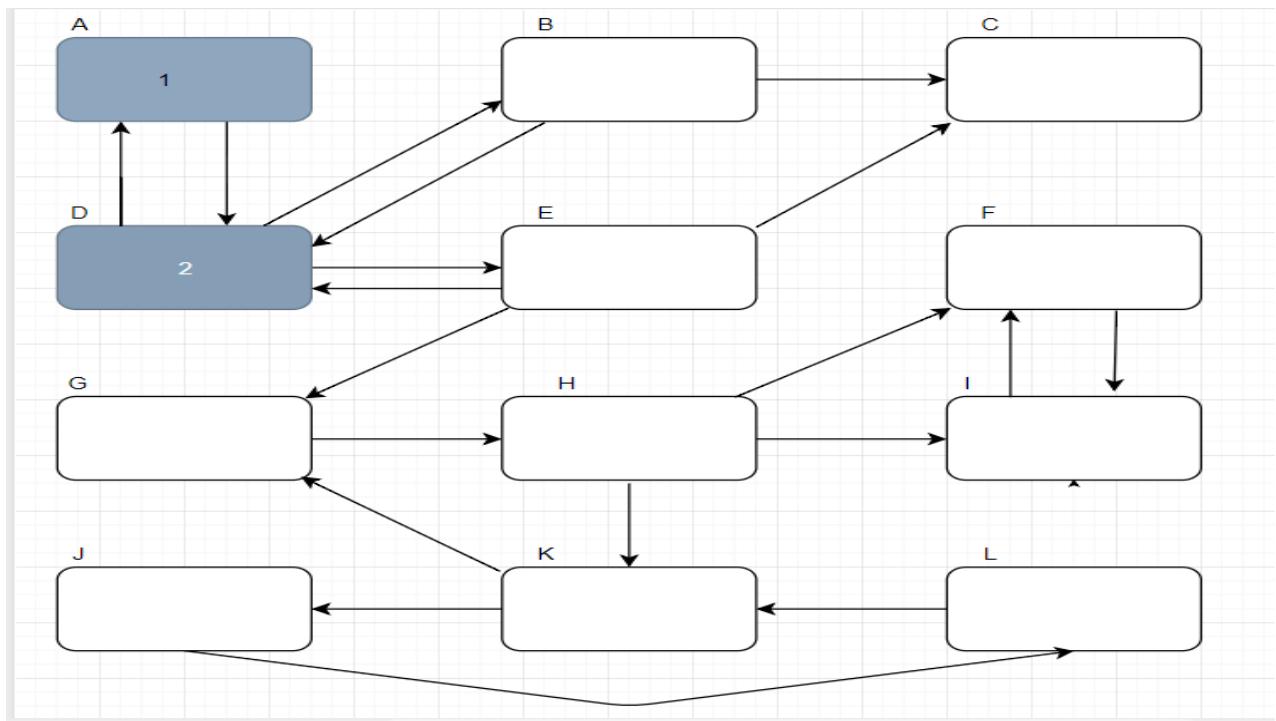
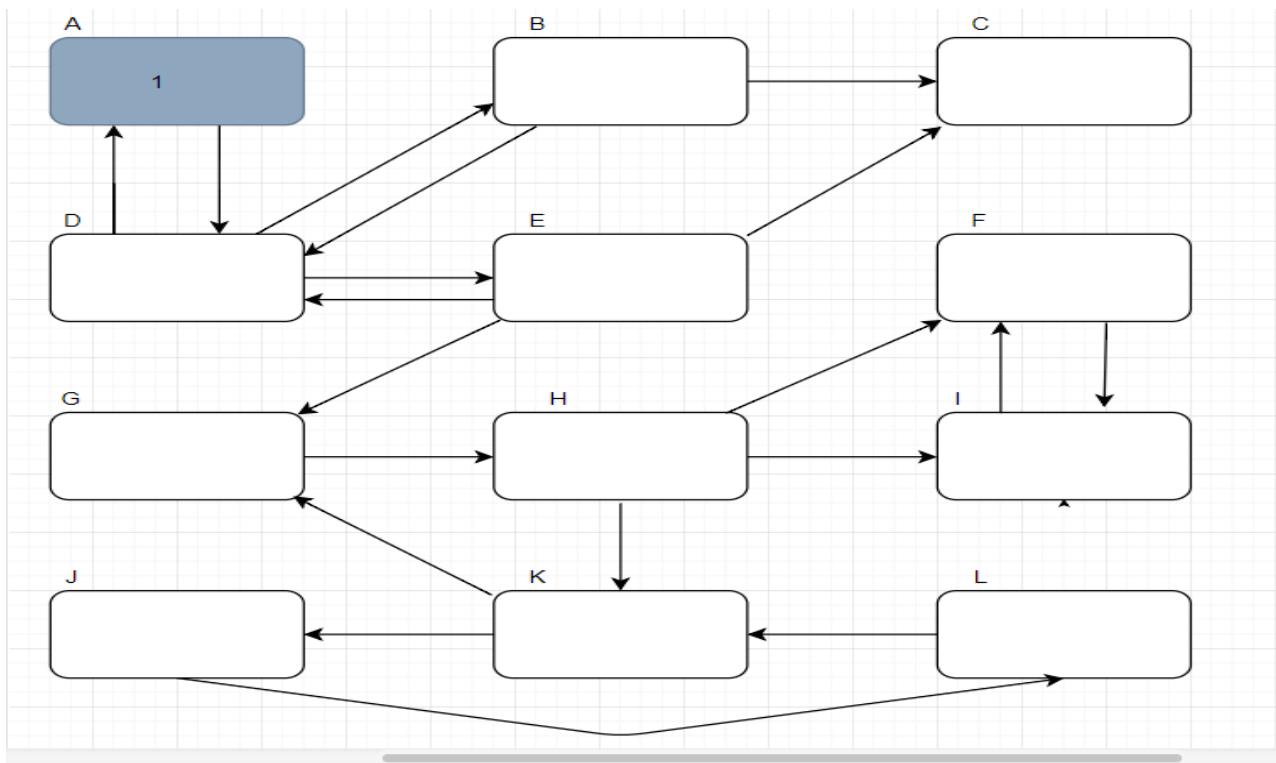
Q1)

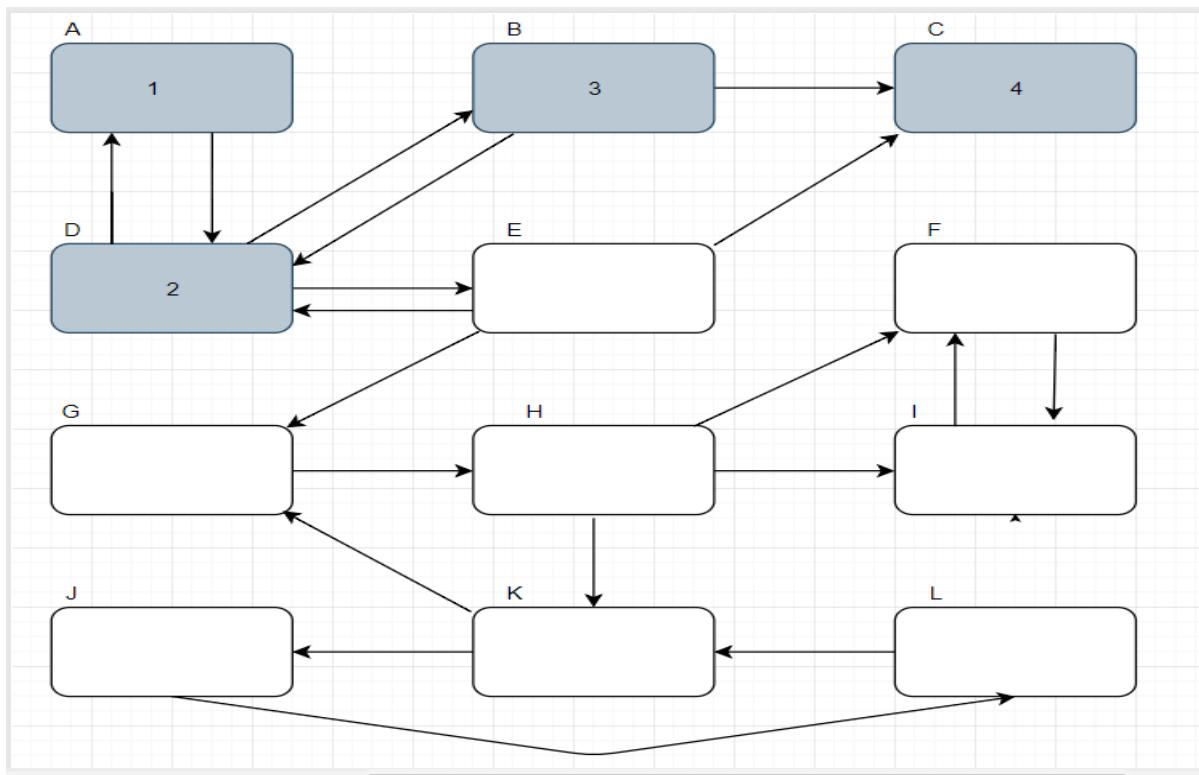
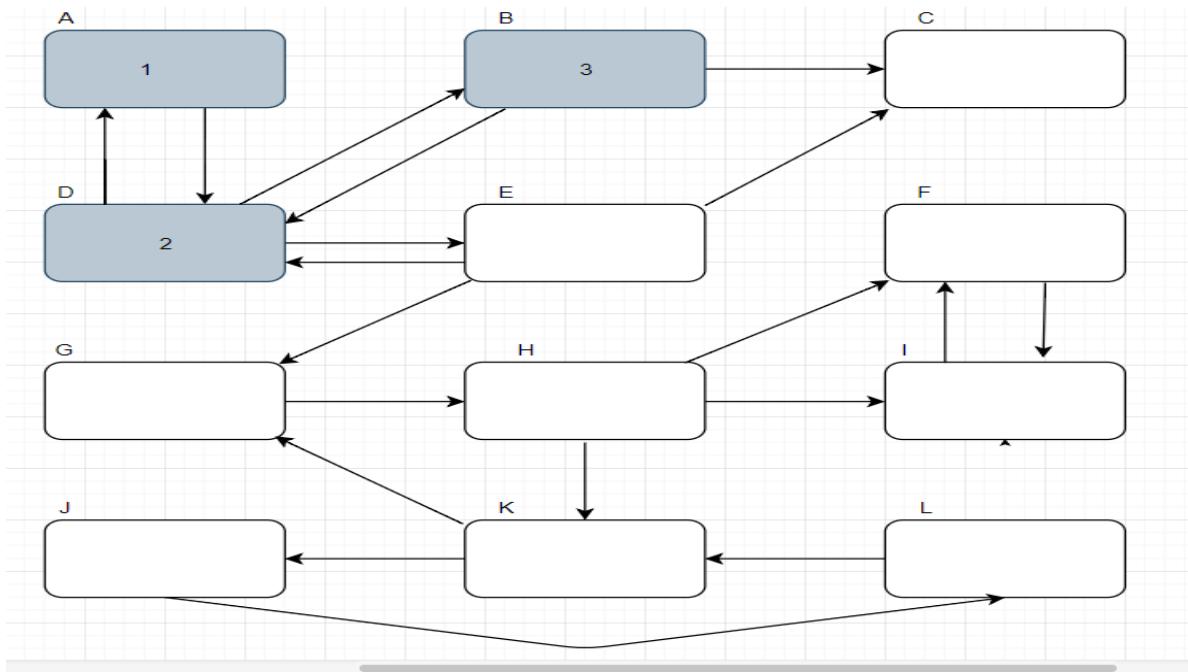
a)

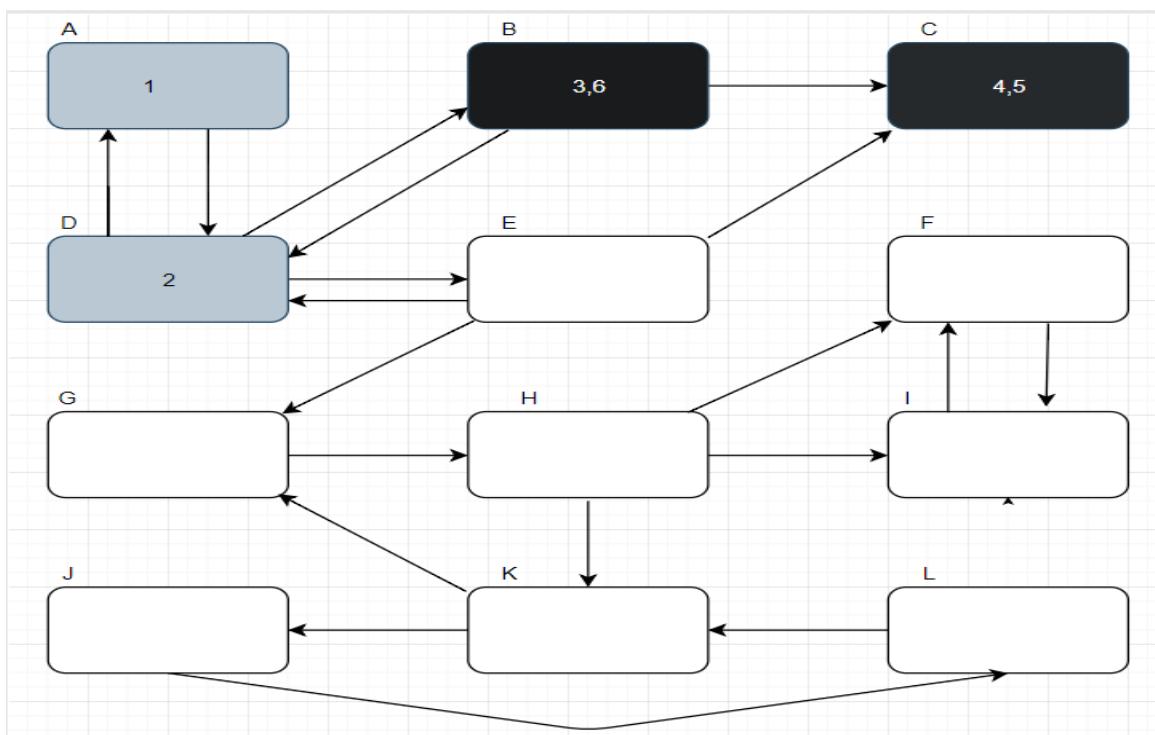
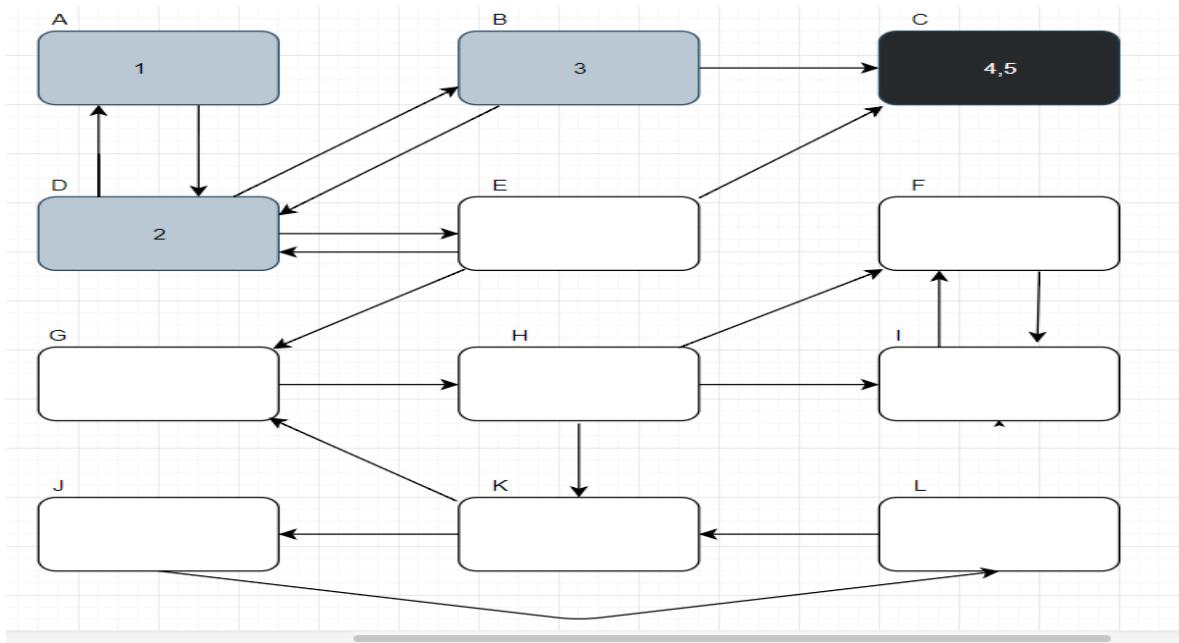
1. You are given the following graph  $G = (V, E)$ :

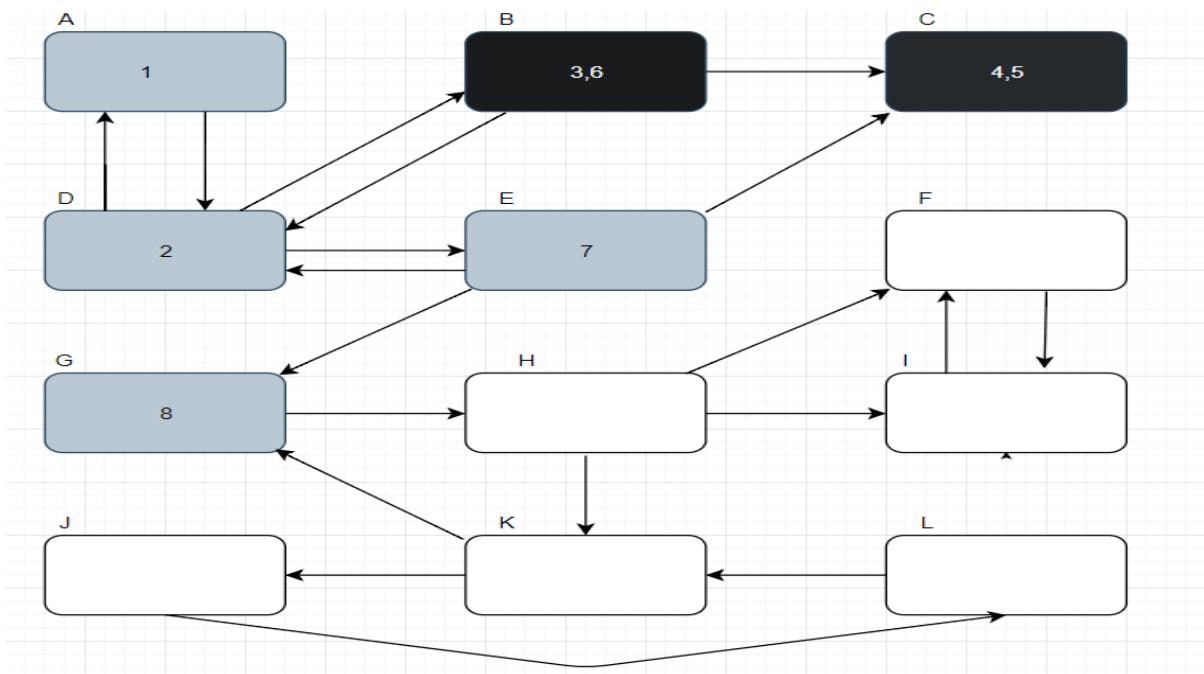
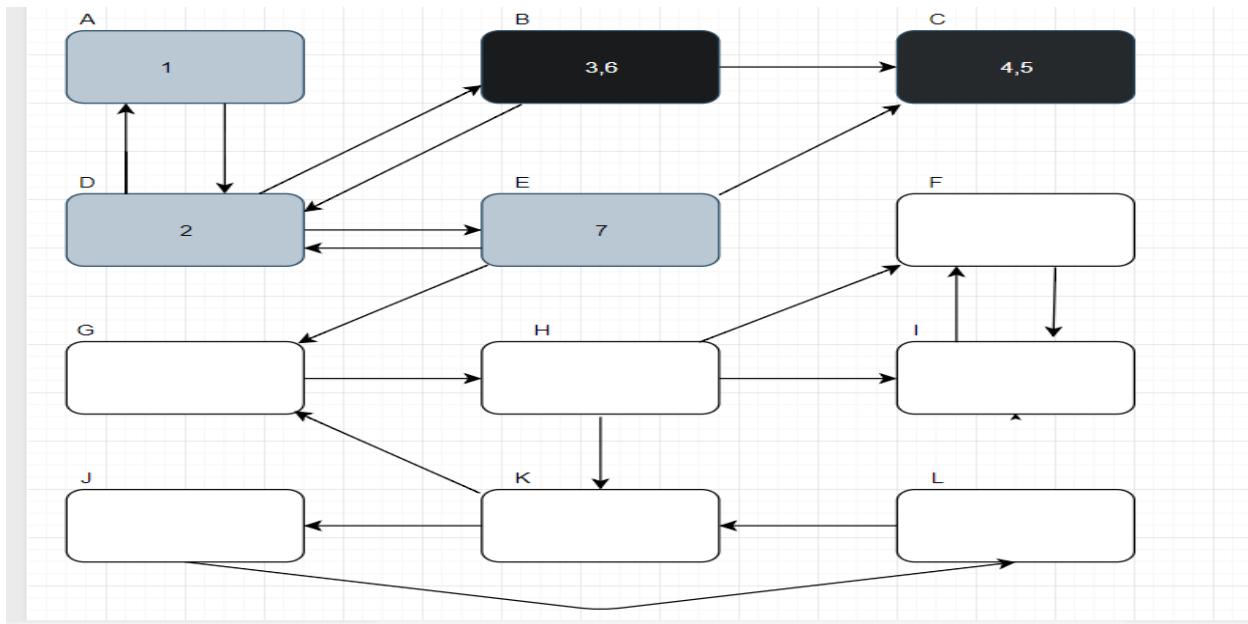


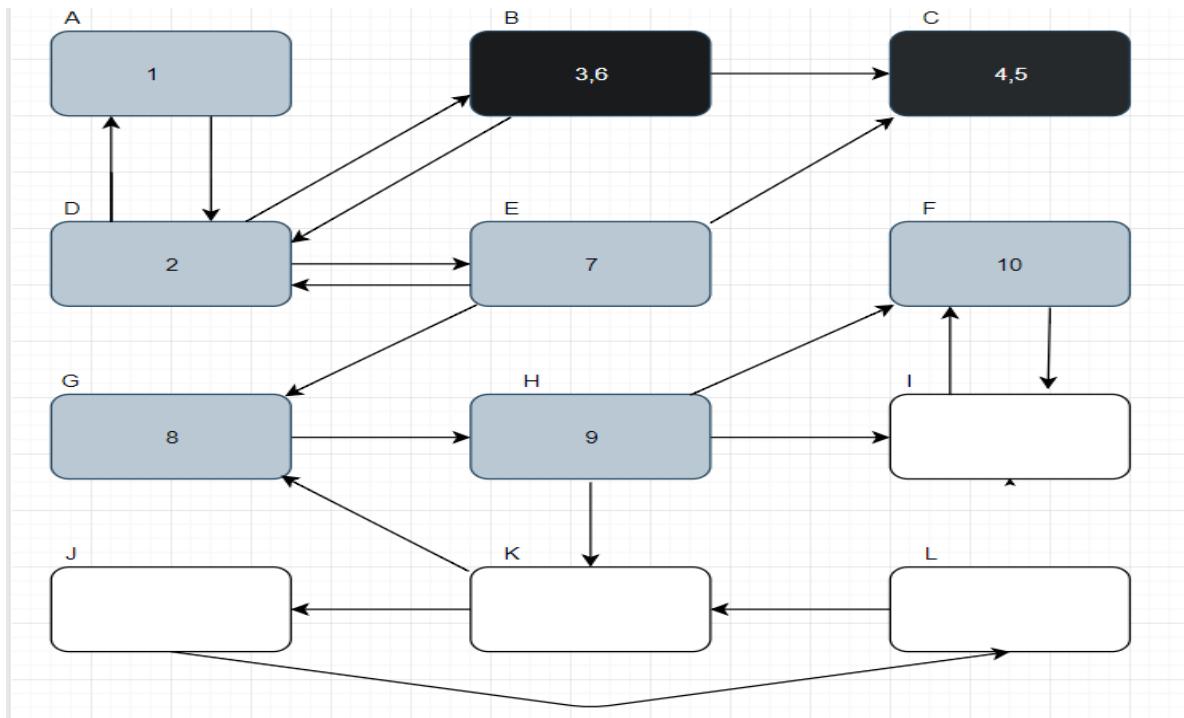
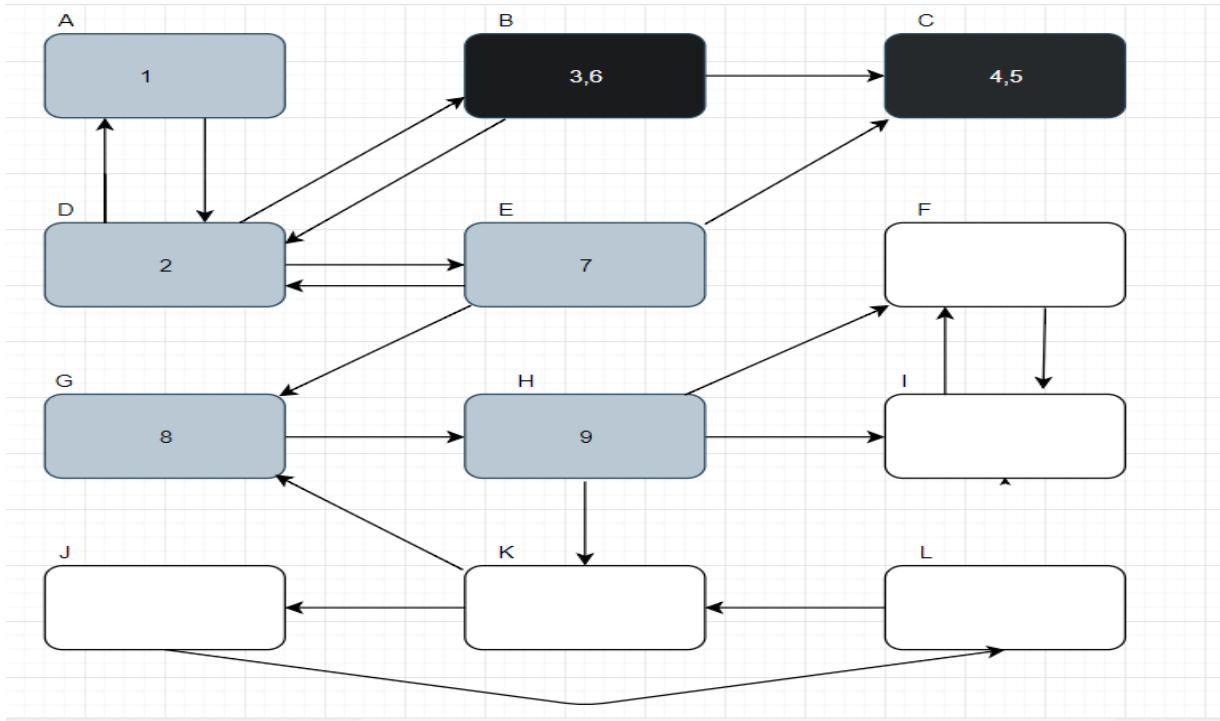
- (a) Perform a DFS (depth-first search) on the given graph. You will visit the nodes in increasing order starting with **A** (increasing from **A** to **L**). Give the discovery  $u.d$  and finishing times  $u.f$  for every vertex  $u$  in the graph. Classify all edges (tree, back, forward, and cross edges).

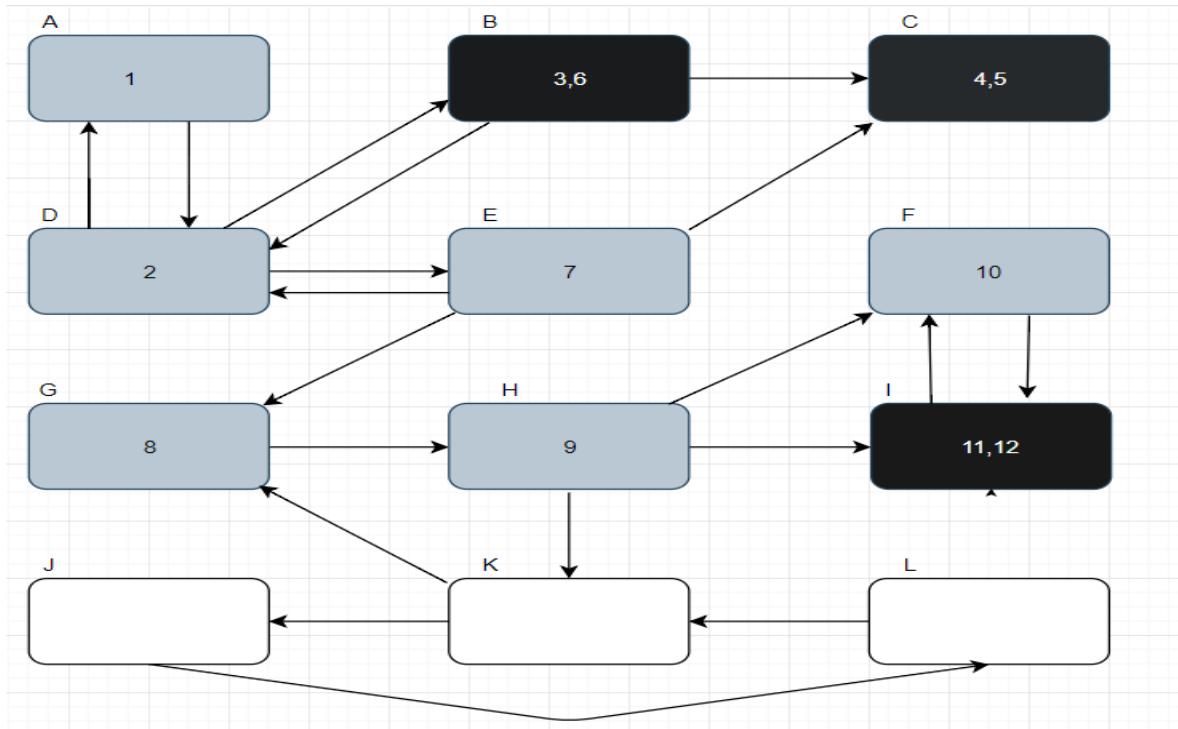
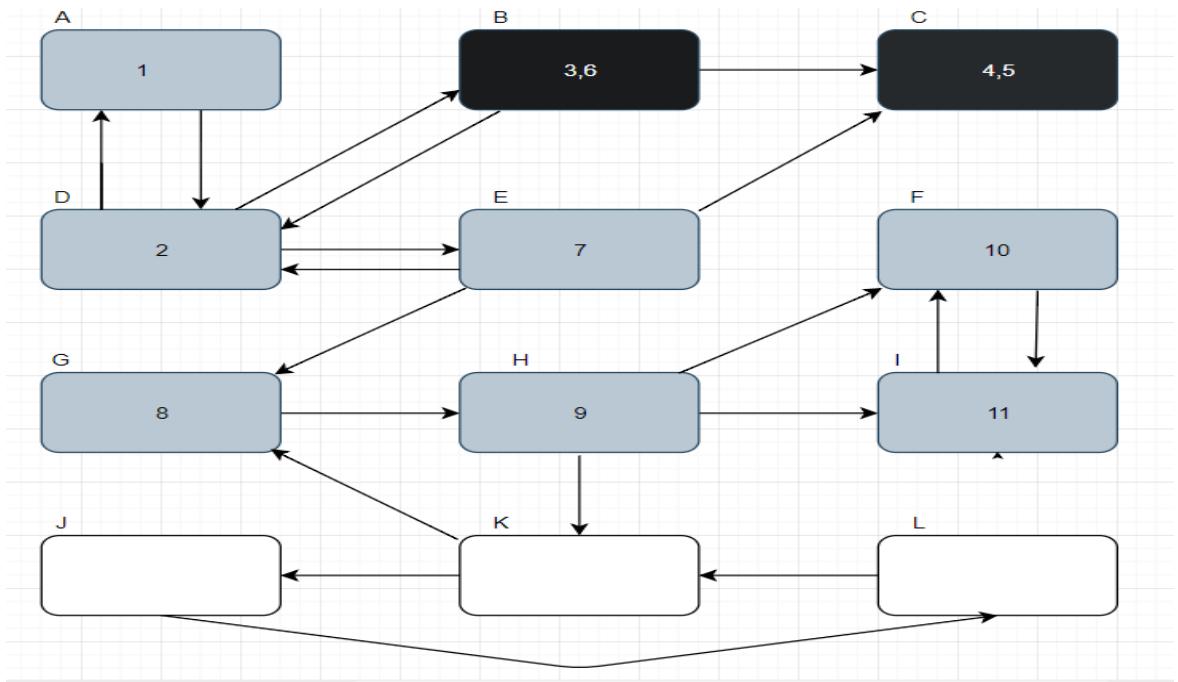


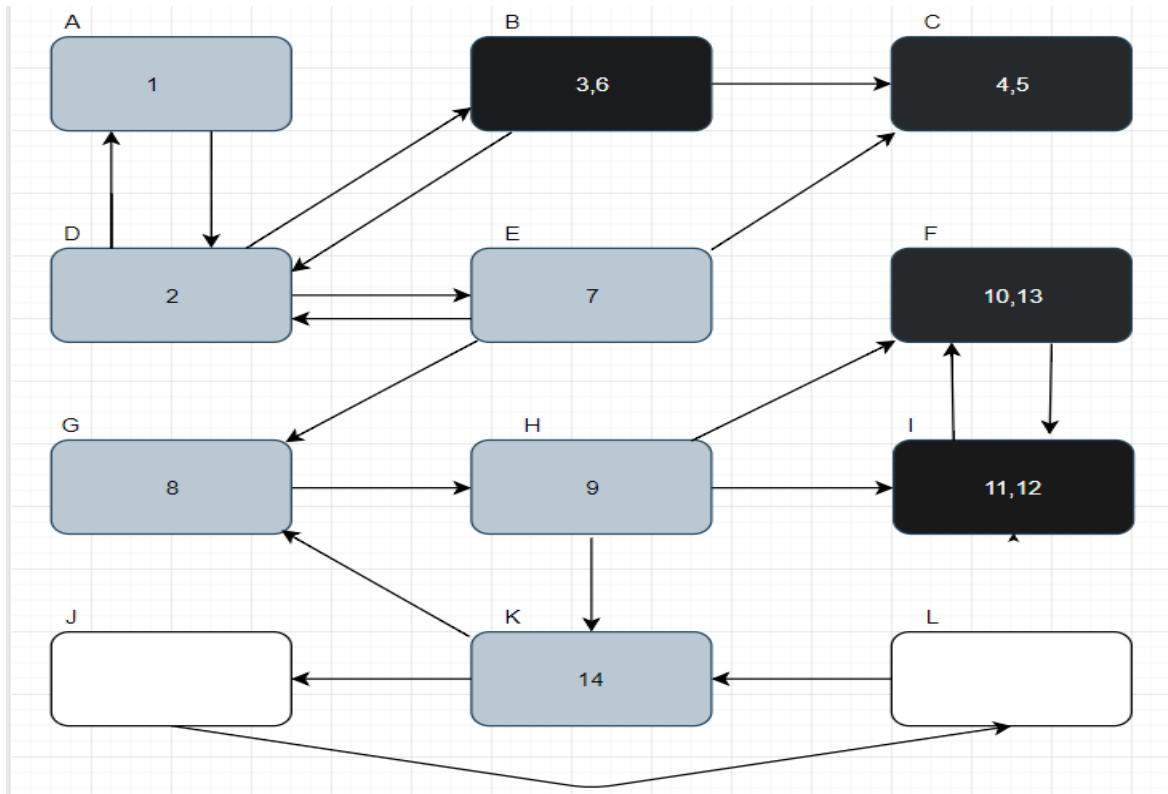
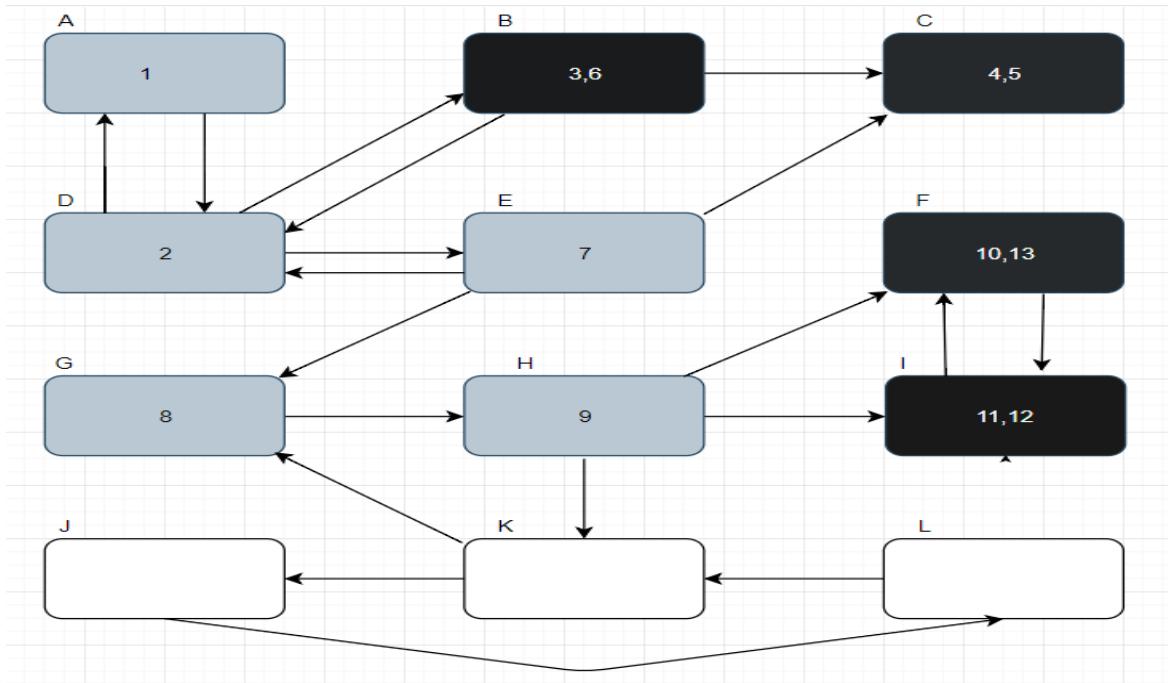


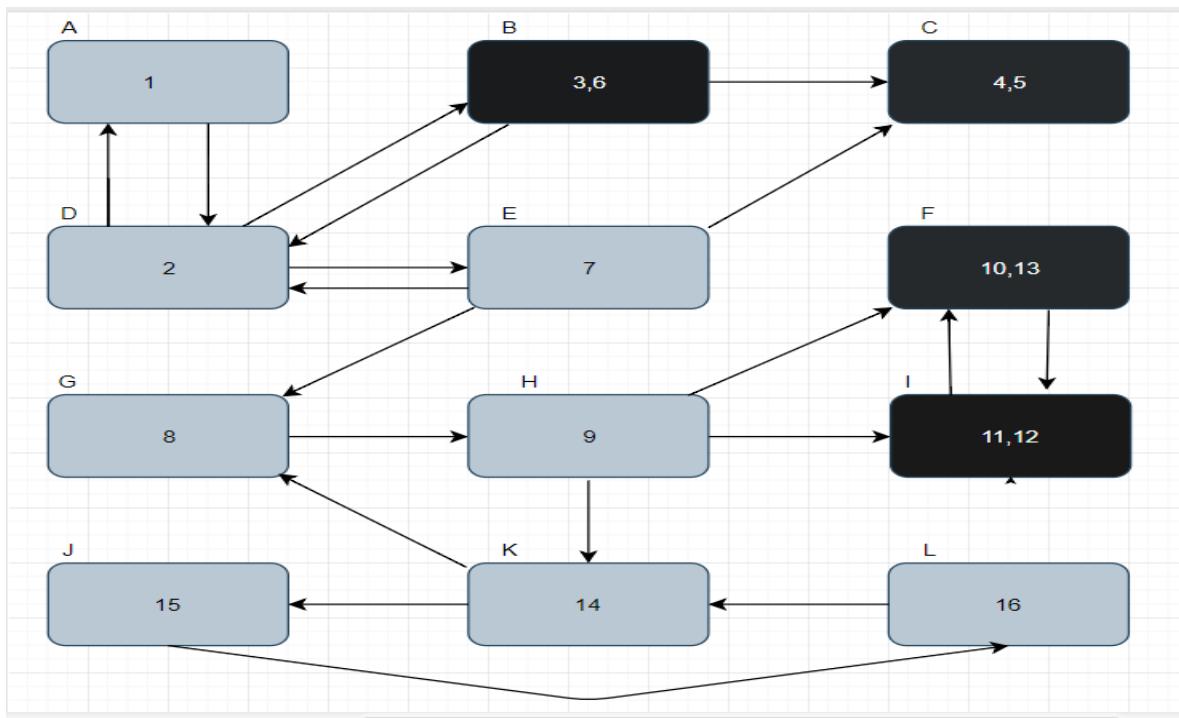
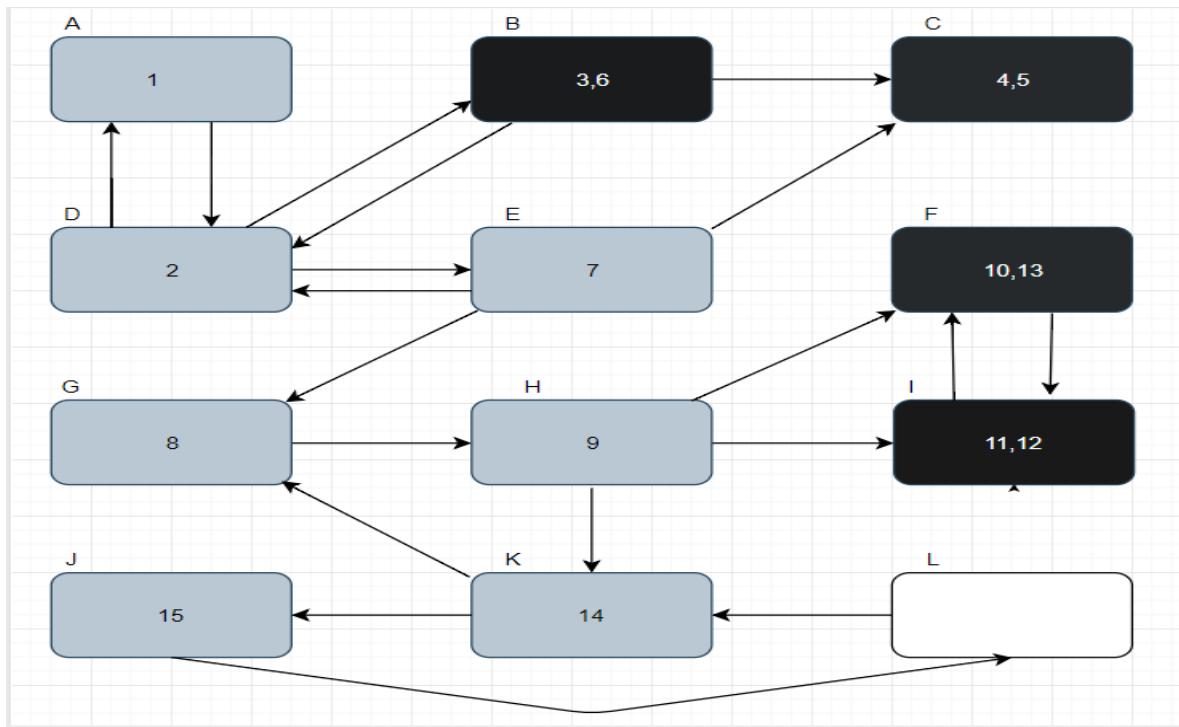


