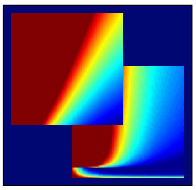
CS/CNS/EE 156a Learning Systems
Caltech - Fall 2022
https://caltech.instructure.com/courses/4778
(Learning From Data campus version)



Homework # 7

Due Monday, November 14, 2022, at 2:00 PM PST

*Definitions and notation follow the lectures. All questions have multiple-choice answers (***[a], [b], [c], ...***). Collaboration is allowed but without discussing selected or excluded choices. Your solutions must be based on your own work. See the initial "***Course Description and Policies***" handout for important details about collaboration and "open book" policies.*

Note about the homework

- Answer each question by deriving the answer (carries 6 points) then selecting from the multiple-choice answers (carries 4 points). You can select 1 or 2 of the multiple-choice answers for each question, but you will get 4 or 2 points, respectively, for a correct answer. See the initial "**Course Description and Policies**" handout for important details.
- The problems range from easy to difficult, and from practical to theoretical. Some problems require running a full experiment to arrive at the answer.
- The answer may not be obvious or numerically close to one of the choices, but one (and only one) choice will be correct if you follow the instructions precisely in each problem. You are encouraged to explore the problem further by experimenting with variations on these instructions, for the learning benefit.
- You are encouraged to take part in the Piazza discussion forum. Please make sure you don't discuss specific answers, or specific excluded answers, before the homework is due.

- **Validation**

In the following problems, use the data provided in the files `in.dta` and `out.dta` for Homework # 6. We are going to apply linear regression with a nonlinear transformation for classification (without regularization). The nonlinear transformation is given by ϕ_0 through ϕ_7 which transform (x_1, x_2) into

$$1 \qquad x_1 \qquad x_2 \qquad x_1^2 \qquad x_2^2 \qquad x_1 x_2 \qquad |x_1 - x_2| \qquad |x_1 + x_2|$$

To illustrate how taking out points for validation affects the performance, we will consider the hypotheses trained on $\mathcal{D}_{\text{train}}$ (without restoring the full $\mathcal{D}$ for training after validation is done).

1. Split `in.dta` into training (first 25 examples) and validation (last 10 examples). Train on the 25 examples only, using the validation set of 10 examples to select between five models that apply linear regression to ϕ_0 through ϕ_k, with $k = 3, 4, 5, 6, 7$. For which model is the classification error on the validation set smallest?

[**a**] $k = 3$
[**b**] $k = 4$
[**c**] $k = 5$
[**d**] $k = 6$
[**e**] $k = 7$

2. Evaluate the out-of-sample classification error using `out.dta` on the 5 models to see how well the validation set predicted the best of the 5 models. For which model is the out-of-sample classification error smallest?

[**a**] $k = 3$
[**b**] $k = 4$
[**c**] $k = 5$
[**d**] $k = 6$
[**e**] $k = 7$

3. Reverse the role of training and validation sets; now training with the last 10 examples and validating with the first 25 examples. For which model is the classification error on the validation set smallest?

[**a**] $k = 3$
[**b**] $k = 4$

[**c**] $k = 5$

[**d**] $k = 6$

[**e**] $k = 7$

4. Once again, evaluate the out-of-sample classification error using `out.dta` on the 5 models to see how well the validation set predicted the best of the 5 models. For which model is the out-of-sample classification error smallest?

[**a**] $k = 3$

[**b**] $k = 4$

[**c**] $k = 5$

[**d**] $k = 6$

[**e**] $k = 7$

5. What values are closest in Euclidean distance to the out-of-sample classification error obtained for the model chosen in Problems 1 and 3, respectively?

