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HOMEWORK - 3

QUESTION 1

(a) State the sample space for the random variable X ! call it S_X.

Sol The number of heads that can occur when flipping the coin three times are:

TTT = 0 heads. and all are tails.

HTT = 1 head and rest are tails.

HHT = 2 heads and one tail.

HHH = 3 heads and zero tail.

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$S_X = \{0, 1, 2, 3\}$ is the sample space of rv X.

So the random variable X counts the number of heads in three coin flips, so the possible values for X are:

$$S_X = \{0, 1, 2, 3\}$$

(b) Find the probablity mass function for X from fundamentals.

Sol To find the probablity of getting 0, 1, 2, 3 heads for a baised coin with the probablity of heads $p = 0.8$ and tails $q = 1 - p = 0.2$, we can use the binomial probability ~~function~~ formula :-

$$P(x = K) = \binom{n}{K} p^K q^{n-K}$$

where

$n = 3$! the number of flips.

K = the number of heads

$p = 0.8$

$q = 0.2$

$\binom{n}{K}$ is the binomial coefficient, calculated as $\binom{n}{K} = \dfrac{n!}{K!(n-K)!}$

$P(X = K)$ for $K = 0, 1, 2, 3$.

1) $P(x=0)$: Probablity of getting 0 heads.

$$P(x=0) = \binom{3}{0}(0.8)^0(0.2)^3 = 1 \times 1 \times (0.2)^3 = 0.008.$$

2) $P(x=1)$: Probability of getting 1 head.

$$P(x=1)\binom{3}{1}(0.8)^1(0.2)^2 = 3 \times 0.8 \times 0.04 = 0.096.$$

3) $P(x=2)$: Probability of getting 2 heads:

$$P(x=2)\binom{3}{2}(0.8)^2(0.2)^1 = 3 \times 0.64 \times 0.2 = 0.384.$$

4) $P(x=3)$: Probability of getting 3 heads:

$$P(x=3)\binom{3}{3}(0.8)^3(0.2)^0 = 1 \times 0.512 \times 1 = 0.512.$$

PMF for X:

$$P(X = K) = \begin{cases} 0.008 & \text{for } K = 0 \\ 0.096 & \text{for } K = 1 \\ 0.384 & \text{for } K = 2 \\ 0.512 & \text{for } K = 3 \end{cases}$$

(C) Find the mean value of X.

Sol The mean of a binomial random variable X with parameters $n = 3$ and $p = 0.8$ can be calculate using the formula.

$$\mu = E(x) = n * p$$

where

$n = 3$ (number of flips)
$p = 0.8$ (probablity of heads)

$$\mu = E(x) = n \times p$$

$$\mu = 3 \times 0.8 = 2.4.$$

The expected number of heads is 2.4.

(d) Find the variance of X.

Sol The variance of a binomial random variable X with parameters $n = 3$ and $p = 0.8$ is given by:

$$Var(x) = n \times p \times (1-p)$$

where

$n = 3$ = number of flips
$p = 0.8$ = probablity of heads

$$Var(x) = \cancel{3 \times 0.8} \times ($$
$$= 3 \times 0.8 \times (1 - 0.8)$$
$$= 0.48$$

The variance of $X = 0.48$.

HOMEWORK - 3

Question 2

(a) Would you buy a raffle ticket? Yes or no? Explain using statistical/probabilistic arguments.

Sol Expected value of winning :-

$$\frac{1}{1,000,000} \times 1000 + \frac{10}{1,000,000} \times 100 + \frac{1000}{1,000,000} \times 2 = 0.001 + 0.001 + 0.002 = 0.004.$$

Since the ticket costs $1, and the expected value is $0.004, the expected loss is significant, so the ration decision is not to buy a ticket.

(b) What is the max price you are willing to pay?

Sol The max price I am willing to pay would be the expected value of $0.004.

Question 3 :

(a) Find the probablity that car A will require a repair and car B will not.

Sol probablity that car A will require an expensive repair: $P(A) = 0.02$

probablity that car B will require an expensive repair: $P(B) = 0.01$

probablity that either car A or B will require a repair: $P(A \cup B) = 0.025$

Probablity that car A will require a repair and car B will not

$$P(A \cap B') = P(A) - P(A \cap B)$$

we need $P(A \cap B)$, which can be found using:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.025 = 0.02 + 0.01 - P(A \cap B)$$

$$P(A \cap B) = 0.005.$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(A \cap B') = 0.02 - 0.005$$

$$P(A \cap B') = 0.015.$$

(b) Are the events that car A will require a repair and that car B will require a repair independent? why or why not

Sol If $P(A \cap B) = P(A) \times P(B)$

$P(A) \times P(B) = 0.02 \times 0.01$

$P(A \cap B) = 0.0002$

It is independent since $P(A \cap B) = 0.0002 \neq 0.005$.

Question 4

(a) $P(0.5 \leq x \leq 1)$

Sol This is the probablity that x lies between 0.5 and 0.1. To find the area under $f(x)$ from 0.5 to 1

The region is a ~~triangle with~~ bigger triangle minus the area of smaller triangle.

Area of bigger triangle:

Base of bigger triangle $= 1 - 0 = 1$

Height of bigger triangle $= 1$

Area $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 1 \times 1 = 0.5$

Area of smaller triangle:

Base of smaller triangle $= 0.5$

Height of smaller triangle $= 0.5$

Area $= \frac{1}{2} \times 0.5 \times 0.5 = 0.125$

Area under $f(x)$ = Area of bigger triangle − Area of smaller triangle

$f(x) = 0.5 - 0.125 = 0.375$

(b) $P(x \geq 1)$

Sol This is the probablity that x is greater than 1. It corresponds to the area under $f(x)$ from 1 to 2. This region forms a right angle triangle

Base $= 2 - 1 = 1$

Height $= 1$

Area $= P(x > 1) = \frac{1}{2} \times 1 \times 1 = 0.5$.