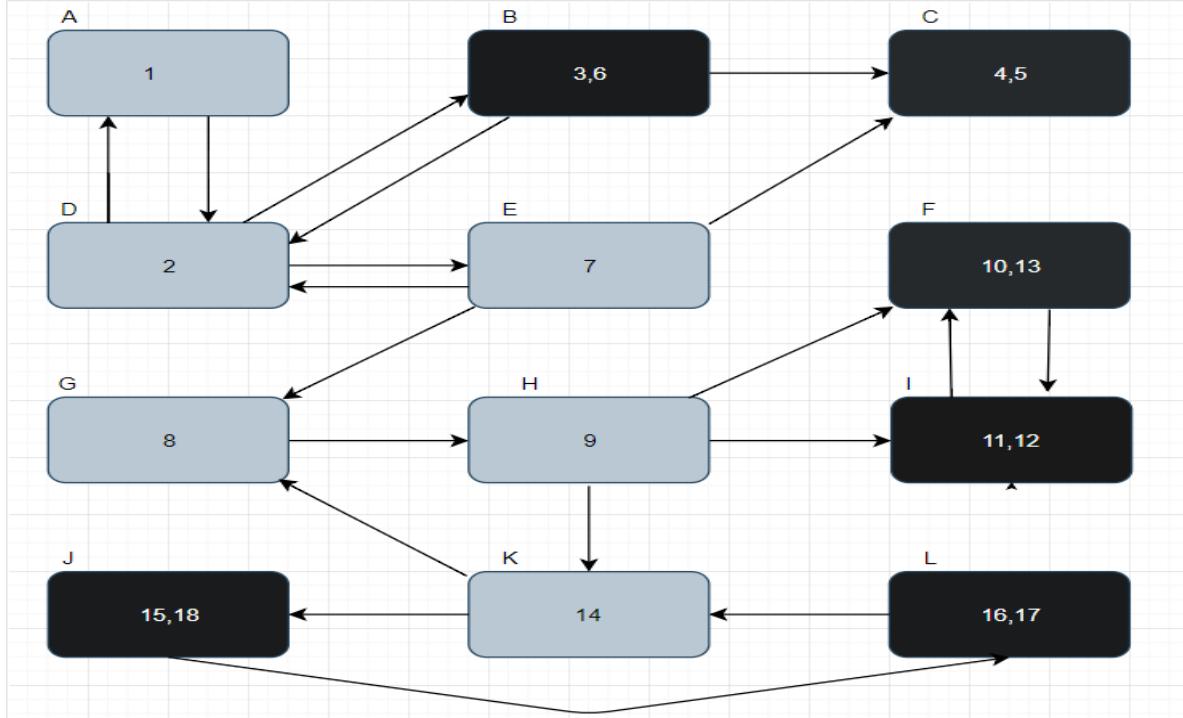
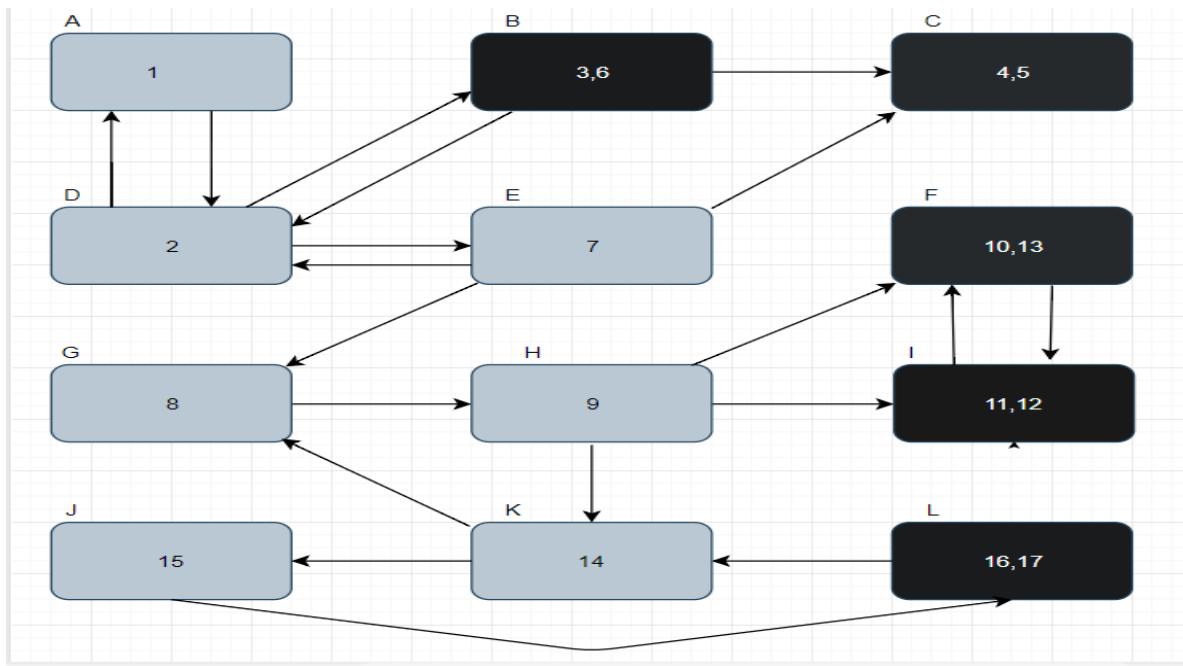


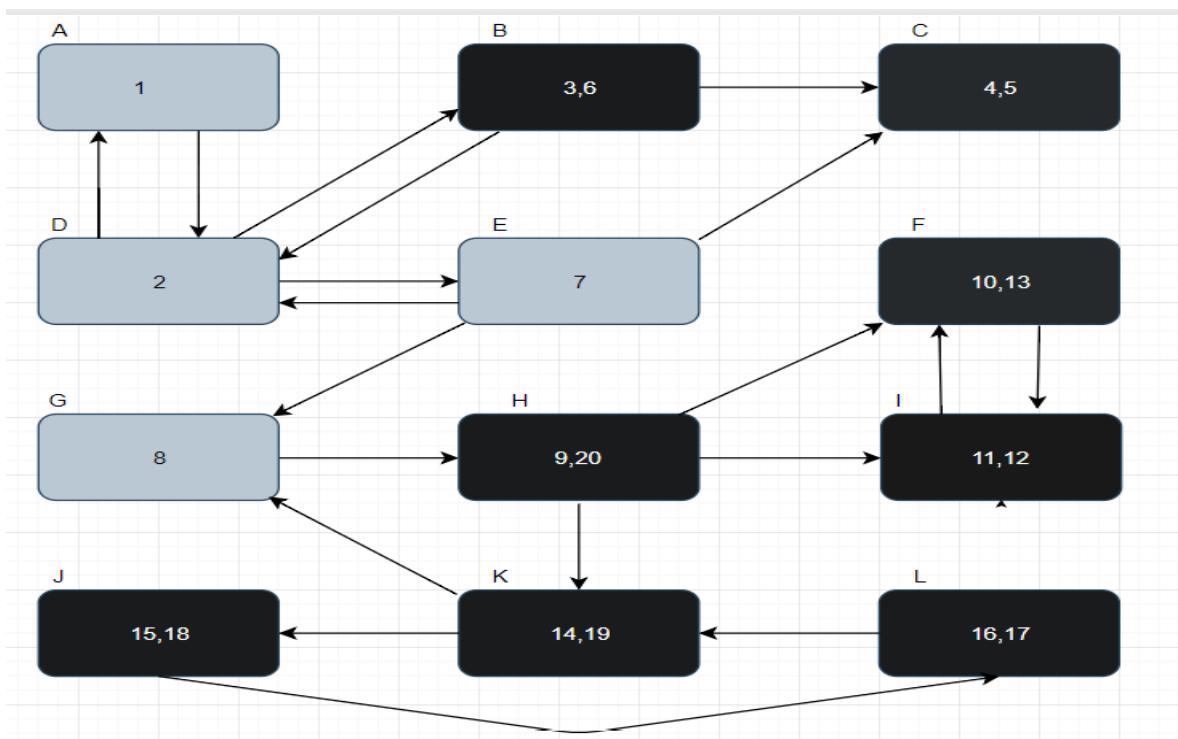
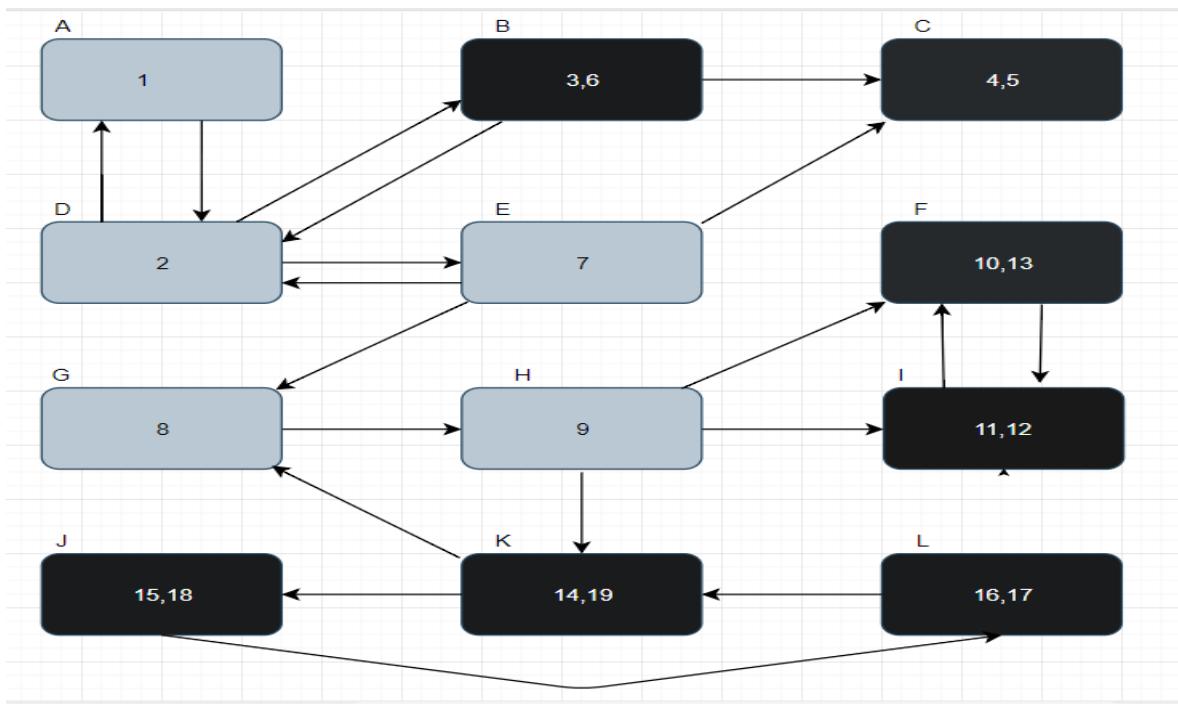


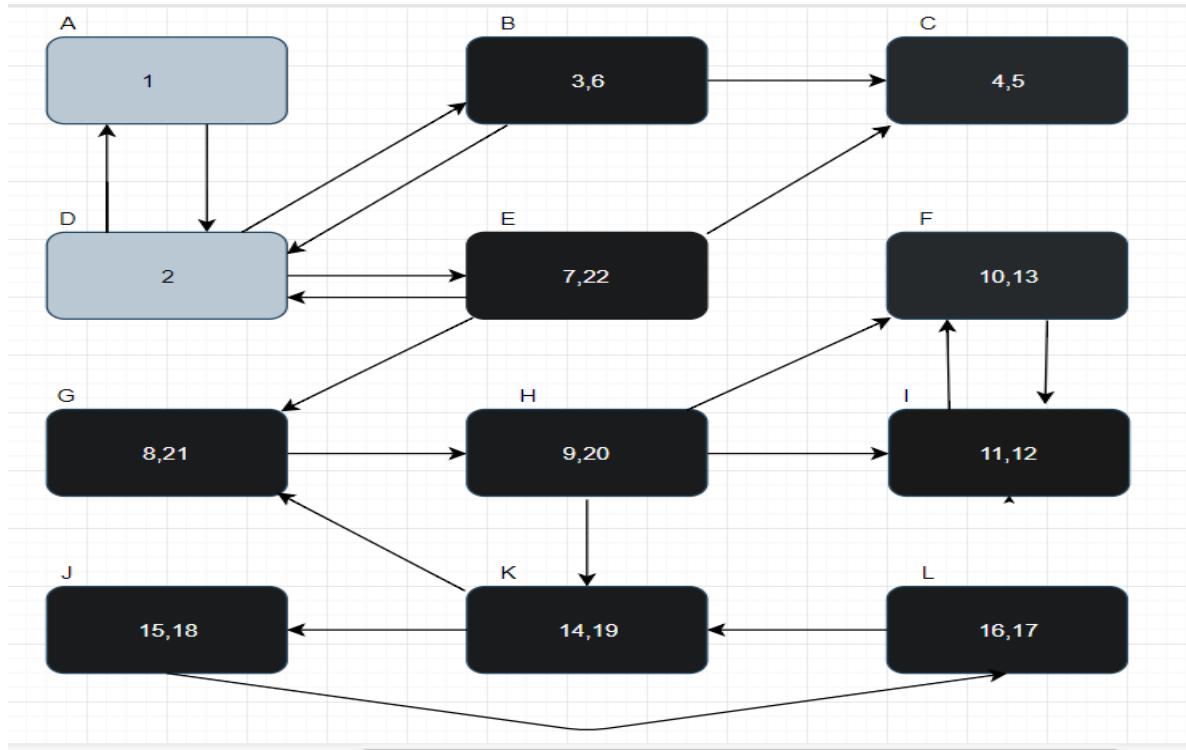
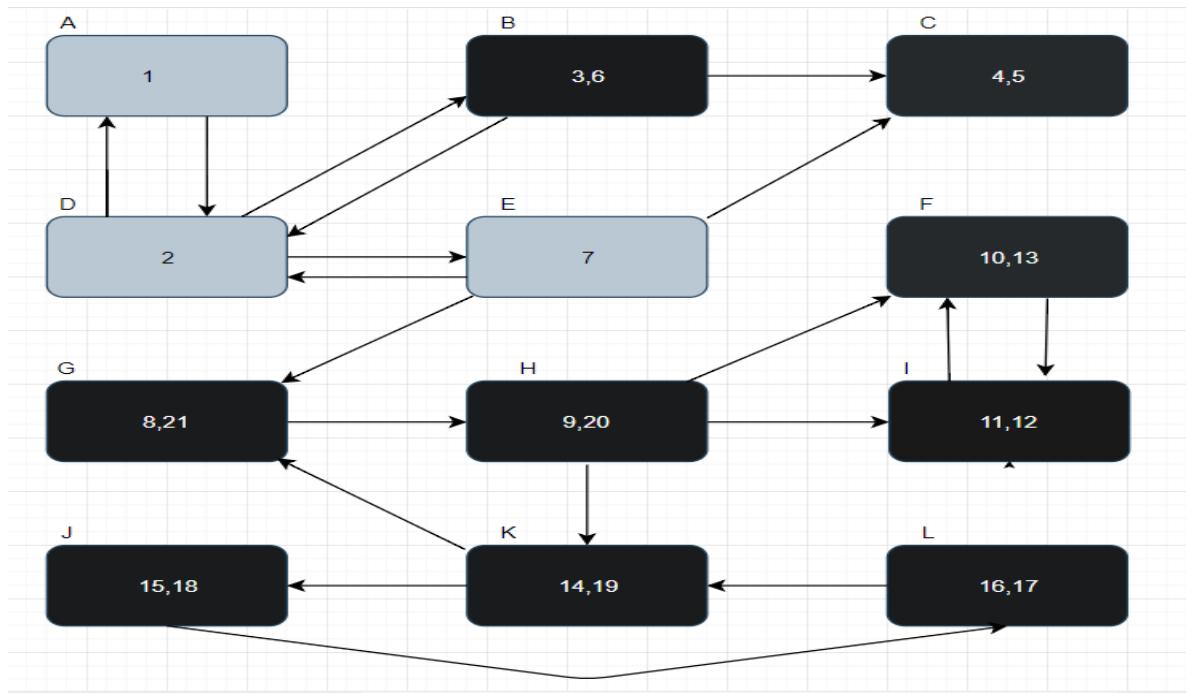


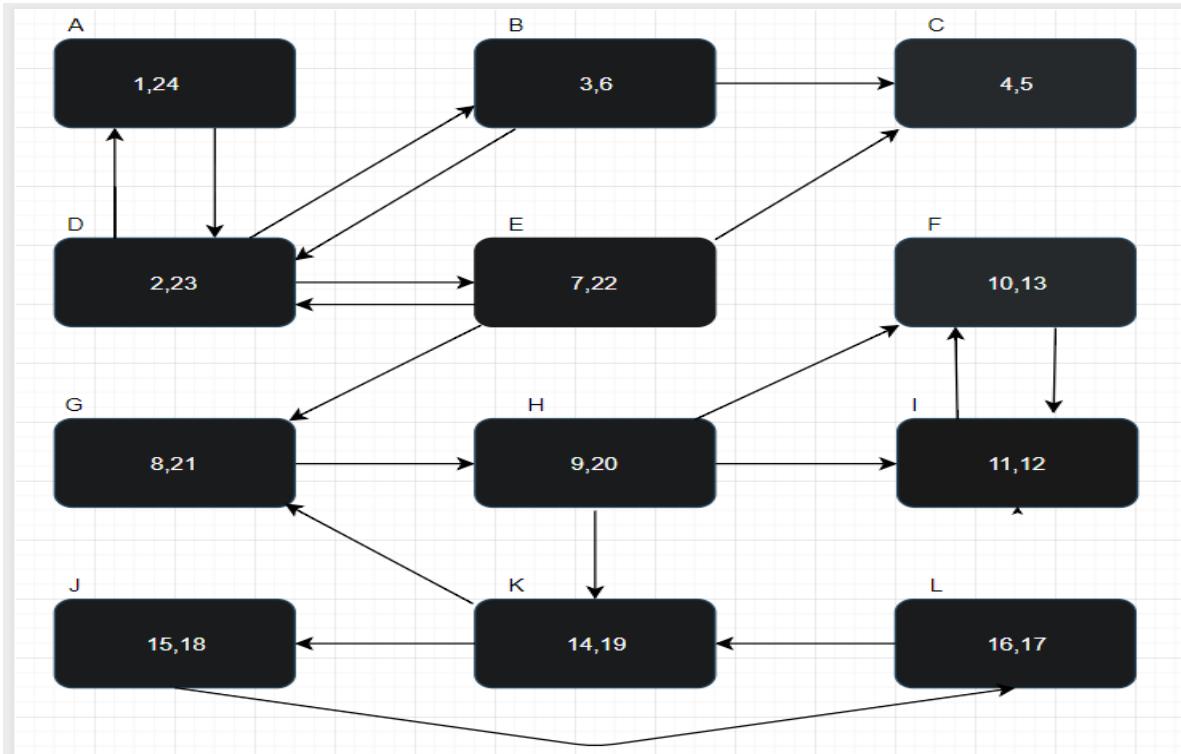
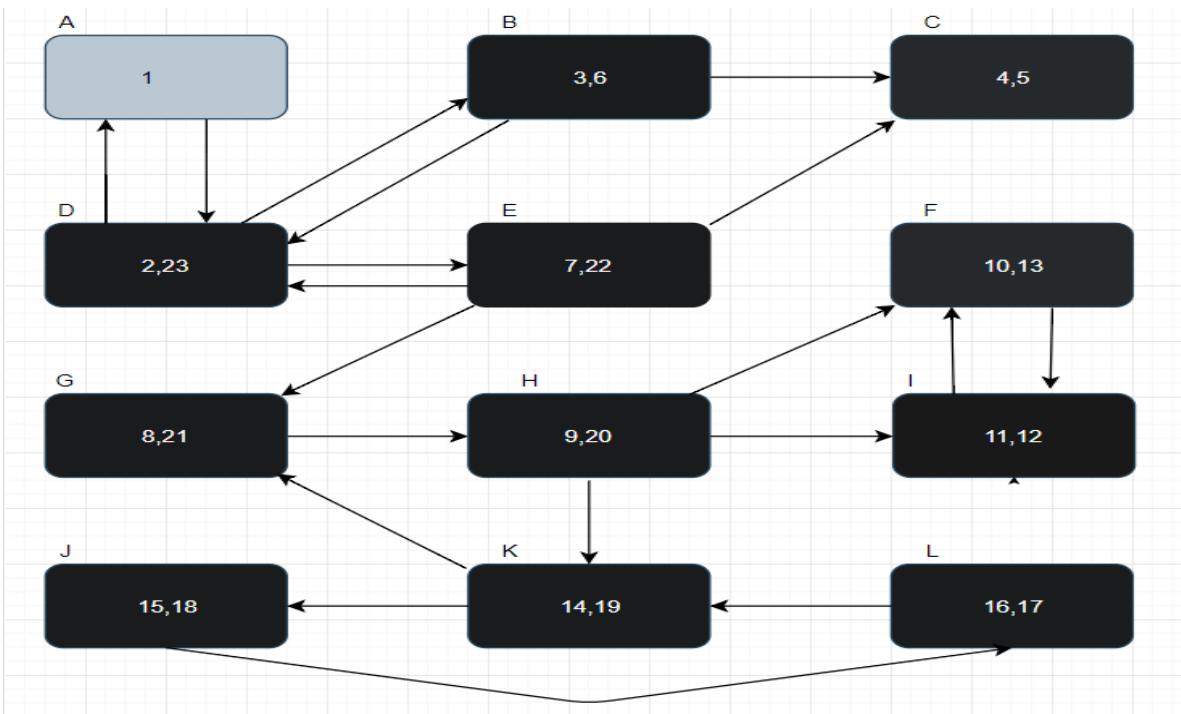












Vertex	Discovery time	Finishing time
A	1	24
B	3	6
C	4	5
D	2	23
E	7	22
F	10	13
G	8	21
H	9	20
I	11	12
J	15	18
K	14	19
L	16	17

**Tree edges:**

A->D

D->B

D->E

B->C

E->G

G->H

H->F

F->I

H->K

K->J

J->L

**Backward edges:**

D->A

B->D

E->D

I->F

K->G

L->K

**Forward edges:**

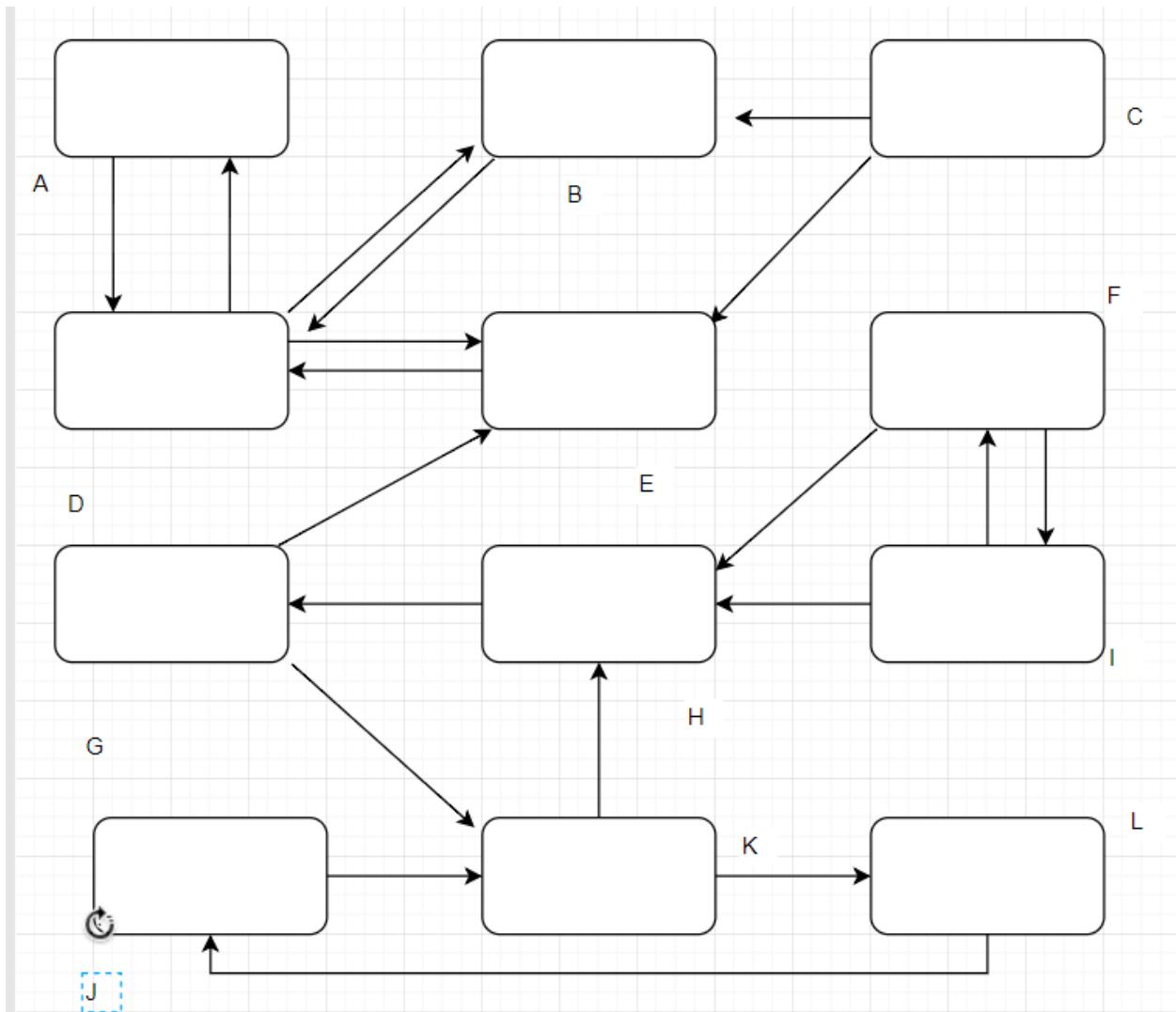
H->I

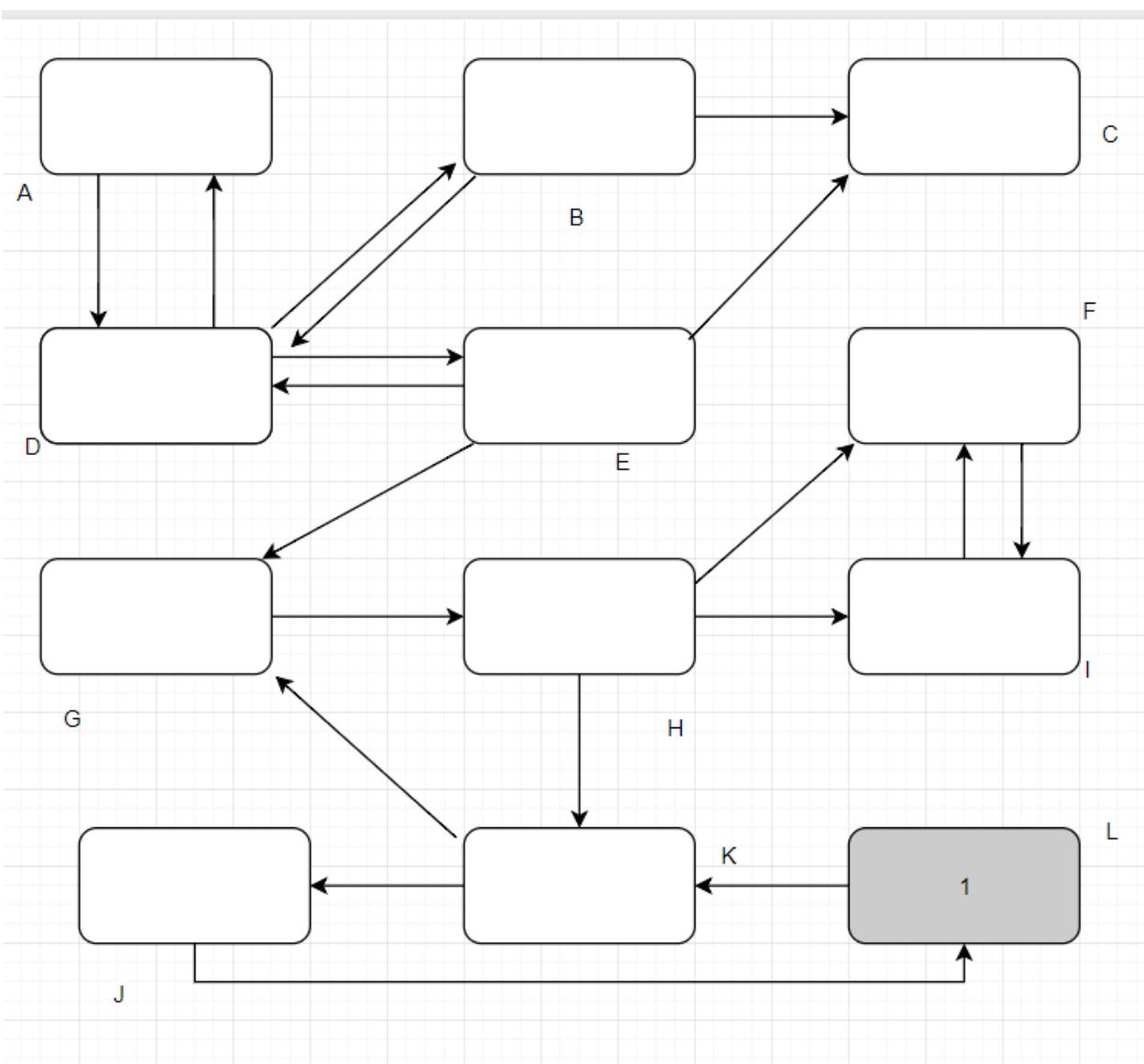
**Cross edges:**

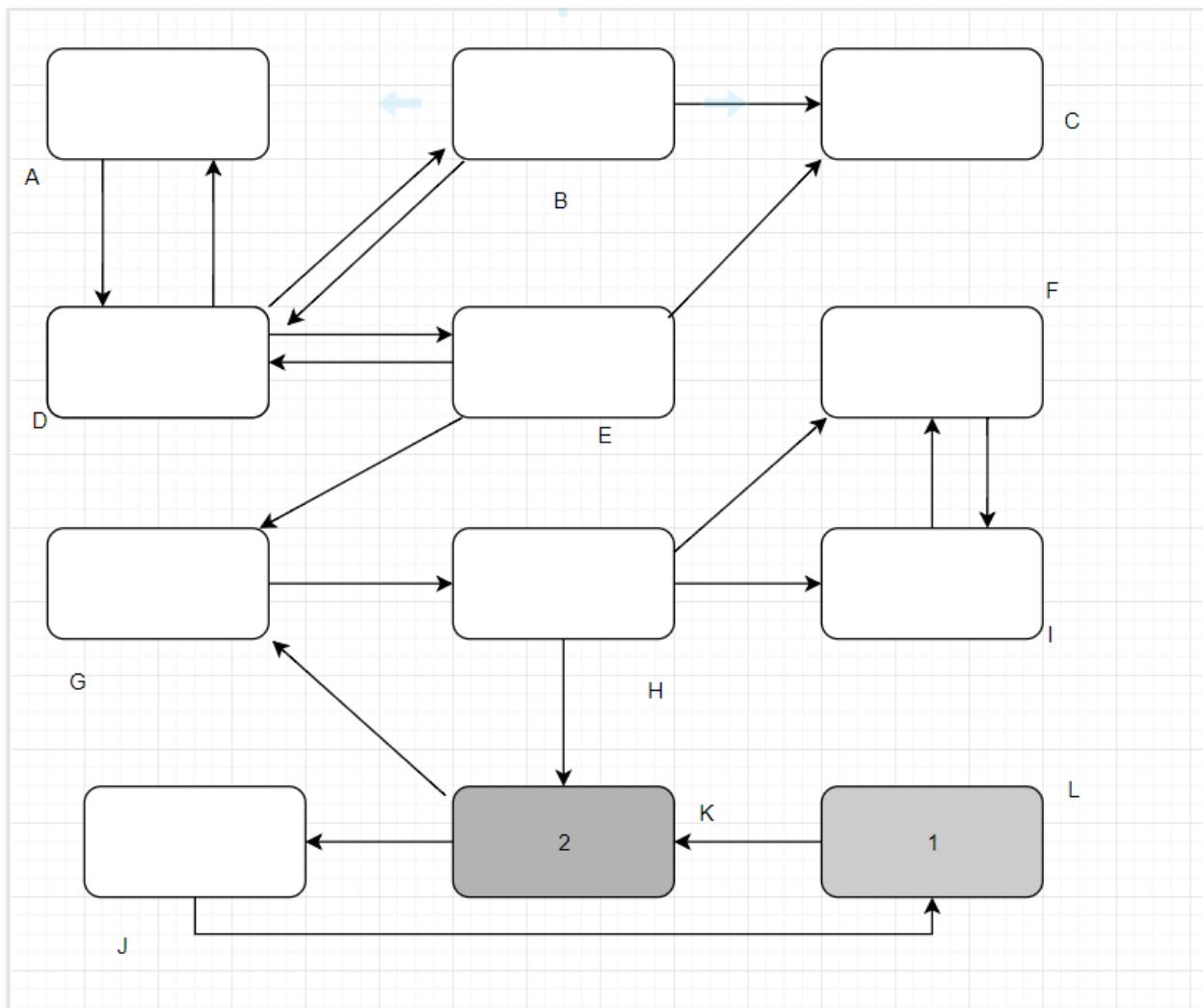
E->C

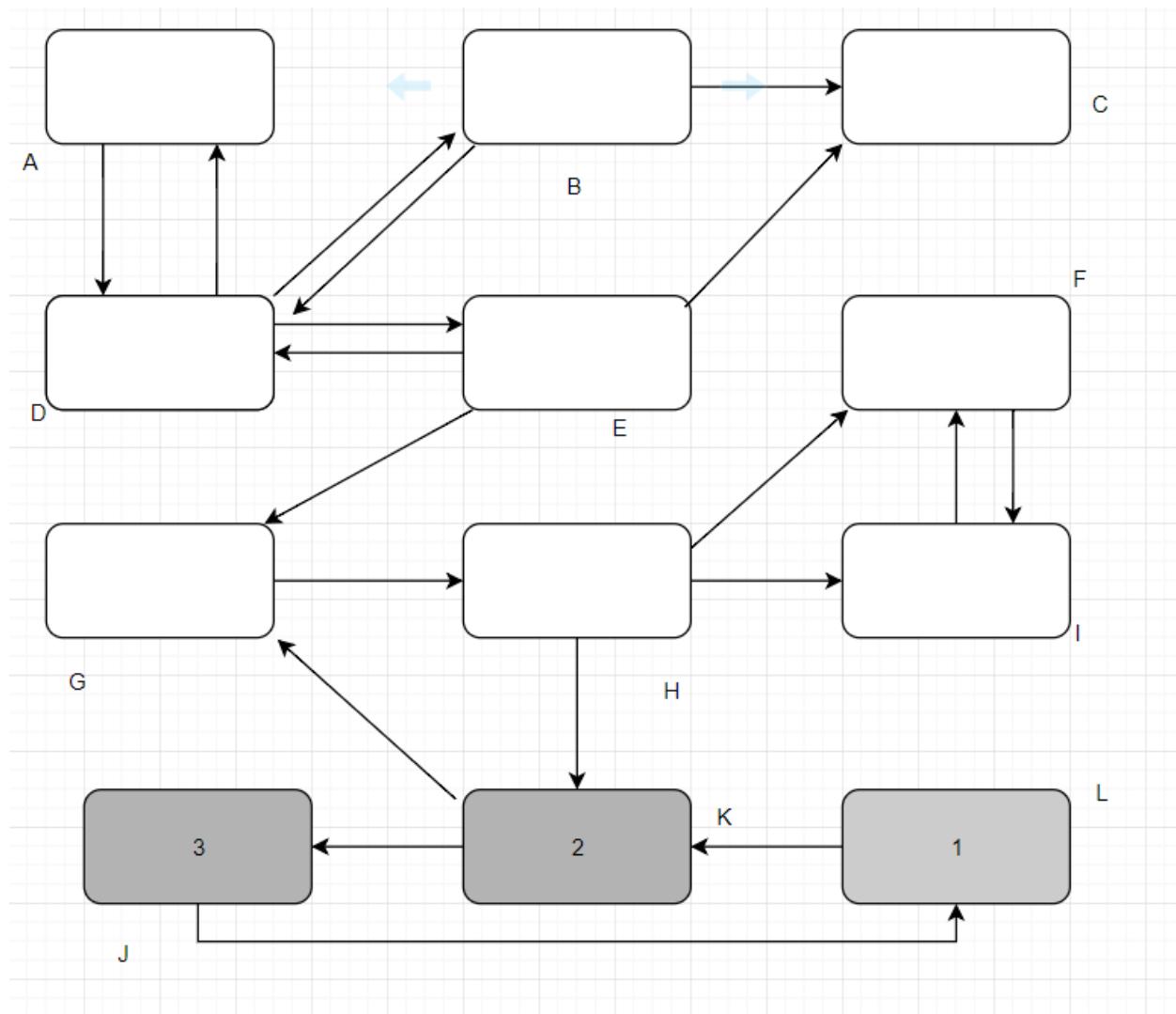
Q 1 b)

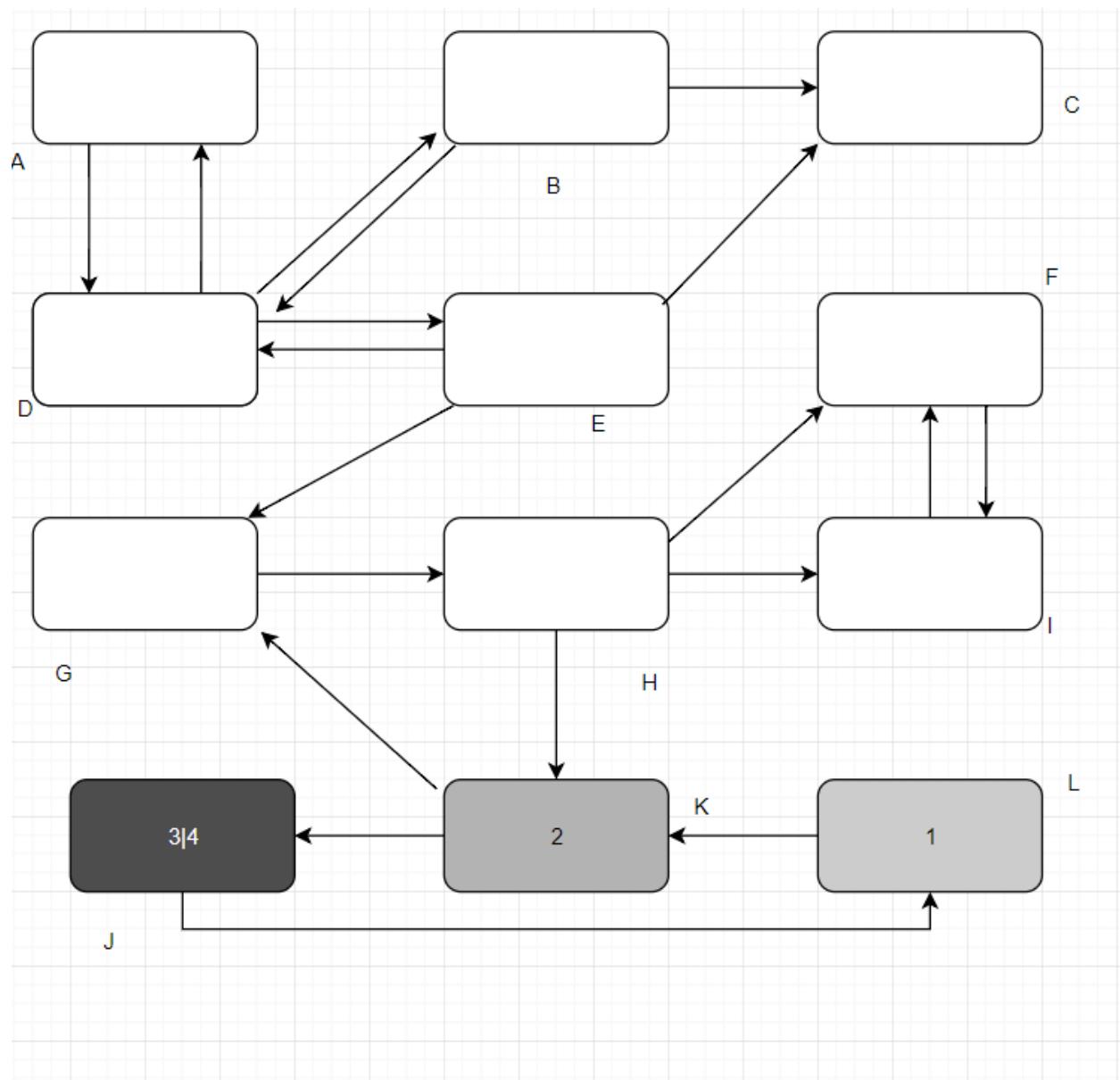
- (b) Perform a DFS (depth-first search) on the given graph. You will visit the nodes in decreasing order starting with **L** (decreasing from **L** to **A**). Give the discovery  $u.d$  and finishing times  $u.f$  for every vertex  $u$  in the graph. Classify all edges (tree, back, forward, and cross edges).