[**a**] 0.0, 0.1

[**b**] 0.1, 0.2

[**c**] 0.1, 0.3

[**d**] 0.2, 0.2

[**e**] 0.2, 0.3

- **Validation Bias**

6. Let e_1 and e_2 be independent random variables, distributed uniformly over the interval $[0, 1]$. Let $\mathsf{e} = \min(\mathsf{e}_1, \mathsf{e}_2)$. The expected values of $\mathsf{e}_1, \mathsf{e}_2, \mathsf{e}$ are closest to

[**a**] 0.5, 0.5, 0

[**b**] 0.5, 0.5, 0.1

[**c**] 0.5, 0.5, 0.25

[**d**] 0.5, 0.5, 0.4

[**e**] 0.5, 0.5, 0.5

- **Cross Validation**

7. You are given the data points (x, y): $(-1, 0), (\rho, 1), (1, 0)$, $\rho \geq 0$, and a choice between two models: constant $\{\, h_0(x) = b \,\}$ and linear $\{\, h_1(x) = ax + b \,\}$. For which value of ρ would the two models be tied using leave-one-out cross-validation with the squared error measure?

[a] $\sqrt{\sqrt{3}+4}$

[b] $\sqrt{\sqrt{3}-1}$

[c] $\sqrt{9+4\sqrt{6}}$

[d] $\sqrt{9-\sqrt{6}}$

[e] None of the above

• PLA vs. SVM

Notice: Quadratic Programming packages sometimes need tweaking and have numerical issues, and this is characteristic of packages you will use in practical ML situations. Your understanding of support vectors will help you get to the correct answers.

In the following problems, we compare PLA to SVM with hard margin[1] on linearly separable data sets. For each run, you will create your own target function f and data set $\mathcal{D}$. Take $d = 2$ and choose a random line in the plane as your target function f (do this by taking two random, uniformly distributed points on $[-1, 1] \times [-1, 1]$ and taking the line passing through them), where one side of the line maps to $+1$ and the other maps to -1. Choose the inputs $\mathbf{x}_n$ of the data set as random points in $\mathcal{X} = [-1, 1] \times [-1, 1]$, and evaluate the target function on each $\mathbf{x}_n$ to get the corresponding output y_n. If all data points are on one side of the line, discard the run and start a new run.

Start PLA with the all-zero vector and pick the misclassified point for each PLA iteration at random. Run PLA to find the final hypothesis g_{PLA} and measure the disagreement between f and g_{PLA} as $\mathbb{P}[f(\mathbf{x}) \neq g_{\text{PLA}}(\mathbf{x})]$ (you can either calculate this exactly, or approximate it by generating a sufficiently large, separate set of points to evaluate it). Now, run SVM on the same data to find the final hypothesis g_{SVM} by solving

$$\begin{aligned} \min_{\mathbf{w},b} \quad & \frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w} \\ \text{s.t.} \quad & y_n\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}_n + b\right) \geq 1 \end{aligned}$$

using quadratic programming on the primal[2] or the dual problem, or using an SVM package. Measure the disagreement between f and g_{SVM} as $\mathbb{P}[f(\mathbf{x}) \neq g_{\text{SVM}}(\mathbf{x})]$, and count the number of support vectors you get in each run.

8. For $N = 10$, repeat the above experiment for 1000 runs. How often is g_{SVM} better than g_{PLA} in approximating f? The percentage of time is closest to:

[1] For hard margin in SVM packages, set $C \to \infty$ and choose 'linear' kernel.

[2] Primal problem is the original formulation in slide 11 of Lecture 14.

[a] 20%

[b] 40%

[c] 60%

[d] 80%

[e] 100%

9. For $N = 100$, repeat the above experiment for 1000 runs. How often is g_{SVM} better than g_{PLA} in approximating f? The percentage of time is closest to:

[a] 5%

[b] 25%

[c] 45%

[d] 65%

[e] 85%

10. For the case $N = 100$, which of the following is the closest to the average number of support vectors of g_{SVM} (averaged over the 1000 runs)?

[a] 2

[b] 3

[c] 5

[d] 10

[e] 20