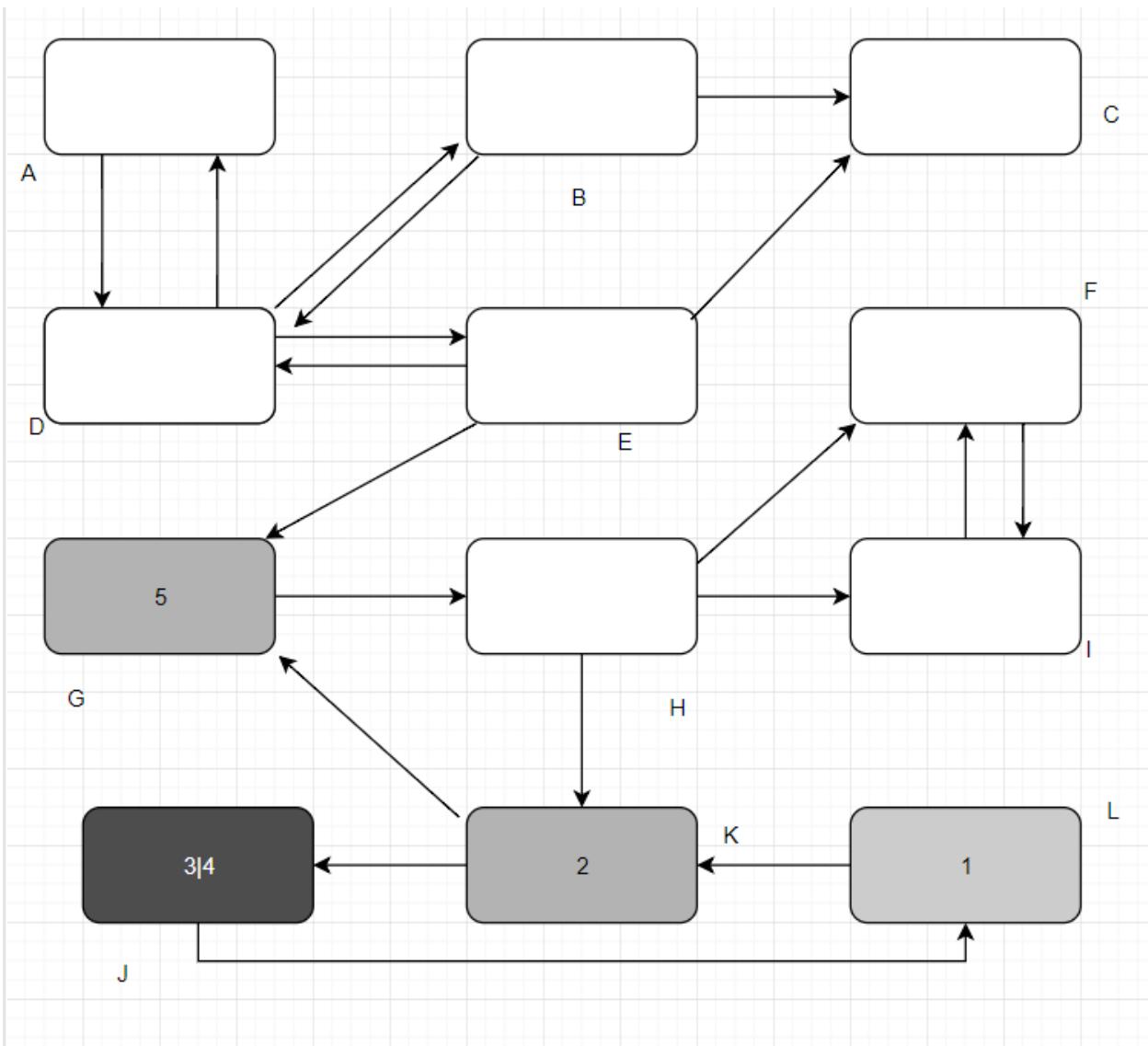


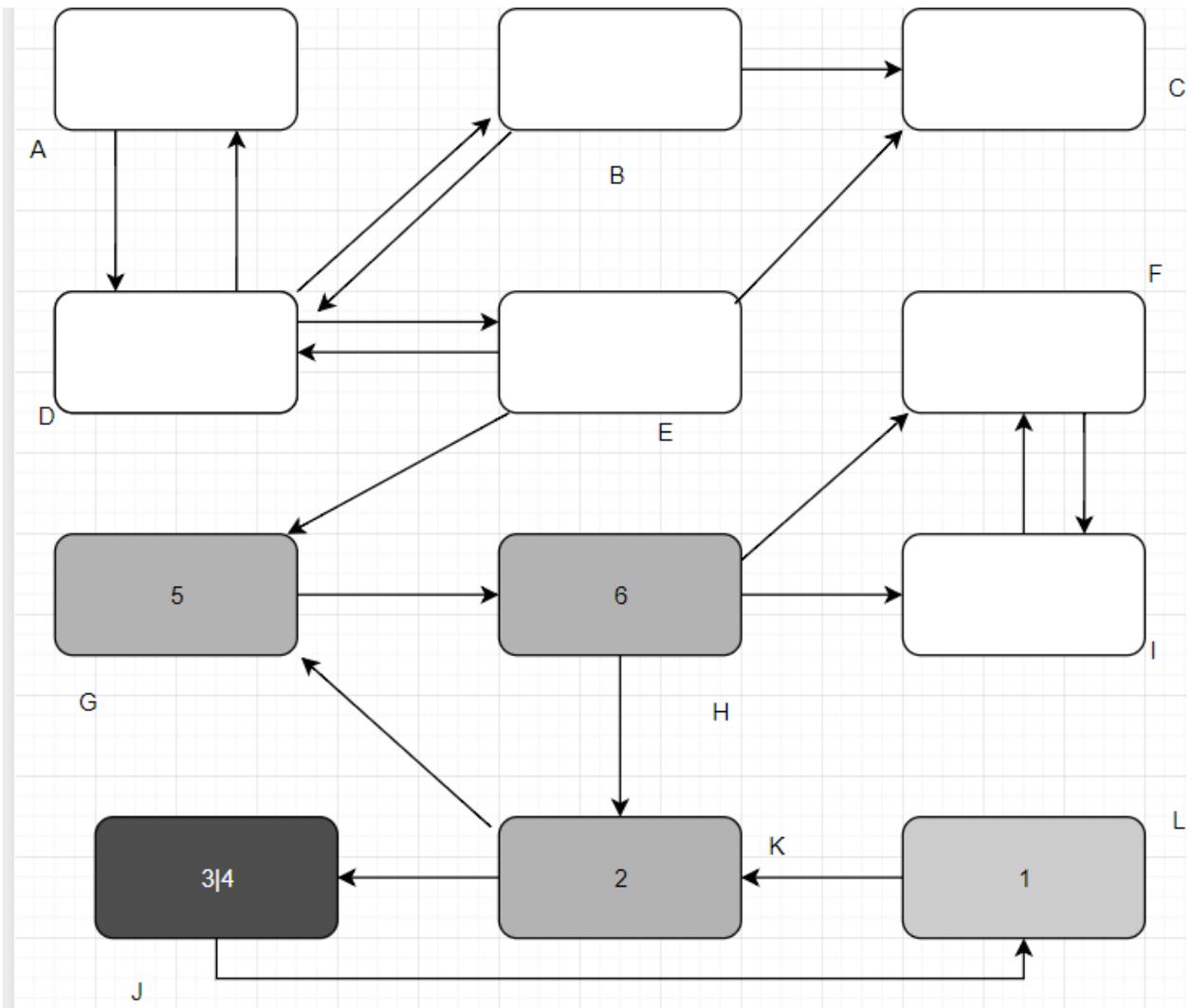


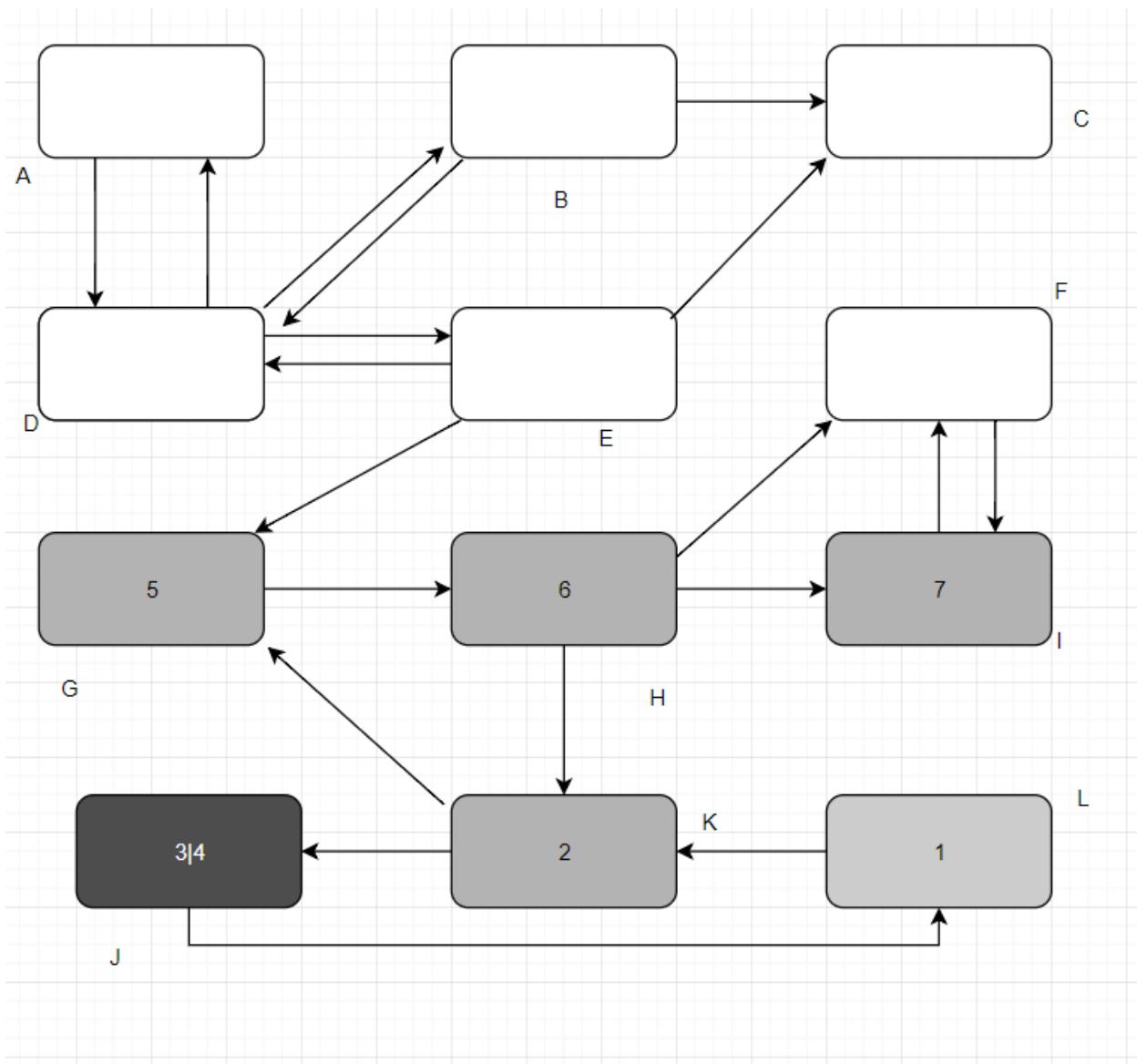


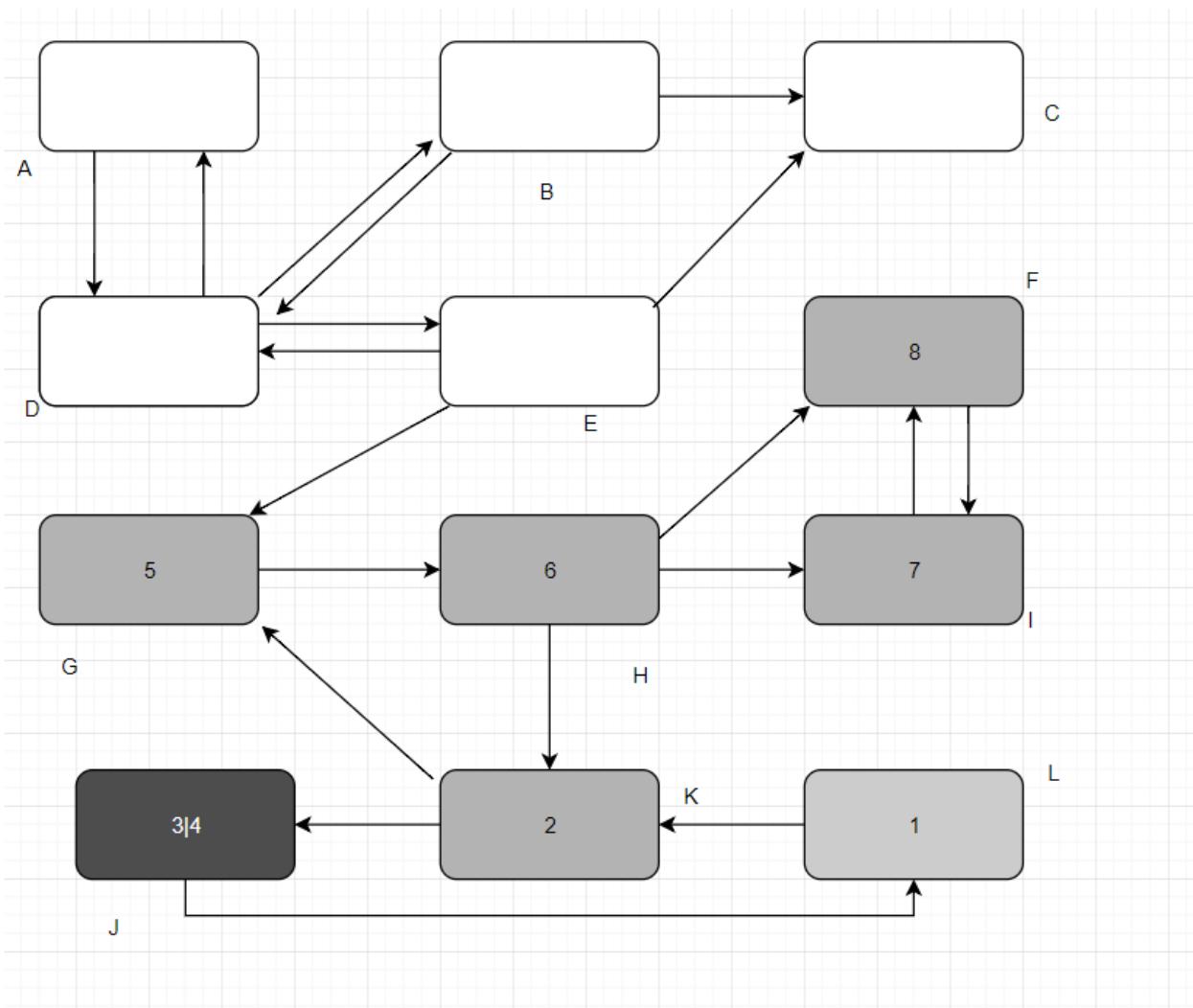


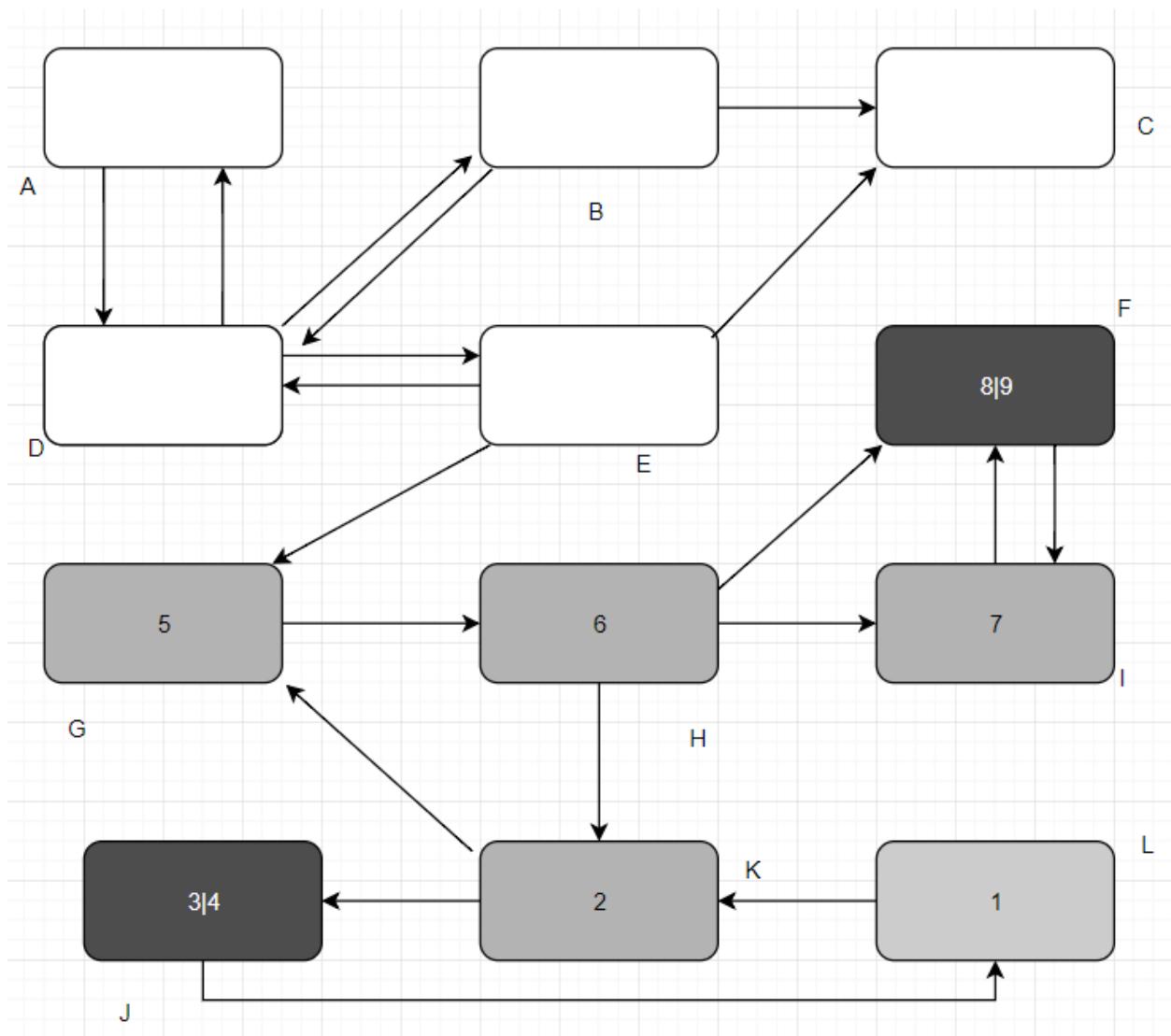


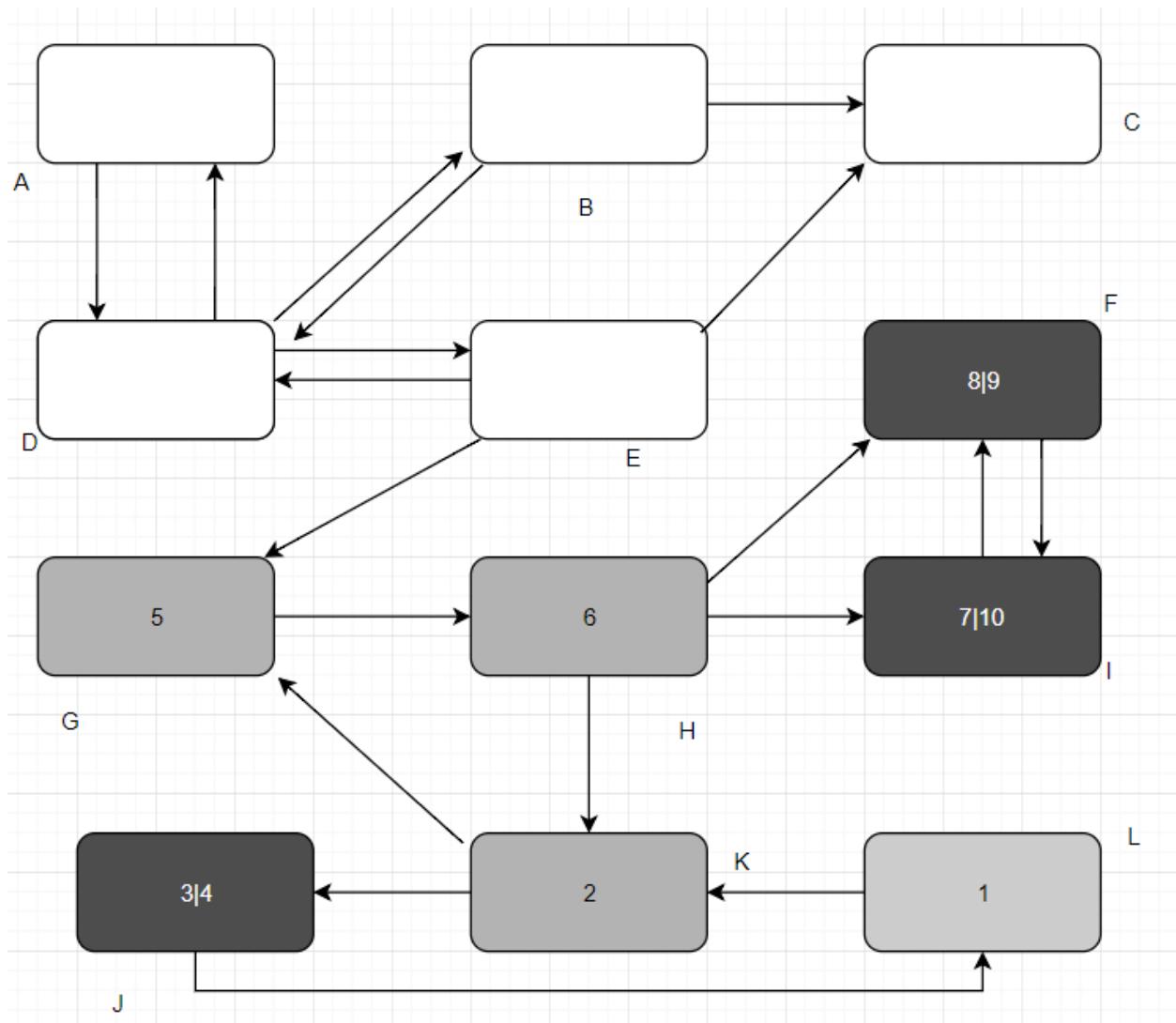


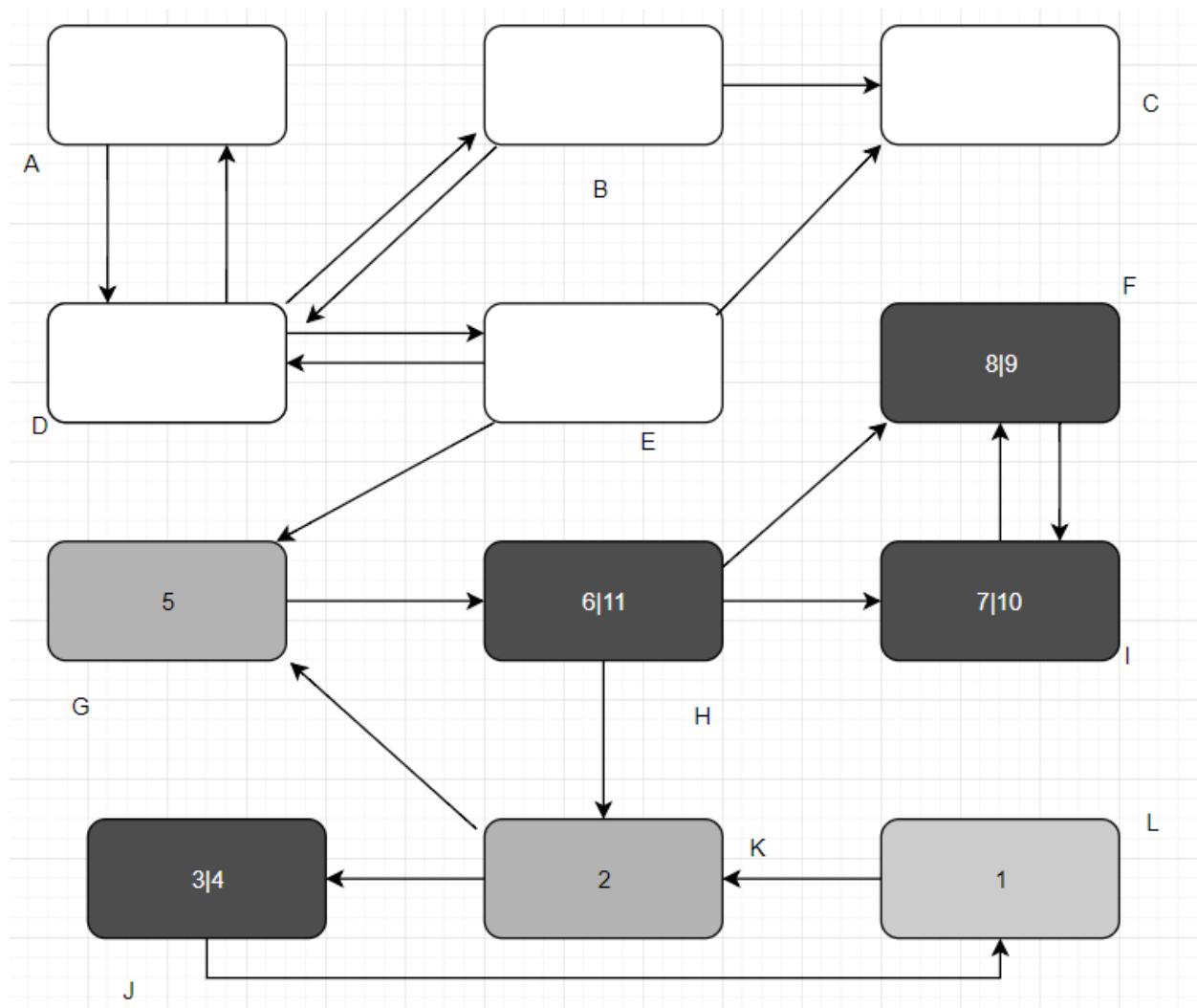


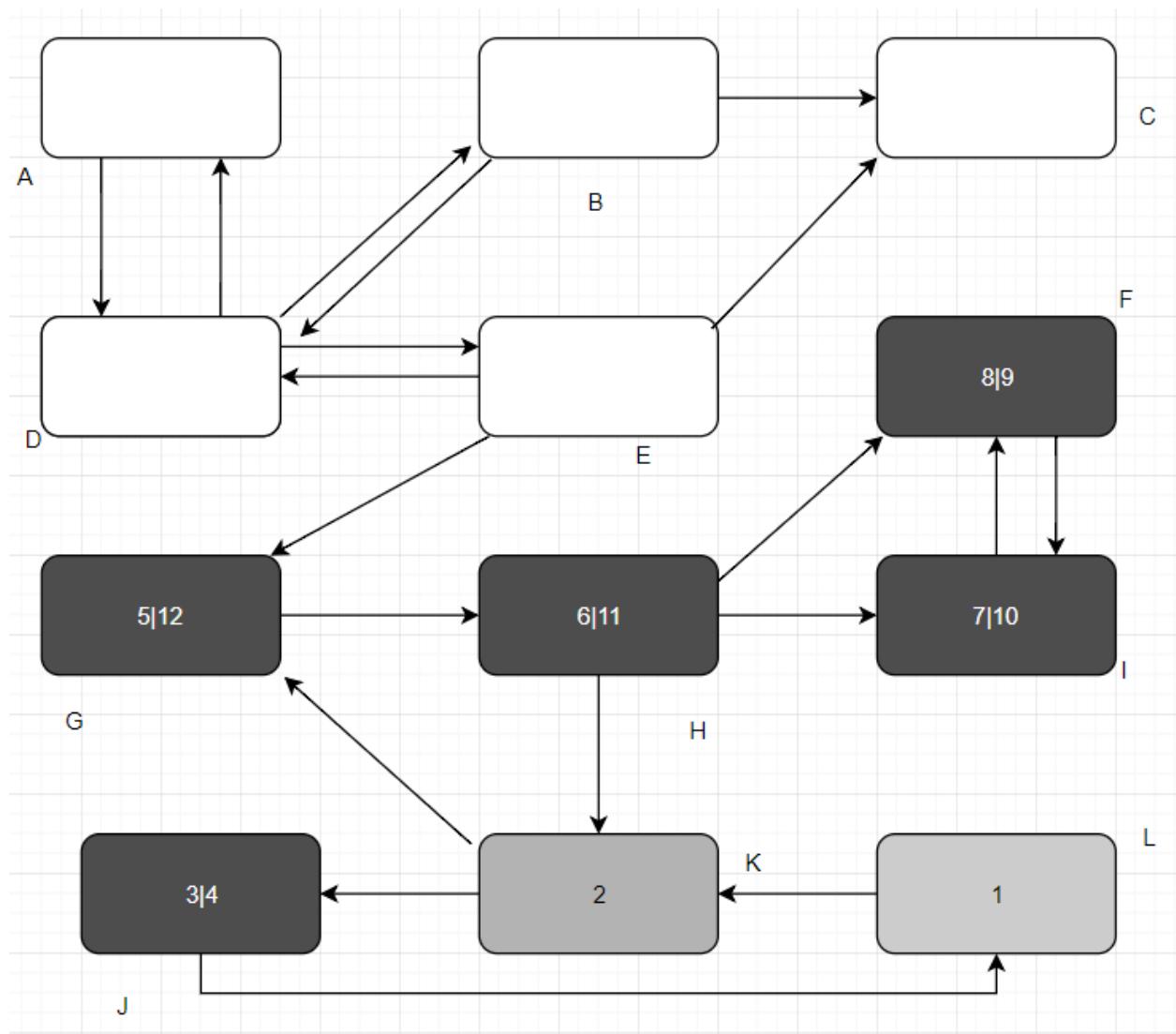


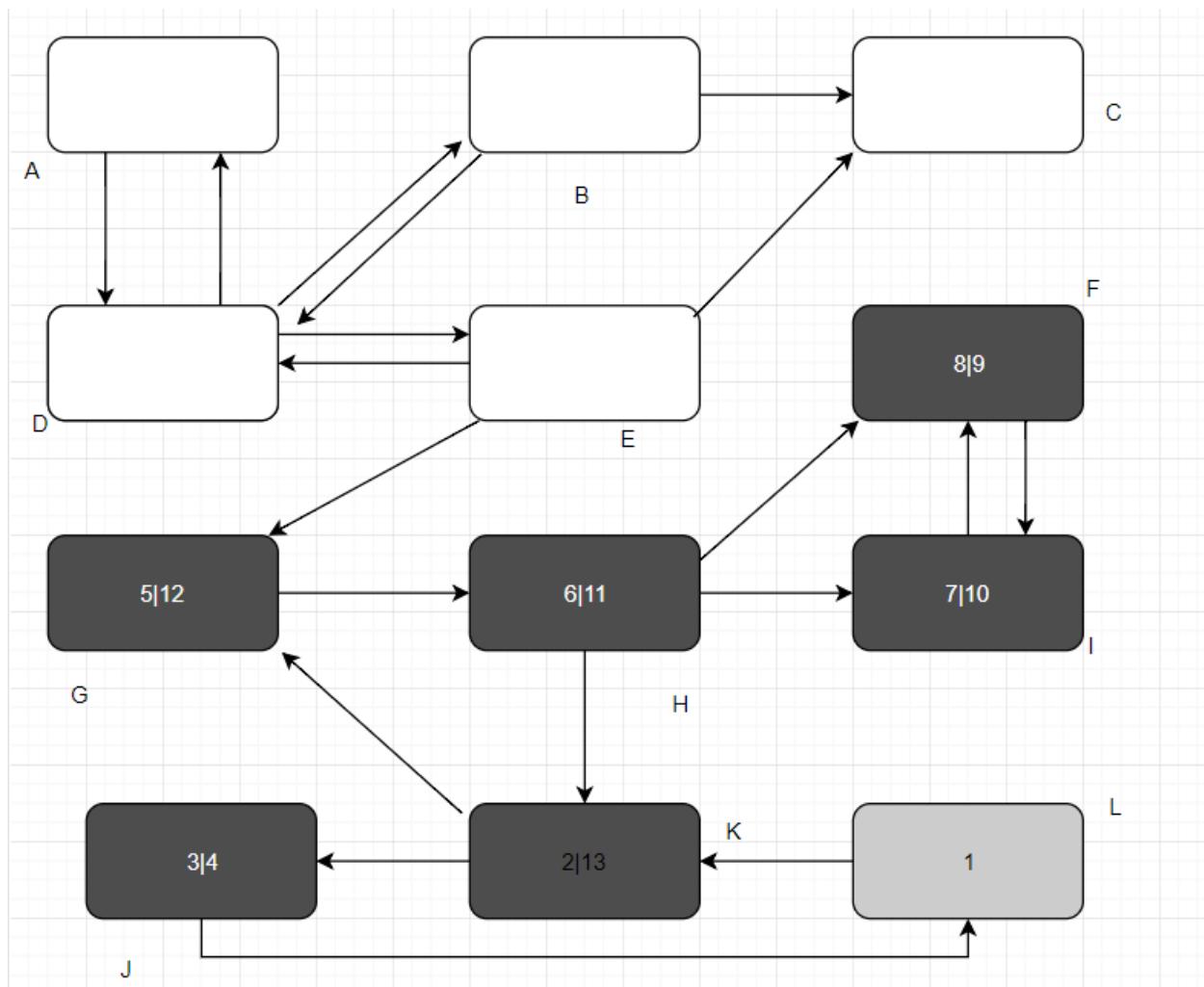


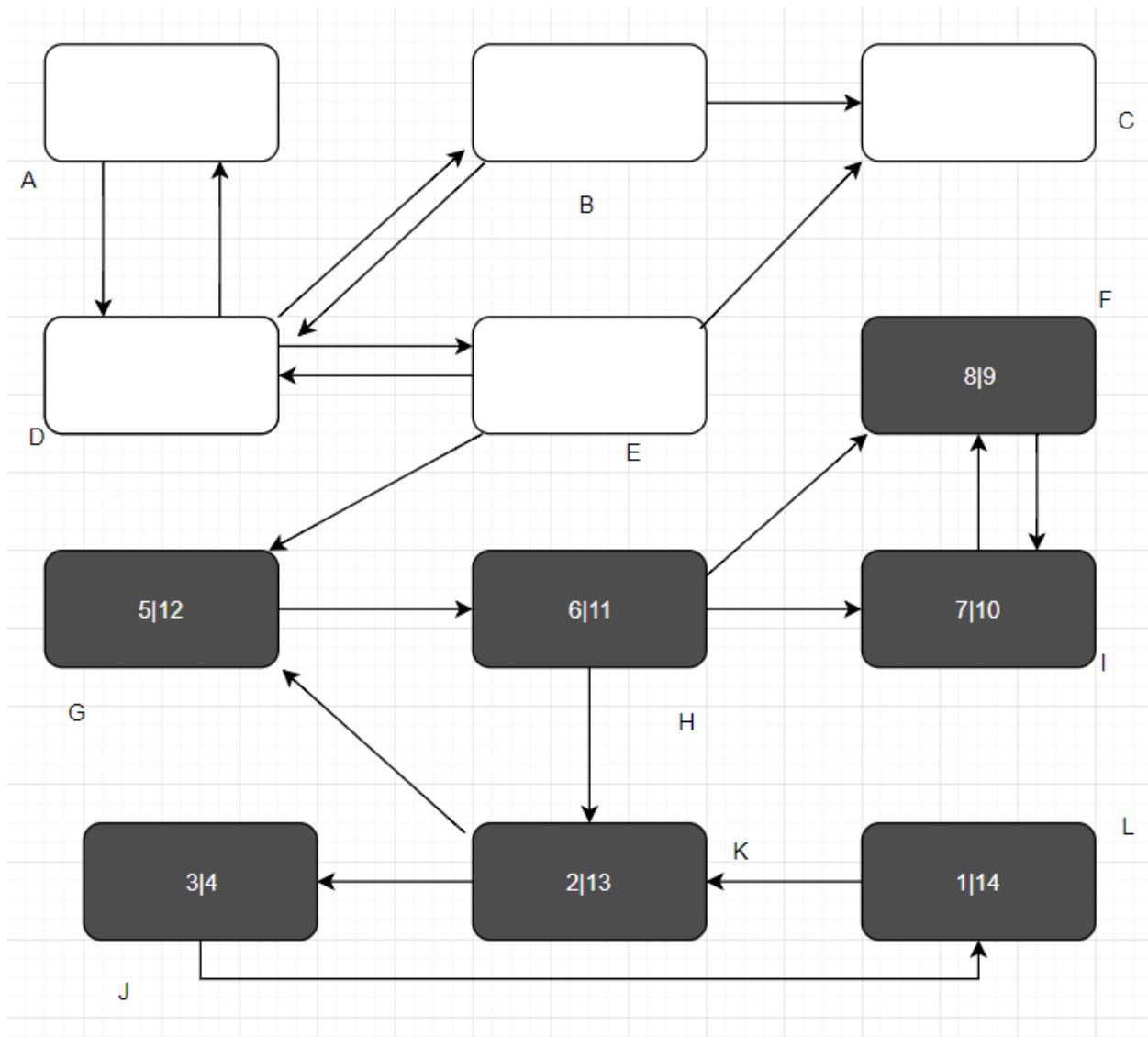


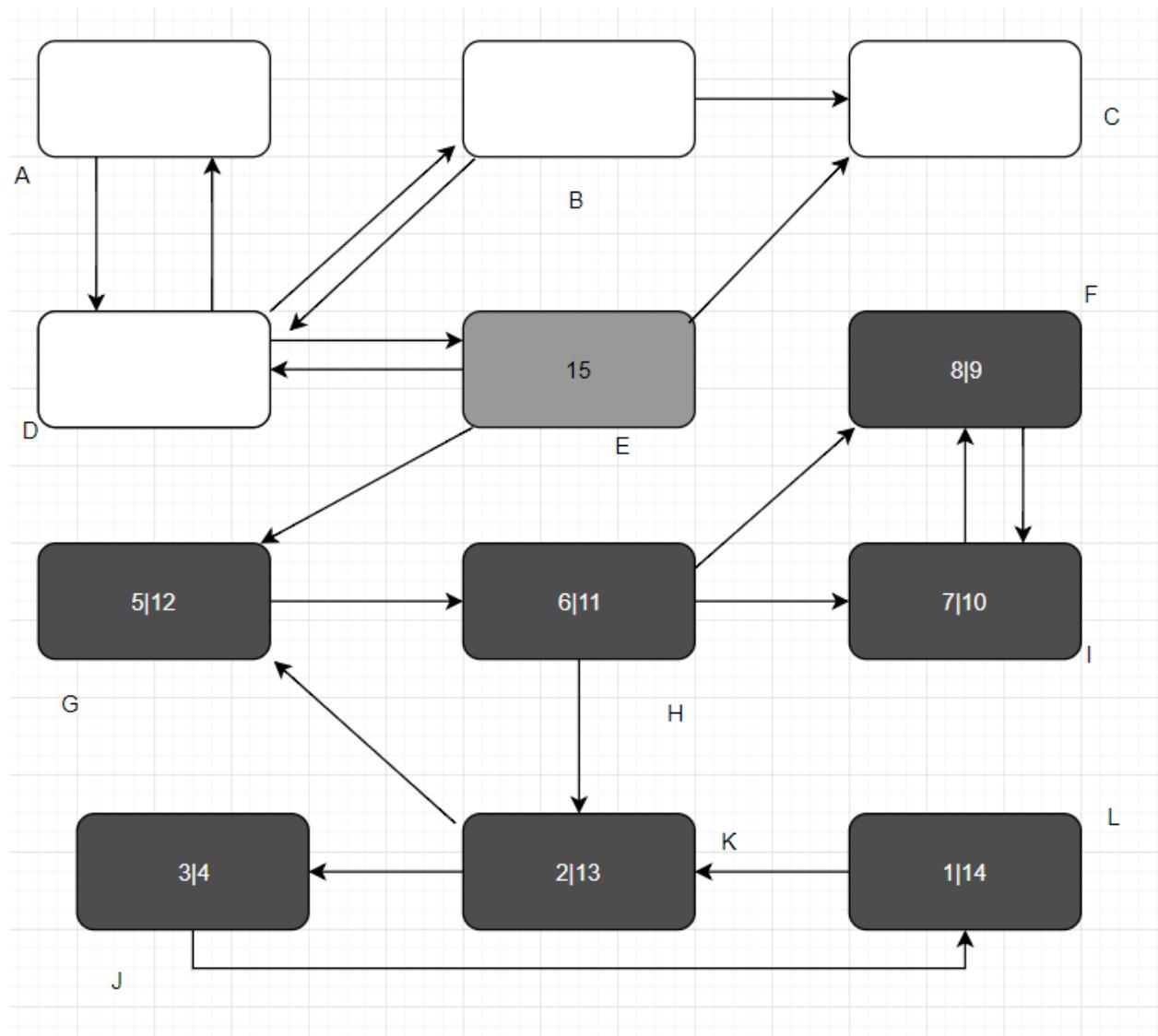


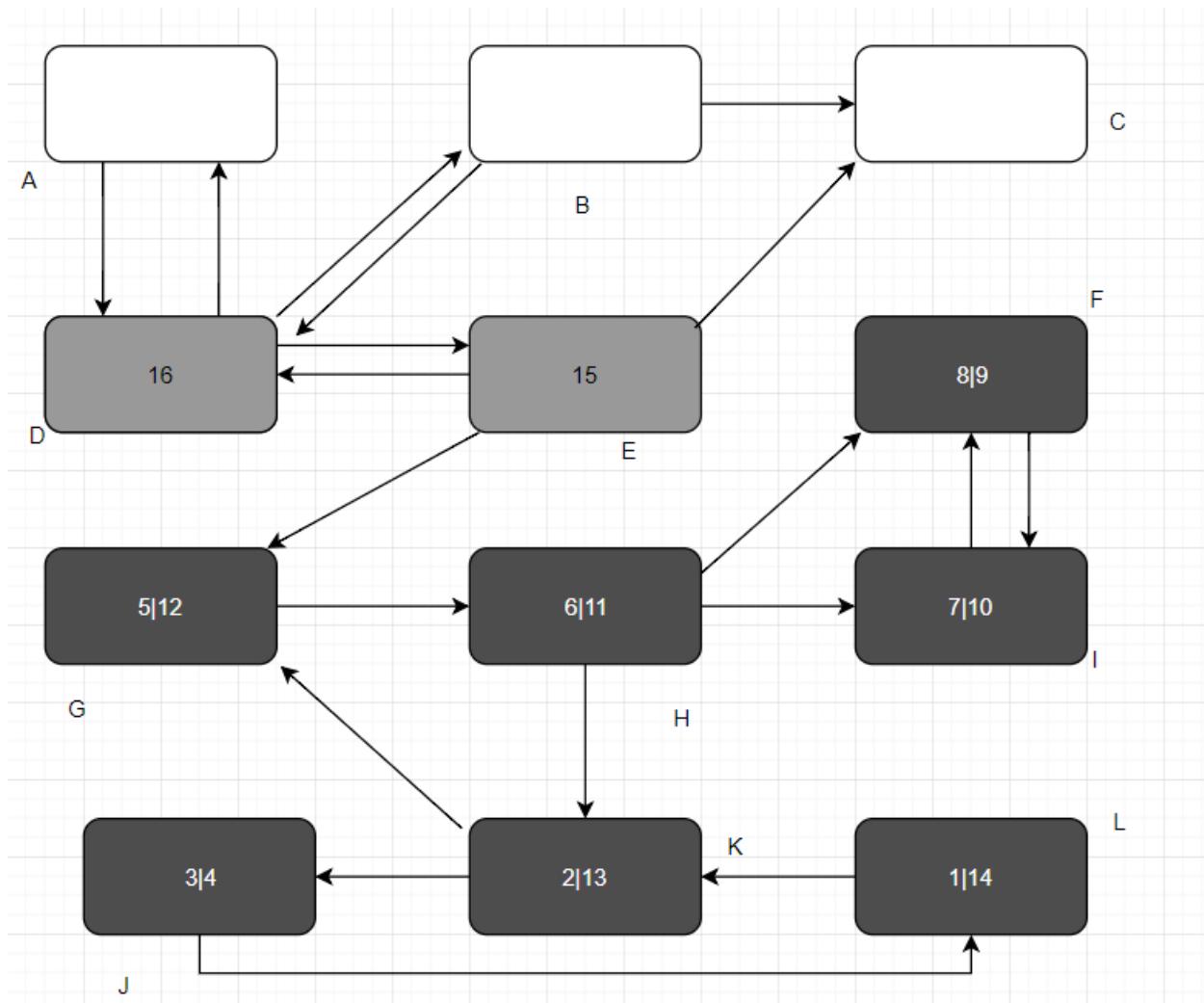


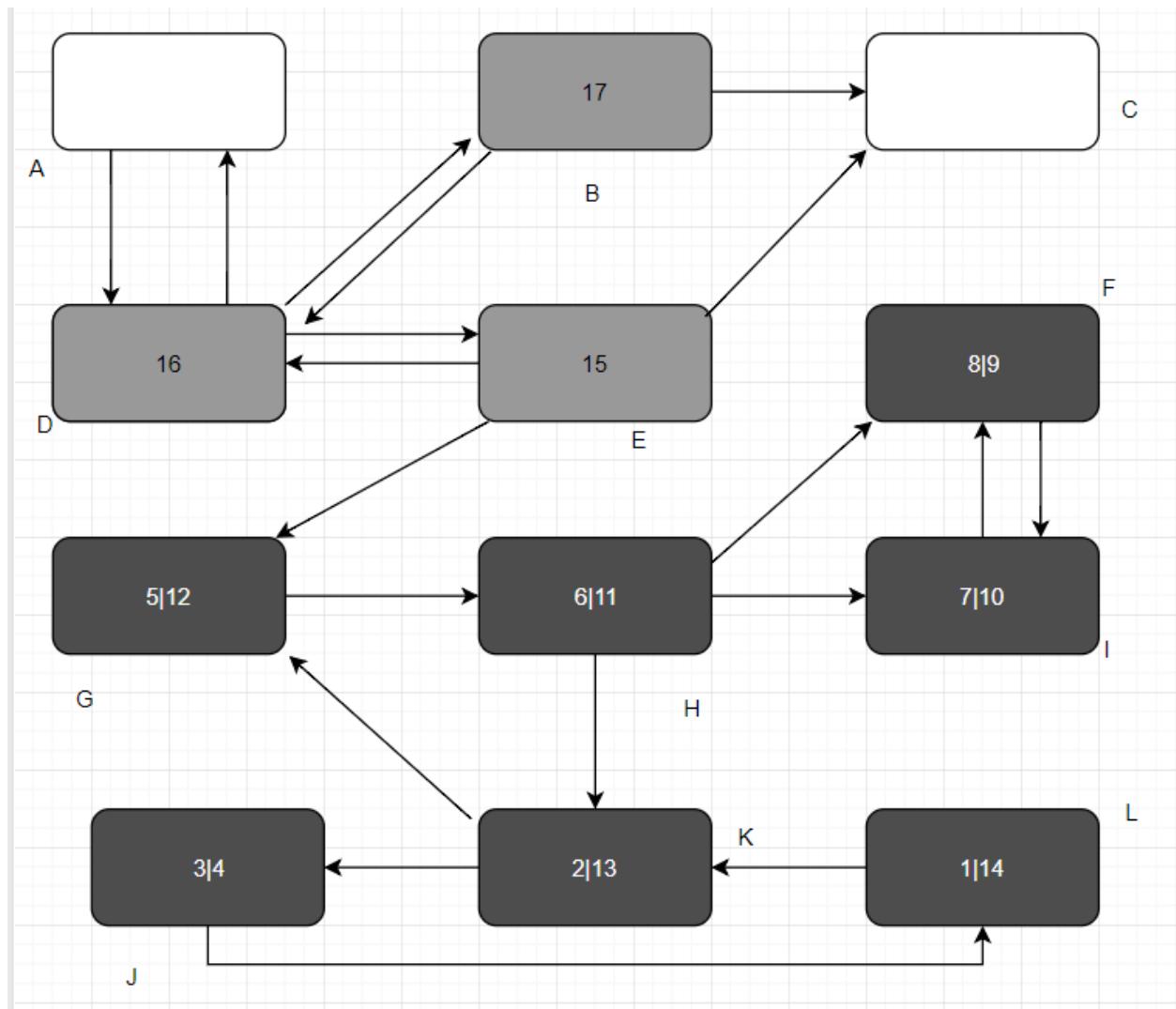


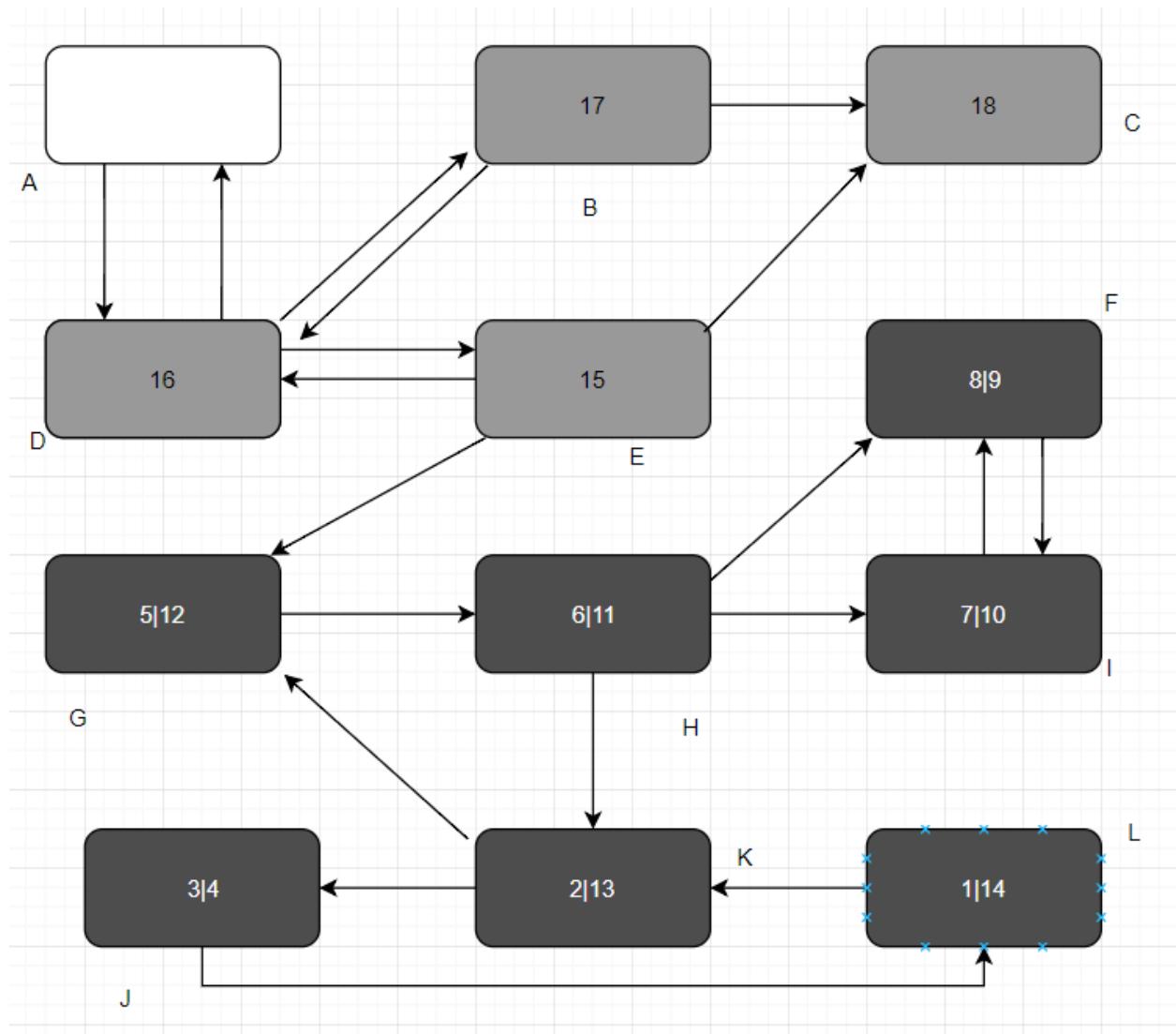


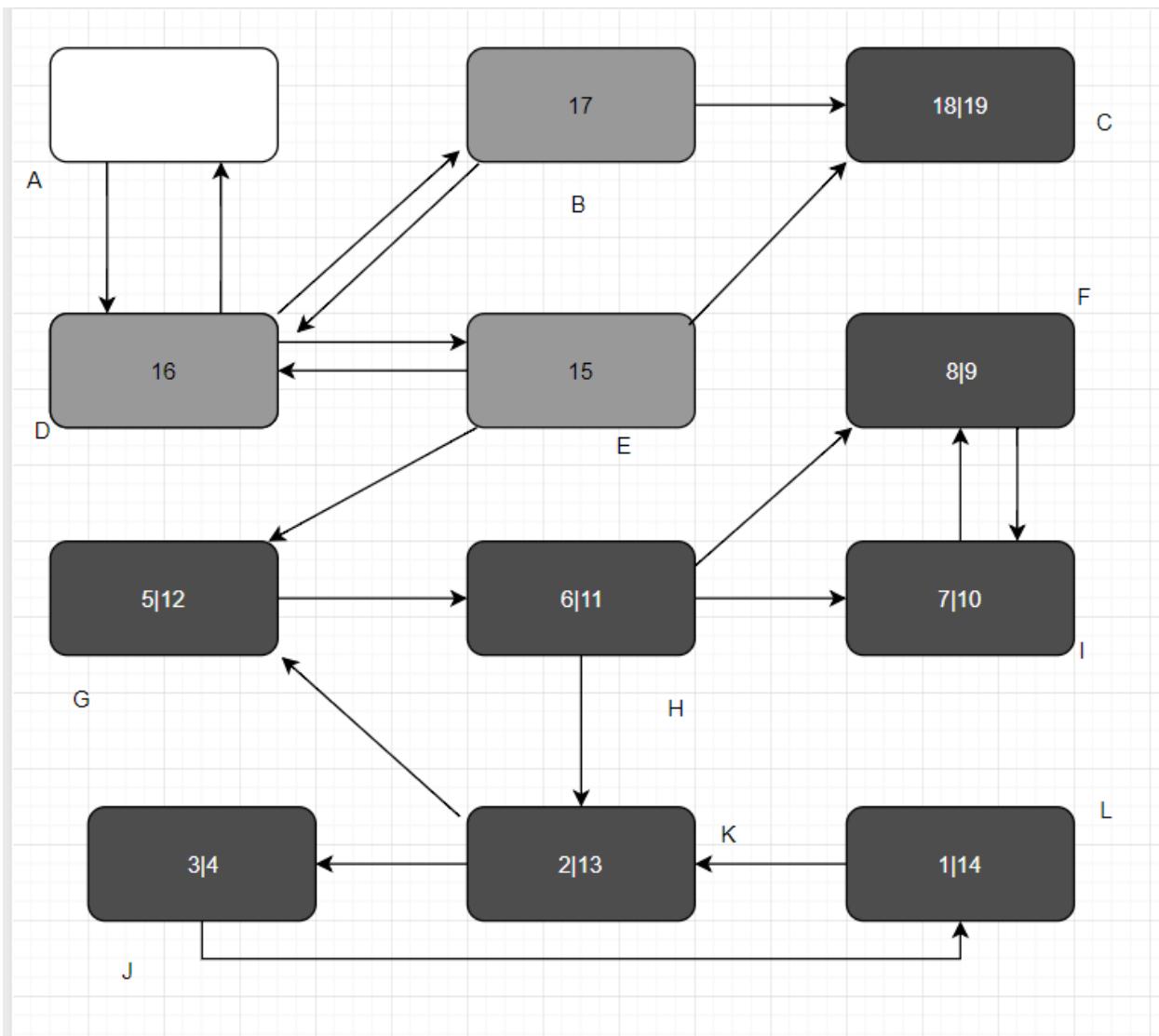


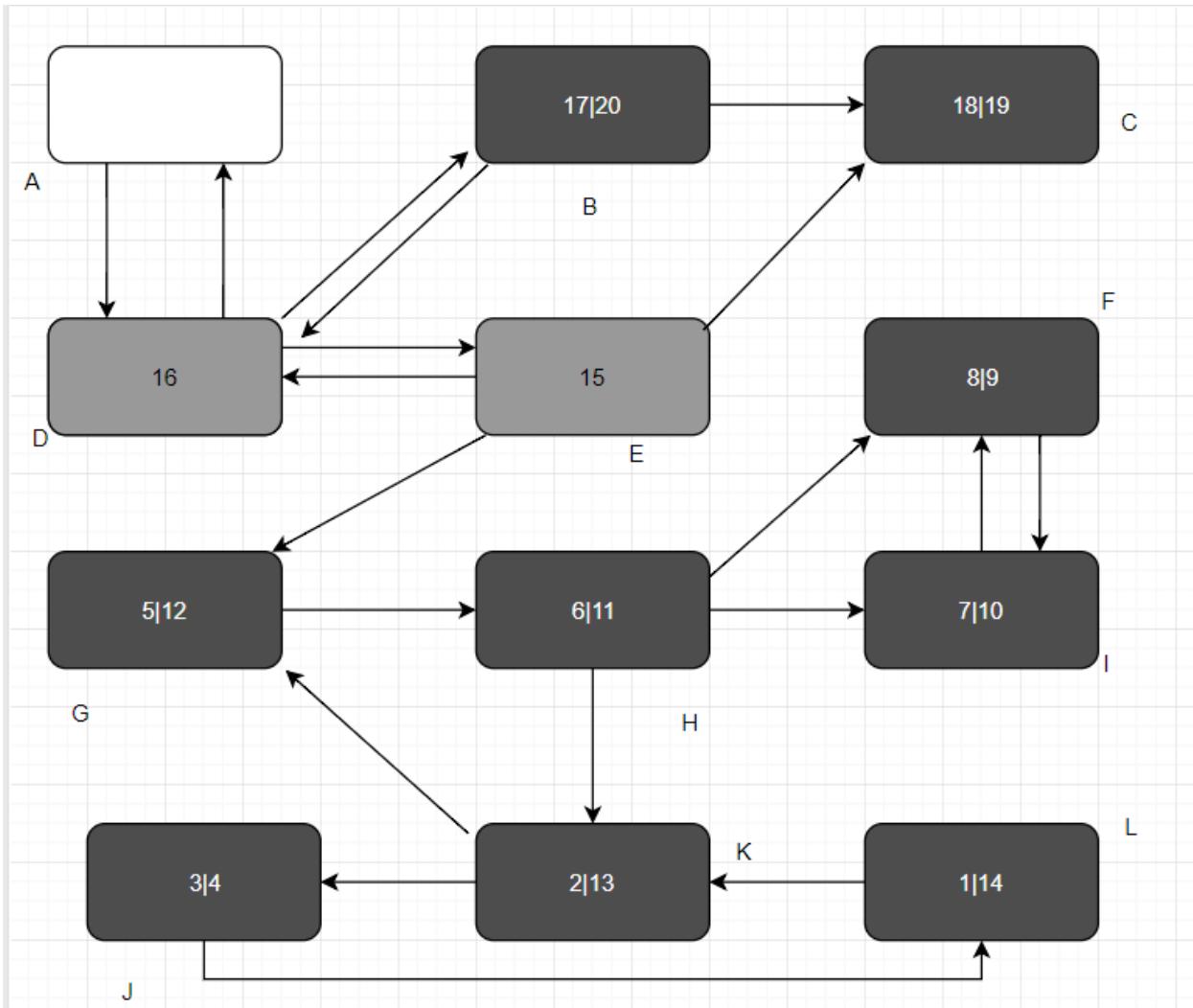


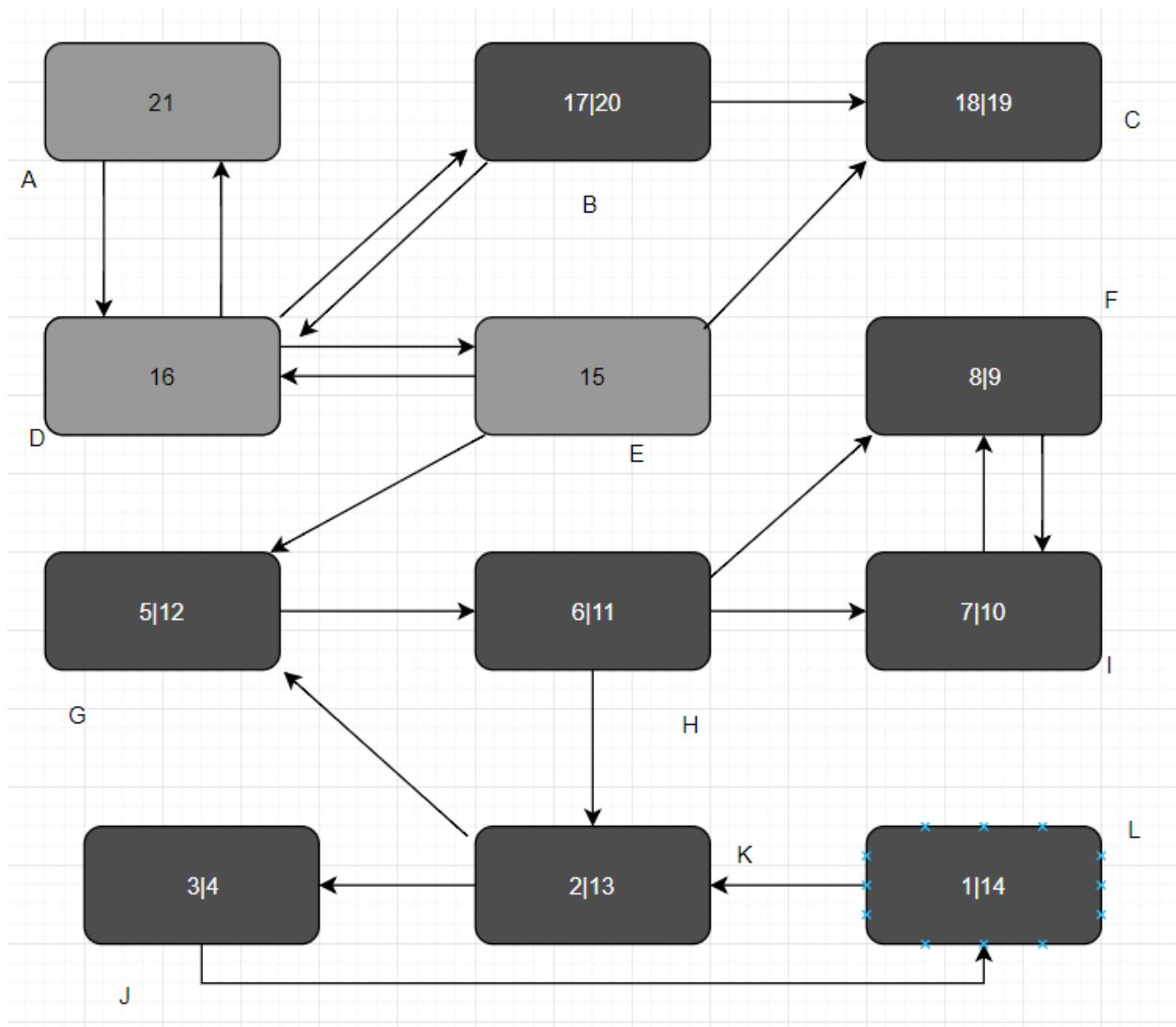


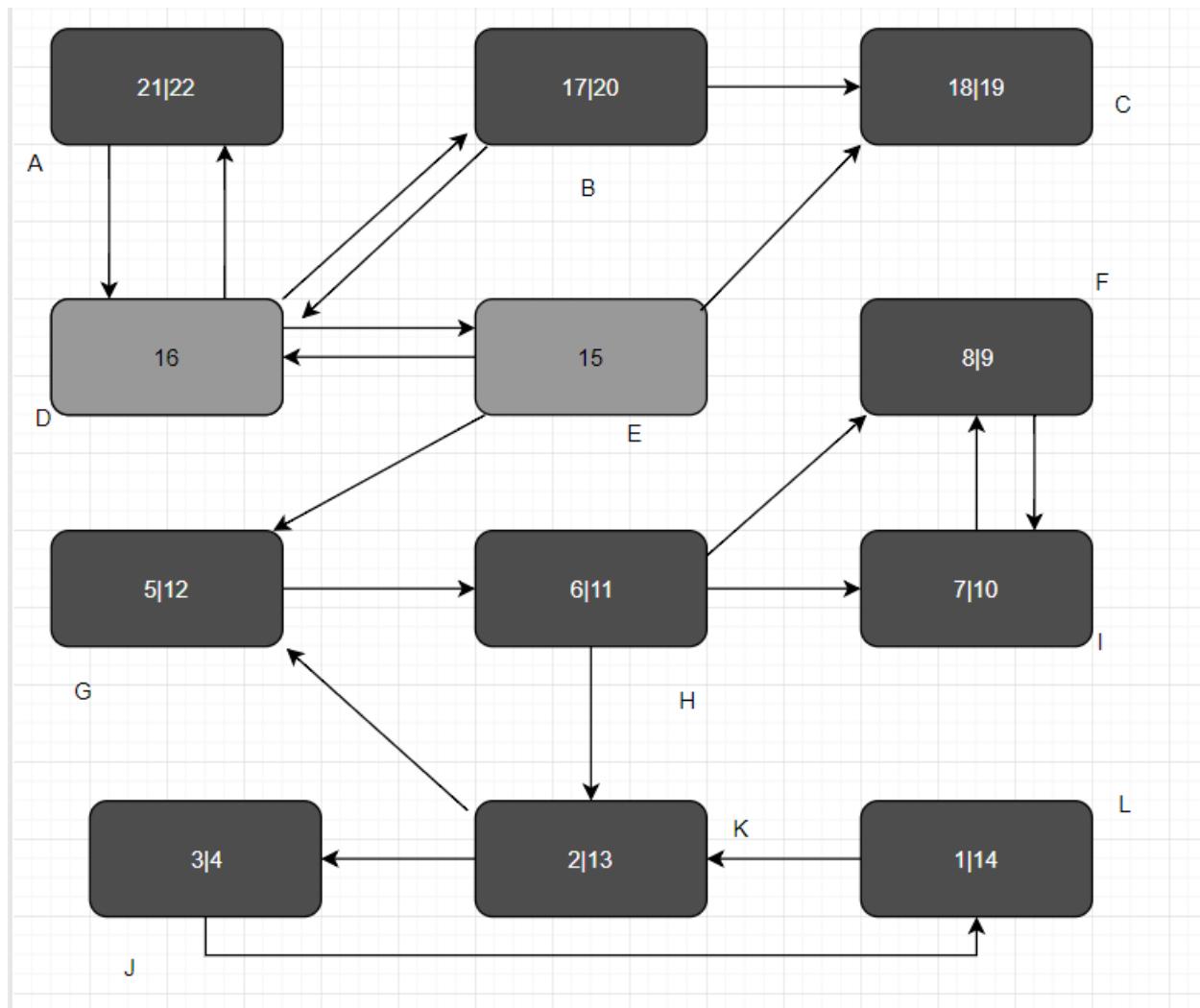


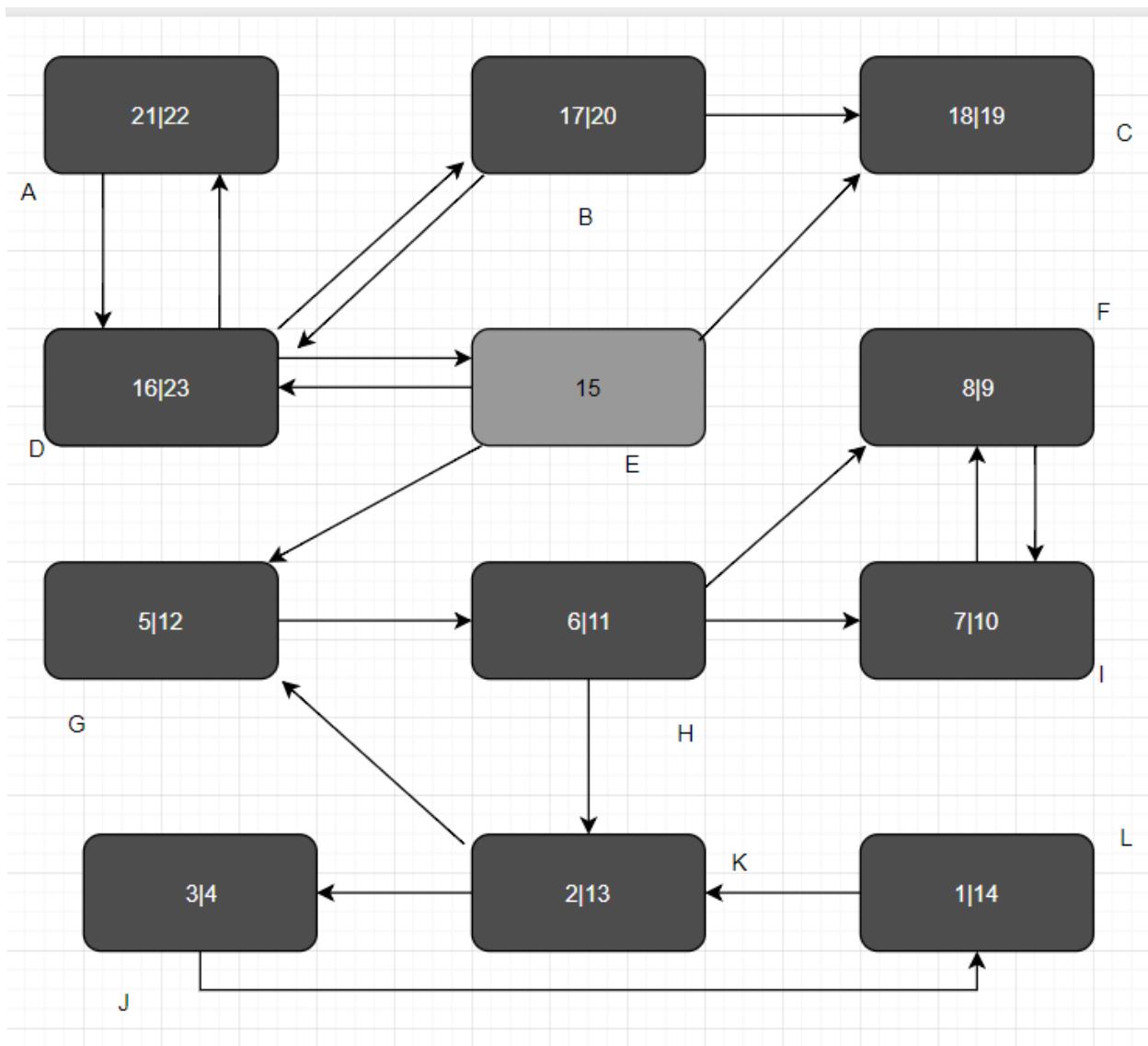


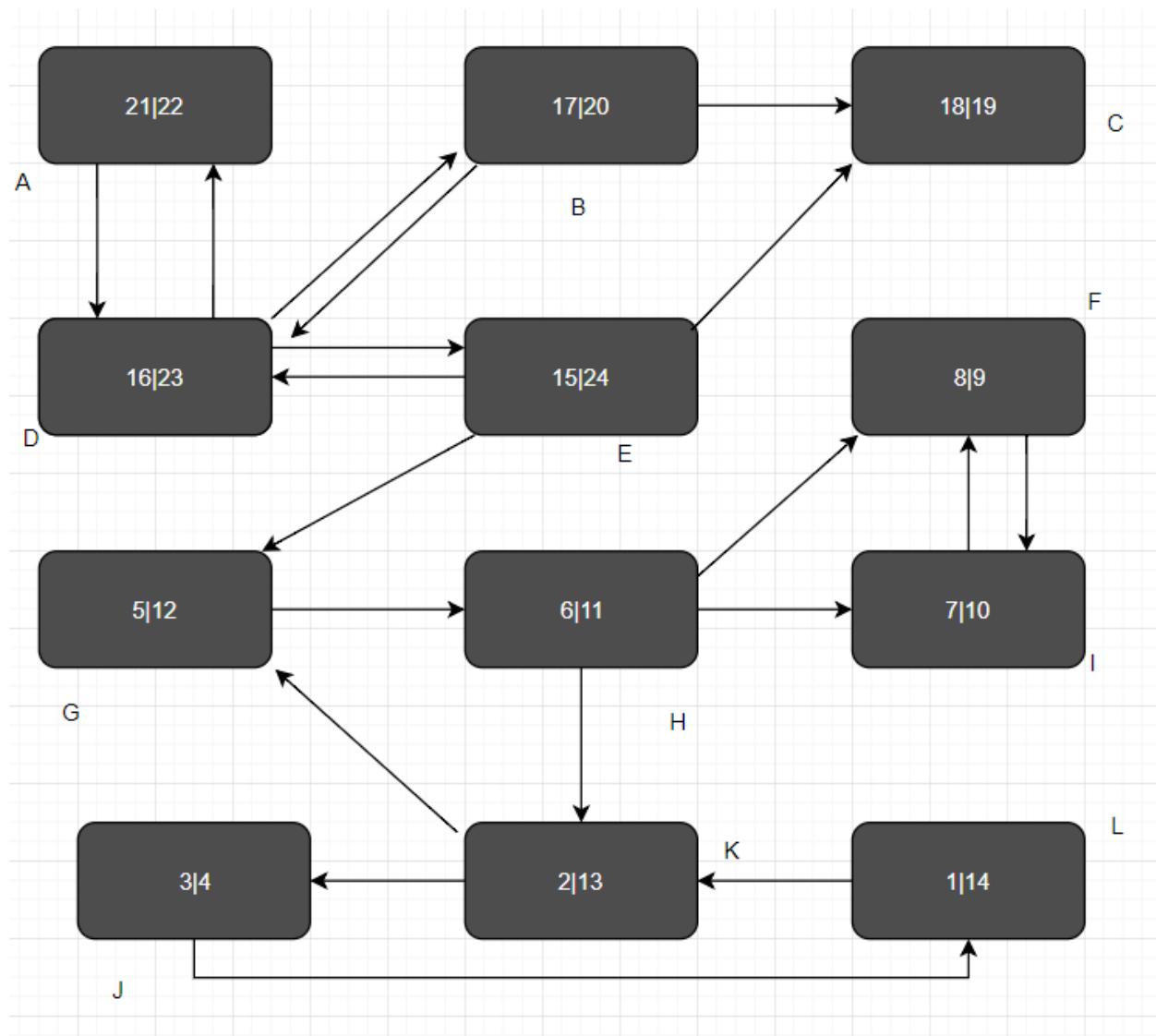


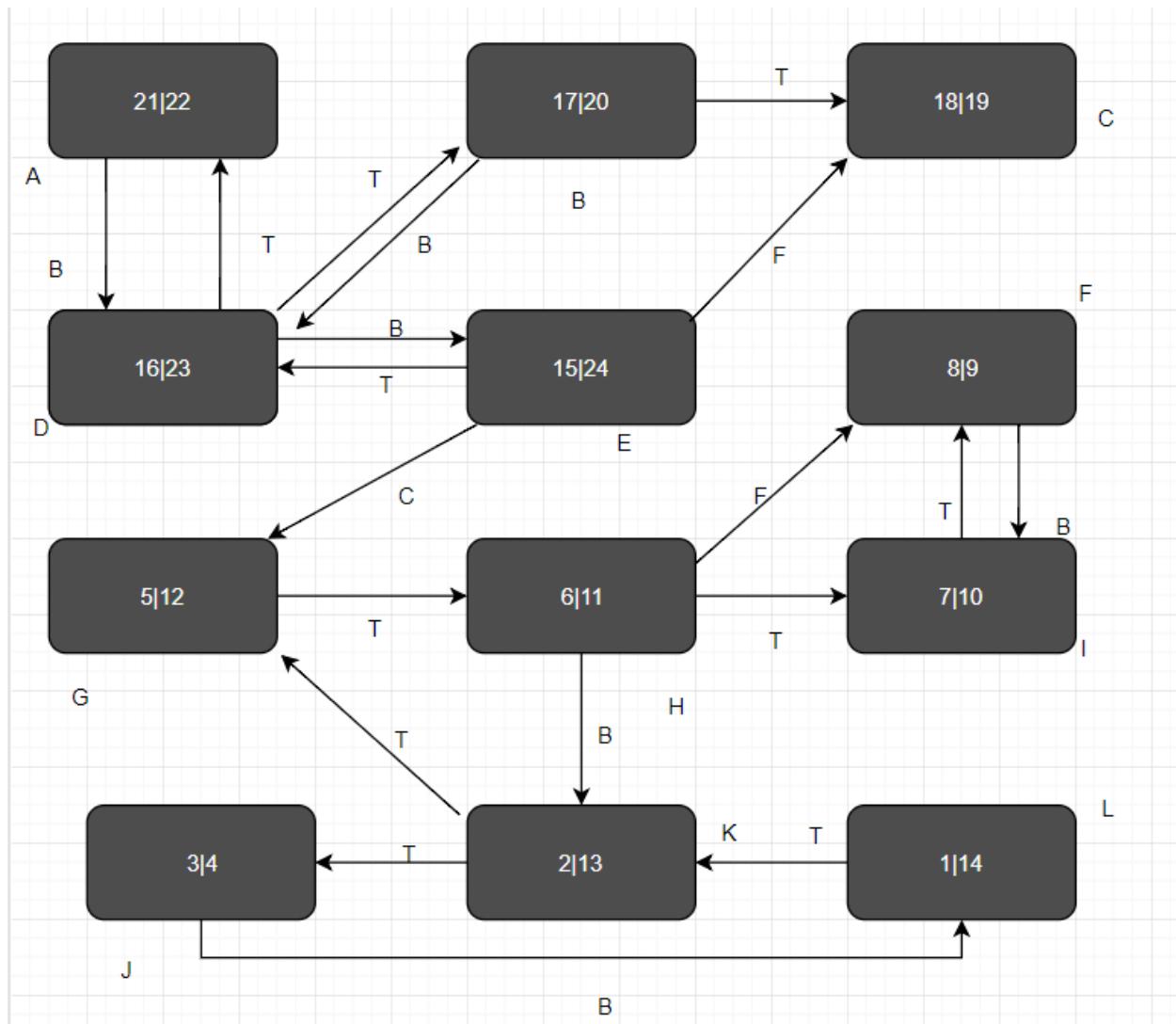












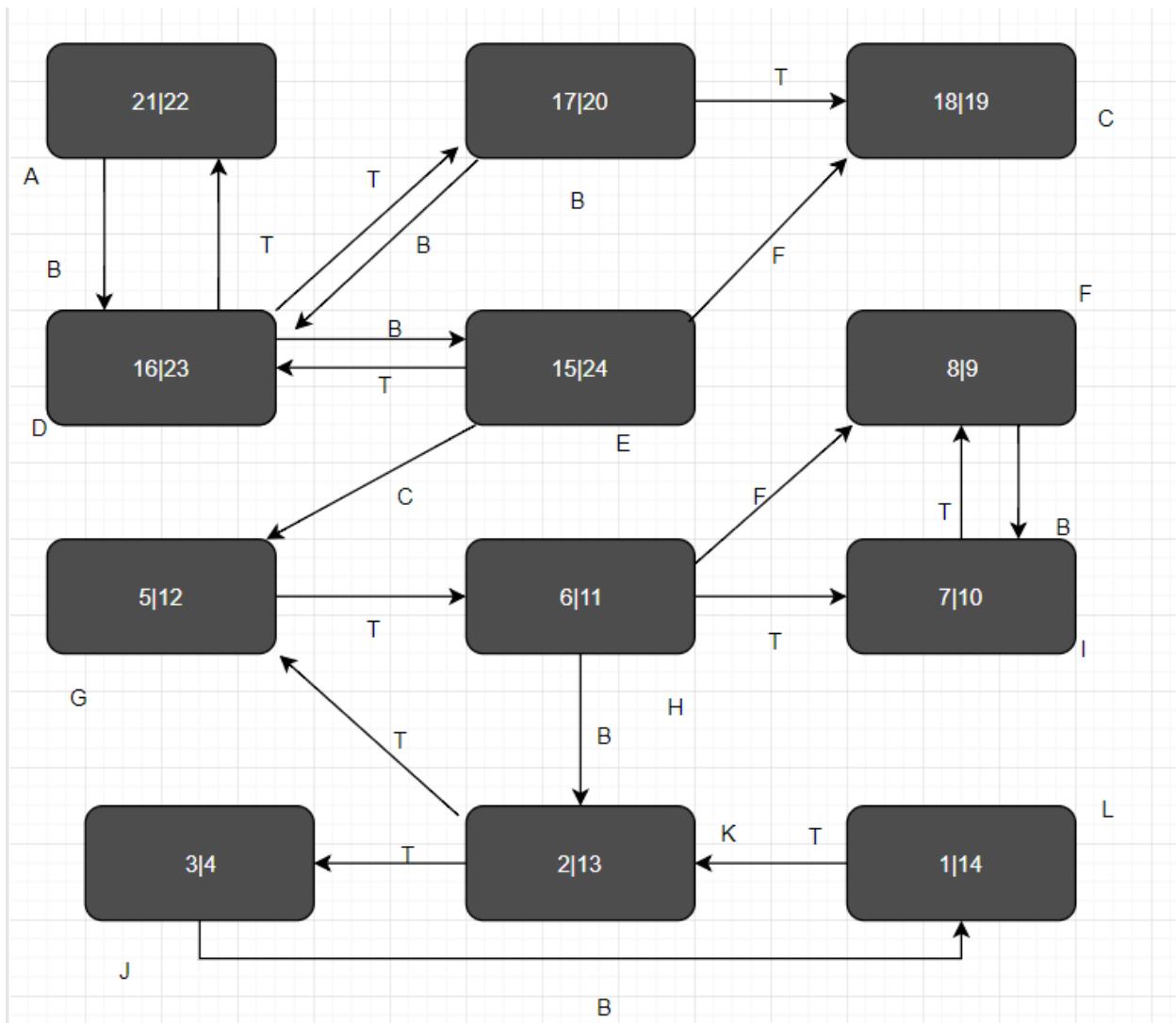
## Classification of edges:

T: Tree

## B: Back

## F: Forward

C: Cross edge



Vertex	Discovery time	Finishing time
A	21	22
B	17	20
C	18	19
D	16	23
E	15	24
F	8	9
G	5	12
H	6	11
I	7	10
J	3	4
K	2	13
L	1	14

**Tree edges:**

L→K

K→J

K→G

G→H

H→I

I→F

E→D

D→A

D→B

B→C

**Backward edges:**

J→L

H→K

F→I

D→E

B→D

A→D

**Forward edges:**

H-->F

E→C

**Cross edges:**

E->G

Q1)

c)

- (c) Determine the strongly connected components (SCC) of the given graph  $G = (V, E)$ . Use your output of (a) as a starting point for the remainder of the SCC algorithm. Give the discovery and finishing times for the second run of DFS. Give the sets of vertices that form an SCC and draw the corresponding SCC graph.

**Solution:**

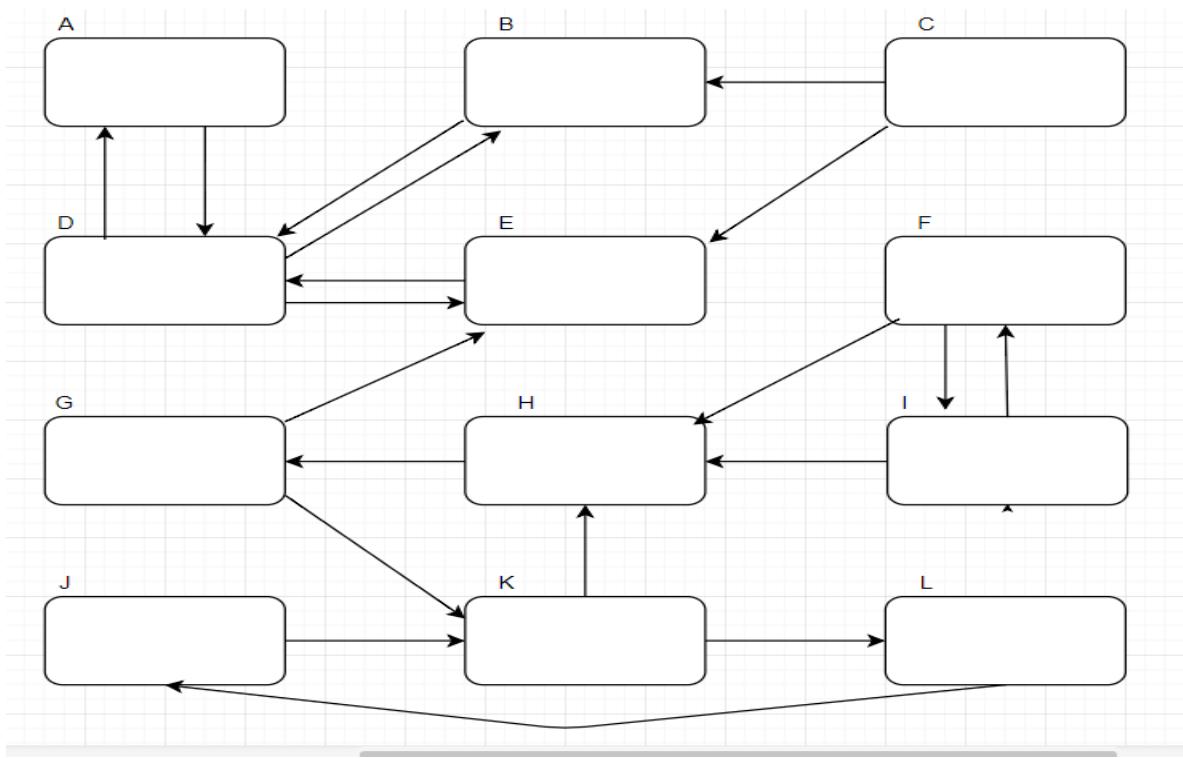
To find out the SCC, we need to take the transpose of the given graph and then perform the second DFS run.

The vertices of the graph in (a) in descending order of their finishing time obtained after performing DFS are as follows:

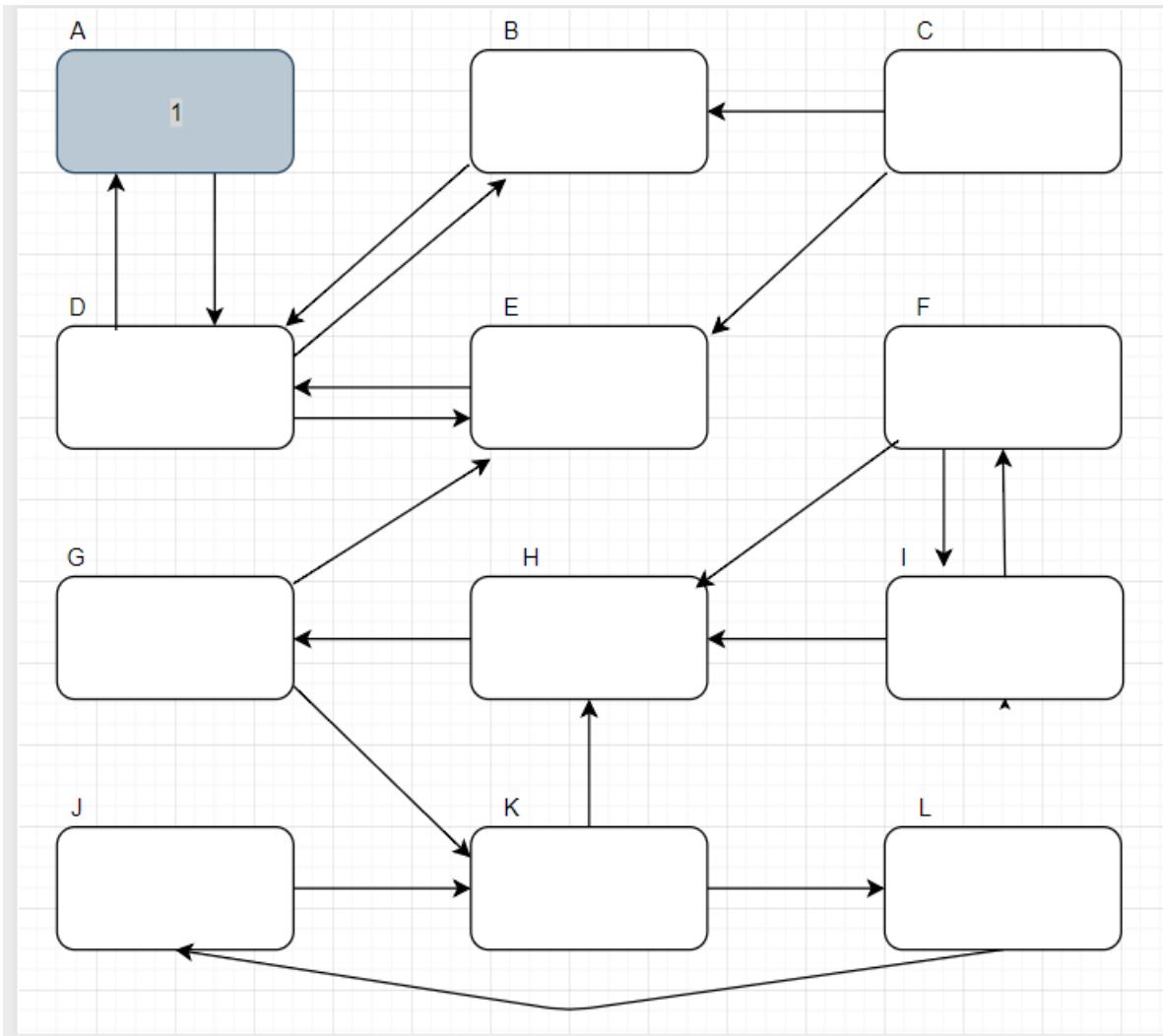
A D E G H K J L F I B C

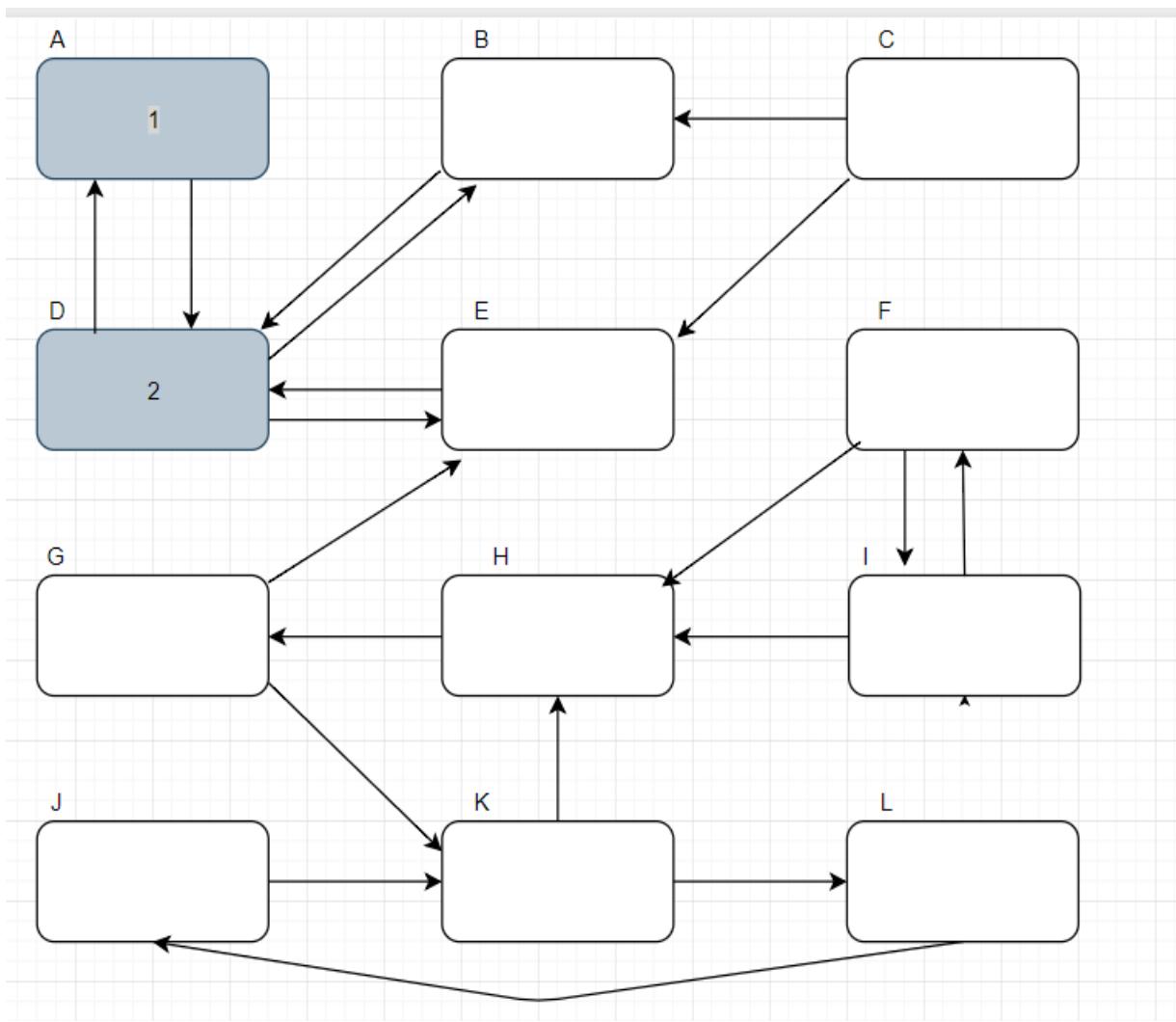
The second DFS traversal is carried out based on this preferred order.

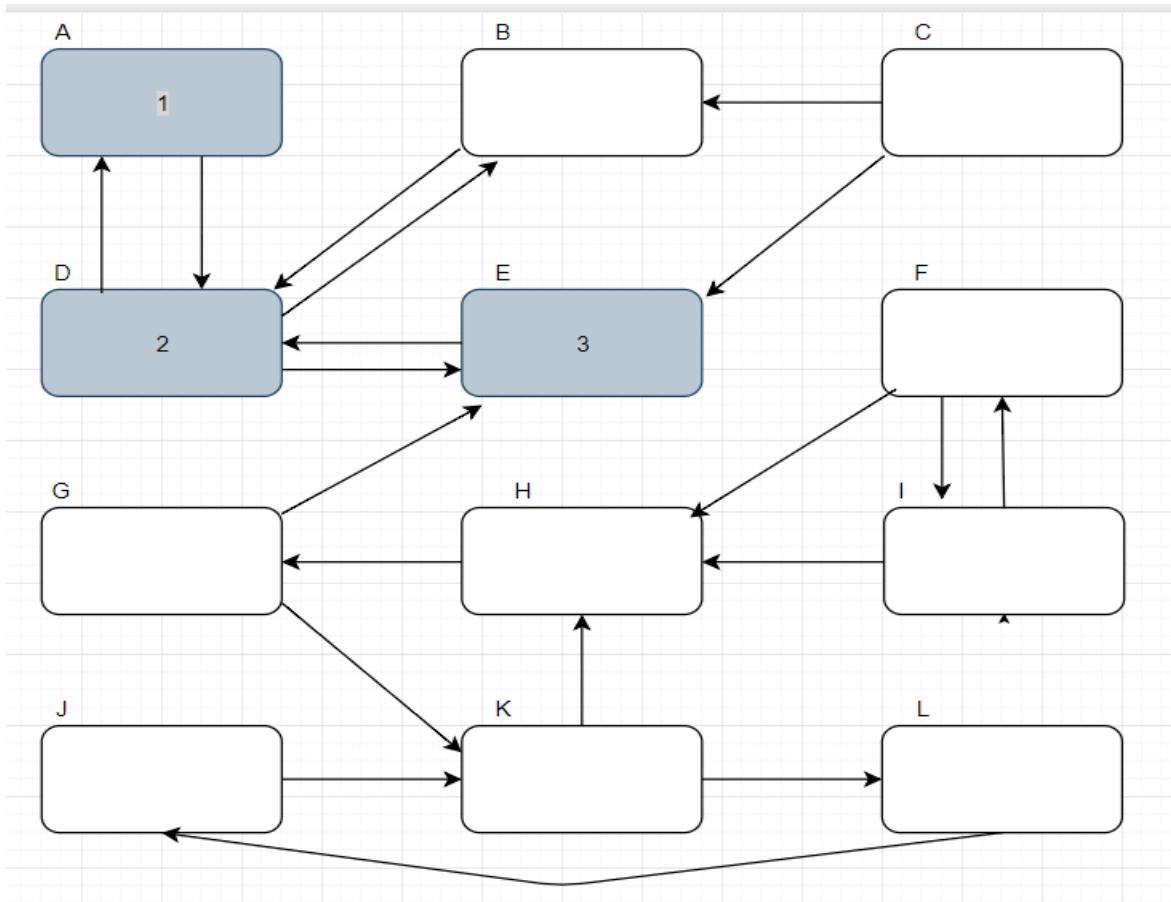
**Following is the transpose of the graph in (a)  $G^T$  :**

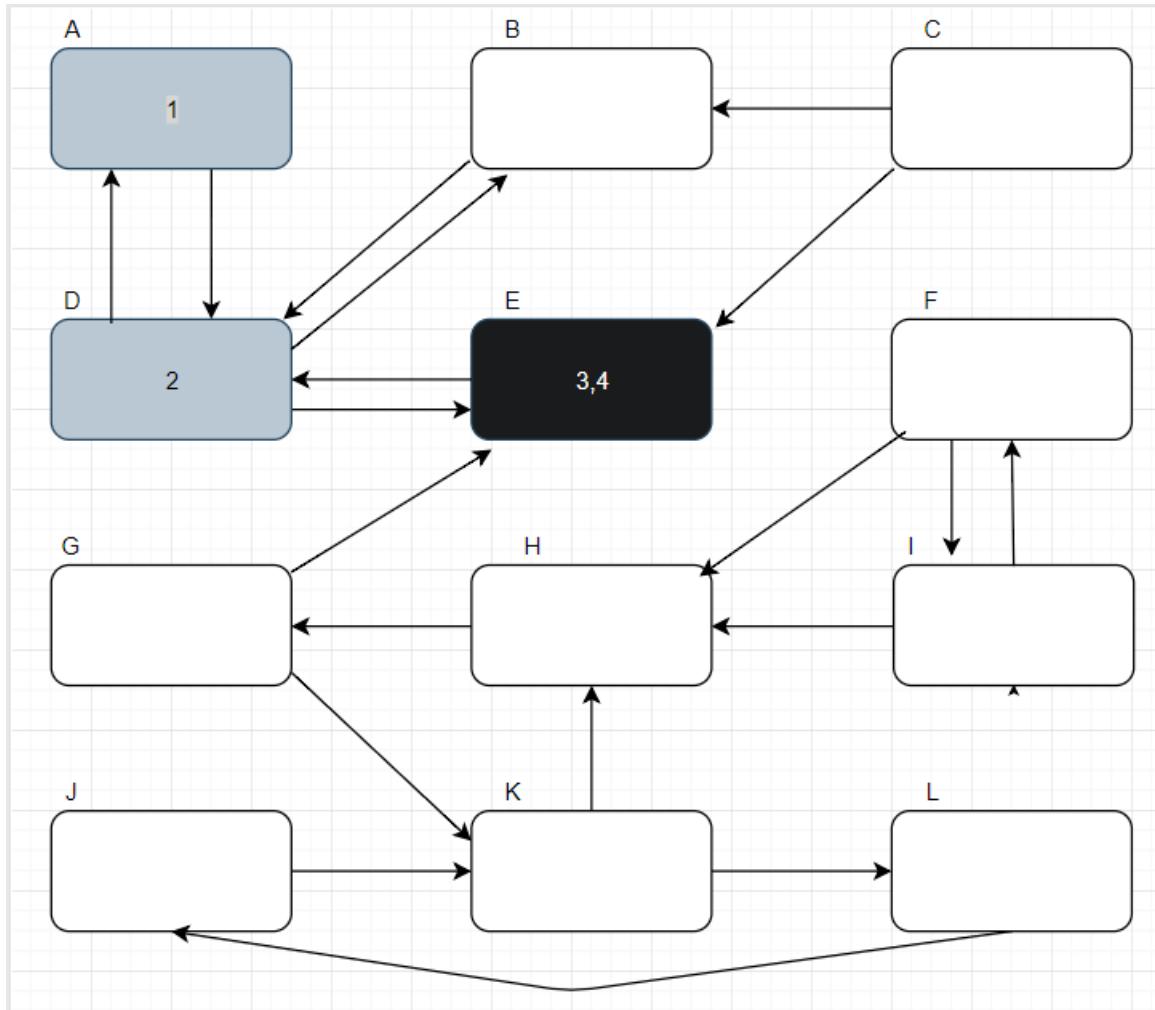


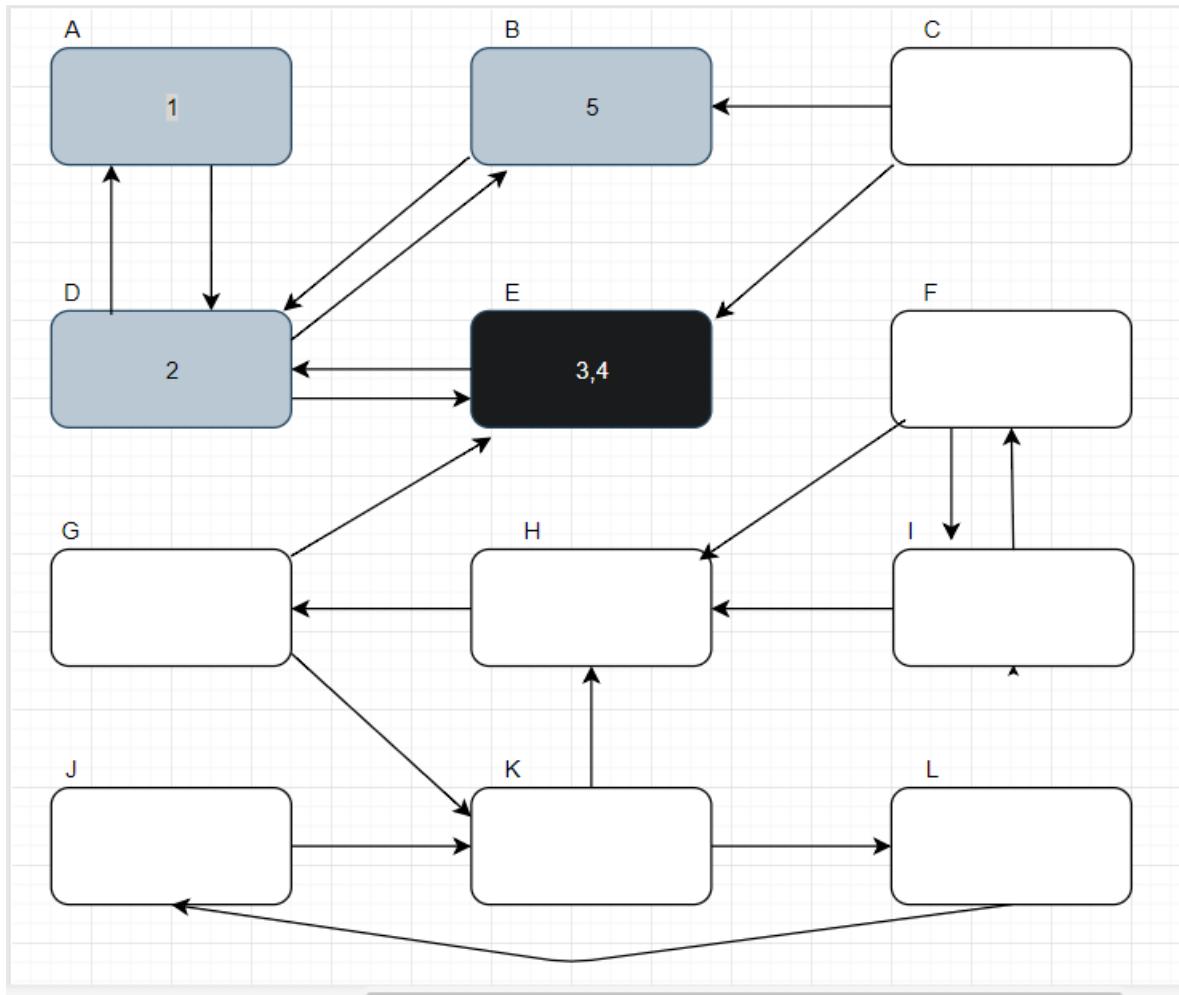
The second DFS traversal :

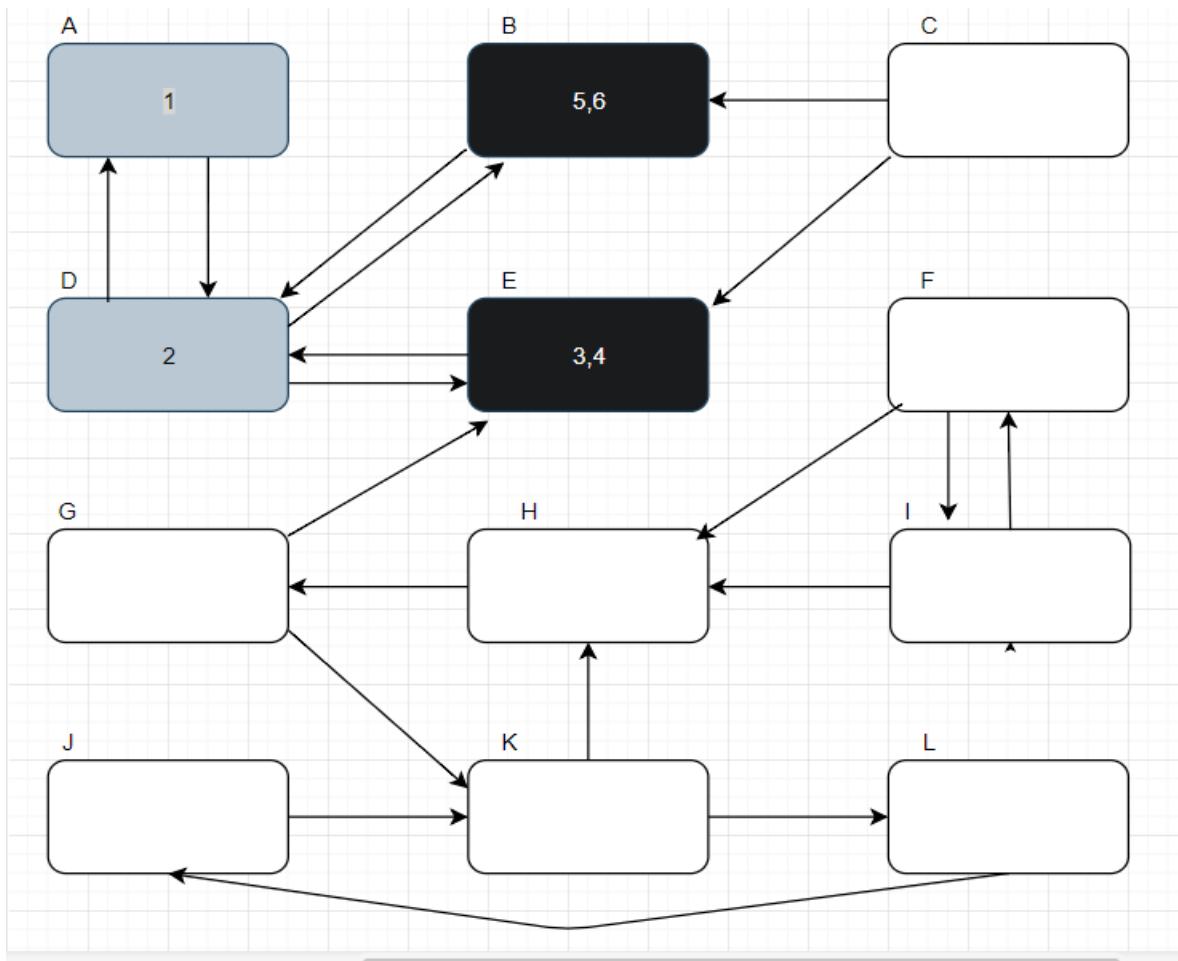


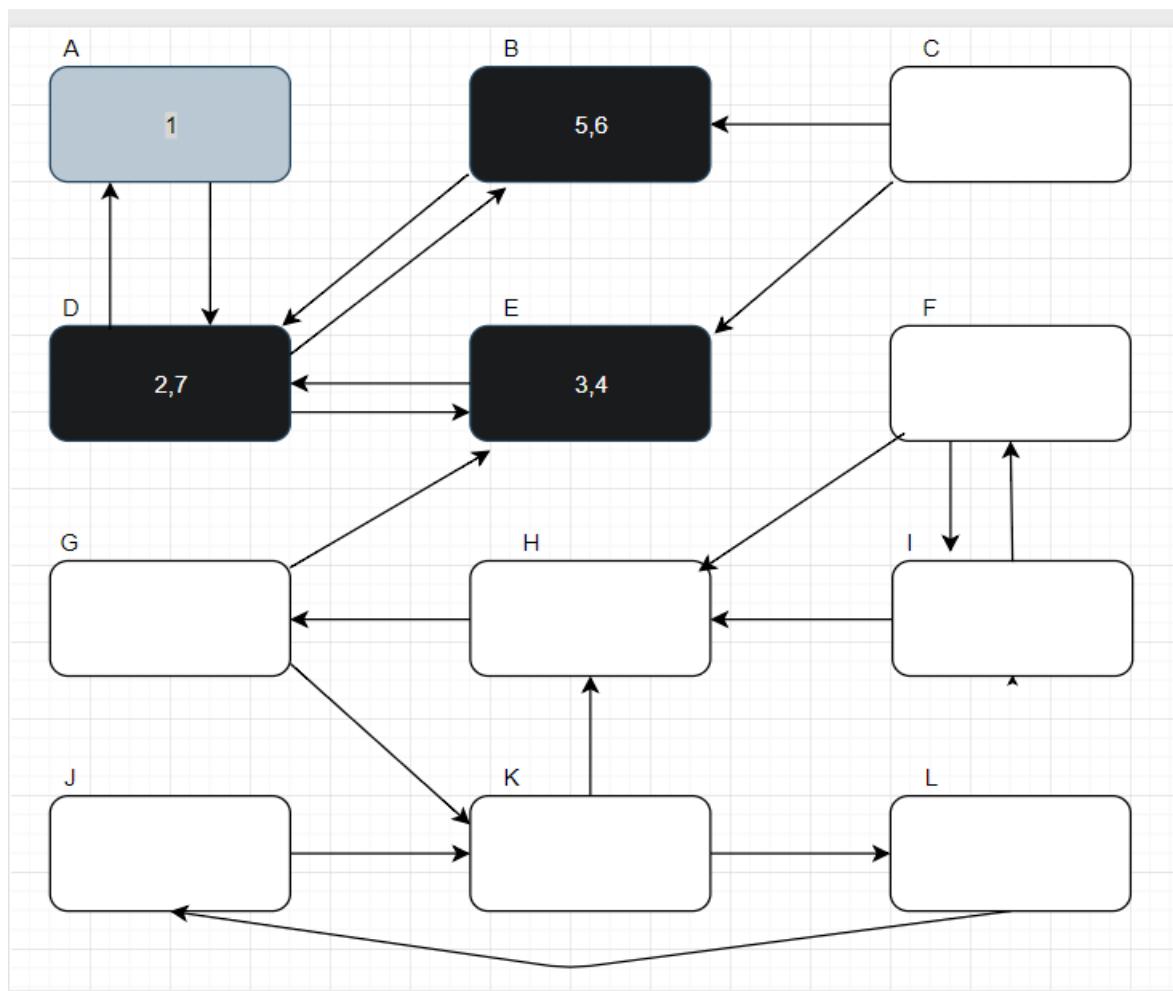


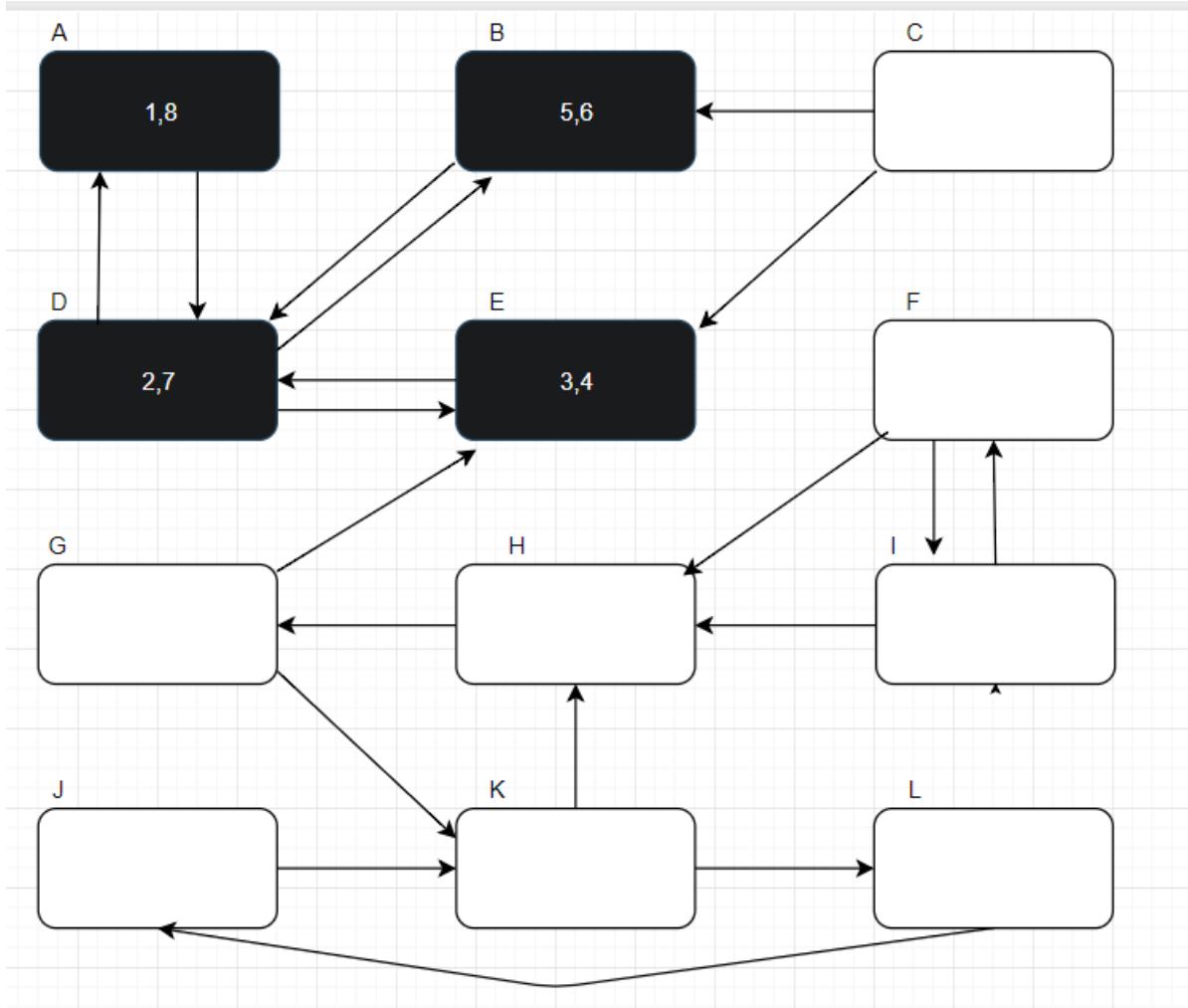


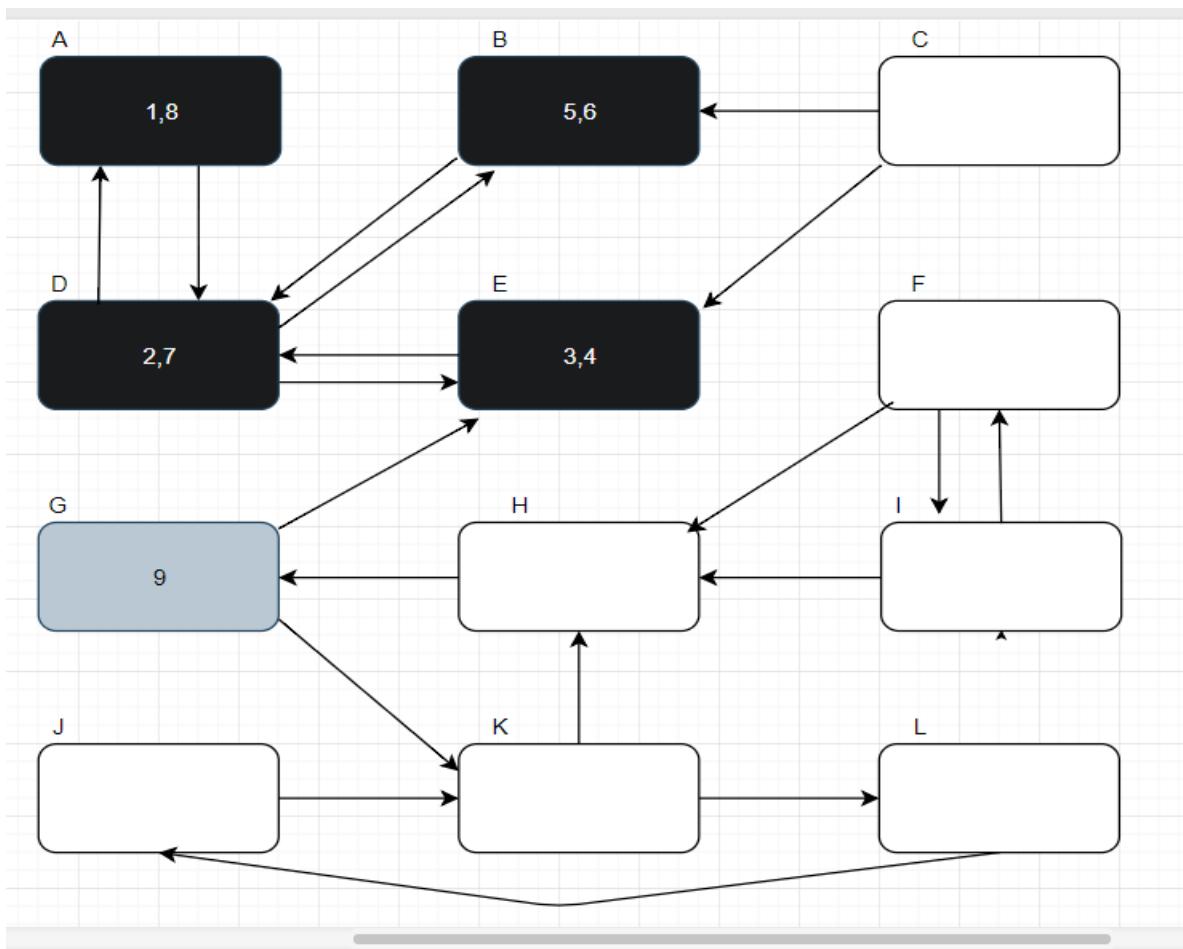


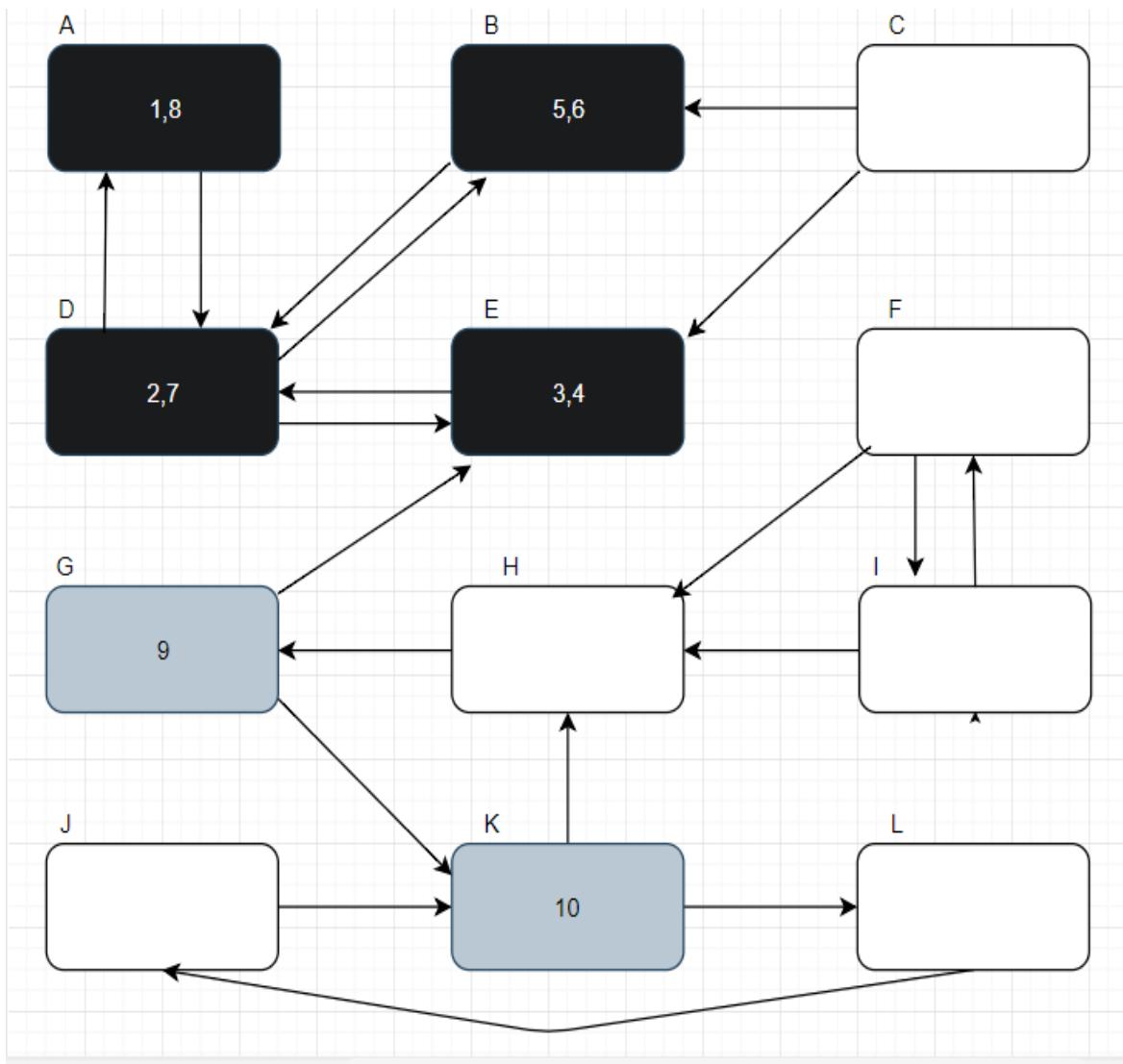


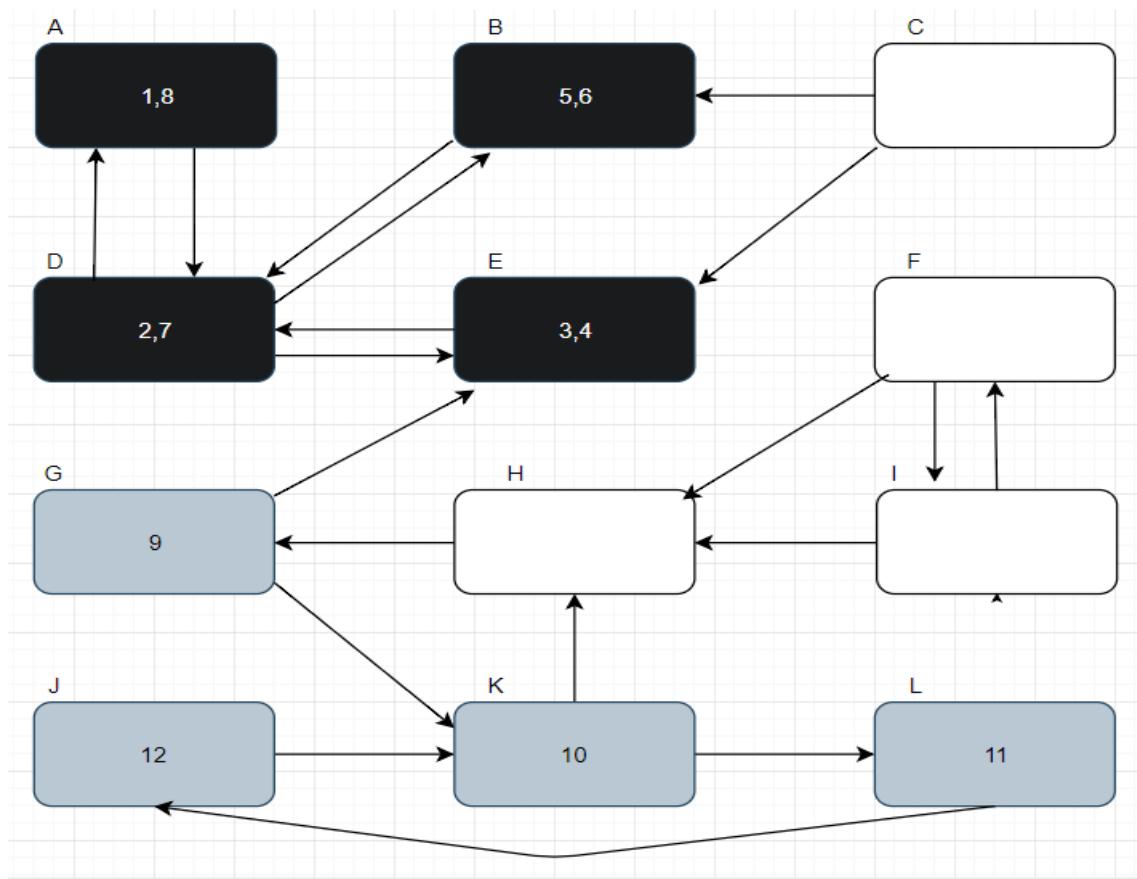


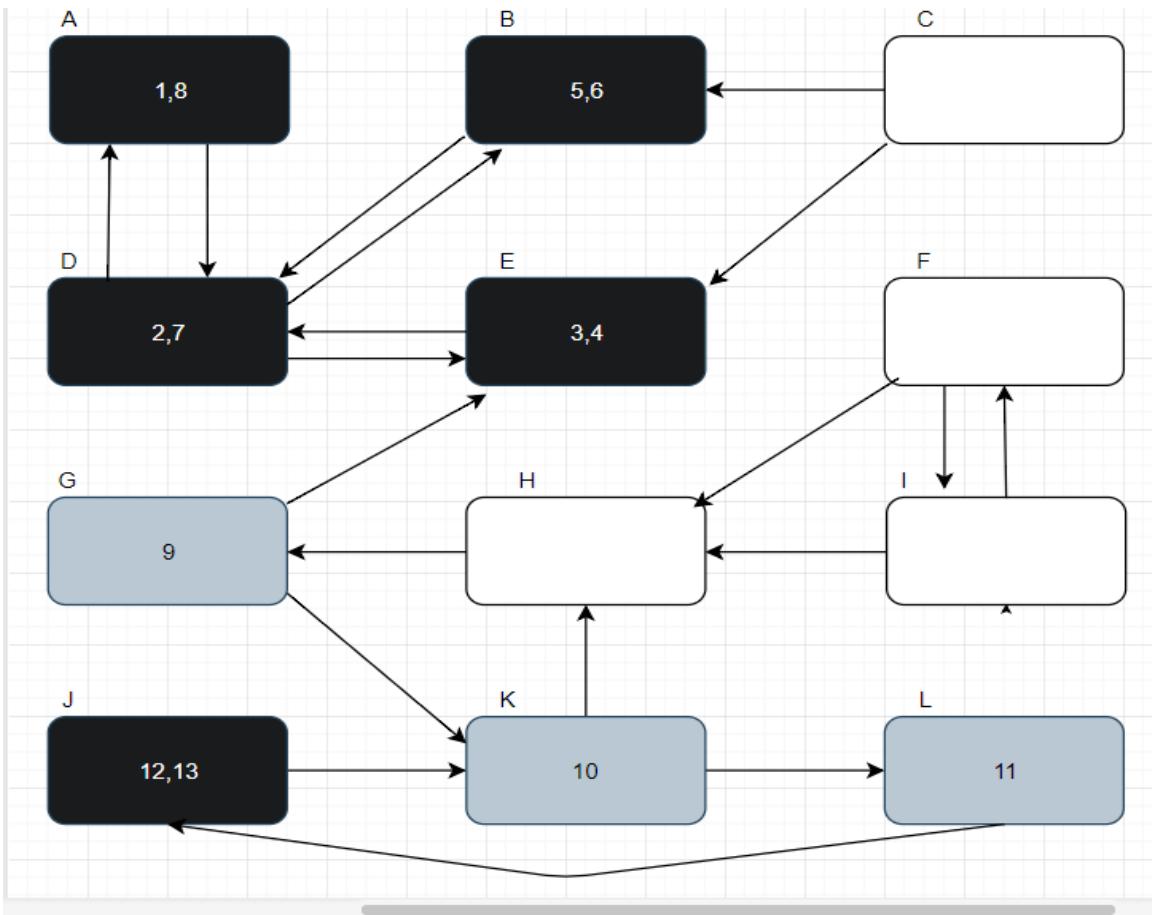


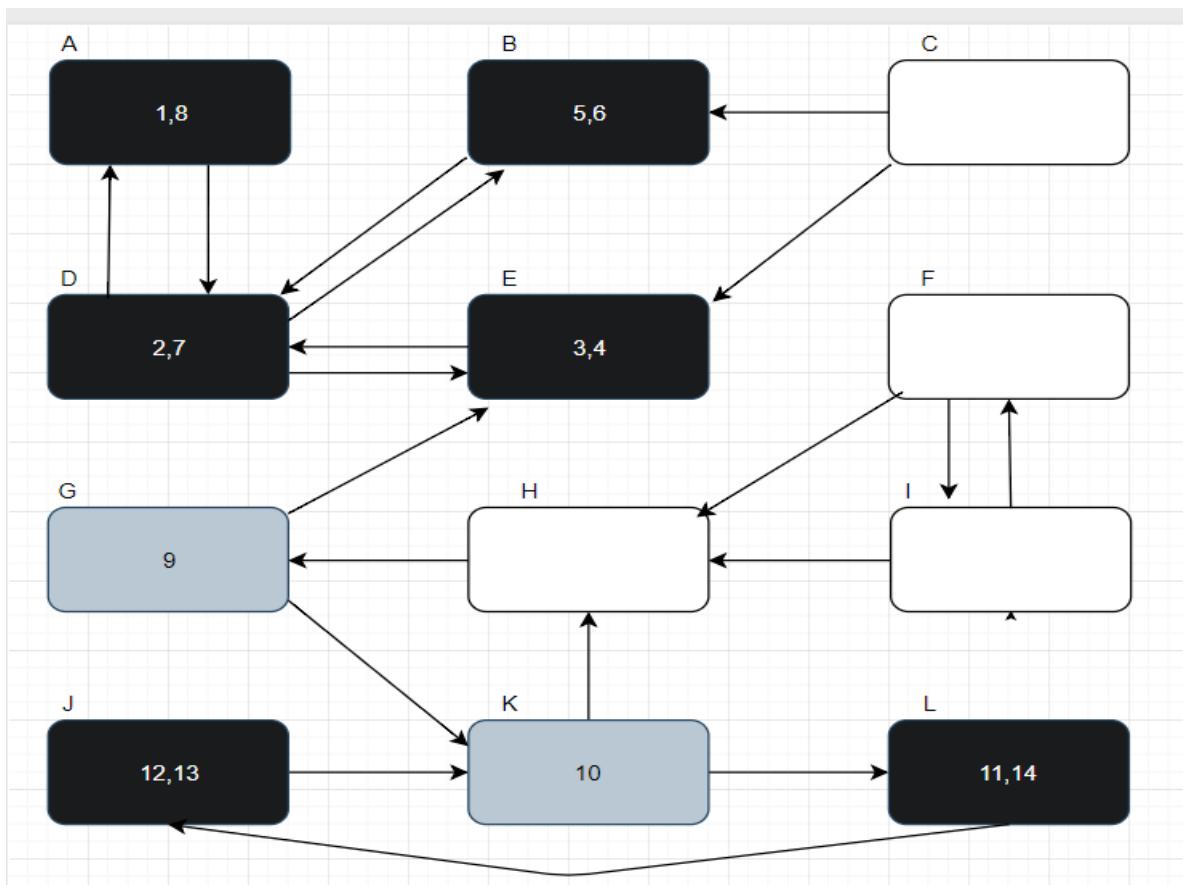


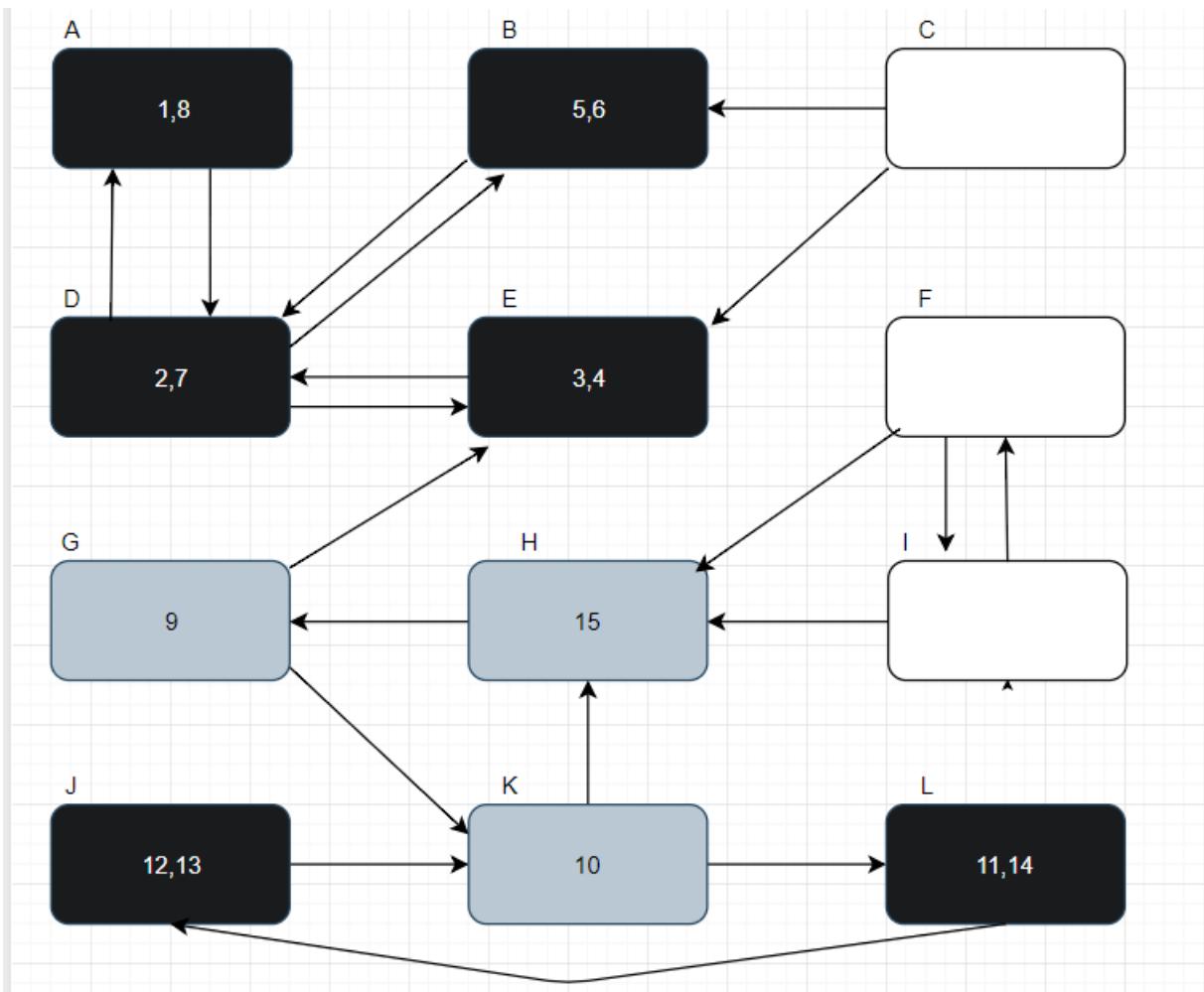


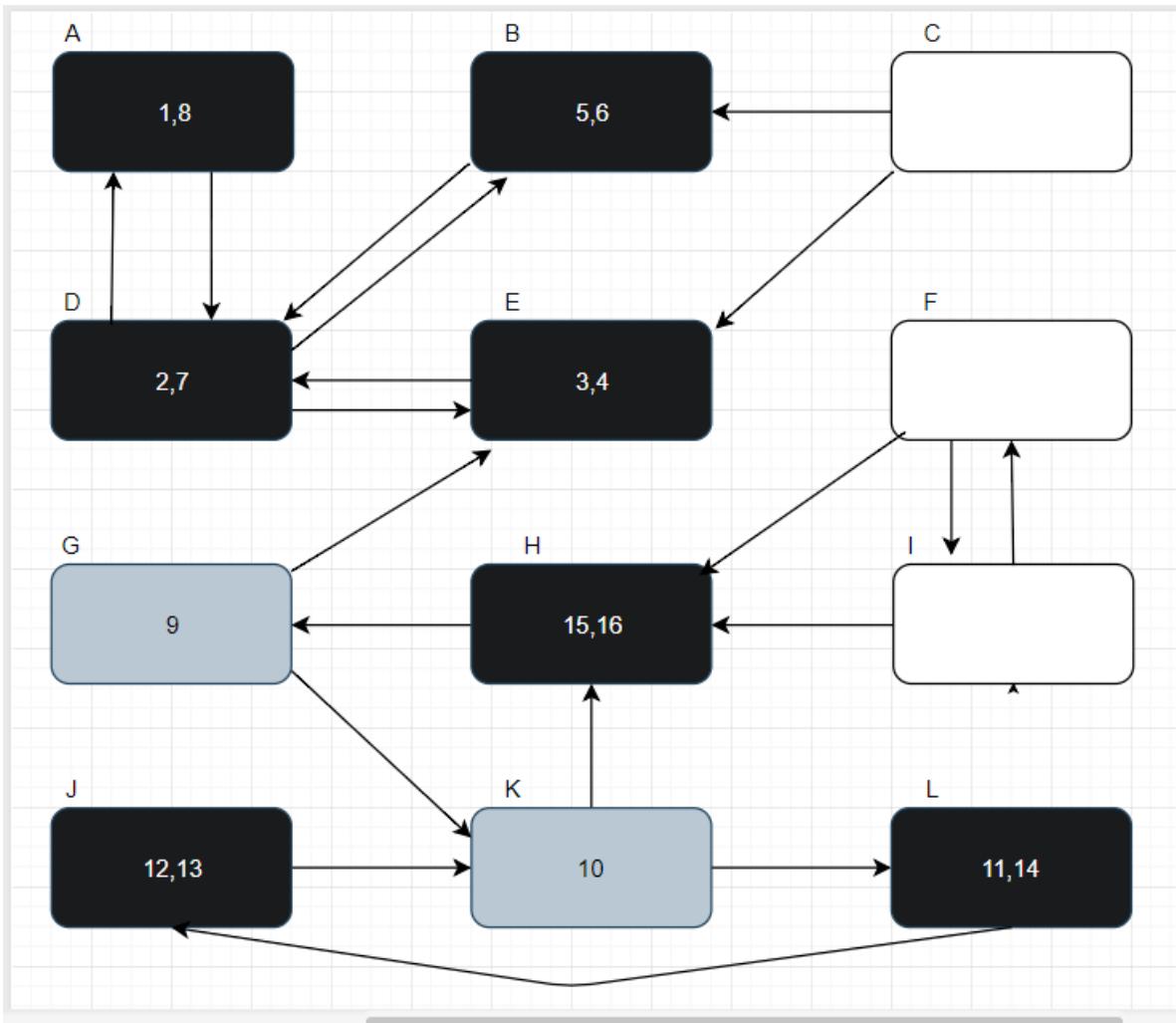


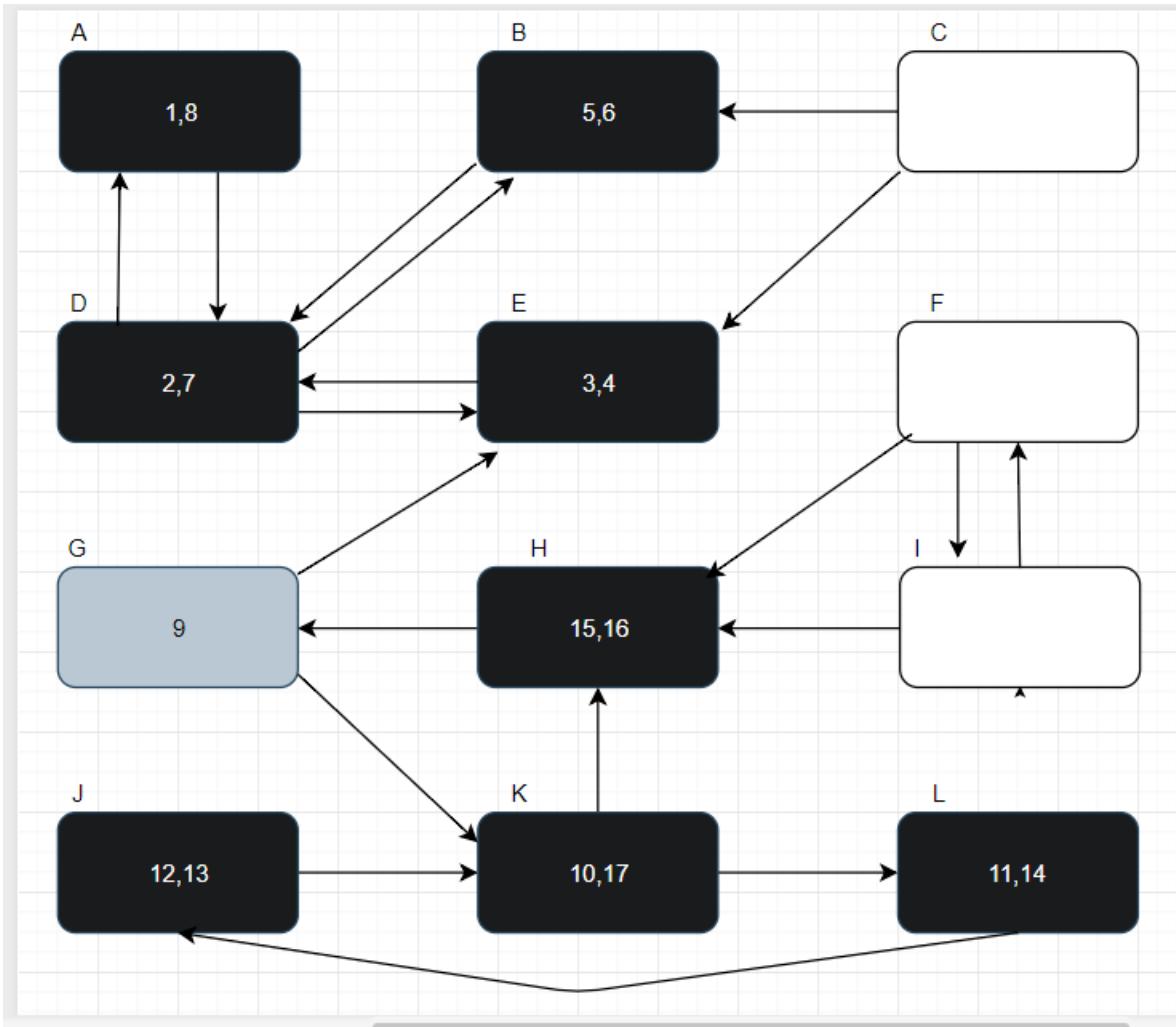


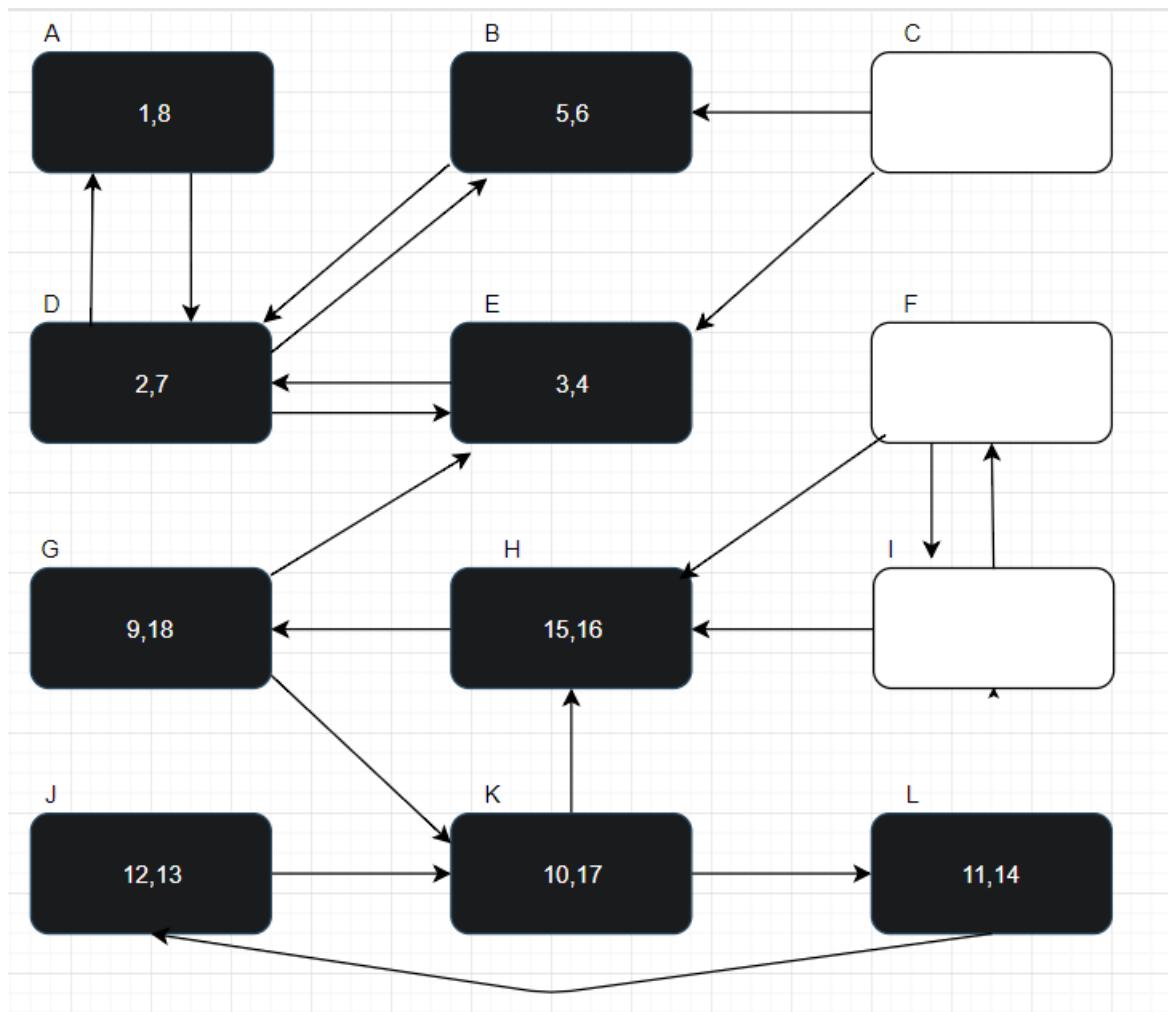


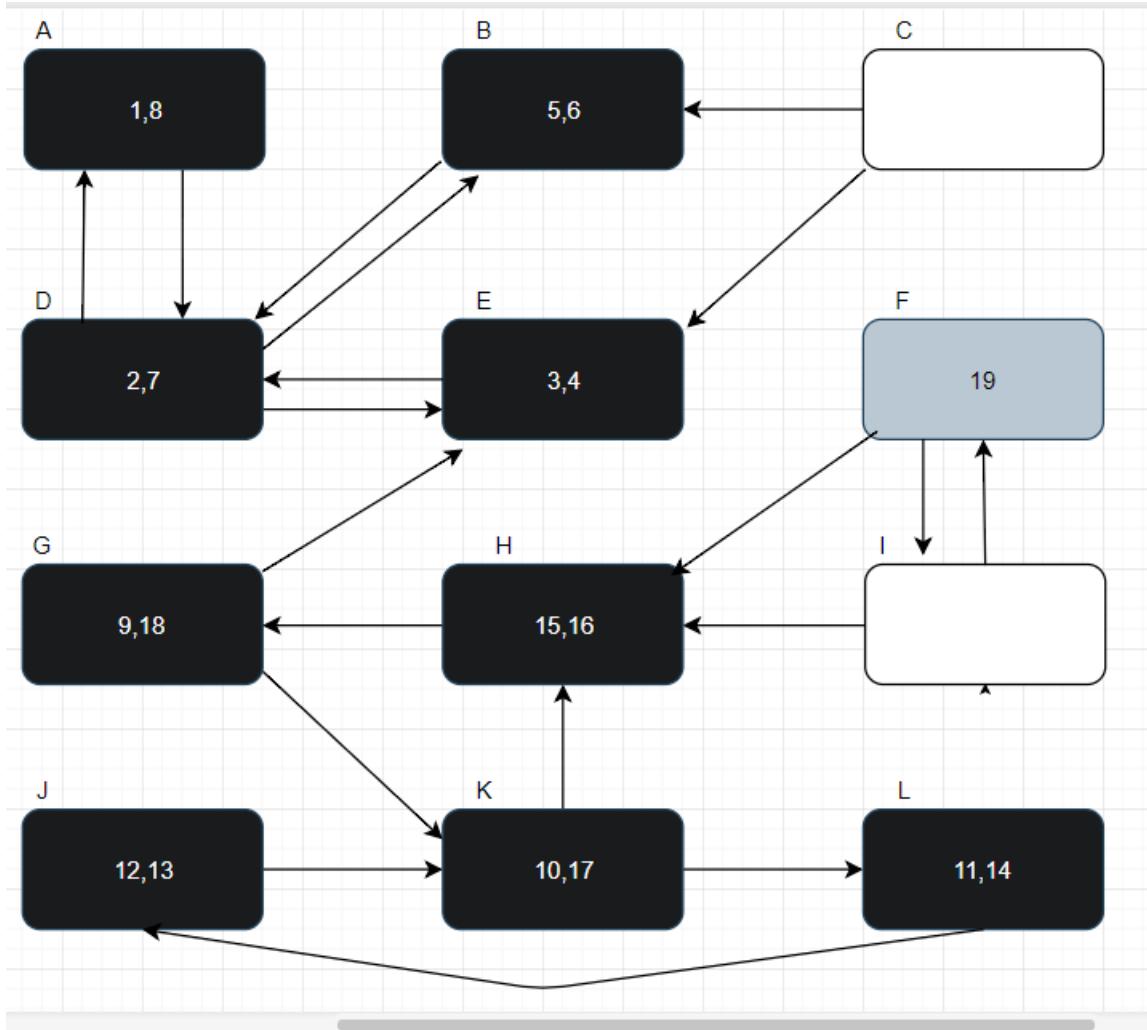


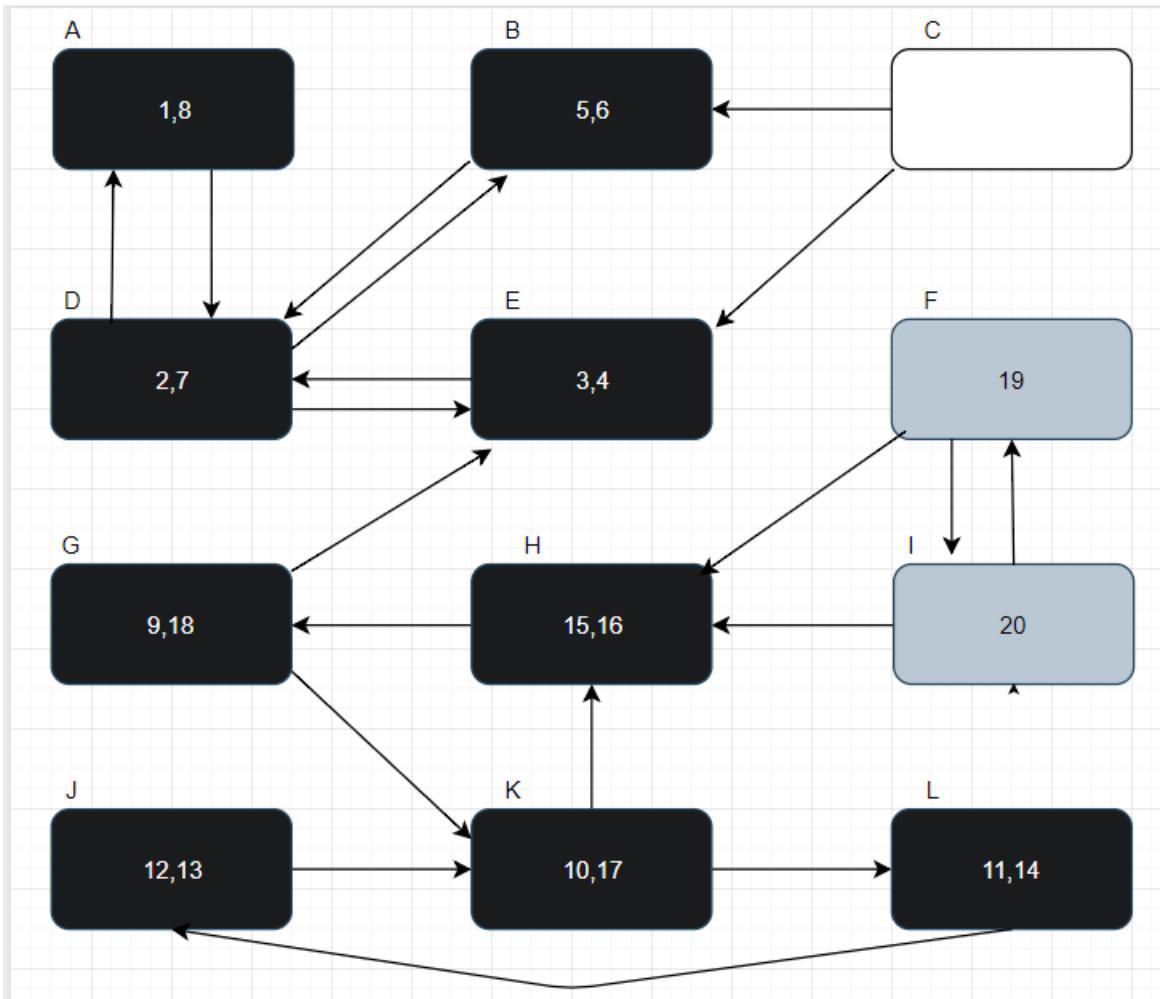


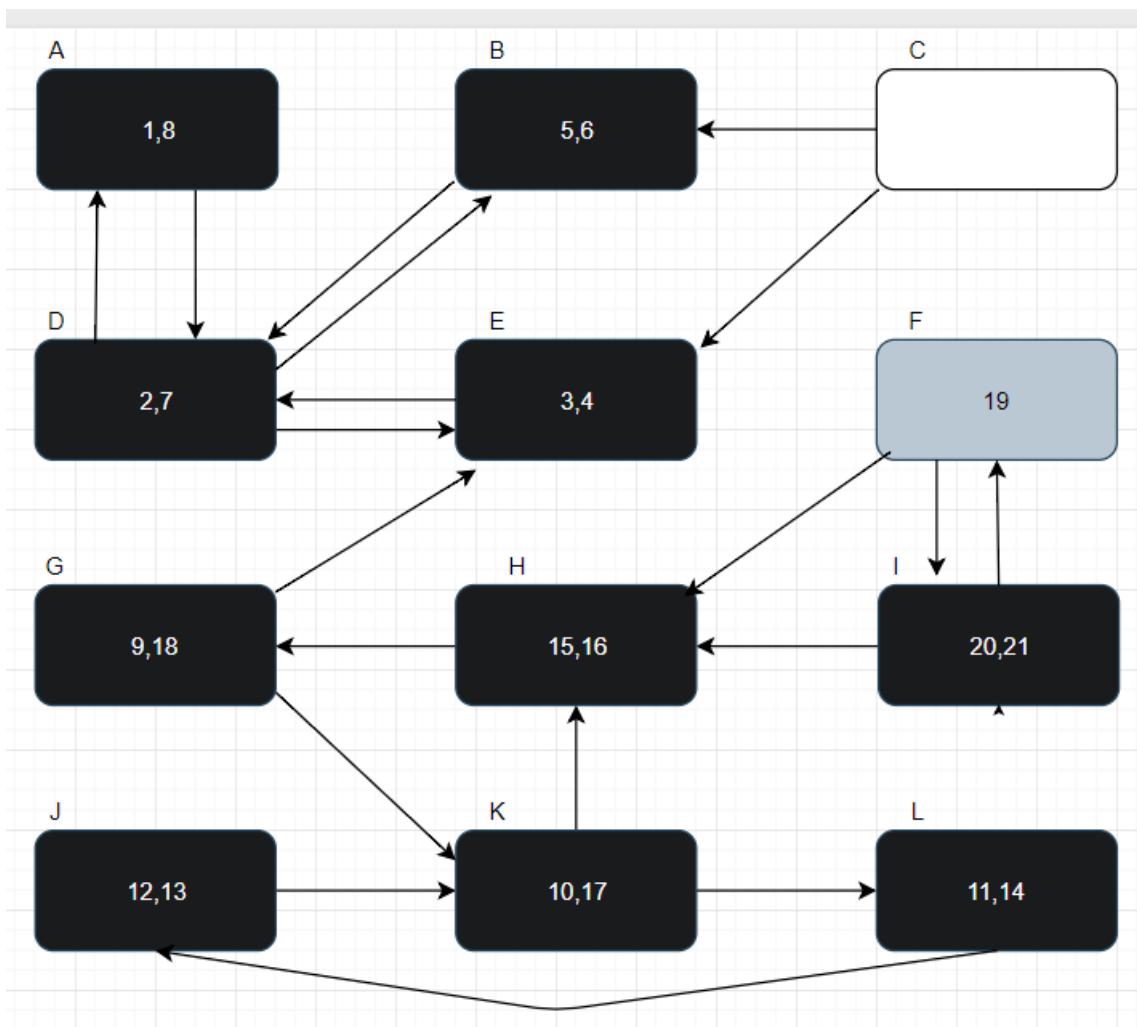


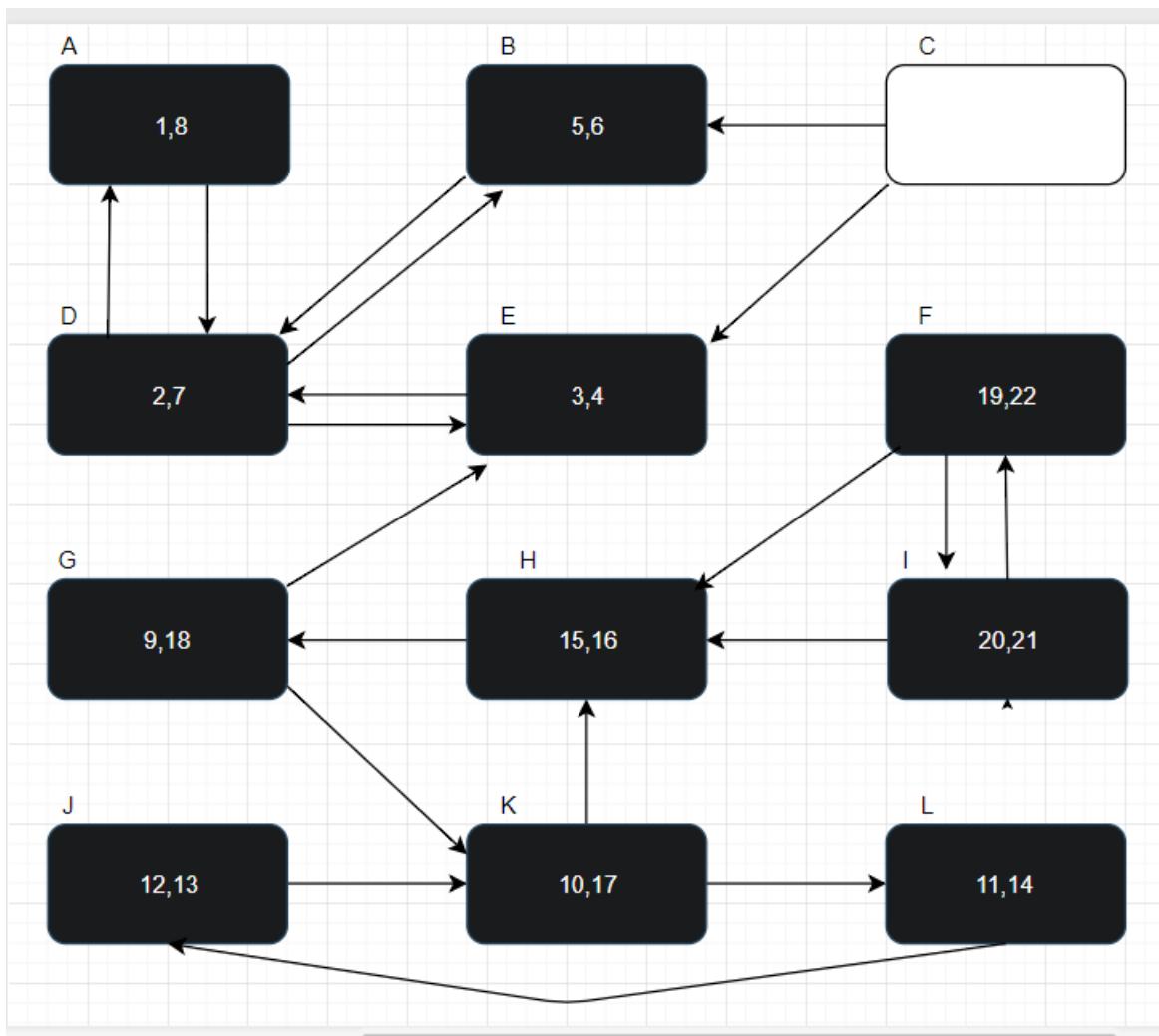


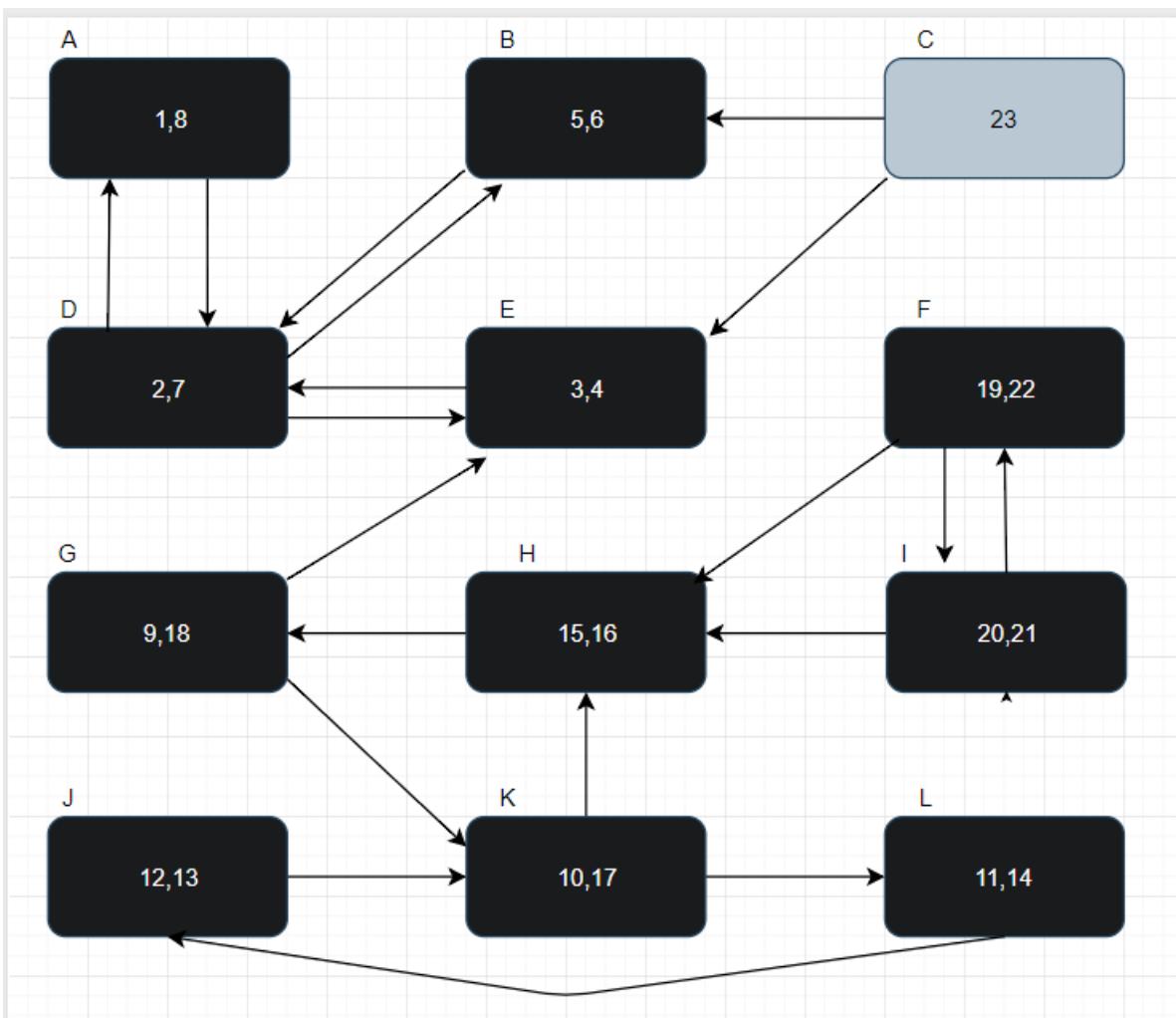


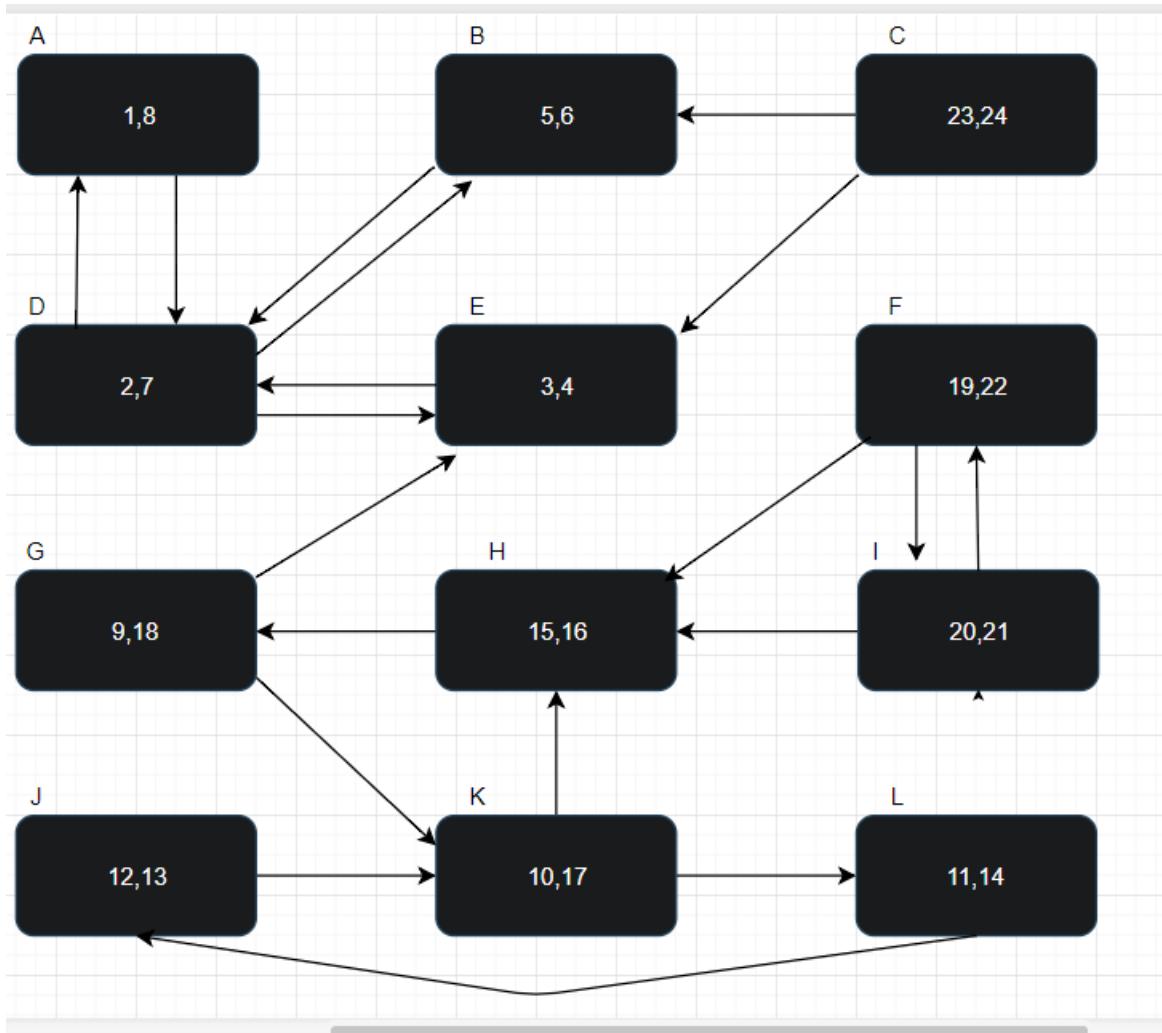




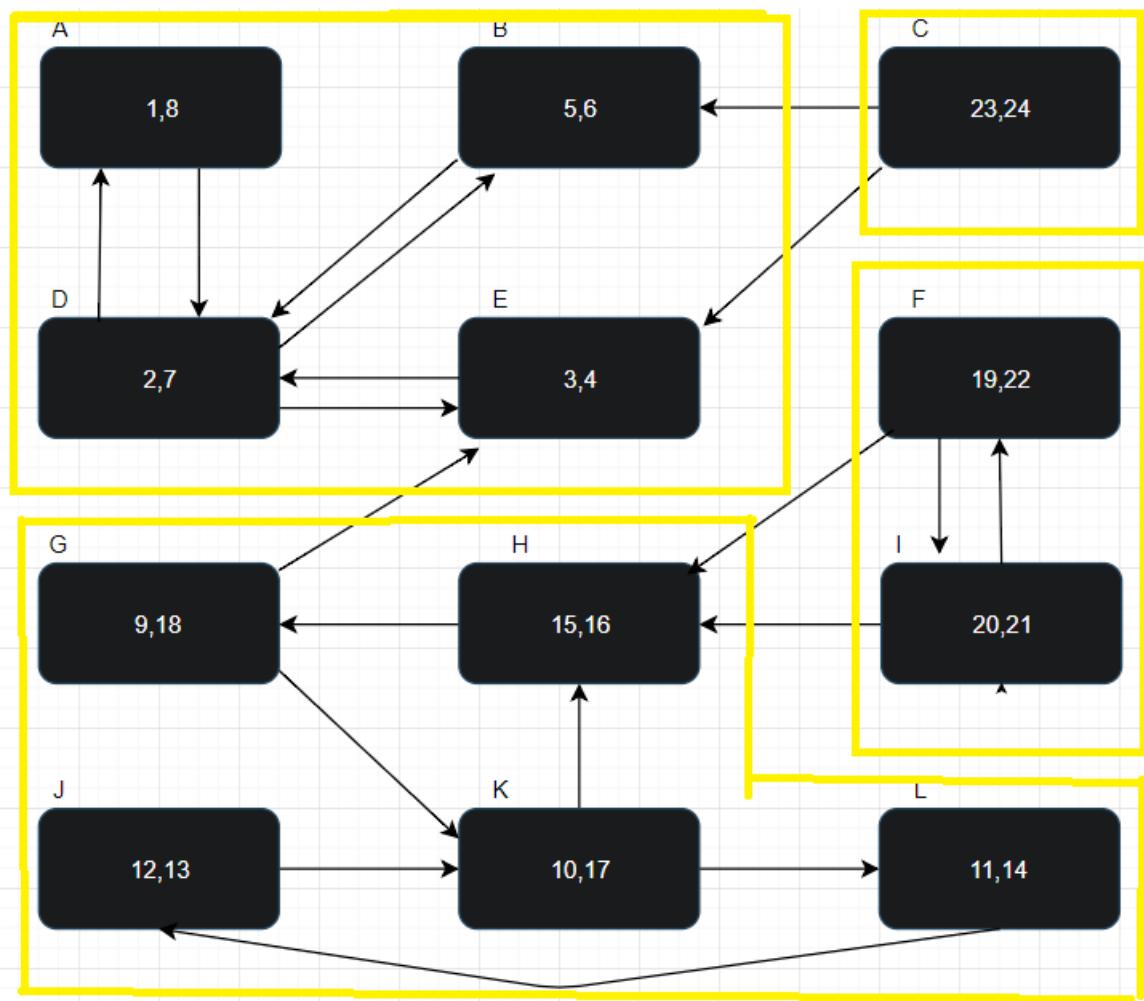


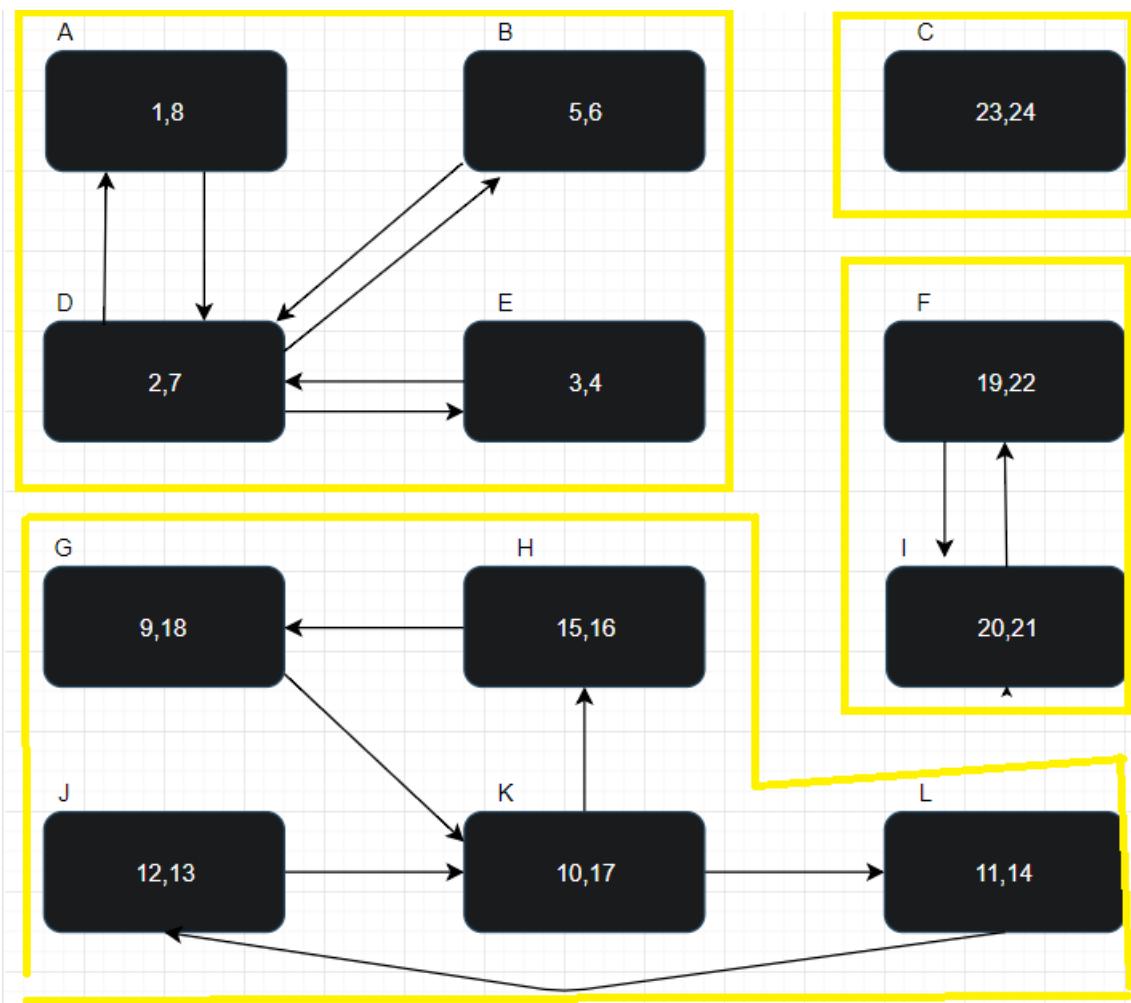






The SCC from the above graph as per the discovery and finishing times of the vertices are shown in the following figure:





The Strongly connected components as per the above solution are

A B D E

G H J K L

F I

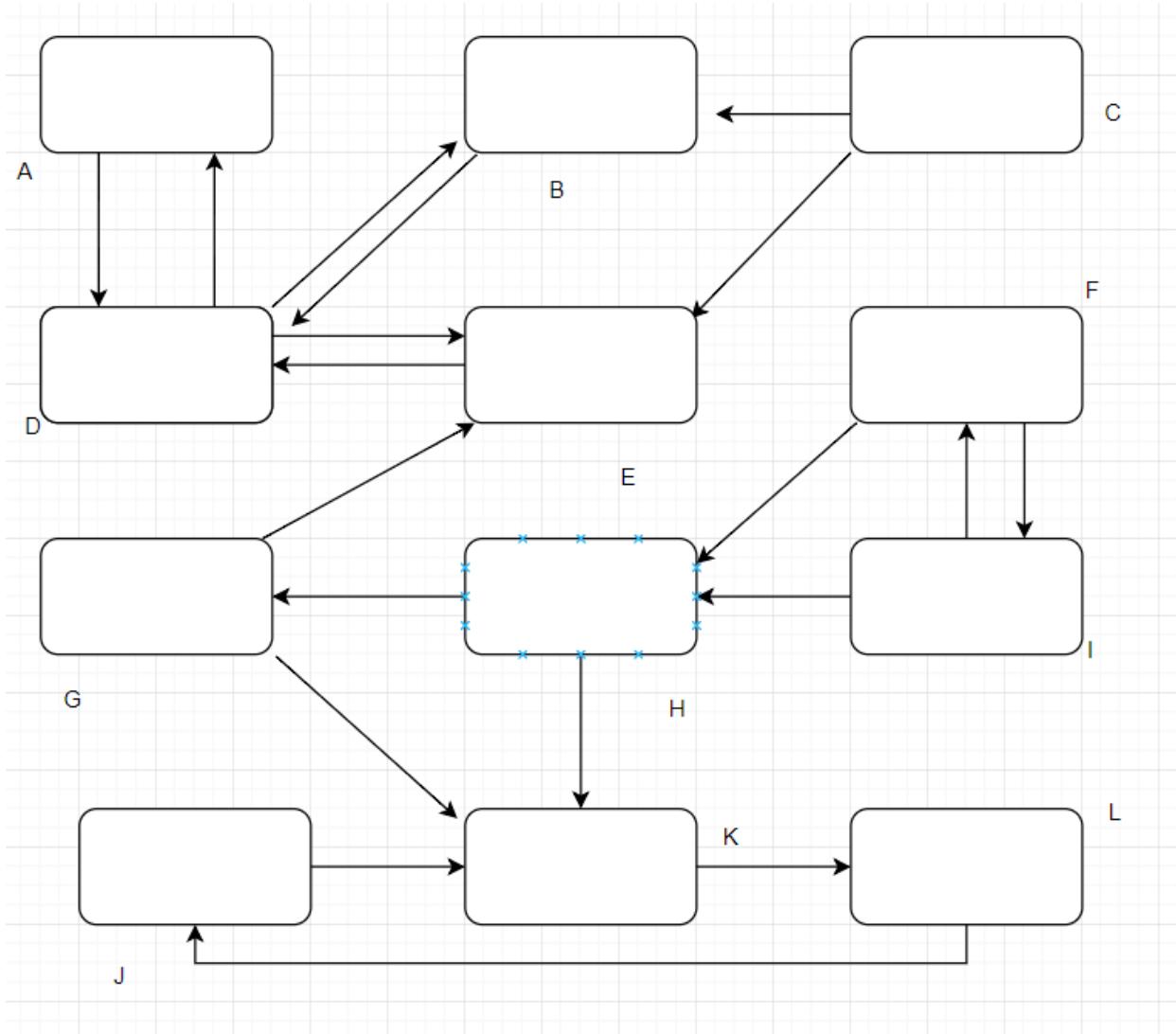
C

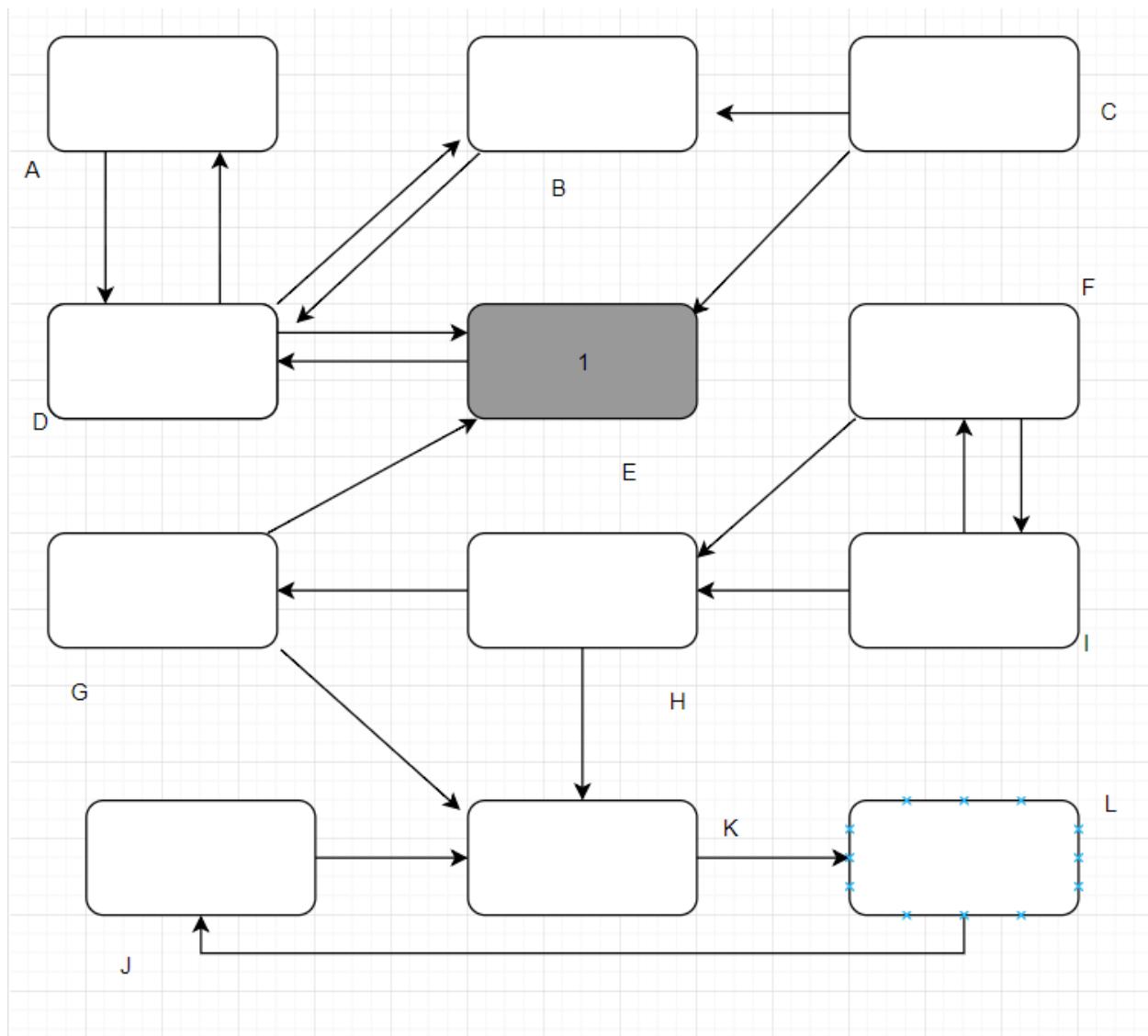
Vertex	Discovery time	Finishing time
A	1	8
B	5	6
D	2	7
E	3	4
G	9	18
H	15	16
J	12	13
K	10	17
L	11	14
F	19	22
I	20	21
C	23	24

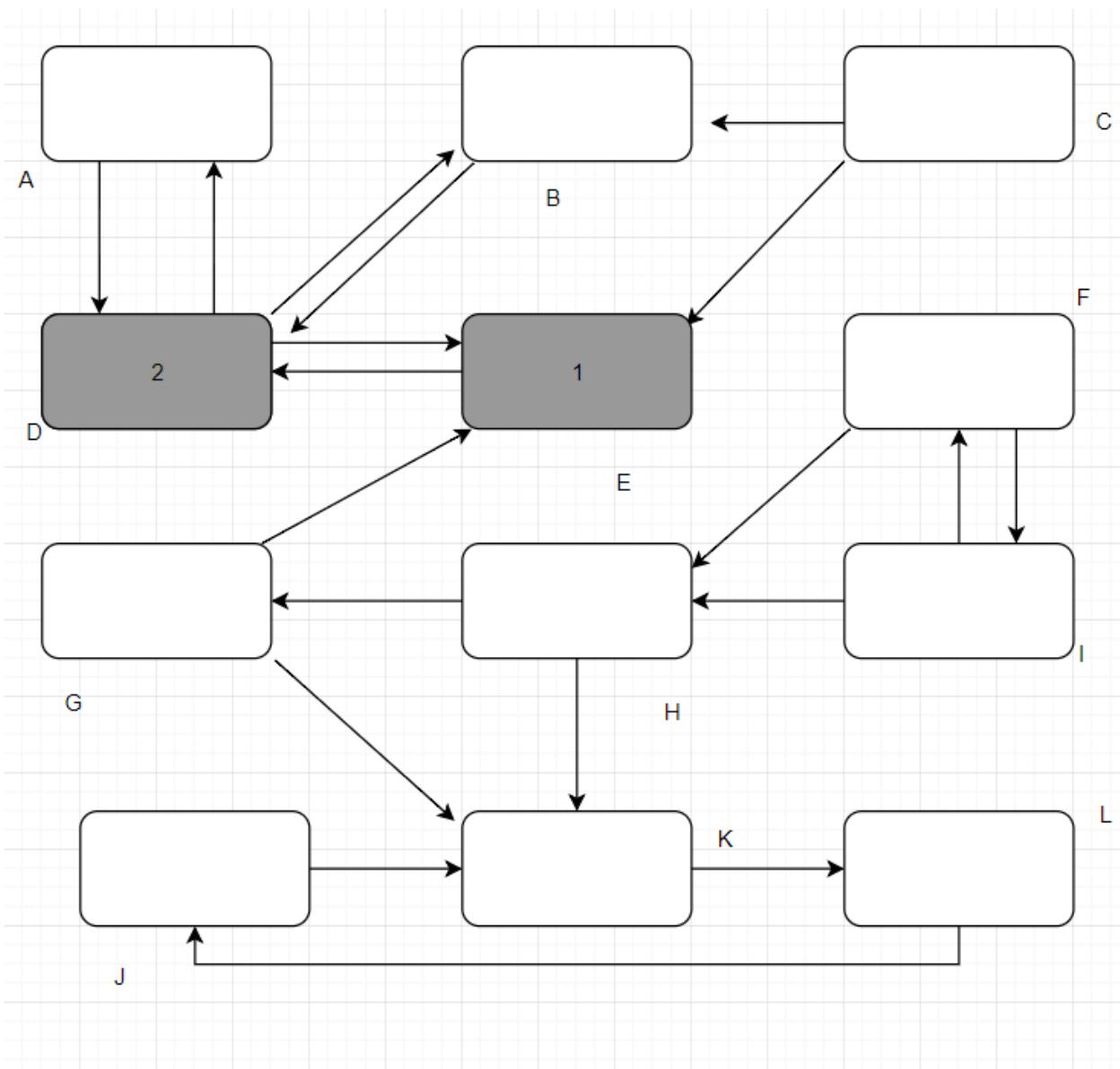
Q 1 d)

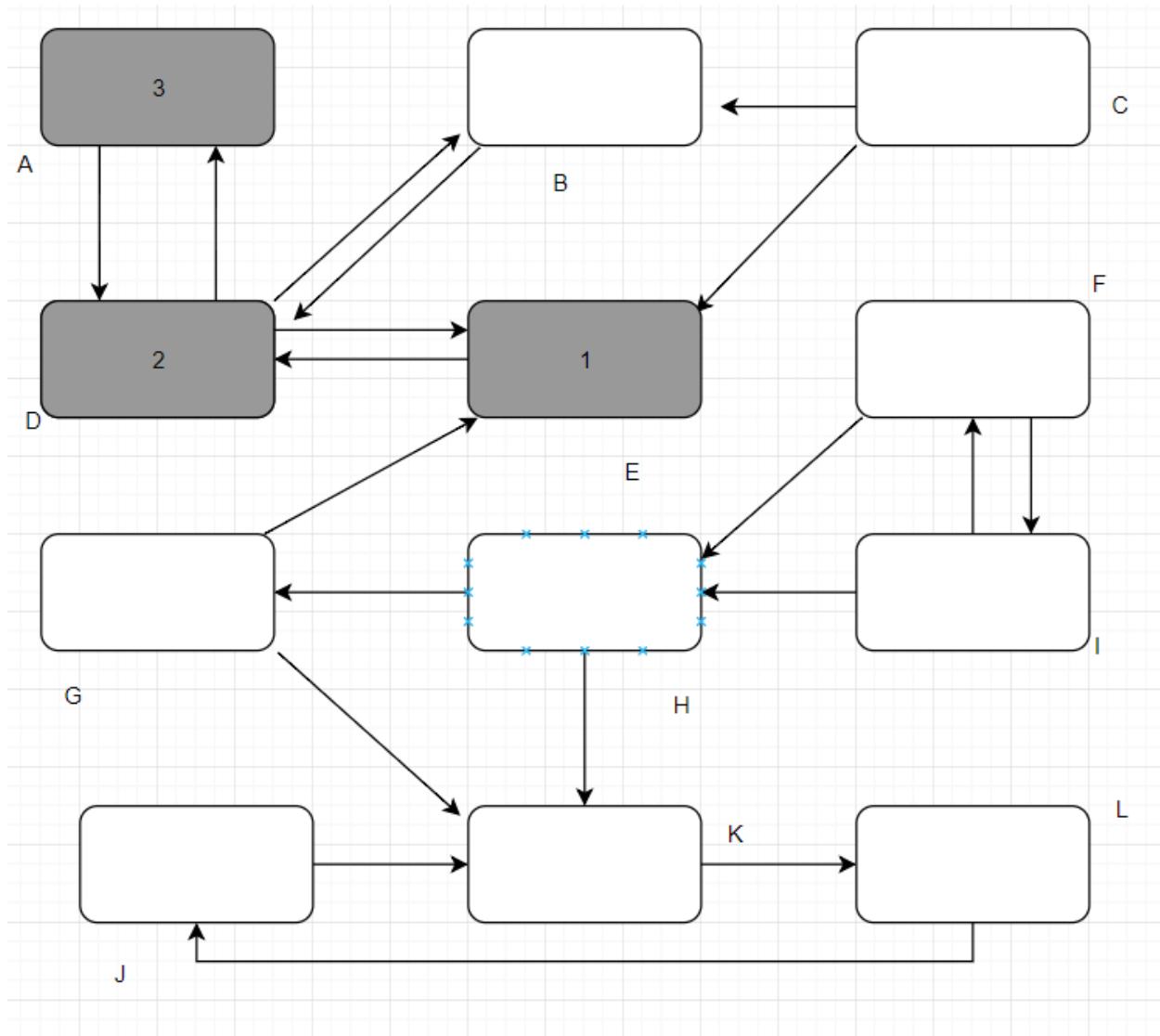
Second run of DFS:

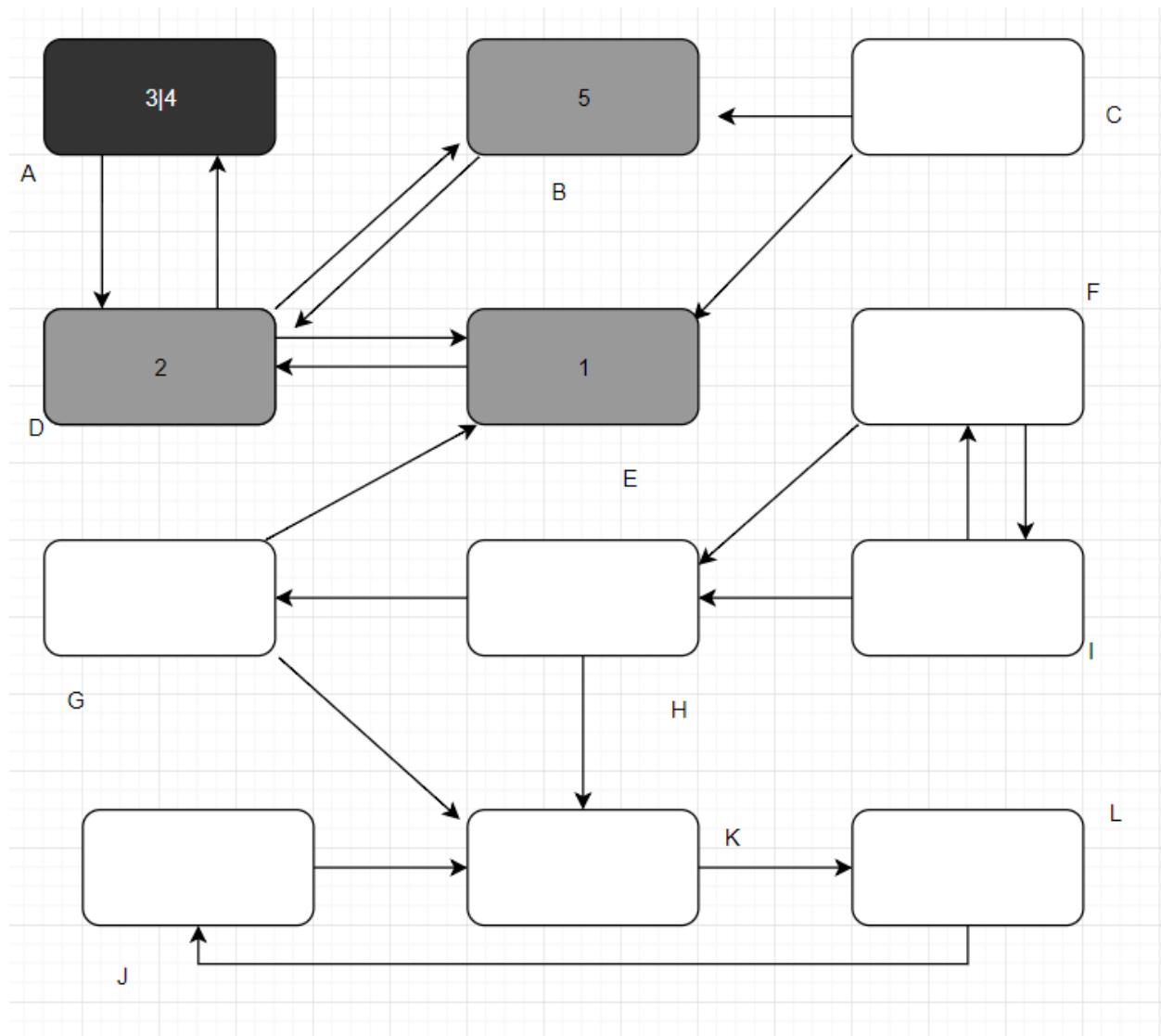
In order of decreasing order time: E D A B C L K G H I F J using output of Q 1 (b)

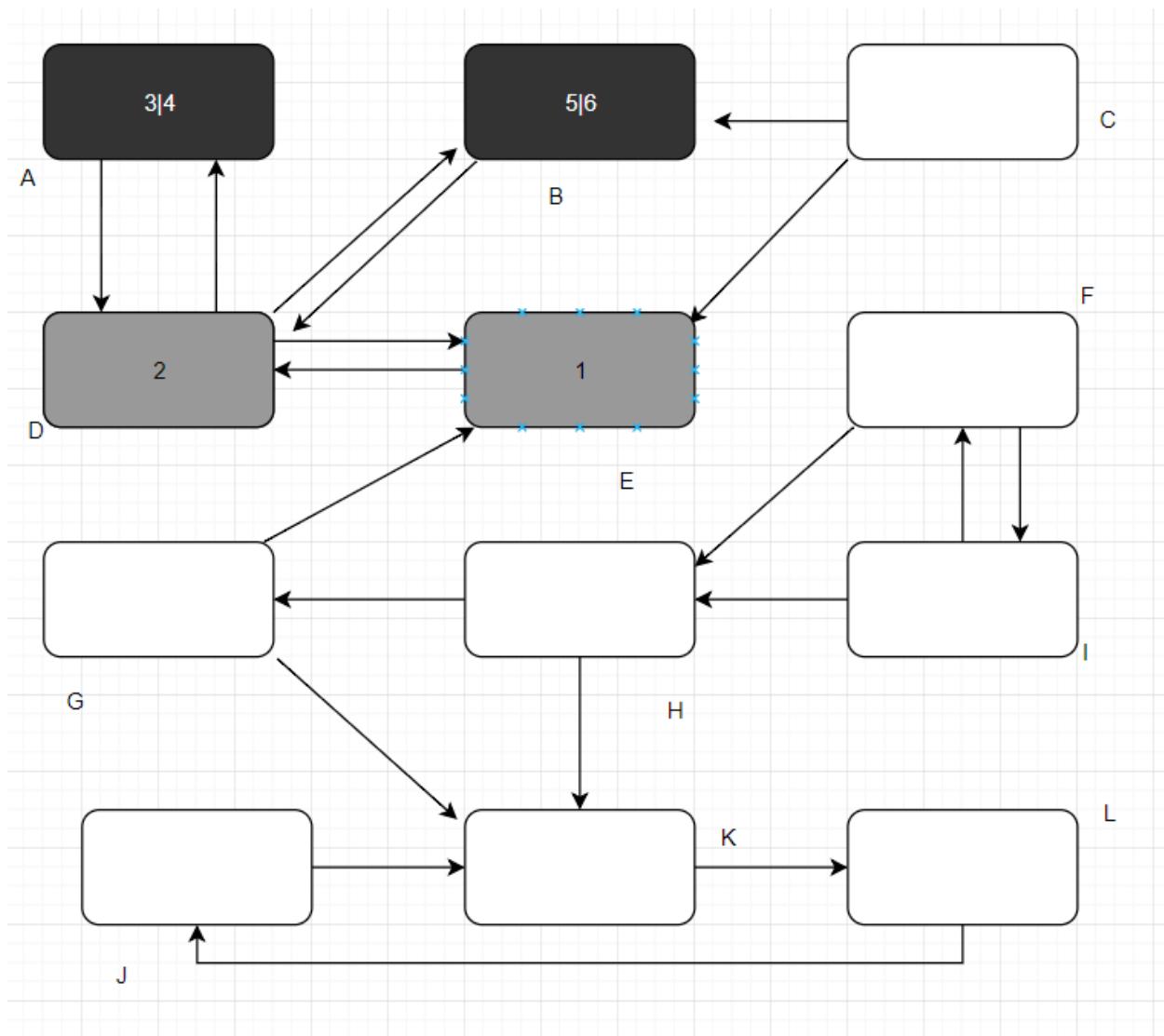


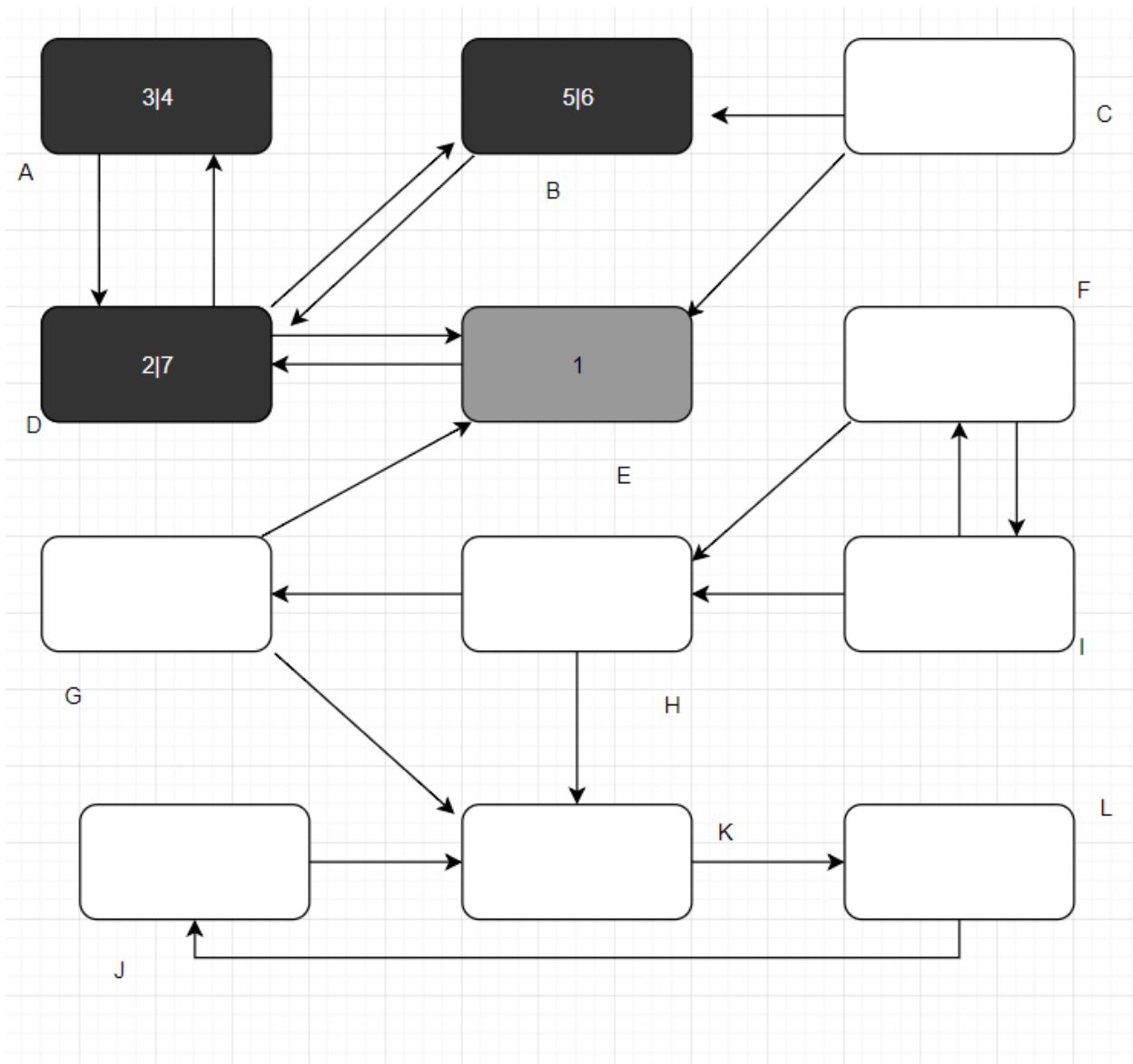


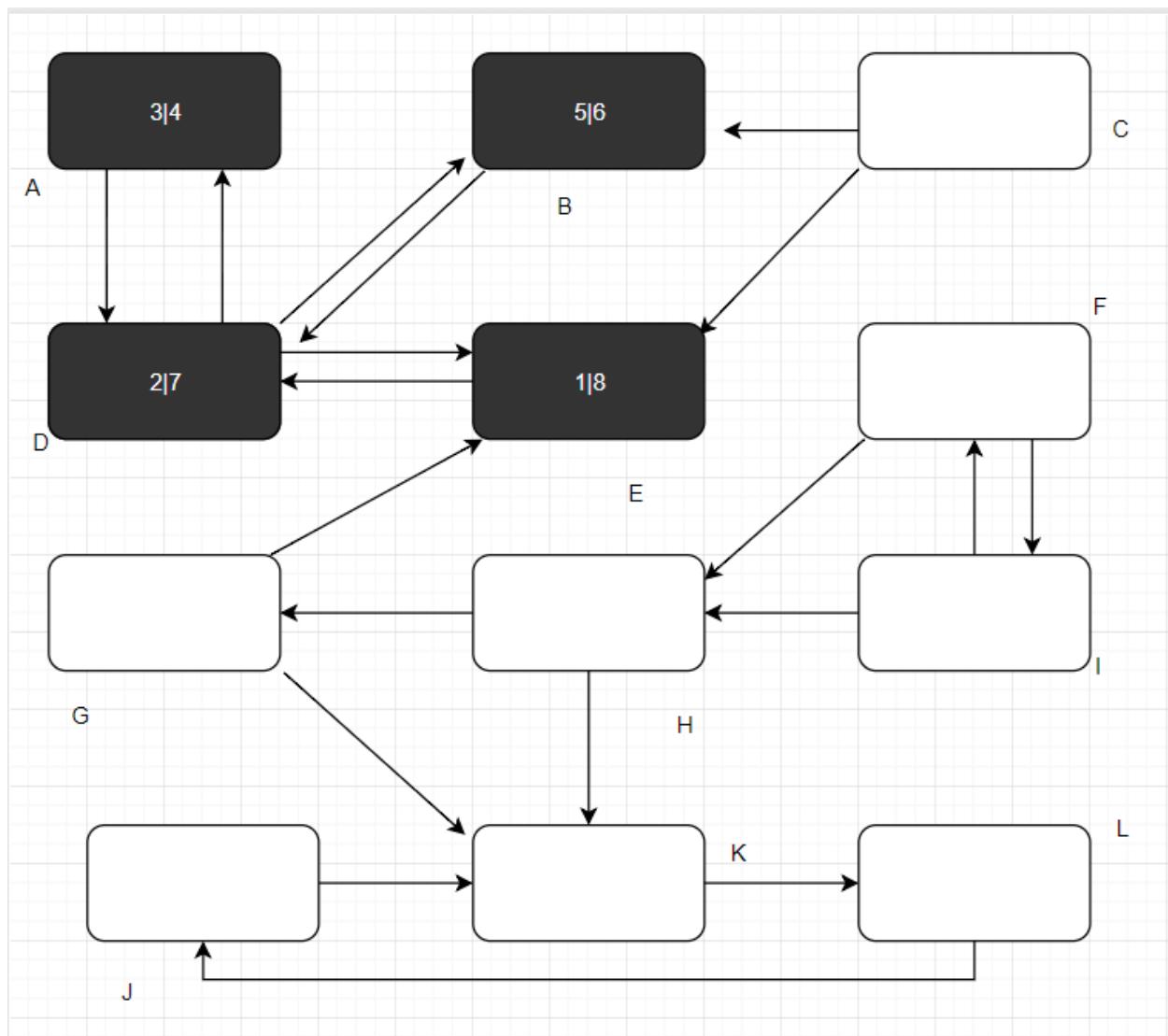




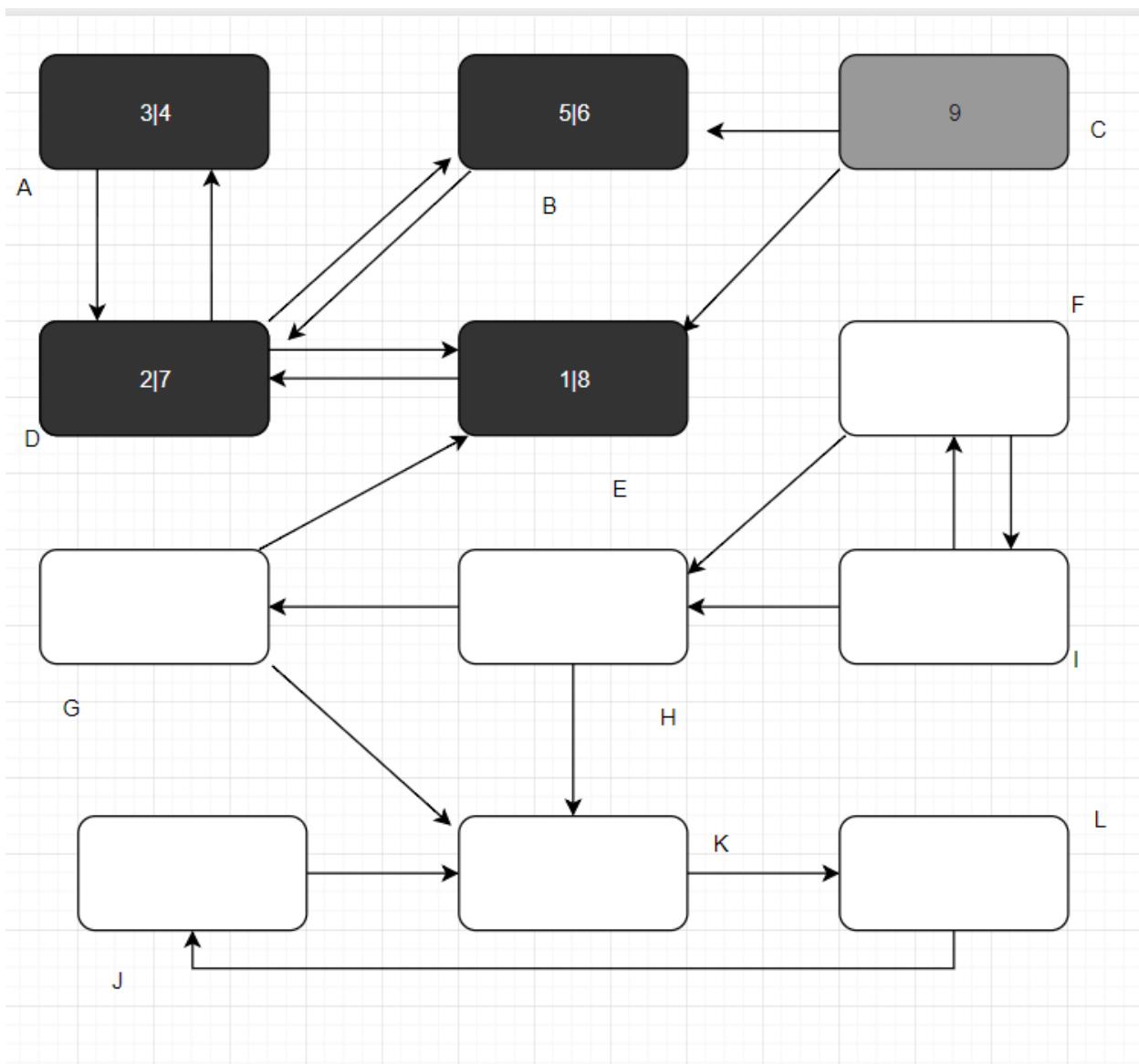


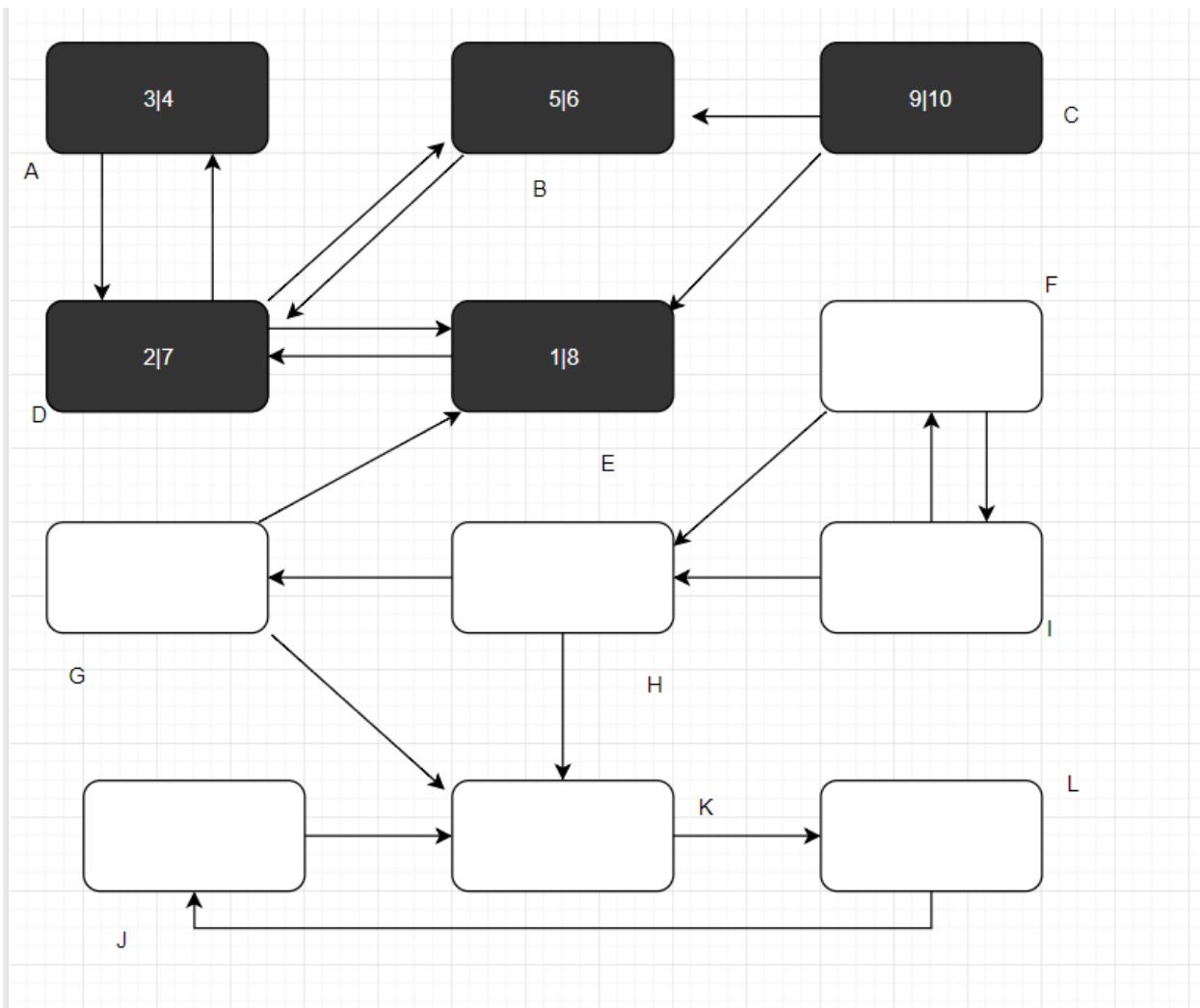




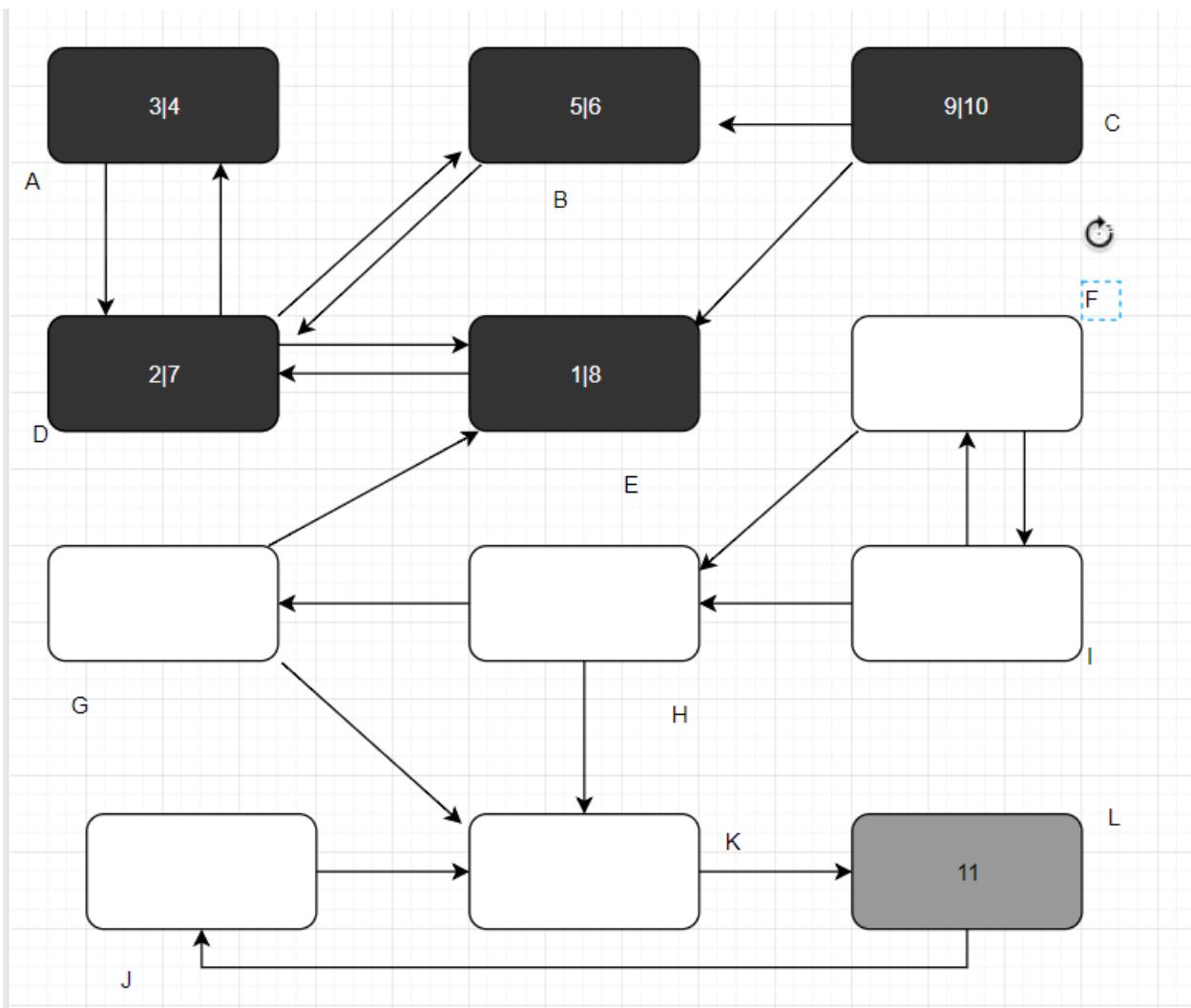


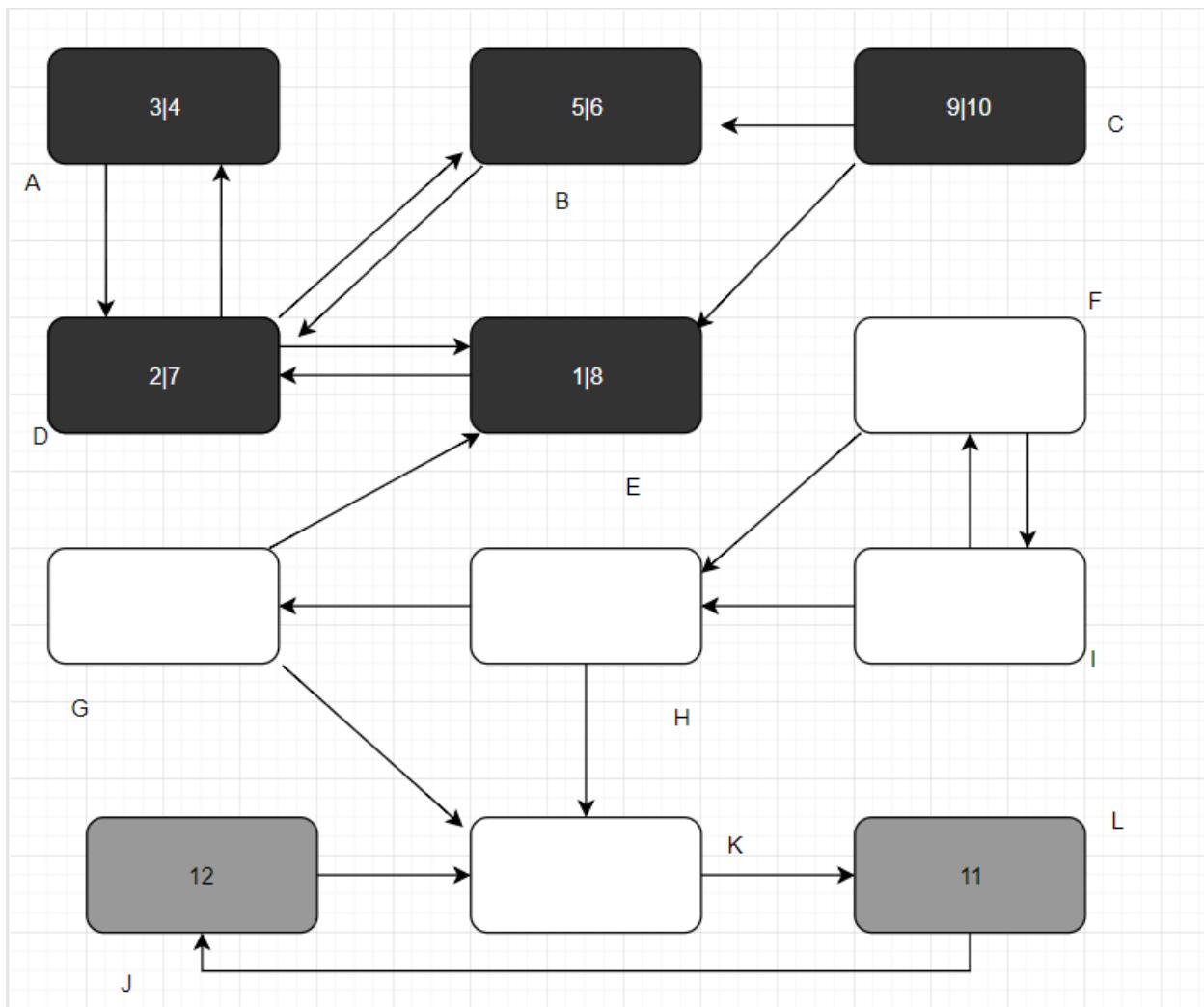
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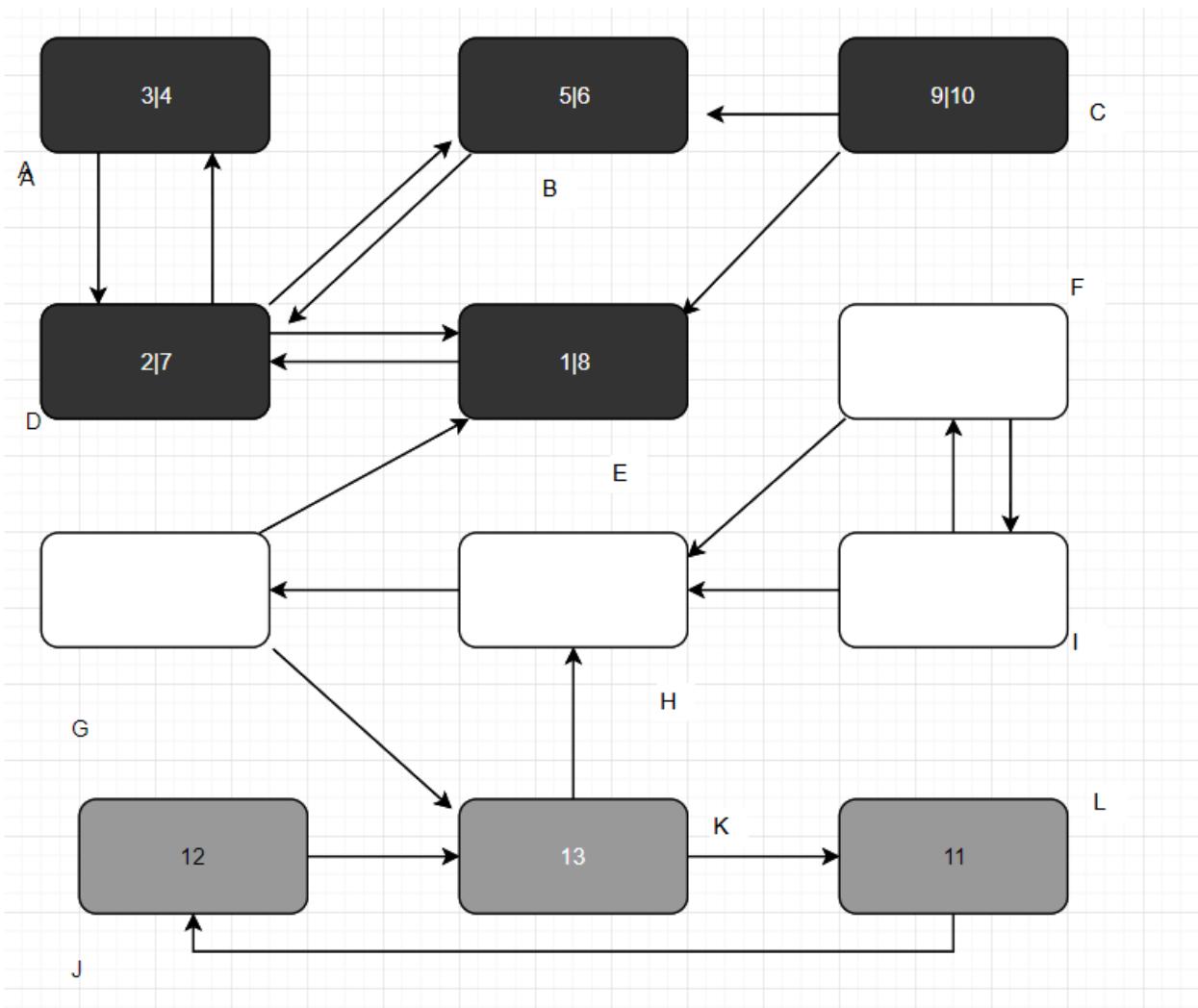


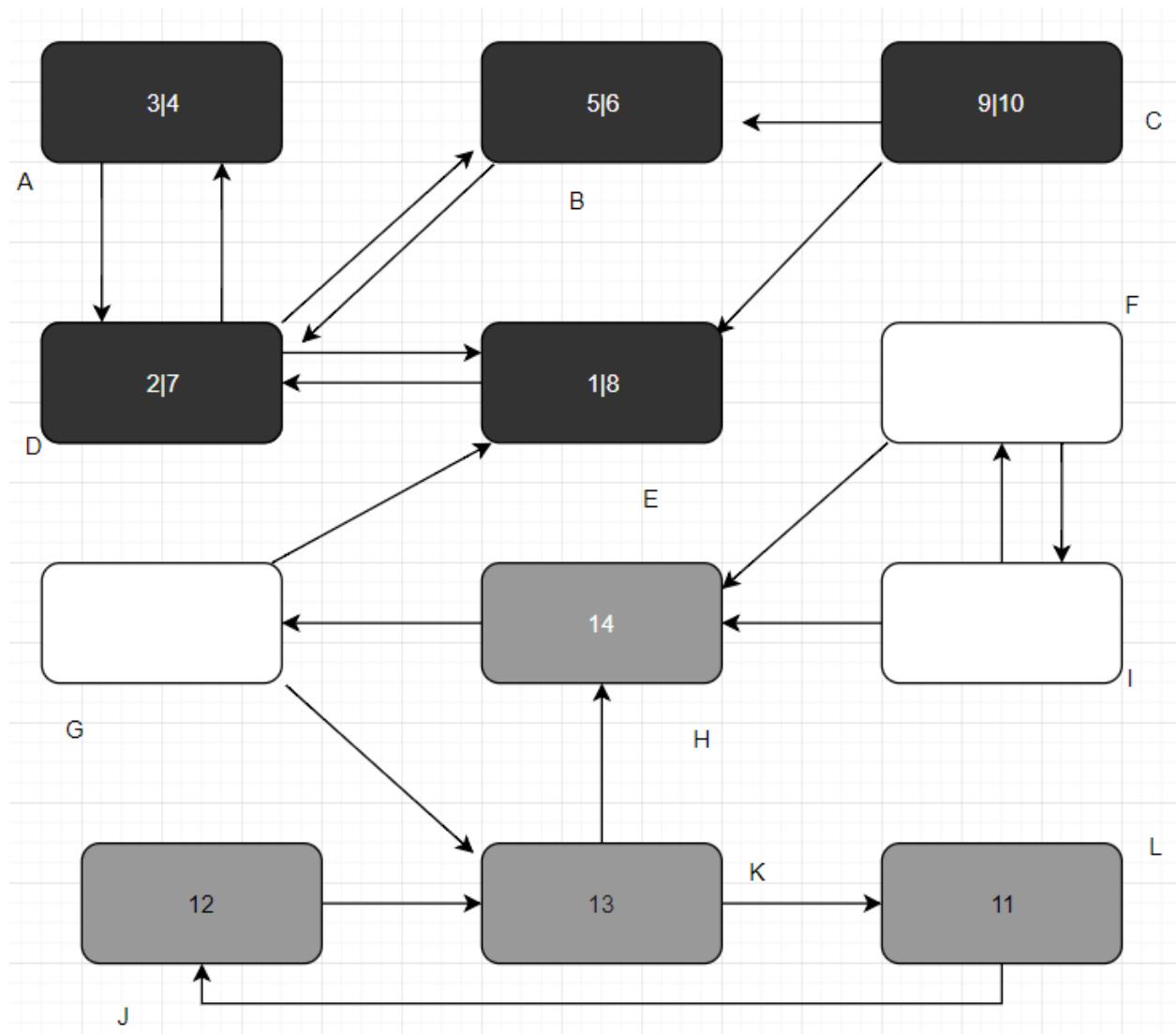


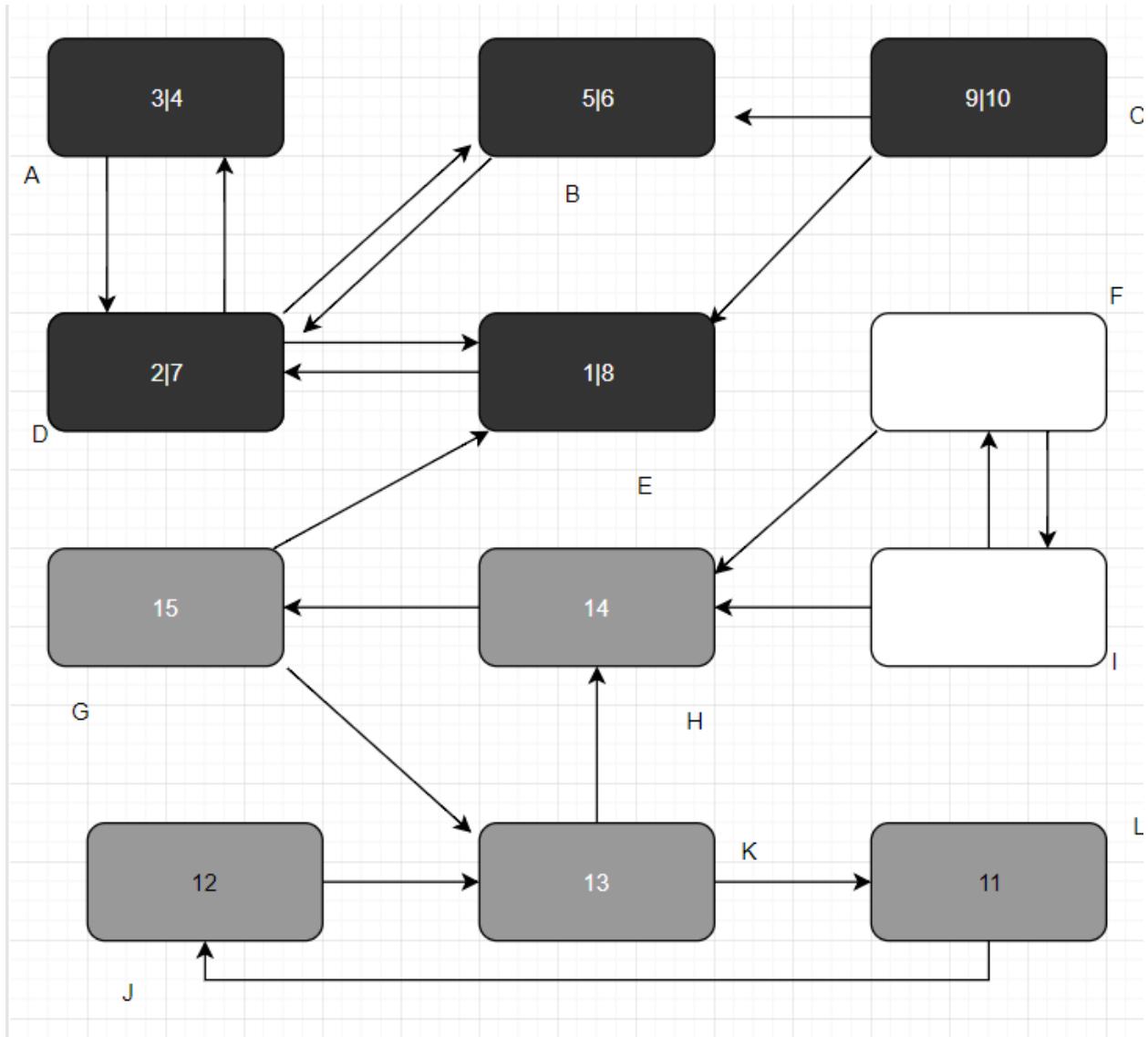
C

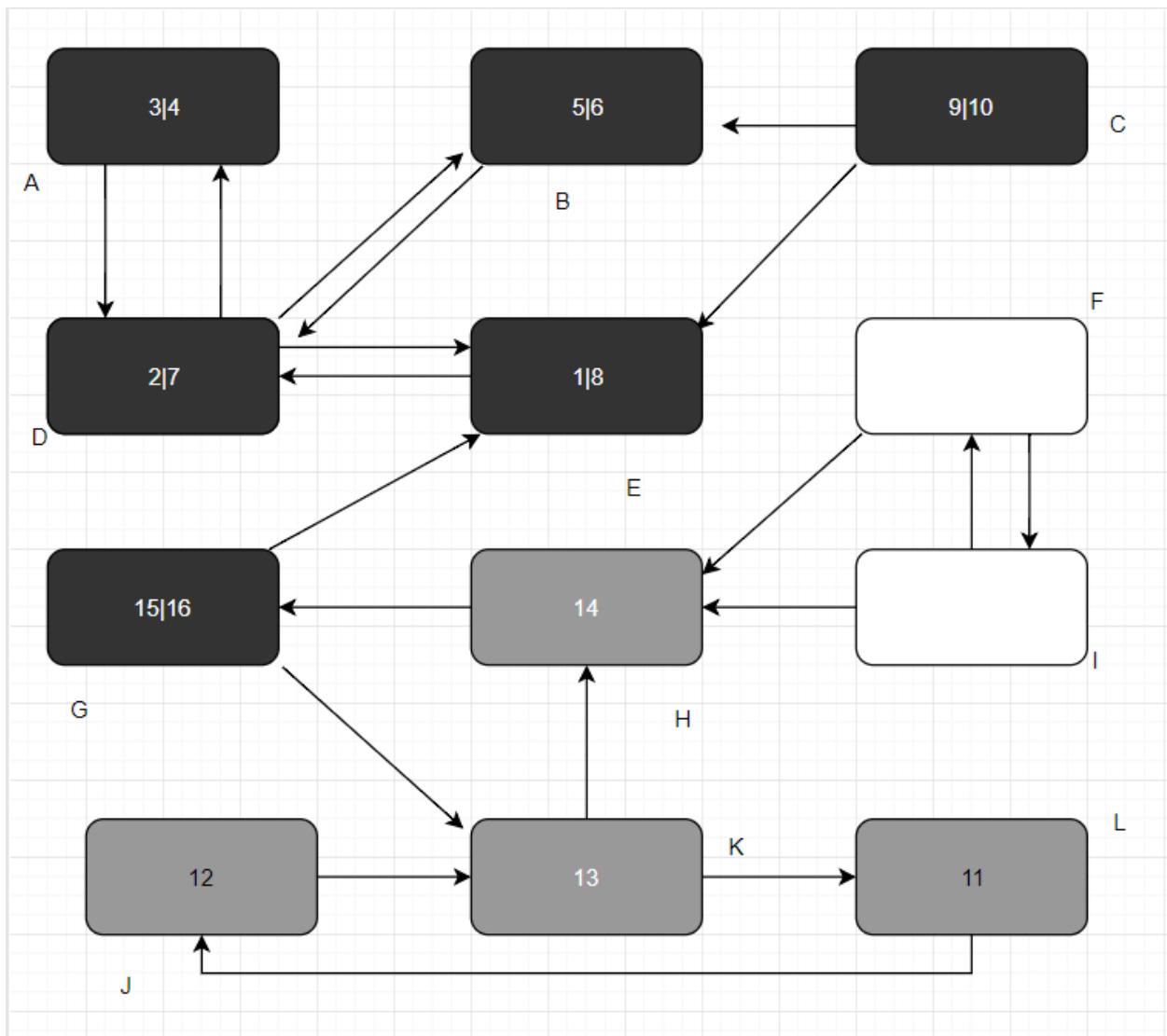


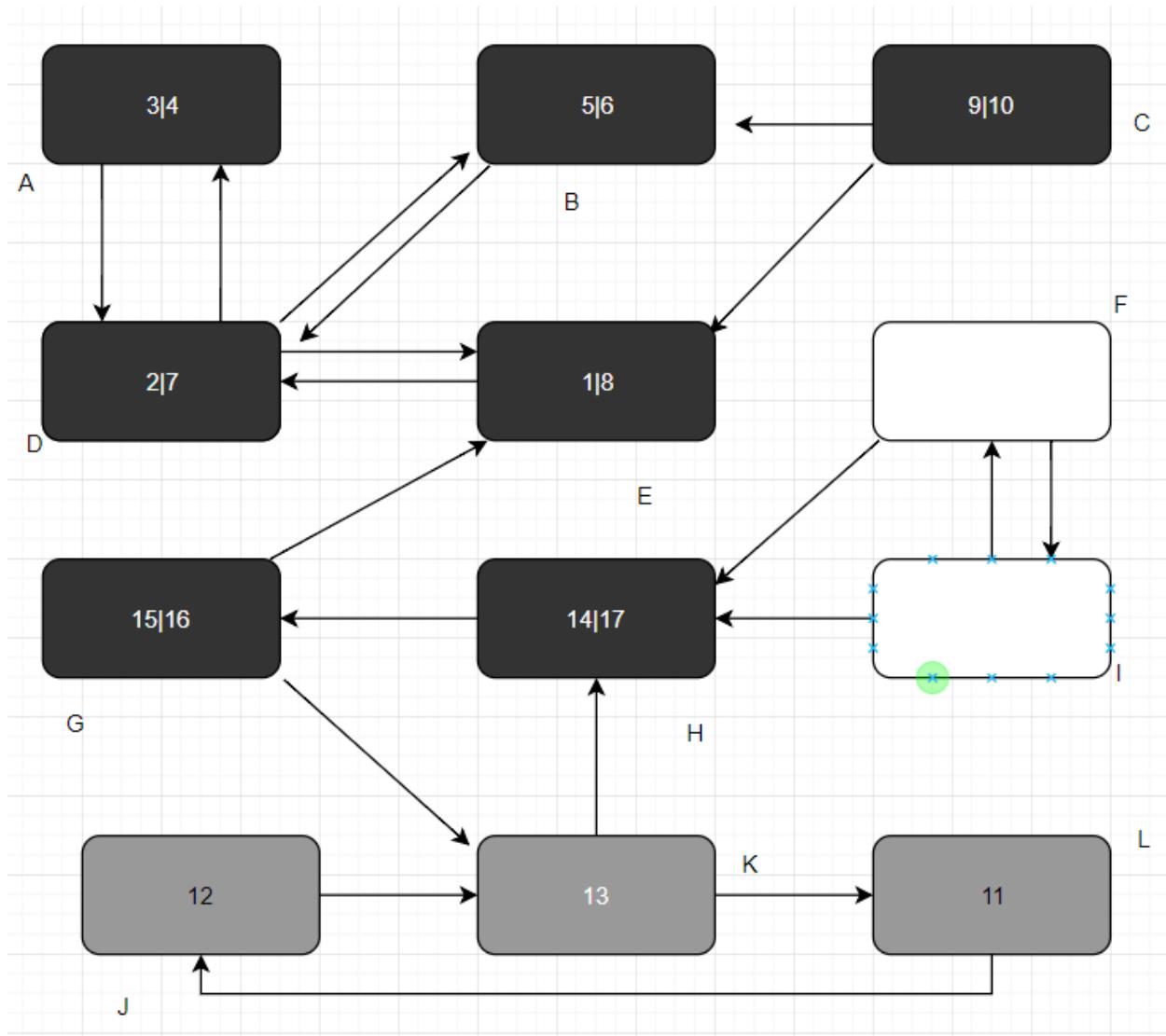


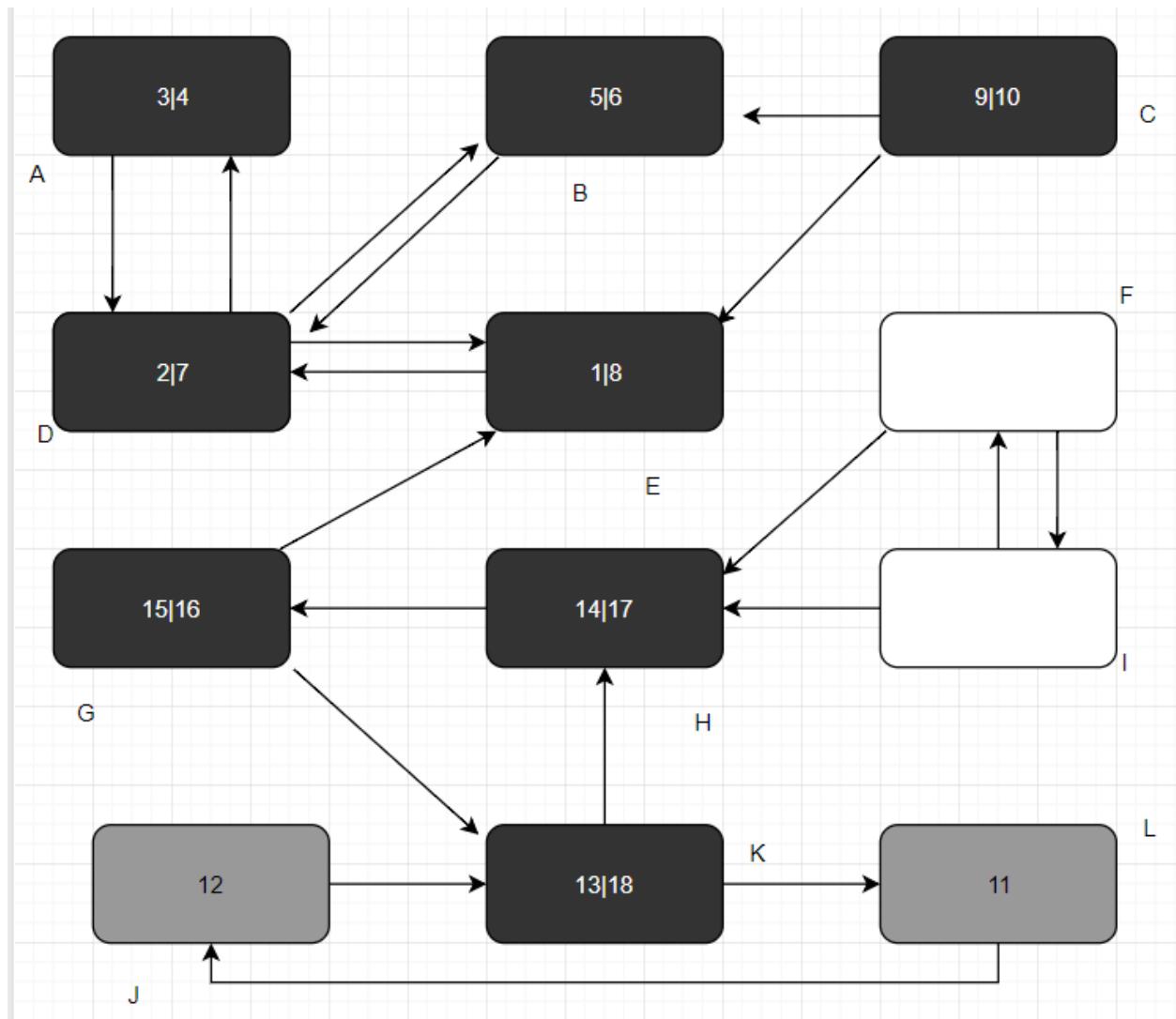


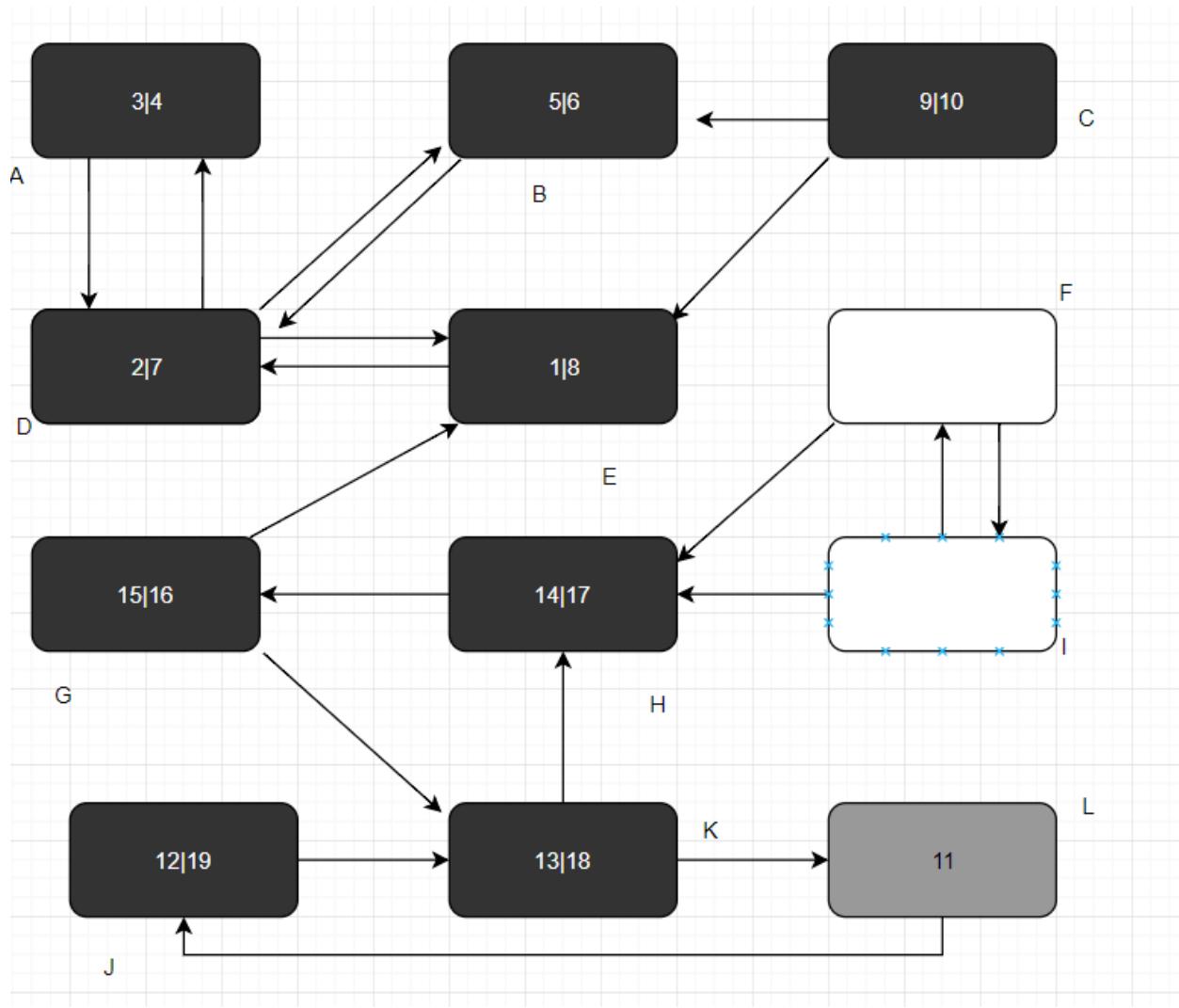


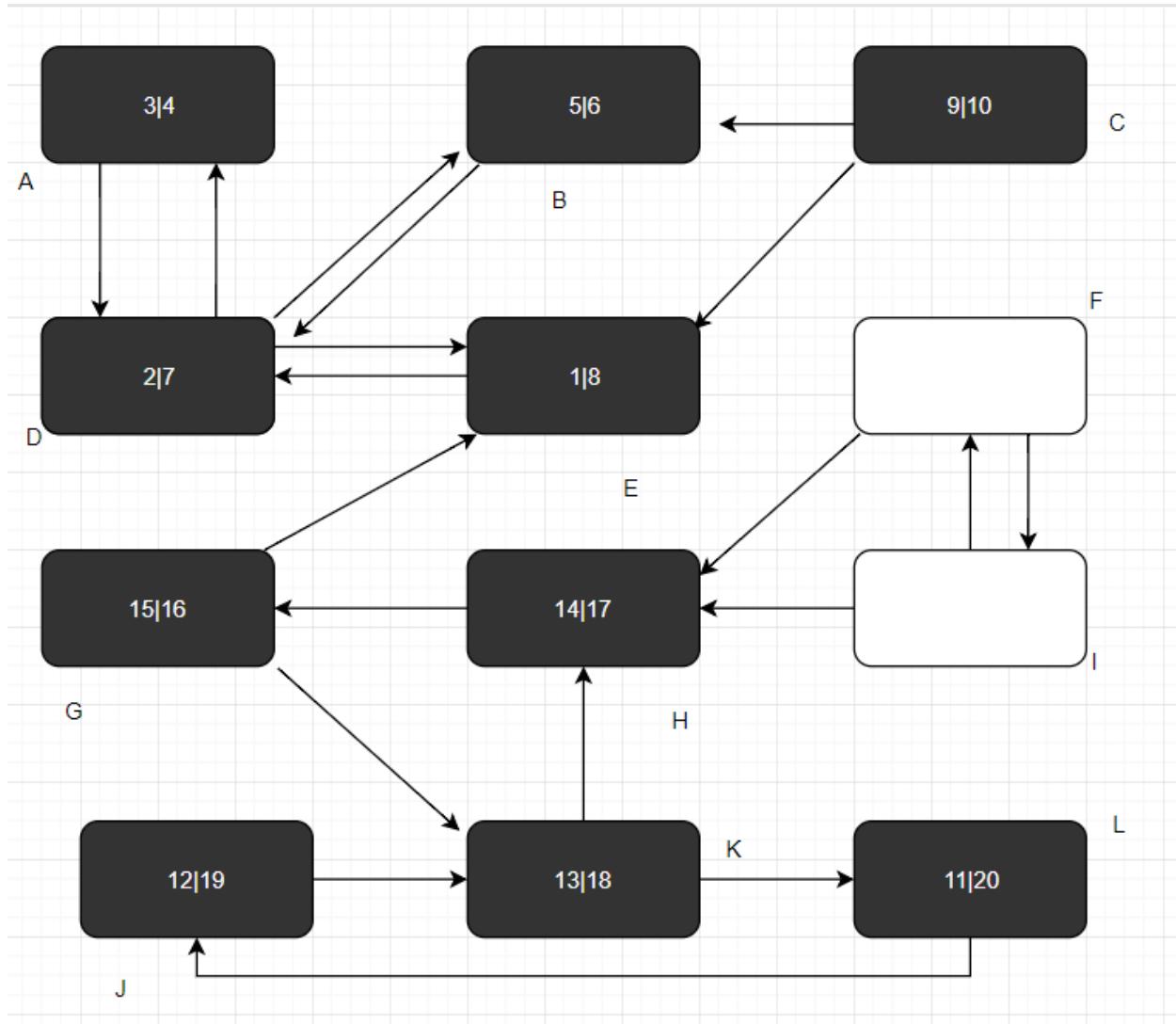




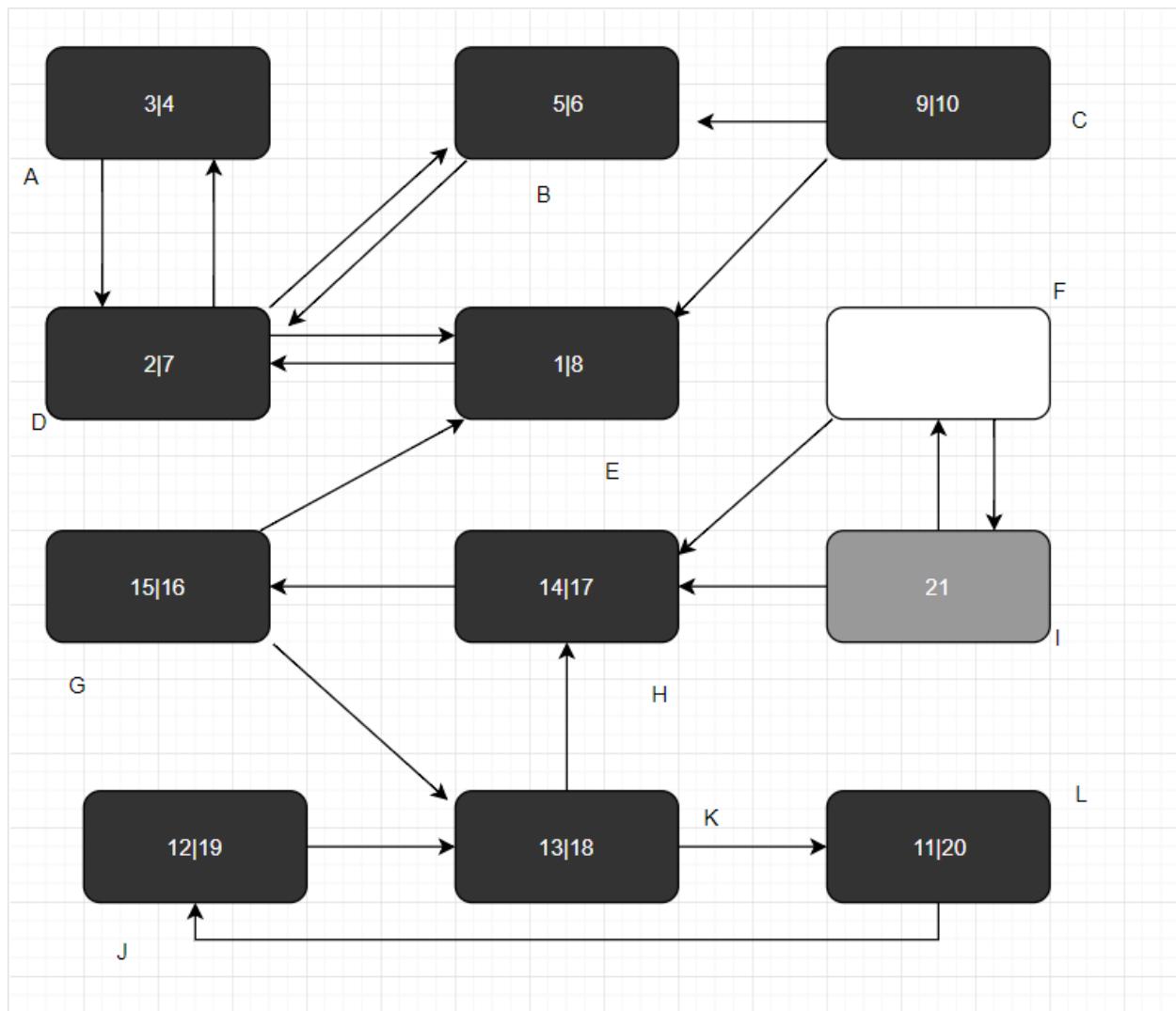


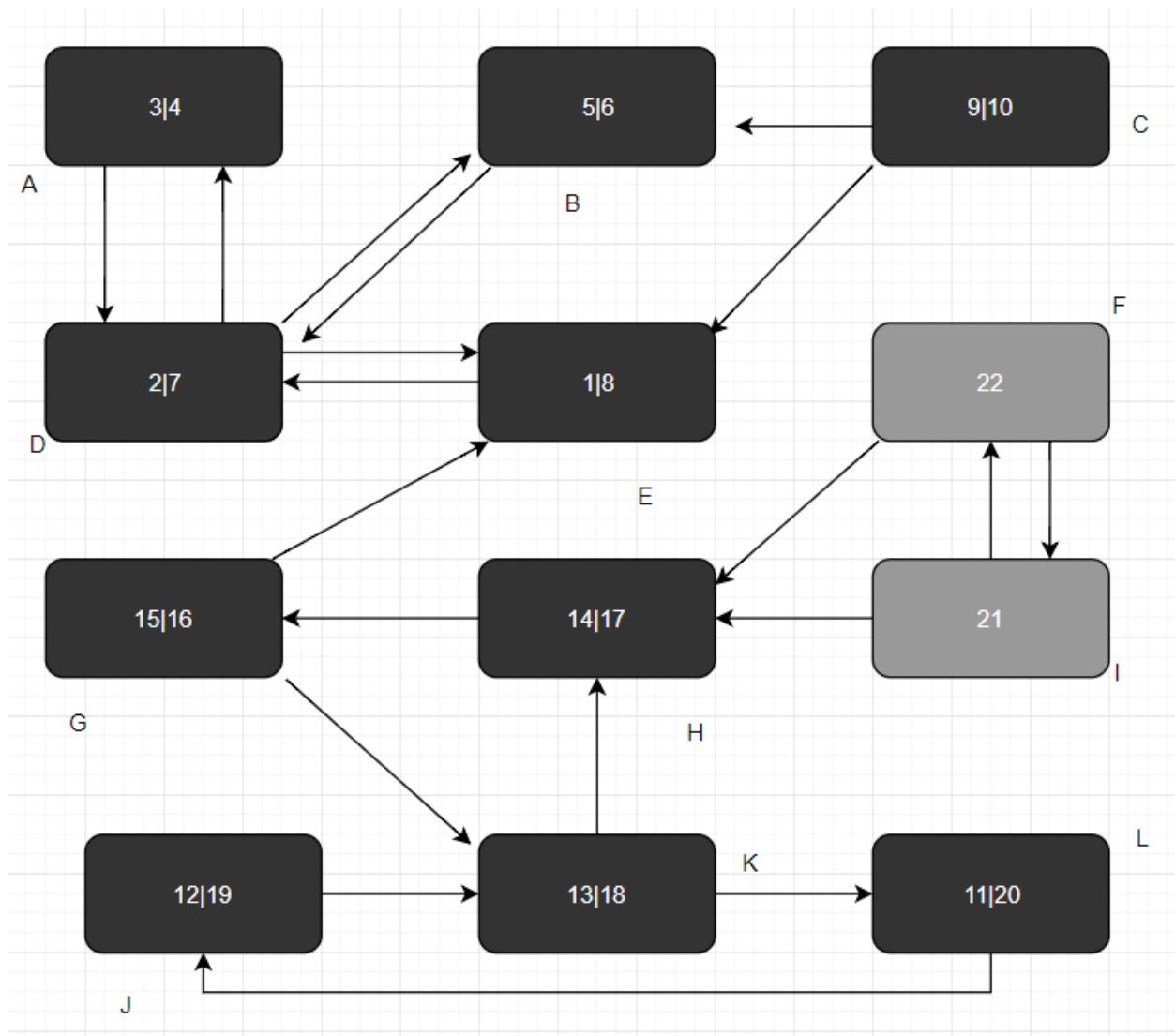


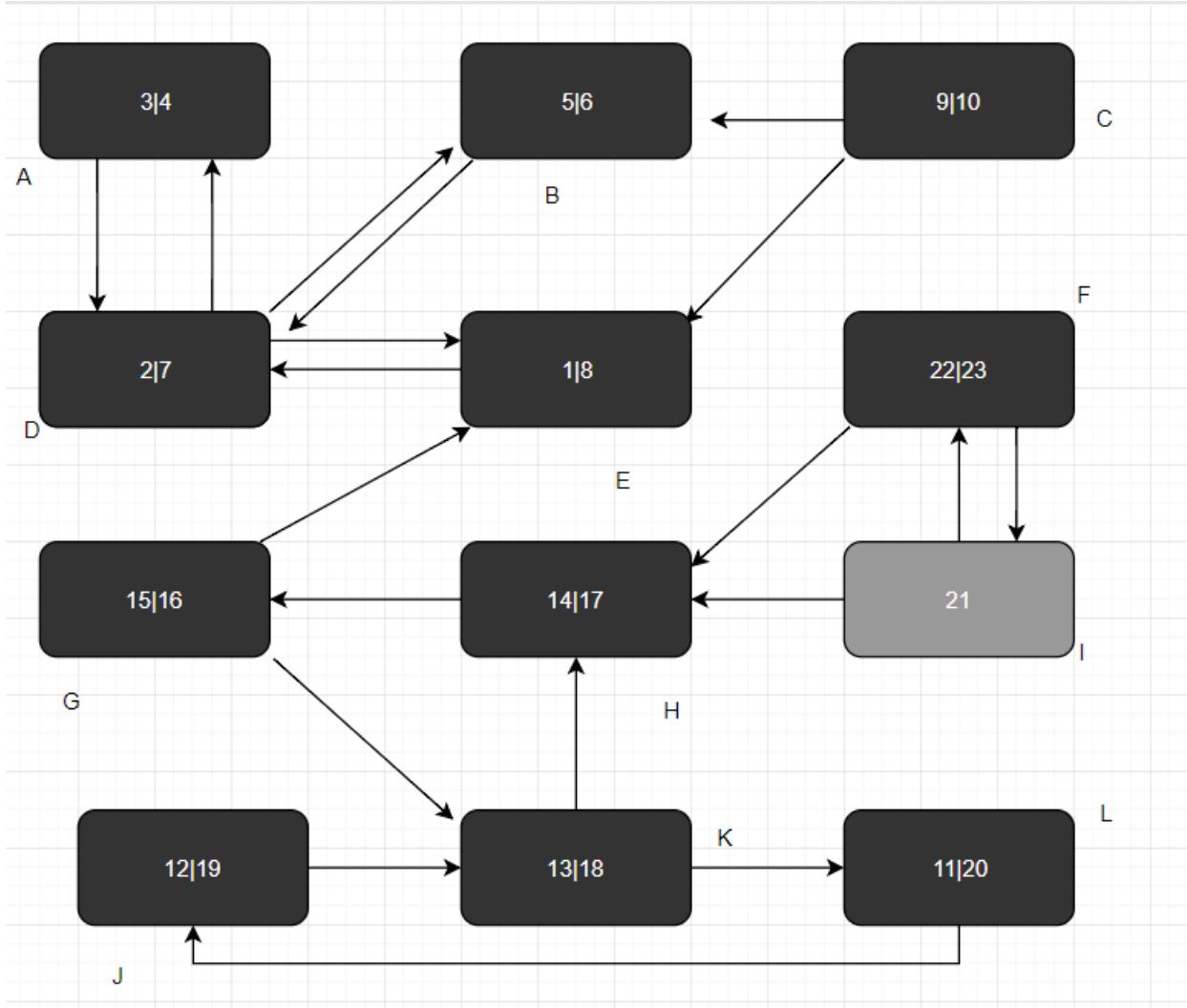


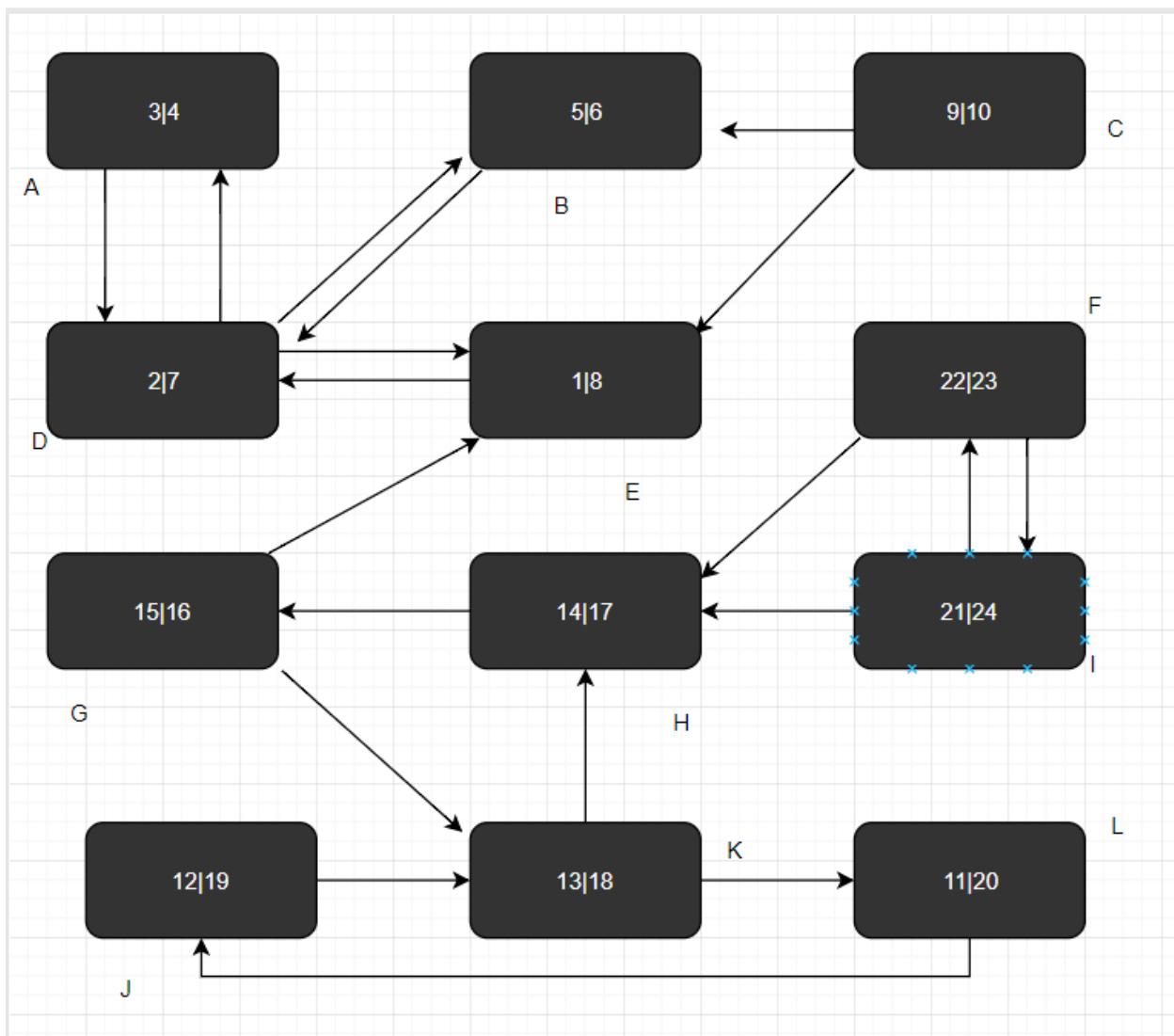


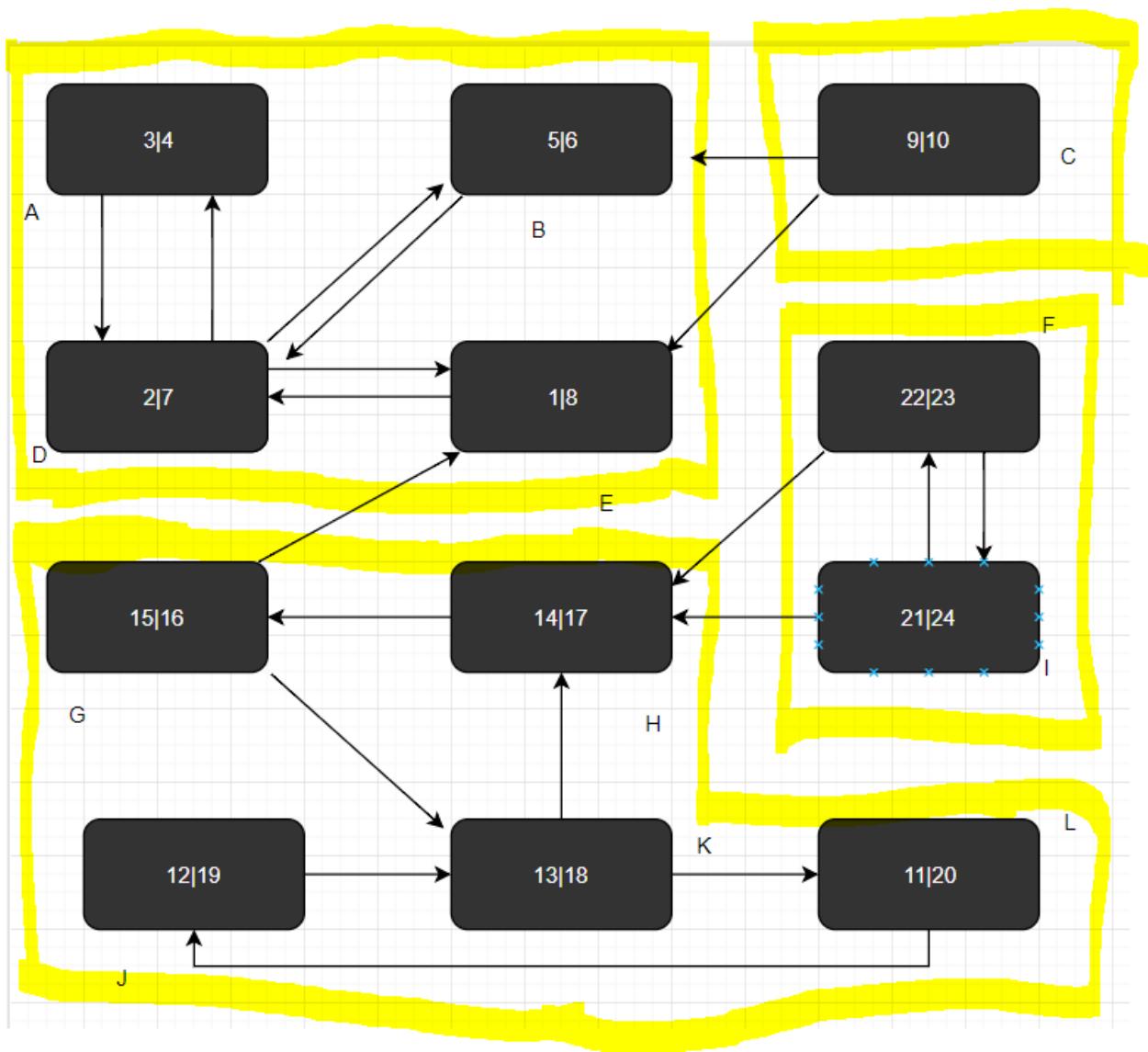
L J K H G











The Strongly connected components as per the above solution are

A B D E

G H J K L

I F

C

Vertex	Discovery time	Finishing time
A	3	4
B	5	6
D	2	7
E	1	8
G	15	16
H	14	17
J	12	19
K	13	18
L	11	20
I	21	24
F	22	23
C	9	10